Determination of the pole and $\overline{\text{MS}}$ masses of the top quark from the $t\bar{t}$ cross section

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We use higher-order quantum chromodynamics calculations to extract the mass of the top quark from the $t\bar{t}$ cross section measured in the lepton+jets channel in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV using 5.3 fb^{-1} of integrated luminosity collected by the D0 experiment at the Fermilab Tevatron Collider. The extracted top quark pole mass and $\overline{\text{MS}}$ mass are compared to the current Tevatron average top quark mass obtained from direct measurements.

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The mass of the top quark (m_t) has been measured with a precision of 0.6%, and its current Tevatron average value is $m_t = 173.3 \pm 1.1$ GeV [1]. Beyond leading-order quantum chromodynamics (LO QCD), the mass of the top quark is a convention-dependent parameter. Therefore, it is important to know how to interpret this experimental result in terms of renormalization conventions [2] if the value is to be used as an input to higher-order QCD calculations or in fits of electroweak precision observables and the resulting indirect Higgs boson mass bounds [3]. The definition of mass in field theory can be divided into two categories [4]: (*i*) driven by long-distance behavior, which corresponds to the pole-mass scheme, and (*ii*) driven by short-distance behavior, which, for example, is represented by the $\overline{\text{MS}}$ mass scheme. The difference between the masses in different schemes can be calculated as a perturbative series in α_s. However, the concept of the pole mass is ill-defined, since there is no pole in the quark propagator in a confining theory such as QCD [5].

There are two approaches to directly measure m_t from the reconstruction of the final states in decays of top-antitop ($t\bar{t}$) pairs. One is based on a comparison of Monte Carlo (MC) templates for different assumed values of m_t with distributions of kinematic quantities measured in data. In the second approach, m_t is extracted from the reconstruction of the final states in data using a calibration curve obtained from MC simulation. In both cases the quantity measured in data therefore corresponds to the top quark mass scheme used in the MC simulation, which we refer to as m_t^{MC}.

Current MC simulations are performed in LO QCD, and higher order effects are simulated through parton showers at modified leading logarithms (LL) level. In principle, it is not possible to establish a direct connec-

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tion between $m_t^{\rm MC}$ and any other mass scheme, such as the pole or $\overline{\rm MS}$ mass scheme, without calculating the parton showers to at least next-to-leading logarithms (NLL) accuracy. However, it has been argued that $m_t^{\rm MC}$ should be close to the pole mass [6, 7]. The relation between $m_t^{\rm MC}$ and the top quark pole mass ($m_t^{\rm pole}$) or $\overline{\rm MS}$ mass ($m_t^{\overline{\rm MS}}$) is still under theoretical investigation. In calculations such as in Ref. [3] it is assumed that $m_t^{\rm MC}$ measured at the Tevatron is equal to $m_t^{\rm pole}$.

In this Letter, we extract the pole mass at the scale of the pole mass, $m_t^{\rm pole}(m_t^{\rm pole})$, and the $\overline{\rm MS}$ mass at the scale of the $\overline{\rm MS}$ mass, $m_t^{\overline{\rm MS}}(m_t^{\overline{\rm MS}})$, comparing the measured inclusive $t\bar{t}$ production cross section $\sigma_{t\bar{t}}$ with fully inclusive calculations at higher-order QCD that involve an unambiguous definition of m_t and compare our results to $m_t^{\rm MC}$. This extraction provides an important test of the mass scheme as applied in MC simulations and gives complementary information, with different sensitivity to theoretical and experimental uncertainties than the direct measurements of $m_t^{\rm MC}$ that rely on kinematic details of the mass reconstruction.

We use the measurement of $\sigma_{t\bar{t}}$ in the lepton+jets channel in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV using 5.3 fb^{-1} of integrated luminosity collected by the D0 experiment [8]. We calculate likelihoods for $\sigma_{t\bar{t}}$ as a function of m_t, and use higher-order QCD predictions based on the pole-mass or the $\overline{\rm MS}$-mass conventions to extract $m_t^{\rm pole}$or $m_t^{\overline{\rm MS}}$, respectively.

The criteria applied to select the sample of $t\bar{t}$ candidates used in the cross section measurement introduce a dependence of the signal acceptance, and therefore of the measured value of $\sigma_{t\bar{t}}$, on the assumed value of $m_t^{\rm MC}$. This dependence is studied using MC samples of $t\bar{t}$ events generated at different values of $m_t^{\rm MC}$ in intervals of at least 5 GeV and is found to be much weaker than the dependence of the theoretical calculation of $\sigma_{t\bar{t}}$ on m_t. The $t\bar{t}$ signal is simulated with the ALPGEN event generator [9], and parton evolution is simulated with PYTHIA [10]. Jet-parton matching is applied to avoid double-counting of partonic event configurations [11]. The resulting measurement of $\sigma_{t\bar{t}}$ can be described by

$$\begin{aligned}\sigma_{t\bar{t}}(m_t^{\rm MC}) \; &= \; \frac{1}{(m_t^{\rm MC})^4}[a + b\;(m_t^{\rm MC} - m_0) \\ &+ \; c\;(m_t^{\rm MC} - m_0)^2 + d\;(m_t^{\rm MC} - m_0)^3]\,,\end{aligned} \quad (1)$$

where $\sigma_{t\bar{t}}$ and $m_t^{\rm MC}$ are in pb and GeV, respectively, $m_0 = 170$ GeV, and a, b, c, d are free parameters. For the mass extraction, we consider the experimental $t\bar{t}$ cross section measured using the b-jet identification technique [8]. This $\sigma_{t\bar{t}}$ determination provides the weakest dependence on $m_t^{\rm MC}$ of the results presented in Ref. [8], which leads to a smaller uncertainty on the extracted m_t, and thereby reduces the ambiguity of whichever convention (here pole or $\overline{\rm MS}$) best reflects $m_t^{\rm MC}$. When using b-tagging, the data sample is split into events with $0, 1$ or > 1 b-tagged jets, and the numbers of events in each of the three categories, corrected for mass-dependent acceptance, yield the measurement of $\sigma_{t\bar{t}}$. The other methods used in Ref. [8] rely on additional topological information that introduces a stronger dependence of the measured $\sigma_{t\bar{t}}$ on $m_t^{\rm MC}$. They are therefore not used in this analysis. The parameters derived from a fit of $\sigma_{t\bar{t}}$ to Eq. (1) are: $a = 6.95 \times 10^9$ pb GeV^4, $b = 1.25 \times 10^8$ pb GeV^3, $c = 1.16 \times 10^6$ pb GeV^2, and $d = -2.55 \times 10^3$ pb GeV. Possible fit shape changes due to the uncertainties on these parameters are small compared to the experimental uncertainties on the $\sigma_{t\bar{t}}$ measurement which are almost fully correlated between different m_t. For $m_t^{\rm MC}$=172.5 GeV, we measure $\sigma_{t\bar{t}} = 8.13^{+1.02}_{-0.90}$ pb [8].

We compare the obtained parameterization to a pure next-to-leading-order (NLO) QCD [12] calculation, to a calculation including NLO QCD and all higher-order soft-gluon resummations in NLL [13], to a calculation including also all higher-order soft-gluon resummations in next-to-next-to-leading logarithms (NNLL) [14] and to two approximations of the next-to-next-to-leading-order (NNLO) QCD cross section that include next-to-next-to-leading logarithms (NNLL) relevant in NNLO QCD [15, 16]. The computations in Ref. [15] were obtained using the program documented in Ref. [17].

Following the method of Refs. [18, 19], we extract the most probable m_t values and their 68% C.L. bands for the pole-mass and $\overline{\rm MS}$-mass conventions by computing the most probable value of a normalized joint-likelihood function:

$$L(m_t) = \int f_{\rm exp}(\sigma|m_t)\;[f_{\rm scale}(\sigma|m_t) \otimes f_{\rm PDF}(\sigma|m_t)]\;d\sigma. \quad (2)$$

The first term $f_{\rm exp}$ corresponds to a function for the measurement constructed from a Gaussian function with mean value given by Eq. (1) and with standard deviation (sd) equal to the total experimental uncertainty which is described in detail in Ref. [8]. The second term $f_{\rm scale}$ in Eq. (2) is a theoretical likelihood formed from the uncertainties on the renormalization and factorization scales of QCD, which are taken to be equal, and varied up and down by a factor of two from the default value. Within this range, $f_{\rm scale}$ is taken to be constant [12–16]. It is convoluted with a term that represents the uncertainty of parton density functions (PDFs), taken to be a Gaussian function, with rms equal to the uncertainty determined in Refs. [12–16]. Table I summarizes the theoretical predictions from different calculations for $m_t^{\rm pole} = 175$ GeV used as an input to the likelihood fit.

In Refs. [12–16] $\sigma_{t\bar{t}}$ is calculated as a function of $m_t^{\rm pole}$ and, consequently, comparing the measured $\sigma_{t\bar{t}}(m_t^{\rm MC})$ to these theoretical predictions provides a value of $m_t^{\rm pole}$. Therefore, we extract $m_t^{\rm pole}$ (i) assuming that the definition of $m_t^{\rm MC}$ is equivalent to $m_t^{\rm pole}$, and (ii) taking $m_t^{\rm MC}$ to be equal to $m_t^{\overline{\rm MS}}$ to estimate the effect of interpreting $m_t^{\rm MC}$ as any other mass definition. For case (i), Fig. 1 shows the parameterization of the measured and the predicted $\sigma_{t\bar{t}}(m_t^{\rm pole})$ [14–16]. The results for the de-

TABLE I: Theoretical predictions for $\sigma_{t\bar{t}}$ with uncertainties $\Delta\sigma$ due to scale dependence and PDFs at the Tevatron for $m_t^{\rm pole}$=175 GeV from different theoretical calculations used as input to the mass extraction. Note that Refs. [12] and [13] use the CTEQ6.6 PDF set [20] while Refs. [14], [15], and [16] use the MSTW08 PDF set [21].

Theoretical prediction	$\sigma_{t\bar{t}}$ (pb)	$\Delta\sigma_{\rm scale}$ (pb)	$\Delta\sigma_{\rm PDF}$ (pb)
NLO [12]	6.39	$^{+0.33}_{-0.70}$	$^{+0.35}_{-0.35}$
NLO+NLL [13]	6.61	$^{+0.26}_{-0.46}$	$^{+0.44}_{-0.34}$
NLO+NNLL [14]	5.93	$^{+0.18}_{-0.17}$	$^{+0.30}_{-0.22}$
Approximate NNLO [15]	6.71	$^{+0.28}_{-0.37}$	$^{+0.17}_{-0.12}$
Approximate NNLO [16]	6.66	$^{+0.11}_{-0.06}$	$^{+0.42}_{-0.35}$

TABLE II: Values of $m_t^{\rm pole}$, with their 68% C.L. uncertainties, extracted for different predictions of $\sigma_{t\bar{t}}$. The results assume that $m_t^{\rm MC} = m_t^{\rm pole}$ (left column). The right column shows the change $\Delta m_t^{\rm pole}$ between these results if it is assumed that $m_t^{\rm MC} = m_t^{\overline{\rm MS}}$. The combined experimental and theoretical uncertainties are shown.

Theoretical prediction	$m_t^{\rm pole}$ (GeV)	$\Delta m_t^{\rm pole}$ (GeV)
MC mass assumption	$m_t^{\rm MC} = m_t^{\rm pole}$	$m_t^{\rm MC} = m_t^{\overline{\rm MS}}$
NLO [12]	$164.8^{+5.7}_{-5.4}$	−3.0
NLO+NLL [13]	$166.5^{+5.5}_{-4.8}$	−2.7
NLO+NNLL [14]	$163.0^{+5.1}_{-4.6}$	−3.3
Approximate NNLO [15]	$167.5^{+5.2}_{-4.7}$	−2.7
Approximate NNLO [16]	$166.7^{+5.2}_{-4.5}$	−2.8

termination of $m_t^{\rm pole}$ are given in Table II. In case (*ii*) the cross section predictions use the pole-mass convention, and the value of $m_t^{\rm MC} = m_t^{\overline{\rm MS}}$ is converted to $m_t^{\rm pole}$ using the relationship at the three-loop level [5, 22]:

$$m_t^{\rm pole} = m_t^{\overline{\rm MS}}(m_t^{\overline{\rm MS}}) \Big[1 + \frac{4}{3}\frac{\overline{\alpha}_s(m_t^{\overline{\rm MS}})}{\pi} + (-1.0414 N_L + 13.4434)\left(\frac{\overline{\alpha}_s(m_t^{\overline{\rm MS}})}{\pi}\right)^2 + (0.6527 N_L^2 - 26.655 N_L + 190.595)\left(\frac{\overline{\alpha}_s(m_t^{\overline{\rm MS}})}{\pi}\right)^3\Big], \quad (3)$$

where $\overline{\alpha}_s$ is the strong coupling in the $\overline{\rm MS}$ scheme, and $N_L = 5$ is the number of light quark flavors. The strong coupling $\overline{\alpha}_s(m_t^{\rm pole})$ is taken at the three-loop level from Ref. [23]. By iteratively rederiving the $\overline{\rm MS}$ mass using Eq. (3) $\overline{\alpha}_s(m_t^{\rm pole})$ is transformed into $\overline{\alpha}_s(m_t^{\overline{\rm MS}})$ leading to a difference of only 0.1 GeV to the final extraction of $m_t^{\overline{\rm MS}}$. For $m_t^{\rm pole} = 173.3$ GeV, the $\overline{\rm MS}$ mass $m_t^{\overline{\rm MS}}(m_t^{\overline{\rm MS}})$ is lower by 9.8 GeV. With this change of

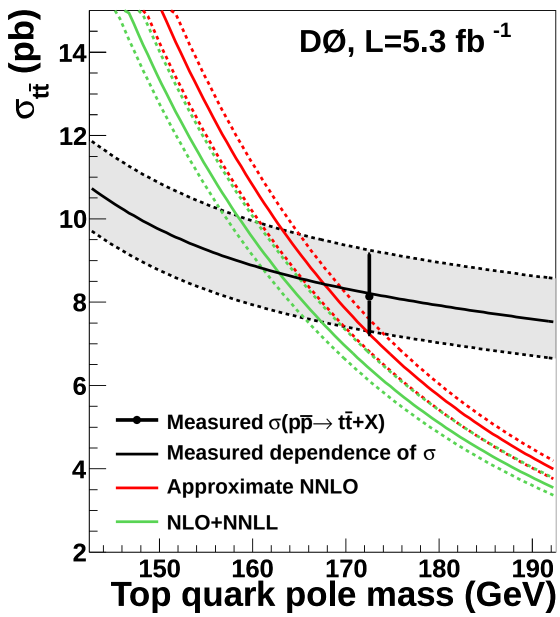

FIG. 1: (Color online) Measured $\sigma_{t\bar{t}}$ and theoretical NLO+NNLL [14] and approximate NNLO [15] calculations of $\sigma_{t\bar{t}}$ as a function of $m_t^{\rm pole}$, assuming that $m_t^{\rm MC} = m_t^{\rm pole}$. The colored dashed lines represent the uncertainties for the two theoretical calculations from the choice of the PDF and the renormalization and factorization scales (added quadratically). The theoretical calculation of Ref. [16] (not displayed) agrees with Ref. [15] within 1% both in mean value and uncertainty. The point shows the measured $\sigma_{t\bar{t}}$ for $m_t^{\rm MC} = 172.5$ GeV, the black curve is the fit to Eq. (1), and the gray band corresponds to the total experimental uncertainty.

the $m_t^{\rm MC}$ interpretation in Eq. (1), we form a new likelihood $f_{\rm exp}(\sigma|m_t)$ and extract $m_t^{\rm pole}$ using Eq. (2). The difference $\Delta m_t^{\rm pole}$ between assuming $m_t^{\rm MC} = m_t^{\rm pole}$ and $m_t^{\rm MC} = m_t^{\overline{\rm MS}}$ is given in Table II. Given the uncertainties, interpreting $m_t^{\rm MC}$ as either $m_t^{\rm pole}$ or as $m_t^{\overline{\rm MS}}$ has no significant bearing on the value of the extracted m_t. We include half of this difference symmetrically in the systematic uncertainties. As a result we extract $m_t^{\rm pole} = 163.0^{+5.4}_{-4.9}$ GeV using the NLO+NNLL calculation of Ref. [14] and $m_t^{\rm pole} = 167.5^{+5.4}_{-4.9}$ GeV using the approximate NNLO calculation of Ref. [15]. Our measurement of $m_t^{\rm pole}$ based on the approximate NNLO cross section calculation is consistent within 1 sd with the Tevatron measurement of m_t from direct reconstruction of top quark decay products, $m_t = 173.3 \pm 1.1$ GeV [1]. The result based on the NLO+NNLL calculation is consistent within 2 sd.

Calculations of the $t\bar{t}$ cross section [14, 15] have also been performed as a function of $m_t^{\overline{\rm MS}}$. Comparing the dependence of the measured $\sigma_{t\bar{t}}$ to theory as a function of m_t provides an estimate of $m_t^{\overline{\rm MS}}$. We note that a previous extraction of $m_t^{\overline{\rm MS}}$ [15] ignored the m_t dependence of the measured $\sigma_{t\bar{t}}$.

We extract the value of $m_t^{\overline{\rm MS}}$, again, for two cases: (i) assuming that the definition of m_t implemented in the MC simulation is equal to $m_t^{\rm pole}$, and (ii) assuming that $m_t^{\rm MC}$ corresponds to $m_t^{\overline{\rm MS}}$. For case (i), $m_t^{\rm pole}$ must first be converted to $m_t^{\overline{\rm MS}}$ using Eq. (3). Figure 2 shows the measured $\sigma_{t\bar{t}}$ as a function of $m_t^{\overline{\rm MS}}$, together with the calculation that includes NLO+NNLL QCD resummation [14] and the approximate NNLO calculation [15].

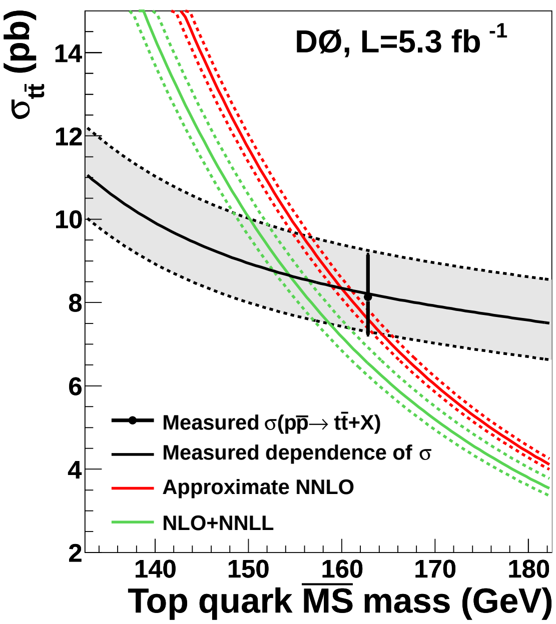

FIG. 2: (Color online) Measured $\sigma_{t\bar{t}}$ and theoretical NLO+NNLL [14] and approximate NNLO [15] calculations of $\sigma_{t\bar{t}}$ as a function of $m_t^{\overline{\rm MS}}$, assuming that $m_t^{\rm MC} = m_t^{\rm pole}$. The colored dashed lines represent the uncertainties for the two theoretical calculations from the choice of the PDF and the renormalization and factorization scales (added quadratically). The point shows the measured $\sigma_{t\bar{t}}$ for $m_t^{\rm MC}$=172.5 GeV, the black curve is the fit to Eq. (1), and the gray band corresponds to the total experimental uncertainty.

The results for the extracted values of $m_t^{\overline{\rm MS}}$ are given in Table III.

In case (ii), we assume that the mass definition in the MC simulation corresponds to the $\overline{\rm MS}$ mass. We set $m_t^{\rm MC} = m_t^{\overline{\rm MS}}$ in Eq. (2), form a new likelihood $f_{\rm exp}(\sigma|m_t)$ and extract $m_t^{\overline{\rm MS}}$ using Eq. (2) for the two calculations of Fig. 2. The difference $\Delta m_t^{\overline{\rm MS}}$ between assuming that $m_t^{\rm MC} = m_t^{\rm pole}$ and assuming $m_t^{\rm MC} = m_t^{\overline{\rm MS}}$ is given in

TABLE III: Values of $m_t^{\overline{\rm MS}}$, with their 68% C.L. uncertainties, extracted for different theoretical predictions of $\sigma_{t\bar{t}}$. The results assume that $m_t^{\rm MC}$ corresponds to $m_t^{\rm pole}$ (left column). The right column shows the change $\Delta m_t^{\overline{\rm MS}}$ between these results if it is assumed that $m_t^{\rm MC} = m_t^{\overline{\rm MS}}$. The combined experimental and theoretical uncertainties are shown.

Theoretical prediction	$m_t^{\overline{\rm MS}}$ (GeV)	$\Delta m_t^{\overline{\rm MS}}$ (GeV)
MC mass assumption	$m_t^{\rm MC} = m_t^{\rm pole}$	$m_t^{\rm MC} = m_t^{\overline{\rm MS}}$
NLO+NNLL [14]	$154.5^{+5.0}_{-4.3}$	-2.9
Approximate NNLO [15]	$160.0^{+4.8}_{-4.3}$	-2.6

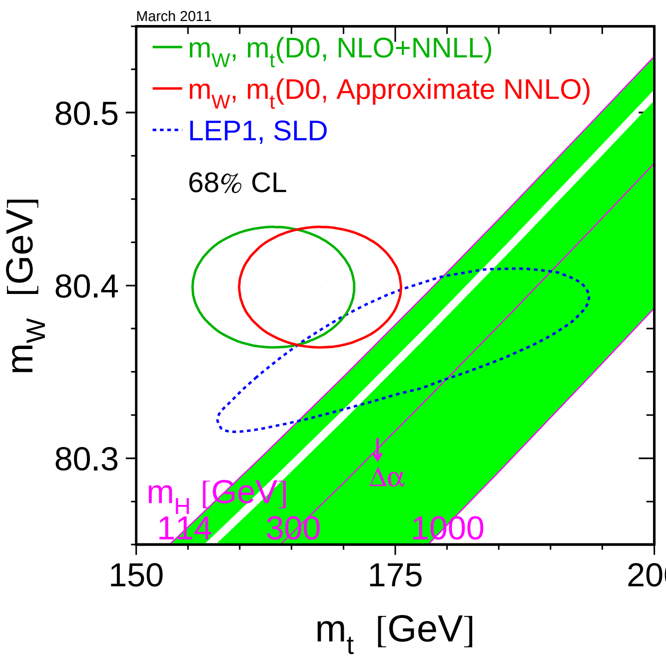

FIG. 3: (Color online) Constraints on the W boson mass from the LEP-II/Tevatron experiments and the top quark pole mass extracted from the $t\bar{t}$ cross section in NLO+NNLL [14] (green contour) and approximate NNLO [15] (red contour). This is compared to the indirect constraints on the W boson mass and the top quark mass based on LEP-I/SLD data (dashed contour). In both cases the 68% CL contours are given. Also shown is the SM relationship for the masses as a function of the Higgs mass in the region favoured by theory ($<$ 1000 GeV) and not excluded by direct searches (114 GeV to 158 GeV and $>$ 173 GeV). The arrow labelled $\Delta\alpha$ shows the variation of this relation if $\alpha(m_Z^2)$ is varied between -1 and $+1$ sd. This variation gives an additional uncertainty to the SM band shown in the figure.

Table III. We include half of this difference symmetrically in the systematic uncertainties and derive a value of $m_t^{\overline{\rm MS}} = 154.5^{+5.2}_{-4.5}$ GeV using the calculation of Ref. [14] and $m_t^{\overline{\rm MS}} = 160.0^{+5.1}_{-4.5}$ GeV using Ref. [15].

To summarize, we extract the pole mass (Table II) and the $\overline{\rm MS}$ mass (Table III) for the top quark by comparing the measured $\sigma_{t\bar{t}}$ with different higher-order perturbative QCD calculations. The Tevatron direct measurements of m_t are consistent with both $m_t^{\rm pole}$ measurements within 2 sd, but they are different by more than 2 sd from the extracted $m_t^{\overline{\rm MS}}$. The results on $m_t^{\rm pole}$ and their interplay with other electroweak results within the SM are displayed in Fig. 3, which is based on Ref. [3].

For the first time, $m_t^{\overline{\rm MS}}$ is extracted with the m_t dependence of the measured $\sigma_{t\bar{t}}$ taken into account. Our measurements favor the interpretation that the Tevatron m_t measurements based on reconstructing top quark decay products is closer to the pole than to the $\overline{\rm MS}$ top quark mass.

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