Adaptive λ-tracking controller for an exothermic chemical plug flow tubular reactor

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ABSTRACT
This paper deals with the tracking of a prespecified profile temperature for exothermic chemical tubular reactor whose dynamics is described by a set of nonlinear partial differential equations where the state variables are the reactor temperature and the reactant concentration. The coolant temperature, the inlet temperature and the inlet concentration are considered as control actions. For practical reasons, it is preferable to consider a non distributed control law to achieve the control objective. In contrast to our previous work that considers fully distributed control actions where the three control inputs are assumed to be distributed along the reactor, here the last two control inputs are applied at the reactor inlet and only the coolant temperature is distributed along the reactor. We show that the temperature of the reactor tends asymptotically to a ball of arbitrary prescribed radius $\lambda > 0$, centered at the given temperature profile.

KEYWORDS
Adaptive control, nonlinear distributed parameter models, exothermic chemical reaction, input saturations, λ-tracking, non isothermal tubular reactor.

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1. Introduction

Tubular reactors cover a large class of industrial processes both in the chemical and biochemical industry as well as in fields like wastewater treatment. The dynamics of such systems are described by partial differential equations (PDE's) that are most often nonlinear due to the coupling between mass and energy balances. In the present instance, we concentrate on plug flow reactors whose dynamics are described by hyperbolic PDE's (Laabissi, Achhab, Winkin, & Dochain, 2001). Due to their industrial impact and to improve their productivity, several control strategies have been proposed for these processes and more generally for PDE systems. A model predictive, PI, PID, LQR and pole placement approaches (Del Vecchio,& Petit, 2005; Dubljevic, Mhaskar, El-Farra, & Christofides, 2005; El-Farra, Armaou, & Christofides, 2003; Nájera,Álvarez, Baratti, & Gutirrez, 2016; Pitarch, Rakhshan, Mardani, Sadeghi, & de Prada, 2016) were based on polynomial approach or on spatial discretization finite differences or Galerkin methods) to obtain a finite dimensional ordinary differential equations'(ODEs') approximation. A linear quadratic optimal control has been designed for tubular reactors based on the model linearization (Aksikas, Alizadeh Moghadam,& Forbes , 2018; Aksikas, Winkin,& Dochain , 2007). Adaptive control techniques have also been developed for this class of systems, for example adaptive extremum seeking control with unknown kinetics for a plug flow tubular reactors (Hudon, Guay, Perrier, & Dochain, 2005), adaptive control for a plug flow reactor used the polynomial approach (Vojtesek & Dostal, 2009) and adaptive controllers based on Lyapunov design (Krstic, & Smyshlyaev, 2007).The novelty of our approach is that they employ the original nonlinear PDE model to design a constrained adaptive controller with a partial access to the state measurements. The only used approximation is to include the boundary controllers, the inlet temperature and the inlet concentration, in the temperature and the concentration evolution equations.

The control objective is to design a constrained control law to maintain the reactant temperature in a neighborhood of a desired profile. In this context, a possible approach is the λ-tracking where the neighborhood is the ball of radius λ centered at the desired profile. This control strategy has been developed by Ilchmann et al. (Ilchmann,& Ryan, 1994; Logemann, & Ilchmann, 1994) for finite and infinite dimensional systems without input saturation. For a nonlinear finite dimensional chemical reactor, a constrained adaptive λ-trackers have been developed to regulate the reactor temperature. This approach has been generalized for a tubular reactor described by nonlinear partial differential equations (Beniich, El Bouhtouri, & Dochain , 2010)-(Beniich, El Bouhtouri, & Dochain , 2019). In (Beniich et al, 2010; Beniich, El Bouhtouri, & Dochain , 2015), constrained adaptive output feedback controllers have been developed to regulate the reactor temperature in a prespecified neighborhood of a given reference profile by assuming a full access to temperature measurements along the reactor via a sufficient number of sensors throughout the reactor. This presents obviously a practical limitation. To overcome it, (Beniich, El Bouhtouri, & Dochain , 2017; Beniich et al, 2019) had developed other adaptive control laws that consider only partial temperature measurements in a zone with length higher than a predetermined level in term of the control parameter.

In this paper, we propose an input constrained adaptive controller based on the $\lambda-$tracker by considering only partial temperature measurements available in a zone Ω of the reactor via a finite number of zone sensors. in (Beniich et al, 2019) we had proposed a controller with three distributed inputs, the heat exchanger temperature $T_c(t, z)$, the inlet temperature $T_{in}(t, z)$ and the inlet reactant concentration $C_{in}(t, z)$, i.e. by assuming that we are able to distribute the inputs all along the reactor. The practical implementation of plug flow reactors is often performed via a cascade of completely stirred tank reactors (CSTR's), that can possibly fed and cooled/heated individually. The fully distributed inputs correspond therefore to the limit case when the number of the reactors in the cascade is large enough (see also Smets, Dochain, Van Impe, 2002). In the present instance, we design the controller for a standard plug flow reactor configuration where the inlet stream is applied at the reactor inlet, i.e. the two control inputs (inlet temperature and inlet reactant concentration) are only functions of time ($T_{in}(t)$ and $C_{in}(t)$). In this work, we propose non distributed control laws for T_{in} and C_{in} by replacing the tracking error in the controller used in (Beniich et al, 2017) with its minimum with respect to z on the measurements zone.

The considered assumptions are given in terms of the input constraints, the desired profile, the reaction kinetics and of the physical properties of the nonlinear terms. We start with the local tracking control when the initial temperature is constrained to be in a fixed domain and only the reactor temperature is controlled. In this case we show that the temperature reactor converges to a ball of the desired profile with radius λ.

In the second part, we show that under an extra control action via the inlet concentration we can obtain the global convergence of the temperature profile towards a ball of arbitrary prefixed radius $\lambda > 0$ centered at the given temperature profile.

The paper is structured as follows. In Section 2, we present the transformation of the mathematical model in the appropriate format in Hilbert space. Then we explicitly

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formulate the hypothesis under which the tracking objective is achievable and we end the section by developing the constrained λ-tracking controller of the reactor temperature. In the third section, we state and prove our main results on the local adaptive and non-adaptive λ-tracking. In section 4, we give our results on the global λ-controller which achieves the desired objective for all initial temperatures. In section 5, numerical simulations are performed to show the performance of our approaches.

2. Dynamical model and State space system framework

In this work, we present a constrained adaptive control law for a plug flow nonisothermal tubular reactor described by the following nonlinear partial differential equations derived from energy and mass balances of all time $t \geq 0$ and for all $z \in [0, L]$, where L is the reactor length:

$$\frac{\partial T}{\partial t} = -v\frac{\partial T}{\partial z} + \alpha f(T, C) - k_0(T - T_c) \tag{1}$$

$$\frac{\partial C}{\partial t} = -v\frac{\partial C}{\partial z} - f(T, C) \tag{2}$$

with the boundary conditions :

$$T(0, t) = T_{in}(t) \tag{3}$$

$$C(0, t) = C_{in}(t) \tag{4}$$

In the above equations, t (>0) and z ($\in [0, L]$ hold for the time and the reactor length, respectively. Without loss of generality, we consider that $L = 1$. $k_0 = \frac{4h}{\rho C_p d}$ (> 0), $\alpha = \frac{-\Delta H}{\rho C_p}$ (> 0), T, C, $v > 0$, $\Delta H < 0$, ρ,C_p, h, d, T_c , T_{in} and C_{in} are the temperature reactor, the reactant concentration, the energy and mass dispersion coefficients, the superficial fluid velocity, the heat of the reaction, the density, the specific heat, the wall heat transfer coefficient, the reactor diameter, the coolant temperature, the inlet temperature, and the inlet concentration, respectively. $f(T(t, z), C(t, z))$ represents the reaction kinetics. It is a nonlinear, positive and locally Lipschitz function. The positivity of f is a direct consequence of the standard kinetics rules (e.g. [19]) for the irreversible reaction $A \longrightarrow bB$ in which the reactant A is consumed and the product B is synthesized. A typical example of $f(T(t, z), C(t, z))$ is the reaction rate of first-order kinetics with respect to the reactant concentration C and characterized by an Arrhenius type dependence with respect to the temperature T

$$f(T(t, z), C(t, z)) = KC(t, z)e^{-\frac{E}{RT(t,z)}} \tag{5}$$

with $K > 0$, $E > 0$ and $R > 0$ the kinetic constant, the activation energy and the ideal gas constant, respectively.

Let us consider the Hilbert state space $H = L^2[0, L]$ of measurable square-integrable function endowed with the usual inner product:

$$< f, g >_H = < f, g > = \int_0^L f(z)g(z)dz \tag{6}$$

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and the usual partial order is defined by

$$f \leq g \text{ if and only if } f(z) \leq g(z) \text{ for almost every } z \in [0, L] \tag{7}$$

where f, g are in H. With respect to this order, the positive cone H^+ of H is defined by:

$$H^+ = \{y \in H,\ s.t\ y \geq 0\} \tag{8}$$

The inlet temperature T_{in} and the coolant temperature T_c (spatially distributed) are considered as the control variables. As in (Beniich et al, 2019; Winkin, Dochain,& Ligarius, 2000), the system equation (1)-(3) are approximated by:

$$\begin{aligned} \dot{x}_1(t) &= A_1 x_1(t) + \alpha f(x_1(t), x_2(t)) + v\Delta_\xi u_1(t) + k_0 u_2(t) && (9)\\ \dot{x}_2(t) &= A_2 x_2(t) - f(x_1(t), x_2(t)) + v\Delta_\xi C_{in} && (10) \end{aligned}$$

with

$$x_1(t) = T(t,.),\ x_2(t) = C(t,.),\ u_1(t) = T_{in}(t),\ u_2(t) = T_C(t,.)$$

and

$$\begin{aligned} A_1 x &= -v\frac{\partial x}{\partial z} - k_0 x, \quad x \in \mathcal{D}(A_1) && (11)\\ A_2 x &= -v\frac{\partial x}{\partial z}, \qquad\quad x \in \mathcal{D}(A_2) && (12) \end{aligned}$$

defined on:(for i=1,2)

$$\mathcal{D}(A_i) = \{x \in H : x \text{ absolutely continuous };\ \frac{dx}{dz} \in H \ \text{ and } x(0) = 0\}$$

and

$$\Delta_\xi(z) = \begin{cases} \dfrac{1}{\xi} & 0 \leq z \leq \xi \\ 0 & \xi < z \leq L \end{cases}$$

It has been shown in (Winkin et al, 2000) that A_1 and A_2 are infinitesimal generators of exponentially stable strongly continuous semigroups $(T_1(t))_{t\geq 0}$ and $(T_2(t))_{t\geq 0}$ of bounded operators in H. Besides, for all $x \in D(A_1)$ the inner product $\langle A_1 x, x\rangle \leq 0$, i.e. A_1 is dissipative.

From (Winkin et al, 2000), $(T_1(t))_{t\geq 0}$ and $(T_2(t))_{t\geq 0}$ are given by:

Lemma 2.1.

$$(T_1(t)(x))(z) = \begin{cases} e^{-k_0 t} x(z - tv) & \text{if } t \leq \frac{z}{v} \\ 0 & \text{else} \end{cases}$$

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$$(T_2(t)(x))(z) = \left\{ \begin{array}{ll} x(z-tv) & \textit{if } t \leq \frac{z}{v} \\ 0 & \textit{else} \end{array} \right.$$

and they satisfy the following properties (Laabissi, Achhab, Winkin, & Dochain (2004)):

Lemma 2.2.

i) $T_i(t)$ *is a positive linear operator*
ii) $(T_i(t)(M))(z) \leq M$ *for all* $M > 0$ *and* $z \in [0,1]$
iii) $(T_i(t)(M))(z) \leq (T_i(t)M)(1)$ *for all positive constant function* M *and* $z \in [0,1]$.

For physical considerations, we assume that u_i (i = 1, 2) are constrained as follows:

$$\underline{u}_1 \leq u_1(t) \leq \overline{u}_1,\ \underline{u}_2 \leq u_2(t) \leq \overline{u}_2 \text{ for all } t \geq 0 \tag{13}$$

where $\underline{u}_i$, $\overline{u}_i$ (i = 1, 2), are positive constants.
Assume that the system (9)-(10) satisfies the following assumptions:

(H_1) The positive cone $H^+ \times H^+$ is positively invariant under (9)-(10) for all non-negative controls u_1 and u_2 satisfying the above constraints, i.e: for all initial conditions $x_1(0) \geq 0$ and $x_2(0) \geq 0$, we have $x_1(t) \geq 0$ and $x_2(t) \geq 0$ for all $t \geq 0$.

For a given $x^* \in H^+$ we assume:

(H_2) The desired profile $x^* \in H^+ \cap \mathcal{D}(A_1)$ and there exist $\rho > 0$, $\underline{x} \in H^+$, $\overline{x} \in H^+$ with $0 < \underline{x} < x^* < \overline{x}$ such that for all x_1, x_2 in H satisfying $\underline{x} \leq x_1 \leq \overline{x}$ and $0 \leq x_2 \leq C_{in}$:

$$\left\{ \begin{array}{lcl} \frac{v}{\xi}\underline{u}_1 + k_0\underline{u}_2 + \rho & \leq & k_0 x_1 - \alpha f(x_1,x_2) - Ax^* \\ & \leq & k_0 u_2^* - \rho \\ A(\overline{x} - x^*) & \leq & 0, \end{array} \right. \tag{14}$$

where $A = A_1 + k_0 I$

(H_3) $0 < \lambda < \overline{x} - x^*$, where λ is a small arbitrary positive constant.

(H_4) There exists a continuous function $\Phi : H \mapsto H$ such that for all $y_1 \geq 0$ and all y_2 satisfying $0 \leq y_2 \leq C_{in}$:

$$f(y_1, y_2) \leq \Phi(y_2) y_1 \tag{15}$$

Remark 1. - Assumption (H_1) means that the concentration and temperature should not become zero once they are positive, if the inlet temperature and the inlet concentration are strictly positive.

If the local solution of (9)-(10) exists, then

$$\begin{array}{rcl} x_1(t) & = & T_1(t)x_1(0) + \int_0^t T_1(t-s)[\alpha f(x_1(s), x_2(s)) + v\Delta_\xi u_1(s) + k_0 u_2(s)]ds \\ & \geq & T_1(t)x_1(0) \end{array}$$

and

$$\begin{array}{rcl} x_2(t) & = & T_2(t)x_2(0) + \int_0^t T_2(t-s)[-f(x_1(s), x_2(s)) + v\Delta_\xi C_{in}]ds \\ & \geq & T_2(t)x_2(0) - \int_0^t T_2(t-s) f(x_1(s), x_2(s))ds \end{array}$$

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We consider the following system:

$$\begin{cases} \dot{z}_1(t) = A_1 z_1(t) \\ \dot{z}_2(t) = A_2 z_2(t) - f(z_1(t), z_2(t)) \\ z_1(0) = x_1(0) \text{ and } z_2(0) = x_2(0) \end{cases} \tag{16}$$

The positivity of the trajectories z_1 and z_2 of (16) has been shown in (Laabissi et al, 2001). We conclude then the positivity of x_1 and x_2 considered in (H_1).

- The upper bound $\overline{u}_2$ does not only depend on the feasibility condition (H_2) but also on the physical limitations of the actuator when both conditions are compatible, i.e. the actuator limit is higher than the bound in (H_2), $\underline{u}_1$ is chosen so that the feasibility condition (H_2) and the saturation bound are verified.
- For a constant desired profile x^* and the input constraints such that the following inequality hold:

  for all $0 \leq x_2 \leq C_{in}$

  $$\frac{v}{\xi}\underline{u}_1 + k_0\underline{u}_2 + \rho \leq k_0 x^* - \alpha f(x^*, x_2) \leq k_0 u_2^* - \rho$$

  By continuity of $f(.,.)$, there exist $\underline{x}$ and $\bar{x}$ such that $0 < \underline{x} < x^* < \overline{x}$ and the inequality

  $$\frac{v}{\xi}\underline{u}_1 + k_0\underline{u}_2 + \rho \leq k_0 x_1 - \alpha f(x_1, x_2) \leq k_0 u_2^* - \rho$$

  is satisfied for all $\underline{x} \leq x_1 \leq \overline{x}$ and $0 \leq x_2 \leq x_2^{in}$
- Assumption H_3 requires to select λ small enough.
- Form (Jadot, 1996), the kinetics function has the following form: $f(T, C) = K(T)\varphi(C)$ where $T \longrightarrow K(T)$ is positive, bounded and globally Lipschitz, and the function $C \longrightarrow \varphi(C)$ is nonnegative, continuous and vanishes if $C = 0$. This ensures the feasibility of (H_4).

The objective is the design of adaptive controller laws for u_1, u_2 that achieve the regulation of the reactor temperature x_1 to a ball of the desired profile x^* with arbitrary radius λ i.e $\limsup\limits_{t \to \infty} \|x_1 - x^*\| \leq \lambda$ and satisfy the imposed input constraints (13).

The tracking error is denoted by

$$e(t) = x^* - x_1(t) \tag{17}$$

The reactor temperature is assumed to be measured, by a limited number of zone sensors, only in a zone $\Omega \subset [0, L]$. The measured tracking error in Ω is denoted by

$$\hat{e}(t)(.) = \mathcal{C}(.)(e(t)(.)) \tag{18}$$

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with

$$\mathcal{C} = \mathbb{1}_{\Omega}(z) = \begin{cases} 1 & \text{if} \quad z \in \Omega \\ 0 & \text{if} \quad z \notin \Omega \end{cases} \tag{19}$$

For a given small $\lambda > 0$, we consider the following adaptive controller laws:

$$\begin{aligned} m(t) &= \min_{z\ \in \Omega}(e(t)(z)) && (20) \\ u_1(t) &= sat_{[\underline{u}_1, \overline{u}_1]}(\beta_1(t)m(t) + u_1^*) && (21) \\ u_2(t) &= sat_{[\underline{u}_2, \overline{u}_2]}(\beta_2(t)\hat{e}(t) + u_2^*) && (22) \\ \dot{\beta}_i(t) &= k_i \begin{cases} (\|\hat{e}(t)\| - \lambda)^{l_i} & \text{if} \quad \| \hat{e}(t) \| > \lambda \quad i = 1,2 \\ 0 & \text{if} \quad \| \hat{e}(t) \| \leq \lambda \end{cases} && (23) \\ \beta_i(0) &= \beta_{0,i}, \ i = 1,2 \end{aligned}$$

The saturation function $sat_{[\underline{u}_i, \overline{u}_i]}y$ is defined for all $y \in \mathbb{R}$:

$$sat_{[\underline{u}_i, \overline{u}_i]}y = \begin{cases} \overline{u}_i & \text{if} \quad y \geq \overline{u}_i \\ y & \text{if} \quad \underline{u}_i < y < \overline{u}_i \\ \underline{u}_i & \text{if} \quad y \leq \underline{u}_i \end{cases}$$

The proposed controller laws consist of a proportional measured error with saturation and time varying proportional gains updated via the measurement error. $\lambda > 0$ is the upper bound for the asymptotic tracking error, $l_i \geq 1$ in the adaptation gain influences the speed of the adaptation. If $\|e(t)\| - \lambda$ is smaller than 1, then a bigger $l_i \geq 1$ gives a slower increase of β_i, if $\|e(t)\| - \lambda$ is bigger than 1, then the bigger l_i gives a faster increase of β_i. Similar effects can be achieved by varying k_i and $\beta_{0.i} > 0$. $u_i^* \in (\underline{u}_i, \overline{u}_i)$ are the constant offset.

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3. Local tracking

In this section, we assume that the initial temperature $x_1(0)$ is constrained to be lower than $\bar{x}$. Similarly to the existing work on the $\lambda-$tracking, we present two feedback strategies that force the temperature into a λ-neighborhood of the given reference profile. The first one is non adaptive while the second one is adaptive.

We consider first the non adaptive version of (20)-(23):

$$\begin{aligned} u_1(t) &= sat_{[\underline{u}_1, \overline{u}_1]}(\beta_1(t)m(t) + u_1^*) && (24) \\ u_2(t) &= sat_{[\underline{u}_2, \overline{u}_2]}(\beta_2(t)\hat{e}(t) + u_2^*) && (25) \end{aligned}$$

Where, for i=1.2, $\beta_i(.) : \mathbb{R}^+ \longrightarrow [\beta_i^*, \infty[$ is a predefined continuous function.
Let us introduce the following constants:

$$
\begin{array}{rcl}
C_1 & = & \max\{\|k_0 x_1 - \alpha f(x_1,x_2) - Ax^* - \frac{v}{\xi}\underline{u}_1 + k_0 u_2^*\|,\ (x_1,x_2)\in\Delta_1\times\Delta_2\} \\
C_2 & = & \max\{\|k_0 x_1 - \alpha f(x_1,x_2) - Ax^* - \frac{v}{\xi}\bar{u}_1 + k_0 u_2^*\|,\ (x_1,x_2)\in\Delta_1\times\Delta_2\} \\
C_3 & = & \max\|\bar{x}-x^*\|, \|\bar{x}\| \\
\varepsilon & = & \dfrac{\rho\lambda}{2C_2}.
\end{array}
$$

where

$$
\begin{array}{rcl}
\Delta_1 & = & \{x_1 \in H \text{ such that } 0 \leq\ x_1(z) < \bar{x}(z)\,\forall z\in[0,1)\} \\
\Delta_2 & = & \{x_2 \in H \text{ such that }\ 0 < x_2(z) \leq\ C_{in}\ \forall z\in[0,1)\}
\end{array}
$$

The following proposition gives the explicit lower bounds for β_1^* and β_2^*, in terms of the system parameters, the control constraints and λ, that achieve our objective.

Proposition 3.1. *Assume that* $(H_1)-(H_3)$ *hold* $(x_1(0),x_2(0))\in(\Delta_1\times\Delta_2)$ *and*

$$\beta_i^* \geq \frac{u_i^*-\underline{u}_i}{\overline{x}-x^*} \tag{26}$$

Then the controllers (24)-(25) applied to (9) (10) yields a unique solution:

$$(x_1(t),x_2(t))\in\ (\Delta_1\times\Delta_2)\ \textit{for all}\ t\geq 0$$

If

$$\beta_i^* \geq \max\left(\frac{u_i^*-\underline{u}_i}{\lambda};\frac{\overline{u}_i-u_i^*}{\lambda};\frac{u_i^*-\underline{u}_i}{\overline{x}-x^*};\frac{2C_2}{\lambda};\frac{\lambda C_1}{\varepsilon^2};\frac{C_2C_3}{(1-k)\lambda^2}\right) \tag{27}$$

then there exists $t_1>0$ *such that:*

$$\|\widehat{e}(t)\| \leq \lambda \qquad \textit{for all}\ \ t\geq t_1$$

and if

$$\omega\ \leq\ \frac{k\lambda^2}{\|\overline{x}-x^*\|_\infty} \tag{28}$$

where w *is the length of* $\Omega' = \{z\in[0,L]$ *such that* $z\notin\Omega\}$ *and* $0<k<1$.
then there exists t_2 *such that:*

$$\|e(t)\| \leq \lambda \quad \textit{for all}\ \ t\geq t_2.$$

Proposition 3.1 is proved in the appendix.

Remark 2. From Proposition 3.1, it is shown that sufficiently large β_1 and β_2 achieve the convergence of the temperature reactor in a finite time but this time is unknown.

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The condition (28) requires that the length of the measurement zone must be larger than a threshold value. For an available measurement zone, we choose suitable control parameters that achieve the objective.

Consider now the adaptive controller (20)-(23). We show that a larger initial gain conditions $\beta_{0,i}$ achieve the convergence of the feedback gain $\beta_i(t)$ and the error norm will approach the interval $[0, \lambda]$ as $t \longrightarrow +\infty$, i.e

$$\limsup_{t \longrightarrow +\infty} \|e(t)\| \leq \lambda$$

Theorem 3.2. *Assume that* (H_1)*-*(H_3) *and (28) hold, and* $(x_1(0), x_2(0)) \in (\Delta_1 \times \Delta_2)$ *and suppose:*

$$\beta_{0,i} \geq \frac{u_i^* - \underline{u}_i}{\overline{x} - x^*} \tag{29}$$

the closed loop system given by equations (9)-(10) and (20)-(23) has the following properties:

(1) $x_1(.), x_2(.), \beta_i(.) : \mathbb{R}^+ \longrightarrow (\Delta_1 \times \Delta_2) \times \mathbb{R}^+$*.*

(2) $\lim\limits_{t \longrightarrow +\infty} \beta_i(t)$ *exists and is finite.*

(3) $\limsup\limits_{t \longrightarrow +\infty} \|e(t)\| \leq \lambda$

Proof. The existence and uniqueness of the maximal solution in a maximal interval $[0, w)$ follows from Theorem [(Cazenave & Haraux, 1998), pp 56-57]. From Proposition 3.1 and the monotonicity of $t \longrightarrow \beta_i(t)$ and (29) for i=1,2 yields that $\beta_i(t) \geq \beta_{0,i}$ with $\beta_{0,i}$ satisfying (26), so $(\Delta_1 \times \Delta_2)$ is positively invariant, $\beta_i(.)$ cannot exhibit a finite escape time on $[0, w)$ which implies that $w = +\infty$.

Now we show that $\beta_i(t)$ i=1, 2 is bounded.
Suppose that, for i=1,2, β_i is unbounded, then there exists $\hat{t}_i$ such that $\forall t \geq \widehat{t_i}$:

$$\beta_i(t) \geq \max\left(\frac{u_i^* - \underline{u}_i}{\lambda}; \frac{\overline{u}_i - u_i^*}{\lambda}; \frac{u_i^* - \underline{u}_i}{\overline{x} - x^*}; \frac{2C_1}{\lambda}; \frac{\lambda C_1}{\varepsilon^2}; \frac{C_2 C_3}{(1-k)\lambda^2}\right)$$

From Proposition 3.1, there exists $t' \geq \sup\{\hat{t}_1, \hat{t}_2\}$ such that $\forall t \geq t' \quad \|\widehat{e}(t)\| \leq \lambda$, whence $\dot{\beta}_i(t) = 0 \quad \forall t \geq t'$, which contradicts the assumption of unboundedness of β_1 and β_2.

The second assertion 2) of the theorem follows from the monotonicity of β_1 and β_2.

To show that $\limsup\limits_{t \longrightarrow +\infty} \|e(t)\| \leq \lambda$, let us define, for $y \in H$,

$$d_\lambda(y) = \begin{cases} (\|y\| - \lambda) & \text{if} \quad \|y\| > \lambda \\ 0 & \text{if} \quad \|y\| \leq \lambda \end{cases}$$

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and $\beta(t)$ as follows:

$$\begin{aligned}
\dot{\beta}(t) &= \begin{cases} k_1(||e(t)|| - \lambda)^{l_1} & \text{if} \quad ||e(t)|| > \lambda \\ 0 & \text{if} \quad ||e(t)|| \leq \lambda \end{cases} && (30)\\
\beta(0) &= \beta_{0,1} && (31)
\end{aligned}$$

The boundedness of β gives:

$$\forall t \geq 0, \quad k_1 \int_0^t d_\lambda(e(t))^{l_1} = \beta(t) - \beta_{0.1}$$

The uniform continuity of $t \longrightarrow d_\lambda^{l_1}(e(t))$ and Barbalat's Lemma (Khalil, 1996) implies that:

$$\lim_{t \longrightarrow +\infty} d_\lambda^{l_1}(e(t)) = 0,$$

then

$$\limsup_{t \longrightarrow +\infty} \|e(t)\| \leq \lambda$$

This completes the proof of the theorem. □

4. Global tracking

The main results of the previous section require a initial temperature $T(0,.) < \bar{x}$. If $T(0,.) > \bar{x}$, the reactor temperature converges to an undesirable profile (see Figure 2 in the numerical simulation section). To overcome this imitation, we have to consider an additional control law $u_3(t) = C_{in}(t)$ that has an indirect cooling effect if the temperature is too large, as in (Beniich et al, 2019; Ilchmann, Thuto,& Townley, 2004), u_3 is given by:

$$\begin{aligned}
u_3(t)(z) &= \begin{cases} 0 & \text{if} \quad \beta_3(t)m(t)(t)(z) \in (-\infty, a] \\ g(\beta_3(t)m(t)) & \text{if} \quad \beta_3(t)\widehat{e}(t)(z) \in (a, a+\delta] \\ \overline{u}_3 & \text{if} \quad \beta_3(t)m(t) \in (a+\delta, +\infty) \end{cases} && (32)\\
\dot{\beta}_3(t) &= k_i \begin{cases} (\parallel \widehat{e}(t)(t) \parallel -\lambda)^{l_i} & \text{if} \quad \parallel \widehat{e}(t)(t) \parallel > \lambda \quad i = 1, 2 \\ 0 & \text{if} \quad \parallel \widehat{e}(t)(t) \parallel \leq \lambda \end{cases} && (33)\\
\beta_3(0) &= \beta_{0,3} && (34)
\end{aligned}$$

with δ a sufficiently small positive constant and:

$$g(\beta_3(t)m(t)) = (\beta_3(t)m(t) - a)\frac{\overline{u}_3}{\delta} \quad \text{and} \quad a = \ \inf\{\underline{u}_1 - u_1^*, \underline{u}_2 - u_2^*\} \qquad (35)$$

Consider first the non adaptive version of (20)-(23) and (32)-(33) where for i = 1 to 3 $\beta_i : \mathbb{R}^+ \mapsto [\beta_i^*, +\infty)$ are a fixed continuous functions and assume that (H_1), (H_2), (H_3) (H_4) hold for $C_{in} = \bar{u}_3$.

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Proposition 4.1. *Assume that* (H_1)*-*(H_4) *hold. Then the feedback control (24),(25) and (32) applied to (9)(10) produces a unique global solution that satisfies:*

$$(x_1(.), x_2(.)) : I\!R^+ \longmapsto H^+ \times \{x_2 \in H : \; 0 \leq x_2 \leq \overline{u}_3\} \tag{36}$$

If for i= 1,2

$$\beta_i^* > \frac{u_i^* - \underline{u}_i}{\lambda} \tag{37}$$

$$\beta_3^* > \max(\frac{u_1^* - \underline{u}_1}{\lambda}, \frac{u_2^* - \underline{u}_2}{\lambda}) \tag{38}$$

and

$$\frac{v}{\xi}\bar{u}_1 + k_0\bar{u}_2 < k_0\overline{x} \tag{39}$$

there exists $t_1 > 0$ *such that* $x_1(t) \leq \overline{x}$ *for all* $t \geq t_1$.

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Proof. - As in the proof of Proposition 3.1, there exists a unique solution $(x_1(t), x_2(t))$ of (9)-(10) in a maximal interval $[0, w)$, $w \in (0, +\infty]$.

From (H_1), we have $x_1(t) \geq 0$ and $x_2(t) \geq 0$ for all $t \in [0, w)$.

- We show now the positive invariance of the set $\{x_2 \in H$ such that $0 \leq x_2 \leq \overline{u}_3\}$. From the positivity of $T_2(t)$, we have:

$$\begin{array}{rcl} x_2(t) & = & T_2(t)x_2(0) + \int_0^t T_2(t-s)v\Delta_\xi u_3(s)ds - \int_0^t T_2(t-s)f(x_1(s), x_2(s))ds \\ & \leq & T_2(t)x_2(0) + \int_0^t T_2(t-s)\frac{v}{\xi}\overline{u}_3 ds \end{array}$$

The last term is the solution of the system:

$$\dot{y}(t) = A_2 y(t) + \frac{v}{\xi}\overline{u}_3 \,; y(0) = x_2(0)$$

By applying Theorem 3.1 (Laabissi et al , 2004) for $E = \{x$ in $D(A_2)$ and $x \leq \overline{u}_3\}$ and $N(y) = \frac{v}{\xi}\overline{u}_3$, we can readily obtain that

$$x_2(t) \leq T_2(t)x_2(0) + \int_0^t T_2(t-s)\frac{v}{\xi}\overline{u}_3 ds \leq \overline{u}_3.$$

- We show that $x_1(t)$ is bounded in $[0, w)$:

$$x_1(t) = T_1(t)x_1(0) + \int_0^t T_1(t-s)(\alpha f(x_1(s), x_2(s)) + v\Delta_\xi u_1(s) + k_0 u_2(s))ds$$

The positivity of $T_1(t)$ and the fact that $f(x_1(t), x_2(t)) \leq \Phi(x_2(t))x_1(t))$ implies that:

$$x_1(t) \leq T_1(t)x_1(0) + \int_0^t T_1(t-s)(\alpha\Phi(x_2(s))x_1(s)) + v\Delta_\xi u_1(s) + k_0 u_2(s))ds$$

The exponential stability of $T_1(t)$ and the Gronwall's lemma implies the boundedness of x_1.

- We show now that there exists a $t_1 > 0$ such that $x_1(t_1) \leq \overline{x}$.
  Suppose that for $\forall\, t \geq 0$ there exists $z_t \in [0, L]$ such that $x_1(z_t) \geq \overline{x}$.
  If $z_t \in \Omega$ then:

$$\begin{array}{rcl} \beta_i(s)\widehat{e}(s)(z_t) + u_i^* & \leq & -\beta_i^*(s)(\overline{x} - x^*)(z_t) + u_i^* \\ & \leq & \underline{u}_i \end{array}.$$

Therefore

$$u_1(t) = \underline{u}_1,\ u_2(t)(z_t) = \underline{u}_2,$$

and if $z_t \in \overline{\Omega}$, $u_1(t) = u_2^*$, $u_2(t)(z_t) = u_2^*$.
From (H_4), we have:

$$\begin{array}{rcl} x_1(t)(z_t) & = & T_1(t)x_1(0)(z_t) + \int_0^t T_1(t-s)(v\Delta_\xi u_1 + k_0 u_2)(z_t)ds \\ & & +\alpha \int_{t_0}^t T_1(t-s)f(x_1(s,z_t), x_2(s,z_t))ds \\ & \leq & T_1(t)x_1(0)(z_t) + \int_0^t T_1(t-s)(\frac{v}{\xi}\bar{u}_1 + k_0\bar{u}_2)(z_t)ds + \alpha M \int_{t_0}^t T_1(t-s)x_1(s)(z_t)ds \\ & \leq & T_3(t)x_1(0)(z_t) + \int_0^t T_1(t-s)(\frac{v}{\xi}u_1 + k_0 u_2)(z_t)ds. \end{array}$$

where M is the upper bound of $\Phi(x_2)$, $A_3 = A_1 + \alpha M I$ and T_3 the positive asymptotically stable C_0-semi group generated by A_3.
From Lemma 2.2, we have $T_1(t-s)(\frac{v}{\xi}\bar{u}_1 + k_0\bar{u}_2)(z) \leq T_1(t-s)(\frac{v}{\xi}\bar{u}_1 + k_0\bar{u}_2)(1)$ for all $z \in [0,1]$. Then,

$$\begin{array}{rcl} \lim_{t \longrightarrow +\infty} \int_{t_0}^t (T_1(t-s)(\frac{v}{\xi}\bar{u}_1 + k_0\bar{u}_2)(z_t)ds & \leq & \int_0^{+\infty} (T_1(s)(\frac{v}{\xi}\bar{u}_1 + k_0\bar{u}_2)(1)ds \\ & \leq & \frac{1}{k_0}(\frac{v}{\xi}\bar{u}_1 + k_0\bar{u}_2) \leq \overline{x} \end{array} \tag{40}$$

Which contradicts our previous assumption.

From Proposition 3.1 if there exists t_1 such that $x_1(t1) \leq \bar{x}$ then $x_1(t) \leq \bar{x}$ for all $t \geq t_1$. □

The following theorem is a direct consequence of the previous propositions.

Theorem 4.2. *Assume that* (H_1)-(H_4) *and (39) hold, and that for i= 1,2 :*

$$\beta_i^* \geq \max\left(\frac{u_i^* - \underline{u}_i}{\lambda}; \frac{\overline{u}_i - u_i^*}{\lambda}; \frac{u_i^* - \underline{u}_i}{\overline{x} - x^*}; \frac{2C_2}{\lambda}; \frac{\lambda C_1}{\varepsilon^2}; \frac{C_2 C_3}{(1-k)\lambda^2}\right) \tag{41}$$

$$\beta_3^* > \max(\frac{u_1^* - \underline{u}_1}{\lambda}, \frac{u_2^* - \underline{u}_2}{\lambda}) \tag{42}$$

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and

$$\omega \;\leq\; \frac{k\lambda^2}{\|\overline{x}-x^*\|_\infty} \tag{43}$$

where w *is the length of* $\Omega' = \{z \in [0, L]$ *such that* $z \notin \Omega\}$ *and* $0 < k < 1$.
Then there exists $t' > 0$ *such that:*

$$\|e(t)\| \leq \lambda \qquad \forall\, t \geq t^{'}$$

Proof. - From Proposition 4.1, there exists $t_1 > 0$ such that $x_1(t) \leq \overline{x}$ for all $t \geq t_1$.

- β_i^* satisfies (26) and (27) of Proposition 3.1 for all $t \geq t_1$, then there exists $t_2 \geq t_1$ such that

$$\|\hat{e}(t)\| \leq \lambda \qquad \forall\, t \geq t_2$$

- The length w of $\Omega' = \{z \in [0, L]$ such that $z \notin \Omega\}$ satisfies 28 of Proposition 3.1, then there exists $t^{'} \geq t_2$ such that

$$\|e(t)\| \leq \lambda \qquad \forall\, t \geq t^{'}$$

□

Consider the adaptive version of the controllers (20)-(23) and (32)-(33) to achieve the asymptotic convergence of the tracking error to the ball with radius λ centered at zero i.e $\limsup\limits_{t\longrightarrow+\infty} \|e(t)\| \leq \lambda$. Proposition 4.1 shows that, for an appropriate $\overline{x}$, there exists t_1 such that $x_1(t) \leq \overline{x}$ for all $t \geq t_1$. Currently, we are placed in the local tracking context.
For all initial positive temperatures we have:

Theorem 4.3. *Assume that* $(H_1) - (H_4)$, *(39) hold and for i=1,2*

$$\beta_{0,i} > \frac{u_i^* - \underline{u}_i}{\lambda}, \tag{44}$$

$$\beta_{0,3} > \max(\frac{u_1^* - \underline{u}_1}{\lambda}, \frac{u_2^* - \underline{u}_2}{\lambda}) \tag{45}$$

then for $i = 1, 2$, *the closed loop system has the following properties:*
(1) $x_1(.), x_2(.), \beta_i(.) : \mathbb{R}^+ \longrightarrow H^+ \times \{x_2 \in H\, 0 \leq x_2 \leq \overline{u}_3\} \times \mathbb{R}^+$
(2) $\lim\limits_{t\longrightarrow+\infty} \beta_i(t)$ *exists and is finite.*
(3) $\limsup\limits_{t\longrightarrow+\infty} \| e(t) \| \leq \lambda$
and if (43) hold, then $\limsup\limits_{t\longrightarrow+\infty} \| e(t) \| \leq \lambda$.

Proof. The proof of this theorem is similar to the proof of Theorem 4.3. □

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5. Numerical simulation results

To illustrate the performance of the proposed adaptive controllers, we consider a tubular reactor with reaction kinetics modeled by a first order with respect to the reactant concentration C and by an Arrhenius-type dependence for the temperature T, $f(x_1, x_2) = Kx_2e^{-E/Rx_1}$. We consider the following system parameters (Table 1) and the following input constraints:

Table 1. Process parameters for numerical simulations

process parameters	numerical values
v	1 m/s
E	11916 cal/mole
K	0.83 1/s
$\frac{4h}{\rho C_p d}$	$13s^{-1}$
R	1.986 cal/(mole.K)

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$$\underline{u}_1 = 51,\ u_1^* = 90,\ \overline{u}_1 = 91,\ \underline{u}_2 = 359,\ u_2^* = 450,\ \text{and}\ \overline{u}_2 = 630$$

For the desired profile $T^* = 500$, the feasibility assumptions (H_1),(H_2) and (H_3) are satisfied for:

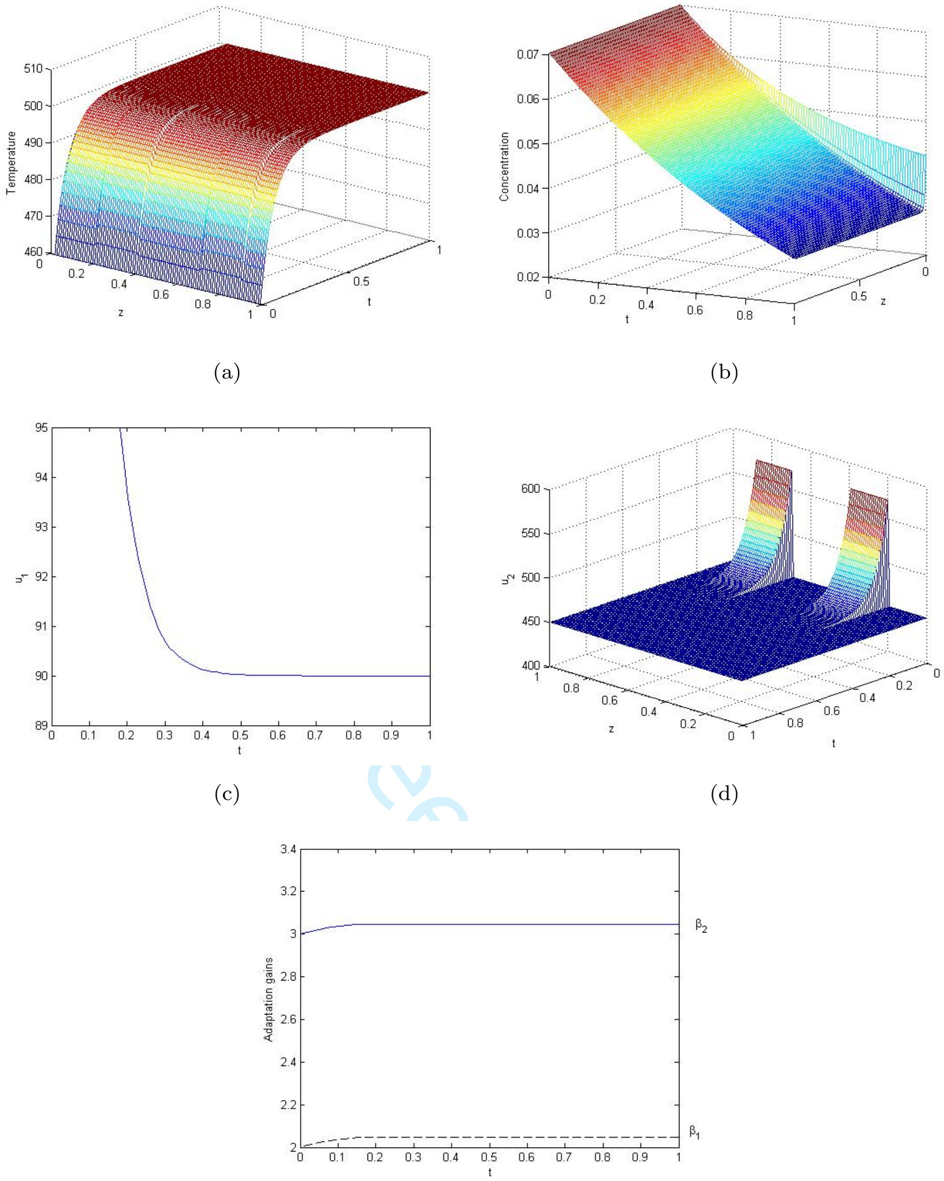
$$\bar{x} = 700,\ x_1(0) = 460k,\ x_2(0) = 0.07,\ \text{and}\ \xi = 0.3.$$

For the local adaptive control, we consider the following parameters:

$$\lambda = 2,\ \delta = 0.1,\ \beta_{0.1} = 2,\ \beta_{0,2} = 3,\ k_1 = k_2 = k_3 = 4, l_1 = 2, l_2 = 3$$

and the measurement zone $\Omega = [0.2, 0.4] \cup [0.7, 0.9]$.
We have considered a finite difference approximation for the space derivatives with 100 spatial discretization steps. We rerun the simulation with an initial temperature $T(0, .) = 800 > \bar{x}$. The results are shown in Figure 1 and 2. In Figure 1 we observe the fast convergence of the reactor temperature to the reference signal $x^* = 500$ (Figure 1a), the convergence of the adaptation gains (Figure 1e), and the controller u_1 and u_2 converge to the reference inputs u_1^* and u_2^* (Figure 1c, Figure 1d). However for an initial temperature larger than $\bar{x}$ (Figure 2), we observe that the temperature T converges to a biased value away from the desired setpoint (Figure 2a), and that the adaptation gains diverge (Figure 2b).

For the global adaptive controller, the results of the simulations are shown in Figure 3 where the same conditions than those considered in Figure 2 have been considered but with the control action $u_3(t)$. From Figure 3, we observe that the temperature in the reactor tends asymptotically to a ball centered at the reference signal $T^* = 500K$ and of radius $\lambda = 2$ (Figure 3a), the concentration remains lower than $\bar{u}_3$ (Figure 3b). We also observer the convergence of u_1,u_2 and u_3 to the reference signals (Figure 3c, Figure 3d and Figure 3e), and the convergence of the adaptation gains (Figure 3f).



(a) (b)

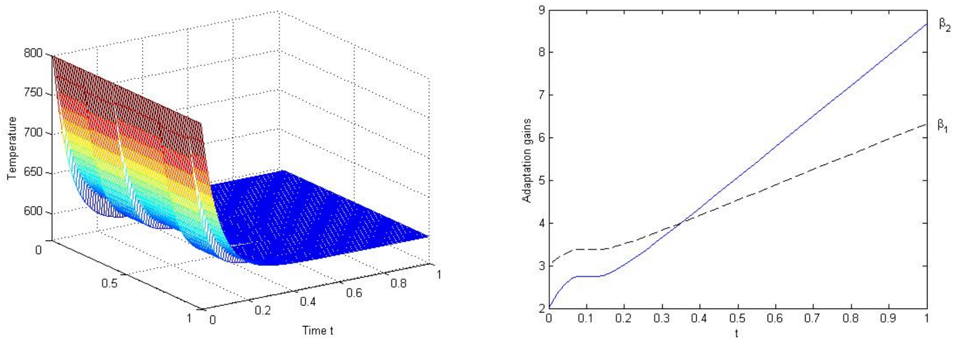
(c) (d)

(e)

Figure 1. Numerical simulation of the adaptive closed-loop system: T(0,.)=460k

6. Conclusion

This paper was devoted to the temperature regulation of the plug flow tubular reactor modeled by a semi linear partial differential equations. A partial temperature measurement zone has been used to regulate the reactor temperature to a ball of arbitrary prefixed radius $\lambda > 0$ centered at the desired temperature profile. The novelty of this work is the use of the non-distributed structures to the inlet temperature and concentration, by replacing the tracking error in the adaptive $\lambda-$ controller used in (Beniich et al, 2010) by its minimum value on the measurement zone with respect to z. The application of this approach required a simple feasibility assumptions that



(a) (b)

Figure 2. Numerical simulation of the adaptive closed-loop system: T(0,.)=800k

depend on natural properties of the chemical reactor, the reaction kinetics and the input constraints. The length of the needed measurement zone depend only on the reference temperature and the control parameters.

7. Appendix

Proof of Preposition 3.1

- The proof of the global existence of the solution and the invariance of the set $(\Delta_1 \times \Delta_2)$ is similar to the proof of the Proposition 3.1 (Beniich et al, 2017)

- We show now that there exists $t_1 > 0$ such that:

$$\|\widehat{e}(t)\| \leq \lambda \qquad \text{for all} \quad t \geq t_1$$

For $\lambda > 0$, we define the following distance function in H:

$$d_\lambda(x) = \max\{\|x\| - \lambda, 0\} \qquad \forall x \in H.$$

For every solution of (9), let us define:

$$V_\lambda(t) = d_\lambda(\widehat{e}(t))^2 \qquad \forall t \geq 0.$$

Let us show that there exists a positive constant c such that:

$$\frac{d}{dt} V_\lambda(t) < -c\sqrt{V_\lambda(t)}.$$

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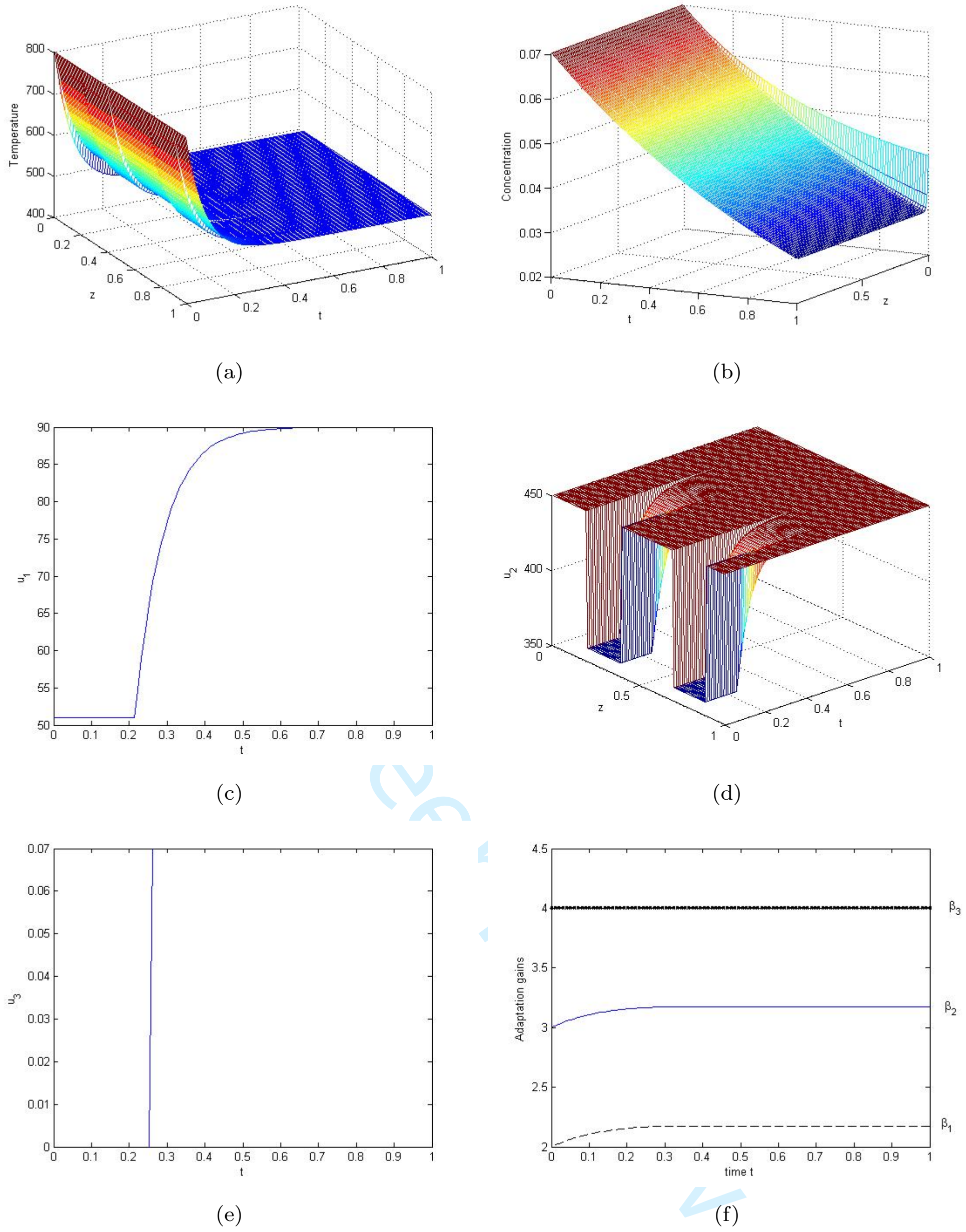

(a) (b)

(c) (d)

(e) (f)

Figure 3. Numerical simulation of the global adaptive closed-loop system

Suppose that $\|\widehat{e}(t)\| > \lambda$, then:

$$\frac{d}{dt}V_\lambda(t) = 2\frac{\sqrt{V_\lambda(\widehat{e}(t))}}{\|\widehat{e}(t)\|}\langle \widehat{e}(t), \dot{\widehat{e}}(t)\rangle$$

$$\begin{aligned}\frac{d}{dt}V_\lambda(t) &\leq n(t)\langle k_0x_1(t) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi u_1(t) - k_0u_2(t) - Ax^*, e(t)\rangle_\Omega \\ &\leq n(t)(\sum_{i=1}^{3}\int_{G_i} e(t)(k_0x_1(t) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi u_1(t) - k_0u_2(t) - Ax^*)(z)dz)\end{aligned}$$

where: $G_1 = \{z \in \Omega \setminus e(t)(z) > \lambda\}$, $G_2 = \{z \in \Omega \setminus e(t)(z) < -\lambda\}$
$G_3 = \{z \in \Omega \setminus -\lambda \leq e(t)(z) \leq \lambda\}$ and $n(t) = 2\dfrac{\sqrt{V_\lambda(e(t))}}{\|e(t)\|}$

If $z \in G_1$, then (27) implies that $u_2(t)(z) = \overline{u}_2$ and by (H_2) :

$$\begin{array}{ll} & \int_{G_1} e(t)(k_0x_1(t) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi u_1(t) - k_0u_2(t) - Ax^*)(z)dz \\ \leq & \int_{G_1} e(t)(k_0x_1(t) - \alpha f(x_1(t), x_2(t)) - k_0u_2(t) - Ax^*)(z)dz \\ < & -\rho\lambda \text{ mes}(G_1) \end{array}$$

If $z \in G_2$, using (27) we find:

$$\beta_1(t)e(t)(z) + u_1^* \leq -\beta_1^*\lambda + u_1^* \leq \underline{u}_1$$

then

$$\beta_1(t)m(t)(z) + u_1^* \leq \underline{u}_1,$$

thus $u_1(t) = \underline{u}_1$ $u_2(t)(z) = \underline{u}_2$. Assumption (H_2) implies:

$$\int_{G_2} e(t)(k_0x_1(t) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi u_1(t) - k_0u_2(t) - Ax^*)(z)dz < -\rho\lambda \text{ mes}(G_2)$$

If $z \in G_3$, and without loss of generality, we suppose that $u_2(t)(z) = \beta_2(t)e(t) + u_2^*$.
First, we assume that $\text{mes}(G_1) + \text{mes}(G_2) < \dfrac{1}{2}$

$$\begin{array}{ll} & \int_{G_3} e(t)(k_0x_1(t) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi u_1(t) - k_0u_2(t) - Ax^*)(z)dz \\ = & \int_{\{z \in G_3 : -\lambda \leq e(t)(z) \leq 0\}} e(t)(k_0x_1(t) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi u_1(t) - k_0u_2(t) - Ax^*)(z)dz \\ + & \int_{\{z \in G_3 : 0 \leq e(t)(z) \leq \lambda\}} e(t)(k_0x_1(t) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi u_1(t) - k_0u_2(t) - Ax^*)(z)dz \end{array}$$

We have:

$$\begin{array}{ll} & \int_{\{z \in G_3 : -\lambda \leq e(t)(z) \leq 0\}} e(t)(k_0x_1(t) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi u_1(t) - k_0u_2(t) - Ax^*)(z)dz \\ \leq & \int_{\{z \in G_3 : -\lambda \leq e(t)(z) \leq 0\}} |e(t)||(k_0x_1(t) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi \bar{u}_1 - k_0u_2^* - Ax^*)(z)|dz \\ & - \int_{\{z \in G_3 : -\lambda \leq e(t)(z) \leq 0\}} k_0\beta_2(t)e^2(t)(z)dz \\ \leq & \lambda C_1 - \int_{\{z \in G_3 : -\lambda \leq e(t)(z) \leq 0\}} k_0\beta_2^* e^2(t)(z)dz, \end{array}$$

and

$$\begin{array}{ll} & \int_{\{z \in G_3 : 0 \leq e(t)(z) \leq \lambda\}} e(t)(k_0x_1(t) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi u_1(t) - k_0u_2(t) - Ax^*)(z)dz \\ \leq & \int_{\{z \in G_3 : -\lambda \leq e(t)(z) \leq 0\}} |e(t)||(k_0x_1(t) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi \underline{u}_1 - k_0u_2^* - Ax^*)(z)|dz \\ & - \int_{\{z \in G_3 : 0 \leq e(t)(z) \leq \lambda\}} k_0\beta_2(t)e^2(t)(z)dz \\ \leq & \lambda C_2 - \int_{\{z \in G_3 : 0 \leq e(t)(z) \leq \lambda\}} k_0\beta_2^* e^2(t)(z)dz. \end{array}$$

Then

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$$\begin{array}{ll} & \int_{G_3} e(t)(k_0x_1(t) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi u_1(t) - k_0u_2(t) - Ax^*)(z)dz \\ \leq & \lambda(C_1 + C_2) - \int_{G_3} k_0\beta_2^* e^2(t)(z)dz \\ \leq & \lambda(C_1 + C_2) - \int_\Omega k_0\beta_2^* e^2(t)(z)dz + \int_{G_1\cup G_2} k_0\beta_2^* e^2(t)(z)dz \\ \leq & \lambda(C_1 + C_2) - k_0\lambda^2\beta_2^* + k_0\lambda^2\beta_2^*(\text{mes}(G_1) + \text{mes}(G_2)) \\ \leq & \lambda(C_1 + C_2) - k_0\frac{\lambda^2}{2}\beta_2^* \\ < & 0. \end{array}$$

Therefore there exists a positive constant θ_1 such that :

$$\frac{d}{dt}V_\lambda(t) < -\theta_1 \frac{\sqrt{V_\lambda(e(t))}}{\| e \|}$$

In the case where $\text{mes}(G_1) + \text{mes}(G_2) \geq \dfrac{1}{2}$

$$\begin{array}{ll} & \int_{G_3} e(t)(k_0x_1(t) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi u_1(t) - k_0u_2(t) - Ax^*)(z)dz \\ \leq & \displaystyle\int_{\varepsilon\leq|e(t)(z)|\leq\lambda} |e(t)(z)|.|[k_0x_1(t) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi \underline{u}_1 - k_0u_2^* - Ax^*](z)| \\ & -k_0\beta_2(t)e^2(t)(z)dz + \displaystyle\int_{0\leq|e(t)(z)|\leq\varepsilon} |e(t)(z)|.|[k_0x_1(t) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi \underline{u}_1 \\ & -k_0u_2^* - Ax^*](z)|dz - \displaystyle\int_{0\leq|e(t)(z)|\leq\varepsilon} k_0\beta_2(t)e^2(t)(z)dz \\ \leq & \displaystyle\int_{\varepsilon\leq|e(t)(z)|\leq\lambda} (-k_0\beta_2^*\varepsilon^2 + \lambda C_2)dz + \displaystyle\int_{0\leq|e(t)(z)|\leq\varepsilon} (\varepsilon C_2)dz \end{array}$$

From (27), we can show that:

$$\int_{G_3} e(t)(k_0x_1(t) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi u_1(t) - k_0u_2(t) - Ax^*)(z)dz \quad \leq \quad \varepsilon C_2$$

Finally, there exists a positive constant θ_2 such that :

$$\frac{d}{dt}V_\lambda(t) < -\theta_2 \frac{\sqrt{V_\lambda(e(t))}}{\| e \|}$$

So in all cases, there exists a positive constant c such that the time derivative of V_λ satisfy for all $t \geq t_1$:

$$\begin{cases} \dfrac{d}{dt}V_\lambda(t) \leq -c\sqrt{V_\lambda(t)} & \text{if} \quad \|e(t)\| > \lambda \\ \dfrac{d}{dt}V_\lambda(t) = 0 & \text{if} \quad \|e(t)\| \leq \lambda \end{cases}$$

We can conclude that there exists $t_1 > 0$ such that: $\forall\ t > t_1, \|\hat{e}(t)\| \leq \lambda$.

- Let us show the third point of the proposition.

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For $t \geq 0$ and $z \in \Omega'$, we have:

$$\dot{e}(t)(z) = Ae(t) + k_0 x_1(t) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi u_1(t) - k_0 u_2(t) - Ax^*$$

The exponential stability and the positivity of $(T(t))$ the C_0-semigroup of A and

$$k_0 x_1 - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi \underline{u}_1 - k_0 u_2^*(t) - Ax^* < -\rho$$

We obtain

$$\begin{array}{rcl} e(t)(z) & = & T(t)e(0)(z) + \int_0^t T(t-s)(k_0 x_1(s) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi u_1(t) \\ & & -k_0 u_2(t) - Ax^*)(z)ds \\ & \leq & T(t)e(0)(z) \end{array}$$

by passage to the limit:

$$\lim_{t \longrightarrow \infty} T(t)e(0)(z) = 0,$$

then there exists t_0 such that:

$$(\overline{x} - x^*)(z) \leq e(t)(z) \leq 0 \qquad \text{for all } t \geq t_0$$

The previous theorem show that there exists $t_1 > 0$ such that $\|\widehat{e}(t)\| \leq \lambda$. So we can assume that $u_2(t)(z) = \beta_2(t)e(t)(z) + u^*$ for all $z \in \Omega$ and $t > t_1$. We consider

$$V_\lambda(t) = d_\lambda(e(t))^2 \qquad \text{for all } t \geq \sup(t_0, t_1)$$

if $\|e(t)\| > \lambda$

$$\frac{d}{dt}V_\lambda(t) = 2\frac{\sqrt{V_\lambda(e(t))}}{\|e(t)\|} < e(t), \dot{e}(t) >$$

$$\begin{array}{rcl} \frac{d}{dt}V_\lambda(t) & \leq & 2\frac{\sqrt{V_\lambda(e(t))}}{\|e(t)\|} < e(t), k_0 x_1(t) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi u_1(t) - k_0 u_2(t) - Ax^* > \\ & \leq & n_1(t) < e(t), k_0 x_1(t) \alpha f(x_1(t), x_2(t)) - v\Delta_\xi \underline{u}_1(t) - k_0 \beta_2(t) e(t)(z) - k_0 u^* - Ax^* >_\Omega \\ & & +n_1(t) < e(t), k_0 x_1(t) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi \underline{u}_1(t) - k_0 u_2^* - Ax^* >_{\Omega'} \\ & \leq & n_1(t)[< e(t), k_0 x_1(t) - \alpha f(x_1(t), x_2(t)) - v\Delta_\xi \underline{u}_1(t) - k_0 u_2^* - Ax^* > \\ & & -k_0 \int_\Omega \beta_2^* e^2(t)(z)dz] \\ & \leq & n_1(t)[C_2 C_3 - k_0 \beta_2^* \lambda^2 + k_0 \beta_2^* \int_{\Omega'} e^2(t)(z)dz] \\ & \leq & n_1(t)[\frac{C_2 C_1}{\lambda} - k_0 \beta_2^* \lambda^2 + k_0 \beta_2^* \omega \|\overline{x} - x^*\|_\infty]. \end{array}$$

Where $n_1(t) = 2\dfrac{\sqrt{V_\lambda(e(t))}}{\|e(t)\|}$.

We have $w < \dfrac{k\lambda^2}{\|\overline{x} - x^*\|_\infty}$ and $\beta_2^* > \dfrac{C_2 C_3}{(1-k)\lambda^2}$.

Consequently, there exists a $\gamma > 0$ such that

$$\begin{array}{ll} \frac{d}{dt}V_\lambda(t) \leq -\gamma\sqrt{V_\lambda(t)} & \text{si} \quad \|e(t)\| > \lambda \\ \frac{d}{dt}V_\lambda(t) = 0 & \text{si} \quad \|e(t)\| \leq \lambda \end{array} \tag{46}$$

Then there exists t_2 such that

$$\forall t > t_2 \ \ \|e(t)\| \leq \lambda$$

□

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