

Analysis of wireless device-to-device networks under Rician fading using stochastic geometry

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Abstract—This paper investigates device-to-device (D2D) communications where two nearby users are enabled to communicate directly with each other. A network analysis under Rician fading is proposed to account for a strong line-of-sight (LOS) component typical for this technology. Semi closed-form expressions for the cellular (CC) and underlaid D2D communication outage probabilities are derived relying on stochastic geometry theory. Schemes where D2D communications reuse the uplink (UL) or the downlink (DL) CC bands are compared. The choice between them is shown to strongly depend on the base stations (BS) and user equipments (UE) transmit powers.

Index Terms—Rician fading, Device-to-device, Stochastic geometry

I. INTRODUCTION

A. Background

In the history of wireless networks analysis, simple and deterministic models such as the Wyner or the grid-based models were considered for a long time as close enough to the reality [1]. Those models were later shown not to correctly take into account the topology of nowadays networks by making too simple or idealistic assumptions [2]. In addition, with the drastic increase of wireless links in the past few years, the signal-to-interference-plus-noise-ratio (SINR) has become the reference variable for the design of such networks. This is where stochastic geometry (SG) jumped in as one of the most promising tools for the set up of closed-form expressions including those based on the SINR. This powerful tool has enabled researchers to capture the randomness of emerging networks by averaging over all possible geometrical patterns and to define and compute macroscopic properties of large-scale networks.

This paper aims to study the integration of D2D communications into a conventional CC network, by means of SG. D2D communications are seen as a promising additional technological features of 5G. The main idea is to exploit the spatial proximity between CC UEs to enable them to communicate directly with each other without passing through the serving BS. This hopefully results in increased throughputs and reduced latencies, and provides the D2D UEs with

a guaranteed quality of service. D2D links generally consist of single-hop links as the transmission distances are very short.

Thanks to the mathematical tractability provided by Rayleigh fading, most prior works related to D2D communications is using that fading model. However, as the D2D technology mainly concerns short-range transmissions, the need to account for LOS components is meaningful.

B. Related Works

A general introduction to the analysis of wireless networks by means of SG can be found in [3]–[5] where all fundamental mathematical tools are covered.

While in the vast majority of cases, D2D communications are said to reuse the UL resources [6]–[11], some authors opted for the reuse of DL resources instead [12]–[14]. Unfortunately, the choice of the underlaid resources (either UL or DL) is still quite unclear. The authors of [15] already provide a comparison between the two cases where interference from CC network in the DL case, which is correlated over different time slots, was shown to promote the D2D link and to permit a lower outage probability. This conclusion was however drawn for equivalent BS and CC transmit powers which might seem unrealistic.

As already said, many publications went for Rayleigh fading for its mathematical tractability [6]–[11]. Several models have however started to account for the typical strong LOS component in D2D links [12–16]. Particularly, the authors of [17] already investigated Rician fading for a similar setup but only considered interferers being at any distance from the source of the link of interest.

C. Contributions

Firstly, this model intends to provide sufficient tools to decide whether to use the UL or DL band for D2D communications and under what conditions.

Secondly, in order to account for inter-cell interference, our model tries to more conveniently consider the interferers as further from the receiver than the source of the useful link, resulting in more realistic and lower outage probability. This hypothesis certainly makes sense for interfering BSs in the DL part of the CC link when intra-cell CC and D2D communications are considered under closest BS association policy. For the UL part of the CC link, this hypothesis makes sense if BSs communicate to ensure that condition. This paper somehow extends the model available in [18] for Rician fading where a similar approach was followed for Rayleigh fading. Rician fading is here considered due to its easily and quickly explainable physical meaning but the proposed model could be extended to generalized fading models following the approach of [16].

Thirdly, unlike [19-20], our model analyzes the impact of a D2D network overlaying the CC network.

D. Structure of the paper

In Section II, the variables and assumptions governing the model are listed. In Section III, mathematical expressions for the CC and underlaid D2D outage probabilities are derived. In Section IV, the analytical results are compared to MC simulation. Finally, Section V proposes several future research directions.

II. SYSTEM MODEL

A. Network model

In this model, a CC UE denoted as m initially tries to communicate with a CC UE m' of the same cell via their closest BS n . D2D communications are then supposed to take place either in the UL band (case 1/C1) or in the DL band (case 2/C2) whereas the cellular communications use both bands as depicted in Figures 1 and 2. This creates additional interference on the CC links. Also, the frequencies of the UL and the DL are considered to be separate bands and each UL or DL frequency can be used twice in each cell: once for a CC link and once for a D2D link.

All co-channel (coc) CC UEs and D2D UEs are respectively distributed following two independent homogeneous Poisson point processes (HPPP) of intensities Λ_{cc} and Λ_{d2d} defined in $\mathbb{R}^2$. Co-channel BSs are randomly chosen among overall BSs which are distributed following a HPPP of intensity Λ_b thereby forming a restricted PPP of intensity Λ_{bs} . CC UEs, D2D UEs and BSs respectively use transmit powers P_{cc} , P_{d2d} and P_{bs} . The D2D receiver is uniformly located in a circle of radius R_{max} around the D2D transmitter. This relies on the same idea as the cut-off frequency mode selection method presented in [21]. Unlike [18], only single-hop D2D communications are considered in this paper owing to mathematical complexification.

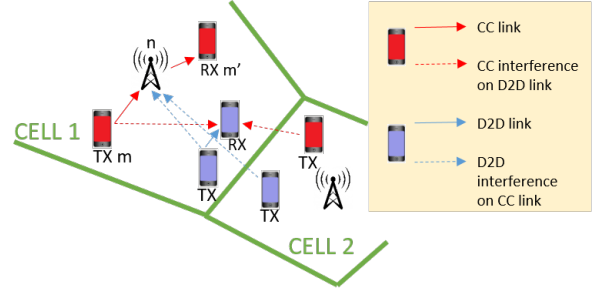


Fig. 1. Case 1 (C1): D2D transmissions in the UL band

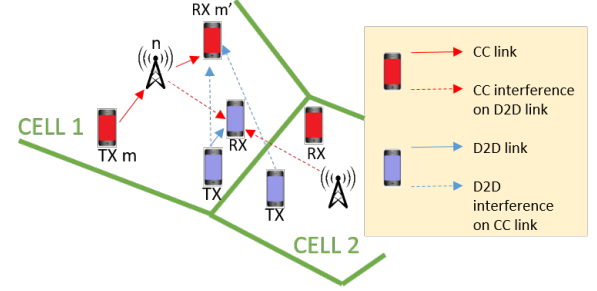


Fig. 2. Case 2 (C2): D2D transmissions in the DL band

TABLE I
INTERFERENCE SCHEME

			Interferers					
			CC UEs		D2D UEs		BSs	
			IaC	IrC	IaC	IrC	IaC	IrC
C1	CC	UL		H1	H2			
		DL						H1
	D2D UL			H2		H1		
C2	CC	UL		H1				
		DL			H2			H1
	D2D DL					H1		H2

Two different hypotheses are considered to match as much as possible the different types of communications:

- **Hypothesis 1 (H1):** interferers are at a distance from the RX which is at least the distance between the useful TX and the RX (e.g. when the interferers produce both intra-cell (IaC) and inter-cell (IrC) interference).
- **Hypothesis 2 (H2):** interferers can be at any distance from the receiver (e.g. when the interferers produce only IrC interference).

A summary of the interference structure is provided in Table I for single-hop D2D communications. The hypothesis that best matches a given source of interference is indicated if it is indeed a source of interference and left empty otherwise.

As shown in [22], the PDF of the distance r_{ub} between a UE and its nearest BS when the BSs are distributed

following a HPPP of density Λ_b in $\mathbb{R}^2$ is given by:

$$f_{R_{ub}}(r_{ub}) = 2\Lambda_b \pi r_{ub} \exp(-\Lambda_b \pi r_{ub}^2). \quad (1)$$

In [9], the PDF of the distance r_{d2d} between two D2D UEs involved in a single-hop D2D link is assumed to be given by:

$$f_{R_{d2d}}(r_{d2d}) = \frac{2r_{d2d}}{R_{max}^2} \quad \text{for } 0 \leq r_{d2d} \leq R_{max}. \quad (2)$$

B. Fading model

As already mentioned, the useful link is here assumed to be affected by Rician fading power gain g which is described by a K-factor. This factor is the ratio between the power in the direct path and the power in the scattered paths. Assuming $\mathbb{E}\{g\} = 1$, the CDF of g is as follows:

$$F_G(g) = 1 - \sum_{k=0}^{\infty} \sum_{j=0}^k \mathcal{A}_{jk} ((K+1)g)^{k-j} e^{-(K+1)g} \quad (3)$$

where

$$\mathcal{A}_{jk} = \frac{K^k}{(k!)^2} e^{-K} j! \binom{k}{j}.$$

Rayleigh fading is however assumed for interferers.

C. Signal-to-interference-plus-noise-ratio model

The SINR of the link between transceivers x and y is defined as:

$$\gamma_{x,y} = \frac{g_{x,y} P_x \lambda r_{x,y}^{-\alpha}}{W + \sum_{\substack{\text{int} \in \Phi_{\text{int}} \\ \text{int} \neq x}} g_{\text{int},y} P_{\text{int}} \lambda r_{\text{int},y}^{-\alpha}} = \frac{g_{x,y} P_x \lambda r_{x,y}^{-\alpha}}{W + I} \quad (4)$$

where $g_{x,y}$ is the fading power gain between transceivers x and y , $r_{x,y}$ is the distance between transceivers x and y , P_x is the transmitted power of transceiver x , Φ_{int} is the point process (PP) of the interferers (int), α is the pathloss coefficient, λ is the frequency dependent pathloss and W is the noise power.

III. ANALYTICAL RESULTS

In the mathematical developments, the following notations are used: "link" refers to the link of interest and "int" to the interferers, whether they are related to cellular nodes (BS or CC) or to D2D nodes. The coverage probability is defined as the probability of the signal-to-interference ratio (SIR) to exceed a certain threshold ξ . The SIR is used instead of the SINR to ease the computations but the reader has to keep in mind that the noise might still have a negative impact if strong enough. D2D communications are initially disregarded. Using expressions (3) and (4), under Rician fading on

the studied link, the coverage probability for link t ($t \in \{\text{UL}, \text{DL}\}$) is given by [17]:

$$P(\gamma_{x,y}^t > \xi; r_{x,y} | \alpha) = \sum_{k=0}^{\infty} \sum_{j=0}^k \mathcal{A}_{jk} (Tr_{x,y}^{\alpha})^{k-j} \times (-1)^{k-j} \frac{\partial^{k-j} \mathcal{L}_I^1(s, \alpha)}{\partial s^{k-j}} \Big|_{s=Tr_{x,y}^{\alpha}} \quad (5)$$

where

$$T = \frac{\xi(K+1)}{\lambda P_{\text{link}}} \quad \text{and} \quad \mathcal{L}_I^i(s, \alpha) = \mathbb{E}[\exp(-sI)]$$

(Laplace transform under hypothesis i).

Then, D2D communications underlying cellular network can be regarded as a 2-tier heterogeneous network [18]. In other words, two independent sources giving respectively rise to interference I_1 and I_2 are considered. By definition, I_1 represents the source of interference under hypothesis 1 and I_2 the source of interference under hypothesis 2. The coverage probability at the distance $r_{x,y}$ from a transmitting node x of the v^{th} tier ($v \in \{\text{CC}, \text{BS}, \text{D2D}\}$) in link t ($t \in \{\text{UL}, \text{DL}, \text{D2D}\}$) is given by:

$$P(\gamma_{v,x,y}^t > \xi; r_{x,y} | \alpha) = \sum_{k=0}^{\infty} \sum_{j=0}^k \sum_{p=0}^{k-j} \mathcal{A}_{jk} \times (Tr_{x,y}^{\alpha})^{k-j} \binom{k-j}{p} (-1)^{k-j} \frac{\partial^p \mathcal{L}_{I_1}^1(s, \alpha)}{\partial s^p} \Big|_{Tr_{x,y}^{\alpha}} \times \frac{\partial^{k-j-p} \mathcal{L}_{I_2}^2(s, \alpha)}{\partial s^{k-j-p}} \Big|_{Tr_{x,y}^{\alpha}}. \quad (6)$$

The computations specific to the Laplace transform $\mathcal{L}_I^1(\cdot)$ of the interference for hypothesis 1 are inspired by [23] and for hypothesis 2 by [3]. Interferers are here supposed to follow a PPP Φ_{int} of intensity Λ_{int} .

Let w be the integration variable. Under hypothesis 1, the Laplace transform of the interference is given by [9]:

$$\mathcal{L}_I^1(s, \alpha) = \exp\left(-2\pi\Lambda_{\text{int}} \left[\frac{1}{2}w^2 {}_2F_1\left(1, \frac{2}{\alpha}; \frac{\alpha+2}{\alpha}; \frac{-w^\alpha}{sP_{\text{int}}\lambda}\right)\right]_{w=r_{x,y}}^{w=\infty}\right). \quad (7)$$

where ${}_2F_1(\cdot)$ is the hypergeometric function.

Under hypothesis 2, the Laplace transform of the interference is given by [24]:

$$\mathcal{L}_I^2(s, \alpha) = \exp(-\mathcal{G}(\text{int})s^\delta) \quad (8)$$

where

$$\mathcal{G}(\text{int}) = \Lambda_{\text{int}} \pi \lambda^\delta P_{\text{int}}^\delta \text{sinc}^{-1}(\pi\delta) \quad \text{and} \quad \delta = \frac{2}{\alpha}.$$

For the unaffected part of the cellular link (UL for C1 and DL for C2), the expectation of the coverage probability at a distance $r_{x,y}$ from a transmitting node x for $\alpha = 4$ is respectively given by (13) and (14) for the UL and the DL. For the affected part of the cellular link, the expectation of the coverage probability at a distance $r_{x,y}$ from a transmitting node x of the v^{th} tier ($v \in \{CC, BS\}$) in the link t ($t \in \{UL, DL\}$) for $\alpha = 4$ is given by (12) where $(a)_b$ is the Pochhammer symbol. The expectation of the coverage probability at a distance $r_{x,y}$ from a transmitting node of the D2D tier ($v = D2D$) to another D2D UE ($t = D2D$) for $\alpha = 4$ is given by Equation 15. Those expressions can be obtained by means of expressions (1), (2) and (5)-(8). The values for "int₁" and "int₂" are respectively associated with I_1 and I_2 which depend on the studied link and which can be deduced from Table I. Note that we will focus on $\alpha = 4$ in our mathematical developments. This value of the pathloss coefficient is not aberrant as it generally considered to be close to 3.5 in urban environments. Other values could also be considered. All posed functions are available in Table 2.

The expectation of the outage probability of the D2D communication between a D2D UE TX and a D2D UE RX for $\alpha = 4$ is given by

$$\mathbb{E}[P_{D2D,out}] = 1 - \mathbb{E}_R [P(\gamma_{D2D,TX,RX}^{D2D} > \xi; r_{TX,RX} | \alpha = 4)] . \quad (9)$$

In case 1, the expectation of the outage probability of the CC communication between CC UE m and CC UE m' via their closest BS n for $\alpha = 4$ is given by :

$$\mathbb{E}[P_{CC,out}] = 1 - \mathbb{E}_R [P(\gamma_{CC,m,n}^{UL} > \xi; r_{m,n} | \alpha = 4)] \mathbb{E}_R [P(\gamma_{n,m'}^{DL} > \xi; r_{n,m'} | \alpha = 4)] . \quad (10)$$

In case 2, the expectation of the outage probability of the CC communication between CC UE m and CC UE m' via their closest BS n for $\alpha = 4$ is given by :

$$\mathbb{E}[P_{CC,out}] = 1 - \mathbb{E}_R [P(\gamma_{BS,n,m'}^{DL} > \xi; r_{n,m'} | \alpha = 4)] \mathbb{E}_R [P(\gamma_{m,n}^{UL} > \xi; r_{m,n} | \alpha = 4)] . \quad (11)$$

IV. SIMULATION RESULTS

A. Discussion

The following results were obtained using the parameters in Table II. Sixth order Taylor approximations were used to simplify higher-order derivatives computations leading to a small lack of precision. The final results for the average CC outage probabilities after the integration of D2D communications are shown in Figures 3 (C1) and 4 (C2). To enable the reader to see the impact on conventional cellular networks, the obtained results for both cases are compared to the case without D2D

communications. The results are also validated using Monte Carlo simulations.

TABLE II
SIMULATION PARAMETERS.

Parameter	Symbol	Value
BS density	Λ_b	2/km ²
Coc CC UE/D2D UE/BS density	$\Lambda_{cc}/\Lambda_{d2d}/\Lambda_{bs}$	1.27/km ²
BS transmit power	P_{bs}	40W
CC/D2D UE transmit power	P_{cc} / P_{d2d}	0.1W
pathloss coefficient	α	4
max. D2D link distance	R_{max}	0.1km

In case 1, D2D UEs are interfering with the considered CC UE for the UL part of the CC communication between the considered user and its associated BS. As CC UEs and D2D UEs are using the same transmit power (0.1W), the added interference on the cellular link is non-negligible, hence a higher outage probability.

In case 2, D2D UEs are competing with the associated BS for the DL part of the CC communication between the associated BS and the target CC UE. As the transmit power of the BS (40W) is strictly higher than the one of D2D UEs (0.1W), the added interference is negligible, hence a similar outage probability.

The final results for the average D2D outage probabilities are depicted in Figures 5 (C1) and 6 (C2). The reciprocal of the discussion on CC communications above can be used to justify a higher outage probability in C2 for D2D communications when compared to C1. The outage probabilities values are proven to be highly acceptable especially if C1 is chosen. This is however the case for a maximum D2D link distance of 100m as shown in Table II. Increasing this maximum distance can lead to a complete outage especially for C2.

This brings us to a general trade-off between D2D communications performances and CC communications performances regarding the fixed parameters values. On the one hand, it is preferable for the D2D communications to be operated in the DL band for an optimal average CC outage probability. On the other hand, it is better for the D2D communications to take place in the UL band for an optimal average D2D outage probability.

B. Model extension

In some cases, the Rayleigh fading model is inconsistent for the studied interferers as there might also be a strong LOS component if they are close enough. It is even more incoherent knowing that the useful link was supposed to be affected by Rician fading.

Opting for the Rician fading model instead makes it incoherent for far interferers. However, if such a choice is

TABLE III
POSED FUNCTIONS

$\mathcal{B}(\text{int}) = \pi \Lambda_{\text{int}} \sqrt{P_{\text{int}} \lambda}$	$\mathcal{C}(\text{int}) = \frac{1}{\sqrt{P_{\text{int}} \lambda}}$
$\mathcal{E}(\text{int}, \text{link}) = \Lambda_{\text{int}} \sqrt{\frac{\xi(K+1)P_{\text{int}}}{P_{\text{link}}}} \arctan \left(\sqrt{\frac{\xi(K+1)P_{\text{int}}}{P_{\text{link}}}} \right)$	$b = 2i' + 2l - 4p + 1 + 4(k - j)$
$\mathcal{D}_{iljk}(\text{int}, \text{link}) = \mathcal{A}_{jk}(-1)^{k-j+i} \frac{\left(-\pi \Lambda_{\text{int}} \sqrt{\lambda P_{\text{int}}} \arctan \left(\sqrt{\frac{\xi(K+1)P_{\text{int}}}{P_{\text{link}}}} \right) \right)^i}{i!(l-i)!} \left(\frac{\xi(K+1)}{\lambda P_{\text{link}}} \right)^{\frac{i}{2}+k-j}$	
$\mathcal{F}_{li}(\text{int}, \text{link}, x, y, n) = \frac{\partial^n \left(-\mathcal{B}(\text{int}) \sqrt{s} \arctan \left(\frac{\sqrt{s}}{\mathcal{C}(\text{int}) r_{x,y}^2} \right) \right)}{\partial s^n} \Big _{s=\frac{\xi(K+1)}{\lambda P_{\text{link}}} r_{x,y}^4}$	
$\mathcal{I}_{iljk}(\text{int}, \text{link}, x, y) = \int_0^\infty \mathcal{F}_{li}(\text{int}, \text{link}, x, y, a) \exp \left(-\pi (\mathcal{E}(\text{int}, \text{link}) + \Lambda_b) r_{x,y}^2 \right) r_{x,y}^{2i+4a+1} dr_{x,y}$	
$\mathcal{J}_{i'l'ipjk}(v, \text{int}_1, \text{int}_2) = \mathcal{A}_{jk} \binom{k-j}{p} (-1)^{k-j+i+i'} \frac{(-\mathcal{G}(\text{int}_1))^l \left(1 - \frac{i}{2} + \frac{l}{2} - p \right)_p}{i!(l-i)!i'!(l'-i')!} \times \left(\frac{\xi(K+1)}{\lambda P_v} \right)^{k-j+\frac{l}{2}-p+\frac{i'}{2}} \left(-\pi \Lambda_{\text{int}_2} \arctan \left(\sqrt{\frac{\xi(K+1)P_{\text{int}_2}}{P_v}} \right) \right)^{i'}$	

$$\mathbb{E}_R [P(\gamma_{v,x,y}^t > \xi; r_{x,y} | \alpha = 4)] = 2\pi \Lambda_b \sum_{k=0}^{\infty} \sum_{j=0}^k \sum_{p=0}^{k-j} \sum_{l=0}^p \sum_{i=0}^l \sum_{l'=0}^{k-j-p} \sum_{i'=0}^{l'} \mathcal{J}_{i'l'ipjk}(v, \text{int}_1, \text{int}_2) \times \int_0^\infty \exp \left(-r_{x,y}^2 \left(\mathcal{E}(\text{int}_2, v) + \mathcal{G}(\text{int}_1) \sqrt{\frac{\xi(K+1)}{P_v \lambda}} + \pi \Lambda_b \right) \right) \mathcal{F}_{l'i'}(\text{int}_2, v, x, y, k-j-p) r_{x,y}^b dr_{x,y}. \quad (12)$$

$$\mathbb{E}_R [P(\gamma_{x,y}^{UL} > \xi; r_{x,y} | \alpha = 4)] = \sum_{k=0}^{\infty} \sum_{j=0}^k \sum_{l=0}^{k-j} \sum_{i=0}^l 2\pi \Lambda_b \mathcal{D}_{iljk}(CC, CC) \mathcal{I}_{iljk}(CC, CC, x, y). \quad (13)$$

$$\mathbb{E}_R [P(\gamma_{x,y}^{DL} > \xi; r_{x,y} | \alpha = 4)] = \sum_{k=0}^{\infty} \sum_{j=0}^k \sum_{l=0}^{k-j} \sum_{i=0}^l 2\pi \Lambda_b \mathcal{D}_{iljk}(BS, BS) \mathcal{I}_{iljk}(BS, BS, x, y). \quad (14)$$

$$\mathbb{E}_R [P(\gamma_{D2D,x,y}^{D2D} > \xi; r_{x,y} | \alpha = 4)] = \frac{2}{R_{max}^2} \sum_{k=0}^{\infty} \sum_{j=0}^k \sum_{p=0}^{k-j} \sum_{l=0}^p \sum_{i=0}^l \sum_{l'=0}^{k-j-p} \sum_{i'=0}^{l'} \mathcal{J}_{i'l'ipjk}(D2D, \text{int}_1, \text{int}_2) \times \int_0^{R_{max}} \exp \left(-r_{x,y}^2 \left(\mathcal{E}(\text{int}_2, v) + \mathcal{G}(\text{int}_1) \sqrt{\frac{\xi(K+1)}{P_v \lambda}} + \pi \Lambda_b \right) \right) \mathcal{F}_{l'i'}(\text{int}_2, v, x, y, k-j-p) r_{x,y}^b dr_{x,y}. \quad (15)$$

Average CC outage probability as a function of the SIR threshold (Case 1)

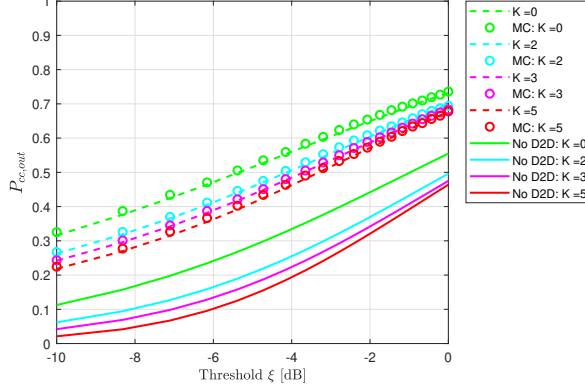


Fig. 3. CC outage probability with D2D communications (case 1).

Average CC outage probability as a function of the SIR threshold (Case 2)

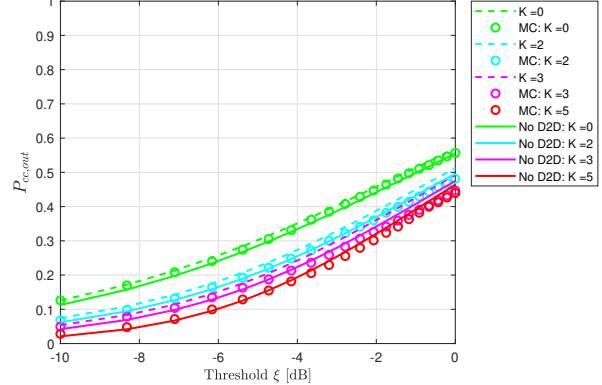


Fig. 4. CC outage probability with D2D communications (case 2).

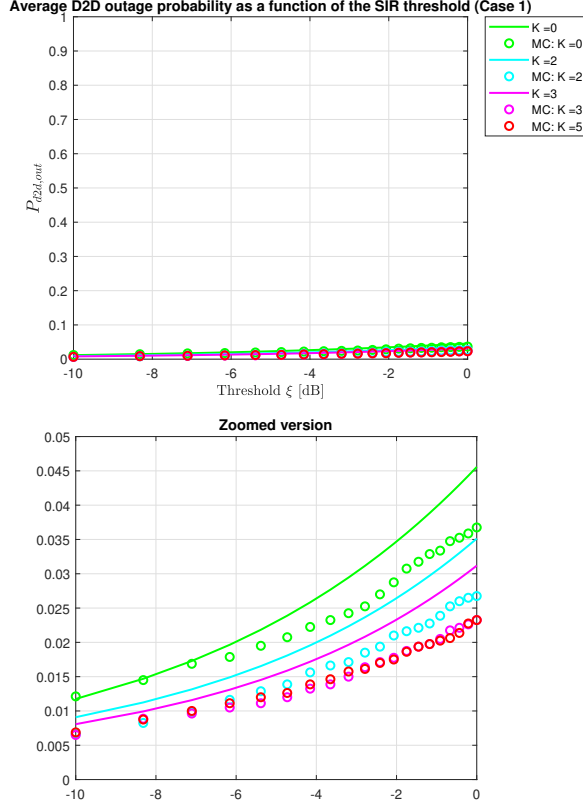


Fig. 5. D2D outage probability (case 1).

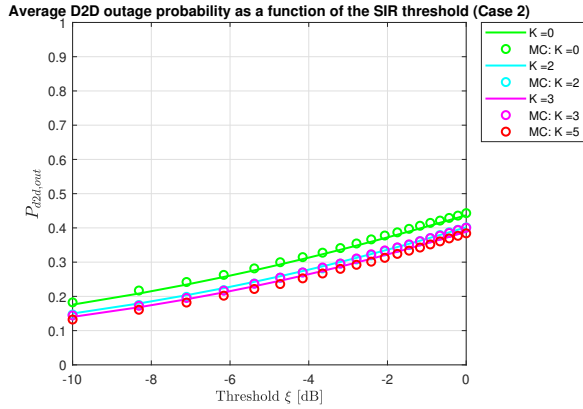


Fig. 6. D2D outage probability (case 2).

made, (8) can be modified using the following expression instead:

$$\mathcal{G}_{bis}(\text{int}) = \Lambda_{int} \pi \lambda^\delta P_{int}^\delta e^{-K} \text{sinc}^{-1}(\pi \delta) \sum_{k'=0}^{\infty} \frac{K^{k'} (K+1)^{-\delta}}{(k'!)^2} (1+\delta)_{k'}. \quad (16)$$

Actually, using the Rayleigh fading model or the Rician fading model leads respectively to a lower and higher outage probability than the real one. Ideally, the fading

model should be adaptively chosen depending on the distance but this is left for a future work.

V. CONCLUSION

In this paper, the integration of underlying D2D communications in a conventional CC network has been studied in two different cases. Based on the outage probability, the choice of the case has been proven to promote either the D2D communications or the CC communications for realistic transmit powers.

Future research directions could include improvements of the high SIR values using control mechanisms, generalization to inter-cell communications or even enhancement of both communications using multihop links.

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