MEMA seminar • 24 October 2024 • 11:00-11:45 AM
room Euler a.002, Euler Bldg., 4 avenue G. Lemaître, LLN

Seiche: the phenomenon that rocked the Earth in September 2023

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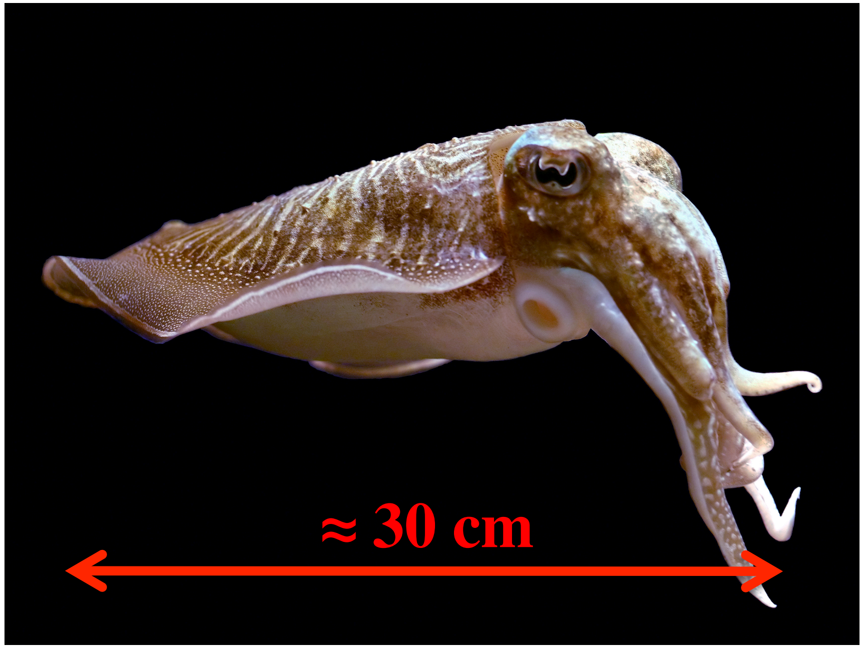
Ishan Deleersnijder, Dr Jaya Naithani and Dr Pierre-Denis Plisnier
and the instigator of this seminar, Ange Pacifique Ishimwe

A little of disambiguation to begin with



In French, *une **seiche** commune* is a marine mollusc (i.e., a “cousin” of octopuses), which is referred to as *a common **cuttlefish*** in English. See photograph opposite[1].

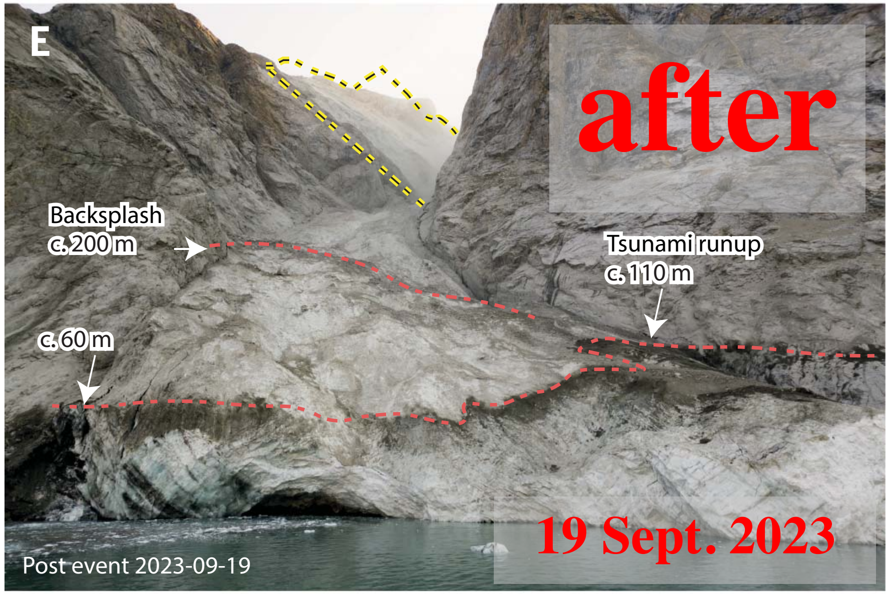
• We focus on the **hydrodynamic phenomenon** called **seiche** in both French and English. It usually is viewed as a **standing wave** triggered by a **sudden disturbance** in a nearly or completely **enclosed water body** (e.g., port, lake, fjord, nearly-enclosed bay or sea).

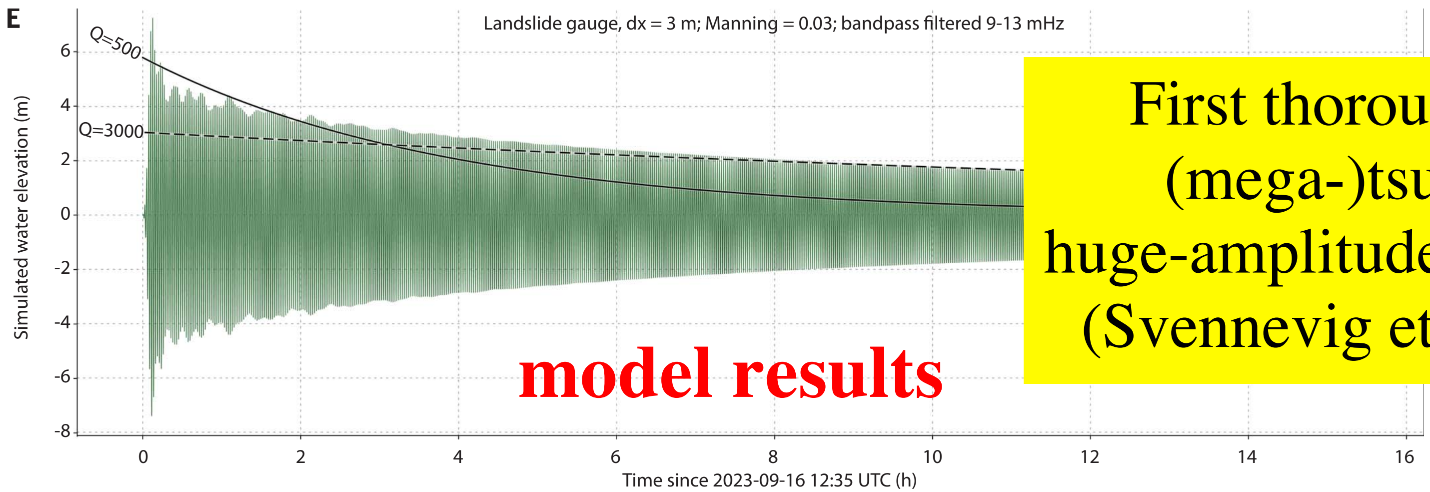
[1] Attribution: Hans Hillewaert. Licence: CC BY-SA 4.0. Source: https://commons.wikimedia.org/wiki/File:Sepia_officinalis_(aquarium).jpg (last viewed on 29 August 2024)

The concept of seiche is rife with misconceptions

Credit: Lakshmi Deleersnijder, October 2024 / Inspiration: the Great Wave off Kanagawa

The Dickson Fjord (Greenland) tsunami-triggered long-lasting seiche





First thoroughly documented (mega-)tsunami-triggered, huge-amplitude, long-lasting seiche (Svennevig et al., *Science*, 2024)

A brief history of megatsunamis to expand our general knowledge

• The GOAT (greatest of all tsunamis): wave height maybe as much as 1.5 km. Cause: an asteroid ($\approx$ 10 km in size) that hit the Earth about 66 Myears ago. Consequence: the end of the dinosaur era (due to climate change).

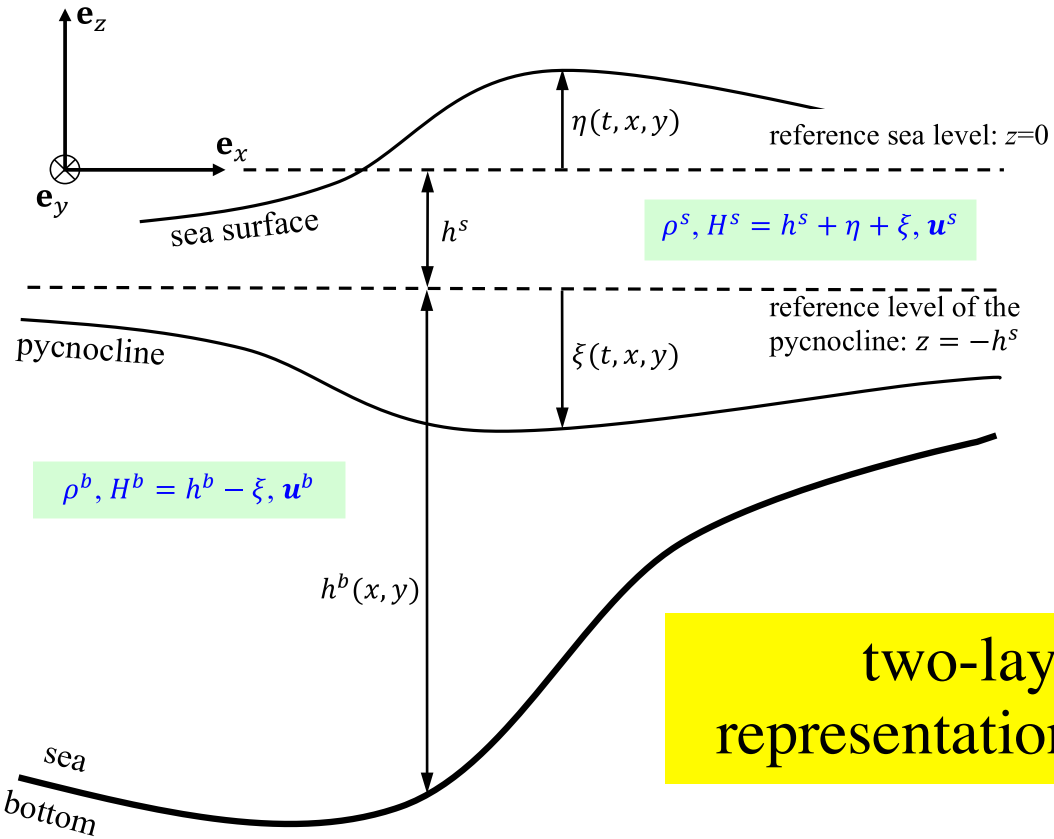
• Lituya Bay (Alaska) on 9 July 1958. Cause: rockslide ($\approx$ 90 Mtonnes). Maximum wave height: $\approx$ 500 m.

• Vajont dam reservoir (Italy) on 9 October 1964. Cause: landslide from *Monte Toc* of $\approx 260\times10^6$ m^3 of earth and rocks. Maximum wave height: $\approx$ 500 m. Death toll: $\approx$ 2000. Partly human made disaster.

• Dickson Fjord (Greenland) on 16 September 2023. Cause: $\approx 25\times10^6$ m^3 avalanche of ice and rock. Maximum wave height: $\approx$ 200 m. Consequence: seiche in the fjord ($\leq$ 7 m in height). Period: 92 s. Duration: $\tilde{>}$ 9 days.

A non-dissipative, non-Boussinesq, two-layer hydrodynamic model (I)

- Two layers of homogeneous density separated by an impermeable pycnocline (free boundary) modelled by means of Saint-Venant (i.e., depth-integrated or shallow-water) partial differential equations.



$$\rho^s < \rho^b$$

$$10^{-3} \ \tilde{<} \ \varepsilon = \frac{\rho^b - \rho^s}{\rho^b} \ \tilde{<} \ 10^{-2}$$

$$|\eta| << |\xi|$$

two-layer model: simplest representation of stratification impact

A non-dissipative, non-Boussinesq, two-layer hydrodynamic model (II)

- Upper layer continuity and (horizontal) momentum equations:

$$\partial_t(\eta+\xi) \;+\; \nabla \bullet \mathbf{U}^s \;=\; 0$$

$$\rho^s\left[\partial_t \mathbf{U}^s + \nabla \bullet \left(\mathbf{U}^s\mathbf{U}^s / H^s\right) + f\,\mathbf{e}_z \times \mathbf{U}^s\right] \;=\; -g\rho^s H^s \nabla\eta \;+\; \boldsymbol{\tau}^s$$

where $\mathbf{U}^s = H^s\mathbf{u}^s$ is the transport (height$\times$velocity), $f\,\mathbf{e}_z \times \mathbf{U}^s$ is the Coriolis term and $\boldsymbol{\tau}^s$ is the surface (wind) stress (i.e., the only forcing term).

- Lower layer continuity and (horizontal) momentum equations:

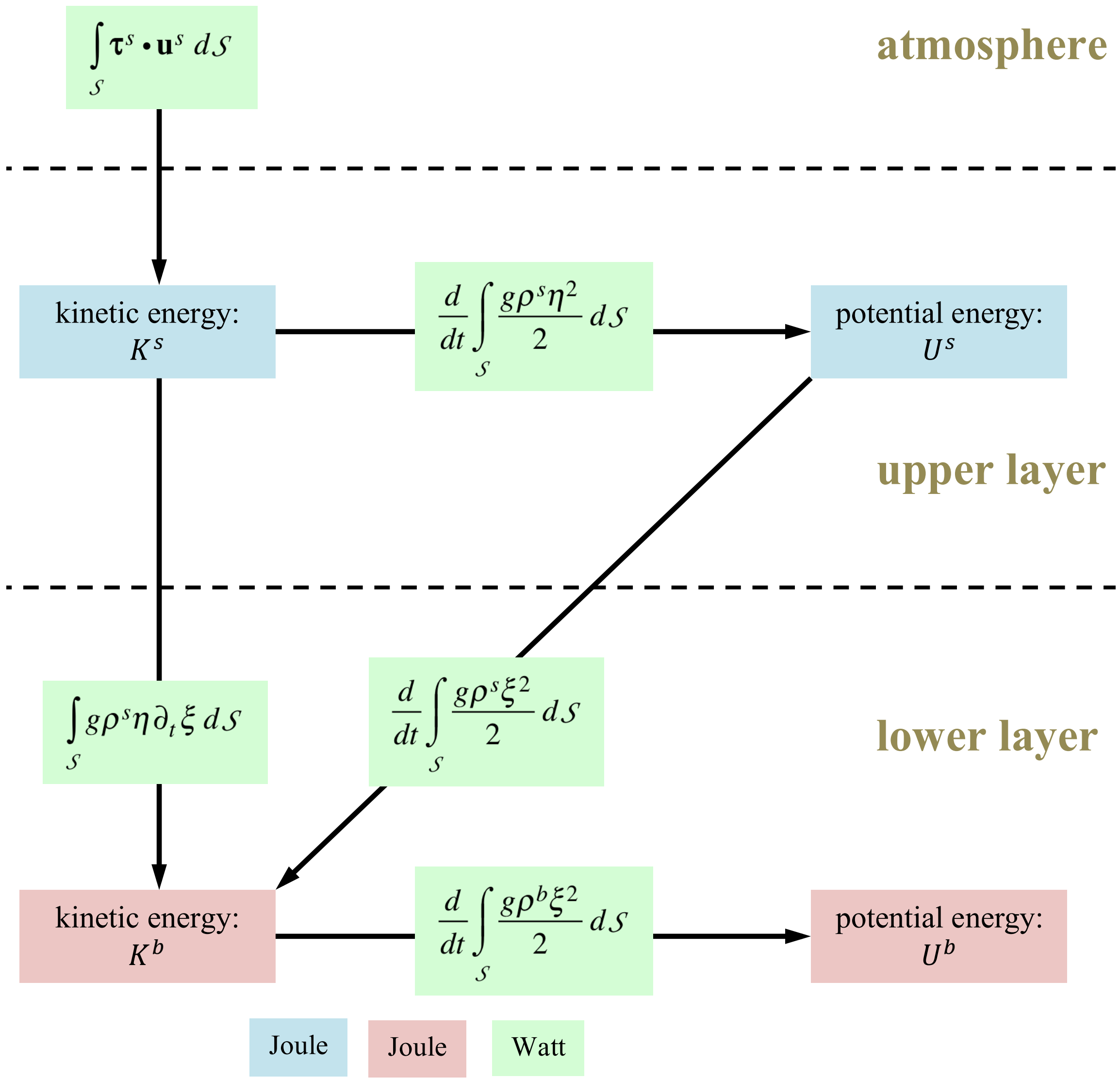
$$\partial_t(-\xi) \;+\; \nabla \bullet \mathbf{U}^b \;=\; 0$$

$$\rho^b\left[\partial_t \mathbf{U}^b + \nabla \bullet \left(\mathbf{U}^b\mathbf{U}^b / H^b\right) + f\,\mathbf{e}_z \times \mathbf{U}^b\right]$$

$$= \; -g\rho^b H^b \nabla\eta \;+\; g(\rho^b - \rho^s) H^b \nabla(\eta+\xi)$$

- The two layers are connected through the pressure alone.

A non-dissipative, non-Boussinesq, two-layer hydrodynamic model (III)



The equations above admit no analytical solution.

They can be solved numerically to a high degree of accuracy (e.g., Vreugdenhil, 1995)

Their **energy budget** may be examined in an **enclosed** domain for validation purposes.

See figure opposite.

Linearised equations for the external and internal modes (I)

• Assume that the domain bottom is flat and horizontal, and all the state variables are rather small. Then, we can linearise the abovementioned governing equations.

• Next, we manipulate the obtained equations so as to derive the equations governing the so-called external and internal mode dynamics.

• The **external mode** focuses on the displacement of the water-air interface:

$$\partial_t \eta + \nabla \bullet \mathbf{U} = 0 \qquad \bullet \qquad \partial_t \mathbf{U} + f\, \mathbf{e}_z \times \mathbf{U} = -gh\nabla \eta + \frac{\boldsymbol{\tau}^s}{\rho^s}$$

where $\mathbf{U} = \mathbf{U}^s + \mathbf{U}^b = h^s \mathbf{u}^s + h^b \mathbf{u}^b$ is the transport related to the whole water column (barotropic transport). These equations can be tackled alone, i.e., without taking into account the pycnocline displacement.

Linearised equations for the external and internal modes (II)

• The **internal mode** focuses on the displacement of the pycnocline (i.e., without taking into account the surface displacement):

$$\partial_t \xi + \nabla \bullet \mathbf{U}' = 0 \qquad \bullet \qquad \partial_t \mathbf{U}' + f\,\mathbf{e}_z \times \mathbf{U}' = -g' h' \nabla \xi + \frac{h^b \boldsymbol{\tau}^s}{\rho^s h}$$

$$g' = \varepsilon g \ , \qquad h' = \frac{h^s h^b}{h^s + h^b} = \frac{h^s h^b}{h} \ , \qquad \mathbf{U}' = h'(\mathbf{u}^s - \mathbf{u}^b)$$

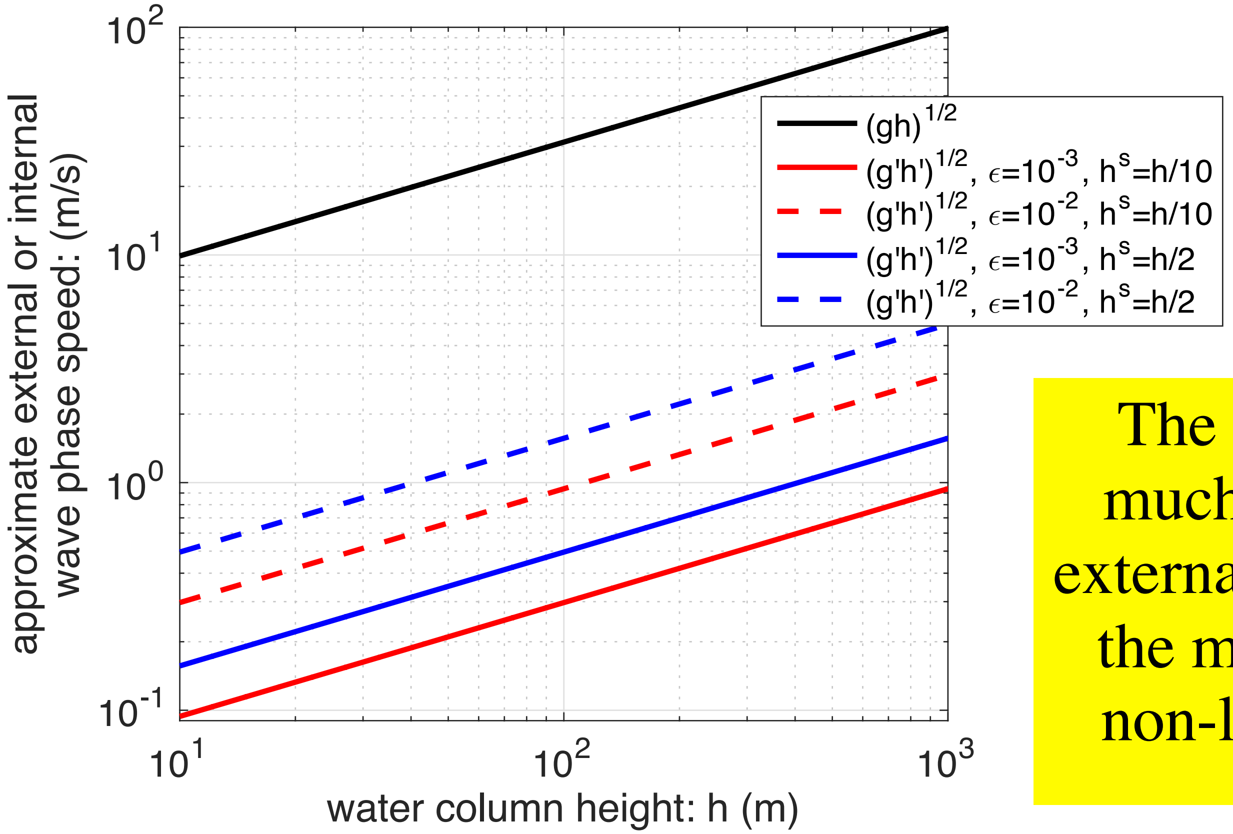
where $\mathbf{U}'$ is the baroclinic transport ($\propto$ velocity difference between the upper and lower layer) and g' is the reduced gravitational acceleration.

• In an unbounded domain, the linearised equations admit external or internal **Poincaré wave** solutions. The phase velocities satisfy:

$$\text{external mode: } \left|\mathbf{C}_p\right| = \sqrt{gh}\,\sqrt{1 + \cancel{\frac{|\mathbf{k}|^{-2}}{R^2}}} \ , \quad |\mathbf{k}|^{-1} << R = \frac{\sqrt{gh}}{|f|}$$

Linearised equations for the external and internal modes (III)

internal mode: $$\left|\mathbf{C}_p'\right| = \sqrt{g'h'}\ \sqrt{1+\cancel{\frac{|\mathbf{k}|^{-2}}{(R')^2}}}\ , \quad |\mathbf{k}|^{-1} \lesssim R' = \frac{\sqrt{g'h'}}{|f|}$$



$$\left|\mathbf{C}_p'\right| << \left|\mathbf{C}_p\right|$$

The internal mode waves are much slower than those of the external mode, hence the need for the mode splitting in (linear or non-linear) numerical models.

A highly-idealised, one-dimensional seiches model (I)

• Using *in situ* data from Lake Geneva, Forel (1876) presumably was the first to identify most key features of (external) seiches:

- period independent of position and oscillation amplitude;
- amplitude maximum at both ends of the lake;
- amplitude negligible in the middle of the lake;
- when elevation is maximum at one end, it is minimum at the opposite end.

• One-dimensional (non-rotating) model for external seiches in a 1D lake:

$$\left.\begin{array}{l}\partial_t \eta + \partial_x U = 0 \\ \partial_t U = -gh\,\partial_x \eta\end{array}\right\} \quad \Leftrightarrow \quad \left\{\begin{array}{l}\partial_{tt} \eta - gh\,\partial_{xx} \eta = 0 \\ \partial_{tt} U - gh\,\partial_{xx} U = 0\end{array}\right.$$

with impermeability conditions

$$\left[U(t,x)\right]_{x=\pm L/2} = 0 \quad \text{or} \quad \left[\partial_x\, \eta(t,x)\right]_{x=\pm L/2} = 0$$

A highly-idealised, one-dimensional seiches model (II)

- The solution may be formulated as a sum of **standing waves**:

$$\eta(t,x) = \sum_{n=1}^{\infty} E_n \cos(\omega_n t)\sin(k_n x)$$

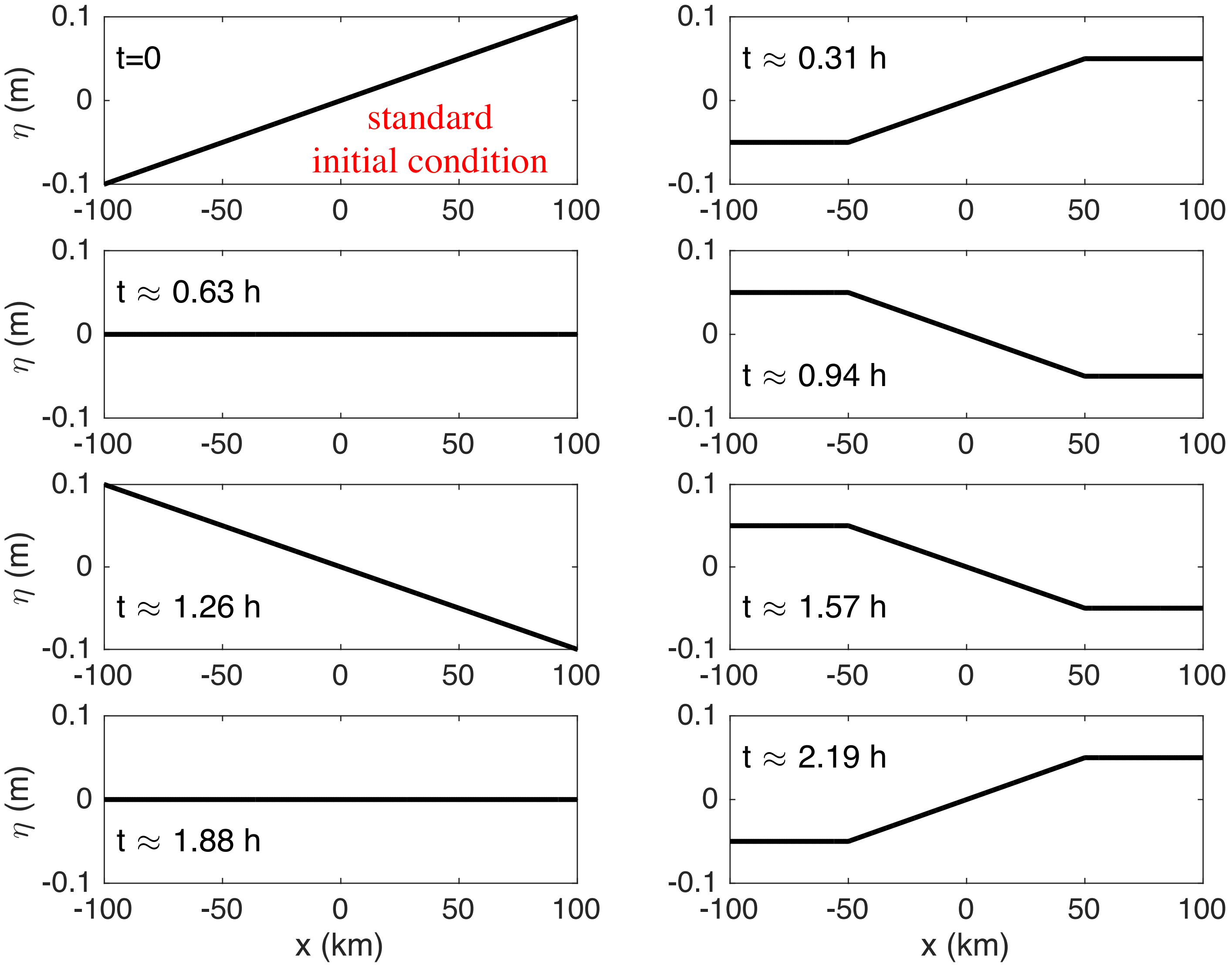
$$U(t,x) = -\sqrt{gh}\sum_{n=1}^{\infty} E_n \sin(\omega_n t)\cos(k_n x)$$

with $k_n = (2n-1)\pi / L$ and $\omega_n = \sqrt{gh}\, k_n$. The **period** reads

$$\tau = \frac{2\pi}{\omega_1} = \frac{2L}{\sqrt{gh}} = \frac{\text{twice the lake length}}{\text{phase speed}}$$

- As is well known, a standing wave may be viewed as the sum of two **travelling waves** propagating in **opposite directions** at phase velocity $\sqrt{gh}$ and $-\sqrt{gh}$.

A highly-idealised, one-dimensional seiches model (III)



lake's length: L = 200 km
lake's depth: h = 200 m

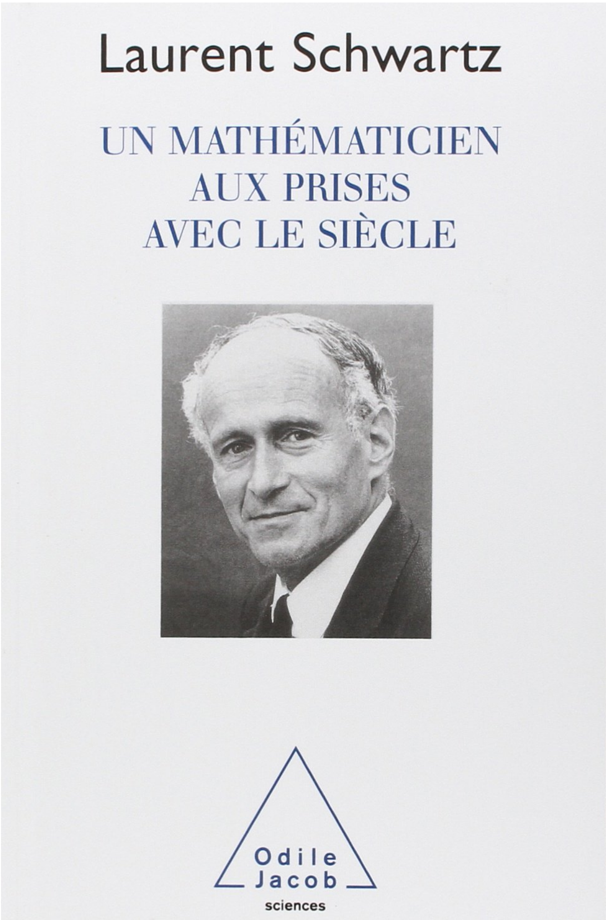
seiche's period:
$\tau \approx$ 2.51 hours

The elevation and transport are C^0!

The internal seiche behaviour is qualitatively similar.

Travelling triangle waves (I)

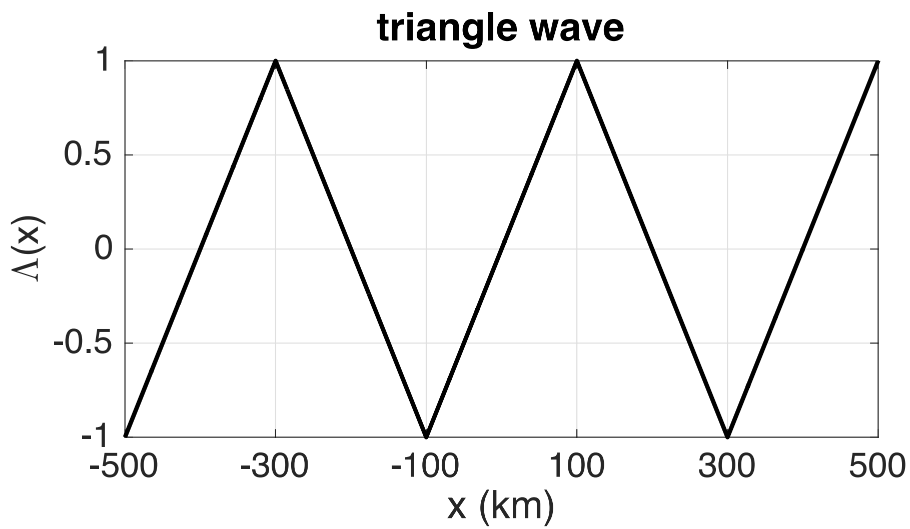
• The presence of **cusps** in the elevation (and transport) field may be interpreted in the framework of the theory of distributions.



• In his memoirs (a must read!), L. Schwartz (1915-2002) sets out one of his motivations for developing the theory of distributions (which earned him the Fields medal), i.e., allowing for the canonical wave equation to have ill-behaved solutions. This is exactly what we are up to here!

• Fortunately, the L^2-norm of the difference between the Fourier series formulation of the solutions and that (very modestly) inspired by Schwartz is... zero!

Travelling triangle waves (II)



Introduce triangle wave

$$\Lambda(x) = 4\left|\frac{1}{4}+\frac{x}{2L}-\left\lfloor\frac{x}{2L}+\frac{3}{4}\right\rfloor\right| - 1$$

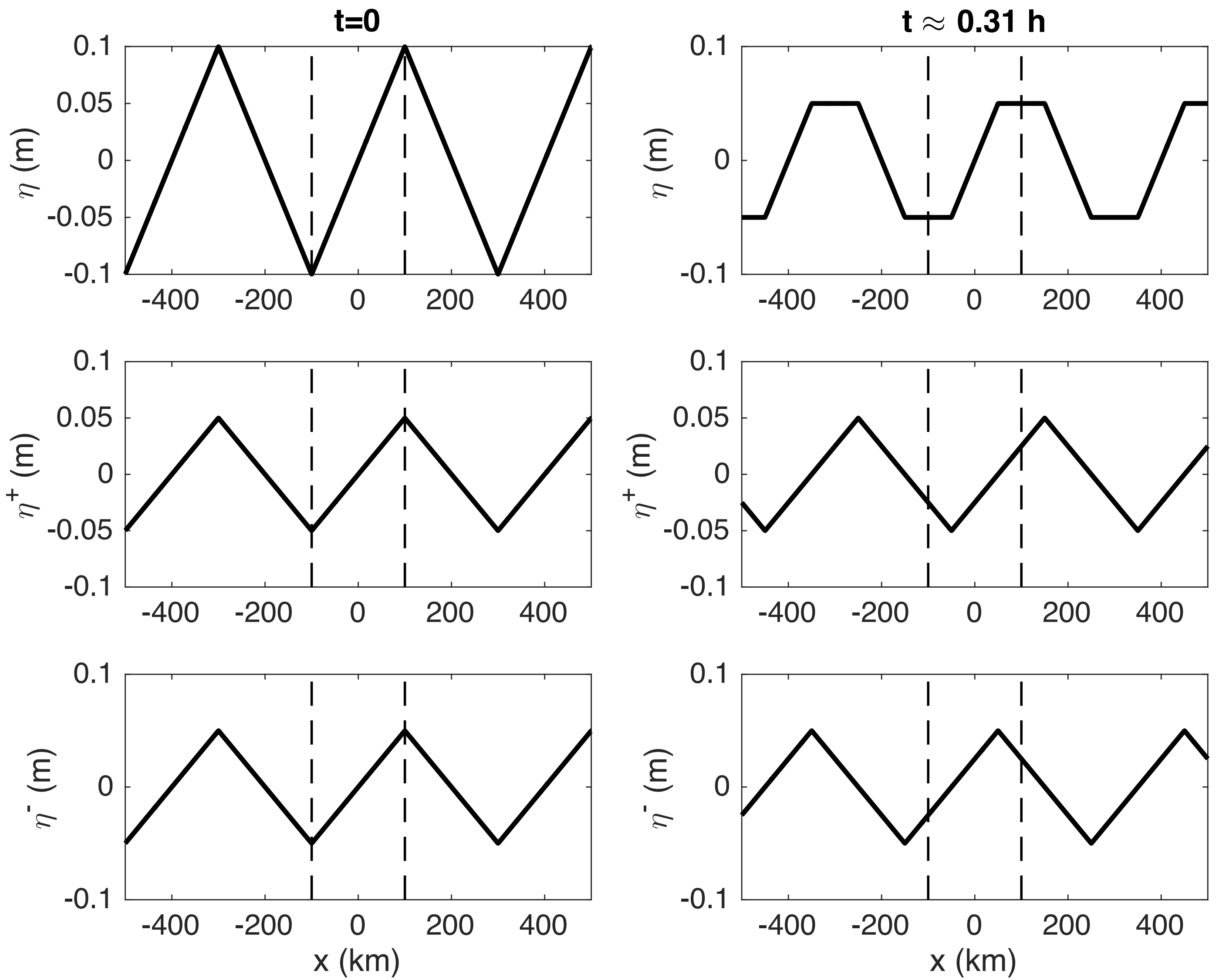
whose wavelength is $2L$.

- Using this C^0 function, the elevation reads

$$\eta(t,x) = \underbrace{\frac{E}{2}\Lambda\left(x-\sqrt{gh}\,t\right)}_{=\eta^+(t)} + \underbrace{\frac{E}{2}\Lambda\left(x+\sqrt{gh}\,t\right)}_{=\eta^-(t)}$$

The sum of η^+ and η^- is a standing wave. A similar formulation holds valid for the transport.

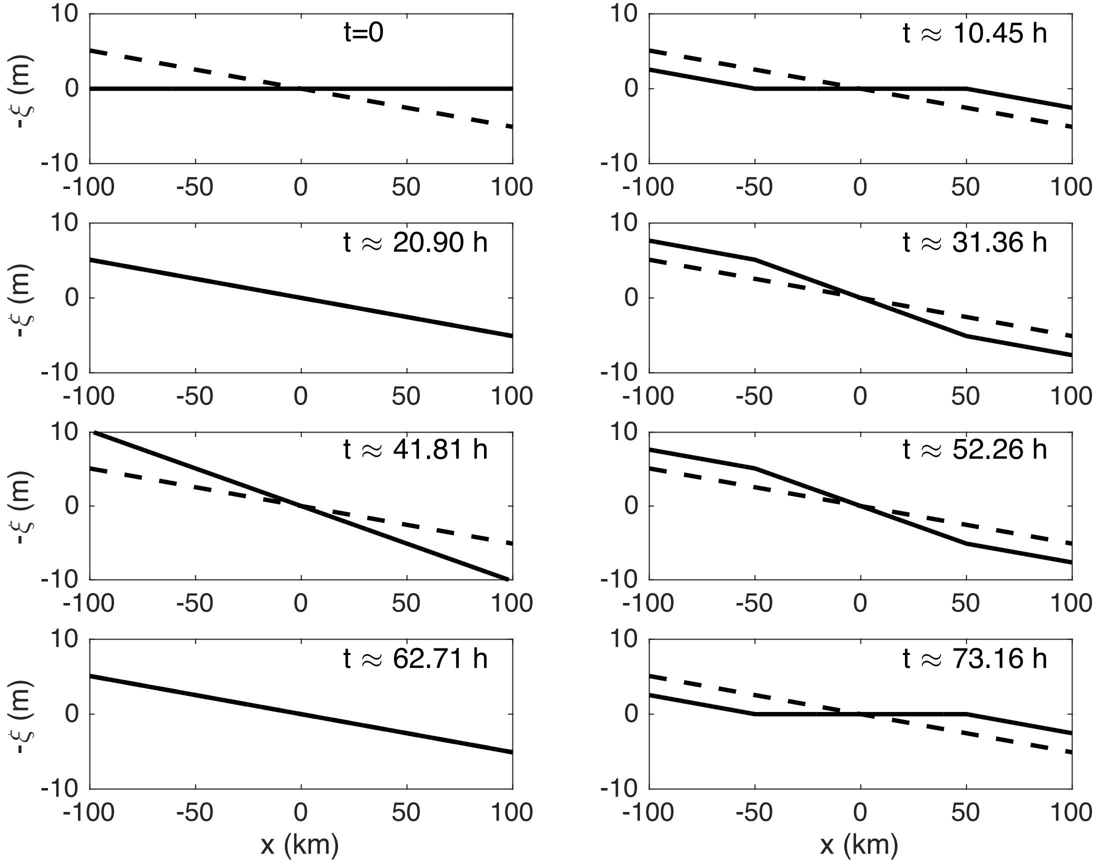
Travelling triangle waves (III)



=> film1

Wind-induced seiching motion

• External or internal seiching motion is triggered by a **"sudden disturbance"** (Baretta-Bekker et al., 1998), e.g., an **abruptly-imposed constant wind stress**.



lake's length: $L = 200$ km

lake's depth: $h = 200$ m

top layer depth: $h^s = 20$ m

relative density diff.: $\varepsilon = 0.01$

wind stress: $\tau^s \approx 0.1$ N/m

internal seiche period ≈ 84 hours

The pycnocline exhibits cusps and oscillates around the steady-state profile (dashed line).

Toward a new schematic depiction of seiches?

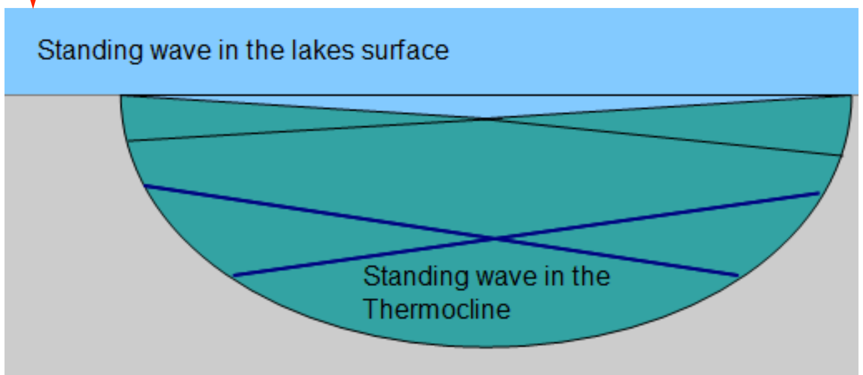
• Many studies of seiches focused on eigenmodes of oscillation (i.e., well-behaved functions), especially the first few modes. However, as was seen above, this paints a somewhat misleading picture of seiching dynamics.

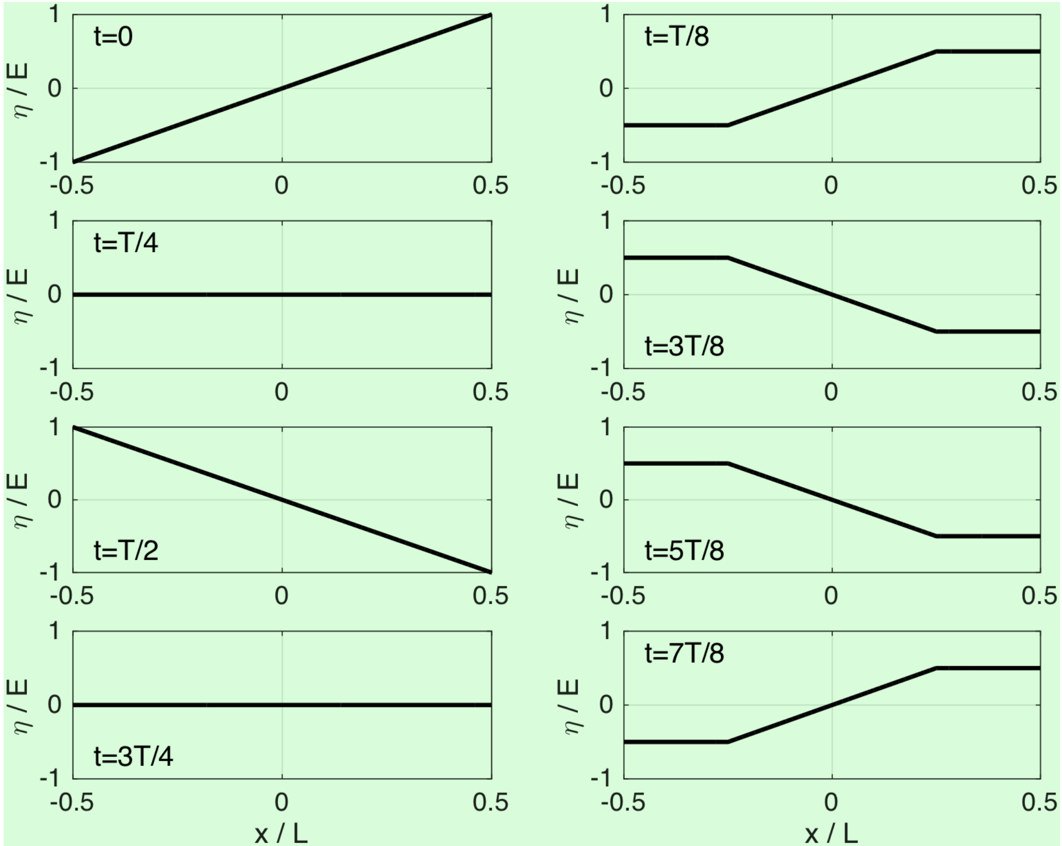
Too complex!

So, what do we do?

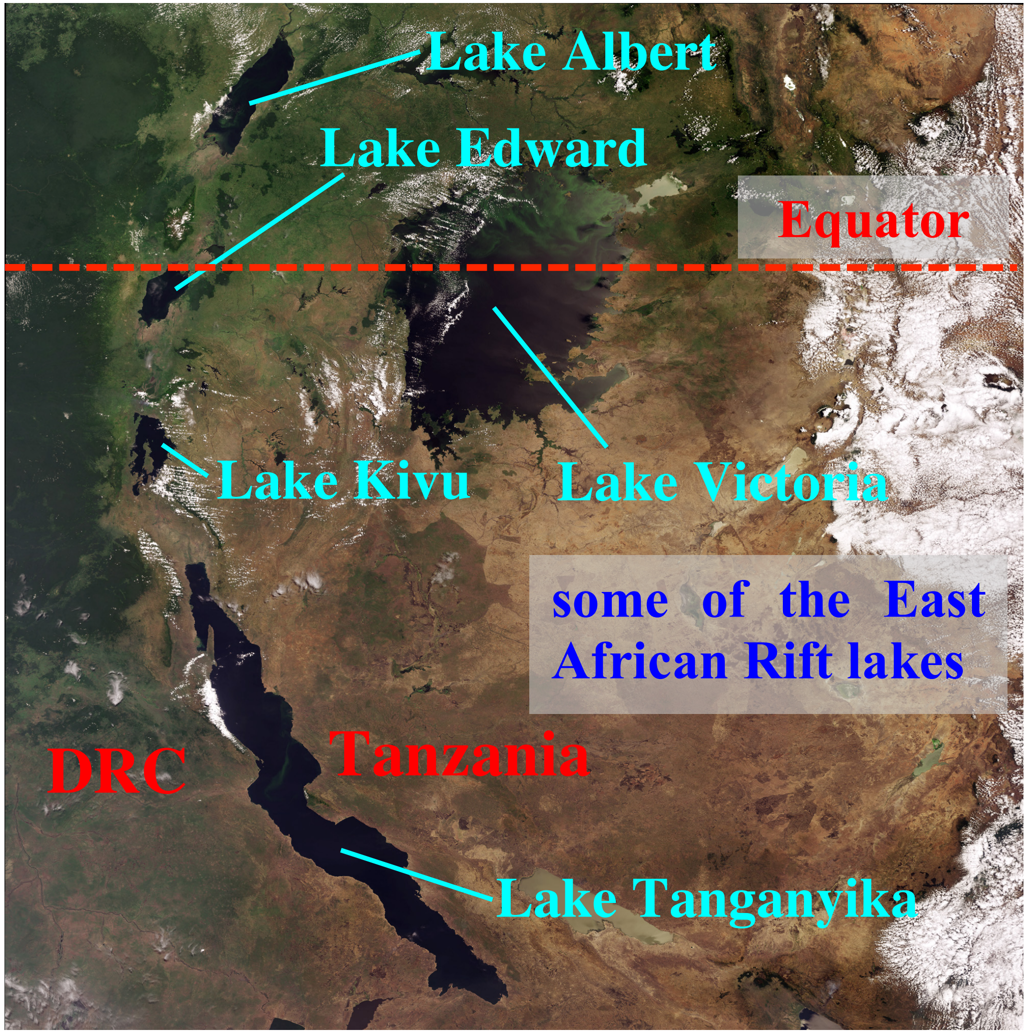
Too simple a cartoon!

Attribution: Frankemann. Licence: CC BY-SA 3.0. Source: lower panel of figure https://commons.wikimedia.org/wiki/File:Illustration_of_the_phenomenon_of_seiches.png (last viewed on 29 August 2024)

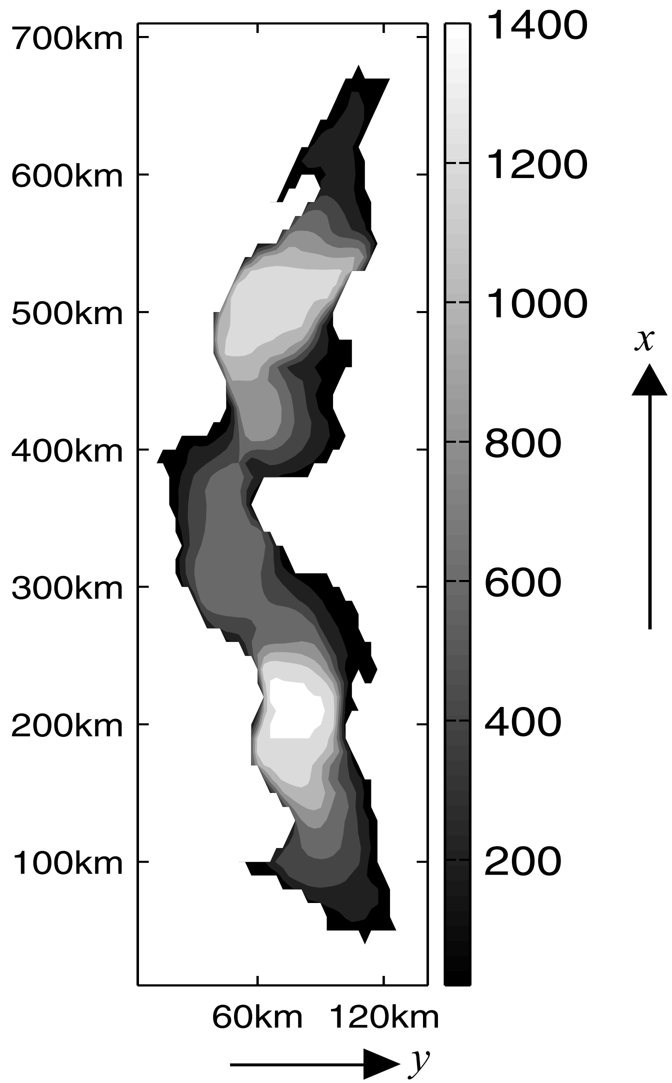




A case study: Lake Tanganyika (I)

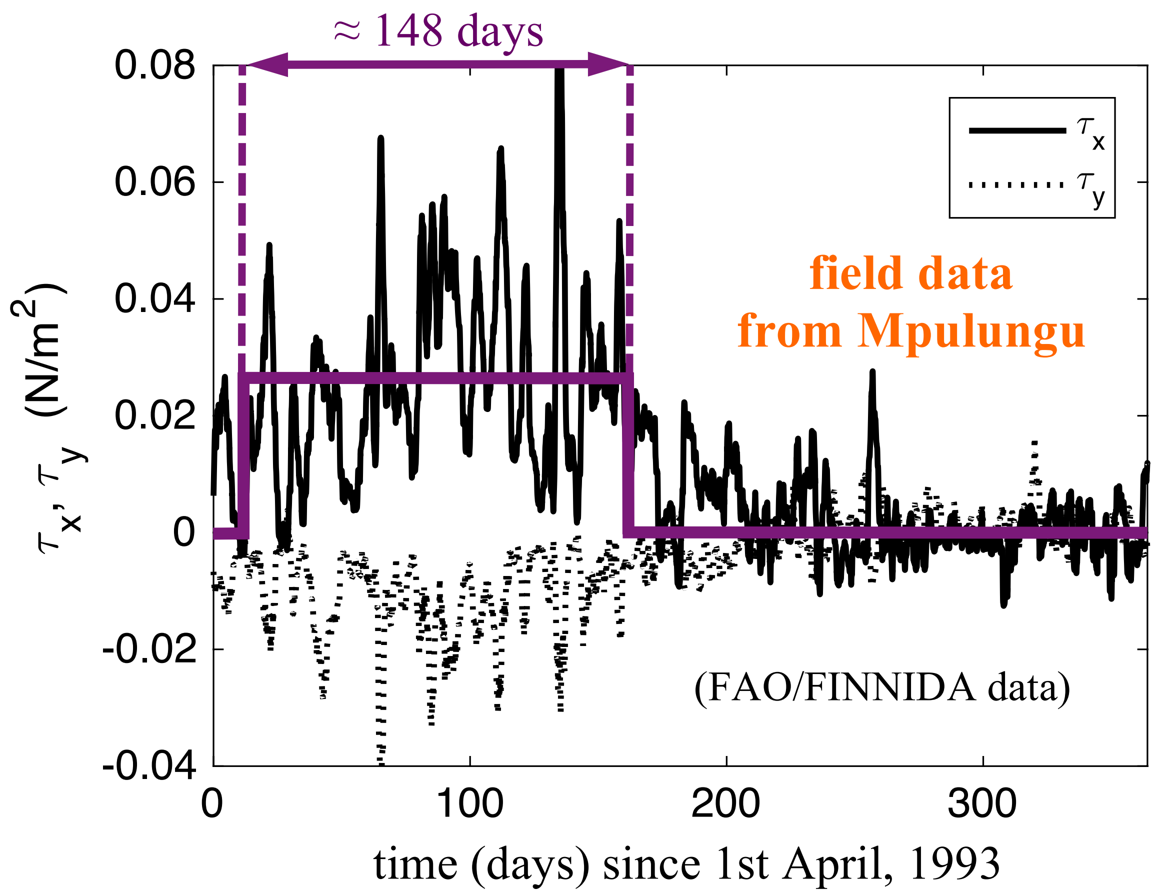


source: http://www.esa.int/spaceinimages/Images/2008/10/Lakes_of_Africa (last viewed on 24 June 2016)



A case study: Lake Tanganyika (II)

- The epilimnion ($h^s \approx 50$ m) is much thinner than the hypolimnion ($h^b >> 10^2$ m) => (non-linear) **reduced-gravity** model ($\eta \approx \varepsilon\xi$).



Two seasons:

1. Dry, windy season (April/May -> Aug./Sept.)
2. Wet, calm season

Large amplitude ($|\xi| \gtrsim 50$ m), wind-induced oscillations of the thermocline.

=> film2

A case study: Lake Tanganyika (III)

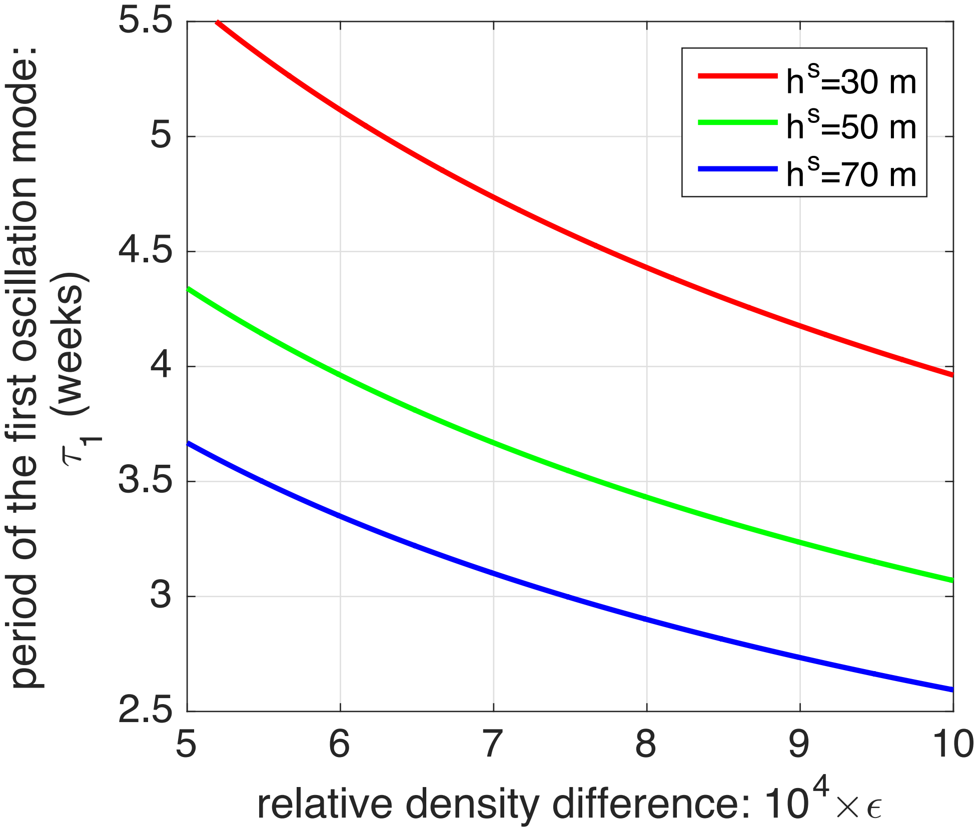
• The **wind stress** is decomposed as follows: $\tau_x = \tau_s + \tau_i$, where τ_s is the **seasona**l wind stress (Morse dash) and τ_i is related to the **intraseasonal** variability. Then, according to the **factor separation method** (Stein and Alpert, *JAS*, 1993), the thermocline response to the surface forcing reads

$$\overbrace{\xi(t,x,y)}^{\text{response to } \tau_x} = \underbrace{\overbrace{\xi_{sno}(t,x,y)+\xi_{so}(t,x,y)}^{\text{response to } \tau_s}}_{=\xi_s(t,x,y)} + \overbrace{\xi_i(t,x,y)}^{\text{response to } \tau_i} + \xi_{syn}(t,x,y)$$

- ξ_{nso}: dry-season non-oscillating response;
- ξ_{nso}: **oscillating** response to the alternation of the seasons (wet, dry, wet, etc.), i.e., **free** oscillations that may be viewed as **seiches**;
- ξ_i: response to the **intraseasonal** variations of the wind stress, i.e., **forced** oscillations (Naithani et al., *GRL*, 2002);
- ξ_{syn}: synergistic term related to the non-linearities of the model.

A case study: Lake Tanganyika (IV)

linearised reduced-gravity model



$$0.5 \times 10^{-3} \leq \varepsilon \leq 10^{-3}$$

(small temperature difference between layers)

Wavelet analysis (Naithani et al., *EFM*, 2003): intraseasonal **variability of the wind stress** has typical **timescales of order 3-4 weeks**.

The **first mode** of thermocline oscillation has a **period of order 3-4 weeks** (see figure opposite).

ξ_i should be seen as a (forced) near-**resonant** response.

Which thermocline oscillation type is the more significant?

A case study: Lake Tanganyika (V)

Signal	Amplitude
$\xi - (\xi_{\mathrm{sno}} + \xi_{\mathrm{i}})$	13.12 m
$\xi - (\xi_{\mathrm{sno}} + \xi_{\mathrm{so}} + \xi_{\mathrm{i}})$	13.4 m
$\xi - \xi_{\mathrm{sno}}$	16.75 m
$\xi - (\xi_{\mathrm{sno}} + \xi_{\mathrm{so}})$	18.39 m
$\xi - \xi_{\mathrm{i}}$	22.74 m
$\xi - \xi_{\mathrm{so}}$	23.27 m
$\xi - (\xi_{\mathrm{so}} + \xi_{\mathrm{i}})$	23.43 m

from Gourgue et al. (2011)

The best approximation of the modelled displacement of the thermocline is $\xi_{\mathrm{sno}} + \xi_i$, i.e., the sum of the **seasonal, non-oscillating** displacement and the **intraseasonal oscillations** (i.e., **forced**, **near-resonant oscillations**). The free oscillations or **seiches** are **less important**. The **synergistic term** cannot be neglected (mean amplitude ≈ 13 m).

Internal waves in three-dimensional models (I)

• In a horizontally unbounded domain of depth h, consider linearised, 3D, hydrostatic, non-dissipative governing equations. Plane wave solutions with angular frequency ω_n and (horizontal) wavenumber vector $\mathbf{k}$ lead to a Sturm-Liouville problem for the vertical velocity amplitude $W_n(\sigma)$:

$$\frac{d^2W_n}{d\sigma^2} + \underbrace{\frac{\delta^2\,\overline{N^2}}{\omega_n^2 - f^2}}_{\substack{=\lambda_n \\ \text{eigenvalue}}} \underbrace{\frac{N^2}{\overline{N^2}}}_{\substack{=\varpi(z) \\ \text{weight}}} W_n = 0 \;, \quad \left[W_n(\sigma)\right]_{\sigma=0,1} = 0 \;, \quad n = 1,2,3...$$

with $\sigma = (z+h)/h$ and $N(z) = \sqrt{-(g/\rho_\bullet)d\rho(z)/dz}$ is the Brunt-Väisälä (or stratification) frequency of the reference (equilibrium) state. The aspect ratio $\delta = h/|\mathbf{k}|^{-1}$ must be << 1, otherwise the hydrostatic balance would not be valid. The angular frequency satisfies $\omega_n^2 = f^2 + \delta^2\,\overline{N^2}/\lambda_n$, an expression similar to the dispersion relation of external Poincaré waves.

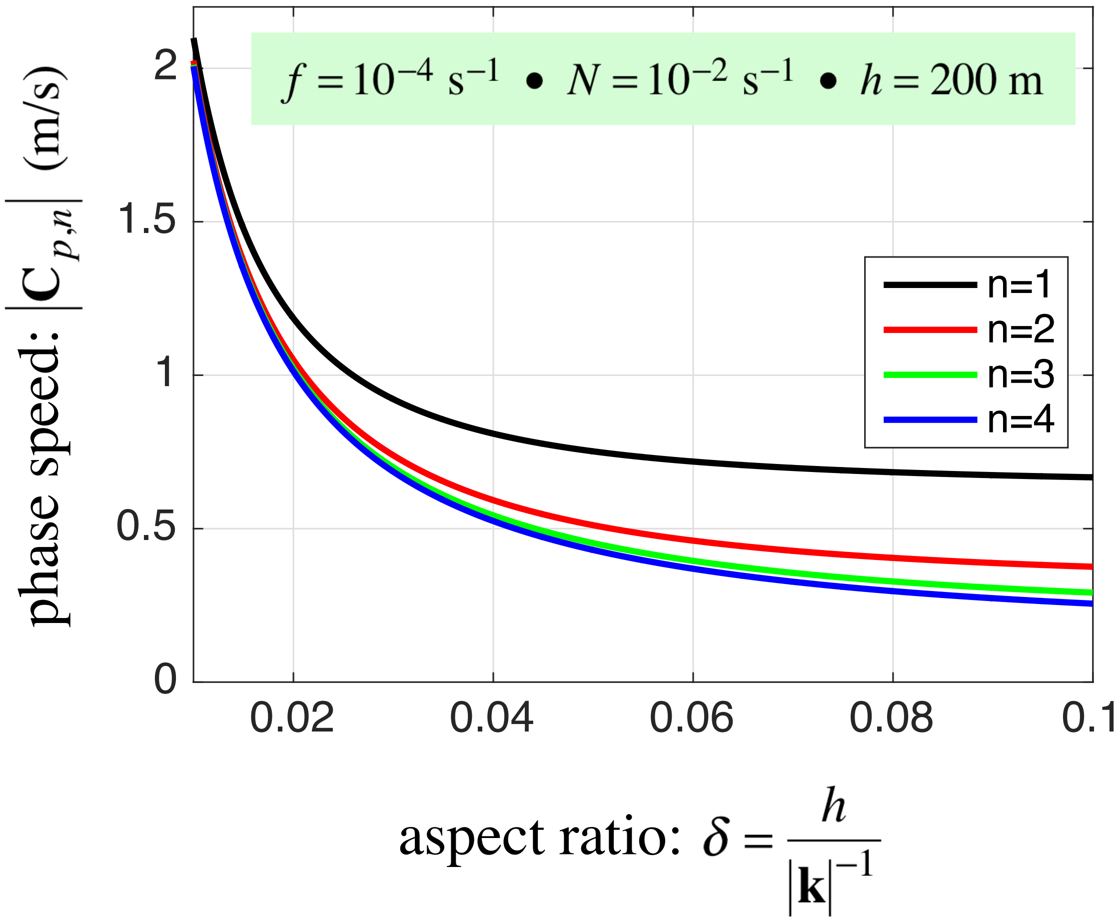
Internal waves in three-dimensional models (II)

- If $N^2 = const.$ (highly idealised case), then $\lambda_n = n\pi$ and the phase speed is

$$\left|\mathbf{C}_{p,n}\right| = \frac{hN}{\lambda_n}\sqrt{1+\frac{\lambda_n^2 f^2}{\delta^2 N^2}}$$

The **phase speed** is $\tilde{>}$ than the **water velocity**!

This should not be disregarded in the analysis of 3D numerical schemes (e.g., Deleersnijder, 1992).



Seiche: a relevant benchmark for mode splitting in a 3D model?

• Most full-fledged **three-dimensional** models rely on some form of a **mode splitting** method, i.e., a different treatment of the (2D) external mode and the (3D) internal one.

• Although the mode splitting concept was first suggested by Gadd (*QJRMS*, 1978), this approach is arguably **not fully mature** (e.g., Ishimwe et al., *OM*, 2024; Ishimwe et al., *JCP*, 2025).

• Preventing a 3D (z- or σ-coordinate) model from **reducing the sharpness of the thermocline** (and possibly destroying it in the long run) remains **uneasy**. Delandmeter et al. (*GSMD*, 2018) somewhat successfully explored the potential of an **adaptive vertical coordinate system** in the Tanganyika.

• Sterckx et al. (*EFM*, 2023) obtained satisfactory numerical results in the Tanganyika **without adaptive meshing**.

Take home message, if any...

• Seiche is a concept (first explored in the 19th century) that is **believed** by many to be **well known**.

• Inexplicably, there are still several **misunderstandings** about seiches.

• Do not invoke "seiche" at every turn, do not rely on turnkey solutions, and, instead, **look into the physics**!

Additional information (i.e., detailed mathematical developments as well as references to published articles and book chapters) may be found in the following working note:

Deleersnijder E., 2024, *Seiches: a tale of misconceptions?*, Working note, Université catholique de Louvain, Louvain-la-Neuve, Belgium, 29 pages, available (on DIAL) at URL http://hdl.handle.net/2078.1/291029

Thank you!