Chapter 7

Conclusions

Throughout this thesis, we have investigated some important aspects of the numerical prediction of elasto-plastic and elasto-viscoplastic inclusion-reinforced composites and proposed some original improvements. To carry this out, two techniques have been examined: mean-field homogenization schemes and finite element simulations.

We first adapted the finite element technique for the analysis of composites. The major advantage of such analysis is the accuracy of its predictions and the possibility of knowing the state of deformation at any point of the representative volume element, although this can only be done at a high computational cost. Furthermore, on the contrary to other methods, it can deal easily with any constitutive behavior for any phase. Based on existing developments for sphere reinforced materials, we extended the technique to inclusions with a spheroidal (ellipsoidal with an axis of revolution) shape which enables to deal with a much wider category of composites. To do this, we first generated geometries with randomly dispersed and non overlapping inclusions. Due to the imposition of periodic boundary conditions, both the geometry and the mesh must also be periodic. This guarantees a limited size of the RVE. A geometrical study proved the required homogeneity and transverse isotropy of the cell in the case of aligned inclusions. The numerical analysis showed that for inclusions with a low aspect ratio, 30 of them were enough to reproduce correctly the behavior of the material. High attention was devoted to the mesh quality so that precision of the microscopic fields was guaranteed, which is of first importance when taking damage into account. Finite element simulations are a fantastic tool for validation purpose of other simplified but much faster homogenization schemes.

Main part of this work is devoted to development of mean-field homogenization schemes for elasto-plastic and elasto-viscoplastic composites. First of all, these are presented in the context of linear thermo-elastic or viscoelastic materials and relies on the Eshelby's result. Accuracy of these models is checked against experimental results and finite element simulations and a good correlation was found between these models. This has a great impact on subsequent developments since this consists of a subproblem of homogenization schemes in the nonlinear regime.

First extension to nonlinear materials is done in the context of two-phase elasto-plastic composites. In order to apply homogenization schemes developed for linear elastic materials, an incremental linearization of the local constitutive equations is carried out. This is done over several time steps and has the advantage of following the loading path so that it can handle any loading history, including cyclic and non proportional ones. However, this general approach is known to be too stiff. In order to circumvent this problem, we suggested several stiffness reduction methods. First of all, this was studied on a theoretical point of view and the dependence of various tensors used during the homogenization step (Hill, Hill's constraint and the effective tangent operators) with respect to the matrix tangent modulus was examined. Previous results have shown that Hill's tensor $\boldsymbol{P}$ decreases (in the meaning of quadratic forms) with the matrix modulus while Hill's constraint tensor $\boldsymbol{C}^*$ and the effective operator $\bar{\boldsymbol{C}}$ are increasing with it. In order to make use of these properties, several matrix modulus tensors are considered: the original anisotropic one $\boldsymbol{C}_0^{ani}$ and the extraction of its isotropic or transverse isotropic parts. The softest modulus is obtained with a special isotropisation technique in a direction orthogonal to the normal to the yield surface in the stress space. Similarly, our proposed transverse isotropisation technique relies on a projection on the subspace of transversely isotropic tensors and a stiffness reduction in the same direction. Dependence of the different tensors on the matrix modulus combined with all the proposed stiffness reduction methods enable to perform various predictions. It has been noted that the previously used evaluation of the Eshelby's tensor $\mathcal{E}$ with the special isotropisation technique $\boldsymbol{C}_0^{IsoSpe}$ can sometimes give non physical predictions, which is due to the loss of major symmetries of the Hill's tensor $\boldsymbol{P}$. After numerous validation tests, we recommend the computation of the Hill's tensor $\boldsymbol{P}$ with the special isotropic matrix modulus $\boldsymbol{C}_0^{IsoSpe}$. Unfortunately, this special method is not applicable to all constitutive models. Spectacular improvements have also been obtained with the transversely isotropic extraction method ($\boldsymbol{C}_0^{TrIsoSpe}$) but this still relies on a fitting parameter which reduces its scope of applications.

Still in the context of elasto-plasticity, a comparison between the secant formulation (per phase reference state defined from either first order or second order moment of the stress tensor), the incremental formulation and finite element simulations has been carried out. This showed that best predictions are obtained with the second order secant homogenization schemes. Incremental formulation also gives satisfying predictions and has the major advantage of remaining valid with general plasticity models (e.g.: kinematic hardening) and non-proportional loadings (e.g.: cyclic). Difference between first and second order methods are especially noticeable when high plastic strain localization occurs and when uniaxial traction test is performed along the direction of the aligned ellipsoids. However, several difficulties for capturing correctly the per phase average strain and stress is still a major problem when dealing with mean-field homogenization schemes. Some of these problems are due to the use of isotropisation technique or the second order secant method.

Major developments of this thesis concern the affine formulation developed for elasto-viscoplastic composites. The main improvement with previous implementations of the method is that it enables to deal with internal parameters. Such formulation transforms the local elasto-viscoplastic constitutive laws into form-similar linear thermo-viscoelastic relations. With the help of the Laplace-Carson transform, these are reduced to linear thermo-elastic ones in the Laplace domain where the homogenization procedure can be applied. In order to solve the localization problem, some numerical inversions are required. Due to the hereditary nature of the constitutive laws, an iterative process must be carried out over several time steps. In order to limit the memory cost, a special attention has been paid to the development of efficient algorithms. Our implementation enables to simulate several loadings including cyclic, relaxation and creep tests, different hardening and viscoplastic laws and several homogenization schemes. Finally, a confrontation with 3D finite element method showed a good accuracy. However, a deterioration of the predictions has been observed as the strain rate increases. This is mainly coming from the isotropisation method.

This thesis showed the general efficiency of mean-field homogenization schemes to capture correctly macroscopic behavior. This has been done for difficult nonlinear behaviors such as elasto-plasticity and elasto-viscoplasticity. Among the future improvements of homogenization methods are:

- Increasing the quality of the microscopic predictions obtained with mean-field methods is a great challenge. This could be done by considering other reference states for the phases of the linear comparison composite. For example, making use of the second order moment of the stress tensor with the incremental formulation is still an open subject.

- Solving some of the limitations of Eshelby-based mean-field homogenization models such as the ellipsoidal shape of the inclusions, the availability of information on the strain localization and the impossibility to model percolation or clustering effects. The latter problem could maybe be solved by considering a decomposition of the two-phase composite into several regions, each one having a different volume fraction of inclusions. Such material can be considered as a multi-phase composite in which each region corresponds to a two-phase composite.

- Modeling size effects which are important when dealing with nanocomposites and are not considered by finite element and mean-field homogenization schemes. A solution could be the introduction of a characteristic length in the model.

- Considering damage (inclusion fracture, interface decohesion,...). This firstly requires a good accuracy of the predictions at the microscopic scale. Anyway, since detailed microscopic fields are not available with these methods, it must be done in a statistical way. Using an additional internal parameter in the affine formulation could be a solution.

- Dealing with the thermal coupling which is especially useful when performing simulations of a complete process of elasto-viscoplastic composites.

Finally, it is stressed out that even if finite element simulations are realizable on a representative volume element of the microstructure, this is far from being possible if this problem is coupled to a simulation on the macroscopic structure. So, even if computational power is increasing every day, the use of mean-field homogenization schemes are the only way to solve such problems and guarantees a bright future to these methods.