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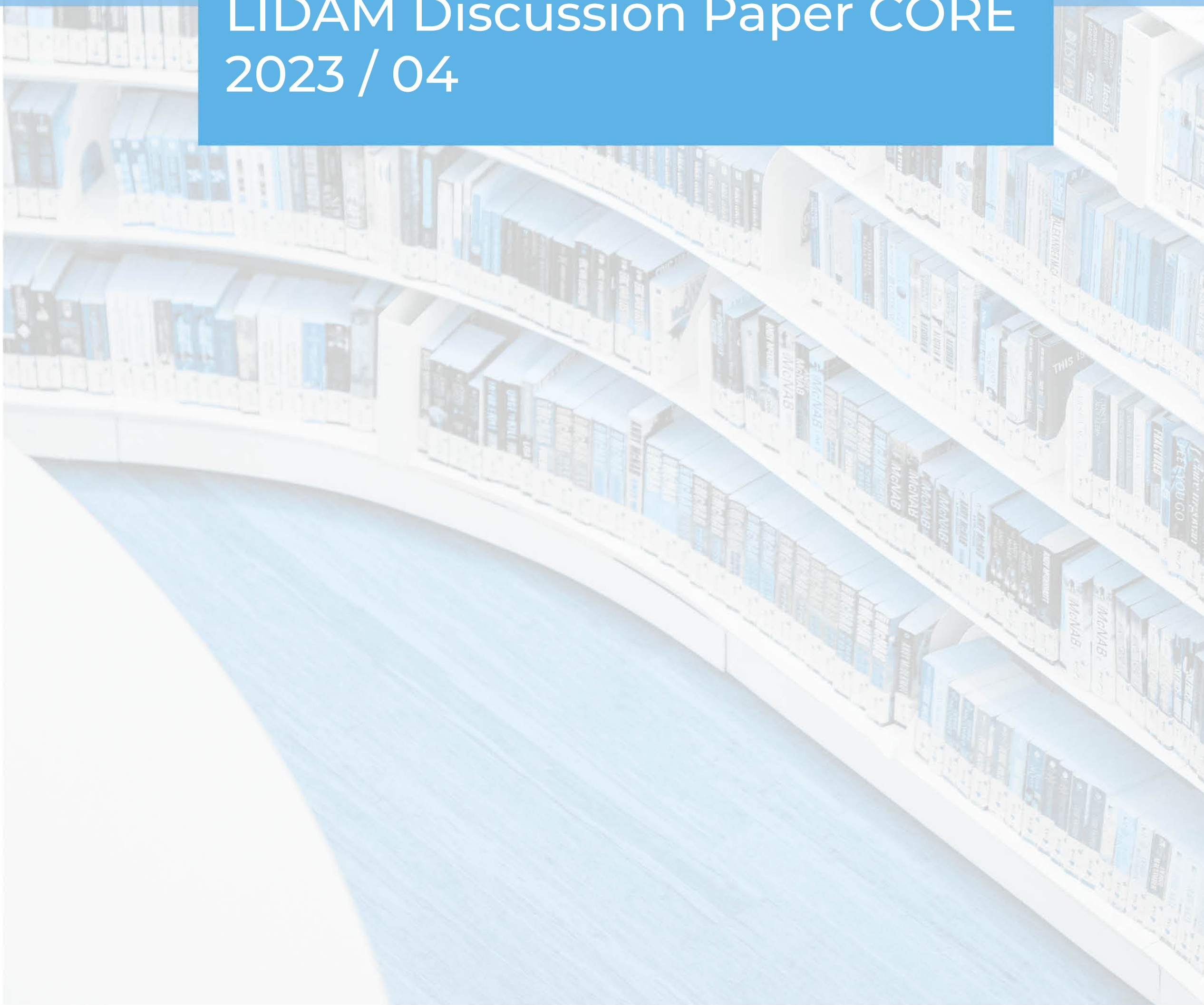

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R&D and Market Sharing Agreements

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Abstract

We analyze the formation of R&D alliances and market sharing (MS) agreements by which firms commit not to enter in each other's territory in oligopolistic markets. We show that R&D alliance structures are stable only in the presence of MS agreements. Thus, long lasting R&D alliances could signal the existence of some MS agreement in the industry. We characterize the set of stable symmetric pairs of coalition structures with identical R&D and MS structure. In addition, we show the stability of a class of asymmetric pairs of coalition structures where the most efficient firms form both an R&D and a MS agreement while the other firms do not form any MS agreement but form two smaller R&D alliances. Even though MS agreements are detrimental for consumers, we show that stable cooperation in terms of R&D is yet a better outcome for consumers than no cooperation at all.

Key words: R&D alliances, Market sharing agreements, Oligopoly, Cournot competition, Stability.
JEL classification: C70, L13, L40.

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1 Introduction

R&D alliances, where members share their R&D resources to reduce the production cost of a product and the risk of innovation,[1] create external effects on competitors who can reply by searching for their own R&D alliances resulting in an R&D alliance structure (i.e., a partition of the set of firms into disjoint R&D alliances). The formation of R&D alliances might also facilitate the development of collusive agreements. If such a correlation between legal and illegal collaborations exists, promoting R&D alliances might enable firms to set higher prices than in perfect competition and finally be harmful for consumers. Among the several types of collusion (price collusion, bid rigging. . .), a scant attention has been given to market sharing (MS) agreements, whereby partners refrain from entering in each other's territory.[2] A market sharing agreement might strongly bolster the competitive advantages generated by an R&D alliance. Firms involved in an R&D alliance are more efficient than the firms outside the alliance in terms of production cost. However, in the absence of a market sharing agreement, the R&D partners continue competing among them in their home markets. By forming a MS agreement, the most efficient firms will only compete with the less efficient firms outside their R&D alliance. Therefore, the competitive advantages generated by the formation of the R&D alliance would be amplified by the formation of the MS agreement. Despite these observations, no work has analyzed the formation of both R&D alliances and MS agreements in an oligopolistic environment. Our paper aims at filling this gap by identifying the R&D alliances and MS agreements that will be formed among oligopolistic firms competing in quantities.

Initially, public authorities were incline to foster R&D cooperation between firms with subsidies and antitrust exemptions towards R&D partners.[3] However, concrete antitrust cases raise the interrogation of the real economic benefits of R&D cooperation.[4] In 2009, a market sharing cartel that took place in the industry of power transformers between 1999 and 2003 was sentenced 67.6 million euros by the European Commission.[5] Prior to the formation of the MS agreements, some of the involved firms created R&D agreements

[1]We only consider R&D aiming at finding more efficient production process (non-drastic innovation). We do not deal with R&D trying to develop new products (drastic innovation).

[2]MS agreements represent a significant part of the cases sentenced by antitrust authorities. In the USA between 1995 and 2017, 53 over 139 antitrust cases fined 10 million USD or more (38%) concern MS agreement. https://www.justice.gov/atr/sherman-act-violations-yielding-corporate-fine-10-million-or-more.

[3]As an example, the National Cooperative Research and Production Act (NCRPA) in the US allows firms involved in a R&D joint venture to recover attorney fees, and to be granted milder antitrust scrutiny (Duso, Röller and Seldeslachts, 2014).

[4]Many empirical evidences validate the theory claiming that R&D agreements facilitate the formation of price collusions : From descriptive analyses (Vonortas, 2000; Snyder and Vonortas, 2005), to laboratory experiments (Cason and Mason, 1999; Suetens, 2008) and quasi-experiments based on the revisions of the US leniency program in 1993 and 1995 (Sovinsky and Helland, 2012). In addition, R&D agreements might provide justifications for the illicit actions of firms involved in MS agreements (e.g. CEOs meetings). This would make the latter agreements harder to detect when R&D cooperation exists.

[5]http://europa.eu/rapid/press-release_IP-09-1432_en.htm?locale=en.

among them: Toshiba and Hitashi, among others, developed a research cooperation on semiconductors along the 1980's.[6] In 1999, Fudji Electrics and Hitachi found Fuji Hitachi Semiconductor Co. to assemble their research efforts on semiconductor materials.[7] It must be noted that semiconductor materials have paramount importance in the efficiency of power transformers. After the detection of the MS cartel, Fudji Electrics and Siemens started a R&D collaboration in the industry of drive systems for metal industry devices in 2005.[8] Drive systems are components of power transformers. Another example concerns the Vitamin cartel where both the US department of justice[9] and the European commission[10] sentenced several pharmaceutical companies including F. Hoffmann-La Roche and BASF AG. These two firms are part of a Chemical and Pharmaceutical research joint venture nammed SISF, created in 2017.[11] Furthermore, Duso, Röller and Seldeslachts (2014) empirically estimate that the size of the R&D alliances as well as the nature of the firms composing them (competitors or not) significantly affect the market share allocation, which is used as a proxy for collusion.[12] In this paper, we analyze the incentives of oligopolistic firms to form both R&D alliances and MS agreements in order to help the antitrust authorities to better assess the efficiency of policies promoting R&D collaborations and policies for the detection of MS agreements.

We provide a game theoretic framework where the n firms in an oligopolistic industry can form R&D alliances and MS agreements before individually deciding the quantities they will sell on each market. Each firm is able to reduce its cost by forming an R&D alliance with other competitors. The reduction of the marginal cost of production for one firm is proportional to the number of firms in its alliance. The collection of all R&D alliances define the R&D alliance structure which in turn determines the marginal costs for the n oligopolists. Each firm is initially present on one market (its home market) and can enter all foreign markets. We model MS agreements as coalitional agreements preventing its members to compete with each other in their respective home markets. However, the members of the MS agreement are still competing with each other on the remaining markets. The collection of all MS agreements define the MS structure which determines the markets in which each firm is active. Once the R&D alliance structure and the MS agreements have been formed, firms compete in quantities in every market where they are active.[13]

[6] OECD Technology Outlook, 2008, p 168.

[7] http://www.referenceforbusiness.com/history2/79/Fuji-Electric-Co-Ltd.htm.

[8] https://www.fujielectric.com/company/news/2005/05022101.htm.

[9] https://www.justice.gov/archive/atr/public/press_releases/1999/2450.htm.

[10] http://europa.eu/rapid/press-release_IP-01-1625_en.htm.

[11] http://www.sisf.info/.

[12] In the literature on R&D agreements, Pogayo-Theotoky (1995) analyses the effect of R&D spillovers on the size of R&D collaborations and shows that the impact of R&D alliances on social welfare depends on its size. Thus, the structure of R&D alliances (and its size) is therefore essential to study the impact of R&D collaborations on economic variables.

[13] Firms collaborate in R&D but do not cooperate on R&D effort choices. For a general background on R&D cooperation in oligopoly the reader is directed to Amir (2000), d'Aspremont and Jacquemin (1988),

The formation of an R&D alliance and of a MS agreement create spillovers of opposite signs on firms outside the alliance and outside the MS agreement.[14] While the formation of a larger competing R&D alliance decreases the profits of firms outside the alliance (there are negative spillovers in the formation of R&D alliances), the formation of a larger competing MS agreement increases the profits of firms outside the agreement (there are positive spillovers in the formation of MS agreements). On the one hand, the members of a larger R&D alliance can produce at a lower cost relative to the firms outside the alliance. On the other hand, the reduction in the number of competitors in the home markets of the members of a larger MS agreement benefits the firms outside the agreement. How the competitive advantage over common competitors gained by a group of firms after becoming R&D partners would be affected by the reduction in the number of competitors if they also become MS partners? The answer to this question will determine the R&D alliance structures and MS structures that will emerge in an oligopolistic industry.

The literature on cooperative R&D has usually assumed that the formation of R&D alliances and of MS agreements takes place separately, i.e., that firms form either R&D alliances or MS agreements but not both types of agreements at the same time. Bloch (1995) has studied the formation of R&D alliances in the absence of MS agreements in the industry. According to Bloch (1995), in an industry where all firms compete with each other in all markets, alliances structures consisting of more than two R&D alliances cannot be stable because the two smallest alliances in the industry have always incentives to merge. However, R&D structures containing several R&D alliances are observed in different industries.[15] We show that once firms can establish both R&D alliances and MS agreements, alliance structures consisting of more than two alliances can be formed in the industry. Hence, it seems that the presence of more than two R&D alliances in an industry could signal the existence of some MS agreement in the industry.

We investigate, given an R&D alliance structure, the incentives of firms to dissolve their MS agreement. We show that only MS agreements of large size relative to the total number of firms in the industry and composed of homogeneous firms in terms of R&D efficiency would not be dissolved. Therefore, MS agreements of moderate size can only emerge on markets where there is high heterogeneity between competitors in terms of R&D efficiency.

We analyze the R&D alliance structures and the MS agreements that one might expect to emerge by examining the equilibrium requirement that no group of firms benefits from altering the R&D alliance structure (given the MS agreements) or the MS structure (given the R&D alliance structure). We follow the notion of γ-stability of Hart and Kurz (1983) according to which the non-deviating players become singletons after the

Kamien, Muller, and Zang (1992), Katz (1986) and Spence (1984), among others.

[14] We are dealing with a theoretical model of overlapping coalition structures where the formation of a coalition has both negative and positive spillovers on the firms outside the coalition.

[15] Among the 17 industries considered in the database of Snyder and Vonortas (2005) between 1985 and 1998, every one of them include strictly more than two R&D joint ventures.

deviation of some coalition members.[16] We propose the concept of γ-stable pair of coalition structures to identify the R&D alliance structure and the MS structure that are going to emerge in the long run.[17] We find that in the absence of MS agreements there is no γ-stable (δ-stable) R&D alliance structure in oligopolistic industries containing more than seven firms. However, there are empirical evidences of many stable R&D alliances in different industries. We show that R&D alliance structures might be stable in oligopolistic industries only in the presence of some MS agreement. Therefore, the ability of firms to form MS agreements stabilize R&D alliances in the long run. This result stems from the fact that a lower competition among firms enable them to capture a larger part of the surplus generated by the R&D cooperation. Thus, our results highlight that the stability of R&D alliances in an oligopolistic industry could also signal the existence of a MS agreement in the same industry.

Our main result establishes that the class of asymmetric pairs of coalition structures where the most efficient firms of the industry in terms of R&D form both an R&D alliance and a MS agreement while the rest of the firms of the industry do not form any MS agreement but form two smaller R&D alliances is γ-stable. We then characterize the set of stable symmetric pairs of coalition structures with identical R&D and MS structure. In these γ-stable pairs of symmetric coalition structures, either the same $n-1$ firms collaborate and form an R&D alliance and a MS agreement or the n firms of the industry form the grand R&D alliance and the global MS agreement. The same pairs of coalition structures are also stable when we use the notion of δ-stability in the formation of R&D alliances and the notion of γ-stability in the formation of MS agreements. However, when δ-stability is used in the formation of MS agreements, no pair of symmetric coalition structures is δ-stable. Regarding the consumer surplus in the stable pairs of coalition structures, we show that larger MS agreements could be detrimental for consumers even if they stabilize larger R&D alliances. However, we show that stable cooperation in terms of R&D between oligopolistic firms is yet a better outcome for consumers than no cooperation at all.

We now discuss the relationship of our article with the existing literature. Starting with d'Aspremont and Jacquemin's (1988) seminal contribution, the relationship between R&D cooperation and the development of collusive agreements has been extensively studied in the literature. d'Aspremont and Jacquemin (1988) highlight that even though collusion in production leads to the formation of more R&D agreements, such agreements erode the consumers surplus by limiting the quantities produced on the market. In a patent race type of game with drastic innovations, Martin (1995) underlines that R&D cooperation makes collusion in quantities more likely and mitigates the social welfare. Regarding non-drastic innovations, Miyagiwa (2009) shows that R&D collaboration elim-

[16]A MS agreement would collapse once, for instance, some members use the leniency program to report the agreement to the antitrust authorities.

[17]Section 6 discusses the notion of δ-stability of Hart and Kurz (1983), where the non-deviating players remain together after the deviation of some coalition members.

inates asymmetries among firms and facilitates price collusions and market allocation. No clear predictions concerning the welfare effect of R&D cooperation are provided. Cooper and Ross (2009) analyze a model where firms cooperate in R&D before competing and can collude on an unrelated market. They show that cooperation in R&D on one market sends pro-cooperation signals that can foster collusion between the two firms on the other market. Moreover, when firms compete before cooperating in R&D, the threat of breaking R&D agreements surges the harshness of the punishment for collusion betrayal. A similar effect is discovered in stochastic R&D race models where firms invest some amount of effort in the R&D cooperation (Cabral, 2000). Nevertheless, all these papers are considering duopolistic models where collusion is sustainable when both firms have no incentives to betray and fail to capture the competitive advantages over common competitors two firms gain by becoming R&D partners. The incentives of competing firms to form R&D alliances and collusive agreements should be analyzed in oligopolistic industries where the formation of alliances and collusive agreements generate spillovers on non-members.

Up to now, the formation of strategic alliances and of MS agreements has been studied using network theory (Goyal and Moraga-Gonzalez (2001), Goyal and Joshi (2003), Belleflamme and Bloch (2004), Mauleon, Sempere-Monerris and Vannetelbosch (2008, 2014)) or coalition theory (Bloch (1995), Mauleon, Sempere-Monerris and Vannetelbosch (2016)), under the assumption that firms are only forming R&D collaborations or MS agreements but not both types of agreements at the same type. We are not aware of any work analyzing the formation of both R&D alliances and MS agreements in an oligopolistic environment. The network approach differs from the group formation approach by focusing on bilateral relationships and allowing for a richer class of collaborations. It also differs in the decision making for establishing collaborations. Mutual consent is needed for forming a new link between two firms,[18] whereas the consent of all members of the association is required when a firm joins the association.[19] The predictions obtain by both approaches do not always coincide.[20] Both approaches lead to similar conclusions only if firms are farsighted and anticipate the reactions of other firms to the decisions they take.[21]

[18]Caulier, Mauleon and Vannetelbosch (2013) and Caulier, Mauleon, Sempere-Monerris and Vannetelbosch (2013) propose the concept of contractual stability to predict the stable networks when the consent of coalition partners is needed for adding or deleting links.

[19]In the open membership game where any firm can form an alliance with another firm simply by announcing the same message, Yi (1997) has shown that the only Nash equilibrium is the grand R&D alliance. However, once firms are asymmetric, the grand R&D alliance may not be a Nash equilibrium outcome of the game. See Belleflamme (2000) and Yi and Shin (2000).

[20]Bloch (1995, 1996) proposes a sequential game for forming associations of firms. In equilibrium, firms form two asymmetric associations, with the largest one comprising roughly three-quarters of industry members. The sizes of the two associations differ from those Mauleon, Sempere-Monerris and Vannetelbosch (2014) obtain under the network approach where pairwise stable R&D networks consist of two components of nearly equal size.

[21]Recent contributions analyze collaboration between farsighted firms: Roketsky (2018), Petrakis and Tsakas (2018), Mauleon, Sempere-Monerris and Vannetelbosch (2014, 2018, 2022).

The paper is organized as follows. Section 2 describes the model. Section 3 analyzes the incentives for the firms active on a market to merge their R&D alliances in an industry containing at least three R&D alliances. Section 4 investigates the incentives of firms to dissolve their MS agreement. Section 5 studies the R&D and MS structures that will emerge in the long run. Section 6 discuses the robustness of the results and concludes.

2 The Model

We consider an oligopolistic setting where each firm can prevent other firms to compete with her in its home market by signing MS agreements, and can decrease its production cost by forming R&D alliances. Bloch (1995) proposes a simple model to analyze the formation of R&D alliances of firms where the benefits from cooperation increase linearly in the size of the alliance. We extend Bloch's (1995) model to analyze the formation of both R&D alliances and MS agreements among firms in an oligopolistic industry.

Consider an industry with n firms that can form R&D alliances and MS agreements before deciding the quantities they will sell on each market. Let $N = \{1, 2, ..., n\}$ be the set of firms with $n \in \mathbb{N}^*$. An R&D alliance structure $R = \{R_1, R_2, \cdots, R_r\}$ is a partition of the set of firms such that $R_i \cap R_j = \emptyset$ for $i \neq j$ and $\bigcup_{i=1}^{r} R_i = N$. Let $r \in \mathbb{N}$ be the cardinality of R (i.e. the number of alliances in R). Let $R(i) \in R$ be the alliance to which firm i belongs and let $r_i \in \mathbb{N}$ denote the size of alliance $R(i)$. Let $R^g = \{N\}$ and $R^s = \{\{1\}, \cdots, \{n\}\}$ be the grand alliance and the singleton R&D alliance structures respectively. We denote by $\mathcal{R}$ the set of all the R&D alliance structures among n firms. In general, let $L = \{1, ..., l\} \subseteq N$ be a set of l firms and let $\mathcal{R}_L$ be the set of all R&D alliance structures that can be formed among the l firms.[22] Let $R_L^s = \{\{1\}, ..., \{l\}\}$ be the singleton R&D alliance structure among the firms in L.

Apart from cooperating on R&D, firms could establish market sharing (MS) agreements, whereby firms refrain from entering each other's territory. We suppose that each firm is initially present on its home market and can enter all foreign markets. We model MS agreements as coalitional agreements not to enter into the markets of the coalitional partners.[23] A MS structure $M = \{M_1, M_2, ..., M_m\}$ is a partition of the set of firms such that $M_i \cap M_j = \emptyset$ for $i \neq j$ and $\bigcup_{i=1}^{m} M_i = N$. Let m be the cardinality of M (i.e. the number of MS agreements in M). Let $M(i) \in M$ be the MS agreement to which firm i belongs and let $m_i \in \mathbb{N}$ denote the size of the MS agreement $M(i)$. Let $M^g = \{N\}$ be the global MS agreement and $M^s = \{\{1\}, \cdots, \{n\}\}$ be the singleton MS structure respectively. We denote by $\mathcal{M}$ the set of all the MS structures among n firms. In general, let $L = \{1, ..., l\} \subseteq N$ be a set of l firms and let $\mathcal{M}_L$ be the set of all MS structures that

[22] Throughout the paper we use the notation $\subseteq$ for weak inclusion and $\subset$ for strict inclusion. Finally, # will refer to the notion of cardinality.

[23] Belleflamme and Bloch (2004) study the stability and welfare implications of bilateral MS agreements that result in a collusive network among firms. Here, we allow for MS agreements involving more than two firms.

can be formed among the l firms. Let $M_L^s = \{\{1\}, ..., \{l\}\}$ be the singleton MS structure among the firms in L.

The pair of coalition structures (R, M) represents an R&D alliance structure together with a MS structure.[24] A pair (R, M) is said to be symmetric if $R = M$; i.e., the R&D alliance structure is identical to the MS structure. The interactions among firms are modelled as a two-stage game. In stage one, R&D alliances and MS agreements are formed. In stage two, given the pair of coalition structures (R, M), firms compete in quantities on the different markets and maximize individual profits.

Each firm is associated to one market: its home market. Each market is endowed with a continuum of consumers. These consumers can only purchase goods from the active firms on the market where they are located. This assumption makes costly the decision for firms to be inactive in a market. An inactive firm in a market cannot have access to the demand located in this market. For any market i, we denote by n_i the number of active firms on the market; i.e., firm i and the firms that do not belong to the MS agreement $M(i)$; $n_i = n - m_i + 1$. Demand captured by firms is linear and is given by $p = \alpha - \sum_{i=1}^{n} q_i$, where α measures the absolute size of the market. Firms have a constant marginal cost of production, which is linearly decreasing in the size of the R&D alliance to which they belong. Formally, the cost of a firm i in an alliance $R(i)$ of size r_i is given by $c_i = \lambda - \mu \cdot r_i$. The parameters α, $\lambda \in \mathbb{R}_+^*$ and $\mu \in \mathbb{R}_+^*$ are chosen in such a way that all firms produce positive quantities in the markets where they are active.

For any given R&D alliance structure $R = \{R_1, R_2, \cdots, R_r\}$ and any given MS structure $M = \{M_1, M_2, ..., M_m\}$, firm i's profit is equal to the sum of the profits in any market where she is active:

$$\pi_i(R, M) = \sum_{h \in (N \setminus M(i)) \cup \{i\}} \left(q_i^h\right)^2 .$$

Firm i's Cournot-Nash equilibrium output on market $h \in N$ is equal to:

$$q_i^h = \frac{\alpha - \lambda + (n_h + 1)\mu \cdot r_i - \mu \sum_{l \in (N \setminus M(h)) \cup \{h\}} r_l}{n_h + 1}$$

where r_i denotes the cardinality of $R(i)$, $(N \setminus M(h)) \cup \{h\}$ denotes the set of active firms on market h, which is of cardinality $n_h = n - m_h + 1$. We assume that, in any R&D and MS structure (R, M), any firm obtains positive profits in any market where she is active; i.e., $\pi_i^h(R, M) = \left(q_i^h\right)^2 > 0$ for any firm i active on market h for all (R, M). Hence, we impose $(\alpha - \lambda) > \mu[(n-1)^2 - n]$.

Notice first that firm i's profit decreases when firms outside its R&D alliance form a larger competing R&D alliance (i.e., there are negative spillovers in R&D alliances). Moreover, firm i's profit increases when firms outside its MS agreement form a larger MS agreement (i.e., there are positive spillovers in MS agreements). Let us formally establish these two spillover effects.

[24] The pairs of structures are ordered pairs or $2-$tuples here.

Property 1. *Negative Spillovers in R&D alliances.* Given any MS coalition structure M, it holds that $\pi_i(R, M) > \pi_i(R \setminus \{R_1, R_2\} \cup \{R_1 \cup R_2\}, M)$ for any firm $i \notin R_1, R_2$.

Proofs can be found in the Appendix. This property states that the formation of a larger R&D alliance by some firms decreases the profit of firms outside the R&D alliance. The profit of a firm decreases when some competitors become more efficient in terms of marginal cost of production.

Property 2. *Positive Spillovers in MS agreements.* Given any R&D alliance structure R, it holds that $\pi_i(R, M) < \pi_i(R, M \setminus \{M_1, M_2\} \cup \{M_1 \cup M_2\})$ for any firm $i \notin M_1, M_2$.

This property states that the formation of a larger MS agreement by other firms increases the profits of firms outside this new MS agreement. Notice that the profits of firm i are only affected in the markets of firms belonging to M_1 and M_2. In the rest of the markets outside M_1 and M_2, the same competitors are still present after the merger of M_1 and M_2. This property implies that a reduction in the number of competitors in a market increases the profit of each firm active in that market, independently of the structure of the R&D alliances.

The intuitions behind these two opposite spillover effects are as follows. On the one hand, the members of a larger R&D alliance can produce at a lower cost relative to the firms outside the alliance. On the other hand, the reduction in the number of competitors in the home markets of the members of a larger MS agreement benefits the firms outside the agreement. How the competitive advantage over common competitors gained by a group of firms after becoming R&D partners would be affected by the reduction in the number of competitors if they also become MS partners? The answer to this question will determine the incentives of oligopolistic firms to establish R&D alliances and MS agreements.

3 Incentives for enlarging R&D alliances

In this section, we analyze, given the MS structure, the incentives for the firms active on a market to enlarge current R&D alliances in an industry composed of at least three R&D alliances. Specifically, for any MS structure and any arbitrary market, we will examine the conditions under which the active firms belonging to the two smallest R&D alliances are willing to form a unique bigger R&D alliance.

Bloch (1995) has done a similar exercise in a model where firms could only create R&D alliances. In the model from Bloch (1995), firms cannot establish MS agreements and then all firms are competing in all the markets. Since all markets have a symmetric competitive structure in Bloch (1995), the incentives of firms to form R&D alliances can be identified through the analysis of a representative market. However, once firms can also establish MS agreements, the number of firms competing in each market could vary and hence the analysis cannot be limited to a representative market.

Given (R, M), the Cournot-Nash profit of firm j on market h is $\pi_j^h(R, M) = \left(q_j^h\right)^2$. Notice that the profit of firm j on market h is a monotonically increasing function of the following valuation $v_j^h(R, M) := \sqrt{\pi_j^h(R, M)} = q_j^h$, with

$$v_j^h(R, M) = \frac{\alpha - \lambda + (n_h + 1)\mu \cdot r_j - \mu \sum_{l \in (N \setminus M(h)) \cup \{h\}} r_l}{n_h + 1}$$

where r_j denotes the cardinality of $R(j)$, $(N \setminus M(h)) \cup \{h\}$ denotes the set of active firms on market h, which is of cardinality $n_h = n - m_h + 1$.

Let $R(i), R(j) \in R$ denote the two smallest R&D alliances in the industry, with R containing more than two R&D alliances. Let $i \in R(i)$ and $j \in R(j)$. A firm j belonging to one of the two smallest R&D alliances can be active in two types of markets. In the first type of market, the domestic firm h is part of a MS agreement that includes all the firms from $R(i)$ and $R(j)$ except j. Thus, j is the only firm belonging to one of the two smallest R&D alliances that is active in market h. In the second type of market, the domestic firm h is part of a MS agreement including some but not all firms from the two smallest R&D alliances. Thus, j together with other firms from the two smallest alliances are active in market h. Formally, we will consider that $n_{R(i)}^h \in [0, r_i]$ firms from the R&D alliance $R(i)$ are active on market h, and $n_{R(j)}^h \in [0, r_j]$ firms from the R&D alliance $R(j)$ are active on market h.[25] The set $A_h \subset N$ contains the firms active in market h that do not belong to $R(i)$ nor $R(j)$; i.e., $A_h := [(N \setminus M(h)) \cup \{h\}] \setminus (R(i) \cup R(j))$. Let $\#(A_h) := n_{A_h}^h \in \mathbb{N}$. The overall number of firms in market h is therefore $n_h = n_{R(i)}^h + n_{R(j)}^h + n_{A_h}^h$.

In order to examine the conditions under which the active firms belonging to the two smallest R&D alliances are willing to merge these two alliances, we should compare the profits they obtain in the initial pair of coalition structures (R, M) with the profits they obtain in the pair of coalition structures $(R^{R(i) \cup R(j)}, M)$. Notice that $R^{R(i) \cup R(j)} = R \setminus \{R(i), R(j)\} \cup \{R(i) \cup R(j)\}$ denotes the R&D alliance structure after the merger of alliances $R(i)$ and $R(j)$.

Proposition 1. *Let (R, M) be given and let $h \in N$ be a given market. Let $R(i), R(j) \in R$ be the two smallest R&D alliances in R, and let $i \in R(i)$ and $j \in R(j)$ two active firms on market h. Then, firms $i \in R(i)$ and $j \in R(j)$ will increase their profits on market h from the merger of $R(i)$ and $R(j)$ if and only if:*

$$\frac{r_i}{r_j} \in \left] \frac{n_{R(i)}^h}{n_{R(i)}^h + n_{A_h}^h + 1}, \frac{n_{R(j)}^h + n_{A_h}^h + 1}{n_{R(j)}^h} \right[\tag{1}$$

Condition (1) in Proposition 1 guarantees the profitability of the merger of the two smallest alliances in R on market h whenever both $n_{R(i)}^h$ and $n_{R(j)}^h$ are positive.[26]

[25] The cases where $n_{R(i)}^h = r_i$ and/or $n_{R(j)}^h = r_j$ embody the situations where all the firms from $R(i)$ and/or from $R(j)$ are active on market h.

[26] In case no firm from $R(i)$ is active on h, i.e. $n_{R(i)}^h = 0$, condition (1) reverts to $\frac{r_i}{r_j} > 0$. Since only some firms from $R(j)$ are active on the market h, we shall notice that these firms have always incentive to merge $R(i)$ with $R(j)$ ($\frac{r_i}{r_j} > 0$ is always verified).

Bloch (1995) has studied the formation of R&D alliances in case no MS agreement has been established; i.e., in case $M^s = \{\{1\}, \cdots, \{n\}\}$ is the MS structure. The absence of MS agreements in the economy implies that all firms of the industry compete in each other home market ($n_h = n$ in any home market h, $n^h_{R(i)} = r_i$ and $n^h_{R(j)} = r_j$). Thus, condition (1) above becomes:

$$\frac{r_i}{r_j} \in \left] \frac{r_i}{n+1-r_j}, \frac{n+1-n_i}{r_j} \right[\tag{2}$$

Bloch (1995) has shown that any two R&D alliances always have incentives to merge if both alliances sizes are smaller than $(n+1)/2$. Thus, if there are at least three R&D alliances in the industry, condition (2) always holds. Hence, according to Bloch (1995), alliance structures consisting of more than two R&D alliances cannot be stable. However, R&D structures containing several R&D alliances are observed in different industries. Here, we propose an explanation for this empirical evidence: the existence of MS agreements in the industry.

Condition (1) depends on the number of firms outside $R(i)$ and $R(j)$ which compete in market h: $n^h_{A_h}$. When firms establish MS agreements, $n^h_{A_h}$ can be very small relative to an economy with no MS agreement (case of Bloch, 1995). In these markets, the merger between the two smallest R&D alliances is not necessarily profitable. For instance, if market h is only composed of firms belonging to $R(i)$ and $R(j)$ ($n^h_{A_h} = 0$), no firm from the largest alliance, $R(i)$ for instance (if $r_i > r_j$), has an incentive to merge these two R&D alliances if $R(i)$ and $R(j)$ have different size.[27] Indeed, since all active firms in h benefit from the merger, the merger does not provide any technological advantage to the active firms relative to their competitors in h (when $n^h_{A_h} = 0$). Further, when the size of the two smallest R&D alliances are different, the merger will make the members from the smallest alliance $R(j)$ as efficient as the members of the other R&D alliance $R(i)$. The firms in $R(i)$ will consequently lose their technological advantages over the firms in $R(j)$ in case of merging. Hence, the firms in $R(i)$ will block the merger. This example reveals that once firms can establish MS agreements, alliance structures consisting of more than two alliances can be stable.

How does the necessary and sufficient condition for the merger to take place (condition (1)) is affected by the competitive structure of the markets? The following remark follows.

Remark 1. The larger the number of firms active in market h that do not belong to $R(i)$ and $R(j)$ (the larger $n^h_{A_h}$) the higher the incentives for merging $R(i)$ and $R(j)$ in market h.

[27] When $n^h_{A_h} = 0$, condition (1) becomes:

$$\frac{r_i}{r_j} \in \left] \frac{n^h_{R(i)}}{n^h_{R(i)}+1}, \frac{n^h_{R(j)}+1}{n^h_{R(j)}} \right[$$

which is true if and only if $r_i = r_j$ since $(r_i, r_j) \in (\mathbb{N}^*)^2$.

The larger $n^h_{A_h}$, the greater the profits obtained by the firms in $R(i)$ and $R(j)$ active on h from the merger of these two R&D alliances. By merging, the firms from $R(i)$ and $R(j)$ become more efficient relative to a larger number of competitors in market h.

The following corollary provides a necessary condition on the minimum number of firms outside the two smallest R&D alliances in every market h for the merger between these two R&D alliances to take place.

Corollary 1. *For any R&D alliance structure with at least three R&D alliances, the two smallest R&D alliances will merge if*

$$n^h_{A_h} > \max\left\{\frac{n^h_{R(i)}(r_j - r_i)}{r_i} - 1, \frac{n^h_{R(j)}(r_i - r_j)}{r_j} - 1\right\} \ \forall h \in N.$$

Note that the only situations considered are those where the merger enables the firms from $R(i)$ and $R(j)$ to increase their profits on all markets where they are active. But, if firms from $R(i)$ ($R(j)$) endures losses in profits within one market because of the merger between $R(i)$ and $R(j)$, these firms can still decide to merge $R(i)$ and $R(j)$ if these losses are overcome by the gains made on other markets.

4 Incentives for dissolving MS agreements

In this section we analyze the incentives of firms belonging to R&D alliances of the same size[28] to dissolve their MS agreement consequently becoming singletons. The dissolution of the MS agreement has two consequences on the profits of the former MS partners. First, these firms will have now access to the home markets of their former partners. Second, a rise in competition will occur in the home markets of these former partners. Notice that the dissolution of the MS agreement only affect the home markets of the initial MS partners and that these markets have the same number and type of competitors. Further, the firms from the initial MS agreement have identical profit functions since they belong to R&D alliances of the same size. It follows that to investigate whether the initial MS agreement will be dissolved, we only need to study the preferences of a representative former partner.

Let $j \in N$ be an initial MS partner. The initial MS structure is denoted by M and $M(j) \in M$ denotes the MS agreement to which j belongs, with $\#(M(j)) = m_j$. The number of active firms in the home market of j is given by $n_j = n - m_j + 1$. The initial R&D alliance structure is denoted by R. $R(j) \in R$, of cardinality r_j, represents the R&D alliance to which j belongs. It is assumed that $r_k = r_j$, $\forall k \in M(j)$ meaning that the MS partners of j are equally efficient (in the sense that they have the same marginal cost).[29] The Cournot-Nash profit function of j is:

[28]A particular case is when firms are part of the same R&D alliance.

[29]In the particular situation where the MS partners are members of the same R&D alliance, we have $M(j) \subseteq R(j)$.

$$\pi_j(R,M) = \left(\frac{\alpha - \lambda + (n_j+1)\mu \cdot r_j - \mu \sum_{l \in (N \setminus M(j)) \cup \{j\}} r_l}{n_j + 1} \right)^2 + \pi$$

where π embodies the profits in all the markets that are not affected by the dissolution of j's MS agreement (the irrelevant markets).
Let $M' = (M \setminus M(j)) \cup_{j \in M(j)} \{j\}$ denotes the MS structure that results after the dissolution into singletons of the MS agreement $M(j)$. Subsequently, the profit function of j in M' is:

$$\pi_j(R,M') = m_j \cdot \left(\frac{\alpha - \lambda + (n_j+m_j)\mu \cdot r_j - \mu[\sum_{l \in (N \setminus M(j)) \cup \{j\}} r_l + (m_j - 1) \cdot r_j]}{n_j + m_j} \right)^2 + \pi$$

$$= m_j \cdot \left(\frac{\alpha - \lambda + (n+1)\mu \cdot r_j - \mu[\sum_{l \in N \setminus M(j)} r_l + m_j . r_j]}{n + 1} \right)^2 + \pi,$$

where we use the fact that $n_j + m_j - 1 = n$. Since the profits generated in the irrelevant markets (π) are not affecting the incentives for j to dissolve its MS agreement, let us consider from now on $\pi_j(R,M) - \pi$ and $\pi_j(R,M') - \pi$ in order to analyze the incentives for dissolving the initial MS agreement $M(j)$.

Proposition 2. *Let (R,M) be given with $r_k = r_j$ for all $k \in M(j) \in M$. Then,*

(i) the firms in $M(j)$ will have incentives to dissolve the MS agreement if $2 \leq m_j \leq n - \sqrt{n + \frac{5}{4}} + \frac{3}{2}$,

(ii) the firms in $M(j)$ will not have incentives to dissolve the MS agreement if $n - \sqrt{n + \frac{5}{4}} + \frac{3}{2} \leq m_j \leq n$.

Proposition 2 studies the incentives of firms belonging to R&D alliances of the same size to dissolve their MS agreement and becoming singletons. It shows that MS agreements of moderate size and composed of homogeneous partners in terms of R&D efficiency will be dissolved. Therefore, MS agreements of moderate size can only emerge among heterogeneous firms in terms of R&D efficiency. On the contrary, MS agreements of large size, relative to the total number of firms in the industry, and composed of homogeneous firms in terms of R&D efficiency would not be dissolved.

5 Stable R&D and MS Agreements

A way to analyze the R&D and MS agreements that one might expect to emerge is to examine an equilibrium requirement that no group of firms benefits from altering the R&D alliance structure or the MS structure. But what is the coalition structure that results from the deviation of a group of firms? What happens to those coalitions from which one or more firms depart? Do they fall apart, or do they still stick together? Hart and Kurz (1983) have introduced the notions of γ-stability and δ-stability to study the stable coalition structures under the assumptions that the non-deviating firms become singletons or remain together, respectively, after the deviation of some coalition members.

We analyze the R&D alliances and MS agreements that will emerge in the oligopolistic industry using the notion of γ-stability.[30] Since firms are able to collaborate forming both R&D alliances and MS agreements, we have to extend the stability concept developed by Hart and Kurz (1983). We propose the concept of γ-stable pair of coalition structures in order to examine the R&D alliance structure and the MS structure that will be formed. Before defining this stability concept, we need to know the coalition structures toward which one firm or a set of firms is able to deviate.

Definition 1. An R&D alliance structure R' (a MS structure M') is γ-obtainable from the R&D alliance structure R (from the MS structure M) via a coalition D, $D \subseteq N$, if (i) $R'(j) = \{j\}$ for any $j \in \{R_i \setminus D \mid R_i \in R,\, R_i \setminus D \neq \emptyset\}$ (ii) $\{R'_i \in R' \mid R'_i \subseteq N \setminus D\} = \{R_i \in R \mid R_i \cap D = \emptyset\}$ and (iii) $D \in R'$.

Condition (i) means that if the firms in coalition D leave their respective alliance(s) in R (their respective MS agreements in M), their previous coalition partners become singletons. Condition (ii) says that R&D alliances (MS agreements) that are not affected by the deviation of coalition D remain together. Condition (iii) imposes the deviating firms in D to form one alliance (one MS agreement) in the new alliance structure R' (in the new MS structure M').[31]

Once all possible deviations from an existing R&D alliance structure (MS structure) are identified, we adopt the notion of γ-stable pairs of coalition structures to predict the pairs of R&D alliance and MS structures that are going to be stable. An R&D alliance structure together with a MS structure is a γ-stable pair if any deviating firm is not better off from the deviation to any γ -obtainable R&D alliance structure (given the MS structure) and to any γ-obtainable MS structure (given the R&D alliance structure).[32]

Definition 2. A pair of coalition structures (R, M) is γ-stable if:

(i) for any $D \subseteq N$, for any γ-obtainable R' from R via D, for $i \in D$ such that $\pi_i(R', M) > \pi_i(R, M)$, there exists $j \in D$ such that $\pi_j(R', M) \leq \pi_j(R, M)$ and

[30] γ-stability seems more adequate to analyze the MS structures expected to emerge since MS agreements collapse when some member(s) use(s) the leniency programs to report the agreement to the antitrust authorities. The power transformer cartel collapsed in late 2002, just after Siemens reported to the antitrust authorities using the leniency law. The Vitamin cartel immediately disappeared after Aventis reported it in 1999. In Section 6 we will comment the results that are still robust to the alternative δ-stability notion.

[31] In general, one could allow the deviating firms in D to form not one but several alliances (several MS agreements) in the new alliance structure R' (in the new MS structure M'). However, our main results in this section remain valid under this alternative assumption. For the sake of presentation, we assume that the deviating firms form only one coalition in the new coalition structure.

[32] Recently, Mauleon, Roehl and Vannetelbosch (2018) have developed a theoretical framework to study the overlapping group structures that are going to emerge at equilibrium, when each group possesses a constitution that contains the rules governing both the composition of the group and the conditions needed to leave the group and/or to become a new member of the group. See also Diamantoudi, Macho-Stadler, Pérez-Castrillo and Xue (2015) for an extension of the Shapley value to games with externalities within and across issues.

(ii) for any $D \subseteq N$, for any γ-obtainable M' from M via D, for $i \in D$ such that $\pi_i(R, M') > \pi_i(R, M)$, there exists $j \in D$ such that $\pi_j(R, M') \leq \pi_j(R, M)$.

Given a MS structure M, the R&D alliance structure R is said to be γ-stable if condition (i) in Definition 2 is satisfied. Given an R&D alliance structure R, the MS structure M is said to be γ-stable if condition (ii) in Definition 2 is satisfied.

Now, we first show in Section 5.1 that in the absence of MS agreements there is no γ-stable pair of coalition structures. Second, in Section 5.2 we show that the class of asymmetric pairs of coalition structures where the most efficient firms of the industry form both an R&D alliance and a MS agreement while the rest of the firms do not form any MS agreement but form two smaller R&D alliances is γ-stable. This class of asymmetric coalition structures encompasses the symmetric pairs of coalition structures that turn to be the only γ-stable symmetric pairs (see Section 5.4). In Section 5.5 we look at policy implications and consumer surplus.

5.1 No stable structures without MS agreement

In their study of the stability of R&D alliance structures with different rules for exiting an alliance, Mauleon, Sempere-Monerris and Vannetelbosch (2016) show that when $n \geq 8$, there is no δ-stable R&D alliance structure in the absence of MS agreements.[33] The following proposition extend this result to γ-stable R&D alliance structures.

Proposition 3. *The pairs of coalition structures* $(R, M^s)_{R \in \mathcal{R}}$ *are not* γ*-stable for* $n \geq 8$.

Proposition 3 and the result obtained by Mauleon, Sempere-Monerris and Vannetelbosch (2016) tell us that, in the absence of MS agreements, there is neither δ-stable nor γ-stable R&D alliance structure in oligopolistic industries containing more than seven firms. However, empirical evidences suggest that many R&D alliances in industries, like the pharmaceutical and the electrical equipments industries, are in fact stable. Here, we provide one explanation for the existence of stable R&D alliances in the economy: the ability of firms to form MS agreements.

5.2 Main result

Notice first that the singleton MS coalition structure M^s cannot be a γ-stable structure because all firms in the industry are willing to deviate toward the global MS agreement.[34] From Section 4, we know that MS agreements of large size relative to the total number of firms in the industry, and composed of homogeneous firms in terms of R&D efficiency would not be dissolved. Let

$$\bar{t} = \sqrt{n + \frac{5}{4}} - \frac{3}{2}$$

[33]The definition of δ-stable R&D alliance structures is similar to the definition of γ-stable alliance structures when, instead of considering all γ-obtainable R&D structures, one considers all δ-obtainable R&D structures where the non-deviating players remain together after the deviation of some coalition members. See Hart and Kurz (1983).

[34]Indeed, $\pi_j(R^s, M^s) - \pi_j(R^s, M^g) = (\alpha - \lambda + \mu)^2 \left[\frac{n}{(n+1)^2} - \frac{1}{4}\right] < 0$.

be the upper bound on the number of firms outside the MS agreement such that the MS partners do not have incentives to dissolve their partnership (See Proposition 2). Notice that $\bar{t} \geq 1$ for $n \geq 5$. From Proposition 2 stems the following corollary:

Corollary 2. *Each pair $(R, \{N \setminus T, M_T^s\})$ such that the firms in $N \setminus T$ are equally efficient is not γ-stable when $t = \#(T) > \bar{t}$.*

Lemma 1 identifies, given the MS structure $M = \{N \setminus T, M_T^s\}$, the possible γ-stable pairs of coalition structures.

Lemma 1. *Given $M = \{N \setminus T, M_T^s\}$, the only possible γ-stable pair of coalition structures are $(\{N \setminus T, X, Y\}, \{N \setminus T, M_T^s\})$ and $(\{N \setminus T, X \cup Y\}, \{N \setminus T, M_T^s\})$, where $N \setminus T$ is the largest R&D alliance and MS agreement of size $n - t$ and (X, Y) are two smaller R&D alliances such that $X \cup Y = T$.*

Our main result asserts that the class of asymmetric pairs of coalition structures where the most efficient firms of the industry form both an R&D alliance and a MS agreement, while the rest of the firms of the industry are not part of any MS agreement but form two smaller R&D alliances $T_{\frac{3}{4}}$ and $T_{\frac{1}{4}}$ of sizes $\lfloor \frac{3t+2}{4} + \frac{1}{2} \rfloor$ and $\lfloor \frac{t-2}{4} + \frac{1}{2} \rfloor$, respectively, are γ-stable.[35]

Proposition 4. *Each pair $(R, M) = \left(\left\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\right\}, \{N \setminus T, M_T^s\}\right)$ such that $t \leq \frac{\bar{t}}{2}$ is γ-stable.*

5.3 Proof of the main result

In order to prove the stability of the pair of coalition structures (R, M), we first examine, given the MS structure $M = \{N \setminus T, M_T^s\}$, whether the R&D alliance structure $R = \left\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\right\}$ is γ-stable.

From Definition 2, the R&D alliance structure $R = \left\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\right\}$ will be γ-stable if any γ-obtainable R&D alliance structure $\overline{R}$ is blocked by at least one of the deviating firms. Three type of possible deviating coalitions should be considered: (i) the deviating coalition $((N \setminus T) \setminus K) \cup S$ contains some firms $((N \setminus T) \setminus K)$ that initially were in the R&D alliance (and in the MS agreement) $N \setminus T$ and possibly some firms S that initially were in the R&D alliances $T_{\frac{3}{4}}$ and $T_{\frac{1}{4}}$; (ii) the deviating coalition $T \setminus S$ contains only some firms that initially were in the R&D alliances $T_{\frac{3}{4}}$ and $T_{\frac{1}{4}}$; and (iii) the deviating coalition is the grand coalition N containing all firms in the industry.

Next lemma shows that a deviation of the coalition $((N \setminus T) \setminus K) \cup S$ to any γ-obtainable R&D alliance structure will always be blocked.

Lemma 2. *The deviation from R in $(R, M) = \left(\left\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\right\}, \{N \setminus T, M_T^s\}\right)$ to any γ-obtainable R&D structure $\overline{R}$ in $\left(\overline{R}, M\right)$ via coalition $((N \setminus T) \setminus K) \cup S$ will always be blocked by the deviating firms that are part of the MS agreement $N \setminus T \in M$, for any $t \leq \bar{t}$.*

[35] $\lfloor x \rfloor$ denotes the floor function that maps any real number x into the greatest integer less than or equal to x.

We prove Lemma 2 using **Property 1** in order to identify the most profitable R&D alliance structure $R^* = \left\{((N \setminus T) \setminus K) \cup S, R^s_{(K \cup T) \setminus S}\right\}$ for the deviating coalition $((N \setminus T) \setminus K) \cup S$. The presence of negative spillovers in the formation of R&D alliances by other firms, implies that at R^*, the deviating coalition is the only non-singleton R&D alliance of the industry. We next show that the deviating firms that are part of the MS coalition $N \setminus T$ in M block the deviation from (R, M) to their most profitable R&D structure (R^*, M). We then conclude that if the deviating firms block the deviation to R^*, they will also block the deviation to any γ-obtainable R&D structure less profitable for them than R^*.

Next lemma shows that a deviation of a coalition $T \setminus S \subseteq T_{\frac{3}{4}} \cup T_{\frac{1}{4}} = T$ to any γ-obtainable R&D alliance structure will also be blocked.

Lemma 3. *The deviation from R in $(R, M) = \left(\left\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\right\}, \{N \setminus T, M^s_T\}\right)$ to any γ-obtainable R&D structure $\overline{R}$ in $\left(\overline{R}, M\right)$ via coalition $T \setminus S \subseteq T_{\frac{3}{4}} \cup T_{\frac{1}{4}}$, will always be blocked by the deviating firms, for any $n \geq 11$ and $t \leq \bar{t}$.*

Notice that the type of deviations described by this lemma can only occur when $n \geq 11$ since $\bar{t} = 1$ for $11 > n \geq 5$. We then prove Lemma 3, using **Property 1** to identify the most profitable R&D alliance structure R^{**} for the deviating coalition $T \setminus S$. Since the deviating firms block the deviation from (R, M) to their most profitable R&D structure (R^{**}, M), they will also block the deviation to any γ-obtainable R&D structure less profitable for them than R^{**}.

Finally, the deviation to the grand coalition $R^g = \{N\}$ will be blocked by the most efficient firms $N \setminus T$ in the initial R&D alliance structure R.

Lemma 4. *The deviation from R in $(R, M) = \left(\left\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\right\}, \{N \setminus T, M^s_T\}\right)$ to the R&D alliance structure R^g in (R^g, M) will always be blocked by the deviating firms in the R&D alliance $N \setminus T$, for any $t \leq \bar{t}$.*

From Lemma 2, Lemma 3 and Lemma 4 it then follows:[36]

Proposition 5. *Given the MS structure $M = \{N \setminus T, M^s_T\}$, the R&D alliance structure $R = \left\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\right\}$ is γ-stable for $t \leq \bar{t}$.*

Next, we examine, given the R&D alliance structure $R = \left\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\right\}$, whether the MS structure $M = \{N \setminus T, M^s_T\}$ is γ-stable.

From Definition 2, the MS structure $M = \{N \setminus T, M^s_T\}$ will be γ-stable if any γ-obtainable MS structure $\overline{M}$ is blocked by at least one of the deviating firms. Five type of possible deviating coalitions should be considered: (i) the deviating coalition $(N \setminus T) \setminus K$ contains only firms from the R&D alliance (that were initially in the MS agreement) $N \setminus T$; (ii) the deviating coalition $((N \setminus T) \setminus K) \cup S$ contains some firms $((N \setminus T) \setminus K)$ from the R&D alliance (that were initially in the MS agreement) $N \setminus T$, some firms S from the R&D alliance $T_{\frac{1}{4}}$ and possibly some firms from the R&D alliance $T_{\frac{3}{4}}$; (iii) the deviating coalition $((N \setminus T) \setminus K) \cup S$ contains some firms $((N \setminus T) \setminus K)$ from the R&D alliance (that were initially in the MS agreement) $N \setminus T$ and some

[36] Among the γ-obtainable R&D alliance structures studied in the proof of Lemma 2, there is also the structure R^s (case $k = n - t - 1$). Thus Lemma 2 shows that the deviation from (R, M) to (R^s, M) is blocked by the firms from the MS agreement $N \setminus T$.

firms S from the R&D alliance $T_{\frac{3}{4}}$; (iv) the deviating coalition $T \setminus S$ contains only some firms from the R&D alliances $T_{\frac{3}{4}}$ and $T_{\frac{1}{4}}$; and (v) the deviating coalition is the grand coalition N containing all firms in the industry.

Next lemma shows that a deviation of the coalition $(N \setminus T) \setminus K$ to any γ-obtainable MS structure will always be blocked.

Lemma 5. *The deviation from M in $(R, M) = \left(\left\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\right\}, \{N \setminus T, M_T^s\}\right)$ to any γ-obtainable MS structure $\overline{M}$ in $\left(R, \overline{M}\right)$ via coalition $(N \setminus T) \setminus K$ will always be blocked by the deviating firms, for any $t \leq \frac{\bar{t}}{2}$.*

In order to prove Lemma 5, we first use **Property 2** to identify the most profitable MS structure M^* for the deviating coalition $(N \setminus T) \setminus K$. The presence of positive spillovers in the formation of MS agreements by other firms, implies $M^* = \{(N \setminus T) \setminus K, K \cup T\}$, where the deviating coalition faces as few competitors as possible on every market; i.e., when all firms outside $(N \setminus T) \setminus K$ are forming a unique MS agreement. We next show that the deviating firms block the deviation from (R, M) to their most profitable MS structure (R, M^*). We then conclude that if the deviating firms block the deviation to M^*, they will also block the deviation to any other γ-obtainable MS structure less profitable for them than M^*. Lemma 5 also shows that the deviation to the singleton MS structure M^s will be blocked by the most efficient firms in the initial R&D alliance structure.

Next lemma shows that a deviation of the coalition $((N \setminus T) \setminus K) \cup S$ to any γ-obtainable MS structure will always be blocked by the deviating firms $j \in S \subseteq T_{\frac{3}{4}} \cup T_{\frac{1}{4}}$ that are members of the R&D alliance $T_{\frac{1}{4}}$ in $R = \left\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\right\}$.

Lemma 6. *The deviation from M in $(R, M) = \left(\left\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\right\}, \{N \setminus T, M_T^s\}\right)$ to any γ-obtainable MS structure $\overline{M}$ in $\left(R, \overline{M}\right)$ via coalition $((N \setminus T) \setminus K) \cup S$ with $S \subseteq T_{\frac{3}{4}} \cup T_{\frac{1}{4}}$ will always be blocked by the deviating firms in coalition $((N \setminus T) \setminus K) \cup S$ that are part of the R&D alliance $T_{\frac{1}{4}} \in R$, for any $t \leq \bar{t}$.*

We prove Lemma 6, using **Property 2** to identify the most profitable MS structure M^{**} for the deviating coalition $((N \setminus T) \setminus K) \cup S$. Since the deviating firms that are part of the R&D alliance $T_{\frac{1}{4}}$ block the deviation from (R, M) to their most profitable MS structure (R, M^{**}), they will also block the deviation to any other γ-obtainable MS structure less profitable for them than M^{**}.

Next lemma shows that a deviation of the coalition $((N \setminus T) \setminus K) \cup S$ to any γ-obtainable MS structure will always be blocked by the deviating firms $j \in S \subseteq T_{\frac{3}{4}}$ that are members of the R&D alliance $T_{\frac{3}{4}}$ in $R = \left\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\right\}$. Notice that $S \subseteq T_{\frac{3}{4}}$ and then no deviating firm belongs initially to the R&D alliance $T_{\frac{1}{4}}$.

Lemma 7. *The deviation from M in $(R, M) = \left(\left\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\right\}, \{N \setminus T, M_T^s\}\right)$ to any γ-obtainable MS structure $\overline{M}$ in $\left(R, \overline{M}\right)$ via coalition $((N \setminus T) \setminus K) \cup S$ with $S \subseteq T_{\frac{3}{4}}$ will always be blocked by the deviating firms in coalition $((N \setminus T) \setminus K) \cup S$ that are part of the R&D alliance $T_{\frac{3}{4}} \in R$, for any $t \leq \bar{t}$.*

We again use **Property 2** to identify the most profitable MS structure M^{***} for the deviating coalition $((N \setminus T) \setminus K) \cup S$. Since the deviating firms that are part of the R&D alliance $T_{\frac{3}{4}}$ block the deviation from (R, M) to their most profitable MS structure (R, M^{***}), they will also block the deviation to any other γ-obtainable MS structure less profitable for them than M^{***}.

Next lemma shows that a deviation of coalition $L \subseteq T_{\frac{3}{4}} \cup T_{\frac{1}{4}}$ to any γ-obtainable MS structure will always be blocked by the deviating firms $j \in L \cap T_{\frac{3}{4}}$ that are members of the R&D alliance $T_{\frac{3}{4}}$ in $R = \left\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\right\}$.

Lemma 8. *The deviation from M in $(R, M) = \left(\left\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\right\}, \{N \setminus T, M_T^s\}\right)$ to any γ-obtainable MS structure $\overline{M}$ in $\left(R, \overline{M}\right)$ via coalition $L \subseteq T_{\frac{3}{4}} \cup T_{\frac{1}{4}}$ will always be blocked by the deviating firms in coalition L that are part of the R&D alliance $T_{\frac{3}{4}} \in R$, for any $t \leq \bar{t}$.*

We prove Lemma 8 by showing that the deviating firms in L that are part of the R&D alliance $T_{\frac{3}{4}}$ block the deviation from (R, M) to the unique γ-obtainable MS structure via L, (R, M^{****}).

Finally, the deviation to the grand coalition $M^g = \{N\}$ will be blocked by the less efficient firms in $T_{\frac{1}{4}}$ in the R&D alliance structure R.

Lemma 9. *The deviation from M in $(R, M) = \left(\left\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\right\}, \{N \setminus T, M_T^s\}\right)$ to the MS structure M^g in (R, M^g) will always be blocked by the deviating firms in the R&D alliance $T_{\frac{1}{4}}$, for any $t \leq \bar{t}$.*

From Lemma 5, Lemma 6, Lemma 7, Lemma 8 and Lemma 9, it then follows:

Proposition 6. *Given the R&D alliance structure $R = \left\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\right\}$, the MS structure $M = \{N \setminus T, M_T^s\}$ is γ-stable for $t \leq \frac{\bar{t}}{2}$.*

The proof of our main result in Proposition 4 establishing that the each pair of asymmetric coalition structures $(R, M) = \left(\left\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\right\}, \{N \setminus T, M_T^s\}\right)$ such that $t \leq \frac{\bar{t}}{2}$ is γ-stable follows then from Proposition 5 and Proposition 6.

5.4 Stable pairs of symmetric coalition structures

Notice that for $t = 1$, the pair of coalition structures from Proposition 4, i.e., $(R, M) = \left(\left\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\right\}, \{N \setminus T, M_T^s\}\right)$, reverts to the following pair of symmetric coalition structures: $(\{N \setminus \{s\}, \{s\}\}, \{N \setminus \{s\}, \{s\}\})$, where $\#(N \setminus \{s\}) = n - 1$. Hence, it follows from Proposition 4 that $(\{N \setminus \{s\}, \{s\}\}, \{N \setminus \{s\}, \{s\}\})$ is γ-stable.

Corollary 3. *Each pair of coalition structures $(\{N \setminus \{s\}, \{s\}\}, \{N \setminus \{s\}, \{s\}\})$ with $\#(N \setminus \{s\}) = n - 1$ is γ-stable.*

Next proposition shows that the pair of grand coalition structures is also γ-stable.

Proposition 7. *The pair of coalition structures $(\{N\}, \{N\})$ is γ-stable.*

The proof of Proposition 7 directly follows from the following lemmas.

Lemma 10. *If $M = M^g = \{N\}$, then $R = R^g = \{N\}$ is the unique γ-stable R&D structure.*

When the global MS agreement has been formed, each firm is a monopoly in its home market. Therefore, the profit of every firm in the economy increases with the size of its R&D alliance (which implies a reduction in its marginal cost of production). Thus, the grand R&D alliance is γ-stable, given the global MS agreement.

Lemma 11. *If $R = R^g = \{N\}$, then $M = M^g = \{N\}$ is γ-stable.*

Indeed, no firm is willing to leave the global MS agreement when this deviation implies the dissolution of the MS agreement into singletons by the non-deviating firms. In that situation, the deviating firm has access to $n-1$ markets after the dissolution of the MS agreement, but all firms in the industry have now access to these markets. Lemma 11 underlines that, when the grand R&D alliance has been formed, the losses of the deviating firm in its home market due to the harsher competition overcome the gains of entering the other $n-1$ markets and then the global MS agreement is γ-stable.

Moreover, it holds that the symmetric pairs of coalition structures found in Corollary 3 and Proposition 7 are the only two γ-stable pairs of symmetric structures.

Proposition 8. *$(\{N\}, \{N\})$ and $(\{N \setminus \{s\}, \{s\}\}, \{N \setminus \{s\}, \{s\}\})$ with $\#(N \setminus \{s\}) = n-1$, are the only γ-stable pairs of symmetric coalition structures.*

Lemma 1 identifies only two possible γ-stable R&D structures when the MS structure is $\{N \setminus T, M_T^s\}$. Proposition 2 establishes that MS agreements of large size and composed of homogeneous firms in terms of R&D efficiency would not be dissolved. Then, using jointly Lemma 1 and Proposition 2, we obtain Proposition 8.

5.5 Policy implications and consumer surplus

Proposition 4, Corollary 3 and Proposition 7 guarantee the existence of γ-stable pairs of R&D alliance structures and MS structures once some MS agreements are present in the industry. However, γ-stable R&D alliances do not exist in the absence of MS agreements (Proposition 3) in industries with at least eight firms. Hence, the ability of firms to form MS agreements stabilize R&D alliances.[37] This result stems from the fact that a lower competition among equally efficient firms enable them to capture a larger part of the surplus generated by the R&D cooperation. We can then conclude that the stability of long-run R&D alliances in an oligopolistic industry could be a signal of the existence of a long-run MS agreement in the same industry.

Which are the implications for the consumer surplus of the stability of R&D alliance structures together with MS agreements? Next proposition studies how the consumer surplus varies with the size of the largest R&D alliance and MS agreement in the stable pairs of coalition structures that have been identified.

Proposition 9. *The consumer surplus of the γ-stable pair of coalition structures $(R^*, M^*) = (\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\}, \{N \setminus T, M_t^s\})$ is greater the smaller the size of the MS agreement $N \setminus T$.*

[37] This echoes the result of d'Aspremont and Jacquemin (1988) according to which MS agreements bolsters the formation of R&D alliances.

This proposition highlights that among the stable pairs of coalition structures, the consumers are better off when the largest MS agreement and then the largest R&D alliance are as small as possible. This result suggests that in the long run large MS agreements could be detrimental for consumers even if they stabilize larger R&D alliances.

Proposition 10. *The consumer surplus of the γ-stable pair of coalition structures $(R^*, M^*) = (\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\}, \{N \setminus T, M_t^s\})$ is strictly greater than the consumer surplus of the unstable pair of singleton structures $(\{R^s, M^s\})$.*

Even though larger MS agreements are detrimental for consumers in the long run as suggested by Proposition 9, Proposition 10 emphasizes that stable cooperation in terms of R&D between oligopolistic firms is yet a better outcome for consumers than no cooperation at all.

6 Discussion

6.1 δ-stability versus γ-stability

Up to now, we have shown the existence of γ-stable pairs of R&D alliances and MS structures. While the use of γ-stability makes sense when studying the formation of MS agreements, nothing precludes the use of δ-stability (Hart and Kurz, 1983) to study the formation of R&D alliances. Contrary to γ-stability, δ-stability assumes that the non-deviating players remain together after the deviation of some coalition members.

Definition 3. An R&D alliance structure R' is δ-obtainable from another R&D alliance structure R via a coalition D, $D \subseteq N$, if (i) $\{R'_i \in R' \mid R'_i \subseteq N \setminus D\} = \{R_i \setminus D \mid R_i \in R, R_i \setminus D \neq \emptyset\}$ and (ii) $D \in R'$.

Condition (i) means that if the firms in D leave their respective alliance(s) in R, the non-deviating firms do not move and remain together. Condition (ii) imposes the deviating firms in D to form one alliance in the new alliance structure R'.[38]

Definition 4. A pair of coalition structures (R, M) is (δ, γ)-stable if:

(i) for any $D \subseteq N$, for any δ-obtainable R' from R via D, for $i \in D$ such that $\pi_i(R', M) > \pi_i(R, M)$, there exists $j \in D$ such that $\pi_j(R', M) \leq \pi_j(R, M)$ and

(ii) for any $D \subseteq N$, for any γ-obtainable M' from M via D, for $i \in D$ such that $\pi_i(R, M') > \pi_i(R, M)$, there exists $j \in D$ such that $\pi_j(R, M') \leq \pi_j(R, M)$.

Mauleon, Sempere-Monerris and Vannetelbosch (2016) show that in oligopolistic industries of at least eight firms there is no δ-stable R&D alliance structure if no MS agreement has been formed. And from Proposition 3, the same result holds when considering γ-stable R&D alliance structures. However, several R&D alliance structures are γ-stable once some MS agreement is present in the industry. One can show that Lemma 1, Proposition 5, Proposition 7, and Proposition 8 remain true when we consider δ-stable R&D alliance structures. Therefore, the

[38] The results of this section remain valid when one allow the deviating firms in D to form not one but several alliances (several MS agreements) in the new alliance structure R' (in the new MS structure M').

stability results that have been exhibited in the last section are robust when we consider δ-stability for R&D alliance structures and γ-stability regarding MS structures.

However, some results obtained before (like Lemma 11) are not robust when considering δ-stable MS structures. When the grand R&D alliance has formed and all firms are equally efficient, one can show that the global MS agreement is not δ-stable. Any firm from the global MS agreement has incentives to unilaterally leave the grand coalition and become singleton. Such a deviation has two opposite effects on the profit of the deviating firm. First, the deviating firm obtains additional profit by accessing $n-1$ new duopolistic markets. Second, the deviating firm will suffer from a harsher competition on its home market as its former MS partners will have access to this market. Let $N \setminus \{i\}$ be the coalition of size $n-1$ containing all firms of the industry except firm i. By comparing the profit of a firm i when $(\{N\}, \{N\})$ is formed with the profit generated in $(\{N\}, \{N \setminus \{i\}, \{i\}\})$, it can be shown that the first effect overcomes the second one and, therefore, the global MS agreement is not δ-stable when the grand R&D alliance has been formed.

Finally, from Proposition 8 and using the argument above, one can show that, when considering δ-stable MS structures, no pair of symmetric coalition structures can be stable.

Proposition 11. *No pair of symmetric coalition structures is* (δ, δ)*-stable.*

6.2 Conclusion

The formation of R&D alliances in order to reduce production costs creates negative external effects on competitors providing a competitive advantage to R&D partners that could be amplified by the formation of some collusive agreement. Up to now, no work has analyzed the formation of both R&D alliances and MS agreements in an oligopolistic environment. In this paper, we fill this gap by analyzing the formation of R&D alliances and MS agreements, by which firms commit not to enter in each other's territory, in oligopolistic markets where firms compete in quantities. In our model, the competitive advantage over common competitors gained by a group of firms after becoming R&D partners would be affected by the reduction in the number of competitors if they are part of some MS agreement. The combination of these two effects would determine the profits of coalition partners and then the R&D alliances and MS agreements that will emerge in the industry.

Bloch (1995) has shown that in an industry where all firms compete with each other in all markets, alliances structures consisting of more than two R&D alliances cannot be stable because the two smallest alliances in the industry have always incentives to merge. We show that once firms can establish both R&D alliances and MS agreements, alliance structures consisting of more than two alliances can be formed in the industry. Hence, it seems that the presence of more than two R&D alliances in an industry could signal the existence of some MS agreement in the industry. We also investigate, given an R&D alliance structure, the incentives of firms to dissolve their MS agreement. We show that MS agreements of large size relative to the total number of firms in the industry and composed of homogeneous firms in terms of R&D efficiency would not be dissolved. Therefore, MS agreements of moderate size can only emerge on markets where there is high heterogeneity between competitors in terms of R&D efficiency.

We extend the γ-stability notion developed by Hart and Kurz (1983) and we propose the concept of γ-stable pairs of coalition structures to identify the stable R&D alliance structures and the MS structures. We prove that R&D alliance structures are stable in oligopolistic industries only in the presence of some MS agreement. Thus, long lasting R&D alliances could signal that the firms involved in the R&D alliances have agreed to share the markets among them. Our main result establishes that the class of asymmetric pairs of coalition structures where the most efficient firms in terms of R&D form both an R&D alliance and a MS agreement while the rest of the firms do not form any MS agreement but form two smaller R&D alliances is γ-stable. We then characterize the set of stable symmetric pairs of coalition structures with identical R&D and MS structure. Regarding the consumer surplus, we show that even though larger MS agreements are detrimental for consumers in the long run, stable cooperation in terms of R&D between oligopolistic firms is yet a better outcome for consumers than no cooperation at all. Our results could be useful for antitrust authorities to better assess the efficiency of policies promoting R&D collaborations and policies for the detection of MS agreements.

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Appendix

Proof of Property 1. Let consider the R&D alliances R_1 and R_2 of respective size r_1 and r_2. The profit $\pi_i(R, M)$ of firms $i \notin R_1, R_2$ is given by:

$$\sum_{j\in(N\setminus M(i))\cup\{i\}} \frac{1}{n-m_j+1}\Bigg(\alpha-\lambda+\mu\Big((n-m_j+1)r_i - \sum_{j\in(R_1\setminus M(j)\cup\{j\}} r_1 \\ - \sum_{j\in(R_2\setminus M(j)\cup\{j\}} r_2 - \sum_{j\in(N\setminus(R_1\cup R_2\cup M(j))\cup\{j\}} r_j\Big)\Bigg)^2.$$

The profit $\pi_i(R \setminus \{R_1, R_2\} \cup \{R_1 \cup R_2\}, M)$ of $i \notin R_1, R_2$ when the R&D alliance $R_1 \cup R_2$ is formed is given by:

$$\sum_{j\in(N\setminus M(i))\cup\{i\}} \frac{1}{n-m_j+1}\Bigg(\alpha-\lambda+\mu\Big((n-m_j+1)r_i - \sum_{j\in((R_1\cup R_2)\setminus M(j))\cup\{j\}} r_1+r_2 \\ - \sum_{j\in(N\setminus(R_1\cup R_2\cup M(j))\cup\{j\}} r_j\Big)\Bigg)^2.$$

Since $\sum_{j\in((R_1\cup R_2)\setminus M(j))\cup\{j\}} r_1 + r_2 > \sum_{j\in(R_1\setminus M(j)\cup\{j\}} r_1 - \sum_{j\in(R_2\setminus M(j)\cup\{j\}} r_2$, we have that $\pi_i(R, M) > \pi_i(R \setminus \{R_1, R_2\} \cup \{R_1 \cup R_2\}, M)$. □

Proof of Property 2. In order to prove that, given R, it holds that $\pi_i(R, M) < \pi_i(R, M \setminus \{M_1, M_2\} \cup \{M_1 \cup M_2\})$ for any firm $i \notin M_1, M_2$, we only need to analyze the profits of firm i in the markets of firms belonging to M_1 and M_2. Let $h \in M_1$ and $i \in M(i) \neq M_1, M_2$. Comparing $\pi_i^h(R, M)$ and $\pi_i^h(R, M \setminus \{M_1, M_2\} \cup \{M_1 \cup M_2\})$ we have:

$$\left(\frac{\alpha - \lambda + (n_h + 1)\mu r_i - \mu \sum_{l \in (N \setminus M_1) \cup \{h\}} r_l}{n_h + 1}\right)^2 \leq$$

$$\left(\frac{\alpha - \lambda + (n_h - m_2 + 1)\mu r_i - \mu \sum_{l \in (N \setminus (M_1 \cup M_2)) \cup \{h\}} r_l}{n_h - m_2 + 1}\right)^2$$

if and only if

$$m_2(\alpha - \lambda) + \mu(n_h + 1) \sum_{l \in M_2} r_l - \mu m_2 \sum_{l \in (N \setminus M_1) \cup \{h\}} r_l \geq 0$$

Substituting $(\alpha - \lambda)$ by $\mu[(n-1)^2 - n]$, its lower bound in order to guarantee that any firm obtains positive profits in any market where she is active; substituting $\sum_{l \in M_2} r_l$ by its lower bound and substituting $\sum_{l \in (N \setminus M_1) \cup \{h\}} r_l$ by its upper bound we obtain:

$$m_2 \mu[(n-1)^2 - n] + \mu(n_h + 1)m_2 - \mu m_2 n_h n \geq 0.$$

Replacing n_h by $n - m_1 + 1$, leads to $(m_1 - 3)(n - 1) \geq 0$. Hence, we obtain that the property of positive spillovers holds for $m_1 \geq 3$.

In order to show that the property of positive spillovers also holds in case $m_1 = 1$ and in case $m_1 = 2$, notice that $\pi_i^h(R, M \setminus \{M_1, M_2\} \cup \{M_1 \cup M_2\}) - \pi_i^h(R, M)$ can be written as follows:

$$m_2(\alpha - \lambda) + \mu(n_h + 1 - m_2) \sum_{l \in M_2} r_l - \mu m_2 \sum_{l \in (N \setminus (M_1 \cup M_2)) \cup \{h\}} r_l \tag{A.0}$$

Again, substituting $(\alpha - \lambda)$ by $\mu[(n-1)^2 - n]$, and assuming that $m_1 = 1$ and then $n_h = n$, we obtain, for any hypothesis regarding the size of the R&D alliances of firms in M_2 and the R&D alliances of the rest of the firms in the industry:[39]

$$m_2(\alpha - \lambda) + \mu(n + 1 - m_2)(m_2 - k) + \mu(n + 1 - m_2)k(n - m_2 + k) - \mu m_2(n - m_2)(n - m_2 + k).$$

It can be verified that the above expression is positive for all $0 \leq k \leq m_2$.

For the case $m_1 = 2$, we have that $n_h = n - 1$ and we obtain:

$$m_2(\alpha - \lambda) + \mu(n - m_2)(m_2 - k) + \mu(n - m_2)k(n - m_2 + k) - \mu m_2(n - m_2 - 1)(n - m_2 + k).$$

It can be verified that the above expression is positive for all $0 \leq k \leq m_2$. □

Proof of Proposition 1. Let $A_h := [(N \setminus M(h)) \cup \{h\}] \setminus (R(i) \cup R(j))$ be the set of active firms on market h that do not belong to $R(i)$ and $R(j)$. Given (R, M), the value function of firm $j \in R(j) \in R$ active on market h is given by:

$$v_j^h(R, M) = \left(\frac{\alpha - \lambda + (n_h + 1)\mu \cdot r_j - \mu[\sum_{a \in A_h} r_a + n_{R(j)}^h \cdot r_j + n_{R(i)}^h \cdot r_i]}{n_h + 1}\right).$$

[39]Among all the R&D structures, the left hand side of (A.0) reaches its minimum value in $R = \{(N \setminus M_2) \cup K, M_2^s\}$, where $\#(K) = k \in \{0, \cdots, m_2\}$.

The value function of $j \in (R(i) \cup R(j)) \in R^{R(i)\cup R(j)}$ active on market h after $R(i)$ and $R(j)$ have merged is given by:

$$v_j^h(R^{R(i)\cup R(j)}, M) = \left(\frac{\alpha - \lambda + (n_h+1)\mu(r_j+r_i) - \mu[\sum_{a\in A_h} r_a + (n^h_{R(j)} + n^h_{R(i)})(r_j+r_i)]}{n_h+1} \right).$$

Then, we have:

$$\begin{aligned} v_j^h(R^{R(i)\cup R(j)}, M) - v_j^h(R, M) &= \frac{1}{n_h+1} \left[\begin{array}{c} \mu(n_h+1)(r_j+r_i) - \mu(n^h_{R(j)} + n^h_{R(i)})(r_j+r_i) \\ -\mu(n_h+1)r_j - \mu\left(n^h_{R(j)} \cdot r_j + n^h_{R(i)} \cdot r_i\right) \end{array} \right] \\ &= \frac{1}{n_h+1} \left[\mu \cdot r_i(n_h + 1 - n^h_{R(j)}) - \mu \cdot r_j \cdot n^h_{R(i)} \right]. \end{aligned}$$

Since $n_h = n^h_{R(i)} + n^h_{R(j)} + n^h_{A_h}$, it follows that for any $j \in R(j)$:

$$\begin{aligned} v_j^h(R^{R(i)\cup R(j)}, M) - v_j^h(R, M) > 0 &\iff r_i(n_h+1-n^h_{R(j)}) - r_j.n^h_{R(i)} > 0 \\ &\iff r_j < r_i \frac{n_h + 1 - n^h_{R(j)}}{n^h_{R(i)}} \qquad (A.1) \\ &\iff \frac{r_i}{r_j} > \frac{n^h_{R(i)}}{n^h_{R(i)} + n^h_{A_h} + 1}. \qquad (A.2) \end{aligned}$$

Since the merger between $R(i)$ and $R(j)$ will occur if and only if all firms belonging to $R(i)$ and those belonging to $R(j)$ agree to merge, let us now investigate the incentive conditions for firm $i \in R(i)$ to merge when she is active on market h. Firm $i \in R(i)$ will have incentives to merge $R(i)$ and $R(j)$ if and only if:

$$\begin{aligned} v_i^h(R^{R(i)\cup R(j)}, M) - v_i^h(R, M) > 0 &\iff r_j(n_h+1-n^h_{R(i)}) - r_i.n^h_{R(j)} > 0 \\ &\iff r_i < r_j \frac{n_h + 1 - n^h_{R(i)}}{n^h_{R(j)}} \qquad (A.3) \\ &\iff \frac{r_i}{r_j} < \frac{n^h_{R(j)} + n^h_{A_h} + 1}{n^h_{R(j)}}. \qquad (A.4) \end{aligned}$$

Therefore, (A.2) and (A.4) provide a necessary and sufficient condition on the relative size of $R(i)$ over $R(j)$ for the two smallest R&D alliances to merge on market h:

$$\frac{r_i}{r_j} \in \left] \frac{n^h_{R(i)}}{n^h_{R(i)} + n^h_{A_h} + 1}, \frac{n^h_{R(j)} + n^h_{A_h} + 1}{n^h_{R(j)}} \right[.$$

□

Proof of Proposition 2. Before proving Proposition 2, we first prove Lemma 12.

Lemma 12. *Let denote $x := \sqrt{m_j}.n_j + \sqrt{m_j} - (n+1)$. Then, it follows that*

$$x > 0 \iff n_j > \frac{n+1}{\sqrt{m_j}} - 1$$
$$x < 0 \iff n_j < \frac{n+1}{\sqrt{m_j}} - 1$$

is equivalent to

$$x > 0 \iff m_j \in \left\{2, ..., n - \sqrt{n + \frac{5}{4}} + \frac{3}{2} - 1\right\} \cup \left\{n + \sqrt{n + \frac{5}{4}} + \frac{3}{2} + 1, ..., +\infty\right\}$$
$$x < 0 \iff m_j \in \left\{n - \sqrt{n + \frac{5}{4}} + \frac{3}{2}, ..., n + \sqrt{n + \frac{5}{4}} + \frac{3}{2}\right\}.$$

Proof of Lemma 12. Knowing that $n_j = n - m_j + 1$, we obtain

$$x > 0 \iff \mathcal{P}[m_j] := (m_j)^3 - (m_j)^2(2n + 4) + m_j(2 + n)^2 - (n + 1)^2 > 0 \tag{A.5}$$

where $m_j \in [0, +\infty]$. As 1 is an obvious root of $\mathcal{P}[m_j]$:

$$(m_j - 1)(\alpha \cdot (m_j)^2 + \beta \cdot m_j + c) = \mathcal{P}[m_j] \iff \begin{cases} \alpha = 1 \\ c = (n + 1)^2 \\ \beta = -(2n + 3). \end{cases}$$

It follows that:

$$\mathcal{P}[m_j] = (m_j - 1)[(m_j)^2 - m_j(2n + 3) + (n + 1)^2] := (m_j - 1)\mathcal{Q}[m_j].$$

Where $\mathcal{Q}[m_j]$ has two positive real roots: $m_j = n \pm \sqrt{n + \frac{5}{4}} + \frac{3}{2} \in \mathbb{R}$. So, $\mathcal{P}[m_j]$ has three roots in $\mathbb{N}^*$: 1 and $n \pm \sqrt{n + \frac{5}{4}} + \frac{3}{2}$. From there, one can easily verify that $\mathcal{P}[m_j]$ is positive between 2 and $n - \sqrt{n + \frac{5}{4}} + \frac{3}{2}$ and above $n + \sqrt{n + \frac{5}{4}} + \frac{3}{2}$. Therefore, the conditions $n_j > \frac{n+1}{\sqrt{m_j}} - 1$ and $n_j < \frac{n+1}{\sqrt{m_j}} - 1$ are equivalent to

$$\begin{aligned} m_j &\in \left\{2, \ldots, n - \sqrt{n + \frac{5}{4}} + \frac{3}{2} - 1\right\} \cup \left\{n + \sqrt{n + \frac{5}{4}} + \frac{3}{2} + 1, \ldots\right\} \\ \text{and} \ \ m_j &\in \{0\} \cup \left\{n - \sqrt{n + \frac{5}{4}} + \frac{3}{2}, \ldots, n + \sqrt{n + \frac{5}{4}} + \frac{3}{2}\right\}. \end{aligned}$$

□

The initial MS coalition will be totally split if the following expression end up being positive:

$$\pi_j(R, M') - \pi_j(R, M)$$

$$= (\alpha - \lambda)\left(\frac{\sqrt{m_j}(n_j + 1) - (n_j + m_j)}{(n_j + m_j)(n_j + 1)}\right) - \mu \sum_{l \in N \setminus M(j)} r_l \left(\frac{\sqrt{m_j}(n_j + 1) - (n_j + m_j)}{(n_j + m_j)(n_j + 1)}\right)$$

$$+ \mu \cdot r_j \left(\frac{n_j \cdot \sqrt{m_j}(n_j + 1) - n_j \cdot (n_j + m_j)}{(n_j + m_j)(n_j + 1)}\right) \tag{A.6}$$

Having in mind that $n_j + m_j = n + 1$, let $y := (n_j + m_j)(n_j + 1) = (n + 1)(n_j + 1) > 0$ and $x := \sqrt{m_j}(n_j + 1) - (n_j + m_j) = \sqrt{m_j}\ n_j + \sqrt{m_j} - (n + 1)$, which can be positive or negative.

We have,

$$x > 0 \iff n_j > \frac{n+1}{\sqrt{m_j}} - 1$$
$$x < 0 \iff n_j < \frac{n+1}{\sqrt{m_j}} - 1$$

which, from Lemma 12, is equivalent to:

$$x > 0 \iff m_j \in \left\{2, ..., n - \sqrt{n + \frac{5}{4}} + \frac{3}{2} - 1\right\} \cup \left\{n + \sqrt{n + \frac{5}{4}} + \frac{3}{2} + 1, ..., +\infty\right\}$$
$$x < 0 \iff m_j \in \left\{n - \sqrt{n + \frac{5}{4}} + \frac{3}{2}, ..., n + \sqrt{n + \frac{5}{4}} + \frac{3}{2}\right\}.$$

Observing that $n + \sqrt{n + \frac{5}{4}} + \frac{3}{2} > n$ and $m_j \in \{1, n\}$ we have:

$$x > 0 \iff m_j \in \left\{1 \ , \ n - \sqrt{n + \frac{5}{4}} + \frac{3}{2} + 1\right\}$$
$$x < 0 \iff m_j \in \left\{n - \sqrt{n + \frac{5}{4}} + \frac{3}{2} + 1 \ , \ n\right\}.$$

Then, (A.6) can be rewritten

$$\pi_j(R, M') - \pi(R, M) = (\alpha - \lambda)\left(\frac{x}{y}\right) + \mu \cdot r_j \left(\frac{n_j \cdot x}{y}\right) - \mu \sum_{l \in N \setminus M(j)} r_l \left(\frac{x}{y}\right). \tag{A.7}$$

If $x > 0$ then $\frac{x}{y} > 0$ and only the last term of (A.7) is negative. Moreover, since $n_j \geq 1$,

$$\mu \cdot r_j \cdot n_j \geq \mu \cdot r_j > \mu \sum_{l \in N \setminus M(j)} r_l \iff r_j \cdot n_j \geq r_j > \sum_{l \in N \setminus M(j)} r_l.$$

is a sufficient condition for (A.7) to be positive as long as $x > 0$, which is always true as $r_j > \sum_{l \in N \setminus M(j)} r_l$ is also a sufficient condition for each firm to produce positive quantities on its home market. So when $x > 0$, firms in $M(j)$ have incentives to dissolve the agreement. On the contrary, when $x < 0$, no firm from the MS agreement has incentives to dissolve the agreement. □

Proof of Proposition 3. In the absence of MS agreements, the non existence of δ-stable structures when $n \geq 8$ is due to Mauleon, Sempere-Monerris and Vannetelbosch (2016).

Let us consider an initial R&D alliance structure R and a deviating coalition D. Let R^δ be the subsequent δ-obtainable R&D alliance structure via coalition D. Let R^γ be the γ-obtainable R&D alliance structure resulting from the deviation of the same coalition D. By definition of γ-obtainable coalition structures, the non deviating firms left behind by some of the deviating firms become singletons in R^γ, but remain together in R^δ. Thus, the non deviating firms in R^γ will be at most as efficient in terms of marginal cost of production as in R^δ. In other words, the deviating firms will face more efficient competitors in R^δ than in R^γ. Thus, if a deviation toward R^δ is profitable for the deviating firms, so does it toward R^γ for the same deviating firms. Hence, if there is no δ-stable R&D alliance structure when $n \geq 8$, there is no γ-stable R&D alliance structure when $n \geq 8$. □

Proof of Corollary 2. All firms in $N \setminus T$ are equally efficient. Therefore, Proposition 2 entails that a MS coalition of size $n-t < n - \sqrt{n+\frac{5}{4}} + \frac{3}{2} \iff t > \sqrt{n+\frac{5}{4}} - \frac{3}{2} := \bar{t}$ will be split into $n-t$ singletons. Therefore, the MS structure $\{N \setminus T, M^s_T\}$ where the most efficient firms of the industry are grouped in the coalition $N \setminus T$ cannot be γ-stable when $t > \bar{t}$.[40] □

Proof of Lemma 1. The MS structure is $\{\overline{M}, M^s_{N\setminus\overline{M}}\}$, where $\overline{M}$ denotes the largest MS agreement and R&D alliance. Let h be an arbitrary market of size $n_h = n - m_h + 1$. Let $X \ni i$ and $Y \ni j$ denote two R&D alliances of firms outside the largest R&D alliance $\overline{M}$. Any R&D alliance cannot be larger than the largest alliance $\overline{M}$, i.e. $\max\{\#(X), \#(Y)\} < \#(\overline{M})$. Moreover, if there are two or more R&D alliances beside $\overline{M}$ in the industry, the number of firms in X and Y is at most the number of firms outside $\overline{M}$:

$$\#(X) + \#(Y) \leq \#(N \setminus \overline{M}) \iff r_i + r_j \leq n - m_h = n_h - 1 < n_h. \tag{A.8}$$

On market h all the firms from X and Y are active since they are not involved in any MS agreement. Rewriting expression (1) of Proposition 1 using the fact that the number of active firms in the market is $n_h = r_i + r_j + n^h_{A_h}$, Proposition 1 tells us that the merger between X and Y will occur if and only if:

$$\begin{cases} r_j < \dfrac{n_h+1}{2} \\ r_i < \dfrac{n_h+1}{2}. \end{cases} \tag{A.9}$$

Assume that the industry contains the R&D alliances $\overline{M}$, X, and Y as well as at least one other R&D alliance. To be stable, there can be at most one R&D alliance having incentives to merge.[41] In particular, there can be at most one of the R&D alliances of firms outside $\overline{M}$ willing to merge. So at most one of the R&D alliances of firms outside $\overline{M}$ can verify (A.9). Moreover, it is not possible that more than one alliance violates (A.9). Indeed, let assume that X and Y violate (A.9). Then, $r_i + r_j \geq n_h + 1 > n_h$, which contradict condition (A.8). Therefore, the R&D structures that are candidate to be γ-stable contain at most two R&D alliances outside $\overline{M}$: $\{\overline{M}, X, Y\}$, or $\{\overline{M}, X \cup Y\}$. If we consider $\overline{M} = N \setminus T$, the only possible γ-stable R&D structures are $\{N \setminus T, X, Y\}$, or $\{N \setminus T, X \cup Y\}$. □

Proof of Proposition 4. The proof of Proposition 4 results from Proposition 5 and Proposition 6 that are proved below by means of several lemmas. □

Proof of Lemma 2. This lemma considers the deviations from $R = \left\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\right\}$ toward any γ-obtainable R&D structures $\overline{R}$ via coalition $((N \setminus T) \setminus K) \cup S$ of cardinality $n - t - k + s$. Let $(n-t-k) \in \{1, \dots, n-t\}$ be the number of firms in $((N \setminus T) \setminus K) \cup S$ from $N \setminus T$ in the initial R&D structure R, and let $s \in \{0, \dots, t\}$ be the number of firms in $((N \setminus T) \setminus K) \cup S$ from $T_{\frac{3}{4}}$ and/or $T_{\frac{1}{4}}$ in the initial R&D structure R. Let $j \in ((N \setminus T) \setminus K) \cup S$ be a deviating firm from the MS agreement $N \setminus T$ in M.

[40] Notice that for t to be larger than 2, we must have $\bar{t} \geq 2$; and $\bar{t} \geq 2 \iff n \geq 11$.

[41] If the members of an R&D alliance do not have incentives to merge with another alliance, the merger between these two R&D alliances will not occur.

From Property 1, the most profitable R&D alliance structure for the deviating coalition $((N \setminus T) \setminus K) \cup S$ is $R^* = \Big\{((N \setminus T) \setminus K) \cup S, R^s_{(K \cup T) \setminus S}\Big\}$ where the deviating coalition is the only non-singleton R&D alliance of the industry. Indeed, since the formation of a larger R&D alliance by firms outside the deviating coalition decreases the profit of firms in the deviating coalition, the profits of the deviating firms in the new R&D alliance structure will be larger the smaller the size of the R&D alliances outside the deviating coalition; i.e., when all non-deviating firms are singleton R&D alliances.

One can compute the profits of firm $j \in ((N \setminus T) \setminus K) \cup S$ in the initial R&D alliance structure (R, M) and the final one (R^*, M):

$$\begin{aligned}\pi_j(R^*, M) =& \frac{t}{(n+1)^2}\Big(\alpha - \lambda + \mu[(t+k-s+2)(n-t-k+s) - n]\Big)^2 \\ &+ \frac{1}{(t+2)^2}\Big(\alpha - \lambda + \mu[(t+1-s)(n-t-k+s) - t + s]\Big)^2 \\ :=& \frac{t}{(n+1)^2}\Big(\alpha - \lambda + \mu \cdot E_2\Big)^2 + \frac{1}{(t+2)^2}\Big(\alpha - \lambda + \mu \cdot E_3\Big)^2\end{aligned}$$

$$\begin{aligned}\pi_j(R, M) =& \Big(\frac{1}{(t+2)^2} + \frac{t}{(n+1)^2}\Big)\Big(\alpha - \lambda + \mu\Big[n \cdot t + n - \frac{13}{8}t^2 - \frac{3}{2}t + \frac{1}{2}\Big]\Big)^2 \\ :=& \Big(\frac{t}{(n+1)^2} + \frac{1}{(t+2)^2}\Big)\Big(\alpha - \lambda + \mu \cdot E_1\Big)^2.\end{aligned}$$

It follows that:

$$\begin{aligned}\pi_j(R^*, M) - \pi_j(R, M) =& 2(\alpha - \lambda)\mu\Big[\frac{t}{(n+1)^2}(E_2 - E_1) + \frac{1}{(t+2)^2}(E_3 - E_1)\Big] \\ &+ \mu^2\Big[\frac{t}{(n+1)^2}((E_2)^2 - (E_1)^2) + \frac{1}{(t+2)^2}((E_3)^2 - (E_1)^2)\Big] \\ :=& 2(\alpha - \lambda)\mu \cdot A + \mu^2 \cdot B.\end{aligned}$$

One can show that $A < 0$ and $B < 0$ for all $n \geq 5$, $t \in [1, \bar{t}]$, $k \in [0, n-t-1]$, $s \in [0, t]$. Therefore, for any $n \geq 5$, and any $(t, k, s) \in [1, \bar{t}] \times [0, n-t-1] \times [0, t]$, the firm j will always block any deviation from (R, M) toward the most profitable R&D alliance structure (R^*, M) and thus, to any other less profitable γ-obtainable R&D alliance structure. □

Proof of Lemma 3. This lemma considers the deviations from $R = \Big\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\Big\}$ toward any obtainable γ-obtainable R&D structures $\overline{R}$ via coalition $T \setminus S \subseteq T_{\frac{3}{4}} \cup T_{\frac{1}{4}} = T$, where no deviating firm is member of the MS agreement $N \setminus T \in M$.

First, let j be one deviating firms that is part of the R&D coalition $T_{\frac{3}{4}} \in R$. From Property 1, the most profitable R&D alliance structure for the deviating coalition $T \setminus S$ is $R^{**} = \{N \setminus T, T \setminus S, R^s_S\}$, with $\#(S) = s \in \{0, .., t-1\}$. One can compute the profits of firm $j \in T \setminus S$ in the initial R&D alliance structure (R, M) and the final one (R^{**}, M):

$$\begin{aligned}\pi_j(R^{**}, M) =& \frac{t}{(n+1)^2}\Big(\alpha - \lambda + \mu[-n^2 + 3nt - 2t^2 + t - s(n - 2t + s + 2)]\Big)^2 \\ &+ \frac{n-t}{(t+2)^2}\Big(\alpha - \lambda + \mu[-n + 3t + s(t - s - 3)]\Big)^2 \\ :=& \frac{t}{(n+1)^2}\Big(\alpha - \lambda + \mu \cdot E_4\Big)^2 + \frac{n-t}{(t+2)^2}\Big(\alpha - \lambda + \mu \cdot E_5\Big)^2,\end{aligned}$$

$$\begin{aligned}\pi_j(R,M) =&\frac{t}{(n+1)^2}\Big(\alpha-\lambda+\mu[-n^2+\frac{11}{4}nt+\frac{n}{2}-\frac{13}{8}t^2+\frac{t}{4}]\Big)^2\\&+\frac{n-t}{(t+2)^2}\Big(\alpha-\lambda+\mu[-n+\frac{t^2}{8}+\frac{5}{2}t+\frac{1}{2}]\Big)^2\\:=&\frac{t}{(n+1)^2}\Big(\alpha-\lambda+\mu\cdot E_6\Big)^2+\frac{n-t}{(t+2)^2}\Big(\alpha-\lambda+\mu\cdot E_7\Big)^2.\end{aligned}$$

It follows that:

$$\begin{aligned}\pi_j(R^{**},M)-\pi_j(R,M)=&\,2(\alpha-\lambda)\mu\Big[\frac{t}{(n+1)^2}(E_4-E_6)+\frac{n-t}{(t+2)^2}(E_5-E_7)\Big]\\&+\mu^2\Big[\frac{t}{(n+1)^2}((E_4)^2-(E_6)^2)+\frac{n-t}{(t+2)^2}((E_5)^2-(E_7)^2)\Big]\\:=&\,2(\alpha-\lambda)\mu\cdot A+\mu^2\cdot B.\end{aligned}$$

One can show that $A<0$ for all $n\geq 5$, $t\in[1,\bar{t}]$, $s\in[0,t-1]$. Moreover, we know that $\alpha-\lambda>\mu((n-1)^2-n)$ to guarantee that any firm produces positive quantities in any market where she is active. Therefore, $(\alpha-\lambda)A>\mu((n-1)^2-n)A$. One can verify that $2((n-1)^2-n)A+B<0$ for $n\geq 5$, $t\in[1,\bar{t}]$, and $s\in[0,t-1]$. This implies that $2(\alpha-\lambda)\mu\cdot A+\mu^2\cdot B<0$ for $n\geq 5$, $t\in[1,\bar{t}]$, and $s\in[0,t-1]$. Therefore, for any $n\geq 5$, and any $(t,s)\in[1,\bar{t}]\times[0,t-1]$, the firm j will always block any deviation from (R,M) toward the most profitable R&D alliance structure (R^{**},M) and thus, to any other less profitable γ-obtainable R&D alliance structure.

Now, let us consider the situation when $T\setminus S\subseteq T_{\frac{1}{4}}$ such that there is no deviating firms from $T_{\frac{3}{4}}$ in $T\setminus S$. Let i be a deviating firm from $T\setminus S$. From Property 1, we know that the most profitable R&D alliance structure for the firms in the deviating coalition $T\setminus S$ is $R^{***}=\Big\{N\setminus T,T_{\frac{3}{4}},T_{\frac{1}{4}}\setminus S,R^s_S\Big\}$, with $\#(S)=s\in\{0,\dots,\lfloor\frac{t-2}{4}+\frac{1}{2}\rfloor\}$ and $t\geq 3$. The profits of firm $j\in T\setminus S$ in the initial R&D alliance structure (R,M) and the final one (R^{***},M) is given by:

$$\begin{aligned}\pi_i(R^{***},M)=&\frac{t}{(n+1)^2}\Big(\alpha-\lambda+\mu[-n^2+\frac{9}{8}nt-\frac{n}{2}-\frac{41}{16}t^2-\frac{t}{2}-\frac{3}{4}-s(n-2t+s+2)]\Big)^2\\&+\frac{n-t}{(t+2)^2}\Big(\alpha-\lambda+\mu[-n-\frac{29}{16}t^2+\frac{3}{4}t+s(t-s-3)]\Big)^2\\:=&\frac{t}{(n+1)^2}\Big(\alpha-\lambda+\mu\cdot E_8\Big)^2+\frac{n-t}{(t+2)^2}\Big(\alpha-\lambda+\mu\cdot E_9\Big)^2,\end{aligned}$$

$$\begin{aligned}\pi_i(R,M)=&\frac{t}{(n+1)^2}\Big(\alpha-\lambda+\mu[-n^2+\frac{9}{4}nt-\frac{n}{2}-\frac{13}{8}t^2-\frac{t}{4}-1]\Big)^2\\&+\frac{n-t}{(t+2)^2}\Big(\alpha-\lambda+\mu[-n-\frac{3}{8}t^2+\frac{t}{2}-\frac{3}{2}]\Big)^2\\:=&\frac{t}{(n+1)^2}\Big(\alpha-\lambda+\mu\cdot E_{10}\Big)^2+\frac{n-t}{(t+2)^2}\Big(\alpha-\lambda+\mu\cdot E_{11}\Big)^2.\end{aligned}$$

It follows that:

$$\begin{aligned}\pi_j(R^{***},M)-\pi_j(R,M)=&\,2(\alpha-\lambda)\mu\Big[\frac{t}{(n+1)^2}(E_8-E_{10})+\frac{n-t}{(t+2)^2}(E_9-E_{11})\Big]\\&+\mu^2\Big[\frac{t}{(n+1)^2}((E_8)^2-(E_{10})^2)+\frac{n-t}{(t+2)^2}((E_9)^2-(E_{11})^2)\Big]\\:=&\,2(\alpha-\lambda)\mu\cdot A_1+\mu^2\cdot B_1\end{aligned}$$

One can verify that (i) $A_1 < 0$ for all $n \geq 11$, $t \in [2, \bar{t}]$, $s \in [0, \lfloor \frac{t-2}{4} + \frac{1}{2} \rfloor]$ and (ii) $2((n-1)^2 - n)A_1 + B_1 < 0$ for $n \geq 11$, $t \in [3, \bar{t}]$, and $s \in [0, t-1]$. Therefore with the same argument as above, this shows that $2(\alpha - \lambda)\mu \cdot A_1 + \mu^2 \cdot B_1 < 0$ for $n \geq 11$, $t \in [3, \bar{t}]$, $s \in [0, t-1]$, and any values of $\alpha - \lambda$ and μ. □

Proof of Lemma 4. Let $j \in N \setminus T$ be a deviating firm member of the MS agreement in the initial MS structure M. One can compute the profits of firm $j \in N \setminus T$ in the initial R&D alliance structure (R, M) and the final one (R^g, M):

$$\pi_j(R, M) = \Big(\frac{t}{(n+1)^2} + \frac{1}{(t+2)^2}\Big)\Big(\alpha - \lambda + \mu\Big(nt + n - \frac{13}{8}t^2 - \frac{3}{2}t - \frac{1}{2}\Big)\Big)^2$$

$$\pi_j(R^g, M) = \Big(\frac{t}{(n+1)^2} + \frac{1}{(t+2)^2}\Big)\Big(\alpha - \lambda + \mu \cdot n\Big)^2.$$

It follows that:

$$\pi_j(R^g, M) - \pi_j(R, M) = \Big(\frac{t}{(n+1)^2} + \frac{1}{(t+2)^2}\Big)\Big[(\alpha - \lambda + \mu \cdot n)^2 - \Big(\alpha - \lambda + \mu\Big(nt + n - \frac{13}{8}t^2 - \frac{3}{2}t - \frac{1}{2}\Big)\Big)^2\Big].$$

Notice that $nt + n - \frac{13}{8}t^2 - \frac{3}{2}t - \frac{1}{2} - n > 0$ for any $n \geq 5$ and any $t \in [1, \bar{t}]$. Thus, $\pi_j(R^g, M) - \pi_j(R, M) < 0$ for any $n \geq 5$ and any $t \in [1, \bar{t}]$, and the firms from the initial MS agreement $N \setminus T$ will always block the deviation from R to R^g. □

Proof of Proposition 5. The proof follows directly from Lemmas 2, 3 and 4. □

Proof of Lemma 5. This lemma considers the deviations from $M = \{N \setminus T, M_T^s\}$ toward any γ-obtainable MS structure $\overline{M}$ via coalition $(N \setminus T) \setminus K$. The deviating coalition is composed of some of the most efficient firms that belong to the R&D alliance $N \setminus T$ in $R = \Big\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\Big\}$. Let $j \in (N \setminus T) \setminus K$ be a deviating firm from the R&D alliance $N \setminus T$ in R.

From Property 2, the most profitable MS structure for the deviating coalition $(N \setminus T) \setminus K$ is $M^* = \{(N \setminus T) \setminus K, K \cup T\}$ where all other firms are grouped together into one MS agreement $K \cup T$. Indeed, since the formation of a larger MS agreement by firms outside the deviating coalition increases the profit of firms in the deviating coalition, the profits of the deviating firms in the new MS structure will be larger the bigger the size of the MS agreement outside the deviating coalition; i.e., when all non-deviating firms are forming a MS agreement. One can compute the profits of firm $j \in (N \setminus T) \setminus K$ in the initial MS structure (R, M) and the final one (R, M^*) with $k = \#(K)$:

$$\pi_j(R,M) = \Big[\frac{t}{(n+1)^2} + \frac{1}{(t+2)^2}\Big]\Big(\alpha - \lambda + \mu\Big(n\cdot t + n - \frac{13t^2+12t+4}{8}\Big)\Big)^2$$

$$\begin{aligned}\pi_j(R,M^*) = &\frac{k}{(n-k-t+2)^2}\Big(\alpha-\lambda+\mu(n-t)\Big)^2\\ &+\frac{3t+2}{4(n-k-t+2)^2}\Big(\alpha-\lambda+\mu\Big(2n-\frac{11t+2}{4}\Big)\Big)^2\\ &+\frac{t-2}{4(n-k-t+2)^2}\Big(\alpha-\lambda+\mu\Big(2n-\frac{9t-2}{4}\Big)\Big)^2\\ &+\frac{1}{(k+t+2)^2}\Big(\alpha-\lambda+\mu\Big(n\cdot t+n-\frac{13t^2+12t+4}{8}\Big)\Big)^2.\end{aligned}$$

One can verify that $\pi_j(R,M^*) - \pi_j(R,M) < 0$ for all $t \in [1, \frac{\bar{t}}{2}]$, all $k \leq n-t$, and all $n \geq 6$. Therefore, firm j will always block any deviation from (R,M) toward (R,M^*) and thus, to any other less profitable γ-obtainable MS structure. □

Proof of Lemma 6. This lemma considers the deviations from $M = \{N \setminus T, M_T^s\}$ toward any γ-obtainable MS structure $\overline{M}$ via coalition $((N \setminus T) \setminus K) \cup S$. The deviating coalition is composed of some of the most efficient firms that belong to the R&D alliance $N \setminus T$ in R, possibly some firms in the R&D alliance $T_{\frac{3}{4}}$ in R, and some firms in the R&D alliance $T_{\frac{1}{4}}$ in $R = \Big\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\Big\}$. Let $j \in ((N \setminus T) \setminus K) \cup S$ be a deviating firm from the R&D alliance $T_{\frac{1}{4}}$ in R. From Property 2, the most profitable MS structure for the deviating coalition $((N \setminus T) \setminus K) \cup S$ is $M^{**} = \{((N \setminus T) \setminus K) \cup S, (K \cup T) \setminus S\}$ where all other firms are grouped together into one MS agreement $(K \cup T) \setminus S$.
Let $x = x_1 + x_2$, where $x_1 \in [1, \lfloor \frac{t-2}{4} + \frac{1}{2} \rfloor]$ and $x_2 \in [1, \lfloor \frac{3t+2}{4} + \frac{1}{2} \rfloor]$ denotes respectively the number of firms from $T_{\frac{1}{4}}$ and from $T_{\frac{3}{4}}$ in $(K \cup T) \setminus S$ in M^{**}. One can compute the profits of firm $j \in ((N \setminus T) \setminus K) \cup S$ in the initial MS structure (R,M) and the final one (R,M^{**}) with $k = \#(K)$:

$$\pi_j(R,M) = \frac{t}{(n+1)^2}\Big(\alpha-\lambda+\mu\Big[\frac{9}{4}n\cdot t - \frac{t}{4} - \frac{n}{2} - \frac{13}{8}t^2 - n^2 - 1\Big]\Big)^2 + \frac{n-t}{(t+2)^2}\Big(\alpha-\lambda+\mu\Big[\frac{3}{4}t^2 + \frac{t}{2} - n - \frac{3}{2}\Big]\Big)^2$$

$$\begin{aligned}\pi_j(R,M^{**}) = &\frac{k}{(n-k-x+2)^2}\Big(\alpha-\lambda+\mu\Big[\frac{9}{4}n\cdot t - \frac{5}{4}k\cdot t + n\cdot k - \frac{13}{8}t^2 - n^2 + t - \frac{3}{2}n + \frac{k}{2} - \frac{3}{2} + x_2(\frac{t}{2}+1)\Big]\Big)^2\\ &+\frac{x_1}{(n-k-x+2)^2}\Big(\alpha-\lambda+\mu\Big[\frac{9}{4}n\cdot t - \frac{5}{4}k\cdot t + n\cdot k - \frac{13}{8}t^2 - n^2 + \frac{t}{2} - \frac{n}{2} + \frac{k}{2} - 1 + x_2(\frac{t}{2}+1)\Big]\Big)^2\\ &+\frac{x_2}{(n-k-x+2)^2}\Big(\alpha-\lambda+\mu\Big[\frac{9}{4}n\cdot t - \frac{5}{4}k\cdot t + n\cdot k - \frac{13}{8}t^2 - n^2 - \frac{3}{4}t - \frac{n}{2} + \frac{k}{2} - 2 + x_2(\frac{t}{2}+1)\Big]\Big)^2\\ &+\frac{1}{(k+x+2)^2}\Big(\alpha-\lambda+\mu\Big[\frac{9}{4}k\cdot t - n\cdot k - \frac{k}{2} + \frac{t}{2} - \frac{1}{2} - x_2(\frac{t}{2}+1)\Big]\Big)^2.\end{aligned}$$

One can show that $\pi_j(R,M^{**}) - \pi_j(R,M) < 0$ for any $n \geq 4$, any $t \in [1, \bar{t}]$, any $x_1 \in [1, \lfloor \frac{t-2}{4} + \frac{1}{2} \rfloor]$, any $x_2 \in [1, \lfloor \frac{3t+2}{4} + \frac{1}{2} \rfloor]$, and any $k \in [2, \frac{n}{2} - x]$. Therefore, firm j will always block any deviation from (R,M) toward (R,M^{**}) and thus, to any other less profitable γ-obtainable MS structure. □

Proof of Lemma 7. This lemma considers the deviations from $M = \{N \setminus T, M_T^s\}$ toward any γ-obtainable MS structure $\overline{M}$ via coalition $((N \setminus T) \setminus K) \cup S$. The deviating coalition is only

composed of some of the most efficient firms that belong to the R&D alliance $N\setminus T$ in R and some firms in the R&D alliance $T_{\frac{3}{4}}$ in $R = \Big\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\Big\}$. Let $j \in ((N \setminus T) \setminus K) \cup S$ be a deviating firm from the R&D alliance $T_{\frac{3}{4}}$ in R. From Property 2, the most profitable MS structure for the deviating coalition $((N \setminus T) \setminus K) \cup S$ is $M^{***} = \{((N \setminus T) \setminus K) \cup S, (K \cup T) \setminus S\}$ where all other firms are grouped together into one MS agreement $(K \cup T) \setminus S$.
Let $x = x_1 + x_2$, where $x_1 \in [1, \lfloor \frac{t-2}{4} + \frac{1}{2} \rfloor]$ and $x_2 \in [1, \lfloor \frac{3t+2}{4} + \frac{1}{2} \rfloor]$ denotes respectively the number of firms from $T_{\frac{1}{4}}$ and from $T_{\frac{3}{4}}$ in $(K \cup T) \setminus S$ in M^{***}. One can compute the profits of firm $j \in ((N \setminus T) \setminus K) \cup S$ in the initial MS structure (R, M) and the final one (R, M^{***}) with $k = \#(K)$:

$$\pi_j(R, M) = \frac{t}{(n+1)^2}\Big(\alpha - \lambda + \mu\Big[\frac{11}{4}n \cdot t + \frac{t}{4} + \frac{n}{2} - \frac{13}{8}t^2 - n^2\Big]\Big)^2 + \frac{n-t}{(t+2)^2}\Big(\alpha - \lambda + \mu\Big[\frac{10}{4}t + \frac{t^2}{8} - n + \frac{1}{2}\Big]\Big)^2$$

$$\begin{aligned}\pi_j(R, M^{***}) = & \frac{k}{(n-k-x+2)^2}\Big(\alpha - \lambda + \mu\Big[\frac{11}{4}n \cdot t - \frac{7}{4}k \cdot t + n \cdot k - \frac{11}{8}t^2 - n^2 + \frac{3}{2}t - \frac{n}{2} - \frac{k}{2} + \frac{1}{2}\Big]\Big)^2 \\ & \frac{\frac{t-2}{4}}{(n-k-x+2)^2}\Big(\alpha - \lambda + \mu\Big[\frac{11}{4}n \cdot t - \frac{7}{4}k \cdot t + n \cdot k - \frac{11}{8}t^2 - n^2 + \frac{t}{4} + \frac{n}{2} - \frac{k}{2} + 1\Big]\Big)^2 \\ & \frac{x_2}{(n-k-x+2)^2}\Big(\alpha - \lambda + \mu\Big[\frac{11}{4}n \cdot t - \frac{7}{4}k \cdot t + n \cdot k - \frac{11}{8}t^2 - n^2 - \frac{t}{4} + \frac{n}{2} - \frac{k}{2}\Big]\Big)^2 \\ & \frac{1}{(k+x+2)^2}\Big(\alpha - \lambda + \mu\Big[\frac{7}{4}k \cdot t - n \cdot k + \frac{k}{2} + \frac{3}{4}t + \frac{t^2}{8}\Big]\Big)^2.\end{aligned}$$

One can verify that $\pi_j(R, M^{***}) - \pi_j(R, M) < 0$ for any $n \geq 4$, any $t \in [1, \bar{t}]$, any $x_2 \in [1, \lfloor \frac{3t+2}{4} + \frac{1}{2} \rfloor]$, and any $k \in [2, \frac{n}{2} - x]$. Therefore, firm j will always block any deviation from (R, M) toward (R, M^{***}) and thus, to any other less profitable γ-obtainable MS structure. □

Proof of Lemma 8. When the deviating coalition contains equally efficient firms; i.e., when $L \subseteq T_{\frac{3}{4}}$ or $L \subseteq T_{\frac{1}{4}}$, the result holds from Proposition 2 since $t \leq \bar{t}$. When the only deviating coalition is $L \subseteq T_{\frac{3}{4}} \cup T_{\frac{1}{4}}$, the only γ-obtainable MS structure from M via the unique deviating coalition L is $M^{****} = \Big\{N \setminus T, L, M^s_{T \setminus L}\Big\}$. Let $\#(L) = k = k_1 + k_2$, where $k_1 > 0$ and $k_2 > 0$ are the number of firms from the R&D coalition $T_{\frac{1}{4}}$ and $T_{\frac{3}{4}}$ respectively. Let i be a firm from the R&D coalition $T_{\frac{3}{4}}$ and from the deviating MS coalition L. We have that:

$$\begin{aligned}\pi_i(R, M^{****}) - \pi_i(R, M) = & \\ & \frac{1}{(n-k+2)^2}\Big(\alpha - \lambda + \mu\Big[\frac{11}{4}nt - n^2 + \frac{n}{2} - \frac{13}{8}t^2 + \frac{t}{4} - k_1\Big(\frac{t}{2} + 1\Big)\Big]\Big)^2 \\ & - \frac{k}{(n+1)^2}\Big(\alpha - \lambda + \mu\Big[\frac{11}{4}nt - n^2 + \frac{n}{2} - \frac{13}{8}t^2 + \frac{t}{4}\Big]\Big)^2.\end{aligned}$$

One can verify that $\pi_i(R, M^{****}) - \pi_i(R, M) < 0$ for any $n \geq 0$, any $t \in \{1, \ldots, \bar{t}\}$, and any $k \in \{1, \ldots, t\}$. □

Proof of Lemma 9. Let $j \in T_{\frac{1}{4}}$ be a deviating firm member of the R&D alliance $T_{\frac{1}{4}}$ in the initial R&D alliance structure R. One can compute the profits of firm $j \in T_{\frac{1}{4}}$ in the initial MS structure (R, M) and in the final one (R, M^g):

$$\begin{aligned}\pi_j(R, M) &= \frac{t}{(n+1)^2}\Big(\alpha - \lambda + \mu\Big[\frac{9}{4}n \cdot t - n^2 - \frac{n}{2} - \frac{13}{8}t^2 - \frac{t}{4} - 1\Big]\Big)^2 \\ &+ \frac{(n-t)}{(t+2)^2}\Big(\alpha - \lambda + \mu\Big[\frac{t}{2} - \frac{3}{8}t^2 - n - \frac{3}{2}\Big]\Big)^2 \\ \pi_j(R, M^g) &= \frac{1}{4}\Big(\alpha - \lambda + \mu \cdot \frac{t-2}{4}\Big)^2\end{aligned}$$

One can show that $\pi_j(R, M^g) - \pi_j(R, M) < 0$ for any $n \geq 6$ and any $t \in [1, \bar{t}]$, and the firms from the initial R&D alliance $T_{\frac{1}{4}}$ will always block the deviation from M to M^g. $\square$

Proof of Proposition 6. The proof follows directly from Lemmas 5, 6, 7, 8 and 9. $\square$

Proof of Corollary 3. One can show that the deviations studied in Lemma 5 are always blocked by the firms in the largest deviating MS coalition when $t = 1$. Moreover, the deviations studied in Lemma 3 do not exist for $t = 1$. Therefore, this result is directly obtained from Proposition 4 when $t = 1$. $\square$

Proof of Proposition 7. The proof follows directly from Lemma 10 and Lemma 11. $\square$

Proof of Lemma 10. When the global MS agreement $M^g = \big\{N\big\}$ is formed, each firm is a monopoly in its home market and then the Cournot-Nash profit function of each firm $j \in N$ is:

$$\pi_j(R, M^g) = \frac{1}{4}(\alpha - \lambda + \mu \cdot r_j)^2.$$

This profit function is strictly increasing in r_j, the size of the R&D alliance of firm j. Moreover, the profit function is not affected by any change in the efficiency of the competitors; i.e. $\pi_j(R, M^g)$ is not affected by changes in r_h, $\forall h \in N \setminus \{j\}$. Therefore, any firm j maximizes its profit by forming the biggest possible R&D alliance $R^g = \big\{N\big\}$. $\square$

Proof of Lemma 11. Let us show that a deviation of coalition K from $\mathcal{M}^g = \big\{N\big\}$ toward some γ-obtainable MS structure will always be blocked. From Property 2, the most profitable γ-obtainable MS structure for the deviating coalition K is $\{K, N \setminus K\}$ where the deviating coalition faces as few competitors as possible on every market; i.e., when all non-deviating firms are forming a MS agreement $N \setminus K$. Let $j \in K$ a deviating firm. It follows that if j has incentives to block the deviation from (R^g, M^g) toward $(R^g, \{K, N \setminus K\})$, then firm $j \in K$ will also block every deviation from (R^g, M^g) toward any other γ-obtainable MS structure $(R^g, \overline{M})$ via coalition K. The profits of firm $j \in K$ in the initial MS structure (R^g, M^g) and the final one $(R^g, \{K, N \setminus K\})$ are given by:

$$\begin{aligned}\pi_j(R^g, M^g) &= \frac{1}{4} \cdot (\alpha - \lambda + n \cdot \mu)^2 \\ \pi_j(R^g, \{K, N \setminus K\}) &= \Big(\frac{1}{(n-k+2)^2} + \frac{n-k}{(k+2)^2}\Big) \cdot (\alpha - \lambda + n \cdot \mu)^2.\end{aligned}$$

Hence, we have:

$$\begin{aligned}\pi_j(R^g, \{K, N \setminus K\}) - \pi_j(R^g, M^g) &= \Big(\frac{1}{(n-k+2)^2} - \frac{n-k}{(k+2)^2} - \frac{1}{4}\Big) \cdot (\alpha - \lambda + n \cdot \mu)^2 \\ &:= h(n,k)(\alpha - \lambda + n \cdot \mu)^2\end{aligned}$$

Since K is the largest MS coalition in $\{K, N \setminus K\}$, we have that $k > n - k \iff k > \frac{n}{2}$. One can verify that $h(n,k) < 0$ for $n \geq 2$ and $k \in]\frac{n}{2}, n-1]$. $\square$

Proof of Proposition 8. According to Lemma 1, the only possible stable pairs of structures when the MS structure is $\{\overline{M}, M_T^s\}$ are $\{\overline{M}, X, Y\}$ or $\{\overline{M}, X \cup Y\}$, where $\overline{M}$ denotes the largest MS coalition. Since we consider symmetric structures, all firms in any MS agreement are equally efficient.
If $\#(\overline{M}) < n - \sqrt{n - \frac{5}{4}} + \frac{3}{2}$, Proposition 2 implies that firms from $\overline{M}$ that are equally efficient are going to dissolve their MS agreement. In that case the symmetric pair of structures containing $\overline{M}$ cannot be stable, as the MS structure is not γ-stable.

If $\#(\overline{M}) > n - \sqrt{n - \frac{5}{4}} + \frac{3}{2}$, then it necessary holds that $(\#(Y), \#(X)) \ll n - \sqrt{n - \frac{5}{4}} + \frac{3}{2}$ because $2 \cdot n - \sqrt{n - \frac{5}{4}} + \frac{3}{2} > n$. Therefore, if $\#(Y) > 1$ and/or $\#(X) > 1$, the members of these MS agreements are going to dissolve their respective MS agreements Y and/or X by Proposition 2.

Moreover, if $(\#(X), \#(Y)) = (1,1)$, we have that $1 < \frac{n_h+1}{2}$ for any market h of size $n_h \geq 2$. But in the industry where the pair of structures $\big(\{\overline{M}, (1), (1)\}, \{\overline{M}, (1), (1)\}\big)$ is formed, all the markets have at least 2 competitors ($n_h \geq 2 \ \forall h$). Therefore, firms from X and Y have incentives to merge their R&D coalitions into $X \cup Y$ as both of them verify condition (A.9) in any market.

Therefore, the only situations where firms do not have incentives to either dissolve their MS agreement or to merge their R&D alliances is when $(\#(X), \#(Y)) \in \{(0,0), (1,0), (0,1)\}$. It follows that the only γ-stable pairs of symmetric structures are:

$$(\{N\}, \{N\}) \text{ and } (\{N \setminus \{s\}, \{s\}\}, \{N \setminus \{s\}, \{s\}\}) \,.$$

$\square$

Proof of Proposition 9. In a Cournot setting with linear demand, the consumer surplus on market j is $\frac{1}{2}(\sum_{i=1}^{h} q_i^j)^2$, where q_i^j is the equilibrium quantities produce by firm i on market j. The consumer surplus associated to a pair of coalition structures (R, M) is the sum of the consumer surpluses on all the markets, where the equilibrium quantities depend upon the coalition structures (R, M).

First of all, let us compute the consumer surplus of the pair of coalition structures $(R^*, M^*) = (\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\}, \{N \setminus T, M_t^s\})$, for $t \in \{2, \ldots, \bar{t}\}$. The overall consumer surplus associated to the pair of coalition structures (R^*, M^*) is given by:

$$\begin{aligned} CS(R^*, M^*, t) = & \frac{(n-t)}{2(t+2)^2}\Big((\alpha - \lambda)(t+1) + \mu\Big[n + \frac{5}{8}t^2 - \frac{t}{2} + \frac{1}{2}\Big]\Big)^2 \\ & + \frac{t}{2(n+1)^2}\Big(n(\alpha - \lambda) + \mu\Big[n^2 - 2nt + \frac{13}{8}t^2 + \frac{t}{2} + \frac{1}{2}\Big]\Big)^2 \end{aligned} \tag{A.10}$$

Let us now investigate whether (A.10) is increasing with respect to t or not. One can compute that:

$$
\begin{aligned}
CS(R^*, M^*, t+1) - CS(R^*, M^*, t) = \\
& \frac{n-t-1}{2(t+3)^2}\Big((\alpha-\lambda)(t+2) + \mu\Big[n + \frac{5}{8}t^2 + \frac{3}{4}t + \frac{5}{8}\Big]\Big)^2 \\
& + \frac{t+1}{2(n+1)^2}\Big(n(\alpha-\lambda) + \mu\Big[n^2 - 2nt - 2n + \frac{13}{8}t^2 + \frac{15}{4}t + \frac{21}{8}\Big]\Big)^2 \\
& - \frac{(n-t)}{2(t+2)^2}\Big((\alpha-\lambda)(t+1) + \mu\Big[n + \frac{5}{8}t^2 - \frac{t}{2} + \frac{1}{2}\Big]\Big)^2 \\
& - \frac{t}{2(n+1)^2}\Big(n(\alpha-\lambda) + \mu\Big[n^2 - 2nt + \frac{13}{8}t^2 + \frac{t}{2} + \frac{1}{2}\Big]\Big)^2 \\
& := x_1 + y_1 - x_2 - y_2.
\end{aligned}
$$

One can verify that (i) $y_1 - y_2 > 0$ for $n \geq 4$ and $t \in \{0, \ldots, \bar{t}\}$, and (ii) $x_1 - x_2 > 0$ for $n \geq 5$ and $t \in \{0, \ldots, \bar{t}\}$. Hence, $CS(R^*, M^*, t)$ is strictly increasing with t for any $n \geq 5$ and any $t \in \{2, \ldots, \bar{t}\}$.

For $t = 0$, we compare $CS(\{N\}, \{N\})$ with $CS(\{N \setminus \{s\}, \{s\}\}, \{N \setminus \{s\}, \{s\}\})$. We have

$$CS(\{N\}, \{N\}) = \frac{n}{8}(\alpha - \lambda + \mu n)^2$$

and

$$
\begin{aligned}
CS(\{N \setminus \{s\}, \{s\}\}, \{N \setminus \{s\}, \{s\}\}) = & \frac{n-1}{18}(2(\alpha-\lambda) + \mu(n))^2 \\
& + \frac{1}{2(n+1)^2}(n(\alpha-\lambda) + \mu(n^2 - 2n + 2))^2.
\end{aligned}
$$

One can verify that for $n \geq 4$ and $(\alpha - \lambda, \mu)$ such that positive quantities are produce in any markets, we have $CS(\{N \setminus \{s\}, \{s\}\}, \{N \setminus \{s\}, \{s\}\}) > CS(\{N\}, \{N\})$.

For $t = 1$, we compare $CS(\{N \setminus \{s\}, \{s\}\}, \{N \setminus \{s\}, \{s\}\})$ with $CS(\{N \setminus \{s_1, s_2\}, \{s_1, s_2\}\}, \{N \setminus \{s_1, s_2\}, \{s_1\}, \{s_2\}\})$. We get:

$$
\begin{aligned}
CS(\{N \setminus \{s_1, s_2\}, \{s_1, s_2\}\}, \{N \setminus \{s_1, s_2\}, \{s_1\}, \{s_2\}\}) = & \frac{n-2}{32}(3(\alpha-\lambda) + \mu(2n-4))^2 \\
& + \frac{2}{2(n+1)^2}(n(\alpha-\lambda) + \mu(n^2 - 4n + 6))^2
\end{aligned}
$$

One can verify that $CS(\{N \setminus \{s_1, s_2\}, \{s_1, s_2\}\}, \{N \setminus \{s_1, s_2\}, \{s_1\}, \{s_2\}\}) > CS(\{N \setminus \{s\}, \{s\}\}, \{N \setminus \{s\}, \{s\}\})$ for all $n \geq 8$.

Thus, for any $n \geq 8$, and any $t \in \{0, \ldots, \bar{t}\}$, the consumer surplus $CS(R^*, M^*) = CS(\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\}, \{N \setminus T, M_t^s\})$ is strictly increasing with t. □

Proof of Proposition 10. Let us compare the consumer surplus of the the pair of grand coalitions relative to the pair of singleton structures. We have that:

$$CS(R^s, M^s) = \frac{n}{2(n+1)^2}(\alpha - \lambda + \mu)^2$$

which is strictly smaller than $CS(\{N\}, \{N\})$ as for any $n \geq 0$: (i) $\frac{n}{2(n+1)^2} < \frac{n}{8}$, and (ii) $\alpha - \lambda + \mu < \alpha - \lambda + \mu \cdot n$. From Proposition 9 we know that $CS(\{N\}, \{N\}) < CS(\{N \setminus T, T_{\frac{3}{4}}, T_{\frac{1}{4}}\}, \{N \setminus T, M_t^s\})$ for all $n \geq 8$, for all $t \in \{1, \ldots, \bar{t}\}$. □

Proof of Proposition 11. From the proof of Proposition 8, the only pairs of symmetric coalition structures that could be stable are $(\{N\}, \{N\})$ and $(\{N \setminus \{s\}, \{s\}\}, \{N \setminus \{s\}, \{s\}\})$. But, these pairs of symmetric coalition structures are not stable when we consider δ-stable MS structures.

From the MS structure $\{N\}$, any firm $j \in N$ has a unilateral incentive to leave the global MS agreement:

$$\pi_j(\{N\}, \{N\}) = \frac{1}{4} \cdot (\alpha - \lambda + n \cdot \mu)^2$$
$$\pi_j(\{N\}, \{N \setminus \{j\}, \{j\}\}) = \Big(\frac{n-1}{9} + \frac{1}{(n+1)^2}\Big) \cdot (\alpha - \lambda + n \cdot \mu)^2$$

and it holds that $\pi_j(\{N\}, \{N \setminus \{j\}, \{j\}\}) - \pi_j(\{N\}, \{N\}) > 0$ for any $n \geq 1$.

Similarly, from the MS structure $\{N \setminus \{s\}, \{s\}\}$, each firm j from the MS agreement $N \setminus \{s\}$ has a unilateral incentive to leave the MS agreement:

$$\pi_j(\{N \setminus \{s\}, \{s\}\}, \{N \setminus \{s\}, \{s\}\}) = \Big(\frac{1}{9} + \frac{1}{(n+1)^2}\Big) \cdot (\alpha - \lambda + n \cdot \mu)^2$$
$$\pi_j(\{N \setminus \{s\}, \{s\}\}, \{N \setminus \{s, j\}, \{s, j\}\}) = \frac{1}{9}(\alpha - \lambda + \mu(3n-5))^2 + \frac{2}{(n+1)^2}(\alpha - \lambda + n \cdot \mu)^2$$

and since $3n - 5 > n \ \forall n > \frac{5}{2}$, we have:

$$\begin{aligned}\pi_j(\{N \setminus \{s\}, \{s\}\}, \{N \setminus \{s, j\}, \{s, j\}\}) \ &- \ \pi_j(\{N \setminus \{s\}, \{s\}\}, \{N \setminus \{s\}, \{s\}\}) \\ &> \frac{1}{(n+1)^2}(\alpha - \lambda + n \cdot \mu)^2 > 0.\end{aligned}$$

So, we have $\pi_j(\{N \setminus \{s\}, \{s\}\}, \{N \setminus \{s, j\}, \{s, j\}\}) - \pi_j(\{N \setminus \{s\}, \{s\}\}, \{N \setminus \{s\}, \{s\}\}) > 0$ for $n \geq 3$. $\square$

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