A Preconditioner for the Fast Analysis of Arbitrarily-Shaped Reflective Intelligent Surfaces

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***Abstract*—Analyzing a reflective metasurface (MTS) is usually simplified by considering the MTS as a continuous sheet of impedance. Thanks to this approach, we avoid the need to describe the fine geometry of the MTS with numerous elementary basis functions. Considering the MTS at the sheet impedance level allows to analyze the structure by solving a linear system of equations with a reduced number of unknowns and, when iterative solvers are used, the convergence is usually very good for conventional highly capacitive MTS. However, the condition number of the system and hence the convergence of the solver dramatically deteriorates when considering impedance ranges spanning both capacitive and inductive domains. In this paper, we present an efficient preconditioner which significantly improves the convergence of the GMRES iterative solver when applied to both inductive and capacitive MTSs . We also show that the method can be easily extended to MTSs with arbitrary shapes.**

***Index Terms*—reconfigurable intelligent surfaces, metasurface, method of moments**

I. Introduction

Over the last few years, the ability to exert some control on propagation channels using reconfigurable intelligent surfaces (RISs) has gained a lot of attention [1]. In practice, these surfaces are often implemented as a wide collection of subwavelength patches printed on a grounded substrate. In order to achieve interesting features, a large aperture is often required. This means that the RIS appears as a large structure with very fine details. Consequently, when analyzing a RIS with the Method of Moments (MoM), it is required to finely describe numerous patches with a huge number of elementary basis functions. This multiscale feature calls for huge computational resources which are often unavailable or lead to simulations lasting for days or weeks, which prevents any optimization procedure. In order to alleviate this issue, the RIS is often considered as a continuous sheet of impedance [2] which can be analyzed with a reduced number of basis functions. This being said, the number of unknowns is usually still important (more than one hundred thousand for realistic size MTSs) and one needs to resort to iterative solvers to efficiently solve the MoM system of equations [3]. The latter is usually well conditioned when considering metasurface (MTS) antennas with highly capacitive impedances. In this case, the number of iterations required for an iterative solver is relatively small and the MoM can be efficiently solved provided that the matrix-vector products are rapidly computed. However, when considering reflective MTSs or RISs, it is often necessary to use a sheet impedance spanning both the capacitive and inductive ranges. When the impedance approaches low capacitance or when it enters the inductive domain, the condition number rapidly deteriorates and the need for a preconditioner arises.

Efficient preconditioners developed for the MoM such as the ones based on the Calderón identities [4], [5] cannot be directly applied in this case due to the presence of an additional matrix accounting for the MTS impedance boundary conditions (IBCs). While such a preconditioner has been derived for the case of constant surface impedance in [6], it has not been extended to the case of a varying surface impedance as required for MTS antennas or RISs. Other preconditioners shuch as shielded-block preconditioners [7] require heavy computational ressources when applied to MTSs both at the patch level [8] and at the sheet impedance level.

In this paper, we propose an efficient and low memory MoM-based iterative method. It satisfactorily applies for MTSs at the sheet impedance level with arbitrary shape and range of impedance. The method is based on a previous publication [9] where highly capacitive arbitrarily-shaped MTSs were efficiently analyzed with a regular grid of rooftop basis functions, which enabled fast matrix-vector products. The method is extended to MTSs with arbitrary ranges of impedance thanks to the introduction of a preconditioner which drastically reduce the number of iterations required by the GMRES solver [10] when applied to weakly capacitive or inductive MTSs.

II. Method of Moments

Let us consider an arbitrarily-shaped MTS modeled as penetrable IBCs lying on a grounded dielectric slab of a given permittivity, as depicted in Fig. 1. In the MoM framework, the structure is first described with elementary basis functions. In order to subsequently enable the use of the FFT algorithm, we choose to represent the MTS with a regular grid of rooftop basis functions. More precisely, the whole MTS is enclosed into a larger rectangular grid of regularly disposed rooftops as shown in [9].

The MoM system of equations to be solved reads [9]:

$$\Big[(\mathbb{I}-\mathbb{I}_s)(\mathbf{Z_G}-\mathbf{Z_{IBC}})(\mathbb{I}-\mathbb{I}_\mathbf{s})+\mathbb{I}_\mathbf{s}\Big]\ \mathbf{I}=\mathbf{V} \tag{1}$$

where $\mathbf{Z_G}$ is the substrate matrix, $\mathbf{Z_{IBC}}$ is the impedance boundary condition matrix and $\mathbf{V}$ is the excitation vector. $\mathbb{I}$ is the identity matrix and $\mathbb{I}_s$ is a diagonal matrix with ones on the entries corresponding to the rooftop basis functions not included in the MTS and zeroes everywhere else. Thanks to the translational symmetries of the initial grid of rooftops, the substrate matrix $\mathbf{Z_G}$ has a Toeplitz-block Toeplitz structure. Hence, a multiplication with the substrate matrix appears as a

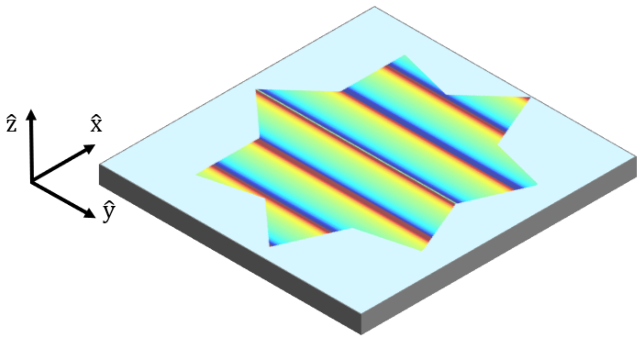

Fig. 1. Schematic illustration of a MTS modeled as penetrable IBCs lying on a grounded dielectric slab.

convolution and can be accelerated using the FFT algorithm. Besides, the impedance boundary conditions matrix $\mathbf{Z_{IBC}}$ is a Gram matrix and thus is very sparse, which means that matrix-vector products with either matrix are fast. The matrix-vector products also come at a low memory cost. Indeed, due to the Toeplitz-block Toeplitz property of the substrate matrix and the sparse nature of the IBC matrix, the filling of both matrices can be achieved with linear complexity. Solving the system of equations (1) gives the equivalent current vector $\mathbf{I}$, from which radiation can be calculated using the appropriate dyadic Green's functions [11].

In this paper, we will consider the MTS as a reflective surface. Therefore, we relate the sheet impedance producing an anomalous reflection to the desired local reflection coefficient [12]:

$$\Gamma(x,y) = \frac{\mathrm{e}^{-\mathrm{j}(k_x^r x + k_y^r y)}}{\mathrm{e}^{-\mathrm{j}(k_x^i x + k_y^i y)}} \tag{2}$$

with $k_x^r = k_0 \sin\theta_r \cos\phi_r$, $k_y^r = k_0 \sin\theta_r \sin\phi_r$, $k_x^i = k_0 \sin\theta_i \cos\phi_i$ and $k_y^i = k_0 \sin\theta_i \sin\phi_i$. (θ_i, ϕ_i) and (θ_r, ϕ_r) are the incident and desired reflection angles, respectively. k_0 is the free-space wavenumber. The opaque impedance producing the anomalous reflection reads:

$$Z_{op,\Psi}(\boldsymbol{r}) = Z_{0,\Psi}(\theta_i, \phi_i)\ \frac{1-\Gamma}{1+\Gamma} \tag{3}$$

where $Z_{0,\Psi}$ is the free-space impedance of the Ψ polarized incident field. Knowing that the opaque impedance can be seen as the addition in parallel of the sheet impedance and the substrate impedance, the sheet impedance can be inferred:

$$Z_{S,\Psi}(\boldsymbol{r}) = \frac{1}{1/Z_{op,\Psi}(\boldsymbol{r}) - 1/Z_{d,\Psi}(\theta_i, \phi_i)} \tag{4}$$

where $Z_{d,\Psi}$ is the substrate impedance considering polarization Ψ.

Besides, the excitation vector will be obtained by projecting the field produced on the grounded slab by the incident field onto the selected rooftop basis functions. Here we will consider a TM polarized incident plane wave. In this case we consider a broadside incidence and a centered Gaussian windowing of the incident field, centered on the MTS, is realized:

$$\mathbf{E}_i(x,y) = G(s_x, s_y)\ \hat{\mathbf{x}} \tag{5}$$

where the standard deviations s_x, s_y are chosen such that the field on the dielectric top surface significantly decays when approaching the edge of the MTS.

The system of equations in (1) is well conditioned when considering conventional highly capacitive MTS antennas. However, the conditioning of the system rapidly deteriorates when the sheet impedance approaches or enters the inductive range [2], [9]. It turns out that the number of iterations required by an iterative solver such as GMRES dramatically increases and so does the computational time when considering weakly capacitive MTSs or RISs. To alleviate this issue, we propose the following preconditioner. First we perform a left multiplication with the conjugate of the initial matrix $\mathbf{A} = (\mathbb{I} - \mathbb{I}_s)(\mathbf{Z_G} - \mathbf{Z_{IBC}})(\mathbb{I} - \mathbb{I}_s) + \mathbb{I}_\mathbf{s}$:

$$\mathbf{A^H A\ I} = \mathbf{A^H V} \tag{6}$$

where H denotes the conjugate transpose operator. Note that a multiplication with $\mathbf{Z_G^H}$ also appears as a convolution and can thus be accelerated with the FFT. Thereafter we apply a block-diagonal preconditioner $\mathbf{D}_b$. Each diagonal block of $\mathbf{D}_b$ is obtained by selecting and inverting the corresponding diagonal block of $\mathbf{A^H A}$. The efficient calculation of $\mathbf{D}_b$ will be the topic of another publication [13]. Consequently, the system of equations solved here with the GRMES [10] iterative solver reads:

$$\mathbf{D_b\ A^H A\ I} = \mathbf{D_b\ A^H V} \tag{7}$$

III. Numerical results

Let us consider a MTS lying on a grounded slab of permittivity $\epsilon_r = 3.66$ and thickness d $= 1.524$ mm. We consider a 24 GHz frequency and a MTS excited with broadside incidence. A square-shaped MTS of size $5\lambda_0 \times 5\lambda_0$ and a circular MTSs of radius $2.5\lambda_0$ are considered, λ_0 being the free-space wavelength. A sheet impedance derived from a local reflection coefficient producing an anomalous reflection at an elevation angle of $\theta = 20°$ is selected. Both MTSs are enclosed in a rectangular grid of rooftop basis functions as done in [9]. The GMRES iterative solver is used to solve both the initial system of equations (1) and the preconditioned system of equations (7). The convergence is shown both for the circular and square-shaped MTSs in Figs. 2 and 3, respectively. In each of the two figures, an inset indicating the shape of the MTS and the range of impedance is provided. One can observe the slow convergence of the initial system of equations, for which the solver has been stopped after 2000 iterations. The preconditioned system however does reach a 10^{-6} norm of the residual after approximately 500 iterations, which takes a little less than 2 minutes using this method on a conventional computer. Besides, approximately the same number of iterations are required for both rectangular and circular domain MTSs. Note that the same initial grid of rooftop basis functions was used for both MTSs. Only matrices

$\mathbb{I}_s$ and $\mathbf{D_b}$, and the excitation vector $\mathbf{V}$ have to be modified to account for the shape of the MTS.

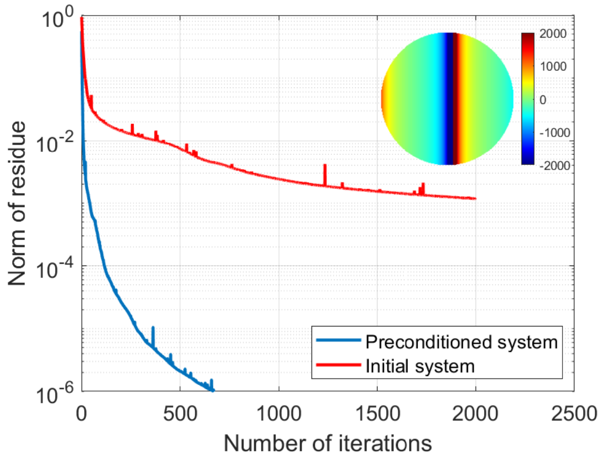

Fig. 2. GMRES convergence for the circular domain MTS. The inset shows the imaginary part of the sheet impedance.

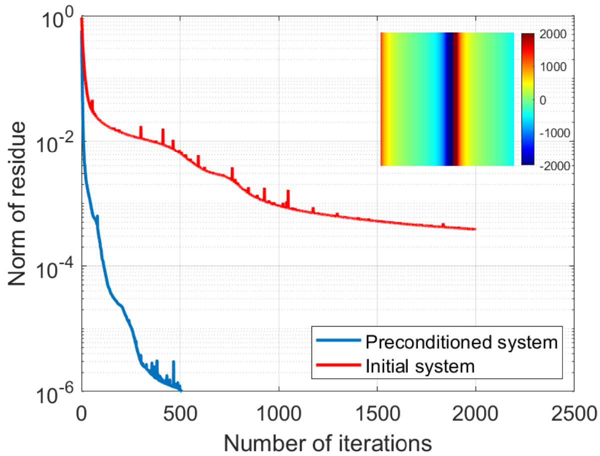

Fig. 3. GMRES convergence for the rectangular domain MTS.

The radiation patterns for both MTSs are also given in Fig. 4, where one can observe the realized anomalous reflection. The specular reflection due to the grounded substrate has been included in the analysis. Subtle differences can be observed between the patterns obtained from circular and rectangular domain MTSs.

IV. Conclusion

The analysis of weakly capacitive MTSs or RISs at the sheet impedance level with the MoM usually requires solving badly conditioned system of equations. Consequently a huge number of iterations is required by iterative solvers. To alleviate this issue, we provide a block diagonal preconditioner which, when combined with the GMRES iterative solver, drastically reduces the required number of iterations. Besides, the MTS is first included in a regular grid of rooftop basis functions which

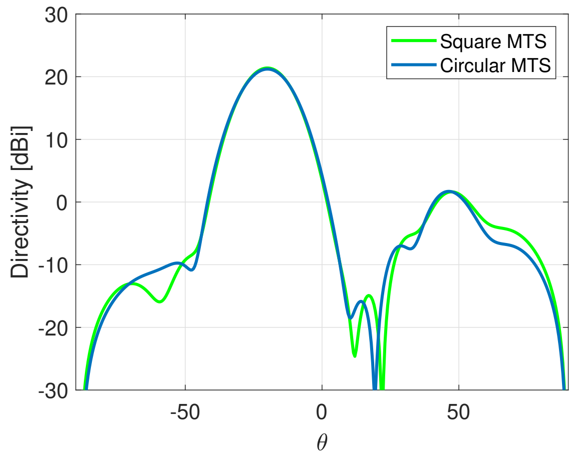

Fig. 4. Radiation patterns of both MTSs, including the specular reflection due to the excitation.

enables the use of the FFT to accelerate matrix-vector products with the substrate matrix. Moreover, by selecting rows and columns of the MoM and IBC matrices, we are able to use this method to efficiently analyse MTSs with arbitrary shapes and an arbitrary range of impedances.

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