ANNALS OF ECONOMICS AND STATISTICS
NUMBER 101/102, JANUARY/JUNE 2011

Bilingualism and Communicative Benefits

Jean GABSZEWICZ
CORE, Université Catholique de Louvain and ECORE

Victor GINSBURGH
ECARES, Brussels, CORE, Louvain-la-Neuve and ECORE

Shlomo WEBER
Southern Methodist University, Dallas and New Economic School, Moscow

We examine patterns of acquiring non-native languages in a model with two languages and heterogenous populations in two countries or regions. We show that both interior and corner linguistic equilibria can emerge, and that the fraction of learners of the foreign language is higher in the country with a larger cost adjusted communicative benefit. We also point out that linguistic equilibria can exhibit insufficient learning which opens the door for welfare-improving policies, which seem to need joint effort by both countries or regions.*

I. Introduction

Proficiency in foreign languages has important consequences on earnings. Job opportunities are more and more often open to candidates who speak several languages, though not all languages are identical in that respect. GRENIER [1985], for example, finds that in Québec, it pays (a six percent wage differential) for a French-speaking Canadian to learn English, but the opposite is not true. For the European Union similar conclusions are obtained by GINSBURGH and PRIETO [2011] whose results show that a second language (in most cases, English) raises wages in the range from five to fifteen percent in Austria, Belgium, Finland, France, Germany, Greece, Italy, Portugal and Spain, much less so in Denmark and the Netherlands (where English is known by 75 and 70 percent of the population, respectively) and has no effect in the United Kingdom, where no second language is in high demand.[1] In addition to effects on earnings, a foreign language makes it possible to be immersed into a different culture and to gain unfiltered access to its history, its arts and its literature.

SELTEN and POOL [1991] formulate a general model of language acquisition. They introduce the notion of "communicative benefits," which cover a wide range of economic, cultural and social advantages gained by learning languages. CHURCH and KING [1993][2] construct a game-theoretic model where every agent is proficient in a single language, but can acquire the other one at a cost which is identical for all agents regardless of their prior language knowledge. Every agent is faced with the following binary choice: to learn the other language at a given cost, or to refrain from learning. The communicative benefit of an individual increases with the number of those with whom she can communicate using a common language. The equilibrium

1. See also MACMANUS *et al.* [1978] and VAILLANCOURT and LACROIX [1985].
2. See also SHY [2001].

*JEL: C72, D83, O52, Z13 / KEY WORDS: Communicative Benefits, Linguistic Equilibrium.

outcome, therefore, depends on a network externality since the strategic decision by an individual to learn the other language expands the communication links for others who speak that language. The larger the number of individuals in the other language group who learn the native tongue of an agent, the smaller the benefit from second language acquisition for that agent. CHURCH and KING [1993] show that the equilibrium patterns of language acquisition depend on the cost of learning and on the number of individuals who initially speak each language. However, only corner solutions exist in equilibrium: either no one learns any language in either country (if the cost of learning is sufficiently high), or everybody learns the foreign language in one country while nobody does in the other. The fact that only corner equilibria exist is due to the assumption that learning costs are homogeneous: once learning is beneficial for one agent initially endowed with some language, it is also so for all those who speak the same language. GINSBURGH *et al.* [2007a] suggest that interior equilibria may exist, but focus on the empirical implications of the model, namely, the derivation of demand functions for languages.[3]

In this paper, we consider a simple model with two languages and heterogenous populations in two countries or regions. Heterogeneity is introduced through the degree of language aptitude which leads agents to bear individual learning costs. Agents opt to maximize their net communicative benefit which is the difference between the communicative benefit discussed above, and the cost of acquiring a new language. This model leads to a complete description of linguistic equilibria as a function of two main parameters, the size of each population and the unit cost of learning in each country. In this framework, both *corner* equilibria (where either all the residents of a country study the foreign language or none of them does), and *interior* equilibria (where some, but not all, residents of each country learn the foreign language) may exist. An important role is played by the ratio of the population of one country, say E, and the unit cost parameter of studying its language in country F. This ratio represents the *cost adjusted communicative benefit* of a citizen in F from learning language E. We show that the fraction of learners of the foreign language is higher in the country with higher cost adjusted communicative benefits, an observation that is consistent with the data on language proficiency in bilingual countries such as Belgium and Canada.

Interior equilibria will exist only if the cost adjusted communicative benefits are either large or small in both countries. However, stability of the interior equilibrium obtains only if those benefits are relatively small. Otherwise, all individuals will be tempted to learn the foreign language, thus making the interior equilibrium unstable.

We also turn to welfare analysis, assuming that the welfare of each country is represented by the aggregate communicative benefits of its residents. Public intervention by each government separately leads to the same outcome as the equilibrium outcome, but joint maximization by both governments may yield more learning in both countries. This raises the possibility that public policies could encourage learning of foreign languages in both countries.

The model is described in SECTION II. SECTION III turns to the characterization of linguistic equilibria and their stability properties, as well as comparative statics results. SECTION IV discusses efficiency of equilibrium solutions. SECTION V concludes and suggests further research avenues. Proofs of the propositions are relegated to the Appendix.

3. See also GABSZEWICZ *et al.* [2011] who study language learning in the framework of a multi-sided market with a monopoly platform, and focus on the problem of language learning tariffs.

II. The Model

We consider two populations (regions, countries) E and F. Each population consists of heterogeneous individuals distinguished on the basis of their learning cost described by a parameter $\theta \in [0,1]$, that can be viewed as the inverse of their ability to learn a foreign language. Those with small θ are more apt to learn than those with high θ, and, in particular, an individual with $\theta = 0$ can learn the language in "her sleep." The distribution of the aptitude parameter is assumed to be uniform in both countries, whose populations are respectively N_E and N_F. Individuals in each population are assumed to be unilingual and to speak their native language, denoted E and F, respectively, but consider acquiring the foreign language.

Let $B_t(x)$ denote the communicative benefit of an individual t, where x represents the number of individuals with whom she can communicate. For simplicity, we assume that

$$B_t(x) = B(x) = x,$$

that is, the communicative benefit function is linear and common to all individuals in both populations.

Every individual in either population is faced with two possible choices: study the language of the other population or refrain from doing so. If a proportion α_F of citizens in population F studies language E, a citizen in population E who refrains from learning language F can communicate with her N_E fellow citizens and with the $\alpha_F N_F$ individuals of the other population who know language E. Thus, her communicative benefit is equal to

$$B(N_E + \alpha_F N_F) = N_E + \alpha_F N_F. \tag{1}$$

Assume that every individual faces a personalized learning cost of acquiring the foreign language, which depends on her ability: an individual in population E, whose (reversed) aptitude is given by θ, incurs a cost $c_E(\theta)$ to learn language F. We assume that for all $\theta \in [0,1]$, the value $c_E(\theta)$ is given by

$$c_E(\theta) = c_E\theta,$$

where c_E is a positive constant. Similarly, $c_F(\theta) = c_F\theta$ represents the cost of learning language E for an individual in population F. The value of c_E may be different from c_F, so that two individuals with the same aptitude in both populations may incur different learning costs.

The net communicative benefit for a citizen of type θ in country E who learns language F is determined by the fact that, if she is willing to incur a cost $c_E\theta$, she will be able to communicate with both populations, *i.e.*,

$$B(N_E + N_F) - c_E\theta = N_E + N_F - c_E\theta. \tag{2}$$

We examine linguistic equilibria in the game whose players are the citizens of both countries, and study the set of pure strategy Nash equilibria where the payoff to every individual is given by her resulting communicative benefit net of learning costs, if any. Note that, in equilibrium, the set of individuals who decide to learn the foreign language in both populations represents a connected interval with respect to θ. Assuming that in the case of indifference, an individual will study the foreign language, the interval of learners' types always includes the type represented by point 0, as individuals with zero learning costs are always willing to study. Thus, every equilibrium choice is determined by the highest cost values of those individuals who study the foreign language in countries E and F, respectively. In other words, the equilibrium is fully described by the population shares α_E and α_F of citizens who study the other language.

Suppose that in equilibrium the measure of individuals who study the foreign language is α_E in country E and α_F in country F. Then, in country E, only those individuals whose θ satisfies

$$N_F - c_E\theta \geq \alpha_F N_F \tag{3}$$

will learn language F. Indeed, since for those individuals, the left-hand side of (3) is at least as large as the right-hand side, learning generates larger benefits than non-learning. If, in addition, $N_F - c_E < \alpha_F N_F$, then the individual with the highest learning cost (θ=1) does not study and there exists a cut-off value $\theta(\alpha_F) \in (0,1)$ such that the individual with type $\theta(\alpha_F)$ is indifferent between studying and foregoing the study of F. Thus, all individuals whose value of the learning cost is lower than $\theta(\alpha_F)$ will study the foreign language. If $N_F - c_E \geq \alpha_F N_F$, then everybody in population E studies language F. Thus, we can define the value

$$\theta(\alpha_F) = \min\left[(1-\alpha_F)\frac{N_F}{c_E}, 1\right], \tag{4}$$

which identifies the highest level of learning costs in population E that still makes it beneficial to study language F. Similarly, the inequality

$$N_E - c_F\theta \geq \alpha_E N_E \tag{5}$$

identifies those types of individuals in population F who gain from studying language E. This defines $\theta(\alpha_E)$ such that

$$\theta(\alpha_E) = \min\left[(1-\alpha_E)\frac{N_E}{c_F}, 1\right]. \tag{6}$$

Under self-fulfilling expectations in equilibrium, the fraction of citizens in country F whom citizens in country E expect to learn their language is equal to the fraction of those who effectively do so. Therefore, in equilibrium, we obtain:

$$\theta(\alpha_F) = \alpha_E \quad \text{and} \quad \theta(\alpha_E) = \alpha_F. \tag{7}$$

Thus, taking into account (4), (6), (7), a linguistic equilibrium is determined by the system of equations

$$\begin{cases} \min\left[(1-\alpha_F)N_F/c_E, 1\right] = \alpha_E \\ \min\left[(1-\alpha_E)N_E/c_F, 1\right] = \alpha_F. \end{cases} \tag{8}$$

We shall focus here on the characterization of equilibria in the linguistic game considered above. In particular, we examine the existence of interior and corner equilibria, where a pair (α_E^*, α_F^*) is an *interior* equilibrium if $0 < \alpha_E^*, \alpha_F^* < 1$, while in a *corner* equilibrium, either α_E^* or α_F^* is equal to 0 or 1.

Notice that both interior and corner equilibria may emerge in our set-up. In an interior equilibrium, there is partial learning of both languages, where, obviously, the fraction of those who speak one or the other may be unequal. This is the case in most European countries which have several official languages spoken by sizeable groups of people (Belgium, Ireland, Finland, Switzerland), as well as in the United States, where Spanish places itself next to English in several states (California, Texas and Florida), and Canada. The same logic works across countries whose populations speak different languages: English is known by 20 percent of the French population, and French is spoken by 9 percent of the British population. On the other hand, the disappearance of many languages points to the possibility of corner solutions.

III. Characterization of Equilibria

Before proceeding with the examination of equilibria in our game, we introduce the notion of *cost adjusted communicative benefit* that will play an important role in our analysis. More specifically, the values b_E and b_F, given by

$$b_E = \frac{N_F}{c_E}, \; b_F = \frac{N_E}{c_F} \tag{9}$$

represent, respectively, the *cost adjusted communicative benefit* of a citizen in E from learning language F and of her counterpart F who learns language E. Then (8) can be rewritten as

$$\begin{cases} \min\left[b_E(1-\alpha_F), 1\right] = \alpha_E \\ \min\left[b_F(1-\alpha_E), 1\right] = \alpha_F. \end{cases} \tag{10}$$

We first analyze interior equilibria, for which system (10) yields:

$$\begin{cases} b_E(1-\alpha_F) = \alpha_E \\ b_F(1-\alpha_E) = \alpha_F, \end{cases} \tag{11}$$

or

$$\begin{cases} \alpha_F + \alpha_E/b_E = 1 \\ \alpha_E + \alpha_F/b_F = 1. \end{cases} \tag{12}$$

An examination of (12) and the sign of the expression $\alpha_E + \alpha_F - 1$ immediately shows that the unique interior equilibrium will emerge if and only if either b_E and b_F are both smaller than 1, or both larger than 1.

In both cases, the interior solution can be written as:

$$\alpha_E^* = \frac{b_E(1-b_F)}{1-b_E b_F}; \; \alpha_F^* = \frac{b_F(1-b_E)}{1-b_E b_F}. \tag{13}$$

Moreover, if $b_E = b_F = 1$, any pair (α_E^*, α_F^*) with $\alpha_E^* + \alpha_F^* = 1$ and $0 < \alpha_E^*, \alpha_F^* < 1$ is an interior linguistic equilibrium. We summarize these results in the following proposition:

Proposition 1. *Interior equilibria.* An interior equilibrium exists if and only if (i) $b_E, b_F < 1$ or (ii) $b_E, b_F > 1$, or (iii) $b_E = b_F = 1$. In cases (i) and (ii) there is a unique interior equilibrium, whereas in case (iii) there is a continuum of interior equilibria.

Proof. Obvious from the examination of (12)-(13). □

Equilibrium (i) and (ii) have different stability properties. Namely,

Proposition 2. *Stability properties of interior equilibria.* If $b_E, b_F < 1$, the interior equilibrium is (globally) stable. If $b_E, b_F > 1$, the interior equilibrium is unstable.

Proof. See Appendix. □

The intuition for this result is quite simple. If the cost adjusted communicative benefits b_E and b_F are not too large (smaller than 1, given the parametrization that is used in the model), both languages will coexist. Only those types of individuals with small enough θ (that is, high learning ability) will acquire the other language. If on the contrary, the communicative benefit, say for population E, is large (larger than 1), then all individuals will learn the foreign language. This also explains why, in that case, the interior solution is unstable.

Note that, by (13), one obtains

$$\alpha_E^* - \alpha_F^* = \frac{b_E - b_F}{1 - b_E b_F}, \tag{14}$$

which implies that in the stable interior equilibrium, the fraction of learners in the country that faces larger communicative benefits is higher than in its counterpart. The proof is obvious.

Proposition 3. *Number of learners and communicative benefits.* Let $b_E, b_F < 1$ and (α_E^*, α_F^*) be a stable interior equilibrium. Then $\alpha_E^* < \alpha_F^*$ if and only if $b_E < b_F$.

Thus if language E yields larger communicative benefits than language F, more individuals with the prior knowledge of F will learn E than the reverse: "the value of assimilation is larger to an individual from a small minority than to one from a large minority group" (LAZEAR [1997]). Belgium is a good example. Though Dutch (Flemish) is nowadays the native language of more inhabitants than French, adding the neighboring French and Dutch populations leads to 2.9 times more speakers of French than of Dutch.[4] Assuming that the learning costs c_F and c_D are equal, the fraction of Dutch-speaking Belgians who learn French should be larger than that of French-speaking citizens who learn Dutch. The Eurobarometer survey (see INRA [2001]) indeed shows that 40 percent of the Dutch-speaking Belgian population claim they know French, while only 12 percent of those who speak French know Dutch. A similar situation prevails in Canada, where only 10 percent of Anglophones know French (BOND [2001]), whereas, according to the 2001 Census results, 41 percent of Francophones know English. Naturally, the roles of English and French are reversed in Québec, where, again according to the 2001 Census, some 43 percent of residents whose mother tongue is English, can communicate in French.

This extends to different countries also. France and the UK have roughly equal populations, and the French distinguished linguist Claude Hagège suggests that the cost of acquiring English for a Frenchman is very high.[5] Then, there should be more British citizens who learn French than the other way round. But this is not so since, worldwide, English is spoken by 1.5 billion citizens, while French is only spoken by some 125 million. The cost adjusted communicative benefits are thus $b_F = N_E/c_F = 1{,}500/c_F$, while $b_E = N_F/c_E = 125/c_E$. Replace this in (14) and note

4. This ratio increases to 5.3 if one includes citizens who speak French and Dutch in the European Union. See GINSBURGH and WEBER [2005] and GINSBURGH *et al.* [2005].
5. See his interview in *Le Figaro*, April 5, 2004: "l'anglais est une des langues les plus difficiles à apprendre... Les choses sont beaucoup moins longues avec le russe ou l'arabe. L'anglais est une langue extrêmement 'traîtresse'."

that for an interior equilibrium, the denominator of this expression is positive. Therefore for English speakers in the UK to become as proficient in French as French speakers of English, we need $c_F = 12c_E$: in our linear model, this implies that learning English for a Frenchman is 12 times more difficult than learning French for an Englishman, which goes in the direction of what Hagège suggests.

Interior equilibria lead to some interesting comparative statics results, summarized in the following proposition.

Proposition 4. *Comparative statics.* The fraction of individuals studying the other language is:

(i) decreasing in the learning cost of the other language;

(ii) increasing in the learning cost of its own language;

(iii) increasing in the population size of the other country;

(iv) decreasing in its own population size.

(v) The relationship between the total number of individuals studying the other language, and the increase in the size of their own population is indeterminate. More specifically, the number of learners of language F in country E always increases in its size if communication benefits in country F are small ($b_F \leq 0.5$). That is, the number of learners of language F in E increases if the citizens in F are not inclined to learn language E. However, if communication benefits in country F are large ($b_F > 0.5$), the number of learners of language F in country E increases in its size if communication benefits in E are sufficiently large. That, is there is an increasing function $h(b_F)$[6] on the interval $[0.5, 1]$ with $h(0.5) = 0, h(1) = 1$, such that an increase in population in E produces new learners of language F only if $b_E \in [h(b_F), 1]$.

Proof. See Appendix. □

We now turn to corner solutions. Note that if b_E and b_F are both larger than 1, or both equal to 1, the two corner solutions, $(1,0)$ or $(0,1)$, also satisfy (13), and both are linguistic equilibria. In some other cases, there exist only corner solutions.

The useful tool to understand corner equilibria is the following Lemma, the proof of which is obvious.

Lemma 1. The pair $(1,0)$ is a linguistic equilibrium if and only if $b_E \geq 1$. Similarly, the pair $(0,1)$ is a linguistic equilibrium if and only if $b_F \geq 1$.

The lemma claims that if communicative benefits faced by one country are sufficiently high, then the situation where all citizens in that country learn the foreign language, whereas nobody in the other one does, is a linguistic equilibrium. Recall that every linguistic equilibrium is a solution of (10). By examining this system, it is easy to distinguish two cases: either b_E or b_F is larger than 1, and the other is smaller than 1; or b_E or b_F is larger than 1, and the other is equal to 1. In the two first cases, there exists a unique corner equilibrium. In the two last cases both pairs $(0,1)$ and $(1,0)$ constitute an equilibrium. We summarize the results in the following proposition.

6. $h(x) = (2x - 1)/x^2$ for all $x \in [0.5, 1]$.

TABLE I. – CHARACTERIZATION OF LINGUISTIC EQUILIBRIA

	$b_F < 1$	$b_F = 1$	$b_F > 1$
$b_E < 1$	stable interior	(0,1)	(0,1)
$b_E = 1$	(1,0)	continuum with $\alpha_E + \alpha_F = 1$	(0,1), (1, 0)
$b_E > 1$	(1,0)	(0,1), (1, 0)	(0,1), (1,0) and unstable interior

TABLE II. – PAYOFFS WHEN CHOICES ARE BINARY

	Learn	Not to learn
Learn	$N - c_E, N - c_F$	$N - c_E, N$
Not to learn	$N, N - c_F$	N_E, N_F

Proposition 5. *Corner equilibria.* (i) If either $b_E < 1 < b_F$, or $b_F < 1 < b_E$, there exists a unique corner equilibrium.
(ii) If either $b_E = 1 < b_F$, or $b_F = 1 < b_E$, there exist two corner equilibria, $(0,1)$ and $(1,0)$.

Proof. Obvious. □

Indeed, if every individual in population E studies language F, then every individual in population F is guaranteed the maximal possible communication benefit at no cost, and there is no reason for any of them to study language E. This also rules out $(1,1)$ as an equilibrium candidate. However, if no citizen of F studies language E, then the entire population of E studies language F only if their communicative benefit is sufficiently large, that is if $b_E \geq 1$. By a similar token, the corner solution $(0,0)$ cannot be an equilibrium either. Indeed, if nobody in population E, say, learns language F, then those with small θ in population F should learn E.

The results presented in Propositions 1 and 6 are summarized in TABLE I.

We conclude this section, by contrasting our set-up with that of CHURCH and KING [1993].[7] In their model, learning costs within each society are homogenous and simply equal to c_E and c_F. This turns the model into a game with two participants E (the row player), and F (the column player), each having a binary choice: to learn or not to learn the other language. The payoff matrix (benefits minus costs) is given in TABLE II.

Here $N = N_E + N_F$. Indeed, if al least one player learns, they both have an access to both communities. If nobody learns, each player is limited to communicate within her own population. If b_E and b_F are "sufficiently large," *i.e.*, are greater than one and $N_E > c_F$ and $N_F > c_E$, this is, in fact a modified Game of Chicken, with two off-diagonal pure strategies Nash equilibria, where only one player learns the other language. There is also a mixed Nash equilibrium in which players E and F learn with probabilities $1 - 1/b_E$ and $1 - 1/b_F$, respectively. This equilibrium is inefficient when compared to the two pure Nash equilibria.

If, say, the value of b_E is less than one, then "no learning" becomes the dominant strategy for E, and, if, in addition, $b_F < 1$, we have no learning as the dominant strategies equilibrium. But if $b_F > 1$, the only Nash equilibrium is the situation where the player with a larger communicative benefit learns, whereas the other does not.

7. As suggested by one of the referees.

IV. Efficiency

We define the welfare of population E, $W_E(\alpha_E, \alpha_F)$, as its aggregate communicative benefit:

$$(1-\alpha_E)N_E(N_E+\alpha_F N_F)+\alpha_E N_E(N_E+N_F)-c_E\int_0^{\alpha_E N_E}\frac{\theta}{N_E}d\theta, \tag{15}$$

where the first term is the welfare of the $(1-\alpha_E)N_E$ citizens who do not learn language F; each of them gets a benefit equal to $(N_E+\alpha_F N_F)$, since he can communicate with that number of E-speakers. The second term describes the net benefit of E-citizens who learn F; their mass is $\alpha_E N_E$, and the benefit that each of them gets is (N_E+N_F); the third term is the total cost of those who learn. The expression can be rewritten as

$$W_E(\alpha_E,\alpha_F)=N_E^2+N_E N_F[(1-\alpha_E)\alpha_F+\alpha_E]-\frac{1}{2}c_E\alpha_E^2 N_E. \tag{16}$$

The welfare of population F, $W_F(\alpha_E,\alpha_F)$ obtains by interchanging E and F in (16).

We examine two scenarios, starting with the one in which the two countries maximize their joint welfare

$$W(\alpha_E,\alpha_F)=W_E(\alpha_E,\alpha_F)+W_F(\alpha_E,\alpha_F). \tag{17}$$

After some simplifications, (17) can be rewritten as

$$W(\alpha_E,\alpha_F)=N_E^2+N_F^2+2N_E N_F(\alpha_E+\alpha_F-\alpha_E\alpha_F)-\frac{1}{2}c_E\alpha_E^2 N_E-\frac{1}{2}c_F\alpha_F^2 N_F,$$

or:

$$N_E^2+N_F^2+c_E c_F\left[2b_E b_F(\alpha_E+\alpha_F-\alpha_E\alpha_F)-\frac{1}{2}\alpha_E^2 b_F-\frac{1}{2}\alpha_F^2 b_E\right]. \tag{18}$$

By differentiating W with respect to each of the variables, the first order conditions yield the following equations:

$$\begin{cases}\min\,[2b_E(1-\alpha_F^0),1]=\alpha_E^0\\ \min\,[2b_F(1-\alpha_E^0),1]=\alpha_F^0,\end{cases} \tag{19}$$

where (α_E^0,α_F^0) are such that

$$W(\alpha_E^0,\alpha_F^0)=\max_{(\alpha_E,\alpha_F)\in S^2}W(\alpha_E,\alpha_F).$$

For every α_E the function $W(\alpha_E,\cdot)$ is concave in α_F, and for every α_F, $W(\cdot,\alpha_F)$ is concave in α_E. However, since $W(\cdot,\cdot)$ is not concave over the unit square $[0,1]^2$, conditions (19) are necessary but not sufficient to determine a maximum of the function W. But if the solution of (19) is unique (which will be the case for an interior optimum), it will also be welfare maximizing.[8] We can state:

Proposition 6. *Efficient allocations.* (i) If $b_E, b_F < 1/2$, the unique efficient allocation is interior.

(ii) If $b_E \le 1/2 \le b_F$ and $b_F > b_E$, then $(0,1)$ is the only efficient allocation.

8. Note that the only difference between (19) and (10) is the coefficient 2 in the left-hand side.

TABLE III. – EFFICIENT NUMBER OF LEARNERS

	$b_F < \frac{1}{2}$	$b_F = \frac{1}{2}$	$b_F > \frac{1}{2}$
$b_E < \frac{1}{2}$	interior solution	$(0,1)$	$(0,1)$
$b_E = \frac{1}{2}$	$(1,0)$	continuum with $\alpha_E + \alpha_F = 1$	$(0,1)$
$b_E > \frac{1}{2}$	$(1,0)$	$(1,0)$	$(0,1)$ or $(1,0)$

(iii) If $b_F \leq 1/2 < b_E$ and $b_E > b_F$, then $(1,0)$ is the only efficient allocation.

(iv) If $b_F = b_E = 1/2$, then there there exists a continuum of efficient allocations, satisfying $\alpha_E + \alpha_F = 1$.

(v) If $b_F > b_E > 1/2$ then the only efficient allocation is $(0,1)$.

(vi) If $b_E > b_F > 1/2$, then the only efficient allocation is $(1,0)$.

(vii) If $b_F = b_E > 1/2$, then both $(0,1)$ and $(1,0)$ are efficient allocations.

Proof. See Appendix. □

Efficient outcomes are given in TABLE III.

It turns out that too little learning may occur in equilibrium, and that policy intervention may be needed.

Proposition 7. *Insufficient learning.* (i) If $b_E, b_F < 1/2$, the number of learners in equilibrium is smaller than in the unique interior efficient allocation.

(ii) If $b_E = b_F = 1/2$, there exists an efficient allocation in which the number of learners in both countries is larger than in the unique interior equilibrium.

Proof. See Appendix. □

It is worth pointing out that joint optimization could be a restrictive assumption. Indeed, one may argue that each country or region will rather maximize the welfare of its own citizens. It is, for example, quite difficult to coordinate the educational systems, and jointly decide on the amount of subsidies needed to reduce the learning cost of the other language. In this case, country E will choose α_E to maximize $W_E(\alpha_E, \alpha_F)$ and country F will likewise set α_F to maximize $W_F(\alpha_E, \alpha_F)$. It is easy to check that this Nash game yields the same solution as the decentralized solution of SECTION III. Indeed, first-order conditions are identical and each country's learning strategy is to have only individuals with positive net communicative benefits learning the foreign language. However, this requirement is consistent with individual incentives that lead to the same outcome, namely the linguistic equilibrium. This implies that only coordination between the countries can lead to more efficient solutions. Thus, the difficulties of cooperation and coordination notwithstanding, some steps in this direction could be rather beneficial.

V. Conclusions

This paper studies a model of language learning and provides a full characterization of linguistic equilibria and their welfare properties for the two-languages case. The heterogeneity assumption on the ability to learn a foreign language generates interior equilibria, thus extending the existing results by CHURCH and KING [1993] and SHY [2001] who have examined corner solutions in a homogenous framework.

We also show that the interior stable equilibrium may lead to insufficient learning, and that subsidies or other forms of intervention may be needed to curb learning costs. But there are examples (LAZEAR [1997]) that show that it may be rational and welfare improving to protect a language by taxing the learning of the foreign language. There are also circumstances where a unique standard (in our case, a corner solution) may be welfare deteriorating.

Transfers from one country to the other[9] may change the equilibrium outcomes. The welfare consequences of those policies[10] could also be tackled in the framework of our model.

An immediate possibility[11] is to generalize our game to N uni-lingual communities, where individuals make their learning choices with respect to the other $N-1$ languages. Even a model with a single individual in each community, where learning costs are represented by a $N \times N$ matrix, leads to a very interesting, challenging but difficult problem of characterizing pure strategies Nash equilibria. Heterogeneous learning costs, though, would make the characterization even more difficult.

One of the referees suggests an extension in the same direction, with patterns of foreign language acquisition costs, depending on the proximity among languages. The issue of impact of linguistic distances on learning patterns has been extensively analyzed in GINSBURGH and WEBER [2011], and in our context it would allow to study how agents coordinate on one (or several) language(s) to be used as *linguae francae*, like Latin in the Middle-Ages or English nowadays. It could also explain the failure of Esperanto to become a world *lingua franca*. Along these lines, BELLEFLAMME and THISSE [2009] and GINSBURGH *et al.* [2007b] examine models involving three languages and two populations in which the two populations would coordinate and learn the third language which is native to none of the two populations.

Another extension could deal with the possibility for agents fluent in one language to acquire the other one with varying proficiency. This would imply that learning costs vary with the proficiency level, while the communicative benefit of an individual in country E would no longer depend on the number of individuals with whom she can communicate, but rather on some average of the proficiency levels chosen by the natives of country F. Since learning still varies with individuals, we conjecture that interior equilibria would exist.

9. For instance, schools financed by the French government in foreign countries, the network of Goethe Institutes which promote the German language and culture outside of Germany. The UK seems to make money on promoting English. This is what Lord Kinnock, the Chairman of the British Council writes in the introduction to GRADDOL's [2006] report: "the English language teaching sector directly earns £1.3 billions (at the time of writing this, the £ was worth 1.3 euros) for the UK as invisible exports and our education related exports earn up to £10 billion a year more."
10. See also POOL [1991] and VAN PARIJS [2004].
11. Suggested by both referees.

ACKNOWLEDGEMENTS

We are grateful to two referees for excellent suggestions which could lead to several extensions of the simple model studied in this paper. Ginsburgh and Weber are grateful to the College of Europe and its Rector, Professor Paul Demaret, for the hospitality of the College during their stay in Bruges. Weber acknowledges support from the Spanish Ministry of Science and Innovation (grant # ECO2008-01300).

Correspondence:
Jean Gabszewicz,
Voie du Roman Pays 34, CORE, *Université Catholique de Louvain,*
1348 Louvain-la-Neuve, Belgium.
Tel +32-10-474346, fax +32-10-474301,
e-mail: jean.gabszewicz@uclouvain.be

Victor Ginsburgh (correspondig author),
ECARES, *Université Libre de Bruxelles C.P.114/04, 50, avenue F. Roosevelt 1050 Brussels, Belgium.*
Tel +32-2-6503846, fax +32-2-6504012,
e-mail vginsbur@ulb.ac.be

Shlomo Weber,
Department of Economics, Southern Methodist University, Dallas, TX *75275,* USA.
Tel +1-214-7683577, fax +1-214-7683911,
e-mail sweber@smu.edu

Appendix

Proof of Proposition 2. To examine the stability of the system (11), assume that the initial conditions $(\alpha_{F,0}, \alpha_{E,0})$ of the following dynamic system do not constitute an equilibrium:

$$\left\{ \begin{array}{l} \alpha_{F,t} = b_F(1-\alpha_{E,t-1}) \\ \alpha_{E,t} = b_E(1-\alpha_{F,t-1}). \end{array} \right. \tag{A.1}$$

This is a linear system which can also be written as:

$$\alpha_t = b + \Gamma \alpha_{t-1},$$

where $\alpha = (\alpha_F, \alpha_E)$ and $b = (b_F, b_E)$ are column vector of two elements each, and Γ is the 2×2-matrix

$$\Gamma = \begin{pmatrix} 0 & -b_F \\ -b_E & 0 \end{pmatrix}.$$

It is easy to check that the characteristic roots of this matrix are equal to $\sqrt{b_F b_E}$ and -$\sqrt{b_F b_E}$. If b_E, $b_F < 1$, the absolute value of both roots is smaller than 1, and this (unique) equilibrium is (globally) stable.

However, if b_E, $b_F > 1$, the interior equilibrium is unstable, and, depending on the initial condition $(\alpha_{E,0}, \alpha_{F,0})$, the dynamic system (20) will converge to one or the other corner solution. □

Proof of Proposition 4. We compute the partial derivatives of α_E^* with respect to $b_E = N_F/c_E$, $b_F = N_E/c_F$. Similar results can be derived for the equilibrium position of the other language.

(i) holds for N_F fixed, and (iii) holds for c_E fixed, since

$$\frac{d\alpha_E^*}{db_E} = \frac{(1-b_F)(1-b_Eb_F)+b_E(1-b_F)b_F}{(1-b_Eb_F)^2} = \frac{1-b_F}{(1-b_Eb_F)^2}.$$

(ii) holds for N_E fixed, and (iv) holds for c_F fixed, since

$$\frac{d\alpha_E^*}{db_F} = \frac{-b_E(1-b_Eb_F)+(b_E)^2(1-b_F)}{(1-b_Eb_F)^2} = \frac{b_E(b_F-1)}{(1-b_Eb_F)^2} < 0.$$

Finally, to prove (v), we compute

$$\frac{d(\alpha_E^*N_E)}{dN_E} = \frac{d(\alpha_E^*b_F)}{db_F} = \frac{b_E(1-2b_F)(1-b_Eb_F)+b_E^2b_F(1-b_F)}{(1-b_Eb_F)^2}$$

$$= \frac{b_E(1-2b_F+b_F^2b_E)}{(1-b_Eb_F)^2}.$$

The sign of the last expression is indeterminate. It is, however, nonnegative within the following set

$$\left\{(b_E,b_F)\in S^2 \,|\, b_E \geq h(b_F)\right\},$$

where S^2 is the unit rectangle $[0,1]^2$ and $h(b_F) = (2b_F-1)/b_F^2$. Thus, the sign of the derivative in (v) is nonnegative if

$$\begin{cases} 0 \leq b_E \leq 1 & if \quad 0 \leq b_F \leq 0.5 \\ h(b_F) \leq b_E \leq 1 & if \quad 0.5 \leq b_F \leq 1. \end{cases}$$

□

Proof of Proposition 6.

(i) follows from the fact that the system (19) has a unique solution, which is an interior allocation.

(ii)-(iii). If either $b_E \leq 1/2 \leq b_F$ with $b_F > b_E$ or $b_F \leq 1/2 \leq b_E$ with $b_E > b_F$, then only solution or solutions of (19) are the corner ones. The choice between $(0,1)$ and $(1,0)$ (if there is one) is determined by the sign of the following expression

$$W(1,0) - W(0,1) = \frac{b_E - b_F}{2}. \tag{A.2}$$

(iv) follows from the fact that every pair (α_E, α_F) with $\alpha_E + \alpha_F = 1$ represents a solution of (11).

(v) Let $b_E, b_F > 1/2$. We claim that the maximal welfare is attained at either $(1,0)$ or at $(0,1)$, depending on the sign of (20).

Let us compare $W(0,1), W(1,0)$ and $W(\alpha_E^0, \alpha_F^0)$, where the latter is the interior solution of (11). Denote

$$B = b_Eb_F, \; b = b_E + b_F.$$

Then

$$\alpha_E^0 = \frac{4B - 2b_E}{4B - 1}, \; \alpha_F^0 = \frac{4B - 2b_F}{4B - 1}. \tag{A.3}$$

Using (22), it suffices to consider the maximization of the following expression:

$$\tilde{W}(\alpha_E, \alpha_F) = 2B(\alpha_E + \alpha_F - \alpha_E \alpha_F) - \frac{1}{2}\alpha_E^2 b_F - \frac{1}{2}\alpha_F^2 b_E.$$

Then

$$\tilde{W}(\alpha_E^0, \alpha_F^0) = 2B\left(\frac{8B - 2b}{4B - 1} - \frac{16B^2 + 4B - 8Bb}{(4B-1)^2}\right) - \frac{(4B - 2b_E)^2 b_F}{2(4B-1)^2}$$

$$-\frac{(4B - 2b_E)^2 b_E}{2(4B-1)^2} = \frac{32B^3 - 8B^2 + 2Bb - 8B^2 b}{(4B-1)^2} = \frac{2B(4B - b)}{4B - 1}.$$

In order to show that

$$\tilde{W}(\alpha_E^0, \alpha_F^0) < \max[\tilde{W}(1,0), \tilde{W}(0,1)],$$

it suffices to demonstrate that

$$\tilde{W}(\alpha_E^0, \alpha_F^0) < \frac{\tilde{W}(1,0) + \tilde{W}(0,1)}{2}.$$

Indeed, $\tilde{W}(1,0) = 2B - \frac{1}{2}b_F$, $\tilde{W}(0,1) = 2B - \frac{1}{2}b_E$, and

$$\frac{\tilde{W}(1,0) + \tilde{W}(0,1)}{2} = 2B - \frac{b}{4}.$$

Thus, it remains to show that

$$\frac{2B(4B - b)}{4B - 1} < 2B - \frac{b}{4},$$

or

$$\frac{b}{4} > (2 - b)B.$$

If $b \geq 2$, this inequality trivially holds. Let $b < 2$. Since $1 > b(2-b)$ for all $1 < b < 2$, we have

$$\frac{b}{4} > (2 - b)\frac{b^2}{4}.$$

Note that if the sum of two positive numbers is b, then their product is maximal if they are equal. Thus,

$$\frac{b}{4} > (2 - b)\,\frac{b^2}{4} \geq (2 - b)B.$$

□

Proof of Proposition 7. (i) Let $b_E, b_F) < 1/2$. Then the linguistic equilibrium (α_E^*, α_F^*) is given by

$$\left(\frac{b_E(1-b_F)}{1-b_E b_F}, \frac{b_F(1-b_E)}{1-b_E b_F}\right),$$

whereas the efficient outcome, (α_E^0, α_F^0), is determined by (22). Consider country E. It easy to verify that $\alpha_E^* < \alpha_E^0$, or

$$\frac{b_E(1-b_F)}{1-b_E b_F} < \frac{2b_E(1-2b_F)}{1-4b_E b_F}.$$

Similar derivations are valid for country F.

(ii) If $b_E = b_F = 1/2$, the linguistic equilibrium is $(1/3, 1/3)$. The welfare optimizing solutions are represented by the pairs (α_E, α_F) with $\alpha_E + \alpha_F = 1$. Then every point in the set

$$\{(\alpha_E, \alpha_F) \in \Re^2 | \alpha_E + \alpha_F = 1, \alpha_E, \alpha_F > 1/3\}$$

and, in particular, $(1/2, 1/2)$ is superior to the interior equilibrium. Thus, the equilibrium indeed could exhibit an insufficient level of learning. □

References

BELLEFLAMME, Paul, and Jacques THISSE (2009): "Sprechen Sie Français? Yes I do. The Economics of Plurilingualism in the European Union," Manuscript. [281]

BOND, Karen (2001): "French as a minority language in bilingual Canada," http://www3.telus.net/linguisticsissues/french.html (accessed on November 4, 2005). [276]

CHURCH, Jeffrey, and Ian KING (1993): "Bilingualism and Network Externalities," *Canadian Journal of Economics*, **26**, 337-345. [271,272,278,281]

GABSZEWICZ, Jean, Victor GINSBURGH, Didier LAUSSEL, and Shlomo WEBER (2011): "Foreign Language Acquisition: Self-Learning and Linguistic Schools," *Review of Network Economics* **10**, Article 1. [272]

GINSBURGH, Victor, Ignazio ORTUÑO-ORTÍN and Shlomo WEBER (2005): "Disenfranchisement in Linguistically Diverse Societies. The Case of the European Union," *Journal of the European Economic Association*, **3**, 946-964. [276]

GINSBURGH, Victor, Ignazio ORTUÑO-ORTÍN and Shlomo WEBER (2007a): "Learning Foreign Languages. Theoretical and Empirical Implications of the Selten and Pool Model," *Journal of Economic Behavior and Organization*, **64**, 337-347. [272]

GINSBURGH, Victor, and Juan PRIETO (2011): "Returns to Foreign Languages of Native Workers in the European Union," *Industrial and Labor Relations Review*, **64**, 599-618. [271]

GINSBURGH, Victor, Alexei SAVATEEV, and Shlomo WEBER (2007b): "Linguistic Equilibria in a Multinational Model," Manuscript. [281]

GINSBURGH, Victor, and Shlomo WEBER (2005): "Language Disenfranchisement in the European Union," *Journal of Common Market Studies*, **43**, 273-286. [276]

GINSBURGH, Victor, and Shlomo WEBER (2011): *How Many Languages Do We Need. The Economics of Linguistic Diversity*. Princeton, N.J.: Princeton University Press, in press. [281]

GRADDOL, David (2006): *English Next. Why Global English May Mean the End of English as a Foreign Language*. London: British Council. [281]

GRENIER, Gilles (1985): "Bilinguisme, transferts linguistiques et revenus du travail au Québec, Quelques éléments d'Interaction," in *Économie et Langue*, ed. by François Vaillancourt. Québec: Éditeur officiel, 243-287. [271]

INRA (2001): "Eurobaromètre 54 Special, Les Européens et les langues," Février. [276]

LAZEAR, Edward (1997): "Culture and Language," *Journal of Political Economy*, **107**, S95-S127. [276,281]

MACMANUS, Walter, William GOULD, and Finis WELSCH (1978): "Earnings of Hispanic Men: The Role of English Language Proficiency," *Journal of Labor Economics*, **1**, 101-130. [271]

POOL, Jonathan (1991): "The Official Language Problem," *American Political Science Review*, **85**, 495-514. [281]

SELTEN, Reinhard, and Jonathan POOL (1991): "The Distribution of Foreign Language Skills as a Game Equilibrium," in *Game Equilibrium Models*, ed. by Reinhard Selten. Berlin: Springer-Verlag, **4**, 64-84. [271]

SHY, Oz (2001): *The Economics of Network Industries*. Cambridge, U.K.: Cambridge University Press. [271,281]

VAILLANCOURT, François, and Robert LACROIX (1985): "Revenus et langue au Québec," http://www.cslf.gouv.qc.ca/publications/PubD120/D120-6.html (accessed on November 4, 2005). [271]

VAN PARIJS, Philippe (2004): "Europe's linguistic challenge," *Archives Européennes de Sociologie*, XLV, 113-154. [281]