

# Load path and prestressing in conceptual design related to Maillart’s Vessy Bridge

Denis ZASTAVNI\*, Jean-François CAP<sup>a</sup>, Jean-Philippe JASIENSKI<sup>b</sup>, Corentin FIVET<sup>c</sup>,

\*<sup>1</sup>Prof. Dr. Arch. Eng, LOCI, UCLouvain,  
Place du levant 1 (5.05.0), B-1348 Louvain-la-Neuve, Belgium  
*denis.zastavni@uclouvain.be*

<sup>a</sup> Civ. Eng, lecturer, EPL, UCLouvain, Louvain-la-Neuve, Belgium

<sup>b</sup> Arch. Eng. PhD candidate, LOCI, UCLouvain, Louvain-la-Neuve, Belgium

<sup>c</sup> Dr. Arch. Eng, LOCI, UCLouvain, Louvain-la-Neuve, Belgium

## Abstract

This paper analyses Maillart’s Vessy Bridge from a design point of view using thrust lines. As it has previously been shown (Fivet and Zastavni [4], Zastavni [7,8]) that Robert Maillart used graphic statics to design the line and the structural features of his bridges, it proves appropriate to reconstruct the working drawings of one of his most complete works. Thrust lines, forces trajectories and funicular polygons within the scope of graphic statics refer to reference loading cases and their choices. Furthermore, it questions the manipulation of forces, with mechanisms such as prestressing and interpreting thrust in geometrical terms.

**Key words:** *design methods, structural morphology, concrete, prestressing, morphogenesis, graphic statics, Maillart (Robert)*

## 1. Introduction: The Vessy Bridge in Maillart’s career

Robert Maillart’s Vessy Bridge (Maillart[6]) was built in 1936. It is one of a series of Maillart’s characteristic three-hinged arch bridges. Excluding unbuilt designs, this series includes 11 bridges, of which the Vessy Bridge is the ninth in the series. Given that Maillart’s career spanned from 1894 to 1940, this bridge was designed by an experienced engineer and his prototype for concrete three-hinged arch bridges has reached a mature state.

Robert Maillart (1872-1940) worked on six major types of bridges: three hinged-arch bridges, massive classical arch bridges, arches with a strongly off-center thrust line, stiffened arch bridges, continuous girder bridges and cantilever bridges. The most symbolic and characteristic ones are the stiffened arch bridges and three-hinged arch bridges (Bill [1], Zastavni [10]).

Maillart’s three-hinged arch bridges can be classified in three stages: early three-hinged arch bridges that included the Stauffacher Bridge (1899), Zuoz Bridge (1901), Billwil Bridge (1904) and ending with the Tavanasa Bridge (1905) as his prototype for this successfully completed and correctly formalized type of bridge; then the second stage in the development of this series was his famous Salginatobel Bridge (1929) – followed a little later by the Schwarzenburg Bridge (1932) – as a magnification of this fully implemented prototype of a large span (90 meters), after having abandoned this model for 24 years. The third stage as later three-hinged arch bridges that are variations of this type, depending on loading, spans, width and rationalization of the geometry. Pointed arches are characteristic of this later period of development.

## 2. Features of the Vessy Bridge

Later three-hinged arch bridges are characterized by a greater width than the earlier ones. Most of the bridges before 1932 are about 3.5 meters wide. The Vessy Bridge is 10.4 meters wide with a span of 56 meters. Live loads to be sustained are larger too: 5kN/m<sup>2</sup> compared to 3 or 3.5kN/m<sup>2</sup> earlier. As has been said before, the geometry of the arch is no longer rounded at its center: the arches are pointed.

Other specific characteristics of the Vessy Bridge are its sag/span ratio and its X-shaped supporting columns. Its sag/span ratio (0.086) is the lowest of the series (mean ratio is about 0.115), except for the Garstatt Bridge (0.073) since it is very characteristic with a shorter span (32 meters) and an arch acquiring straight lines as the borders of its geometry, and therefore no longer curved.

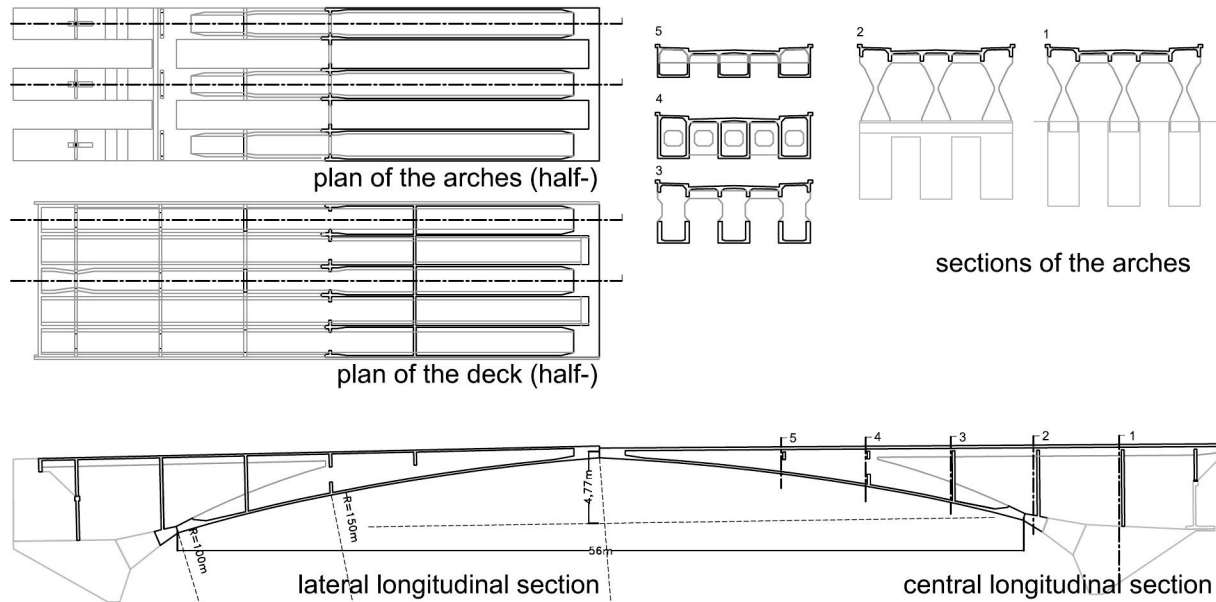


Figure 1: Maillart's Vessy Bridge (1936), 1:500

The arches of the Vessy Bridge are also made of three parallel boxed sections. This is the only bridge with three arches since the early three-hinged arch bridge: Salginatobel and Schwarzenburg Bridges have a single arch, while Felsegg and later ones have just two parallel arches. The section of the boxes has no overhanging lower slab either side of the arch, which makes them look very contemporary. It is the first time that Maillart simplified the geometry of his arches in this way. Finally, Maillart explored different ways of gaining material in the way he designed the shear walls bearing the deck by resting on the arches. Portals were used in the Felsegg Bridge, but only for the approach spans, and these bearing walls became more or less non-existent in the two subsequent bridges in the three-hinged arch bridges of the series. At Vessy, Maillart explored X-shaped columns/bearing elements, and this is also the only occurrence of this.

### 3. Design method

This section describes first the approach taken by Maillart to design his three-hinged arch bridges. Then it describes in more detail the steps in the design of the Vessy Bridge. It then concludes with the advantages and drawbacks of the design choices made.

#### 3.1. General approach to the design of three-hinged arch bridges

Maillart's method of designing the Vessy Bridge is not specific to this structure. From design documents that still exist, the approach remains quite similar to that used in the Salginatobel Bridge seven years earlier. Only two graphic statics working drawings of the Vessy Bridge are contained in the archives, and they show almost the same line each time. The Salginatobel Bridge working drawings show multiple attempts to sketch the thrust lines via funicular polygons and geometries of the transversal sections and the lateral elevation of the arches [Zastavni [7], Fivet and Zastavni[4]). What was specific and unique in them is Maillart's approach to the columns having taken the form of an X. Maillart expected a larger expansion of the deck compared to the supporting arches. He therefore implemented a (potentially plastic) hinge at the junction of the part of the

support connected to the deck, and the one connected with the foundation, at the column’s mid-height, and calculated the forces implied by the relative deformation of the deck and the supports due to temperature loading.

Maillart’s bridges are made of concrete. Unreinforced concrete is similar to masonry and the same techniques have been used to design them. It involves studying the possible trajectories of forces in the form of thrust lines, and ensuring that they preferably remain close to the center of the successive sections making the arch or the structure. Depending on its relative position, the material sustains uniform stresses in compression and in a dissymmetrical stress state or has to sustain tension forces that have to be avoided with masonry.

Reinforcement of concrete opens up the possibility of managing traction forces, and therefore of managing bending. The deviation in thrust lines compared to the locus of the center of gravity of the successive sections gives the bending forces, enabling stresses to be computed.

Maillart’s method is logical and simple. Firstly, he uses the logic of the thrust line found with masonry, which enables him to design structures where concrete primarily remains compressed, guaranteeing good long-term behavior (Zastavni [9]). Instead of sketching a possible bridge and then calculating it to adapt the geometry of the successive section through the dimensioning – the final geometry resulting from the dimensioning of the successive section – Maillart designs the sections around the expected thrust line. The thrust line is simply obtained by sketching the funicular polygon corresponding to the design loads. The challenge is therefore to devise a regular geometry fitting the trajectory of forces while matching the geometrical constraints of the structure (relative position of the deck compared to the arch, connected or unconnected relative positioning of the arch and the deck structure).

### 3.2. The Vessy Bridge design

Archive documents show that the design started with a competition in which Maillart presented two versions for the proposed bridge. One version is the sketch of the Vessy Bridge as a three-hinged arch, and the other as an arch with a strongly off-center thrust line. The latter was an arch with high bending forces and was Maillart’s preference. There is only one bridge of this type built by Maillart in Innertkirchen (1934). However the contracting authority preferred the three-hinged arch bridge.

A quick analysis of the former shows that the span is a little bit shorter (52 meters instead of 56 meters), the bridge is lower, and the geometry more simplified. Indeed, the geometry is heaviest. The lower line of the arch was a straight line along the first quarter span, then became an arc of a circle. The straight line joined the supporting hinge to the top of the bridge, over the central hinge. The two extending arcs of a circle, with a radius of 150 meters centered just below the quarter span of the opposite half-arch, met at the center at an angle, forming a pointed arch. The vertical height of the arch is higher; the cut at the junction between the arch and the deck is less deep. The connection is made with a straight line, also just below the quarter span. The section of the bridge is a unique box and on both sides close to the supports is a unique U-shaped arch instead of three arches.

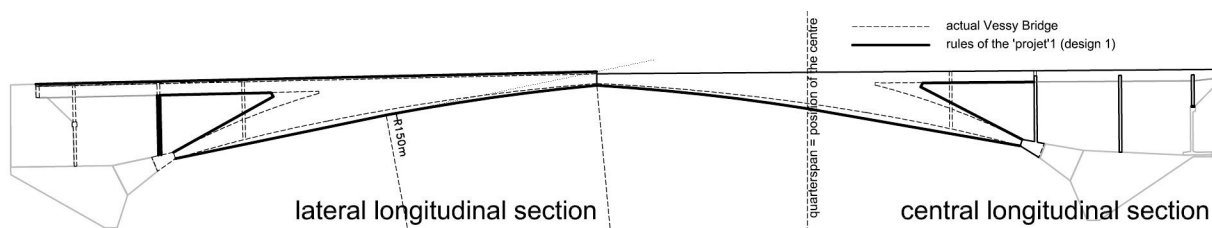


Figure 2: Geometrical rules of Maillart’s ‘projet 1’ sketch for the Vessy Bridge applied to actual dimensions

This sketch was heaviest but simplest, and since it is nearly the copy of a bridge to be built three years later in Garstatt (32-meter span only) – but with two arches reduced to two straight lines intersecting at the crown hinge – it can be questioned whether it would have been an interesting design.

As in every design by Maillart, simple ratio and geometries are used to draw the bridge. Maillart’s method remains the same: a sketch of an informed geometry enables him to compute the weight of the successive sections making the arch. These loads are used to build a thrust line as a funicular polygon using graphic statics.

As in Maillart’s other design (Maillart [5]), a trial funicular with the extreme rays at an angle of 45 degrees is drawn first, then used to obtain the final thrust line in a second step. Maillart generates a partial graphic statics drawing corresponding to punctual loading or asymmetrical loading to generate the orientation and magnitude of reaction forces. The extreme values enable the foundations of the structure to be designed.

The lines of the profile of the bridge are drawn to maintain the locus of the center of gravity close to the thrust line. This has to be done with geometries that are as regular as possible, such as straight lines or arcs of circles, and considering the average appearance of the whole profile of the bridge. Working drawings show that Maillart worked on the geometries as they were simplified as simple rectangular boxes, therefore neglecting the interaction with the deck’s slab. At the same time that the lines of the arch were outlined, the successive corresponding sections are sketched and their mechanical properties were calculated. Maillart was able to play with the different parameters of the geometries to adjust the mechanical properties of the successive sections. These parameters are the thickness of the concrete wall of the box, the thickness of the horizontal slabs, the width or the height of the boxes forming the central part of the arch, or the U-shapes making the sides of the arch.



Figure 3: Centers of gravity and mechanical properties of Maillart’s Vessy Bridge (1936), 1:500

Maillart explained with the Felsegg Bridge that a logical form for the arch of a bridge should be an ogive [Billington [2] p. 115, [3]) since asymmetric live loads lead to a strong variation in the trajectory of the thrust line. Maillart explains that the domain of variation of the thrust line makes a lenticular geometrical domain between two hinges. This implies that the extreme lower thrust line joins the central hinge at an angle and so should the two halves of the arch. Therefore the logic of the geometry making the profile differs from earlier three-hinged arch bridges. In the Salginatobel Bridge, a parabolic curve is used to carve the lower line of the arch to make it sufficiently high to connect an arc of a circle spanning the central half of the bridge with a continuous line. In the Vessy Bridge, a flat curve (or a straight line in the early version of the design) is used to lower the limit of the arch and to prepare the broken connection below the central hinge.

Although in the case of the Vessy Bridge the extreme thrust lines due to live loads are not drawn, the question can be asked of how Maillart proceeded to choose the final geometry for implementation on the side view of the bridge. This will be analyzed below.

## 4. Analysis and behavior

### 4.1. Loadings, forces and stresses on the actual structure

Considering the reference loading case, a reference thrust line can be obtained: it should be middle line of the geometry. Variations of the loading case are linked to live loads.

A first question is the nature of the loading used to draw the thrust line: the definition of the reference loading case. Depending on that choice, stresses can be made regular to minimize creep, or extreme forces are minimized to optimize the amount of concrete or reinforcement or the cracks can be closed for most of the time to improve the bridge’s life.

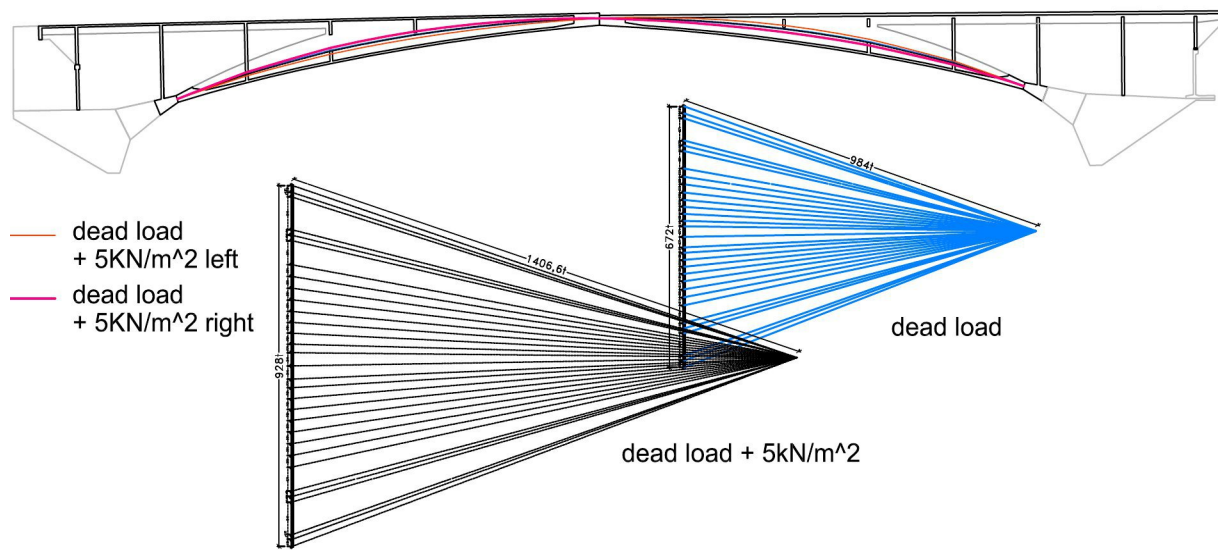


Figure 4: Thrust lines of various loading cases of Maillart's Vessy Bridge (1936), 1:500

On Maillart's drawing managing simplified rectangular sections while neglecting the deck's slab, the thrust lines under the dead load and corresponding to the dead load+a live load of  $5\text{kN/m}^2$  are nearly superimposed. A more carefully analysis made on a CAD system shows a variation between positions of about 4 centimeters. The locus of the centers of gravity are closest to the lower thrust line corresponding to the live+dead load, with a maximum distance of 3.5 centimeters (therefore a 7.5 cm distance of the thrust line of the dead loads alone). Therefore when Maillart designed the bridge, the dead load or the dead load joined to the live load, since they were superimposed on his drawing, were its reference loading case. This means that the dead load was not far from being uniformly distributed. Unlike a structure with imposed fixed loading, mobile live loads on a three-hinged arch bridge lead to variations of loading in both directions, therefore increasing or reducing accompanying bending forces. This justifies the mean position of the thrust line being taken as a reference trajectory of forces.

Maillart evokes an improper geometry of arches rounded at their centers, as in his Salginatobel Bridge. It is not so straightforward, since in 1929 live loads represent 11 % of dead loads, and almost 38 % in the case of the Vessy Bridge in 1936. For both bridges, the geometry has been designed to cause almost no traction in the arches. It could be considered that the relative extent of live loads compared to dead loads influences the tolerance of the suited geometry. A detailed calculation of actual stresses has been made from the available document. In Maillart's approach when designing the mechanical properties of the successive sections of the arches, Maillart considered the boxes only, neglecting the deck's slab. Maillart's calculations show the concrete remaining in compression. Calculations show stresses spanning  $-0.1$  to  $10.5\text{ N/mm}^2$  in the axes of the bridge, *i.e.* at the higher position according to Maillart's simplified model of the isolated boxes. However, stresses at the upper limit of the border of the pedestrian sidewalk are lower:  $-0.7\text{ N/mm}^2$ . It could be encountering cracks that have a negative impact on the life of the work, but it is not certain that traction stresses exceed the resistance of concrete in traction.

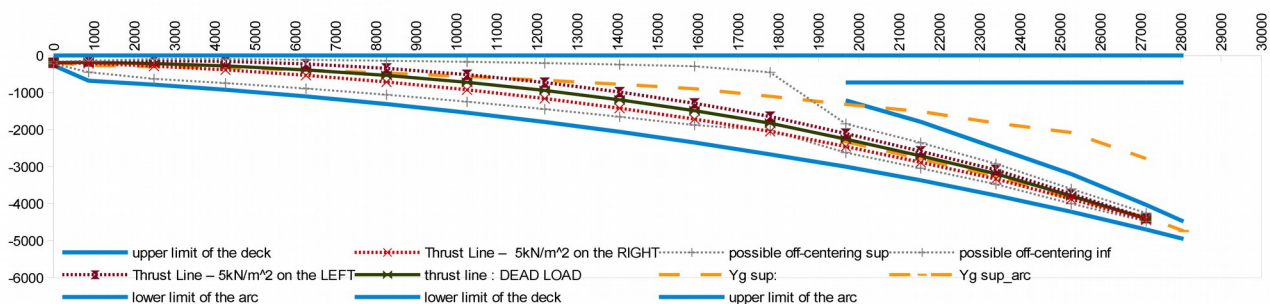


Figure 5: Analysis of thrust and center of gravity of Maillart's Vessy Bridge (1936)

#### 4.2. Equilibrating forces in the actual structure

In the middle of the bridge, due to the necessary connection of the central box-arch to the supporting U-shaped arches, the position of the center of gravity is quite remote from the ideal thrust line. But for most of the cases, the possible off-centering of the resulting forces compared to the focus of the centers of gravity is quite wide due to the high box-sections. A closer analysis of these respective dimensions shows that due to the widening of the thickness of the side walls of the box to connect the stronger U-shape arches, the admissible off-centering for no tension in the concrete is reduced. In fact, the increase of the area of the section lowers the value of the stresses due to the centered compression forces that balance tensions due to bending forces. In other words, the increasing area of the sections lowers the amount of prestressing due to dead loads and symmetrical live loads, while increasing the bending resistance of the section.

Could this situation be corrected by reworking the geometrical arrangement? The following actions are possible: reworking the section to suppress the excess of material at the top part of the bridge that is too far from the center of gravity; managing the section to raise the center of gravity, for instance in manipulating the height of the box; managing the section better to lower its area at the connection and then increase the amount of ‘prestressing’ due to the dead loads; adding some prestressing above the center of gravity to lift the mean position of the resulting thrust line.

It can be noted that the action of a prestressing tendon following the thrust line will be the following: the abutment anchoring forces would replace the supporting forces at the hinges and deviation forces along the cable will replace the loadings along the bridge. It will simply replace the actions and consequences of the loading on the structure, generating a thrust line. Acting against loading forces means using an inverted trajectory of the thrust line that cannot be implemented inside the geometry of the arch. Other actions using prestressing mean modifying the structural behavior of the bridge and the structural action of the supports.

Therefore the only way of introducing prestressing to the Vessy arch is to use the effect of dead loads to lead to distributed stabilizing compression forces.

#### 4.2. The simplified version from early sketches

The early design for the three-hinged version of the Vessy Bridges used a straight line for the upper and lower limits of the arch along the first quarter span. Having lifted the geometrical rules being used to sketch the early prototype (see 3.2), these rules can be applied to the drawing with the actual dimension and this first prototype recreated at real scale. The central thickness is scarcely higher than the final version, but the straight lines of the early version make an outer widened envelope to the arch that increases its resistance. Boxes and U-shapes are higher and stronger. The locus of the center of gravity of the arch at its central part is lower, so the stresses are better balanced. At the side part of the arch, off-centering is greater, but stresses are lower. In the whole bridge, stresses span 0.5 to 7.5 N/mm<sup>2</sup>. This early design would have been suited to higher loading at the cost of a slightly more robust appearance close to that of the Felsegg Bridge. The advantages of straight lines are the simpler formworks and possible straight reinforcement bars or tendons. Therefore this is an alternative geometry for designing a bridge of this kind, as can be seen from its use subsequently in the Garstatt Bridge. It would have been perfectly adapted to the Vessy Bridge if the loadings had been higher.

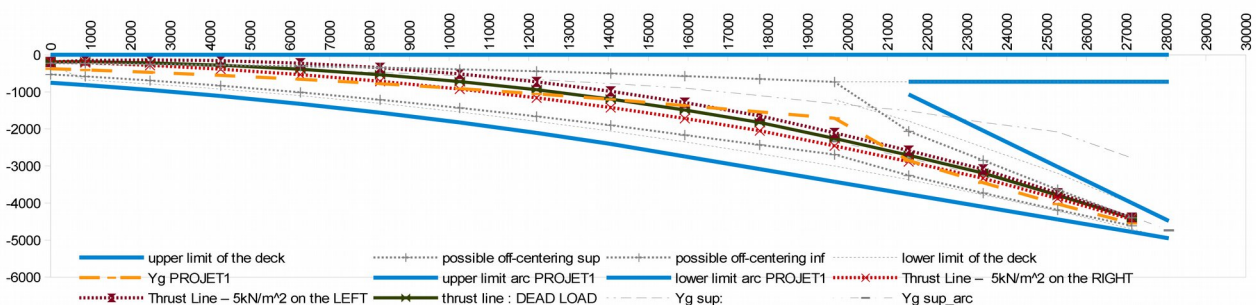


Figure 6: Analysis of thrust and center of gravity of Maillart’s Vessy Bridge (1936)

## 5. Conclusions

This paper studied the design of Maillart’s Vessy Bridge and explained the geometrical rules considered for the early design and for the actual design, and the use of thrust lines built with graphic statics. The choices of reference loading cases to build the reference thrust lines have been discussed. The magnitude of respective loadings compared to Maillart’s previous designs has been analyzed. Respective geometries have also been compared.

The two versions of the design of the Vessy Bridge have been analyzed and mechanical stresses evaluated. It shows that both designs are valuable, with no or very few defects. Maillart’s great experience is deducible from the very few drawings made to design the Bridge. The positive use of dead loads has been shown as a kind of prestressing along the arch and a factor guaranteeing proper behavior. As shown above, manipulating the geometry is the most appropriate way to manage its magnitude and effects. In contrast, the classical approach of prestressing using tendons has been shown to be inadequate in this context.

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