DESIGN AND MANAGEMENT OF NETWORKS WITH FIXED TRANSPORTATION COSTS FOR THE REVERSE FLOWS OF REUSABLE PACKAGES

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A good plan violently executed now
is better than a perfect plan executed next week.

— George S. Patton

This thesis is dedicated to the three women of my life :
my grandmother, my mother and my wife,
who have all taught me and share the core values I live by daily.

ABSTRACT

Reusable packages are logistic items used for the shipments of goods from a producer to its customers and which, once the goods have been consumed by the customers, are to be returned to the producer. This doctoral thesis addresses the design and the management of large independent reverse networks for their return flows.

We address the issues of

- the allocation of the customers to the production points (to which factory should a customer send its empty packages back?)
- the determination of the ideal shipment sizes (at what frequency should a customer return its empty packages?),
- the size of the fleet of reusable packages (how many reusable packages should we hold in the network?),
- the use and locations of intermediary facilities (where should we locate local accumulation depots?), and
- the problem of coordinating the return flows on a day-to-day basis (how should the arrivals of empty packages be coordinated at the factories?).

Return shipments are subject to a fixed transportation cost and packages held in the network bear a linear holding cost. Our methodology relies on the distinction between strategic and operational models.

At the strategic level, we make use of an approximation about how the flows are to be coordinated at the operational level. The strategic models then become tractable and, in most cases, solved analytically. We identify the economic logic by which the allocation of the customers should be made and the shipment sizes determined. We analyze the impact of having a standard fleet of packages when compared to a fleet in which different types of packages coexists with different levels of substitution and the impact of capacity limits on transportation. We evaluate the benefit of standardizing the fleet and show that it can reach up to 30% of the costs.

At the operational level, we tackle the problem of the coordination of the flows. Although it is a well known hard combinatorial problem, our main contribution is to show that, in large granular networks, finding very good solutions to this problem is relatively easy and natural. We suggest a myopic heuristic procedure to solve this problem. We also derive lower bounds on the optimal cost which are then used to assess the performance of the heuristic. The heuristic proves to perform very well on large networks as the optimality gap rapidly falls under a few percent when the network size increases. Additionally, the results achieved at the operational level demonstrate the relevance of the strategic approximation we have used. We evaluate the cost impact of using a discrete review policy instead of a continuous policy. We finally discuss the impact of variability and lead times in transportation.

The study has a strong industrial and managerial focus originating from a collaboration with a large company of the glass industry. Although the insights are derived for reverse networks, the assumptions used in this thesis are relatively general. The models could easily be applied on delivery networks or integrated forward and reverse networks.

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CONTENTS

1

INTRODUCTIVE CHAPTER

1.1 RESEARCH AMBITION

This thesis presents the scientific and managerial contributions of the research that we carried out on the management of reusable packages. The objective of the study was to design models and their solution procedures to derive managerial insights for the implementation and the management of independent reverse networks for returnable packages when fixed costs characterize the transportation process. Our research was carried out in the context of the TransLogisTIC project. A collaboration with *AGC Flatglass Europe* supported the direction taken by our study. Our aim was to identify the dominant economic logics driving the problem and the interaction between the main cost factors and parameters. These managerial insights had to be clearly identified in order to provide logistic managers with a set of guidelines helping them evaluate and implement their system for returning reusable items.

1.1.1 *The Case of AGC Flatglass Europe*

AGC Flatglass Europe produces glass panes, mainly for the automotive and construction industries, in a set of factories in Europe. It delivers its products to a large set of customers across Europe. Figure 1.1[1] illustrates such an industrial network. To perform the expeditions by truck, the glass panes must be fixed to a triangular metallic structure, called a trestle, in order to guarantee the safety of the product during transportation. The loaded trestles cannot be transported by standard trucks. Specially designed trucks are needed to carry the loaded trestles. Third party logistic partners have invested in a fleet of these special trucks and provide the transportation service for the glass industry. Naturally, the price charged for delivering the loaded trestles

1 For confidentiality reasons, the exact locations of the factories and customers has been altered. The network bears however similar characteristics with the network of AGC.

is above the standard transportation rate in the industry. The special trucks are usually loaded with a single trestle, sometimes two, and perform a direct delivery from a single factory to a single customer. Naturally, once the customers have consumed the glass panes, the empty trestles must be returned to the production facilities to be reused for further expeditions. Empty trestles may be folded. Once folded, a truck can be loaded with twenty empty trestles. Moreover, empty trestles do not require special trucks to be transported. They can be loaded into standard trucks.

In the current situation, the return flows are managed by the transporters them-

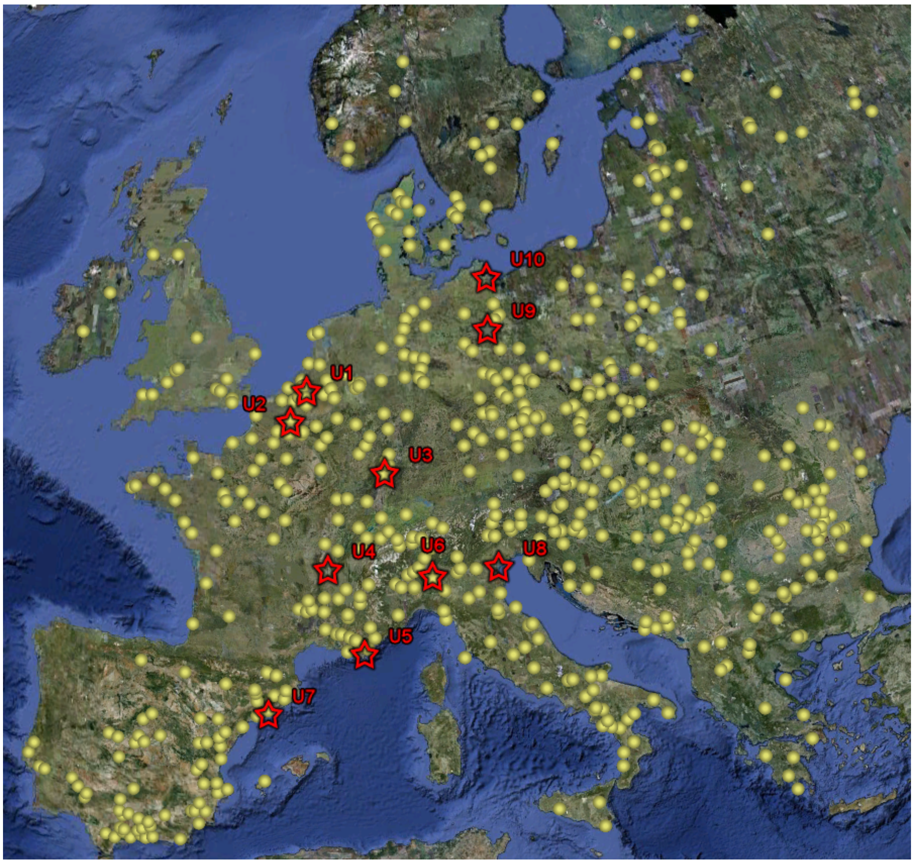

Figure 1.1: An industrial network similar to the network of AGC.

selves. But, for various reasons which will be discussed later, AGC started a study about how to design and manage the reverse network for the return flows of the empty trestles itself. We collaborated in this matter with the Supply Chain Division of AGC by providing the models, the methods and the managerial insights that are presented in this thesis. We focus on the design of a network for the return flows of the empty reusable packages which is independent from the delivery network. We also study the daily management of the reverse network.

1.1.2 *Scope of the Models*

We define the scope of the models we study by clarifying the following four elements:

A. The type of networks studied,
B. The flows that are considered,
C. The costs that are included, and
D. The timing elements

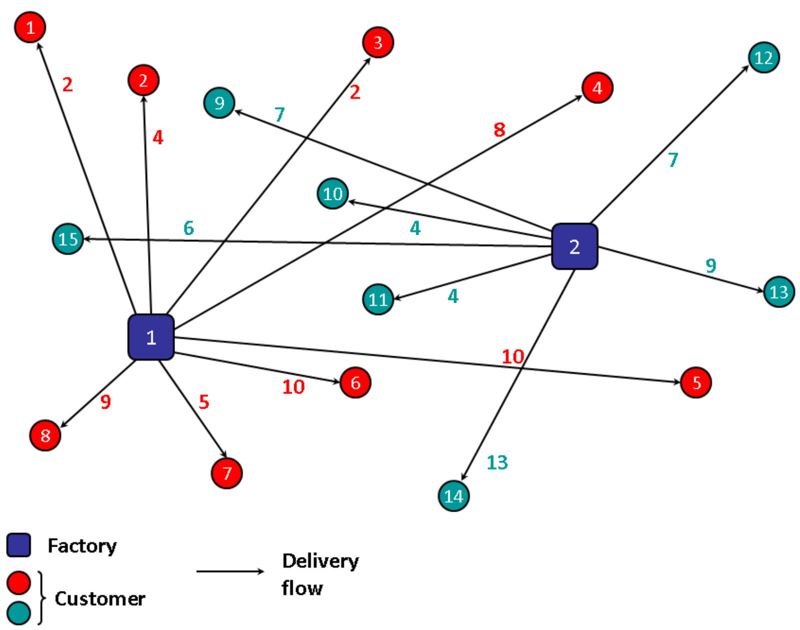

Figure 1.2: An example of the delivery flows in a 2-factory 15-customer network.

A. NETWORK We consider general networks made of F factories and C customers. The index f is used to refer to the factories while the index c refers to the customers. The factories produce goods that are loaded on reusable packages to be shipped to the customers. The set of customers served by each factory may depend on various factors, such as product specificities, regional clustering of the network or technical considerations. Figure 1.2 shows an example of a network with 2 factories and 15 customers. Factory 1 serves 8 customers while Factory 2 has 7 customers. The numbers indicated on the delivery arrows are the average number of loaded packages sent to each customer per period of time, say, one week.

The ambition of the study is to tackle networks of industrial sizes. In the case of AGC, the network includes 10 factories and approximately 600 customers. The methodological choices made in this research are directly linked to this requirement for the models to be suitable for dealing with large networks.

B. FLOWS The object of this study is to elaborate models for the design and the management of independent reverse networks for the return flows of empty packages. The independence of the reverse network originates in the fact that the distribution policy may fundamentally differ from the return policy in terms of, for example, transportation means, transportation frequency or transportation strategy. In the reverse network, the customers having received loaded packages will progressively produce empty packages as they consume the goods. The factories symmetrically consume empty packages as they perform expeditions of loaded packages. Consequently, an inventory of empty packages is built at each customer location and an inventory of empty packages is consumed at each factory. This configuration is illustrated in Figure 1.3.

Accordingly, we define

- Λ_c: the average rate at which the inventory of empty packages is built at customer c.

and

- Γ^f: the average rate at which the inventory of empty packages is depleted over time at factory f.

Although the empty packages "created" by the customers must have been delivered loaded with goods in the distribution process at some point, we make no assumption about delivery flows. In the case where the characteristics of the delivery process lead to modeling assumptions that are similar to the ones we used, the models we present could be applied on the distribution network as well. In this case, the development of an integrated model for forward and reverse flows could be designed by the extension of the models presented.

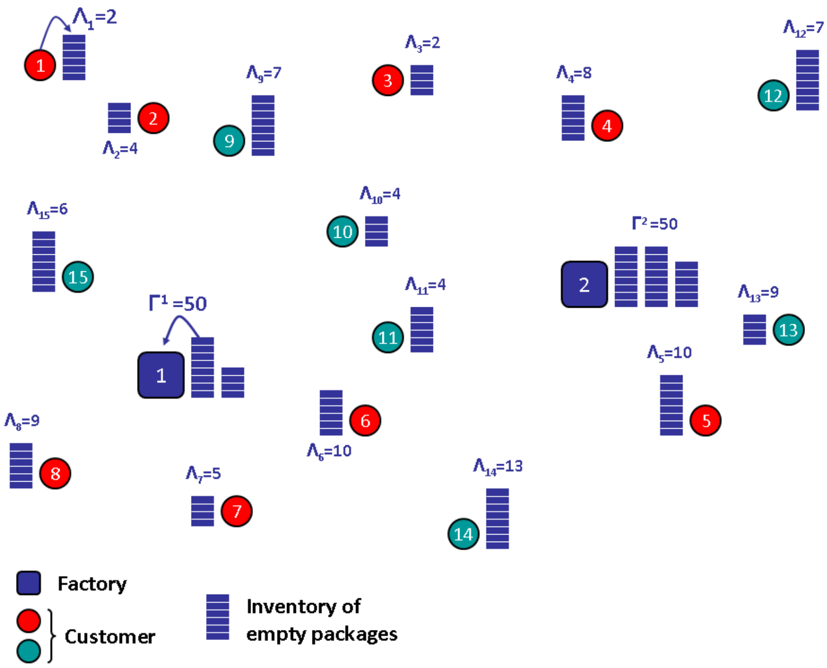

Figure 1.3: The flow input parameters of the reverse network.

C. COSTS Each return flow is performed directly from one customer to one factory or, in some cases, to a regional depot. This point-to-point return flow is subject to a fixed transportation cost which depends on the specific travel for which the truck is ordered but not on the actual load of the truck. This fixed cost includes the cost of the distance traveled, but may also include any ordering charge, travel taxes, tolls,... We define

- O_c^f: the fixed transportation cost charged for a shipment from customer c to factory f.

In practice, some transporters could charge an additional variable cost depending on the load of the truck, but it is not the most common case. Moreover, in this case, the variable cost is much lower than the fixed cost charged by these transporters. Adding variable transportation costs in our models could be done easily, since it does not affect the structure or the complexity of the problems.

The inventory of empty packages held at each location is subject to a holding cost:

- H: the periodic holding cost of one package.

In most of the models presented, the holding cost is assumed to be constant and identical for all locations. This simplifies the presentation of the models. The models would straightforwardly generalize with location-dependent holding costs as they do not affect the structure of the problem. In the case of AGC, the holding cost reflects the investment cost made in the fleet of packages. In this context, the holding cost corresponds to the depreciation value of the packages. In practice, it is likely to also include storage costs. Figure 1.4 displays the cost parameters for our small example.

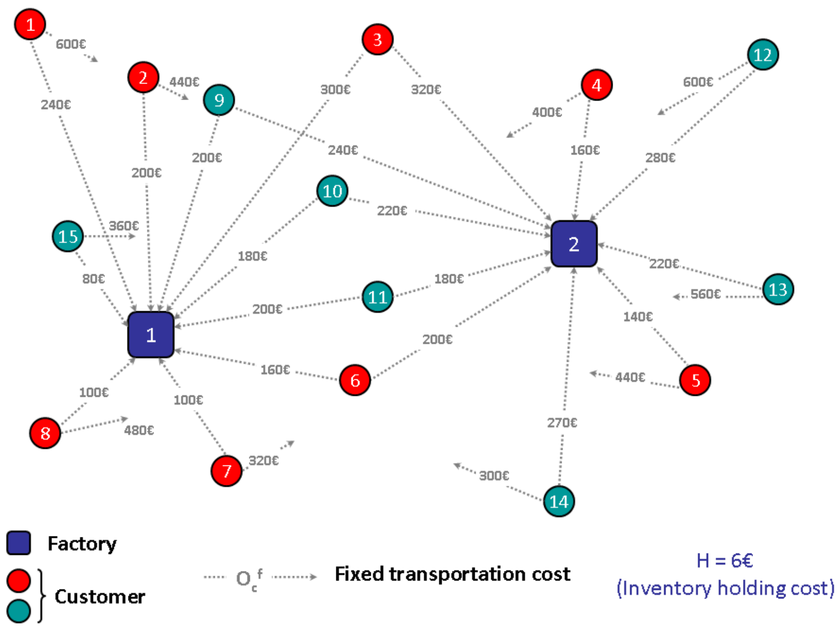

Figure 1.4: The cost parameters of the models.

D. TIMING We consider both continuous and periodic review policies of the inventories in the network. Figure 1.5 illustrates the evolution of a customer's inventory of empty packages in both cases. When a continuous review policy is considered, the exact inventory levels are assumed to be known at each moment. Return flows can be ordered at any time. When a periodic review policy is considered, the evolution of the inventories of empty packages is not impacted. However, the exact inventory level is only known at the moment of the periodic review. Return flows can only be ordered at a review moment. We refer to this case as the discrete case. In both the continuous and the discrete cases, the average inventory of empty packages is identical.

We define

- P: the review periodicity.

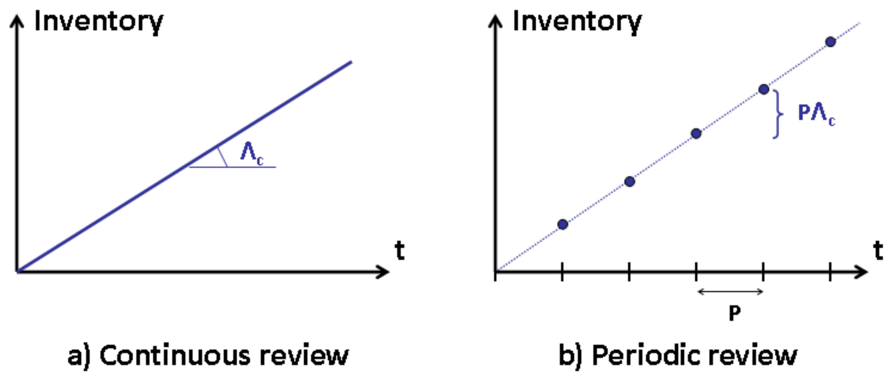

Figure 1.5: Modeling of the inventory position in the continuous and discrete cases.

In this research, we generally model the return flows with zero transportation lead time for the sake of simplicity. This also corresponds to the situation of AGC for which the planning period is one week while the transportation time is of one day. We consequently assume that an inventory of empty packages at a customer can be returned and used at the factory in the same planning period.
A lead time of one period would mean that the return flows ordered at one moment would only be available for consumption at the factory one period later. Considering lead times would not fundamentally impact the models presented. Basically, it would first imply that the decisions about return flows should be based on the forecasted requirements of the factories of a future period (the current period + the lead time) instead of the observed requirement of the current period. In this case, the decision about the number of packages that will be returned in a specific period will be final a certain number of periods earlier. If the flows are not deterministic in this interval, safety stocks should be calculated in a classical way (Chopra and Meindl, 2003). Second, transportation lead times would also imply that a certain number of packages will be "in transit" in the reverse network. These packages should be taken into account in the computation of the total holding cost of the fleet of packages. We do not address these issues in detail, as they do not alter the structure of the problem and because corrective measures can be added afterwards. They will be briefly discussed in Chapter 5.

1.1.3 *Issues Addressed*

The following issues are addressed by our study:
A. How to allocate the flows from the customers to the factories? (λ_c^f)
B. What average shipment sizes should be used? (Q_c^f)
C. What is the optimal size of the fleet of packages? (L)
D. What is the impact of pooling the packages? (standardization of the fleet)
E. How should the flows be coordinated at the factories?
F. What options could allow us to share the fixed transportation costs?

A. ALLOCATION OF THE CUSTOMERS The first question raised in our work is the allocation of the return flows from the customers to the factories. Although customers have been allocated to the different factories in the distribution network, an alternative allocation can be designed in the reverse network as long as the packages are standardized between the distribution clusters of the factories. The objective of this allocation is to reduce the average distances that must be traveled in the network. The variables used to model this allocation are

- λ_c^f: the average flow per period allocated from customer c to factory f.

The allocation is naturally subject to flow constraints ensuring that the total flow allocated to each factory corresponds to its average consumption. In our example, as it can be seen on Figure 1.6, this allocation process reallocates, among others, Customer 15 to Factory 1 and Customer 5 to Factory 2 which decreases the average distances that must traveled in the network. It may be that some customers are allocated to more than one factory, as it is the case with Customer 14. The average flow of this customer is split in order to meet the exact requirement of each factory.

B. RETURN FREQUENCIES The second issue is to determine the average frequencies at which the customers send their empty packages, or equivalently, the average shipment sizes to use in the network. This choice impacts both the transportation and the holding costs. On the one hand, high frequencies would imply high transportation costs along with low truck utilization. On the other hand, low frequencies would allow for lower transportation costs but would immobilize the packages for a longer period at the customers, therefore slowing their rotation rate. It follows that more packages would be required in the network to accommodate the accumulation of packages. Choosing the right frequencies fundamentally consists in balancing the fixed transportation costs and the variable holding costs. We use the variables

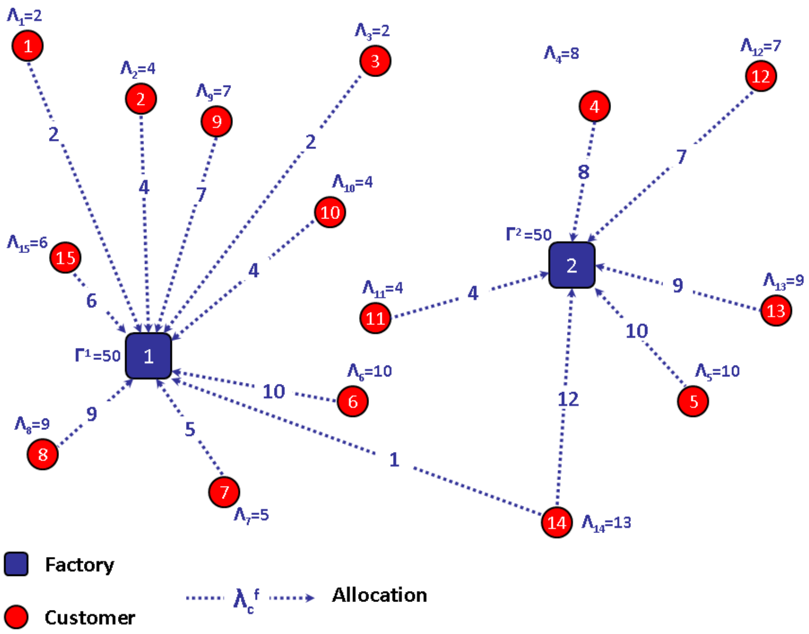

Figure 1.6: An example of the allocation of the customers.

- Q_c^f: the shipment size used for the return flows performed between customer c and factory f.

The return frequencies may be limited by a truck capacity. We denote by

- CAP: the transportation capacity limit, i.e. the maximal truckload.

Figure 1.7 displays the shipment sizes in our example.

C. FLEET SIZE The shipment sizes directly affect the number of packages that must be held by the producer. Letting the packages accumulate in the network requires that a sufficient number of packages be available to sustain the building of the shipment sizes. We distinguish two parts in the fleet of packages:

1. **logistic inventory of packages:** It covers the empty packages present in the inventories of the customers and of the factories. At the customers, these packages are available for being returned and await to be picked up. At the factories, these empty packages have been returned and are available to be used in the production/distribution process. The logistic inventory consequently represents the number of additional packages needed in the network to sustain the accumulation of the chosen shipment sizes. It represents the average number of empty

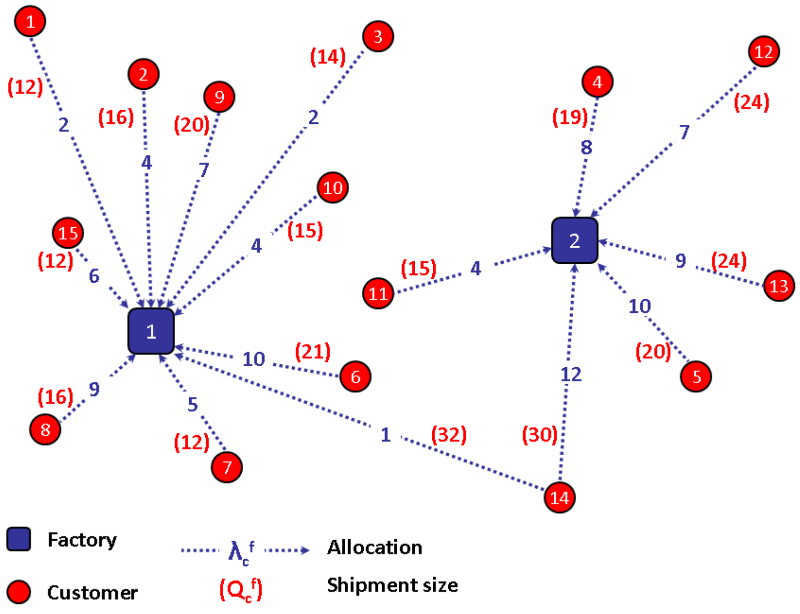

Figure 1.7: An example of the shipment sizes.

packages used in the reverse network. The decisions handled by our models directly impact this logistic inventory. We define

- L: the logistic inventory, i.e. the average number of empty packages required in the reverse network.

2. **in-process inventory of packages:** It covers all other packages held by the producer, such as:

  - the packages held at the factories to sustain the production and storage processes (packages loaded with raw materials or intermediary products in production buffers, packages used in the production line, packages held to store finished goods, packages kept as safety stocks, packages being repaired,...).

  - the packages in transit in the distribution process (packages loaded with goods being delivered).

  - the loaded packages at the customers (packages having been delivered and waiting to be emptied from their goods).

The rationale for distinguishing these two parts of the fleet of packages is that the decisions taken in our models only impact the logistic inventory. The in-process inventory depends on the production and distribution processes and on the time it takes for the customers to empty the packages they receive. It is not affected by the way return flows are organized. As a consequence, we only consider the logistic inventory in our models.

D. POOLING In industrial supply chains, the fleet of packages is rarely fully standardized. Certain types of packages may be dedicated to specific products, or certain production points could be restricted to the use of certain types of packages for technical reasons. Similarly, some customers may impose constraints on the types of packages they are willing to receive. Dealing with different transporters may also lead to the use of different types of packages. Standardizing the fleet of packages is likely to provide various benefits in terms of cost, but also in terms of efficiency of the reverse chain. In order to estimate the significance of a possible investment to suppress the barriers to a standardized fleet of packages, we must identify exactly how pooling the fleet of packages generates savings.

E. COORDINATION OF THE FLOWS Finding the allocation of the customers and the shipment sizes cannot, unfortunately, be done independently of the coordination problem. The coordination problem relates to the way we manage the arrivals of the return flows at the factories. Indeed, this coordination affects the average inventory at the factory, and, in turn, the choice of the shipment sizes and the allocation of the customers.

F. SHARING OF FIXED TRANSPORTATION COSTS The fundamental costs in our models are the fixed transportation costs. We initiated two extensions based on the idea of sharing transportation.

The first extension relies on the consolidation of neighboring customers to share the fixed transportation cost. There are different alternatives for implementing a local consolidation logic. We could design vehicle tours or use consolidation facilities. In this study, we have chosen to consider the use of local accumulation depots where empty packages of nearby customers accumulate together as illustrated in Figure 1.8.

Although the model and the method that would be used for vehicle tours would be significantly different, the logic of consolidation is similar, and we would expect similar in both cases. The idea behind the use of depots is to share, between various customers, the fixed transportation cost incurred on the large distance between the depot and the factory. However, the benefit generated by this consolidation should be higher than the additional costs linked to the implementation of the policy. Indeed, short haul transportation will be necessary to move the packages from the customers to the local depot. Moreover, the implementation of depots may lead to fixed costs and the flows transiting through a depot could be subject to handling costs.

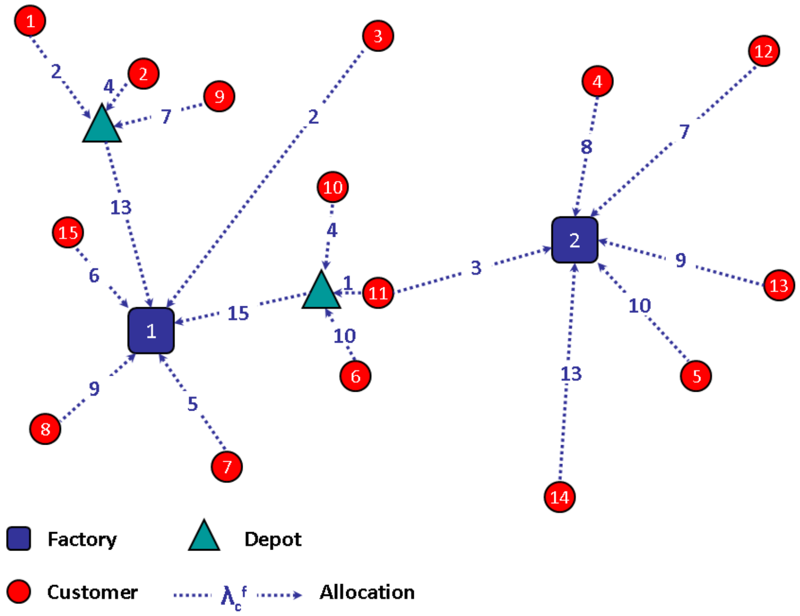

Figure 1.8: An example of a reverse network with local accumulation depots.

A second extension deals with heterogeneous fleets of packages. Although we will demonstrate that the pooling of the fleet leads to important savings, the investment required to achieve a standard fleet of packages may prevent the complete homogenization of the packages. In this second extension, we consider how the flows of different types of packages may share the fixed transportation costs. We assume that the packages are not substitutable, but we explore how the network must be designed if these flows were to share the same transportation means and the benefit that could be realized in this way.

1.2 METHODOLOGICAL APPROACH

The scope of the models studied, as well as the questions that are addressed in this research, situates our work in the category of EOQ-extensions and of discrete lot-sizing. However, a hard combinatorial problem, the coordination of the return flows, is embedded in our problems. We first illustrate how the coordination of the flows leads to a difficult problem. Second, we explain how our methodology can rely on a two levels analysis, based on strategic and operational models, by the identification of an ideal coordination from which an important assumption is derived that allows us to solve the strategic models easily.

1.2.1 *The Coordination Problem*

Let us consider, for example, a single factory having two customers as displayed in Figure 1.9. The shipment sizes from Customer 11 and 15 are 12 and 18 units respectively, i.e. both customers ship their packages at a common frequency of 3 periods. The way these return flows are coordinated at the factory impacts its average inventory. If both shipments arrive at the same time, the average inventory at the factory is maximal. This results from the fact that the shipments received have to wait some time before being consumed. A better coordination could be achieved by delaying the shipments that are not needed. If each shipment arrives at the factory exactly when its inventory drops to zero, we achieve the best possible coordination. One immediatly

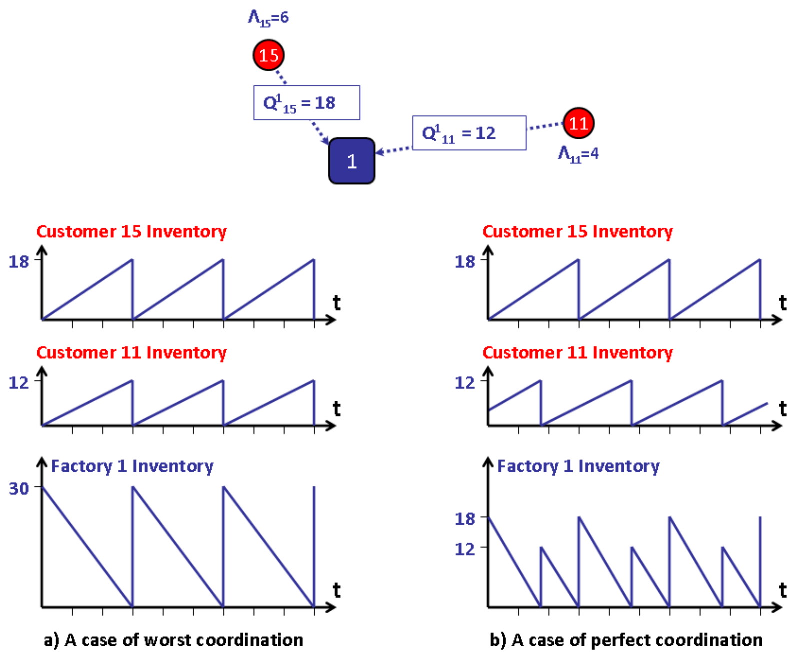

Figure 1.9: Coordination problem - an example of the a) worst coordination b) best coordination.

observes, however, that achieving a perfect coordination with fixed shipment sizes, is only possible if all customers send their empty packages at the exact same frequency. Let us consider, such as in Figure 1.10 a), that the shipment size of Customer 15 would be 12 units. It is straightforward that there will necessarily be a point in time where both shipments arrive at the same time at the factory.

At an operational level, it is likely that optimal solutions will not rely on fixed ship-

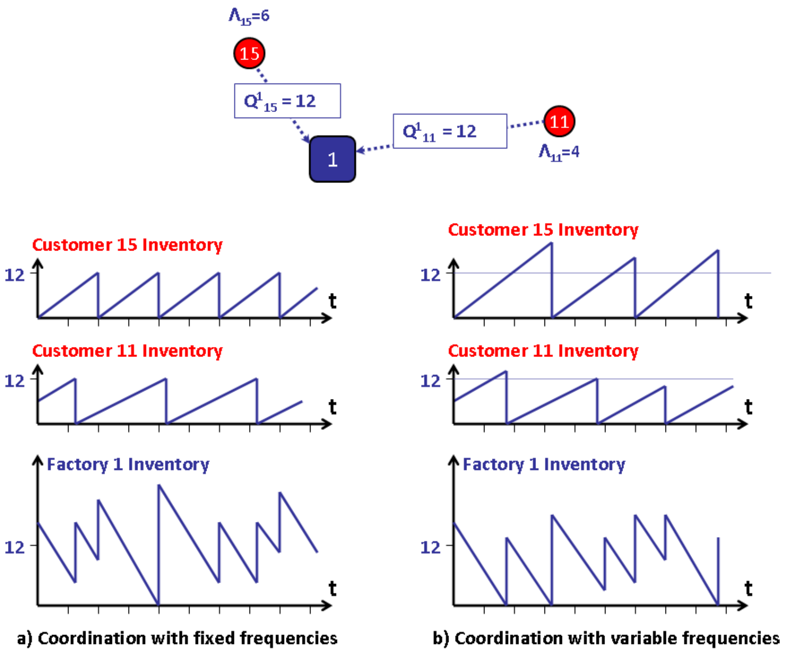

Figure 1.10: Coordination problem - an example with a) fixed frequencies b) variable frequencies.

ment sizes. Instead, they would use variable shipment sizes in an attempt to limit the inventory at the factory. However, this must be done while ensuring that the actual shipment sizes used are not too different from optimal sizes balancing transportation and holding costs. This is illustrated in Figure 1.10 b). This operational problem is particularly difficult, and was referred to as the *staggering problem* (Gallego et al., 1992) in literature.

1.2.2 *The Perfect Coordination Assumption*

The methodology on which we rely develops two fundamental levels of analysis. At a strategic level, we address the network design which includes the customers allocation problem, the determination of optimal frequencies and the possible depot location problem. At an operational level, we handle the problem of coordinating the daily return flows at the facilities.

The most important methodological choice made in this research is the use of the *perfect coordination assumption*. When we use this assumption, we state that the choice of the return frequencies is made without taking the coordination problem into account. We assume that it will always be possible to reach a perfect coordination of the return flows. Using this assumption actually implies that we do not aim at solving the exact coordination problem, but that we make an approximation of the cost solutions that could be achieved in the coordination problem. This choice relies on the fact, that for large networks, which precisely fall into our scope, we will show that

- although the coordination problem is difficult, describing an ideal coordination, the perfect coordination, is very natural;
- moreover, in practice, achieving a coordination of the flows granting cost solutions which are very close to those of perfectly coordinated flows is also very natural and easy, at least when the network is large. This indicates that the assumption is a very reasonable approximation to use at a strategic level;
- finally, under this assumption, the difficulty created by the coordination problem is suppressed and most models become easy to solve.

This methodological choice allows us to connect the two levels of analysis. Indeed, we use the perfect coordination assumption at the strategic level as an approximation of what will be achieved at the operational level. At the operational level, the solutions of the strategic model provide a reference to an ideally coordinated network on which our heuristic relies.

At the strategic level, a continuous infinite timeline is used to determine the optimal design of the reverse network. The rates at which customers make reusable packages available or the rates at which the factories use reusable packages to perform their expeditions are assumed to be constant over time. The aim of the strategic models is to find the optimal allocation, return frequencies and depot locations in a static context representing a long term equilibrium between the different cost factors. The working hypothesis are classical at the strategic level and allow us to clearly underline how the different decisions and parameters impact the cost factors.

The operational level deals with the coordination of the flows at the factories. At this level, the network design decisions have been taken and we are concerned with the daily management of the network. The customer allocation, the number of reusable packages available in the network and the optimal return frequencies are known. Consequently, each factory has a specific set of customers which have been assigned to its cluster and from which it must be replenished with empty reusable packages. The operational problem considers the coordination of the daily flows in each factory cluster separately. We suggest a heuristic which uses as a reference the optimal frequencies determined at the strategic level, since they represent the perfect coordination that we try to achieve.

1.3 RESEARCH BACKGROUND

The research presented in this thesis has been largely influenced by the industrial context in which it was performed due to its insertion in the TransLogisTIC project. This section describes how this specific background has led to the research topic and how it has driven the choice of the models and assumptions. We explain how this industrial context, supported by the collaboration with *AGC Flatglass Europe*, motivated the objectives pursued by the study and the managerial focus adopted throughout the presentation.

In this section, we also relate our work to two important fields of the literature. Our research topic situates our work in the *Reverse Logistics* literature, while the modelling on which we rely relates it to the field of *Inventory Control*.

1.3.1 *The TransLogisTIC Project and the Collaboration with AGC Flatglass Europe*

The TransLogisTIC project was a major collaboration between 23 partners, industrials and academics, which has been financed by the Walloon Region of Belgium. The project was part of the *Plan Marshall* of which the goal was to inspire a development phase in five industrial sectors identified as central for the economic growth and perenity of the Region. One of these five areas was the Walloon logistic infrastructure and knowledge. The investment granted by the Region to the logistic pole of development took the form of the TransLogisTIC project. The objective of the project was to promote the Region's logistic infrastructure by the development of new technologies and the realization of selected studies in the field of logistic management. The *Center of Excellence in Supply Chain Management* (CESCM) of the *Louvain School of Management* (LSM) was one of the four academic partners participating in the project. The CESCM was given the responsibility of several academic studies which were carried out over a period between September 2007 and February 2010.

In a preliminary stage of the project, a research group was given the responsibility to determine the most promising areas in which to perform academic studies by interviewing supply chain managers in the industry in order to identify their current concerns in logistic management. One of the research areas retained by this early exploration phase of the project was the management of the reverse flows of reusable packages[2], with a special focus on options for resource sharing and collaboration. The CESCM was given the responsibility of the studies in this field. In order to guarantee the relevance of the academic study for the industrial actors, a collaboration with the *Supply Chain Division* of *AGC Flatglass Europe* (AGC) was organized throughout the project.

The return flows of the empty trestles of AGC are currently managed by the transporters carrying out the deliveries. They manage these return flows independently so as to provide a sufficient number of trestles at the production points. The price charged by the transporters mainly consists of a fixed cost per truck depending on the distance traveled but independent of the actual load of the truck. It is based on the following transportation pattern. A truck performing a delivery at a customer's site loads an empty trestle at this site and ships it back to the original production site of AGC. Various reasons led AGC to reevaluate the management of these reverse flows.

First, although special trucks are required for the deliveries of loaded trestles, return flows do not specifically require such trucks and could be handled by the use of cheaper standard transportation. Moreover, empty trestles can be folded and, consequently, occupy less space in a truck. Once folded, a regular truck could load up to 20 trestles. Under these assumptions, the transportation scheme charged to AGC implies a very low truck utilization. In practice, the transporters only charge a proportion of the fixed transportation cost for the return flows, based on the fact that the transportation pattern would allow to complete the truck's load with additional orders from other customers of the transporters. However, the proportion of the price that is charged remains much larger than the truck space utilized for the return flows of trestles.

Second, as the special trucks are exclusively used by the glass industry, the investment of transporters in these special trucks has been limited. It is natural that the special transportation capacity is based on the level requirements of the glass industry. As a consequence, during peak periods, competition for this transportation capacity

2 The terms used in the literature to refer to logistic items used for the expeditions of goods and which are reused after they have been emptied are numerous. In this work, we use the term *reusable/empty packages* to refer to these items.

emerges between glass producers, resulting in additional pressure on the transportation rates. Relying on this special truck capacity for the return flows is an additional and unnecessary exposure to this competition.

Third, having subcontracted the management of these flows implies a loss of control and visibility on the disseminated fleet of trestles. It is likely that the transportation scheme used for pricing the service offered by the transporters does not correspond to the actual transportation flows carried out by these transporters. Transporters are bound to perform some optimization of these flows. Due to this loss of visibility, difficulties arise in locating the empty trestles in the network, forecasting exact replenishments of the factories or evaluating the current size and state of the fleet of trestles. This loss of control and visibility makes it hard to assess the efficiency of the return system and creates an environment in which it would be laborious to evaluate the impact of possible collaboration with competitors to share the fleet of trestle or to manage collaboratively the scarce transport capacity.

For these main reasons, AGC was willing to evaluate the opportunity of an alternative self-managed return system. In this alternative system, empty trestles would not be returned by the special trucks performing the deliveries, but would instead accumulate at the customers' sites or at a regional accumulation depot. Based on the requirements of the factories, direct return flows from customers to factories with standard transportation would be ordered. This alternative return system would provide cost benefits by allowing for better truck utilization and the use of standard transportation. It would enable the company to reassess the required fleet size and to gain visibility and control over the process.

The insertion of the study in the TransLogisTIC project promoted a strong industrial and managerial focus reinforced by the collaboration with AGC which guided the whole research. This collaboration had two purposes. First, it allowed us to design models integrating the right assumptions about the industrial processes and tackling the real concerns of the logistic management. On the one hand, we had to provide models and solution methods which were appropriate for solving industrial problems. On the other hand, we aimed at building knowledge which would highlight the fundamental economic logics acting in the management of these return flows. We had to communicate this knowledge about these basic mechanisms to the supply chain managers to help them design their reverse flows management policy. Second, this collaboration gave us an excellent opportunity to test our models on real industrial data and to evaluate our ability to use the solutions to communicate this managerial knowledge to the industrial sector. However, the research carried out by the CESCM could not be specific to the case of AGC. The models which were developed had to be applicable to a large spectrum of industrial contexts. Accordingly, the models studied are general models dealing with large scale networks where fixed transportation costs apply to the return flows. The research accomplished in the TransLogisTIC project produced a series of reports including managerial, technological and scientific elements. This thesis will present the scientific contributions that supported this research.

To sum up, this thesis presents the scientific and managerial contributions of our research on the design and the management of an independent network for the return flows of reusable packages. The objective of the study is to design models and their solution procedures in order to derive managerial insights to help logistic managers evaluate their policy for returning the reusable packages. Although the collaboration with AGC Flatglass Europe led to the various assumptions used in the study, the objective is to provide models that remain generic and, consequently, would be applicable to various industrial contexts. The aim is to identify the dominant economic logics driving the problem and the interaction between the main cost factors and parameters.

1.3.2 *The Reverse Logistics Field of Research*

The issues addressed in this study establish it in the field of reverse logistics which has become an important area of research since the early nineties. Until two decades ago, most efforts in logistic management had been focused, both by the academics and the industrials, on the optimization of the distribution flows and the inventory management of final products and raw materials. The management of the reverse flows had usually been relegated to a secondary position for various reasons (Kroon and Vrijens, 1995; Jayaraman et al., 2003; Dekker et al., 2004).

From a scientific point of view, it was assumed that the models and methods of the distribution literature would be straightforwardly applicable to the symmetric reverse cases. From an industrial and economical point of view, the reverse activities have mobilized less managerial interest because of their status of secondary non-core activities. In this perspective, their management was less fundamental than the strategic distribution management. As it is merely supporting production and distribution activities, the efficiency of the return flows was less critical. It was frequent to simply rely on important buffers of empty packages, such as safety stocks at the factories, implying an oversized fleet of packages. These buffers create an additional cost, but guarantee the perenity of the core processes while allowing for a reduced effort in the management of the secondary activities. This policy was supported by the fact that return flows were representing a small proportion of the total logistic cost, because they handle less critical items with a lower required quality. Consequently, it has become frequent for companies to subcontract these activities to actors of the logistics industry. The third party logistic providers would manage the return flows, while even offering processing activities of the returned items when desired. This was a convenient opportunity to discard the need to manage these activities in-house.

However, several factors have emerged which increased the economic burden and the strategic impact of reverse activities. First, as the rationalization wave infiltrated the processes of the companies in mature markets, it progressively reached the boundaries of core activities and the need to consider support activities has become more present. The lessons learned from the optimization of the classical value-adding activities have shown that the larger part of the savings were achieved by the first improvements, whereas implementing highly efficient processes requires a more intensive effort in optimization. The observation of decreasing yields of the time and resources dedicated to the system improvement has been reinforced by the massive implementation of information systems in logistic management. This implementation offers new opportunities for improvement by making large amounts of quality information available regarding the processes and granting new computing capabilities.

Second, an increasing environmental concern has emerged in the western societies' consciousness, leading to a growing need to justify the use of CO_2 emitting transportation. This societal concern impacting the image of companies has further extended to more concrete constraints as green legislation is voted in the markets in which these companies operate. This green legislation often acts out via taxes and, consequently, increase the relative cost of these return activities.

Thirdly, another recent trend in the scientific literature, as well as in the industry, is to consider the sharing of resources or activities, especially, but not exclusively, when these resources or activities are not of core strategic importance. In order to consider the implementation of collaborative sharing, a careful analysis of the economical implications of the activities to be shared is required. The design of such collaboration will be an important challenge requiring good insights into the logics behind the processes. Even the decision to consider such collaboration requires a thorough analysis in terms of potential gains and losses.

Consequently, over the last two decades, an extended literature has flourished in the field of reverse logistics. Unsurprisingly, this literature includes numerous case studies. It appears largely true that models from the distribution literature have covered most of the important aspects needed in reverse logistics. In terms of models, classical decisions in transportation, network design, fleet sizing, routing, planning and inventory control are found in the reverse logistic literature. The specificities of reverse logistics have not required much development in models and methods. However, they have highlighted some characteristics leading to a different focus when considering reverse flows.

First, the strategic position of reverse activities remains clearly different from the position of distribution activities. The clients of reverse activities are usually internal to the company and the levers for determining the efficiency of these activities are less focused on the markets and more on the production processes. Second, it remains true that reverse activities are secondary support activities. As such, the resources that can be mobilized for the implementation and for the management of the reverse network must be adapted to the economical relevance of these activities. More simple systems are expected, focusing on the main cost drivers and requiring a limited amount of internal management resources. Third, some adaptations of distribution models must be made to take into account the convergent aspect of reverse networks. Beyond some modeling variation, this convergent aspect of the reverse network implies that most of the uncertainty will be moved from the demand side of the network to the supply side, thus from the few large destinations to the many small sources. The sources are usually external to the producer and, consequently, the most important uncertainty will be found in the sources' inventories and their quality.

Although some early publications date back to the seventies and eighties, the true emergence of the reverse logistics literature can be found in the nineties. The literature is dominated by business cases for the reasons that we have previously discussed. This literature has a very extensive scope. It considers many different industrial contexts, different levels and types of decision, and relies on various methods of resolution. We refer to Fleischmann et al. (1997) and de Brito et al. (2002) for reviews of the quantitative models and of case studies respectively. These papers also provide insightful historical contexts and useful categorization of the work presented in this literature. Using the vocabulary used in the reverse logistics literature, we may classify our work as the design of an independent reverse distribution network for returnable distribution items, or reusable packages, with direct reuse by the original producer. We take network design decisions and consider the inventory control. The models are deterministic and consider a transfer system, i.e. a system where the packages belong to the sender who is responsible for managing their return flow.

1.3.3 *The Inventory Control Literature*

The decision structure of our problem locates our continuous models in the field of EOQ-extensions. These extensions are based on the EOQ cost structure dating back to Harris (1915) and are classified in the field of *Inventory Control/Management*. Inventory management is a classical and important field of management science. Its results are presented in many academic reference manuals (Axsäter and Boston (2000) among many others).

In our research, the complexity of the problems we address lies in the coordination of the flows. We must determine the size of all the shipments of empty packages from the customers to the factories. This decision is based on the EOQ cost structure and, consequently, attempts to balance holding and transportation costs. The holding cost depends on the average inventory at the customer locations as well as on the average inventory at the factories. However, the average inventory at each factory is linked to the way the arrivals of empty packages at this factory are coordinated. Let us assume that all return flows arrive at the factory at the same time. In this case, the average inventory of the factory will be relatively large. In order to reduce this average inventory, we could think of a better coordination in which the return flows are spread over time. Ideally, to minimize the average inventory at a factory, each return flow would be received by the factory when its inventory has just dropped to zero. It would then be immediately consumed. But, as discussed previously in Section 1.2, this ideal coordination is rarely accessible.

In this sense, the problem of the coordination of the flows could be seen as finding the best possible coordination of the flows, i.e. the one that minimizes the average inventory at the factory, that can be achieved given that the shipment sizes of each customer are given. Literature has shown that this problem is very hard to solve. The actual coordination problem, however, is even more complex. Indeed, the shipment sizes themselves depend on the average inventory of the factory and, consequently, on the level of coordination that can be achieved. It is even not likely that the shipment sizes used for the return flows of a customer will be constant. The decision about the size of each shipment must assess the punctual impact of this shipment on the inventory of the factory. The optimal solution of this problem would be a sequence of shipments with varying sizes that minimizes the periodic holding costs (at the customers and at the factory) and transportation costs. This optimal sequence could be very long and complex. It will then repeat itself over time. This coordination problem has been largely studied in a literature field referred to as the *Single Resource Multi-Item Inventory System* (SRIMS). Interestingly, the works presented in this literature have generally abandonned the idea of finding the optimal solution.

The SRMIS considers a warehouse storing different items and facing a constant demand for each of these items. The timeline is continuous and infinite. At some point, the inventory of each item must be replenished by a supplier. A replenishment consists of a batch of product delivered by the supplier. A fixed ordering cost is due for each replenishment. The inventory of products held at the warehouse is subject to a linear variable holding cost. The capacity of the warehouse corresponds to an investment that is amortized over time. This is modeled by a linear periodic unit capacity cost. The discrete version of the problem is the *Warehouse Scheduling Problem* (WSP).

The core challenge in this problem is similar to our coordination problem. There are replenishment quantities that would be individually optimal with respect to the balance between the holding and ordering costs, but the coordination of the replenishments impacts the maximum inventory level, which must be accommodated by the capacity of the warehouse. It is obvious that, in the worst case, if no effort is made on coordination, all replenishments could arrive at the same time. It could be preferable to design a coordination pattern where the replenishments are spread over time, in order to limit the maximum inventory level (which relates to our *logistic inventory*). This coordination has been referred to as the *staggering problem* in the literature and has been shown to be NP-complete by Gallego et al. (1992).

Two versions of the SRMIS are found in the literature. The *tactical* SRMIS integrates the capacity limit of the warehouse as a constraint. The maximum inventory level is constrained by this fixed warehouse capacity. The *strategic* SRMIS considers that the warehouse capacity is a decision variable. In our models, we will use a similar distinction between models in which the logistic inventory is chosen without restriction (*variable L models*) and models in which the logistic inventory is constrained by a fixed investment (*fixed L models*).

The first category of solution procedures that appeared in the SRMIS literature actually ignores the staggering problem. It must, consequently, assume a worst case scenario with respect to the coordination of the flows. These methods are referred to as *Independent Solutions* (IS), because the choice of the batch sizes becomes independent for each item if the coordination problem is ignored. It follows that the individual lot sizes are constant for each customer over time and that the maximum inventory level is the sum of these lot sizes. These IS solutions were developed for example by Churchman et al. (1957), Holt (1958), Hadley and Whitin (1963) and Parsons (1966).

These procedures mainly rely on the use of lagrangian multipliers in the tactical version of the problem and are classical in inventory management (Nahmias, 2009).

A second category of procedures attempts to take the coordination into account. It relies on the fact that the optimal coordination is easily determined if all replenishments occur at the same frequency. Procedures relying on a common replenishment frequency for all items are named *Rotation Cycle Policies* (RC). It is a widely used approach for tackling the SRMIS/WSP. It was developed namely in the works of Krone (1964), Maxwell (1964), Parsons (1965), Hommer (1966), Goyal (1974), Silver (1976), Zoller (1977) and Hall (1988). These procedures must handle two problems. First, the common frequency must be determined, then second, the items must be staggered. In the second step, the authors either determine the optimal staggering or prove that no feasible solution exists (due to the warehouse capacity) for the chosen common frequency. In this last case, the first step must be reiterated to find a smallest frequency. The solutions provided under this category are usually optimal for the common frequency, but the authors have noted that this policy is not generally better than IS (Rosenblatt, 1981). Moreover, Page and Paul (1976) note that this category of procedures could only be expected to generate good solutions if the characteristics of the items do not differ much. Otherwise, a common frequency would bring individual batch sizes too far away from individual optimum. They suggest a refinement of these methods by creating groups of items with similar characteristics. Each group has a common frequency and the items in each group are then staggered optimally. The groups themselves are treated as IS.

A more recent category of procedures has increased the space in which solutions are sought. This category, named *Stationary Order Sizes and Intervals* (SOSI), relies on a fixed frequency for each item. However, for this category, the complexity of the staggering problem typically does not allow one to derive optimal solutions and the literature is dominated by heuristics. Gallego et al. (1996) falls in this last category of methods. In this category, more papers can be found referencing works dedicated to the discrete version of the problem such as Hariga and Jackson (1996) and Roundy (1985). These approaches usually use integer multiples for the determination of replenishment frequencies. Haksever and Moussourakis (2005) suggest an MIP model for finding SOSI policies with additional constraints.

Other models are focused on the discrete case when demands faced by the factory are not constant, but are deterministic and known in advance. These models address an issue which is more related to the discrete lot-sizing literature originating in the

Wagner and Whitin (1958) model. Dynamic heuristic procedures for lot-sizing are reviewed in Wemmerlöv (1982) and Zoller and Robrade (1988). Among these dynamic procedures for the WSP, we underline three important references. The method of Günther (1990) starts from a lot-for-lot policy for each item and then tries to increase the lot-sizes in each successive period based on a benefit criterion. Dixon and Poh (1990) first solve the single-item problems independently then consider anticipating or postponing replenishments when the capacity constraint is violated. Axsäter (1980) starts from a lot-for-lot policy, then considers the savings gained by combining every two consecutive replenishments.

In addition to the development of heuristic procedures to solve the staggering problem, the SRMIS literature has focused on deriving lower bounds on the maximum inventory level and on the cost solutions. First, Anily (1991) provided a lower bound for the strategic SRMIS with respect to all SOSI policies. Later, Gallego et al. (1996) showed that this lower bound was actually valid for any feasible solution of the strategic SRMIS. They further extended the lower bound for the tactical SRMIS for any policy.

It appears that the SRMIS/WSP literature has, however, focused primarily on solving the combinatorial problem itself. But, at the same time, due to the complexity of the problem, it has relied on assumptions about the replenishment frequencies (common frequencies, constant frequencies,...) which are very unlikely to be optimal (because optimal solutions will typically not be characterized by constant replenishment frequencies). A very interesting exception is the paper by Güder et al. (1995) in which the replenishment quantities vary between the IS quantities and the unconstrained EOQ for each item.

Our approach in this thesis is somewhat different. We actually do not aim at solving the combinatorial coordination problem but, instead, we strive to find a very intuitive and simple approximation of optimal cost solutions in the long term. This results from the fact that we wish to tackle networks for which the size (industrial size) and the complexity (multi-factory) would make the actual combinatorial problem intractable. Instead of focusing on finding a sequence that would be a good solution to the combinatorial problem, we aim at finding the underlying logic for organizing a good coordination in practice. Moreover, remaining focused on the combinatorial problem with constant rates is likely to produce solutions that are not adapted to deal with an environment where the flows would be variable. Our method relies on the identification of a long term equilibrium of the system, derived from a strategic model, which is then applied to the operational coordination problem. Our objective is to use the long term economic logic to derive a robust procedure which has the flexibility to adapt to the state of the network. In other terms, in its structure, our problem bears many similarities with the SRMIS/WSP but our focus is very different. For example, the lower bound derived by Anily (1991) and Gallego et al. (1996) appears to be applicable to our continuous problem, after making some adjustments to take the differences between the problems into account. However, no interpretation of the underlying economic logic has been presented by these authors. In this work, we analyze the interpretation that lies behind the lower bound and use this interpretation to derive a robust heuristic for handling the problem on a larger scale and with flows which may be variable.

It is worth mentioning a recent work by Huang et al. (2005) who has also dealt with an extension of EOQ-like models with multiple factories and multiple customers. In this problem, lot sizing and customer allocation decisions must be taken. The objective function includes variable transportation costs and the main focus is on production costs. Batch production campaigns have a fixed setup cost so that the allocation of customers to a factory would reduce the amount of fixed costs to be paid. In this problem, all items are produced at the same frequency at a factory and the setup cost is paid for the production of the batches of all items at the factory. The problem addressed by these authors is significantly different than the problem we consider. However, their solution procedure relies on a decomposition that bears some resemblance with our solution approach in Chapter 2.

1.4 STRUCTURE OF THE THESIS

The thesis is divided into two parts. The first part deals with the strategic models and contains three chapters. The second part tackles the operational problem and contains two chapters.

Strategic Level

The second chapter is dedicated to the development of the fundamental strategic models, which will show easy to solve. It presents the analysis of networks with several factories and several customers but without local depots. It first deals with the trivial case of one factory and one customer, then with the 1-factory C-customer case where the pooling issue is largely discussed. Capacity constraints are considered in each case. Finally, it tackles the F-factory C-customer case. As previously discussed, we demonstrate in this chapter that optimal solutions can be found easily for all cases. Analytical solutions enable us to clearly identify the main economic logics and their trade-off to provide the essential managerial insights of the study. The content of this second chapter was originally presented in a paper submitted under the title *Design of a network of reusable logistic containers* by Jean-Charles Lange and Pierre Semal. The paper was accepted as Core Working Paper 2010/9.

In the third chapter, we consider the logic of locally aggregating customers' inventories in order to share the fixed transportation costs. We present the non-convex model for determining the allocation of the customers, the optimal fleet size, the location of the depots and the optimal frequencies on each link of the network. We solve the model heuristically by using the fact that only some variables prevent the model from being linear. This third chapter corresponds to a paper presented to the *3rd International Conference on Information Systems, Logistics and Supply Chain* (ILS 2010) held in Casablanca in April 2010 under the title *Should reusable items be accumulated in Regional Depots?* by Jean-Charles Lange, Jean-Sébastien Tancrez, Pierre Semal, Géraldine Strack and Kevin Evrard.

The fourth chapter considers the extension of the strategic model to a heterogeneous fleet of packages sharing transportation. In this chapter, we show how the problem can be solved as a convex optimization problem and we focus on the managerial interpretation of the results. Some straightforward recommendations are derived which demonstrate that important managerial lessons can be immediately identified. This chapter is based on a paper originally presented to the *3rd International Conference on Information Systems, Logistics and Supply Chain* (ILS 2010) under the title *Design of a network for the return flows of a multi-type fleet of logistic containers* by Jean-Charles Lange, Pierre Semal and Kevin Evrard.

Operational Level

The fifth chapter addresses the operational problem. We suggest a myopic heuristic determining which return flows to perform in each planning period. It is based on the minimization of a global value attributed to the network state. In this fifth chapter, we also present important lower bounds which allow us to assess the efficiency of the heuristic procedure. The problem is considered under constant flows and with variable flows. We will show that, in some particular cases, the variability can lead to an improvement of the operational cost of the system. This chapter corresponds to a paper submitted to the *European Journal of Operational Research* in June 2010 with the title *Daily management of the flow of empty containers*, by Jean-Charles Lange and Pierre

Semal.

In Chapter 6, we consider the operational problem over a finite discrete horizon with known flows formulated as a mixed integer program. We will show the equivalence of our problem with a multi-item lot-sizing with joint inventory bounds and discuss the instances that we could expect to solve after using reformulations and deriving valid inequalities. This short chapter will highlight the difficulty of the operational problem and the fact that we could only solve small instances to optimality.

In the concluding chapter, we focus on highlighting the main results of the study and discuss the applications of the methods which were developed. We discuss the perspectives opened by the study and summarize the methodological and managerial contributions of our research.

Part I

STRATEGIC MODELS FOR THE DESIGN OF THE REVERSE NETWORK

2

FUNDAMENTAL STRATEGIC MODELS

2.1 INTRODUCTION

In this chapter, we present strategic models for designing an independent reverse network for empty packages in which the fleet of packages is fully standardized. We consider a general network with several customers (sources) and several factories (destinations). In this network, empty packages progressively accumulate at the customer locations as the customers consume the goods loaded on the packages they have previously received. At the factories, the inventories of empty packages are depleted over time as they consume empty packages in their production/distribution processes. At some point, the inventory of empty packages of each customer will be returned to a factory. The return flow is realized directly from a single customer to a single factory and is subject to a fixed transportation cost which is independent of the load. A linear holding cost is considered for each package held in the network.

The questions that are handled in the strategic models presented are the following:

Q1: To which factory should each customer return its empty packages?
Q2: At what frequency should they be returned ?
Q3: How many packages are needed in the network?

The first question addresses the allocation of the flows originating at the customers to the different factories. Because the packages are all identical, each customer may return its packages to any of the factories, without consideration for the factory that had sent these packages to the customer in the first place. The managerial logic behind this allocation problem is to design a network in which the average distances that are traveled are as small as possible. In other words, customers should ideally be allocated to the closest factory. However, the total average number of empty packages received by each factory has to correspond exactly to its average consumption.

The second question deals with the sizes of the shipments of empty packages. The transportation costs decrease with increasing shipment sizes. But, the holding costs increase if the shipment sizes become larger, as more packages are needed in the network to compensate for the immobilization of empty packages at the customer locations.

This immediately raises the third question about the total number of packages needed to sustain the choice of the shipment sizes.

These questions are interdependent. The link between Q3 and Q2 is straightforward. For example, if we have few containers (Q3) then it is clear that they must rotate quickly. This means that the return flows will be, at least on short distances, more frequent (Q2). The dependence with Q1 is more complex. But we will show that, given a total inventory in the system (Q3), the optimal allocation of customers to factories varies with the capacity constraint on the return flows, if any. However, we will also show that, in some cases, Q1 can be solved independently of Q2 and Q3. This problem is, at the same time, a transportation problem (Q1) and an inventory control problem (Q2 and Q3).

We use the model to identify the managerial insights that must be highlighted in the design of the reverse network, such as the benefit that is offered by a standardized fleet of packages, the impact of transportation capacity limits or the logic that lies behind the allocation problem.

As we adopt a strategic perspective, we aim at the characterization of a static long-term equilibrium which minimizes the logistic costs. The assumptions that we use are common for strategic models. We rely on constant average flows and a continuous timeline. Shipments may be ordered at any time. They are performed immediately and we assume, for simplicity, that the transportation lead time is zero[1]. We also rely on an important assumption, the *perfect coordination assumption*, for the coordination of the flows at the factories which impacts their average inventories.

The chapter is structured as follows. In Section 2.2, we present the general mathematical model and clarify the variables and parameters. The solutions of the problem for increasingly complex cases are then detailed in Sections 2.3 to 2.5. Section 2.3 focuses on the "1-factory 1-customer" case while Section 2.4 deals with the "1-factory

1 We refer the reader to Section 1.1 for discussions about the general modeling assumptions. Adding positive transportation lead times in the models can easily be done.

C-customer" case. Since neither of these cases raises the issue of allocating customers to factories, we simply focus on the analysis of the optimal inventory of packages and on the optimal return frequencies, that is, Q2 and Q3. Not only can these two questions be analytically solved but the benefit of pooling the packages among all the customers can be quantified. Section 2.5 deals with the solution of the general "F-factory C-customer" case. We there show under which circumstances Q1 may (or may not) be solved independently of Q2 and Q3. In each of these sections, we also consider the impact of a potential capacity constraint on transportation. Finally, we conclude the chapter by underlining the main contributions of the study.

2.2 THE MODEL

Let us, first, define the **parameters** of the model. We let $NF = \{1, ..., F\}$ denote the set of factories and $NC = \{1, ..., C\}$ the set of customers[2].

The formulation is based on constant average rates. In other words, we assume deterministic stable demand and supply:

- Λ_c: the average constant rate at which the inventory of empty packages is built at customer c, and
- Γ^f: the average rate at which the inventory of empty packages is depleted at factory f.

The network has balanced flows, i.e. we have that $\sum_c \Lambda_c = \sum_f \Gamma^f$.

The costs included in the model are

- O_c^f: the fixed transportation cost charged for a shipment from customer c to factory f, which does not depend on the load of the shipment, and
- H: the periodic holding cost of one package.

The horizon is assumed to be continuous and infinite. We focus on the average cost per period. The return shipments are performed between a single customer and a single factory. We do not consider vehicle tours or intermediary consolidation facilities (such facilities are dealt with in Chapter 3). The holding cost is assumed to be identical for all locations[3].

2 When not otherwise specified, all operations with the index c (respectively f) correspond to operations over all elements of NC (NF).

3 This choice was discussed in Section 1.1.

Second, we define the **variables** of the model. These variables are used to represent the allocation of the customers and the choice of the shipment sizes.

- λ_c^f: the average number of empty packages sent from customer c to factory f per period, and
- Q_c^f: the size of the shipments used for transportation between customer c and factory f.

Additionally, we define

- L: the logistic inventory, i.e. the average number of empty packages required in the network[4].

This variable, however, depends explicitly on the other variables as indicated in the mathematical formulation below. In this chapter, we consider two different versions of the problem. In the first version, the logistic inventory may be chosen without any restriction. This would correspond to a longer term vision of the problem in which the investment in the fleet of packages can be adapted. In the second version, representing a more medium term vision of the problem, the logistic inventory cannot exceed a certain amount $\bar{L}$ implied by the actual investment made in the fleet of packages. $\bar{L}$ is the number of empty packages actually available to be used in the reverse network. The corresponding versions of the problem are named respectively *variable L* or *fixed L* models.

The two versions of the model can be stated in their general form as follows:

4 Note that this number does not correspond to the total number of packages held in the network. Section 1.1 discusses the composition of the fleet of packages. We indicated that the decisions integrated in the model only impact a part of this fleet, which is the logistic inventory. The logistic inventory is defined as the additional number of packages that are required to allow the accumulation of empty packages prescribed by the choice of the shipment sizes.

VARIABLE L MODEL

$$\text{(VAR)} \quad \min_{(L,\lambda_c^f,Q_c^f)} \quad LH + \sum_{c,f} \frac{\lambda_c^f O_c^f}{Q_c^f} \tag{2.1}$$

$$\text{s.t.} \quad \sum_c \lambda_c^f = \Gamma^f \qquad \forall f \tag{2.2}$$

$$\sum_f \lambda_c^f = \Lambda_c \qquad \forall c \tag{2.3}$$

$$L = \sum_{c,f} \frac{Q_c^f \lambda_c^f}{2\Lambda_c} + \sum_{c,f} \frac{Q_c^f \lambda_c^f}{2\Gamma^f} \tag{2.4}$$

$$Q_c^f \leq CAP \qquad \forall c,f \tag{2.5}$$

$$\lambda_c^f, Q_c^f \quad \geq 0 \qquad \forall c,f \tag{2.6}$$

FIXED L MODEL

$$\text{(FIX)} \quad \min_{(\lambda_c^f,Q_c^f)} \quad \sum_{c,f} \frac{\lambda_c^f O_c^f}{Q_c^f} \tag{2.7}$$

$$\text{s.t.} \quad \sum_c \lambda_c^f = \Gamma^f \qquad \forall f$$

$$\sum_f \lambda_c^f = \Lambda_c \qquad \forall c$$

$$L = \sum_{c,f} \frac{Q_c^f \lambda_c^f}{2\Lambda_c} + \sum_{c,f} \frac{Q_c^f \lambda_c^f}{2\Gamma^f}$$

$$L \leq \bar{L} \tag{2.8}$$

$$Q_c^f \leq CAP \qquad \forall c,f$$

$$\lambda_c^f, Q_c^f \quad \geq 0 \qquad \forall c,f$$

The objective (2.1) for the variable L model is to minimize the sum of the periodic holding and transportation costs. In the fixed L model, the holding cost is a constant, and we minimize the periodic transportation cost (4.6). Note that λ_c^f / Q_c^f is the number of shipments performed during one period from customer c to factory f.

The two first constraints are flow conservation constraints. A factory f requires for its further expeditions Γ^f empty packages per time unit. Constraint (2.2) expresses that this demand must be exactly met by the sum of the contributions of the customers to this factory. Similarly, each customer c releases packages at the rate Λ_c. Constraint (2.3) expresses that this global supply must be partitioned among the different factories.

Constraint (2.5) expresses the fact that there could be some limit on the transport capacity. In this chapter, we systematically address our problem with and without this constraint.

Constraint (2.4) is more complex. It expresses the fact that the total number L of empty packages used in the system is directly determined by the sum of the average number of packages held at each location of the network.

Let us consider a customer c. Empty packages accumulate at the speed Λ_c and are then shipped to a given factory f with a shipment size equal to Q_c^f. The quantity (λ_c^f/Λ_c) represents the portion of time customer c is accumulating packages for factory f. During this portion of time, the inventory varies from 0 to Q_c^f. The average inventory during this time is $Q_c^f/2$. The average inventory at customer c is therefore

$$\sum_f \frac{Q_c^f}{2} \frac{\lambda_c^f}{\Lambda_c}$$

The first term on the right hand side of constraint (2.4) thus expresses the sum of the average inventories over all the customer sites.

We apply a similar reasoning to the factories. Let us assume a factory f has no empty packages left. At that time, it receives the shipment Q_c^f sent by customer c. It will consume the received packages at the speed Γ^f. During this time, its average inventory will be $Q_c^f/2$. Since, the quantity (λ_c^f/Γ^f) represents the portion of the time the factory f lives from the packages sent by customer c, the average inventory at factory f is given by

$$\sum_c \frac{Q_c^f}{2} \frac{\lambda_c^f}{\Gamma^f}$$

By summing up all the factories we obtain the total average inventory on the factory sites. This is the last term of constraint (2.4). Because the number of packages emptied by the customer per period is exactly equal to the number of empty packages used at the factory, we have that the number of empty packages used in the network is constant over time. It equals the sum of the average inventories of all locations.

The crucial assumption in the model is that we assume "perfect coordination". It means that a customer c sends its packages to factory f when it has reached its shipment size Q_c^f. At the same time, the factory f receives this shipment size Q_c^f exactly when needed, that is, when its inventory has just dropped to zero. This perfect coordination is discussed in more detail in Section 2.4. This assumption leads to the fact that the suggested solutions are optimistic and are sometimes referred to as lower bounds. At an operational level, special attention should thus be given to the issue of coordination. However, at the strategic level, since we aim at comparing different solution schemes and at drawing lessons from these comparisons, this assumption presents the advantage of keeping the problem tractable without introducing obvious biases. Chapter 5 will be dedicated to the operational coordination problem and to the evaluation of the strength of the perfect coordination assumption.

In the fixed L model, the number L of packages used in the network may be constrained if the investment in the fleet of packages cannot be adapted on the medium term. This fixed investment allows for $\bar{L}$ empty packages to be used in the reverse network as indicated by constraint (2.8). Because the objective function (4.6) decreases with increasing shipment sizes, and because L increases with the shipment sizes, we will always have that $L = \bar{L}$ if no transportation capacities are included. However, due to capacity constraints, it is possible that the number of empty packages used in the network does not reach the limit $\bar{L}$. In this case, there are some excess packages that are never used in the network.

2.3 1-FACTORY AND 1-CUSTOMER

The simplest case arises when a single factory deals with a single customer. The customer accumulates empty packages at a constant rate while the factory consumes empty packages at the same rate. Referring to our model notation, we have here:

$$\lambda_1^1 = \Lambda = \Gamma;$$
$$Q_1^1 = Q$$

The variable L uncapacitated version of the problem $(VAR - U)$ reduces to a classical economic order quantity problem with the following cost function:

$$\min_{(Q)} \quad \frac{Q}{2}H + \frac{Q}{2}H + \frac{\Lambda}{Q}O \tag{2.9}$$

where the first term is the holding cost linked with the average inventory at the customer, the second term is the holding cost linked with the average inventory at the factory and the third term of the sum is the transportation cost associated with Q. The holding cost grows linearly with the shipment size, while the transportation cost is a monotone decreasing function of the shipment size. Figure 2.1 shows the classical inventory evolution at the customer and at the factory, while Figure 2.2 depicts the evolution of the total logistic cost with the shipment size. The optimal solution Q^* is

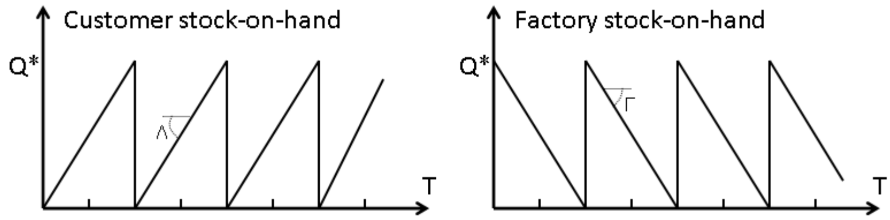

Figure 2.1: Inventory of empty packages at the customer and at the factory.

found by setting the first derivative of the convex objective function to zero:

$$Q^* = \sqrt{\frac{\Lambda O}{H}} \tag{2.10}$$

This quantity differs from the classical economic order quantity by a factor $\sqrt{2}$. This is

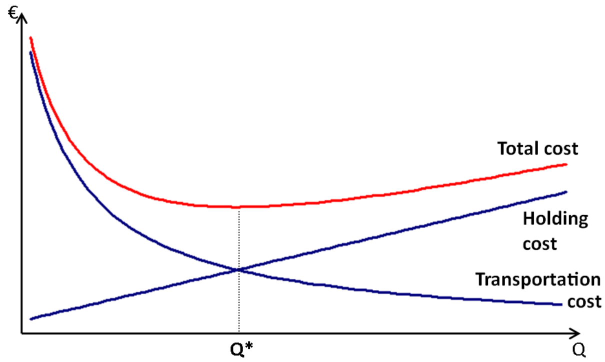

Figure 2.2: Cost structure and optimal shipment size.

explained by the fact that we consider holding costs for the inventory at the customer and at the factory. Otherwise, this quantity shares similar characteristics with the classical EOQ, among others, the equality of the holding and transportation costs and the relatively flat slope around the optimal quantity. This leads to a total logistic cost of

$$V(Q^*) = 2\sqrt{\Lambda OH} \tag{2.11}$$

If there is a constraint CAP on the transported quantities $(VAR - C)$, since the total cost (2.9) is a decreasing function of Q up to Q^*, the optimal solution becomes

$$Q^* = min\{\sqrt{\Lambda O/H}, CAP\} \tag{2.12}$$

Because the transportation cost function is a decreasing function of Q, the optimal solution of the fixed L uncapacitated problem (FIX-U) is

$$Q^* = \bar{L} \tag{2.13}$$

For the capacitated problem (FIX-C) it is

$$Q^* = min\{CAP, \bar{L}\} \tag{2.14}$$

2.4 1-FACTORY AND C-CUSTOMER

In this section, we consider a single factory having a set NC of C customers. Each customer produces empty packages at a rate Λ_c and the factory consumes them at a rate $\Gamma = \sum_c \Lambda_c$. Referring to our model notation, we have here:

$$\lambda_c^1 = \Lambda_c$$
$$Q_c^1 = Q_c$$

2.4.1 *Variable L model*

NON-POOLED PACKAGES

Let us first consider the uncapacitated problem in which each customer has its own specific type of packages. The factory may only send this specific type to the corresponding customer. In this case, let us assume that the consumption of the factory for each type of package is constant and equal to $\Gamma_c = \Lambda_c$. In this case, it is straightforward to conclude that the problem decomposes into C independent "1-factory 1-customer" problems. For each customer, we have the optimal shipment size given by

$$Q_c^{[np]*} = \sqrt{\frac{\Lambda_c O_c}{H}} \tag{2.15}$$

where $[np]$ refers to the fact that the packages are not pooled among the customers. The optimal logistic inventory is the sum of the average inventories at the customer and at the factory

$$L^{[np]*} = \sum_c \frac{Q_c^{[np]*}}{2} + \sum_c \frac{Q_c^{[np]*}}{2} = \sum_c Q_c^{[np]*} \tag{2.16}$$

and the optimal total logistic cost is

$$V^{[np]}(\{Q_c^{[np]*}\}) = 2\sqrt{H}\sum_c \sqrt{\Lambda_c O_c} \tag{2.17}$$

In this case, the factory must hold C different inventories, each being consumed at a constant rate Γ_c, as illustrated on Figure 2.3 for 2 customers having optimal shipment sizes of 20 units. Blue packages of Customer 1 and red packages of Customer 2 are not substitutable.

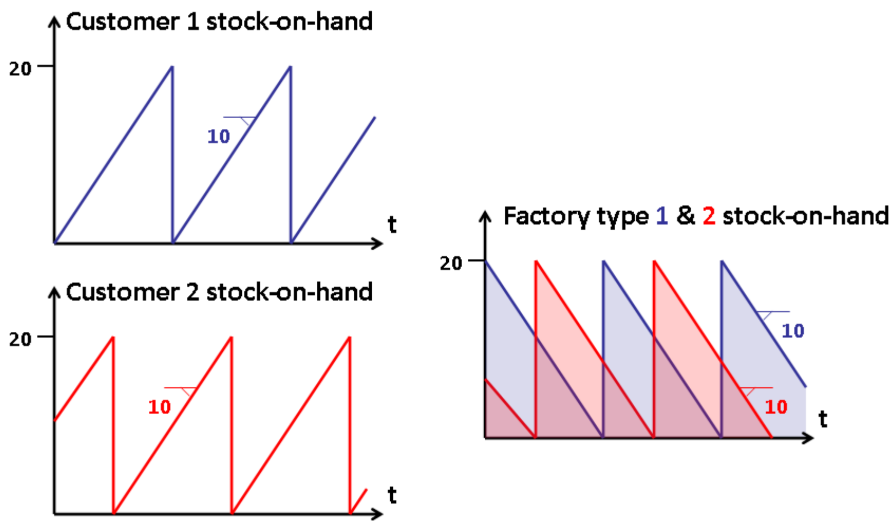

Figure 2.3: 2-customer non-substitutable packages flows.

SUBSTITUTABLE PACKAGES

Let us now assume that all the packages in the network are identical and interchangeable. It does not change anything for the customers. However, the factory may now pool all the received packages into a single inventory that can be used for the delivery flows to both customers. The factory now consumes a whole shipment Q_c at rate Γ before requiring another shipment to be supplied. Figure 2.4 illustrates a case with 2 customers supplying lots of size 20 in a coordinated way. Compared to the non-substitutable packages' case, this results in a reduction of the average inventory at the factory of 10 units, while keeping unchanged the transportation costs and the inventory costs at the customers.

In our example, each return flow arrives at the factory when its inventory reaches 0. This is what we refer to as *perfect coordination*. In theory, there is no guarantee that such

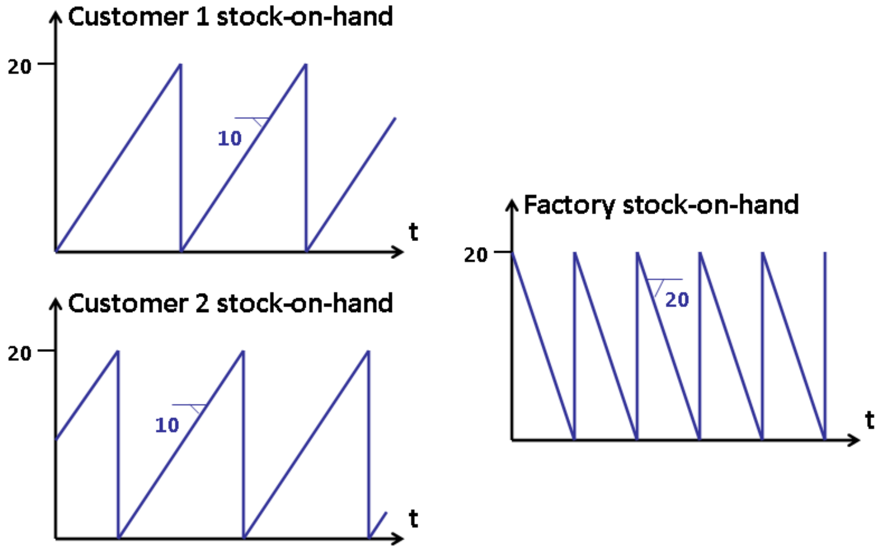

Figure 2.4: 2-customer perfectly coordinated return flows.

a coordination is achievable in all cases. However, our own experiments, described in Chapter 5, have shown that almost perfectly coordinated strategies can generally be found at the operational level. This is in line with the results in the SRMIS literature that have shown that there generally are cost solutions which are reasonably close to that of the ideal perfectly coordinated case. We refer the reader to the literature on SRMIS for more insight on the coordination issue.

In order to measure the potential benefit of pooling the packages, let us consider three cases. In the first case, the packages are not substitutable. In this case, a lot Q_c induces an average inventory of $Q_c/2$ at the customer site and at the factory site. The total inventory at the customers is thus $\sum_c Q_c/2$. The factory also holds $\sum_c Q_c/2$ packages on the average. In the second case, let us assume that the packages are pooled but the lots are systematically shipped simultaneously to the factory. The inventory at the factory evolves then from $\sum_c Q_c$ to zero, leading to the same average inventory $\sum_c Q_c/2$ at the factory. There is therefore no gain in this situation compared to the non-substitutable case. Finally, let us consider pooled packages and perfect coordination. Here, a shipment size Q_c still induces an average inventory of $Q_c/2$ at the customer site but an average inventory of only $(Q_c/2)(\Lambda_c/\Gamma)$ at the factory site. Indeed, (Λ_c/Γ) is the fraction of the time it takes the factory to consume the packages from the lot Q_c.

The cost function in this case then becomes

$$V^{[ppc]}(\{Q_c\}) = \sum_c \frac{Q_c}{2} H + \sum_c \frac{Q_c}{2} \frac{\Lambda_c}{\Gamma} H + \sum_c \frac{\Lambda_c O_c}{Q_c} \tag{2.18}$$

which leads to the following optimal shipment sizes

$$Q_c^{[ppc]*} = \sqrt{\frac{2\Lambda_c O_c}{H(1 + \frac{\Lambda_c}{\Gamma})}} \tag{2.19}$$

where $[ppc]$ refers to the *pooled with perfect coordination* case. This leads to a total inventory of

$$L^{[ppc]*} = \sum_c \frac{Q_c^{[ppc]*}}{2}(1 + \frac{\Lambda_c}{\Gamma}) \tag{2.20}$$

and an optimal cost of

$$V^{[ppc]}(\{Q_c^{[ppc]*}\}) = \sum_c \sqrt{2\Lambda_c O_c H(1 + \frac{\Lambda_c}{\Gamma})}$$

Finally, note that the reasoning remains perfectly valid if some capacity constraint is imposed on the shipment sizes. In this case, we would simply have

$$Q_c^{[ppc]*} = min\{CAP, \sqrt{\frac{2\Lambda_c O_c}{H(1 + \frac{\Lambda_c}{\Gamma})}}\} \tag{2.21}$$

COMPARISON BETWEEN THE POOLED AND NON-POOLED CASES

We have seen that the non-pooled and the pooled cases constitute two references for the total logistic cost in our network. Both cases are decomposable into independent subproblems for each $c \in NC$. It is the assumption made on the average inventory at the factory which differentiates the two problems.

In the $[np]$ case, the optimal shipment sizes are

$$Q_c^{[np]*} = \sqrt{\Lambda_c O_c / H}$$

leading to the optimal cost of:

$$V^{[np]}(\{Q_c^{[np]*}\}) = 2\sum_c \sqrt{\Lambda_c O_c H}$$

In the $[ppc]$ case, the optimal shipment sizes are

$$Q_c^{[ppc]*} = \sqrt{\frac{2\Lambda_c O_c}{H(1+\frac{\Lambda_c}{\Gamma})}} \approx \sqrt{\frac{2\Lambda_c O_c}{H(1+1/C)}}$$

where the last approximation assumes that all customers have equal rates. This leads to a total cost of

$$V^{[ppc]}(\{Q_c^{[ppc]*}\}) \approx \sum_c \sqrt{2\Lambda_c O_c H(1+1/C)}$$

The ratio of the two costs is thus

$$\frac{V^{[ppc]*}}{V^{[np]*}} \approx \sqrt{\frac{1+1/C}{2}}$$

which is close to 70% if the number of customer C is not too small.

The managerial lessons are obvious. First, pooling can only be cheaper. Indeed, a factory can delay its calls for packages as long as some are available. This means that the inventory at the factory side will remain very low at all times. Delaying the calls for the shipments will allow the shipment sizes to be larger on the average, the transportation to be less frequent and the cost to be lower.

In terms of profit, pooling the packages can spare up to 30% of the cost if perfect coordination can be achieved and if the network is not too small. Such saving is obtained by increasing the shipment sizes by a factor of $\sqrt{2}$. This increase allows to reduce the transportation frequencies by the same factor $\sqrt{2}$. On the inventory side, at the customer, the inventory is increased by $\sqrt{2}$ but this is compensated by the fact that a lot stays almost no time at the factory side. Altogether, the inventory level is also reduced by a factor $\sqrt{2}$.

In the case in which the transportation is constrained by some capacity limit, the profit of pooling could be reduced to the reduction of the holding cost of the inventory at the factory. Indeed, if all the lots sizes were already at their limit CAP in the non-pooled case, then the only saving realized is on the inventory at the factory.

2.4.2 *Fixed L model*

UNCAPACITATED TRANSPORTATION

Let us consider the case where the total logistic inventory L is limited by the actual investment made in the fleet of packages. We then aim at minimizing the transportation cost:

$$(FIX-U) \qquad \min_{Q_c} \quad \sum_c \frac{\Lambda_c O_c}{Q_c} \tag{2.22}$$

$$s.t. \quad L = \sum_c Q_c \qquad [np] \tag{2.23}$$

or

$$L = \sum_c \frac{Q_c}{2} + \sum_c \frac{Q_c}{2}\frac{\Lambda_c}{\Gamma} \qquad [ppc] \tag{2.24}$$

$$L \leq \bar{L} \tag{2.8}$$

$$Q_c \geq 0 \qquad \forall c$$

The link between the shipment sizes and the logistic inventory L is determined by Constraint (2.23) in the non-pooled case and (2.24) in the pooled case with perfect coordination. Intuitively, this problem is not too difficult. The objective function is strictly decreasing with the shipment sizes. The optimal shipment sizes would be determined independently if L was not constrained. Since L is limited by $\bar{L}$, the shipment sizes are chosen so that their marginal contributions to the objective function are equal. However, we know that (2.8) will be active at optimality, and we may replace (2.23) and (2.8) by

$$\bar{L} = \sum_c Q_c \tag{2.25}$$

Let us solve this problem mathematically for the $[np]$ case in order to illustrate the solution mechanism. The procedure can be repeated for the $[ppc]$.

We first build the Lagrangian function

$$\mathcal{L} = \sum_c \frac{\Lambda_c O_c}{Q_c} - \beta[\bar{L} - \sum_c Q_c] \tag{2.26}$$

and then derive the first order optimality conditions

$$\begin{cases} \frac{\Lambda_c O_c}{(Q_c)2} = \beta \quad \forall c \\ \\ \bar{L} = \sum_c Q_c \end{cases} \tag{2.27}$$

which leads to the optimal shipment sizes

$$Q_c^{[np]*} = \frac{\bar{L}}{\sum_{d \in NC} \sqrt{O_d \Lambda_d}} \sqrt{O_c \Lambda_c} \tag{2.28}$$

These optimal shipment sizes are now linked together by the available number of empty packages $\bar{L}$. Since the ratios between the shipment sizes are independent of $\bar{L}$,

$$\frac{Q_c^{[np]*}}{Q_d^{[np]*}} = \frac{\sqrt{\Lambda_c O_c}}{\sqrt{\Lambda_d O_d}}$$

it is easy to define a first set of shipment sizes on an arbitrary basis, e.g. fixing $Q_d^{[np]*} = 1$. Then, the resulting logistic inventory can be computed. If it is not equal to $\bar{L}$, all the shipment sizes are scaled in order to reach the given $\bar{L}$ value. The same process is also valid for the value $\bar{L}$ equal to $L^{[np]*}$, the optimal logistic inventory of the variable L model. In this case, the shipment sizes are the EOQ quantities given by (2.15) resulting from the minimization problem (VAR-U).

The fact that the shipment sizes remain proportional for different values of $\bar{L}$ results from the optimality conditions (2.27) that require that the gradients of the individual cost functions at the optimal shipment sizes Q_c^*

$$-\frac{\Lambda_c O_c}{(Q_c^*)^2} = -\frac{\Lambda_d O_d}{(Q_d^*)^2} \qquad \forall c, d \in NC \tag{2.29}$$

are all equal. This is illustrated on Figure 2.5 for three different customer cost functions.

The case of pooled packages with perfect coordination, subject to constraint (2.24), can be solved by the same procedure. The optimality conditions

$$\frac{\Lambda_c O_c}{(Q_c)^2} \frac{1}{(1 + \Lambda_c/\Gamma)} = \frac{\Lambda_d O_d}{(Q_d)^2} \frac{1}{(1 + \Lambda_d/\Gamma)} \qquad \forall c, d \in NC \tag{2.30}$$

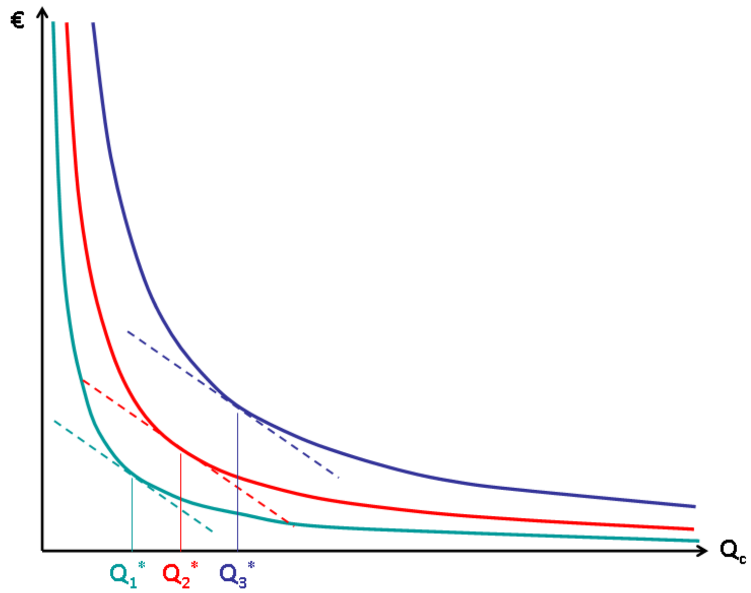

Figure 2.5: Individual transportation cost functions of 3 customers and optimal shipment sizes based on the gradients.

lead to the following optimal shipment sizes

$$Q_c^{[ppc]*} = \frac{2\bar{L}}{\sum_{d \in C} \sqrt{O_d \Lambda_d (1 + \Lambda_d / \Gamma)}} \frac{\sqrt{O_c \Lambda_c}}{\sqrt{(1 + \Lambda_c / \Gamma)}} \tag{2.31}$$

The "1-factory C-customer" case thus leads to shipment sizes that are easy to compute. In the [np] case, with fixed or variable L, they are proportional to

$$Q_c^{[np]*} \propto \sqrt{O_c \Lambda_c} \propto \sqrt{\frac{O_c}{1/\Lambda_c + 1/\Lambda_c}}$$

while in the [ppc] case, they are proportional to

$$Q_c^{[ppc]*} \propto \sqrt{\frac{O_c}{1/\Lambda_c + 1/\Gamma}}$$

This clearly indicates that, in the [ppc] case, the factory consumes the packages at the speed Γ instead of Λ_c. In both cases, a large Λ_c still favors a large shipment size since this shipment rotates quickly. However the advantage is reduced in the [np] case since the advantage of a high rotation takes place at the customer site only.

In practice, because every unit of a shipment rotates almost twice as fast in the [ppc] case as in the [np] case, the shipment sizes will be larger in the [ppc] case. In the fixed L model, they will be almost doubled, leading to a reduction of 50% in the transportation cost.

CAPACITATED TRANSPORTATION

Let us now complete the "1-factory C-customer" problem with (FIX-C), the fixed L case with some capacity constraints on the shipment sizes, i.e. $Q_c \leq CAP \quad \forall c$.

The solution of the problem with this additional constraint is more complex analytically. Intuitively, the approach remains the same. The optimal shipment sizes are determined so that their marginal contributions to the objective function are all equal, while satisfying the $\bar{L}$ constraint. If now, some capacity constraints are exceeded, then these shipment sizes are limited to CAP. This limitation makes an additional number of packages available allowing all the other shipment sizes to be increased while keeping the same marginal contributions. The process is reiterated until either $\bar{L}$ is reached or all the capacity limits have been reached. Another simple method consists in considering the packages of the logistic inventory one by one and in allocating them to the customer with the steepest marginal contribution. If, in this process, a customer reaches the transportation capacity limit, then this customer is not allowed to receive any additional unit. Again, the process ends when the last container has been allocated or when all the capacity limits have been reached.

Figure 2.6 represents the typical evolution of the total logistic cost (holding + transportation) for both cases, pooled or non-pooled, with a small or a large capacity on transportation. For low levels of $\bar{L}$, the situation typically remains unchanged as all optimal shipment sizes are smaller than the capacity limit. When $\bar{L}$ becomes larger, the $[ppc]$ curve with capacity constraints separates from the $[ppc]$ curve without capacity constraints as soon as some shipment sizes reach the capacity limit. Additional packages allocated to the system still bring a profit, but this profit is lower than in the uncapacitated case because these units are allocated to customers with flatter slopes. As $\bar{L}$ further increases, we come to a point where all shipment sizes reach the capacity limit. The capacitated curve becomes a straight line, indicating the linear increase of the holding cost linked with each additional unit. As from this point, the logistic inventory in the capacitated case becomes smaller than $\bar{L}$. The same behavior is to be observed in the $[np]$ case. However, for any given level $\bar{L}$, the $[ppc]$ optimal quantities are larger than the $[np]$ optimal quantities. Consequently, the level of $\bar{L}$ for which the

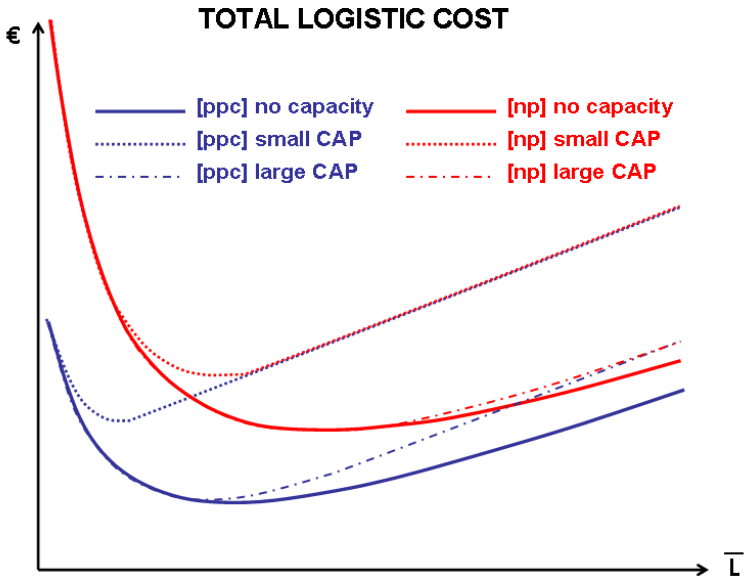

Figure 2.6: Impact of a small or a large capacity limit.

capacitated curve separates from the uncapacitated curve in the $[np]$ case is larger than in the $[ppc]$ case.

2.5 F-FACTORY C-CUSTOMER

In this section, we consider the general network composed of F factories and C customers. If the reusable packages are specific to each factory, the problem degenerates into F 1-factory C-customer problems. Such problems were studied in the Section 2.4. So, we will assume here that all packages in the network of F factories and C customers are identical and can be pooled. We will also assume perfect coordination as in Sections 2.4.

The general model introduces the λ_c^f variables, that is the average flow each customer sends back to a specific factory f. The determination of these variables is called the *customer allocation problem*. Along with the customer allocation problem, we must also determine the shipment sizes Q_c^f by which a given customer c will send back its flow λ_c^f to the factory f. Finally, the shipment sizes will also provide the total inventory of packages L.

The analysis will show that the general problem is rather easy since it can, in most cases, be solved by a decomposition into 2 subproblems: a customer allocation problem that determines which customer sends packages to which factory and a shipment sizing problem that defines how these flows are grouped. However, in some cases, these two problems have to be solved jointly. As in the previous sections, we will consider the cases of fixed and variable L. The impact of capacity constraints will also be studied.

2.5.1 *Variable L model*

DECOMPOSITION OF THE PROBLEM

Let us rewrite the general variable L model (VAR) in which we introduce constraint (2.4) on the logistic inventory into the objective (2.1) by substitution of the L variable.

$$
\begin{array}{llll}
(VAR) & \min\limits_{(\lambda_c^f, Q_c^f)} & [\sum\limits_{c,f} \frac{Q_c^f \lambda_c^f}{2} (\frac{1}{\Lambda_c} + \frac{1}{\Gamma^f})] H + \sum\limits_{c,f} \frac{\lambda_c^f O_c^f}{Q_c^f} & (2.32) \\
& s.t. & \sum\limits_{c} \lambda_c^f = \Gamma^f & \forall f \\
& & \sum\limits_{f} \lambda_c^f = \Lambda_c & \forall c \\
& & Q_c^f \leq CAP & \forall c, f \\
& & Q_c^f, \lambda_c^f \geq 0 & \forall c, f
\end{array}
$$

Rewriting the objective function (2.32) as

$$
\sum_{c,f} \lambda_c^f \quad [\frac{Q_c^f H}{2} (\frac{1}{\Lambda_c} + \frac{1}{\Gamma^f}) + \frac{O_c^f}{Q_c^f}] \tag{2.33}
$$

allows us to observe that the problem decomposes into two subproblems:

1) the shipment sizing problem

$$
\begin{array}{lll}
\min\limits_{(Q_c^f)} & \frac{Q_c^f H}{2} (\frac{1}{\Lambda_c} + \frac{1}{\Gamma^f}) + \frac{O_c^f}{Q_c^f} & (2.34) \\
s.t. & Q_c^f \leq CAP & \forall c, f \\
& Q_c^f \geq 0 & \forall c, f
\end{array}
$$

2) the customer allocation problem

$$
\begin{aligned}
\min_{(\lambda_c^f)} \quad & \sum_{c,f} \lambda_c^f \quad [\frac{Q_c^{f*} H}{2}(\frac{1}{\Lambda_c} + \frac{1}{\Gamma^f}) + \frac{O_c^f}{Q_c^{f*}}] && (2.35) \\
s.t. \quad & \sum_c \lambda_c^f = \Gamma^f && \forall f \\
& \sum_f \lambda_c^f = \Lambda_c && \forall c \\
& \lambda_c^f \geq 0 && \forall c, f
\end{aligned}
$$

THE SHIPMENT SIZING PROBLEM

The shipment sizing problem exhibits C independent EOQ-like subproblems that are solved independently:

$$
Q_c^{f*} = min[\sqrt{\frac{2O_c^f}{H(\frac{1}{\Lambda_c} + \frac{1}{\Gamma^f})}}, CAP] \tag{2.36}
$$

which becomes

$$
Q_c^{f*} = \sqrt{\frac{2O_c^f}{H(\frac{1}{\Lambda_c} + \frac{1}{\Gamma^f})}} \tag{2.37}
$$

if the problem is uncapacitated.

This implies that the shipment sizes that would be used on each link $c - f$ of the network can be determined beforehand. These optimal shipment sizes do not depend on the actual flow that will be directed on each specific $c - f$ link.

THE CUSTOMER ALLOCATION PROBLEM

Let us first consider the uncapacitated problem (VAR-U). Using the optimal shipment sizes given by (2.37) in the objective function of the allocation problem (2.35), we obtain the following problem:

$$
\begin{aligned}
\min_{(\lambda_c^f)} \quad & \sum_{c,f} \lambda_c^f \sqrt{O_c^f H(\frac{1}{\Lambda_c} + \frac{1}{\Gamma^f})} && && (2.38)\\
s.t. \quad & \sum_{c} \lambda_c^f = \Gamma^f && \forall f \\
& \sum_{f} \lambda_c^f = \Lambda_c && \forall c \\
& \lambda_c^f \geq 0 && \forall c, f
\end{aligned}
$$

The customer allocation problem is independent of the shipment sizing problem. It is a simple linear program which is an instance of the transportation problem with a cost function that exhibits some specificities.

First, the model is not based on the direct transportation cost O_c^f but on its square root. This comes from the EOQ-like general model. If a customer is more distant than another, then its shipment size Q_c^f will be made larger in order to smooth its impact on the transportation costs. This results in the square root function. Secondly, the transportation cost O_c^f under the square root is corrected by a term $(\frac{1}{\Lambda_c} + \frac{1}{\Gamma^f})$. Indeed, the faster the inventory along a $c - f$ relationship rotates, the less costly this inventory is and the larger the shipment size can be. This comes from the contribution of Q_c^f to L

$$L = \sum_{c,f} \frac{Q_c^f \lambda_c^f}{2} (\frac{1}{\Lambda_c} + \frac{1}{\Gamma^f})$$

that decreases as Λ_c and Γ_f increase.

Second, let us now consider the capacitated problem (VAR-C). In this case, the objective coefficients of the λ_c^f variables for which the optimal shipment size is not limited by the transportation capacity will remain identical. The other objective coefficients become the sum of the holding and transportation costs associated with a shipment size equal to CAP. We obtain the following problem:

$$
\begin{aligned}
\min_{(\lambda_c^f)} \quad & \sum_{c,f:Q_c^{f*}<CAP} \lambda_c^f \sqrt{O_c^f H(\frac{1}{\Lambda_c}+\frac{1}{\Gamma^f})} \\
& \sum_{c,f:Q_c^{f*}=CAP} \lambda_c^f (\frac{CAP\ H}{2}(\frac{1}{\Lambda_c}+\frac{1}{\Gamma^f})+\frac{O_c^f}{CAP}) && (2.39) \\
s.t. \quad & \sum_c \lambda_c^f = \Gamma^f && \forall f \\
& \sum_f \lambda_c^f = \Lambda_c && \forall c \\
& \lambda_c^f \geq 0 && \forall c, f
\end{aligned}
$$

In this case, the two subproblems are nested. We need to solve the shipment sizing problem first in order to derive the objective coefficients of the allocation problem.

An interesting feature of the problem decomposition is that the fixed transportation costs have been replaced, along with the holding costs, by variable costs.

A quick analysis of the linear customer allocation problem allows us to identify $C+F-1$ linearly independent constraints. It follows that, among the $C.F$ variables, $C+F-1$ are basic. Since each customer must have a least one allocation, we can conclude that there will be, at most, $F-1$ customers who are allocated to more than one factory.

THE OPTIMAL LOGISTIC INVENTORY

Finally, finding the optimal logistic inventory is immediate once the optimal allocation is known:

$$L^* = \sum_{c,f} \frac{Q_c^{f*}\lambda_c^{f*}}{2}(\frac{1}{\Lambda_c}+\frac{1}{\Gamma^f}) \tag{2.40}$$

2.5.2 *Fixed L model*

UNCAPACITATED CASE

In the fixed L model, if no capacity constraints are considered, we may rely on a similar decomposition.

Indeed, the two subproblems are independent. The allocation problem remains unchanged when compared to the previous uncapacitated variable L case. The allocation for (FIX-U) is based on the same objective (2.38) as for (VAR-U). This also means that the allocation of the customers is independent of the number of available packages to be used in the network.

Regarding optimal shipment sizes, they always appear in the same proportions, which are the proportions given by the optimal shipment sizes of the variable L case (2.37). These shipment sizes are simply scaled in such a way that the logistic inventory exactly meets its bound ($\bar{L} = L$).

Let us demonstrate these two statements mathematically. We assume that we have fixed the values of the λ_c^f variables, i.e. that an allocation of the customers is given. We obtain the following problem:

$$\min_{(Q_c^f)} \quad \sum_{c,f} \frac{\lambda_c^f O_c^f}{Q_c^f} \tag{2.41}$$

$$s.t. \quad \bar{L} = \sum_{c,f} \frac{Q_c^f \lambda_c^f}{2\Lambda_c} + \sum_{c,f} \frac{Q_c^f \lambda_c^f}{2\Gamma^f} \tag{2.4}$$

$$Q_c^f \geq 0 \qquad \forall c, f$$

We define the associated Lagrangian function:

$$\mathcal{L} = \sum_{c,f} \frac{\lambda_c^f O_c^f}{Q_c^f} - \beta[\bar{L} - \sum_{c,f} \frac{Q_c^f \lambda_c^f}{2}(\frac{1}{\Lambda_c} + \frac{1}{\Gamma^f})] \tag{2.42}$$

then derive the first order conditions

$$\begin{cases} \dfrac{\lambda_c^f O_c^f}{(Q_c^f)^2} + \beta\dfrac{\lambda_c^f}{2}(\dfrac{1}{\Lambda_c} + \dfrac{1}{\Gamma^f}) = 0 \quad \forall c, f & (2.43) \\ \bar{L} - \displaystyle\sum_{c,f} \dfrac{Q_c^f \lambda_c^f}{2}(\dfrac{1}{\Lambda_c} + \dfrac{1}{\Gamma^f}) = 0 & (2.4) \end{cases}$$

Using (2.43) we obtain to the following optimal quantities:

$$Q_c^{f*} = \sqrt{\frac{2}{\beta} \frac{O_c^f}{(\frac{1}{\Lambda_c} + \frac{1}{\Gamma^f})}} \tag{2.44}$$

This result demonstrates that the optimal quantities appear in the same proportions as in the variable L case (2.37). These quantities are scaled such as to meet constraint (2.4).

Using (2.4), we may determine the scaling factor.

$$\sqrt{\frac{2}{\beta}} = \frac{2\bar{L}}{\sum_{d \in NC, g \in NF} \lambda_d^g \sqrt{O_d^g (\frac{1}{\Lambda_d} + \frac{1}{\Gamma^f})}} \tag{2.45}$$

Finally, reinserting these optimal quantities into the objective function (4.6) of the general model leads to the following allocation problem:

$$\begin{aligned} \min_{(\lambda_c^f)} \quad & \frac{[\sum_{c,f} \lambda_c^f \sqrt{O_c^f (\frac{1}{\Lambda_c} + \frac{1}{\Gamma^f})}]^2}{2\bar{L}} \\ s.t. \quad & \sum_c \lambda_c^f = \Gamma^f \quad \forall f \\ & \sum_f \lambda_c^f = \Lambda_c \quad \forall c \\ & \lambda_c^f \geq 0 \qquad \forall c, f \end{aligned} \tag{2.46}$$

Minimizing (2.46) is equivalent to minimizing (2.38) which proves that the allocation is unique and independent of the logistic inventory.

CAPACITATED CASE

When capacity constraints are active in the fixed L model, the problem becomes more complex. In this case, we may not decompose the problem as in the previous cases. In the previous cases:

1) When the logistic inventory is variable, the possibility to decompose the problem originates in the fact that each customer selects its shipment size independently of all other customers. It realizes the trade-off between transportation and holding costs independently.

2) When the logistic inventory is fixed but no capacity constraints are included, the ability to decompose the problem comes from the fact that we know that all shipment sizes used in the network will exhibit identical marginal contributions, as indicated by (2.43):

$$\beta = \frac{2O_c^f}{(Q_c^f)^2 (\frac{1}{\Lambda_c} + \frac{1}{\Gamma^f})} \quad \forall c, f$$

However, when capacity constraints are active, this condition no longer holds, because some shipment sizes will be limited by the capacity.

In the fixed L capacitated case, the allocation of the customers depends on the logistic inventory. This is easily observed by considering different range of $\bar{L}$.

If we consider extremely small values for $\bar{L}$, the capacity constraints are inactive. In this case, our problem is identical to the uncapacitated problem. And the λ_c^f can be determined by minimizing

$$\sum_{c,f} \lambda_c^f \sqrt{O_c^f \left(\frac{1}{\Lambda_c} + \frac{1}{\Gamma^f}\right)}$$

If we consider very large values for $\bar{L}$, then the capacity limits will be reached and all the shipment sizes will be equal to CAP. In this case, we just aim at minimizing the transportation costs

$$\sum_{c,f} \frac{\lambda_c^f O_c^f}{CAP} \quad \Leftrightarrow \quad \sum_{c,f} \lambda_c^f O_c^f$$

However, the problem remains easy to solve numerically. Let us consider the following problem

$$(VAR(\omega) - C) \quad \min_{(\lambda_c^f, Q_c^f)} \quad \left[\sum_{c,f} \frac{Q_c^f \lambda_c^f}{2} \left(\frac{1}{\Lambda_c} + \frac{1}{\Gamma^f}\right)\right]\omega + \sum_{c,f} \frac{\lambda_c^f O_c^f}{Q_c^f} \qquad (2.47)$$

$$s.t. \quad \sum_c \lambda_c^f = \Gamma^f \qquad \forall f \qquad (2.2)$$

$$\sum_f \lambda_c^f = \Lambda_c \qquad \forall c \qquad (2.3)$$

$$Q_c^f \leq CAP \qquad \forall c, f \qquad (2.5)$$

$$\lambda_c^f, Q_c^f \quad \geq 0 \qquad \forall c, f \qquad (2.6)$$

This problem is similar to the capacitated variable L problem (VAR-C) and may be solved for a given value of the multiplier ω by the same decomposition procedure. The optimal solution obtained will be characterized by a certain logistic inventory $L^*(\omega)$.

PROPERTY 2.1 The function $L^*(\omega)$ is monotonically decreasing. It has a maximum L^{MAX} which is achieved for small values of ω.

The maximum of the function corresponds to small values of the multiplier, interpreted as a unit holding cost, for which all shipment sizes are limited by the capacity constraints (as observed in (2.36)). To prove that the function is decreasing, it suffices to consider any two values ω^1 and ω^2 of the multiplier ω such that $\omega^2 \geq \omega^1$. Let hc^{*1}

and hc^{*2} be the total holding cost for the optimal solution of each case, while tc^{*1} and tc^{*2} are the total transportation cost of these optimal solutions.

If $L^*(\omega^2) > L^*(\omega^1)$, then we must have that $(L^*(\omega^2) - L^*(\omega^1))\omega^2$ (the increase in the holding cost due to the additional units of logistic inventory used by the optimal solution with ω^2) is smaller than $tc^{*1} - tc^{*2}$ (the decrease of the transportation cost that can be achieved thanks to these additional units). But, this would also mean that $(L^*(\omega^2) - L^*(\omega^1))\omega^1 < tc^{*1} - tc^{*2}$. This would imply that the solution found for ω^1 is not optimal. Consequently, we must have that $L^*(\omega^2) \leq L^*(\omega^1)$ proving that the function $L^*(\omega)$ is decreasing.

The optimal solution of $(VAR(\omega) - C)$ is also the solution that minimizes the transportation costs for a level of the logistic inventory which is equal to $L^*(\omega)$. Consequently, solving the fixed L capacitated problem is equivalent to finding the value $\widetilde{\omega}$ such that $L^*(\widetilde{\omega}) = \bar{L}$ or showing that $\bar{L} > L^{MAX}$. Consequently, we may solve the fixed L capacitated problem by the following procedure:

1. **Check if $\bar{L} \geq L^{MAX}$**
   - Set all shipment sizes equal to CAP
   - Determine the optimal allocation (using (2.32) subject to (2.2)-(2.6))
   - Derive the optimal logistic inventory, which is L^{MAX}
   - If $\bar{L} \geq L^{MAX}$, the solution is optimal for the fixed L capacitated problem, otherwise go to step 2.
2. **Find the value $\widetilde{\omega}$ that leads to $L^*(\widetilde{\omega}) = \bar{L}$ by a bisection method**
   - Set $\omega_{LOW} = 0$, $\omega^{HIGH} = \textit{very large value}$ and $\widetilde{\omega} = 1$
   - Solve the variable L capacitated problem with $\widetilde{\omega}$
   - Compute the optimal logistic inventory $L^*(\widetilde{\omega})$
   - If $L^*(\widetilde{\omega}) = \bar{L}$ the solution is optimal for the fixed L capacitated problem
   - Otherwise, update the value of $\widetilde{\omega}$ and go back to step 2:
     - if $L^*(\widetilde{\omega}) > \bar{L}$, set $\omega_{LOW} = \widetilde{\omega}$
     - if $L^*(\widetilde{\omega}) < \bar{L}$, set $\omega^{HIGH} = \widetilde{\omega}$
     - go back to step 2

2.5.3 *Managerial Implications*

Let us consider the allocation of the customers in the example used in the introductive chapter. If we allow the packages to accumulate at the customers, we showed that we obtain the following objective function to minimize in the uncapacitated case

$$min \sum_{c,f} \lambda_c^f \sqrt{O_c^f (\frac{1}{\Lambda_c} + \frac{1}{\Gamma^f})} \tag{2.38}$$

The solution is illustrated on Figure 2.7. In this case, the flow of Customer 14 is split between the two factories.

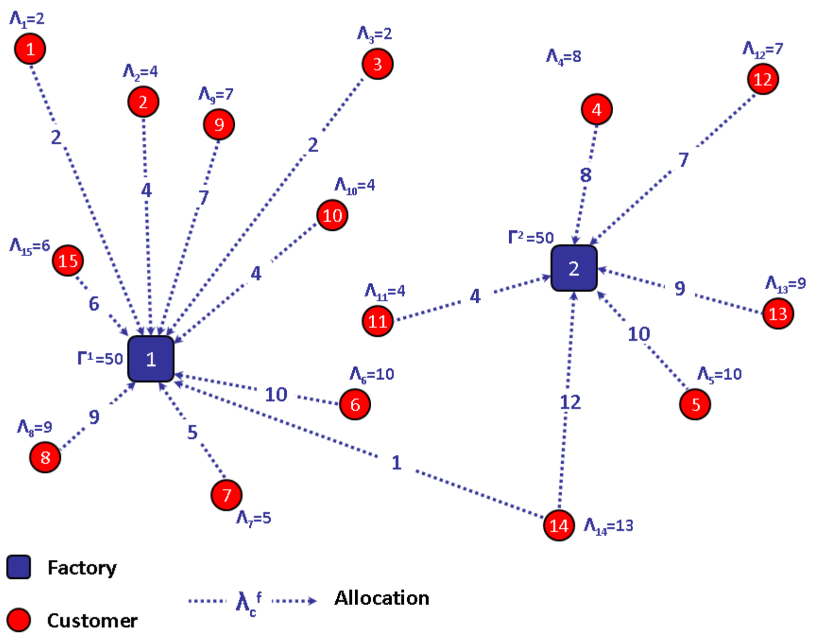

Figure 2.7: Allocation of the customers in the uncapacitated case.

Our objective function differs from the more classical objective based on transportation costs:

$$min \sum_{c,f} \lambda_c^f O_c^f \tag{2.48}$$

Figure 2.8 shows the difference between the optimal solutions obtained by using each of these objective functions.

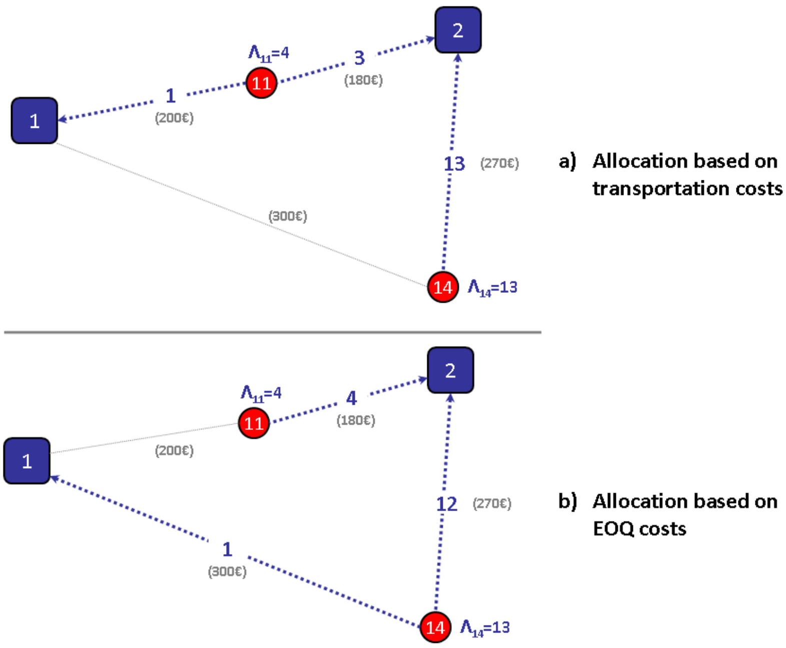

Figure 2.8: Difference in the allocation with the two objective functions.

If the allocation is based on transportation costs, we choose to allocate one unit from Customer 11 to Factory 1. The cost increase of allocating this unit to Factory 1 instead of Factory 2 is 20€, whereas it would be 30€ if the unit was taken from Customer 14.

However, when the objective (2.38) is used, the cost increase for allocating one unit from Customer 11 to Factory 1 instead of Factory 2 becomes

$$\sqrt{200(\frac{1}{4}+\frac{1}{50})} - \sqrt{180(\frac{1}{4}+\frac{1}{50})} = 0.377$$

while for Customer 14 it equals

$$\sqrt{300(\frac{1}{13}+\frac{1}{50})} - \sqrt{270(\frac{1}{13}+\frac{1}{50})} = 0.27$$

which explains why Customer 14 will be chosen to send one unit to Factory 1.

This difference originates in (i) the existence of an inventory which allows the use of larger shipment sizes for distant customers, and (ii) the rotation rate of that inventory.

We showed, however, that the effects allowing us to use the first objective function in the uncapacitated case may be tempered by certain capacity limits. In the capacitated case, we have seen that, if the logistic inventory is small, no shipment size would reach the capacity limit and the solution found for the uncapacitated case, represented on Figure 2.7, remains valid. However, when the logistic inventory progressively increases, some capacity limits will be reached, and the proportion between the quantities will begin to change. Eventually, if the logistic inventory is very large, all quantities are limited by the capacity, and we retrieve the classical objective based on transportation costs. In our example, with a transport capacity limit of 20 units, if the logistic inventory reaches 96 units, the solution depicted on Figure 2.8 a), based on a classical allocation on the transportation costs, becomes the optimal solution.

In the multi-factory multi-customer case, the gain of pooling appears at two levels. First, resource pooling allows the customers to send their packages back to any factory. This reallocation provides a benefit in terms of total traveled distance. This benefit cannot be measured analytically. It depends on the actual data and can only be computed numerically. Secondly, within each factory cluster, pooling provides an additional benefit. This benefit can amount up to $1/\sqrt{2}$. When capacity constraints are active, they can limit this benefit. In the worst case, capacity constraints are all binding and pooling provides no benefits on the transportation costs. However, a reduction of the logistic stock due to an increased rotation rate of the units at the factories is nevertheless obtained.

2.6 CONCLUSION AND FUTURE WORK

The objective of this chapter was to answer three strategic questions related to the design of an independent reverse network for the return flows of empty reusable packages:

Q1: To which factory should each customer return its empty packages?
Q2: At what frequency should they be returned ?
Q3: How many packages are needed in the network?

and to get some insights on the benefit of pooling such packages among the players. To tackle all of these questions, we considered cases of increasing complexity from the "1-factory 1-customer" case, to the "F-factory C-customer" case. We also considered the impact of capacity constraints and of a fixed/variable logistic inventory. Optimal solutions were provided in all cases.

At first, in Section 2.3, we showed that the single-factory single-customer case with variable L was easily solved by an EOQ-like formula. The optimal quantity, however, was smaller than a classical EOQ by a factor $\sqrt{2}$ since we consider the inventory both at the customer sites and at the factory sites.

The case with a single-factory and several customers analyzed in Section 2.4 reduces to C independent single-factory single-customer problems if the logistic units are not pooled. Each customer will select its own optimal shipment size. Dependence between the customers appears if we suppose that the reusable packages can be pooled at the factory. However, the benefit of pooling depends on the coordination of the return flows. If they all occur at the same moment, then we have no coordination and no benefit. In the other extreme, namely perfect coordination, each return flow arrives at the factory when the inventory has just fallen to zero. This allows the quickest rotation of the inventory at the factory and thus the best use of the inventory of packages. The pooled and perfectly coordinated case leads to shipment sizes that are $\sqrt{2}$ larger and a total benefit of up to $1/\sqrt{2}$, that is, about 30 percent. A set of experiments presented in Chapter 5 will show that the perfect coordination assumption is very realistic on large networks.

The multi-factory multi-customer case, developed in Section 2.5, introduced the problem of allocating the customers to the factories. The first important result is that the allocation problem, in the uncapacitated case, is independent of the actual level of the logistic stock. It defines an allocation of the customers and relative frequencies for all customer-factory pairs. As before, the logistic inventory will be used to scale these relative frequencies such as to meet the constraint implied by the number of units in the network. The allocation problem appears as a transportation problem with cost coefficients including features of the EOQ-like model. It was shown that the cost comparison between the customer-factory relationships is based on a square root of the distances weighted by a factor depending on the consumption rates at the customer and at the factory.

If a transport capacity limit is added to the model, then the allocation problem is not independent of L anymore. For a low level of logistic inventory, when the capacity has no impact, the EOQ-like costs remain valid, as well as the uncapacitated allocation. However, for large values of L, the holding cost effect tends to disappear and a classical allocation based on the distances becomes optimal. We have suggested an iterative procedure in order to determine, for the given value of the logistic stock, which shipment size will be limited by the capacity and which ones will not. On the basis of this result, the optimal customer allocation can be found.

Our analysis allowed us to identify the economic logic behind the problem, the trade-offs that appear in the management of the return flows of empty packages and how the costs would react to variations of the parameters. It also allows us to identify the different problems and the procedures for solving them. This analysis provides a good basis for tackling strategic comparisons when deciding for certain network structures.

The strategic analysis relies on the perfect coordination assumption. The next two chapters extend the strategic models presented, and also rely on the perfect coordination assumption. We will consider how the fixed transportation costs can be shared between customers by means of local aggregation of the flows (Chapter 3) or between distinct fleets of packages which are not substitutable (Chapter 4).

3

STRATEGIC MODEL WITH DEPOTS

3.1 INTRODUCTION

In Chapter 2, we presented the fundamental strategic models for taking allocation and shipment sizes decisions for the reverse network of the empty packages used by the producer. This chapter's objective is to extend these models to more complex networks. We demonstrate that some interesting properties of the models, identified in Chapter 2 and allowing us to decompose the problem, can still be used to tackle more complex cases. The model presented in this chapter is a strategic model which relies on the same assumptions as the fundamental models and which has the same strategic focus. It aims at the characterization of a static long term equilibrium between the cost factors.

In the fundamental strategic models, we assumed that the return flows were performed directly from one customer to one factory. If the transportation fixed costs are significant, it immediately raises the question of the possibility to share this fixed cost between neighboring customers. In other words, when a truck is ordered to return the empty packages of a customer, loading empty packages from neighboring customers, even with some additional cost, is likely to provide an important improvement of the solution.

There are two classical ways of implementing such local consolidation of the shipments. We can either rely on vehicle tours or on local accumulation depots. In the first case, a truck will visit several customers before traveling to the factory. In the second case, the customers will send their empty packages to a local depot which will be visited later by a truck that will return these empty packages to the factory. These two cases, illustrated on Figure 3.1, fundamentally rely on a similar logic. In a local cluster, i.e. a group of neighboring customers allocated to the same depot or visited by the same vehicle, the empty packages accumulate faster. This allows the shipment sizes used on the longest part of the distance between the cluster and the factory to be larger than the individual shipment sizes used otherwise. Moreover, this shipment size is built faster which increases the rotation rate of the packages.

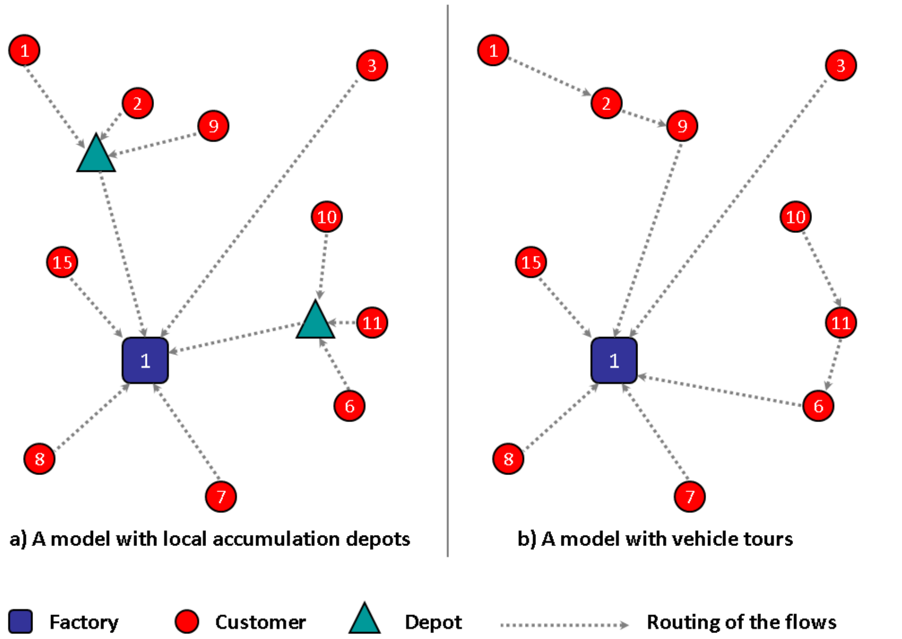

Figure 3.1: Local consolidation by the use of local accumulation depots or of vehicle tours.

However, additional costs will typically be incurred by the implementation of local consolidation. In both cases, the distance traveled by an empty package is increased. The packages are routed with a detour to a local depot or to other customers. Packages routed through a depot are also handled (unloaded and loaded) one additional time. Moreover, implementing and managing a depot is likely to generate some costs. Vehicle tours do not require any investment. But, they do not allow for the use of different transportation means for short- and long-haul travels. The scheduling of the tours will probably require more effort and coordination between the partners than the use of depots. Consequently, the advisability of a local consolidation logic must be assessed by taking into consideration the benefits, the additional costs and the characteristics of the type of consolidation.

In this thesis, we opted for a model with local accumulation depots. This choice results from our discussions with AGC, and the model presented in this chapter has been applied to the specific case of AGC. It follows that we face a quite classical and difficult problem: the network design of a supply chain with depots. This design integrates the inventory management of empty packages. The network design of supply chains and the facility location problem are classical subjects treated in reference books (see Ghiani et al. (2004) for example). However, the analysis of the return flow of reusable packages has been more scarcely studied. One of the first papers dealing with empty containers is the one by Crainic et al. (1993). It offers a quite torough analysis of inventory management and of shipments planning, involving dynamic and stochastic models. However, it covers the full truck load case and does not deal with network design and clustering. Two interesting case studies focus on reusable items. Duhaime et al. (2001) studies the flow of standardized logistical packagings at Canada Post. Del Castillo and Cochran (1996) examines the reusable bottle production and distribution at a Mexican soft drink manufacturer. Both case studies mainly analyze the impact of planning rules and the optimal number of reusable items.

Our research is also related to the facility location literature. Facility location problems aim to find the optimal placement of facilities, and their links with other layers of the network, so that the customers' demand is satisfied at minimum cost. As noted by Melo et al. (2009), facility location models and methods encountered for forward and reverse networks are similar. Melo et al. (2009) provide a thorough literature review of facility location related publications with a focus on supply chain management. The interested reader is also referred to (Klose and Drexl, 2005) and (ReVelle and Eiselt, 2005).

From our literature review, we identify four papers which are closest to our work. Two papers deal with returnable containers. Inspired by the case of a Dutch logistics service organization, Kroon and Vrijens (1995) offer a classical facility location model (transportation and opening costs) to choose the depots in a network aiming to bring returnable containers back to their original sender. Quite similarly, Jayaraman et al. (2003) suggest a facility location model for the design of reverse distribution networks. Due to the complexity of such mixed integer programs, and thus the impossibility to solve large instances exactly, they focus on the introduction and deep analysis of an elaborated heuristic to solve the suggested model. These two papers focus on pure network design (location of depots), while the present work also deals with inventory management (number of packages, shipment sizes and dispatch frequencies). Moreover, our model avoids a mixed integer programming formulation, so that large realistic problems can be solved.

The two other relevant papers are part of the distribution literature and integrate facility location with inventory management. In a forward network, the model of Shen et al. (2003) aims at choosing the location of distribution centres (DCs) and the assign-ment of the customers to theses DCs. The costs taken into account are fixed costs for opening the DCs, variable transportation costs from the DCs to the customers, ordering costs for the replenishment of the DCs from the factory, and non-linear costs for the inventory at the DCs. These inventory costs include the costs of a working inventory based on a EOQ model and of a safety stock held at the DCs. The customers have normally distributed stochastic demands. Lead times are considered on the factory-DC link. In Shen et al. (2003), the model is solved to optimality for instances with up to 150 customers via column generation on a set-covering formulation. In Daskin and Coullard (2002), the model is solved by an algorithm based on a lagrangian relaxation. Our model does not consider safety stock, but includes several plants to which the customers must be allocated. We additionally consider the inventory management at the plants and at the customers using an EOQ model and consider capacity limits on transportation.

In this chapter, we suggest a model and a solution procedure in order to assess the advisability of aggregating reusable empty packages in depots, and to guide the possible implementation of this option. The evaluation of this reverse logistics option covers the short-haul and long-haul transportation costs, the location-dependent inventory holding costs, the handling costs at the depots and fixed costs for opening the depots. The approach offers solutions concerning the network design and clustering (positioning of the depots and links with the customers and factories) as well as the inventory management. The targeted level of decision is strategic. Moreover, in order to be able to analyze real-life size problems, we suggest, based on reasonable assumptions, a continuous optimization formulation (avoiding integer variables), which then decomposes into a simple EOQ-like problem and a close-to-linear program, basis of the proposed iterative heuristic. The final goal of the study is to be able to advise a company (AGC in particular) about the implication of changing its policy from direct return flows to a two-stage reverse policy with accumulation. We evaluate the cost of the second alternative (the first alternative was analyzed in Chapter 2) and give insights on how it should be put in place.

The remaining of the chapter is structured as follows. In Section 3.2, we present the problem, the assumptions we make, and the mathematical model that follows. A simple heuristic approach is then described to solve it, in Section 3.3. In Section 3.4, we illustrate the application of the method on an example based on the case of AGC, and show the impact of various parameters of the problem. Finally, we conclude in Section 3.5 and discuss the methodology's possible extensions.

3.2 PROBLEM AND MODEL

3.2.1 *Problem description*

As explained in the previous section, the main goal is to evaluate the total logistic cost of the return flows of reusable packages if depots are included in the network and to compare this result with the cost achieved with direct shipments (calculated with the model of Chapter 2). The reverse network, including depots, is illustrated in Figure 3.2. The model has the following characteristics:

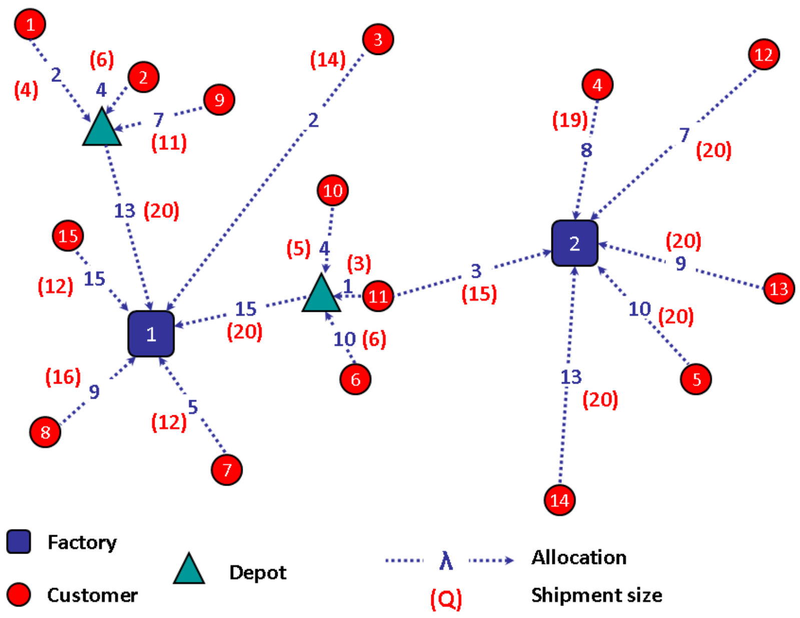

Figure 3.2: The reverse network with accumulation depots.

1. **Multi-factory**: we consider a network with several factories and several customers. The flows are directed from the customers to the factories.
2. **Standardized fleet**: all packages are identical. Consequently, the empty packages from a customer may be returned to any of the factories, regardless of the factory that had sent the loaded packages to the customer in the first place.

3. **Capacitated transportation**: the shipment sizes used on each link of the network may be limited by truck capacity. Transportation capacities may be different on each link.

4. **Zero transportation lead time**: We consider that the shipments are performed instantaneously[1].

5. **Variable L model**: in this chapter, we consider that the logistic inventory, i.e. the average number of empty packages used in the reverse network, may be chosen without any restriction.

6. **Continuous single-period model**: the model has a strategic perspective. As such, we rely on a continuous timeline with constant average flows. This is a quite classical assumption, notably in order to apply an EOQ-like inventory control. Even if based on simplifying assumptions, the EOQ policy has been shown to be quite robust, and valid at the strategic level of decision (see Nahmias (2009) for example).

7. **Perfect coordination assumption**: the perfect coordination assumption used in Chapter 2 is used for the model with depots. The perfect coordination assumption is used for the average inventory at the factories, but also for the average inventory at the depots. This corresponds to the ideal case and simplifies the computations. This assumption is reasonable as we focus on strategic decisions and because the depots are centrally managed. It allows us to avoid dealing with the coordination problem, which is not relevant at the strategic level.

8. **Discrete facility location**: we assume that a set of potential depots has been previously determined.

The strategic decisions integrated in the model are:

1. **Depot selection**: we must determine which ones of the potential depots will be used in the reverse network.

2. **Customers and depots allocation**: the flows of customers must be allocated to the factories and depots. The flows allocated from the customers to the depots must then be allocated to the factories.

3. **Shipment sizing**: the shipment size used on each active link of the network must be determined.

1 This methodological choice has been discussed in Section 1.1. In the case of AGC, the planning period is one week, whereas the transportation time is one day.

The model aims at the minimization of the total logistic cost, which includes:

1. **Transportation fixed costs**: each shipment (customer-depot, customer-factory, depot-factory) is subject to a fixed transportation cost. This fixed cost depends on the specific distance traveled, but not on the load of the truck.

2. **Holding costs**: the inventory held at each location of the network is subject to a variable unit holding cost. This cost may include the investment cost resulting from the detention of one package as well as the storage cost depending on the location where the inventory is held.

3. **Handling costs**: each package that transits through a depot will be unloaded (when coming from the customer) and loaded (when transported to a factory) one additional time. A variable unit handling cost is consequently added to the logistic cost.

4. **Opening costs**: opening a depot requires an investment that is reflected as a fixed periodic cost.

3.2.2 *Variables and Parameters*

The problem and the modeling assumptions are now well established. Before presenting the mathematical model, we list the variables and parameters, and present their notation. The factories are given an index $f = 1 \dots F$ and the customers an index $c = 1 \dots C$. The potential depots are given an index $d = 1 \dots D$.

We first list the variables we wish to estimate on the reverse logistics system. These variables eventually reflect the network design and the inventory management of the network.

λ_c^f is the (return) flow from customer c to factory f, i.e. the average number of reusable packages sent per time period from customer c to factory f, in nb. packages/period. Empty packages may thus be routed through a depot, but also directly to a factory (this has been seldom addressed in literature, as revealed by Melo et al. (2009)).

λ_c^d is the (return) flow from customer c to depot d in nb. packages/period.

λ_d^f is the (return) flow from depot d to factory f in nb. packages/period.

Λ_d is the flow in, and out, of depot d from all customers and to all factories in nb. packages/week. It is easily seen that $\Lambda_d = \sum_f \lambda_d^f = \sum_c \lambda_c^d$.

Q_c^f is the size of the shipments sent from customer c to factory f (according to the EOQ policy), i.e. the load of the trucks going from c to f, in nb. packages/truck.

Q_c^d is the size of the shipments sent from customer c to depot d in nb. packages/truck.

Q_d^f is the size of the shipments sent from depot d to factory f in nb. packages/truck.

L is the total number of empty packages used in the reverse logistics network, i.e. the logistic inventory[2].

Note that every potential depot d is not necessarily opened. Real depots correspond to the potential depots for which the flow Λ_d is strictly larger than zero.

Second, we list the parameters and their notations. They correspond to data, and thus depend on the application.

Γ^f is the flow (out) of reusable packages of factory f in nb. packages/period. It is the average number of empty packages that the factory requires per period for its production/distribution processes.

Λ_c is the flow (in) of reusable packages of customer c in nb. packages/period. It is the average number of packages per period that the customer makes empty, by consuming the goods stored in the packages he has previously received. Note that the network flows are balanced and that we have the following equality: $\sum_c \Lambda_c = \sum_f \Gamma^f$.

O_d^f, O_c^d, O_c^f are the fixed transportation costs, in €/vehicle, between potential depot d and factory f, between customer c and depot d and between customer c and factory f, respectively.

H_f, H_d, H_c are the unit holding costs, in €/(item·period), at factory f, at potential depot d and at customer c, respectively.

F_d is the fixed periodic cost to use depot d, in €.

M_d is the cost for handling one package at potential depot d, in €/item.

CAP_d^f, CAP_c^d, CAP_c^f is the capacity of a vehicle, in nb.items/vehicle, between potential depot d and factory f, between customer c and depot d and between customer c and factory f, respectively.

2 The logistic inventory was defined in Section 1.1 as the additional number of packages that are required to allow the accumulation of empty packages prescribed by the choice of the shipment sizes.

3.2.3 *Mathematical Model*

$$\min \quad \sum_{d,f} O_d^f \cdot \frac{\lambda_d^f}{Q_d^f} + \sum_{c,d} O_c^d \cdot \frac{\lambda_c^d}{Q_c^d} + \sum_{c,f} O_c^f \cdot \frac{\lambda_c^f}{Q_c^f} \tag{3.1}$$

$$+ \sum_f \frac{H_f}{2\Gamma^f} \cdot \Big(\sum_c \lambda_c^f Q_c^f + \sum_d \lambda_d^f Q_d^f\Big) \tag{3.2}$$

$$+ \sum_{d,\Lambda_d>0} \frac{H_d}{2\Lambda_d} \cdot \Big(\sum_f \lambda_d^f Q_d^f\Big) \tag{3.3}$$

$$+ \sum_c \frac{H_c}{2\Lambda_c} \cdot \Big(\sum_f \lambda_c^f Q_c^f + \sum_d \lambda_c^d Q_c^d\Big) \tag{3.4}$$

$$+ \sum_{d,\Lambda_d>0} \frac{F_d}{\Lambda_d} \cdot \Big(\sum_c \lambda_c^d\Big) \tag{3.5}$$

$$+ \sum_d M_d \cdot \Big(\sum_f \lambda_d^f\Big) \tag{3.6}$$

$$\text{s.t.} \quad \sum_d \lambda_d^f + \sum_c \lambda_c^f = \Gamma^f \qquad \forall f \tag{3.7}$$

$$\sum_f \lambda_d^f = \sum_c \lambda_c^d = \Lambda_d \qquad \forall d \tag{3.8}$$

$$\sum_d \lambda_c^d + \sum_f \lambda_c^f = \Lambda_c \qquad \forall c \tag{3.9}$$

$$Q_d^f \le CAP_d^f \qquad \forall f, d \tag{3.10}$$

$$Q_c^d \le CAP_c^d \qquad \forall d, c \tag{3.11}$$

$$Q_c^f \le CAP_c^f \qquad \forall f, c \tag{3.12}$$

$$\lambda_d^f, \lambda_c^d, \lambda_c^f, Q_d^f, Q_c^d, Q_c^f \ge 0 \qquad \forall f, d, c \tag{3.13}$$

The objective is to minimize the total average logistic cost of the network, which consists of the following costs:

Transportation costs - the transportation cost between two facilities (3.1) is given by the cost per vehicle (O_d^f, O_c^d, O_c^f) times the number of vehicles. The latter number is equal to the container flow (λ_d^f, λ_c^d, λ_c^f) divided by the number of containers per vehicle (Q_d^f, Q_c^d, Q_c^f).

Holding costs - the holding costs depend on the average inventory at each location. The computation of the average inventory relies on the perfect coordination assumption used in Chapter 2. Let us detail the computation of the average inventory under this assumption for a customer, a depot and a factory.

A customer may be allocated to more than one facility. The perfect coordination assumption states that, in this case, it will build the shipment size for one of these facilities, send this shipment, then start building the shipment size for the next facility, and so on. In other terms, it builds one shipment at a time. The portion of time the customer is building a shipment for a facility a is λ_c^a/Λ_c. During this time, its average inventory is $Q_c^a/2$. The average inventory of a customer is consequently given by

$$\bar{I}_c = \sum_f \frac{\lambda_c^f}{\Lambda_c}\frac{Q_c^f}{2} + \sum_d \frac{\lambda_c^d}{\Lambda_c}\frac{Q_c^d}{2}$$

This average inventory is multiplied by the unit holding cost at customer c (3.4).

Applied on the inventory of a factory, the perfect coordination assumption states that the factory receives one shipment at a time. It receives the shipment from a customer or from a depot, entirely consumes this shipment before receiving the next shipment. The average inventory at a factory is consequently given by

$$\bar{I}_f = \sum_c \frac{\lambda_c^f}{\Gamma^f}\frac{Q_c^f}{2} + \sum_d \frac{\lambda_d^f}{\Gamma^f}\frac{Q_d^f}{2}$$

leading to (3.2).

Finally, the average inventory at a depot is given by

$$\bar{I}_d = \sum_f \frac{\lambda_d^f}{\Lambda_d}\frac{Q_d^f}{2}$$

leading to (3.3). The perfect coordination states, in this case, that the depot builds the shipment for one factory to which it is allocated, then sends this shipment and starts building the next shipment. The size of the shipments sent from the depots to the factories are expected to be significantly larger than the shipments from the customers to the depots. As a consequence, we approximate the flow coming into a depot as a continuous flow equal to $\sum_c \lambda_c^d$.

The logistic inventory is thus computed as

$$L = \sum_{c,f} \lambda_c^f \frac{Q_c^f}{2}\left(\frac{1}{\Lambda_c} + \frac{1}{\Gamma^f}\right) + \sum_{d,f} \lambda_d^f \frac{Q_d^f}{2}\left(\frac{1}{\Lambda_d} + \frac{1}{\Gamma^f}\right) + \sum_{c,d} \frac{\lambda_c^d}{\Lambda_c}\frac{Q_c^d}{2} \tag{3.14}$$

Fixed opening costs - the fixed cost of using a depot (3.5) is incurred if the depot is opened in the solution. However, we want to avoid the use of binary variables to model the opening of the depots. Instead, we observe that, for each depot d which is opened in the solution, we will have that $\Lambda_d = \sum_c \lambda_c^d$.

Handling costs - a unit handling cost is incurred for each package that flows through a depot. Constraint (3.6) multiplies this unit handling cost by the total flow of the depot.

The constraints state that the average flow sent to each factory must cover its requirement of empty packages (3.7), that the average flow at each depot is made of the average flow that is sent to the depot by the customers which is also equal to the average flow sent by the depot to the factories (3.8) and that the average flow sent by each customer must be equal to the average number of packages that it makes available (3.9). Capacity constraints (3.10), (3.11) and (3.11) apply on each link of the network. All the variables are continuous and positive (3.13).

3.3 SOLUTION PROCEDURE

We suggest a solution procedure which exploits the continuous formulation of the problem and the decomposition approach used in Section 2.5. In Chapter 2 we demonstrated that, in the variable L case, the shipment sizing problem could be solved independently of the allocation problem, i.e. that the optimal shipment sizes for each link of the network could be derived without knowing the actual flow allocated to this link, if any.

We first show that this is still true for the direct shipments between the customers and the factories and for the local shipments between the customers and the depots. However, the optimal shipment sizes between the depots and the factories depend on the variable Λ_d representing the flow transiting through the depot. If the values of the Λ_d variables were fixed, all optimal shipment sizes could be determined *a priori*. As in Section 2.5, the remaining allocation problem in the λ variables would be a simple linear continuous problem. In order to exploit this closeness to the easily solved fundamental model, we suggest a heuristic which will use estimates of the Λ_d variables that are iteratively updated.

3.3.1 *Shipment Frequencies*

Let us assume that the values of the λ variables are fixed, i.e. an allocation is given to us. Then, we can easily see that the problem of finding the optimal shipment sizes on each link of the network decomposes into independent subproblems bearing an

EOQ-like structure subject to a capacity constraint. For example, for a specific variable Q_c^d, we obtain the independent subproblem

$$\begin{array}{ll} min & \lambda_c^d [\frac{O_c^d}{Q_c^d} + \frac{H_c}{2\Lambda_c} Q_c^d] \\ s.t. & Q_c^d \leq CAP_c^d \end{array}$$

We then derive the optimal shipment sizes on each link c-d of the network:

$$Q_c^{d*} = min\left[CAP_c^d, \sqrt{\frac{2\Lambda_c O_c^d}{H_c}}\right] \quad \forall c, d \tag{3.15}$$

This expression indicates that the optimal shipment sizes on the links between customers and depots are independent of the allocation variables.

Similarily, we can determine

$$Q_c^{f*} = \min\left[CAP_c^f, \sqrt{\frac{2\,O_c^f}{\frac{H_f}{\Gamma^f} + \frac{H_c}{\Lambda_c}}}\right] \quad \forall f, c \tag{3.16}$$

$$Q_d^{f*} = \min\left[CAP_d^f, \sqrt{\frac{2\,O_d^f}{\frac{H_f}{\Gamma^f} + \frac{H_d}{\Lambda_d}}}\right] \quad \forall f, d \tag{3.17}$$

Consequently, the optimal shipment sizes used for the shipments from the customers to the depots and to the factories are independent of the allocation. However, the shipment sizes used for the shipments from the depots to the factories depend on Λ_d, the average flow at the depot.

3.3.2 *Allocation of the Flows*

Let us now consider the allocation problem in the λ variables. It is a more difficult problem, but turns out to be close to a linear program. Indeed, only the average depot flow variables Λ_d prevent the model of being linear. The depot flow variables are found in the objective terms (3.3) and (3.5) preventing the objective function from being convex, and, less obviously, in the optimal shipment sizes from depots to factories (see equation (3.17)). If the values of these Λ_d were fixed, the problem would solve easily. The simple heuristic we offer takes advantage of this closeness to a linear program. Note that, of course, our formulation of the problem has also been driven by this goal.

The heuristic iteratively estimates the depot flow Λ_d (*a priori*), solves the linear program based on this estimation, and then uses the solution to improve the depot flow estimation. In the following, we call Λ_d^{est} the estimation of the depot flow Λ_d. Using this estimation, the linear program is solved and gives a feasible solution in terms of λ_d^f, λ_c^f and λ_c^d (feasible as Λ_d is not present in the constraints). From the flow variables λ, the other variables ($\Lambda_d = \sum_f \lambda_d^f$, Q^* and L), using (3.14), (3.15), (3.16) and (3.17), are easily deduced in order to obtain a complete feasible solution of the original problem. The depot flows of a current real feasible solution are denoted Λ_d^{sol}. In the next iteration, this real depot flow is then used to find an improved estimation Λ_d^{est}.

To start the algorithm, an initial estimation Λ_d^{est} of the depot flows is needed. This estimation depends on the instance we want to solve. Finding a reasonable initial estimation requires to carefully analyze the structure of the instance beforehand. We will illustrate this step in the next section when we apply our heuristic to the case of AGC.

The heuristic then aims to iteratively get closer to the optimal solution of the problem. The real depot flow Λ_d^{sol} potentially offers a better estimation of the optimal flow through the depots. However, with this solution, some depots are closed, with $\Lambda_d^{sol} = 0$, while they could be open in the optimal solution, and some depots are open while they should not be. It would thus be misleading to directly equal the new estimation Λ_d^{est} to the values Λ_d^{sol} computed. The latter rather give a good direction for the decrease (we start with a relatively large estimate Λ_d^{est}). The heuristic thus progressively decreases the depots flows values Λ_d^{est}, based on the direction offered by the flows Λ_d^{sol} computed in the previous step of the algorithm (i.e. a fraction of the difference observed between the previous approximation Λ_d^{est} and the observed Λ_d^{sol}). It is stopped when the solution found does not change anymore.

The heuristic is presented in Algorithm 1. We present a simplified version, in order to show the main ideas. At each iteration, the solution is computed via the linear program, using the depot flow estimation Λ_d^{est}. Then, the depot flow estimations Λ_d^{est} are iteratively and carefully decreased, and the linear programs are solved with these values. To update the depot flows Λ_d^{est}, a fraction (parameter *decFac*) of the difference between the previous value Λ_d^{est} and the value Λ_d^{sol} computed from the linear program is removed from Λ_d^{est}. Furthermore, the best solution (*BestSol*) is saved if necessary. We recall that this solution is of course a real solution: the depots flows, Λ_d, the cost function, the shipment sizes Q, and the number of reusable packages L are recomputed from the variables λ of the solution. When the algorithm does not improve the

Algorithm 1 : Simple Heuristic

Derive an initial estimate Λ_d^{est} and then the shipment sizes $Q^*(\Lambda_d^{est})$.
Solve the linear program using $\Lambda_d = \Lambda_d^{est}$.
From the solution, compute $\Lambda_d^{sol} = \sum_f \lambda_d^f$.
Save solution as $BestSol$.
while $NbConsecutiveNotChange < MaxNotChange$ **do**
  $\Lambda_d^{est} = \Lambda_d^{est} - decFac \cdot (\Lambda_d^{est} - \Lambda_d^{sol})$.
  Compute $Q^*(\Lambda_d^{est})$ from (3.15), (3.16) and (3.17).
  Solve the linear allocation program using Λ_d^{est} and Q^*.
  From the solution, compute $\Lambda_d^{sol} = \sum_f \lambda_d^f$.
  if the solution is better than $BestSol$ **then**
    Save solution as $BestSol$.
    $NbConsecutiveNotChange = 0$
  else
    $NbConsecutiveNotChange = NbConsecutiveNotChange + 1$.
  end if
end while
The best solution found is given by $BestSol$.

solution anymore (parameter $MaxNotChange$), it stops and the best solution found is given.

The idea of the suggested heuristic is thus very simple: give the problem a formulation which is very close to a linear program, and then iteratively solve easy linear problems with an improving approximation for the nonlinear term, until no improvement can be reached.

3.4 THE REVERSE NETWORK OF AGC

In this section, we apply the heuristic on an illustrative example based on the industrial network of AGC Flatglass Europe and then provide some experiments to show the impact of various parameters.

3.4.1 *Illustrative Example*

For confidentiality reasons, we only provide a similar case. We consider a network of 10 factories and 500 customers, spread all over Europe. The total flow, among all factories, is equal to 500 trucks (charged with glass) per week. It corresponds to 1000 trestles sent per week. A factory's requirement Γ^f ranges from 10 to 200 trestles per week, while a customer's rate Λ_c ranges from 2 trestles per year to 80 trestles per week. The network is displayed on Figure 3.3.

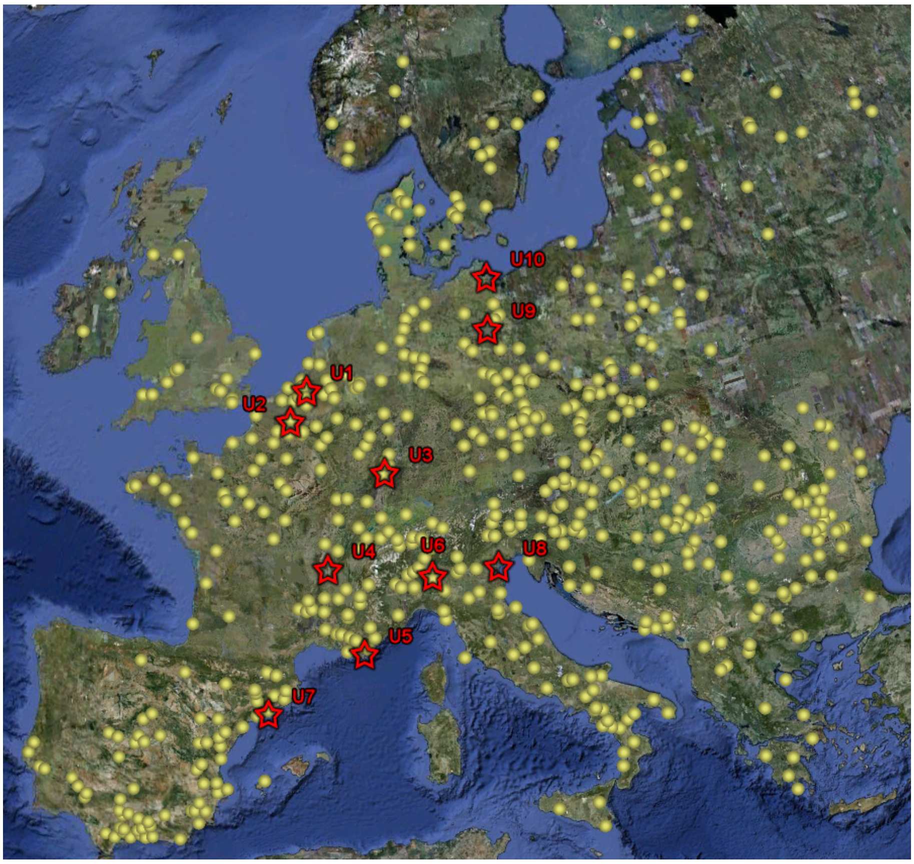

Figure 3.3: An industrial network similar to the network of AGC.

We largely discussed the exact problem faced by AGC with its supply chain division[3]. Based on the specific challenge faced by AGC for designing its reverse network, the general model presented in Section 3.2 was adapted.

1. Set of potential depots: AGC wanted us to assume that the set of potential locations for the depots was the set of its customers and factories. The objective of AGC was to assess the relevance of a three-layer reverse network. At this stage, a detailed map of potential locations was not considered to be necessary. The map given by the set of customers was assumed to be sufficiently large and flexible to assess the advisability of a network with depots. We thus have $ND = NC \cup NF$.

2. All flows are routed through depots: For commercial reasons, AGC would not consider letting empty trestles accumulate at the customers. As a consequence,

3 The case is described in more detail in Sections 1.1 and 1.3

each customer must be allocated to a depot for the return flow of its empty trestles.

3. Local transportation costs are not integrated: AGC estimated that the transportation between the customers and the depots would be performed by the transporters in charge of the deliveries. In this configuration, a special truck performing the delivery of two loaded trestles to a customer, would then load two empty trestles and bring them to a depot. It will be negotiated with the transporters that this local flow would be part of the delivery service the transporters provide. Consequently, the cost of these flows was not included in the model. However, customers must still be allocated to the depots.

4. Maximal distance customer-depot: The negotiation with the transporters would rely on a maximal distance that must be traveled from any customer to the depot to which it is allocated. This maximal guaranteed distance is denoted D, while $|cd|$ is the distance in kilometers from customer c to depot d. AGC estimates this maximal distance to be 150 kilometers.

5. No fixed costs for using depots: AGC plans to rely on existing locations (customers, AGC facilities, transporter sites,...). At this stage, they want to evaluate the cost of the network without any fixed costs for using the depots.

6. No handling costs at the depots: AGC also estimated that the handling costs at the depots would be negligible and should not be included in the model.

7. Constant holding cost: The cost of holding the packages is dominated by the value of the investment made in the fleet of packages. Storage costs, considering the expected volumes at the different locations, are assumed to be negligible. The unit holding cost has been fixed to 6€/week.

8. Capacity: The truck capacity is identical for all shipments from depots to factories. A standard truck can accommodate up to 20 empty folded trestles.

It is important to note that, because the factories are potential depots, and because the local transportation costs are not included in the model, most of the customers within a distance D of a factory will most likely be directly allocated to the factory. It means that the return flows from these customers are managed by the transporters performing the deliveries and induce no cost in the reverse model suggested.

The transportation cost matrix O_d^f is deduced from the distance matrix, multiplied by a 1.1€/km factor.

The model is significantly simplified for the reverse network of AGC. It reduces to

$$\min \quad \sum_{d,f} O_d^f \cdot \frac{\lambda_d^f}{Q_d^f} + H \sum_{d,f} \frac{\lambda_d^f Q_d^f}{2} \cdot \left(\frac{1}{\Lambda_d} + \frac{1}{\Gamma^f}\right) \tag{3.18}$$

$$s.t. \quad \sum_d \lambda_d^f = \Gamma^f \qquad \forall f \tag{3.19}$$

$$\sum_f \lambda_d^f = \sum_c \lambda_c^d = \Lambda_d \qquad \forall d \tag{3.8}$$

$$\sum_d \lambda_c^d = \Lambda_c \qquad \forall c \tag{3.20}$$

$$Q_d^f \leq CAP \qquad \forall d, f \tag{3.21}$$

$$\lambda_c^d = 0 \qquad \text{if } |cd| > D,\ \forall c, d \tag{3.22}$$

$$\lambda_d^f, \lambda_c^d, Q_d^f \geq 0 \qquad \forall c, d, f \tag{3.23}$$

The objective (3.18) minimizes the sum of the holding cost associated with the logistic inventory, which equals the sum of the average inventory at the depots and at the factories, and of the transportation cost linked with the shipments from the depots to the factories. The minimization is subject to flow conservation constraints (3.19), (3.8) and (3.20). Shipment sizes are limited by the capacity of the trucks (3.21). Finally, constraint (3.22) forces the allocation of the customers to depots that are at a distance which is smaller than the maximal distance negotiated with the transporters.

3.4.2 *Results*

The solution procedure described in Section 3.2 was applied to solve the model with the data provided by AGC. The best solution found is displayed on Figure 3.4. The white squares represent the depots used in the solution.

Constraint (3.22) allows us to derive an interesting initial estimate, denoted Λ_d^{max}, for the depot flow variables Λ_d. We use an upper bound on each depot flow defined as the sum of the flow Λ_c of all the customers that are within a distance D of the depot. This value is the initial estimate used in the algorithm. Using this estimate, the resulting linear program is solved in the first iteration. This provides a feasible solution and an upper bound on the total cost, computed by evaluating the objective function with the solution of the λ and Q variables, as well as the value of Λ_d^{sol}. Moreover, this first iteration additionally allows us to compute a lower bound on the optimal total cost. Indeed, it can be observed that each coefficient of the cost function in which Λ_d

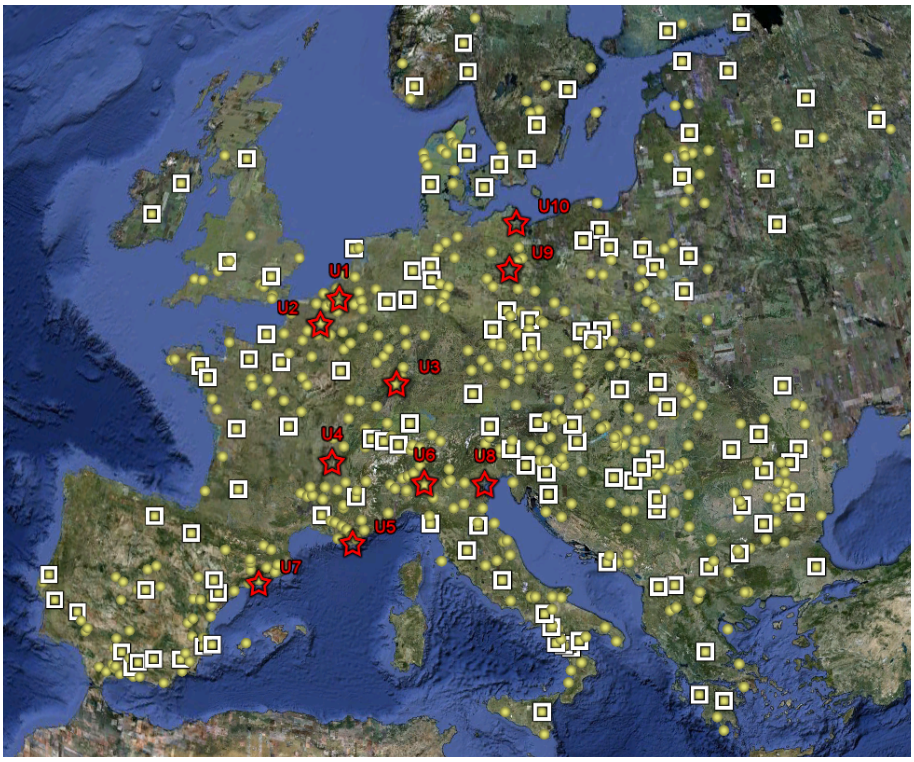

Figure 3.4: The best solution found.

has been replaced by its upper bound is a lower bound on the possible value of that coefficient. It follows that if we evaluate the objective function with the solution of the λ and Q variables and the value Λ_d^{max}, we obtain a lower bound on the cost of the optimal solution. The lower bound was calculated to be approximately 8% in our instance.

The algorithm was run for 22 iterations, taking about 8 minutes (on a 2.66 GHz PC, with $decFac = 1/3$ and $MaxNotChange = 5$). This is quite fast in view of the size of the problem and knowing that we face a strategic problem. Moreover, most of the improvement from the initial solution is made in the first iterations, in less than 3 minutes.

In Figure 3.5, we show how the heuristic behaves when solving this example. The cost of the initial solution, which offers an upper bound, is 27,539 €/week. It then decreases to the best solution found, 25,795 €/week (we remind that this is a real solution, with updated depot flows Λ_d^{sol}). This cost includes the transportation cost (depots to factories), 21,474 €/week, and the inventory holding cost, 4,321 €/week. The fact that the transportation cost is more important than the inventory holding cost essentially comes from the maximum capacity put on the shipments. Indeed, this constraint is often tight in our case (see below), and prevents the two terms to be equal at optimality. Figure 3.5 also depicts the evolution of the number of depots

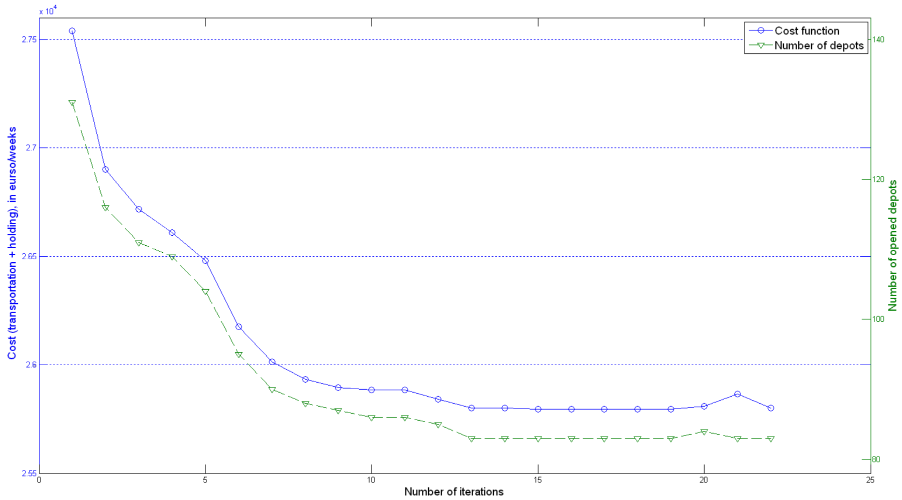

Figure 3.5: Evolution of the cost function and the number of depots, according to the number of iterations.

introduced (right-hand side axis). The initial solution involves 131 opened depots. As the depot flows Λ_d^{est} are first overvalued, this is equivalent to an underestimation of the unit holding cost which tends to favor the creation of depots (more depots decrease the transportation costs which have overvalued weight, see equation (3.18)). With the best solution found, only 83 depots are opened, out of 510 possible. Among them, 44 depots send more than 2 reusable packages per week. The 39 other depots are small depots, which often cover an isolated area with few small customers.

It can also be observed in Figure 3.5 that the evolutions of both values (cost function and number of opened depots) are relatively parallel. One reason for that is the following. The iterations of the algorithm mainly aim to improve the approximation of the depot flows estimations Λ_d^{est}, and thus the inventory holding cost (see equation (3.18)). The impact on the transportation cost is a side effect (balancing the costs). The transportation cost varies from 21,852 to 21,474 €/week, while the inventory holding cost decreases from 5,686 to 4,321 €/week from the start to the end of the heuristic. The decrease of the inventory holding cost is directly linked to the decrease of the number of opened depots. Note that the impact of the unit holding cost H on the network properties is analyzed in Section 3.4.3.

In the best solution found, 720 reusable packages (L) are necessary for a good inventory management of the return flows. The average shipment size from depots to factories (Q_d^{f*}) is 19.7 empty containers, and most (93 percent) of the trucks are full, i.e. with a charge of $CAP = 20$ items.

The distance covered by the trucks, by week, to return the reusable packages from depots to factories is 19,522 kilometers. In addition to this, in order to allow the comparison with the network without depots, it is interesting to compute the distance covered from customers to depots. It adds up to 33,169 km, which leads to a total distance, from customers to factories, of 52,691 km. It may seem surprising that more kilometers are covered from customers to depots but this comes from the fact that a truck only ships two trestles in this travel, while a truck carries twenty trestles from depots to factories. Ten times more trucks are thus necessary for the travels from the customers to the depots. This is an important motivation for the implementation of depots. In order to analyze more in depth the impact of the shipments from customers to depots, they could be included in the cost function.

In comparison, the network without depots would lead to a total distance of 214,642 km (this is the optimal value, solution of a classic transportation problem). This total distance is far greater than the one computed for the network with depots. Again, here on the whole network, this is due to the fact that in the network without depots, only two reusable packages are carried by trucks at one time. Note that, from a green supply chain point of view, i.e. for environmental concerns, the reduction of the total distance covered, and thus the pollution discharged, is of course an important benefit. However, the big difference in total distance is smoothed over by the fact that, in the network without depots, the trucks can ship an additional charge, if one can be found in the area, and if the distance is long enough. This explains that the return flows of AGC in the current situation are only charged at 30% of the price of a full truck. It remains however that, in the case of AGC, the benefit identified by the three-layer network configuration, when compared to the current two-layer configuration, was around 20% of the total logistic costs.

The proposed methodology allows us to compute a good estimation of the cost of the return flows if depots are included in the network. The company may compare these to the present costs (possibly charged by a transportation provider), and assess the advisability to introduce depots in the reverse logistics network. Moreover, the method offers several other insights on the network and on how it should be managed: how many depots should be opened and where; to which customers and factories the depots should be linked (i.e. the clustering); what should be the frequency and shipment sizes for each link; how many reusable packages are needed; what are the distances that are covered; etc.

3.4.3 *Parameters' Impact*

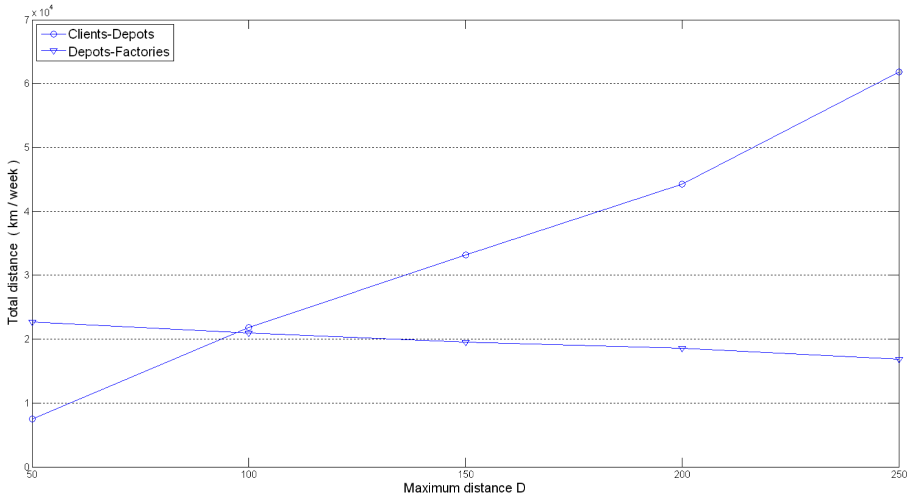

Figure 3.6: Evolution of the total distances, customers to depots and depots to factories, in function of the maximum distance D from customers to depots.

In this subsection, we further explore the impact of two parameters: the maximum customer-depot distance D and the unit holding cost H.

D	50	100	150	200	250
Total Cost (trans+holding)	32847	28725	25795	24469	21778
Nb.Depots (total)	221	124	83	74	53
Nb.Depots (>2 items/week)	66	54	44	43	39
Number Items in Network	1308	953	720	667	540
Batch Size (average)	18.8	19.4	19.7	19.8	19.9
Batch Size (% full trucks)	0.82	0.91	0.93	0.97	0.98
Total Dis customers-Depots	7499	21826	33169	44258	61873
Total Dis Depots-Factories	22725	20918	19522	18602	16850

Table 3.1: Impact of the maximum distance D from customers to depots, on the network characteristics.

In the previous subsection, we observed that the distance to be covered per week from customers to depots is in fact greater than that from depots to factories (due to the different number of reusable packages carried). This may cause extra costs on the forward flow, which includes the shipments of the empty packages to the depots. One could be tempted, in order to avoid this problem, to reduce the maximum distance D covered for this part of the shipment. The impact of this bound on the distance is given in Table 3.1. Naturally, when D decreases, the total distance covered from customers to depots decreases. On the other hand, the total distance from depots to factories increases, as more open depots are required to cover the market area, and, as thus, the shipments are more frequent and less charged. Figure 3.6 further shows that the impact on the customer-depot distances is stronger than on the depot-factory distances. This partly comes from the fact that ten times more empty trestles are shipped in a truck going from a depot to a factory. This observation may motivate the choice for a quite short distance from customers to depots, in order to get a good balance in terms of distance and costs.

However, it would be difficult to determine the adequate value for the maximal distance D without including a measure of its cost impact on local transportation (customer-depot) in the model. When the results were discussed with AGC, one of our main recommandation was to consider more precisely how the cost of the local transportation was affected by D. Indeed, excluding this cost from the model lead to solutions that may significantly increase local distances to achieve very small reductions of the distances between the depots and the factories. This has been observed in the solutions we generated. In a depot cluster, the location of the depot is generally not central. It is at the edge of the cluster, as close as possible to the factory to which it is allocated. This significantly increases the local distances and is very likely to impact the negotiation of AGC with its transporters. To illustrate this drawback, we modified the best solution found in the following manner: for each depot cluster, we relocated the depot at the barycenter of its cluster. This simple adaptation of the solution reduces the local distances (customer-depot) by 28%, while increasing the depot-factory distances by only 6%.

The evolution of several other characteristics of the network are given in Table 3.1. It can be seen that, when D increases, the total cost of the return flow (transportation from depots to factories and holding) decreases. This is expected, as the problem is less constrained. As the number of open depots decreases, the logistic inventory decreases, and the average shipment size increases (a given depot accumulates items more quickly).

H	2	4	6	8	10	15	20
Total Cost	22643	24341	25795	27166	28396	31341	34151
Transport Cost	20970	21263	21474	21840	22160	22903	23656
Holding Cost	1673	3078	4321	5326	6236	8438	10495
Nb. Depots	84 / 47	84 / 43	83 / 44	82 / 42	81 / 40	82 / 38	82 / 38
Nb. Items	837	769	720	666	623	562	525
Batch Size	19.9 / 0.99	19.8 / 0.97	19.7 / 0.93	19.5 / 0.92	19.4 / 0.91	19.1 / 0.88	18.7 / 0.77

Table 3.2: Impact of the unit holding cost H, on the network characteristics.

The holding cost H is acknowledged to be a parameter which is very difficult to accurately estimate (it should include carrying costs, opportunity cost, devaluation, obsolescence, etc.). It is thus particularly important to analyze the sensitivity of the model's results to the holding cost. Table 3.2 shows the impact of the unit holding cost H on the reverse logistics network characteristics. It can be seen that the total cost of the return flow increases with the unit holding cost. Unsurprisingly, this increase comes from the global inventory holding cost and, to a lesser extent, from the transportation cost. Table 3.2 also shows that the number of depots is stable, while the number of reusable packages used in the system and the shipment sizes significantly decrease. This tends to show that, in order to adapt to higher unit holding cost, the shipments frequency, in other words the packages turnover, is increased so that less items need to be stored. Note that this also explains why the transportation cost tends to increase.

3.5 CONCLUSION

In this chapter, we suggested a methodology to answer a question which is quite natural in practice: "Should reusable packages be accumulated in regional depots?" This question has been raised by AGC, concerned with the best management of the return flows of its empty trestles. We thus face a quite classical, strategic, problem in reverse logistics: the design of a three-stage network and its inventory management.

In order to be able to solve real-life cases, we suggested a mathematical model based on well justified simplifying assumptions. It allowed us to introduce a simple heuristic. Some assumptions are driven by the case at hand (standard fleet, capacity,...) while other assumptions come from the strategic perspective of the model (constant rates, continuous model,...).

We provided a continuous mathematical model, which aims to minimize the total logistic cost of the three-stage network in a EOQ-like inventory management. We showed that the decomposition of the problem used in Chapter 2 between a shipment sizing problem (consisting in finding the optimal shipment sizes) and an allocation problem (which objective is the routing of the flows of empty packages) could be used for this more complex model. However, the optimal shipment sizes from depots to factories depend on the average depot flow variables. Additionally, these average depot flow variables also prevent the allocation problem from being linear. The idea of the suggested heuristic is to fix the value of these variables using an estimation which is updated in an iterative procedure. Having fixed the values of these variables, the problem becomes easy to solve, as it reduces to a simple continuous linear program. This decomposability and the closeness with a linear program are among the main originalities of the suggested methodology. They have motivated our assumptions and the proposed model. They allow to offer a simple heuristic which quickly provides good solutions to large realistic problems.

We illustrated our methodology on an example inspired by the case of AGC, with 10 factories and 500 customers all over Europe. The suggested method was able to achieve a good solution for this large instance in less than ten minutes. The iterations of the heuristic, i.e. the improvement of the depot flow approximations, lead to decrease the number of depots and the total inventory holding cost in particular. When computing the total distances covered per week, the shipments from customers to depots appear to be important. One way to avoid this is to bind more strictly the maximum distance D allowed from a customer to a depot. A more interesting approach would be to include these shipments into the model. Our experiments also show that the impact of a decrease in D is stronger on the total distance from customers to depots than on the one from depots to factories. One reason for that lies in the fact that ten times as many reusable packages are shipped in a truck from a depot to a factory. Furthermore, we analyzed the impact of the unit holding cost H. Increasing it leads to larger costs, unsurprisingly, and to a higher turnover of the reusable packages (which are thus less numerous). All in all, the proposed methodology allows us to compute a good estimation of the cost of the return flows if depots are included in the network, in order to assess the advisability of the introduction of depots. Moreover, the method offers several other insights on the network, its design, its clustering and its inventory management.

The heuristic procedure, when applied on the case of AGC, implies a simplified version of the general three-layer model. Future research will analyze the performance of the heuristic on the more general model presented in Section 3.2. Assessing the performance of the general model with depots will be done by deriving lower bounds on the optimal cost, by comparing its solutions with the solutions of models in which the space of solution is restricted by additional constraints or with the solutions which could be obtained using other approaches for solving the embedded facility location problem (metaheuristics, column generation such in Shen et al. (2003),...) and by testing various instances including extreme cases.

Additionally, exploring how vehicle tours could be integrated in the model in order to be able to evaluate, in a specific context, the best strategy for local consolidation would be of strong managerial relevance.

4

STRATEGIC MODEL FOR A HETEROGENEOUS FLEET

4.1 INTRODUCTION

The fundamental strategic models presented in Chapter 2 dealt with the design (allocation of the customers to the factories and optimal shipment sizes) of an independent reverse network for the return flows of empty packages of a standardized fleet of packages. A first extension was presented in Chapter 3 in which intermediary depots were used to aggregate the flows of neighboring customers in order to share the fixed transportation costs, at least on the longest part of the distance between these customers and the factory.

In this chapter, we develop a second extension of the strategic models, which also exploits the possibility to share the fixed transportation costs. So far, we have always assumed that the fleet of packages was standardized, i.e. that the packages used in the network were all identical, or at least completely substitutable. Clearly, this represents an ideal scenario. It provides all the benefits offered by the pooling of the packages that we discussed in Section 2.4. In practice, especially in large industrial networks, it is likely that standardizing the fleet of packages completely would require very important investments which may not be justified by the benefits granted by pooling the packages.

The existence of different types of packages in the network of a producer may result from various factors.

1) The packages may be specific to products, some products having to be loaded on certain specific packages.
2) The packages may be specific to factories, the production facilities being equipped for the use of certain types of packages.
3) The packages may be specific to customers, some customers having technical con-

straints restricting the types of packages they are willing to receive.
4) The packages may be specific to some transporters, for trucks may be only adapted to transport certain types of packages.

Naturally, the use of some types of packages may be specific to a combination of these factors. A complex matrix of substitutability rules then probably drives the management of the empty packages. Standardizing the fleet is likely to require important investments at the production facilities and long negotiations with customers and transporters. Additionally, if these barriers could be suppressed, an additional investment would be required in the fleet of packages itself in order to replace the packages types made obsolete by new standard packages.

In practice, the investment and managerial effort needed to achieve a complete harmonization of the fleet of packages may appear disproportionate. The producer would probably, at least on the medium term, focus on reducing the number of different types of packages and rely on a few main types.

This was the case of AGC. After the results of the fundamental strategic model were analyzed (standardized fleet), they concluded that a harmonization of the fleet was necessary. However, in regard of the investments that would have been necessary at the production facilities and in the packages' fleet, a realistic objective was to reduce the number of packages types to two, or three, main types. Accordingly, they required a strategic assessment of the logistic cost of a reverse network with few different types of packages. The objective of this chapter is to suggest a strategic model for the design of the reverse network with different types of non-substitutable packages. The assumptions we use are linked to the strategic perspective of the model and to the case of AGC. The issues addressed by the model are the following:

1) How should the return flows of the empty packages be organized?
2) What is the optimal sequence of replenishments?
3) How many packages of each type should the producer ideally hold?
4) How does a transportation capacity limit affects the design?

The first question is specific to the case of a heterogeneous fleet. It is related to the composition of the shipments. When a truck visits a customer, there will be different inventories of empty packages ready to be returned, possibly one for each type considered in the network. The question is to know which types of packages are to be loaded in the truck and in which proportions. Should we use the possibility to share the trucks between the different types?

This immediately leads to the second question about the repeating sequence of shipments that we must determine to characterize the long term equilibrium of the network. As in the previous chapters, the choice of the sequence of shipments impacts the number of packages that are used in the network, i.e. the logistic inventory. In addition, Question 3 also addresses the issue of the composition of the logistic inventory. Finally, Question 4 considers a transportation capacity limit and how it modifies (or not) the answers to the previous questions.

The chapter is structured as follows. In Section 4.2, we detail the problem and present the mathematical formulation on which we rely. We discuss the assumptions used for this extension of the strategic models. The structure of the problem is then analyzed in Section 4.3. We highlight some important features of the model and of its solutions. Because the formulation of the problem shows that it is a continuous convex optimization problem, we do not provide a detailed solution procedure, but rather rely on classical methods for convex optimization. We focus on the managerial insights derived from the analysis of the solutions in Section 4.4. Finally, we conclude the chapter in Section 4.5 by summarizing the most important insights gained from the analysis and by underlining the implications of the assumptions we have used.

4.2 PROBLEM AND MODEL

4.2.1 *Problem Description and Assumptions*

We consider a network made of a single factory having a set $NC = \{1, ..., C\}$ of customers to which it delivers goods loaded in certain reusable packages. For various reasons, there are different types of packages. The set of the packages types is $NP = \{1, ..., P\}$. The factory has a constant periodic requirement for each type of packages, which is based on its production and delivery processes. It thus consumes its inventory of empty packages of each type p at a specific constant rate. The customers receive the loaded packages, consume the goods, then build an inventory of empty packages which has to be returned to the factory at some point. The customers may receive different types of packages, for example because they receive different types of products. In this case, they build an inventory of empty packages of each type they receive.

The return flows are performed directly from one customer to the factory. They are subject to a fixed transportation cost which is independent of the load. A truck visiting a customer may load packages of different types. This allows the fixed transportation costs to be shared between the different return flows of packages of each type. But, the capacity of the truck may be limited. The packages held in the network are subject to a variable linear periodic holding cost. In this respect, we only consider the average number of empty packages needed in the reverse network, i.e. the logistic inventory, as discussed in Section 1.1.

The objective is to determine how the return flows should be organized in order to minimize the total logistic cost (transportation and holding costs). Additionally, we determine the ideal size and composition (in terms of types of packages) of the logistic inventory.

The assumptions on which we rely are specific to the strategic perspective adopted and to the case of AGC. They are listed below. We refer to Section 1.1 for a discussion of the methodological choices.

1. **Single factory**: The network contains only one factory and C customers from which it is resupplied with empty packages.
2. **Capacitated transportation**: The shipment sizes used in the network may be limited by a truck capacity. Without loss of generality, it is assumed that all empty packages occupy the same volume in a truck.
3. **Zero transportation lead time**: We consider that the shipments are performed instantaneously. This corresponds to the case of AGC in which the planning period is one week whilst the transportation time is one day.
4. **Variable and Fixed L versions**: In this chapter, we solve both the variable L version of the problem (in which the logistic inventory may be chosen without any restriction) and the fixed L version of the problem (in which the logistic inventory is limited by the current investment). The number of empty packages of each type p used in the network is named the *logistic inventory of type p*. In the fixed L case, the limit applies on each type of packages separately, i.e. it limits the size of each logistic inventory.
5. **Continuous single-period model**: We rely on a continuous timeline with constant average flows. This results from the strategic perspective adopted. We aim at finding a long term static equilibrium between the cost factors.
6. **Perfect coordination assumption**: We use the perfect coordination assumption to model the average inventory at the factory. It implies that each replenishment

of the factory with empty packages of a specific type p is received at the factory exactly when its inventory of empty packages of type p drops to zero.

7. **Constant holding cost**: The unit periodic holding cost is identical for all locations and all types of packages. In the case of AGC, the holding cost mainly consists of a depreciation cost, and is identical for the different types of trestles[1].

8. **No local consolidation**: Shipments are made from one single customer to the factory. We do not consider the use of depots or vehicle tours.

It can be observed that the assumptions used in this chapter are more restrictive than those used in the previous chapters. The relaxation of some of these assumptions would significantly increase the complexity of the model, and consequently, of the methods that would be required for solving it. This narrows the scope of the model presented in this chapter. Nevertheless, using these assumptions allows us to derive important managerial insights about the composition of the fleet and provides a basis for the assesment of reverse networks with a heterogeneous fleet. They also correspond to the case of AGC. We will discuss the impact of these assumptions in more detail at the end of the chapter.

Relying on the perfect coordination assumption, used in the strategic fundamental models of Chapter 2, similarly allows us to approximate the average inventory at the factory without taking into account the difficult coordination problem. This approximation corresponds to a best-case scenario in which the arrivals of the return flows are perfectly coordinated at the factory. The quality of this approximation will be discussed in Chapter 5.

4.2.2 *Parameters of the Model*

The parameters of the model first represent the flows (in and out) of the network:

- Λ_c^p is the constant rate at which customer c makes packages of type p available (empty)
- $\Lambda_c = \sum_p \Lambda_c^p$ is the global rate at which customer c makes packages available
- Γ^p is the constant rate at which the factory uses empty packages of type p

1 We use identical holding costs for all types of packages for the simplicity of the presentation. Using holding costs dependent on the types of packages does not alter the model nor the solution procedure. However, as it will be discussed later, relying on holding costs that are identical at the customers and at the factory is necessary in order for the strategic model to adequately approximate the management of the return flows, even if the strategic model would solve easily with location dependent holding costs.

- $\Gamma = \sum_p \Gamma^p$ is the global rate at which the factory uses empty packages

The network flows are balanced, i.e. we have that $\sum_c \Lambda_c^p = \Gamma^p \quad \forall p \in NP$.

Second, we define the cost parameters:

- O_c: the fixed transportation cost of a shipment from customer c
- H: the unit periodic holding cost of a package

Additionally, we have

- CAP: the capacity of the trucks
- $\bar{L^p}$: the number of packages of type p available to be used in the reverse network, i.e. a bound on the value of the logistic inventory of type p

4.2.3 *Variables of the Model*

Defining the variables of the model requires that the first question (about the way return flows are managed) is answered beforehand. This is easily done, using the perfect coordination assumption, by deriving the following two statements about optimal solutions:

Statement 1: Once a return flow is performed from a customer, it is optimal to load all empty packages (all types together) available at the site of this customer.

Statement 2: The optimal solution relies on periodic return flows from each customer, i.e. each customer sends its empty packages at a constant frequency. In other words, the shipment sizes used from a customer c to the factory are constant.

Let us show first the validity of Statement 1. As illustrated on Figure 4.1, a customer accumulates empty packages of each type p at a constant rate Λ_c^p. The problem is to minimize the sum of the fixed transportation costs (incurred for each shipment) and of the variable holding costs (linked with the number of empty packages used in the network). Note that

1. Because the flows are balanced in the network, the number of empty packages of each type is constant over time. It equals the sum of the average inventory of this type of packages over all locations.
2. The perfect coordination assumption implies that packages of a specific type p sent by customer c are consumed at the factory during a portion of time which is

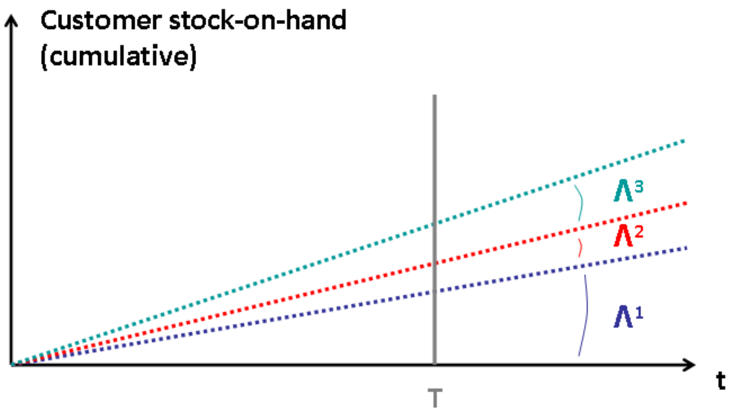

Figure 4.1: The evolution of a customer's inventory of empty packages.

Λ_c^p / Γ^p. The size of the shipments from customer c does not impact this portion of time.

Let us assume that a truck arrives at time T at a customer to pick up some empty packages. We must prove that it is optimal to load all empty packages in the truck.

If we ignore the truck capacity, then loading the empty packages of a specific type, say the green packages of type 3,

1. Does not impact the transportation costs, because the truck has already been paid for,

2. Minimizes the average inventory of this type of packages at the customer,

3. Minimizes the average inventory of this type of packages at the factory. Indeed, the average inventory of green packages (type 3), for example, is made out of the average inventory created by the shipments of green packages coming from each customer. The portion of time that shipments coming from a customer c are used at the factory is not affected by the size of the shipments. Consequently, the smaller these shipments are, the smaller the average inventory that they create at the factory is.

In other terms, because loading the empty packages of a specific type does not impact the transportation cost (because it is a sunk cost once the truck has been ordered), increasing their rotation rate can only improve the total cost.

This also holds when transportation capacity limits are considered. Indeed, once the global inventory of a customer reaches the capacity limit, we have the opportunity to return the empty packages of this customer at the lowest transportation cost possible (full truckload). In this case, letting the inventory of empty packages accumulate further at the customer will not reduce the transportation costs, but will slow the rotation rates of the packages.

From this first statement, it follows that the shipments from a customer will always consist of all packages (all types together) that are available at the customer when it is visited. Each shipment will consist of each type of packages in proportion to its accumulation rate. In other words, each shipment of customer c will contain Λ_c^3/Λ_c packages of type 3.

The validity of Statement 2 immediately follows. Because of the perfect coordination assumption, the timing of the shipments from a customer does not impact the average inventory these shipments create at the factory. Consequently, there is a single shipment size that is optimal with regard to the balance between transportation and holding costs for this customer.

These two statements characterize the way the return flows are managed. Each customer uses a constant shipment size in which all available packages are loaded. The problem can thus be modeled with the following variables:

- Q_c: the size of the shipments of customer c
- L^p: the logistic inventory of type p

4.2.4 *Mathematical Model*

COMPUTATION OF THE LOGISTIC INVENTORIES The way return flows are managed implies that a customer c builds a quantity Q_c which is then shipped to the factory where it is immediately consumed at a rate Γ.

The average inventory of packages of a type p at a customer c is consequently given by

$$\frac{Q_c}{2}\frac{\Lambda_c^p}{\Lambda_c}$$

At the factory, the packages of type p of this shipment are immediately consumed. But, the factory uses the shipments from customer c only part of the time. This leads to an average inventory which is

$$\frac{Q_c}{2}\frac{\Lambda_c^p}{\Lambda_c}\frac{\Lambda_c^p}{\Gamma^p}$$

The logistic inventory of type p is consequently given by

$$L^p = \sum_c Q_c(\frac{1}{2} + \frac{\Lambda_c^p}{2\Gamma^p})\frac{\Lambda_c^p}{\Lambda_c} = \sum_c U_c^p Q_c \tag{4.1}$$

where

$$U_c^p = (\frac{1}{2} + \frac{\Lambda_c^p}{2\Gamma^p})\frac{\Lambda_c^p}{\Lambda_c} \tag{4.2}$$

is the number of packages of type p required in the network for each unit composing the shipment size of customer c. The variables L^p are thus explicitly linked to the Q_c variables and can be replaced by substitution.

VARIABLE L VERSION In the variable L version of the model, there is no restriction on the values which may be taken by the L^p variables. The variable L version of the model is

$$\text{(VAR-HET)} \quad \min_{(Q_c)} \quad \sum_c \frac{\Lambda_c O_c}{Q_c} + \sum_p (H \sum_c U_c^p Q_c) \tag{4.3}$$

$$s.t. \quad Q_c \leq CAP \quad \forall c \tag{4.4}$$

$$Q_c \geq 0 \quad \forall c \tag{4.5}$$

The problem is to minimize the sum of the fixed transportation costs and of the variable holding costs (4.3). The first term of the objective function multiplies the number of shipments from each customer (Λ_c/Q_c) by the fixed transportation cost. The second term of the objective represents the sum of the logistic inventories of each type of packages which is multiplied by the constant holding cost. Capacity constraints (4.4) apply on the shipment Q_c.

FIXED L VERSION In the fixed L version of the model, the value of each logistic inventory is limited by the current number of empty packages that are available in the network (due to historical investment decisions). The model is

$$\text{(FIX-HET)} \quad \min_{(Q_c)} \quad \sum_c \frac{\Lambda_c O_c}{Q_c} \tag{4.6}$$

$$s.t. \quad \sum_c U_c^p Q_c \leq \bar{L}^p \quad \forall p \tag{4.7}$$

$$Q_c \leq CAP \quad \forall c$$

$$Q_c \geq 0 \quad \forall c$$

In this version, Constraint (4.7) represents the bound on each logistic inventory.

4.3 PROBLEM'S STRUCTURE AND SOLUTIONS

In this section, we first analyze the structure of the problem and indicate how it can be solved. Then, we illustrate the procedure on the case of a fleet with two types of packages. Third, we comment its generalization to networks with P types of packages.

4.3.1 *Problem Structure*

VARIABLE L UNCAPACITATED VERSION

The variable L version of the model generalizes the fundamental strategic model (VAR) of Chapter 2 when only one factory is considered. Similarly to the approach used for the fundamental (VAR) problem, the (VAR-HET) model decomposes into C independent EQO-like subproblems. The optimal solution is immediately computed to be

$$Q_c^{*[VAR-HET-U]} = \sqrt{\frac{\Lambda_c O_c}{\sum_p HU_c^p}} \quad \forall c \tag{4.8}$$

by setting the first derivative of the strictly convex objective function to zero.

FIXED L UNCAPACITATED VERSION

The fixed L version without capacity constraints on transportation (FIX-HET-U) has a structure close to the fundamental model (FIX-U) of Section 2.4. Similarly to the approach used for the problem (FIX-U), we build the Lagrangian function

$$\mathcal{L} = \sum_c \frac{\Lambda_c O_c}{Q_c} - \sum_p [\omega^p (\bar{L^p} - \sum_c U_c^p Q_c)] \tag{4.9}$$

and derive the first order conditions:

$$1: \qquad (\frac{\Lambda_c O_c}{(Q_c)^2}) = \sum_p w^p (U_c^p) \quad \forall c \tag{4.10}$$

$$2: \qquad w^p \geq 0 \quad \forall p \tag{4.11}$$

$$w^p (\sum_c U_c^p Q_c - L^p) = 0 \quad \forall p \tag{4.12}$$

Let us name the first set (4.10) *optimality conditions* and the second set (4.11)-(4.12) *complementarity conditions*.

It follows from the optimality conditions that

$$Q_c^* = \sqrt{\frac{\Lambda_c O_c}{\sum_p w^p U_c^p}} \quad \forall c \tag{4.13}$$

Comparing this equation with (4.8) provides an intuitive interpretation of the problem. The solution of the fixed L model can be seen as the solution of a variable L model with adequately chosen "fictitious" value w^p for the holding cost of each type of packages. This also provides an intuitive indication of how the problem is solved. For a given problem, let us assume that the limit on the logistic inventory for a container type r appears to be relatively small, while for another type s it seems to be quite large. One could start by assigning some low fictitious value to w^s and some high value to w^r. The optimal solution given by (4.13) is likely to use a small amount of type r and a large amount of type s. Based on the logistic inventories used in this solution, one can update the chosen values w^p for each type by increasing the values corresponding to the types of packages that are excessively used in the current solution and by decreasing the values of the types for which there remains a slack in the availability constraint. However, we should not expect to saturate all availability constraints in all cases. There may be some cases where a type of packages is in excess. In this event, even when the corresponding value w^p is set to zero, the constraint remains not binding, and could, eventually, be removed from the problem.

This intuition actually illustrates the lagrangian approach which may be used to solve this problem. The problem of minimizing the lagrangian function (4.9) is a lagrangian relaxation of the problem (FIX-HET-U). Because the problem is continuous and convex, the approach will converge easily to the optimal solution.

The complementarity conditions state that the "fictitious" holding cost value of a type of packages is either positive (in this case, all available packages of this type p, $\bar{L^p}$, are used) either null (in this case, the available packages of this type are in excess in the network and some of them cannot be used).

Let us consider the optimality conditions more in detail. They state that the gradient of each individual transportation cost function must be equal to a weighted sum of the gradients of the availability constraints. For a given vector w, the proportions between the optimal shipment sizes are fixed. If the vector w is multiplied by a constant α, the proportions between the Q_c remain unchanged.

Let us consider a problem A with $\bar{L_A} = \bar{L_A^1}, \bar{L_A^2}, ..., \bar{L_A^P}$ and the optimal solution given by the vector $w_A = w_A^1, w_A^2, ..., w_A^P$. We then consider problem B with $\bar{L_B}$ and optimal solution w_B. It is easy to verify that if $\bar{L_B} = \alpha \bar{L_A}$, then $w_B = \frac{1}{\alpha^2} w_A$ and $Q_B^* = \alpha Q_A^*$. This proportionality rule was already identified for the case of a standardized fleet in Chapter 2.

CAPACITATED PROBLEMS

For the variable L model with capacity constraints, we immediately obtain that the optimal shipment sizes are given by

$$Q_c^{*[VAR-HET-U]} = min[\sqrt{\frac{\Lambda_c O_c}{\sum_p HU_c^p}}, CAP] \quad \forall c \tag{4.14}$$

For the fixed L model, the capacity constraints may prevent the proportionality rule to apply. Indeed, with capacity constraints, we do not have the guarantee that the marginal contributions of the customers to the objective are identical. The problem, however, may still be solved to optimality using any convex continuous optimization technique, such as the lagrangian approach discussed earlier.

4.3.2 *Illustration with Two Types of Packages*

Let us illustrate the uncapacitated problem with only two types of packages.

The solution to the variable L model is given by

$$Q_c^{*[VAR-HET-U]} = min[\sqrt{\frac{\Lambda_c O_c}{H(U_c^1 + U_c^2)}}, CAP] \quad \forall c \tag{4.15}$$

For the fixed L model, the KKT conditions are

$$1: \qquad (\frac{\Lambda_c O_c}{(Q_c)^2}) = w^1 U_c^1 + w^2 U_c^2 \quad \forall c \tag{4.16}$$

$$2: \qquad w^1, w^2 \geq 0 \tag{4.17}$$

$$w^1(\sum_c U_c^1 Q_c - L^1) = 0 \tag{4.18}$$

$$w^2(\sum_c U_c^2 Q_c - L^2) = 0 \tag{4.19}$$

The problem may be solved by considering the different complementary cases.

CASE 1: $w^1 = w^2 = 0$

This case generates no solution. It creates an impossibility in (4.13). This is natural because using the available packages reduces the transportation cost. In the uncapacitated case, at least one availability constraint must be saturated.

CASE 2: $w^1 > 0$ AND $w^2 = 0$

In this region, the type 2 packages are in excess in the network. The minimization of the transportation costs must, consequently, focus on the packages of type 1 which are limited by $\bar{L^1}$. The availability constraint (4.7) on type 2 packages can actually be dropped. Consequently, this case becomes identical to the case with one type of packages described in Chapter 2.

Mathematically, by setting $w^2 = 0$, we determine using (4.13) that optimal solutions in the Q_c space are generated by a vector

$$(\sqrt{\frac{\Lambda_1 O_1}{U_1^1}}, \sqrt{\frac{\Lambda_2 O_2}{U_2^1}}, ..., \sqrt{\frac{\Lambda_C O_C}{U_C^1}})$$

scaled to meet the active constraint exactly. We refer to this case as the case $[onL^1]$.

We obtain fixed proportions between the optimal quantities

$$Q_c^{*[onL^1]} = \sqrt{\frac{\Lambda_c O_c}{U_c^1}} (\frac{L^1}{\sum_{d \in NC} \sqrt{\Lambda_d O_d U_d^1}}) \quad \forall c \tag{4.20}$$

This optimal solution implies that a certain amount of the other, non-limiting, packages type is required. The logistic inventory of type 2 packages is in this case

$$L^{2[onL^1]} = \sum_c U_c^2 Q_c^{*[onL^1]} \tag{4.21}$$

which defines a constant ratio

$$\frac{L^{2[onL^1]}}{L^1} = R^{[onL^1]} = \frac{\sum_c U_c^2 \sqrt{\frac{\Lambda_c O_c}{U_c^1}}}{\sum_c \sqrt{\Lambda_c O_c U_c^1}} \tag{4.22}$$

Consequently, the optimal solutions given by this case $[onL^1]$ are feasible for the region of the $\bar{L}$ space where

$$\frac{\bar{L^2}}{\bar{L^1}} \geq R^{[onL^1]}$$

In this region, we have more packages of type 2 available to be used ($\bar{L^2}$) than the number of packages of type 2 actually used (the logistic inventory of type 2, L^2). The additional packages of type 2 ($\bar{L^2} - L^{2[onL^1]}$) are simply left unused.

The optimal transportation cost for the case $[onL^1]$ is given by

$$V(Q^{*[onL^1]}) = \frac{(\sum_c \sqrt{\Lambda_c O_c U_c^1})^2}{\bar{L^1}} = w^1 \bar{L^1}$$

CASE 3: $w^1 = 0$ AND $w^2 > 0$

This case is symmetrical to the previous case. It provides solutions for a region in the $\bar{L}$ space where

$$\frac{\bar{L^2}}{\bar{L^1}} \leq \frac{\bar{L^2}}{L^{1[onL^2]}} = R^{[onL^2]}$$

Figures 4.2 and 4.3 illustrate the cost solutions for the region covered by these two cases in which one type of packages is in excess in the network. We can immediately observe that the third case involving two saturated availability constraints will typically covers a non-empty region in the $\bar{L}$ space.

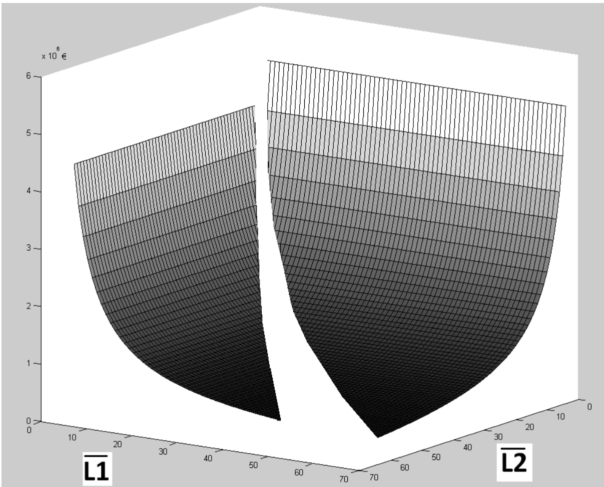

Figure 4.2: Transportation cost for solutions with one saturated constraint.

CASE 4: $w^1 > 0$ AND $w^2 > 0$

This case provides solutions for the region where $\bar{L^2}/\bar{L^1}$ is comprised between $R^{[onL^2]}$ and $R^{[onL^1]}$. In this region, both types of packages have a positive value indicating that the minimization must integrate the contribution of the variable Q_c to the transportation cost reduction while taking the utilization of both types into account.

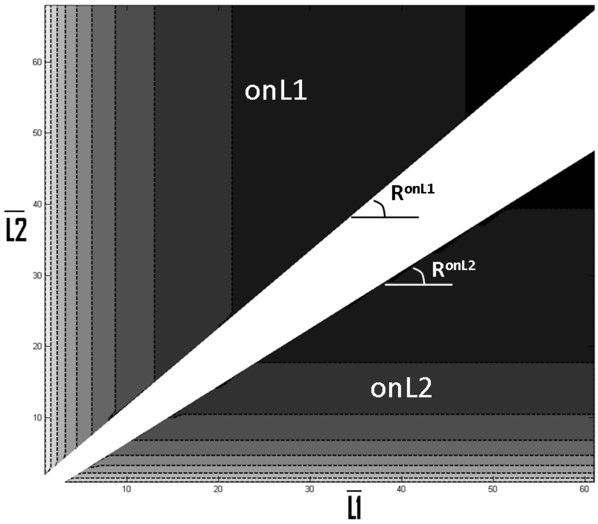

Figure 4.3: Contour lines of the transportation cost with one saturated constraint.

The optimality conditions provide a two-dimensional non-linear relation between the Q_c. The complementary conditions provide two one-dimensional relations between the Q_c. Consequently, the optimal solution is a unique point in the Q_c space

$$Q_c^* = \sqrt{\frac{\Lambda_c O_c}{w^{1*} U_c^1 + w^{2*} U_c^2}} \quad \forall c$$

of which the optimal transportation cost is given by

$$V(Q^*) = w^{1*} \bar{L}^1 + w^{2*} \bar{L}^2$$

When moving from a point in the $\bar{L}$ space where $\bar{L}^2/\bar{L}^1 = R^{[onL^1]}$ to a point where $\bar{L}^2/\bar{L}^1 = R^{[onL^2]}$, the ratio between w^2/w^1 will take strictly increasing values in the range $]0, \infty[$. The proportions between the Q_c will evolve from those shown by the case $[onL^1]$ to those of the case $[onL^2]$. However, we could not derive an analytical expression for the relation between $\bar{L}$ and w. The solutions in this region have consequently been calculated numerically.

Figure 4.4 represents the total optimal transportation cost of the problem with two types of packages. Full lines show the cost of the uncapacitated problem and dotted lines display the cost of the capacitated problem. In this case, for low levels of investment in the fleet ($\bar{L}$), the shipment sizes are not limited by the capacity and contour lines are identical to the uncapacitated solutions. As $\bar{L}$ increases, some return flows are limited by the capacity. A cost reduction is still achievable by the allocation of the remaining packages to customers with a smaller marginal contribution. As $\bar{L}$ further increases, we reach a level where all shipment sizes are equal to CAP and no further decrease in transportation cost is achievable.

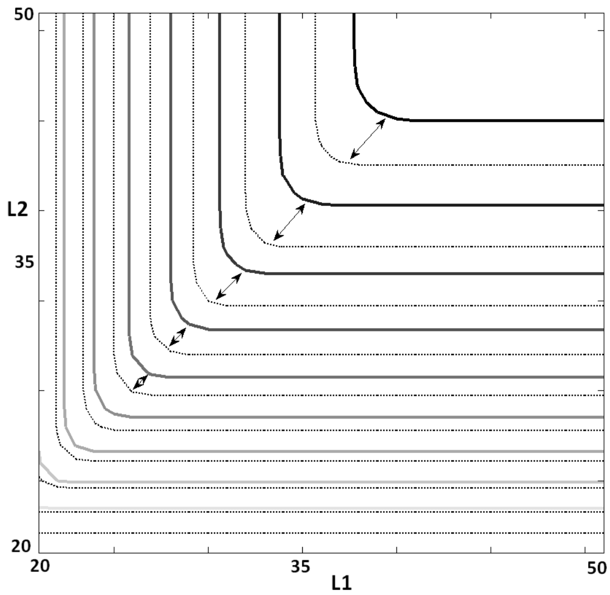

Figure 4.4: Contour lines of the transportation cost for the uncapacitated (full lines) and the capacitated (dotted lines) cases.

4.3.3 *P Types of Packages*

The general problem with P types of packages and a capacity limit is a continuous convex optimization problem which can easily be solved numerically. We will make two observations on the solutions.

First, it is possible, based on the solutions of the minimization performed on each type p individually ($\omega^p > 0$ and all other multipliers set to zero), to decompose beforehand the $\bar{L}$ space into different regions corresponding to the different lagrangian cases. Given specific limits $\bar{L}$, it is straightforward to identify the multipliers which can be set to zero in the problem. The corresponding inequality constraints can be eliminated from the problem. The other constraints may be transformed into equality constraints. The procedure considers each availability constraint p successively. We compute the equation of the hyperplane in the $\bar{L}$ space containing the solutions of the one-type problems for every other type of packages (each type $s \in NP \setminus p$). This is easily done knowing that this hyperplane contains the origin and that the solutions of the one-type minimization are proportional to

$$(\sqrt{\frac{\Lambda_1 O_1}{U_1^s}}, \sqrt{\frac{\Lambda_2 O_2}{U_2^s}}, ..., \sqrt{\frac{\Lambda_C O_C}{U_C^s}}) \quad \forall s \in NP \setminus p$$

In the equation of this hyperplane, let the coefficient of each $\bar{L}^s$ be A^s. We then immediately check if the availability constraint of type p is active at optimality. That is the case if

$$\bar{L}^p \leq \frac{\sum_{s \in NP \setminus p} A^s \bar{L}^s}{A^p} \tag{4.23}$$

A second observation is that, when capacity constraints are active for some customers at the optimal solution, the optimal shipment sizes for these customers in the unconstrained problem would be larger than or equal to CAP. Consequently, an alternative method to solve the capacity constrained problem is by iteratively solving the unconstrained problem and by removing at each iteration the customers of which the quantity is larger than or equal to CAP until the unconstrained problem provides a solution where all, or none, of the return quantities are larger or equal to CAP.

4.4 MANAGERIAL INSIGHTS

The aim of the study was to offer a model for the strategic managerial analysis of the return flows of a heterogeneous fleet of empty packages. Let us illustrate the managerial conclusions that emerge from the model with two types of packages (it generalizes easily). Figure 4.5 represents, in the $\bar{L}$ space, the typical shape of the contour lines of the optimal transportation cost (dotted lines). Point A represents the optimal solution of the uncapacitated variable L model. The dotted line that goes through point A is the set of points where the shipment sizes are in the same proportions as in point A. On Figure 4.5, the contour lines of the total logistic cost, i.e. the sum of the transportation and holding costs, are also represented (full lines).

Let us consider a small example. Figure 4.6 displays a small network with 6 customers along with their rates and the cost parameters.

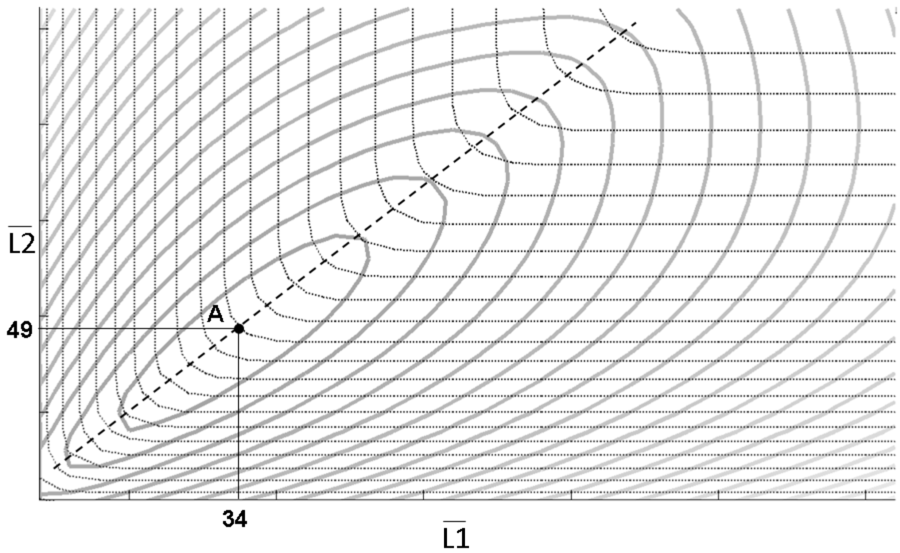

Figure 4.5: Contour lines of the transportation cost (dotted) and of the total cost (full).

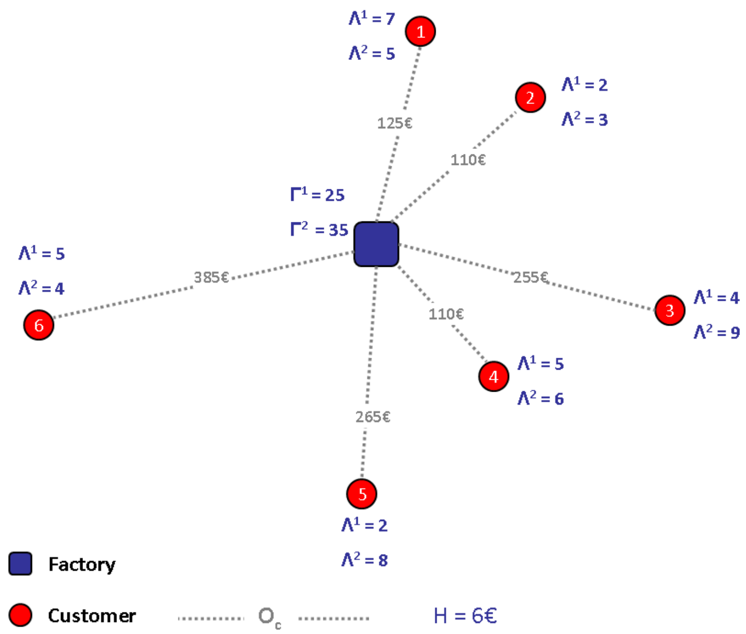

Figure 4.6: An illustrative network with 2 types of packages.

The long term optimal solution is obtained by solving the variable L model. Using (4.8) we derive the optimal shipment sizes[2]:

$$Q^{*A} \approx \{20, 13, 30, 18, 27, 32\}$$

2 We rounded the results to the closest integer for simplicity. Consequently, we use approximation signs.

The proportions between these shipment sizes are

$$Prop^A \approx \{1, 0.64, 1.48, 0.91, 1.34, 1.56\}$$

These shipments consist of empty packages of both types, in proportion to their rates at the customer. Using these shipment sizes require a certain amount of each type of packages. The logistic inventories of each type are computed using (4.1):

$$L^1 \approx 34 \text{ packages of type 1}$$
$$L^2 \approx 49 \text{ packages of type 2}$$

The cost of holding these logistic inventories is

$$V^{*A}_{HOLD} = H(L^1 + L^2) \approx 500€$$

The periodic transportation cost is

$$V^{*A}_{TRANS} = \sum_c \frac{O_c \Lambda_c}{Q^*_c} \approx 500€$$

In the variable L model without capacities, it is natural that both costs are equal at optimality. The total cost is consequently approximately 1000€.

By computing the optimal solution that would be obtained if all packages in this network were standardized, we reach our first managerial insight. In this case, we would compute the optimal shipment sizes based on Equation (2.19) used for the single-type single-factory perfectly coordinated case. The optimal shipment sizes are practically identical to the shipment sizes obtained at point A. This is an immediate conclusion if we compare Equations (2.19) and (4.8). This, actually, is a direct consequence of the perfect coordination assumption which we have used. If it is assumed, at the strategic level, that all packages types accumulate together at a customer, are shipped together in a truck and are all used immediately when arriving at the factory, we basically deal with the heterogeneous fleet as if it was standardized. Does this mean that there is no impact in having a heterogeneous fleet?

At the strategic level, if we have the opportunity to adapt the logistic inventory to the optimal long term solution computed by the variable L model, then we do indeed obtain an optimal long term cost of the reverse network which is almost identical to that of the case of a standardized fleet.

However, the difference is to be found at the operational level. Indeed, with different types of packages, the perfect coordination assumption is a stronger assumption to use at the strategic level. Consequently, this conclusion about the cost impact of a heterogeneous fleet is valid as long as almost perfectly coordinated solutions can be found at the operational level when having different packages types. The operational problem is adressed in the next chapter. Anticipating certain results derived in Chapter 5, we can state that the approximation provided by the perfect coordination will be reasonable if the network is sufficiently large, if the number of different packages types is limited and if the different types are used in a rather uniform manner in the network. The long term strategic analysis of the model presented in this chapter thus reveal that, on large network and up to a certain number of different types, an adequate investement in the fleet of packages and a careful operational management of the return flows may signigicantly limit the cost impact of using a heterogeneous fleet.

The second most interesting managerial insight derived from the model presented is that the proportions between these optimal shipment sizes of the variable L case are dominating in many cases. On Figure 4.5, these solutions appear on the dotted line spanning through point A. Let us illustrate how these proportions dominate by considering adjustments of the fleet composition along an investment line or along an axis of the $\bar{L}$ space.

ALONG INVESTMENT LINES - The solutions with shipment sizes proportionate to the optimal long term sizes dominate any solution with an identical investment in the fleet of packages. Let us, for example, consider point B on Figure 4.7. The proportions between the shipment sizes are different than the proportions used at point A. The solution at point B tries to use as many packages of type 2 as possible, because these packages are in excess in the system, but are paid for. Then, if we draw an investment line (the set of points with identical holding cost), we can observe that point C, where the proportions between the shipment sizes are identical to the proportions of point A, is the best solution one can achieve on this investment line. In other words, the dotted line going through point A is the set of points where the transportation cost contour lines are tangent to the investment lines.

We illustrate this observation using our example. Let us assume that the current fleet of empty packages available to be used in the reverse network is

$$\bar{L} = \{20, 55\}$$

The investment in these packages is

$$V_{HOLD}^{*B} = 75 \cdot 6€ = 450€$$

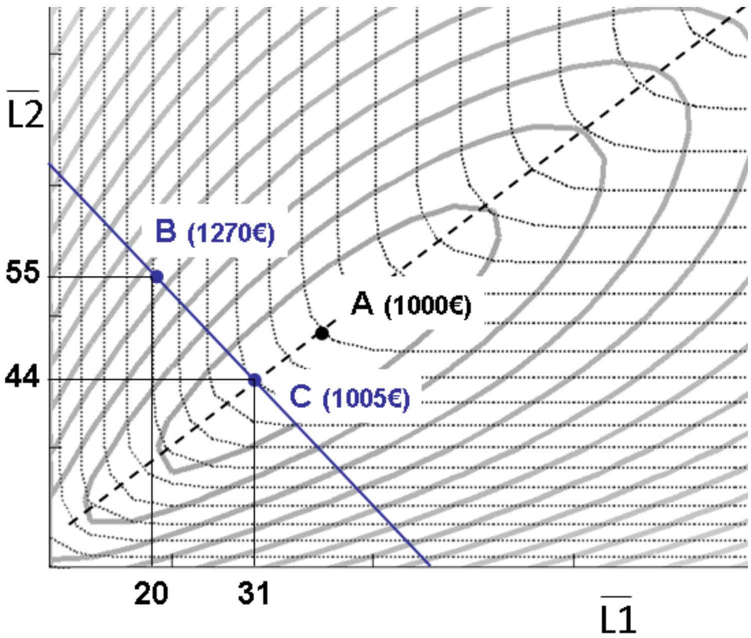

Figure 4.7: Adaptation of the fleet composition along an investment line.

We may immediately check, using (4.23), that the packages of type 2 are in excess in the system. The optimal shipment sizes are then calculated using (4.20) to be

$$Q^{*B} \approx \{10, 8, 21, 10, 24, 16\}$$

implying a transportation cost of approximately 820€. The total cost at point B is thus approximately 1270€.

The solution at point B, however, only uses 35 empty packages of type 2 out of the 55 available. If the investment can be adapted, we should move from point B to point

C. Point C is reached by replacing 11 packages of type 2 by packages of type 1. We obtain the following solution

$$Q^{*C} \approx \{18, 12, 27, 17, 24, 28\}$$

$$Prop^{C} \approx \{1, 0.64, 1.48, 0.91, 1.34, 1.56\}$$

$$L^{*C} \approx \{31, 44\}$$

$$V^{*C}_{HOLD} \approx 450€$$

$$V^{*C}_{TRANS} \approx 555€$$

The total cost at point C is consequently equal to 1005€, improving the cost achieved at point B by 265€.

ALONG AXIS - The proportion between the shipment sizes at point B are

$$Prop^{B} = \{1, 0.8, 2.15, 1.05, 2.47, 1.6\}$$

and are different than the long term optimal proportions $Prop^{A}$. To understand why we do not use these long term optimal proportions, let us consider the solutions at point D (with the optimal long term proportions) on Figure 4.8.

$$Q^{*D} \approx \{12, 8, 18, 11, 16, 18\}$$

$$L^{*D} \approx \{20, 29\}$$

$$V^{*D}_{HOLD} \approx 294€$$

$$V^{*D}_{TRANS} \approx 850€$$

This solution provides a total cost of 1144€, which is better than the total cost achieved at point B (1270€). This solution does not use more packages than the packages available in the network. However, if we were to use this solution in our case with $\bar{L} = \{20, 55\}$, even if only 29 packages of type 2 are used in the solution, we would have to pay the holding cost of the 55 packages of type 2. The total cost would then be 1300€.

This explains why the solution in B changes the long term optimal proportions. It tries to use more packages of type 2, because their holding cost is incurred anyway. We can observe on Figure 4.8 that moving from point D in the direction of point B reduces

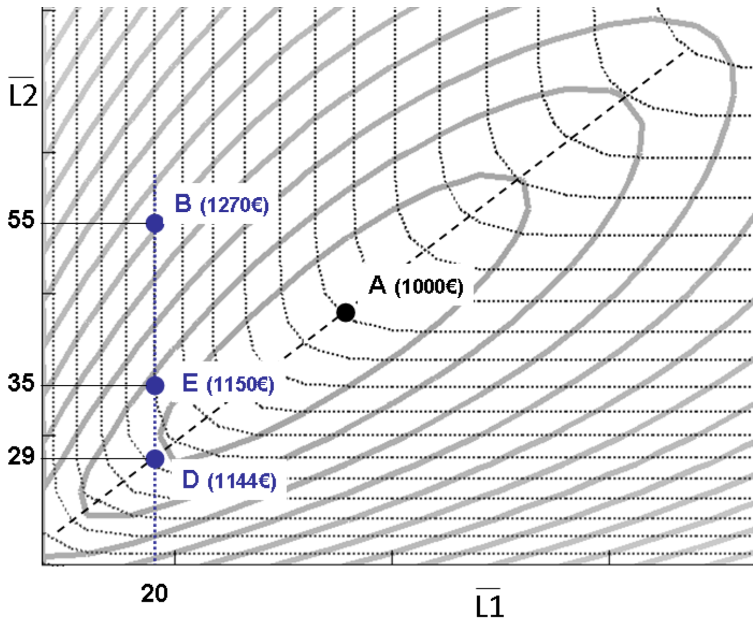

Figure 4.8: Adaptation of the fleet composition along an axis of the $\bar{L}$ space.

the transportation cost. By modifying the proportions between the shipment sizes, it is possible to use some additional packages of type 2 to reduce the transportation cost. This is true up to a certain point. Indeed, once we reach point E, we obtain the following solution:

$$Q^{*E} \approx \{10, 8, 21, 10, 24, 16\}$$

$$L^{*E} \approx \{20, 35\}$$

$$V^{*E}_{HOLD} \approx 330€$$

$$V^{*E}_{TRANS} \approx 820€$$

$$V^{*E} \approx 1140€$$

We have decreased the transportation cost by 30€, using 6 more packages of type 2. All types together, 55 packages are used in the solution, which would imply a holding cost of 333€. However, in our case, we must still pay for the 75 packages held in the network. Our total cost would be of 1270€. This is the same cost as calculated in point B. Indeed, once we have reached point E, it is not possible to use any additional package of type 2 to reduce the transportation cost without requiring more packages of type 1. The solution achieved at point E is consequently identical as the solution at point B, except that, at point B, 20 more packages of type 2 are available in the network which are left unused.

If it is possible, on the medium term, to get rid of some packages of type 2, then it is obvious that we should decrease the number of packages of type 2 from 55 to 35 units. However, even if it increases the transportation cost, we should also get rid of 6 additional packages of type 2. These 6 packages allows us to reduce the transportation cost by 30€, but have an holding cost of 36€.

To sum up, the model we presented allows us to determine, for each level of investment $\bar{L}$, the optimal shipment sizes and the total cost of the reverse network. Second, it allows us to derive some very straightforward managerial recommendations about adaptations of the fleet of packages.

We first compute the optimal long term solution with the variable L model. This is done by a simple EOQ computation for each customer. On the long term, the fleet of packages should be adapted to match this case.

Second, on the medium term, if the total level of investment cannot be adapted, we will recommend adjustments of the fleet composition such as to achieve the same proportions as the long term optimal proportions. If the level of investment is smaller than the optimal level of investment, these adjustments should ideally be done along an investment line. However, if this is not possible, simply decreasing the number of empty packages that are in excess proportion in the network provides an immediate benefit.

4.5 CONCLUSION

The objective of this chapter was to provide strategic insights in the management of the return flows of reusable packages when different non-substitutable packages types are used in the network. This analysis relies on a continuous strategic model with constant rates in the network under the perfect coordination assumption which allows us to keep the problem tractable. The link between the different types of packages is the ability to share the same transportation means for being returned.

The most immediate question is how the return flows should be organized. We have shown that, at the strategic level, the dynamic of the network relies on periodic shipments of each customer. These shipments consist of packages of all the types used by the customer in proportion to their accumulation rates. In other words, when a customer is visited, all the empty packages are loaded into the truck.

Consequently, the remaining problem is to determine the optimal return frequencies for each customer. When the size of the fleet of packages can be chosen without any restriction (variable L model), we demonstrated that the problem of finding the optimal frequencies decomposes into independent EOQ-like subproblems for each customer. When each component of the fleet (the number of packages of each type available) is limited by an investment limit (fixed L model), the problem can be viewed as the allocation of the available resources to the customers in order to decrease their transportation costs. At optimality, when no truck capacities are considered, the marginal contributions of all shipment sizes should be identical. When one type of packages is scarce while the other types are in excess, the problem is solved analytically, as it reduces to a single type problem dealt with in Chapter 2. When the availability constraints of several types of packages are active, the problem is trickier, but can be numerically solved as a convex optimization problem. An important result was to show that the proportionality rule derived for the fundamental models of Chapter 2 still applies in this case. It states that the proportions between the optimal shipment sizes of an instance with investment limits remain unchanged if these limits are all modified proportionally.

We showed how to compute the optimal solution for any given combination of the investment limits. The optimal solutions are clustered into different regions according to the investment limit constraints that are saturated. We demonstrated that it is possible to determine beforehand which of these constraints would be active given specific limits. There is typically a region where all of these constraints are active, which is the region containing the solution with the lowest total cost. But, there are also regions in which some of these constraints are not saturated. In these cases, some packages are available but are not used in the solution, although they bear a holding cost.

The issue of finding the optimal number of packages of each type to use in the network is immediately answered by considering the solution of the variable L model. The optimal size and composition of the logistic inventory is directly computed using the optimal shipment sizes.

Additionally, we demonstrated that this long term optimal composition of the logistic inventory provides an important managerial reference. Indeed, if, on the medium term, the optimal size of the logistic inventory cannot be reached, adapting the current logistic inventory such as to achieve the optimal long term proportions between its components always provides an immediate benefit. In other words, we showed that packages which are in excess proportion, when compared to the optimal long term composition, always have a holding cost which is larger than the marginal reduction of the transportation they can allow to achieve.

But, more importantly, we showed that this optimal composition of the fleet is the composition that allows us to consider, at the strategic level, that the heterogeneous fleet is managed as if there was only one type of packages. It leads to optimal shipment sizes and an optimal total cost which are almost identical to those of a standardized fleet. However, the difference between the standardized and the heterogeneous fleet is to appear at the operational level. Perfectly coordinating the flows of a heterogeneous fleet is likely to be more difficult. This conclusion thus remains valid for the cases where the perfect coordination assumption is a reasonable approximation of the operational coordination of the return flows of a heterogeneous fleet. This will be the case for large networks in which the number of different types of packages is small and in which these types are used in a relatively uniform manner in the network.

Finally, we noted that the assumptions used in this chapter were more restrictive than those used in the previous chapters presenting strategic models. Using these assumptions allowed us to solve the problem easily and to identify the main economic logic in the management of a heterogeneous fleet as well as some interesting managerial insights about the composition of the fleet. It remains, however, that this study should be taken further in order to be applicable to real industrial cases.

In particular, we relied on two strong assumptions. First, we considered the case of a single factory. The allocation problem that was addressed in Chapters 2 and 3 would become significantly more complex in the case of a heterogeneous fleet, as balancing the flows between the factories would require splitting the flows of many customers between the clusters of the factories. Second, we have used identical holding costs for all packages types and for all locations. Even if the model presented in this chapter would generalize easily to holding costs dependent on the type of packages or on the location, we must bear in mind that the strategic models approximate the solutions of the operational problem in which the coordination of the flows will have to be realized. With location dependent holding costs, it is not guaranteed that the organization of the flows we use in the strategic model (all of the empty packages are loaded when a customer is visited) would be a good policy at the operational level. If we assume that holding costs are lower at the customers, for example, we might want to leave some unrequired packages at a visited customer. Further research is consequently required if we want to determine precisely when the strategic model presented in this chapter adequately approximates the operational level when certain assumptions are relaxed.

Part II

OPERATIONAL MANAGEMENT OF THE REVERSE NETWORK

5

OPERATIONAL PROBLEM AND PORTFOLIO HEURISTIC

5.1 INTRODUCTION

In the first part of this thesis, we adopted a strategic perspective of which the aim was to determine the design of the reverse network for reusable packages resulting from a static long term equilibrium between the cost factors. The strategic focus of the models presented in Part I implied relying on various assumptions. Most of these assumptions are common for strategic models, such as the use of constant average flows and a continuous timeline. But, an additional assumption has been largely used, which allowed us to ignore the coordination problem at the strategic level. This assumption, called the perfect coordination assumption, states that the timing of each shipment of empty packages to a factory does not impact the average inventory of this factory. This results from the fact that the perfect coordination assumption supposes that each shipment is immediately used at the factory, as it arrives when the factory's inventory exactly drops to zero. This represents an ideal case in which the inventory at the factory is minimal. As a consequence, the choice of the shipment sizes could be done without integrating the timing element. It allowed the strategic models to be tractable.

As discussed in Section 1.2, in an operational perspective adressing the day-to-day management of the return flows, it is obvious that this perfect coordination does not systematically hold. The fact that a shipment arrives at a factory exactly when the inventory of this factory drops to zero certainly depends on the timing of the shipment. The question is to know whether the choice of the shipment sizes made in a strategic model allows the design of an operational policy with perfectly coordinated shipments.

This issue was largely studied in the SRMIS literature presented in Section 1.3. The works of this field of literature highlight three main results.

First, if the shipments from each customer have a constant size (as it is the case in the solutions of the strategic models we presented), then there is only one particular case in which the flows can be perfectly coordinated. It is when the shipment frequencies from all customers are identical. In all other cases, it is not possible to coordinate the shipment perfectly.

Second, given a set of shipment sizes for each customer, finding the best possible coordination, i.e. the one that minimizes the average inventory at the factory, is a hard combinatorial problem, referred to as the *staggering problem* (Gallego et al., 1992) in the SRMIS literature.

Third, if we integrate the coordination problem with the other design decisions, then it is very unlikely that constant shipment sizes would provide optimal solutions. Instead, the decision about the size and the timing of a shipment would depend on the current state of the inventory at the factory. Considering variable shipment sizes makes the coordination problem even more difficult. A model taking network design decisions along with coordination decisions would be untractable.

However, the real question raised by our methodological choice of using the perfect coordination assumption at the strategic level, is not as much whether it can be reached in practice, but rather whether it is a reasonable approximation of what can be achieved at the operational level. In other words, can we thus operationally manage the return flows in such a way that the cost solution achieved is sufficiently close to the cost solution that would be implied by the perfect coordination? In this chapter, we provide an answer to this question by demonstrating that, as long as the network is sufficiently large and granular, it is rather easy and quite natural to achieve a good coordination in practice. In this sense, our approach differs from the SRMIS literature. We do not focus on the concept of the "best" possible coordination. Instead, we show how a "good" coordination is easily achieved in practice, under certain network size and granularity conditions that correspond to the scope we defined for our research.

The operational problem which is analyzed in this chapter applies to a single factory network. Indeed, we consider that the strategic design of the network has been previously realized. A specific set of customers has been allocated to each factory. We address the coordination of the return flows for each factory cluster independently. The factory uses empty packages over time, while the customers accumulate them. The rates at which empty packages are consumed or accumulated are not necessarily constant, but may be variable around the specific average rate of each location. We must determine a schedule of shipments from the customers to the factory over an infinite horizon, such that the factory never gets short of empty packages. Each shipment is subject to a fixed transportation cost. The number of empty packages available in the reverse network is given at the beginning of the horizon.

We consider both the continuous and the discrete version of the problem. In the continuous operational problem, the shipments may be performed at any time. In the discrete version, the shipments are only performed at the review periods. We show that a simple myopic procedure is sufficient to provide a schedule of which the transportation cost is very close to the transportation cost that would be achieved under the perfect coordination assumption. To illustrate this result, we derive lower bounds on the continuous and discrete cases with constant rates. The lower bounds are used to benchmark the results achieved on instances with constant rates by the myopic heuristic suggested, the *portfolio heuristic*. We then analyze how the variability of the flows (around the average rates) affects these results. The variability generates several impacts. Most impacts negatively affect the cost of the solutions. However, we also identify an impact which tends to decrease the total cost. We demonstrate that the portfolio heuristic suggested is appropriate to take advantage of this specific impact.

The chapter is structured as follows. In Section 5.2, we detail the exact problem we study in its continuous and discrete versions. Then, in Section 5.3, we provide a lower bound on the optimal cost of each version when the flows in the network are constant. In Section 5.4, we present the myopic portfolio heuristic used to solve the operational problem. We present a series of numerical results on instances with constant rates, which are compared with the lower bounds. In Section 5.5, we analyze how variability impacts the solutions of the problem. We finally conclude this chapter by summarizing the main implications of the good performance we achieve on the operational problem.

5.2 THE OPERATIONAL PROBLEM

The operational problem is considered in a context in which the strategic design for a standard fleet of packages has been realized, using the models presented in Chapter 2. This design has created separate clusters of customers for each factory[1]. The problem thus applies to the cluster of each factory independently.

1 In Section 2.5 we indicated that some customers may be allocated to more than one factory. However, the number of these customers was shown to be limited to $F-1$. For these customers, we simply set that Λ_c in the operational problem is equal to the flow that was allocated from this customer to the factory we consider, that is λ_c^f.

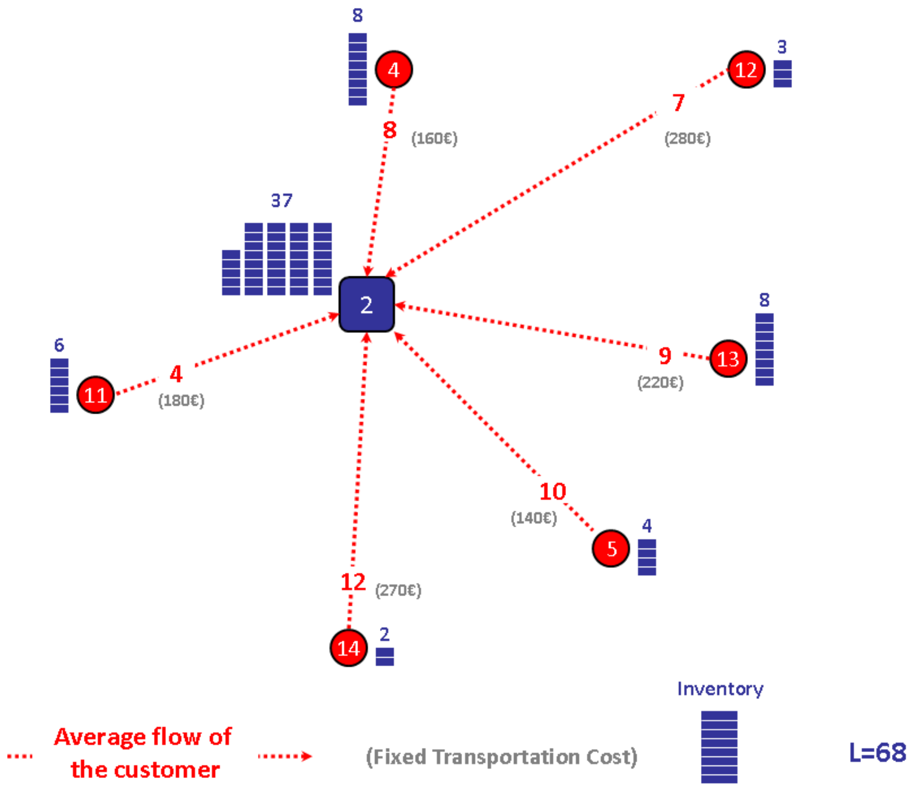

Figure 5.1: The state of the cluster of Factory 2 at the beginning of the horizon.

Let us consider the cluster of Factory 2 of the example used in Section 1.1. Its current state, at the beginning of the horizon, is displayed on Figure 5.1. Six customers are allocated to Factory 2. The average rate at which these customers build their inventory of empty packages is indicated on the red arrows. The factory consumes, on average, 50 empty packages per period of time.

At the beginning of the horizon, some inventories of empty packages are present at the customers and at the factory. In total, we can count 68 empty packages in this example ($\bar{L}$). This is the number of packages used in the reverse network, i.e. the logistic inventory (L). This number is fixed for the problem. At the operational level, the logistic inventory cannot be adapted.

Over time, the factory will consume its inventory of empty packages. At the same time, the customers will continue to build their inventory of empty packages. The problem is to determine when and which shipments will be performed from the customers to the factory. The factory must never gets short of empty packages. A fixed transportation cost is incurred for each shipment and the shipments are assumed to be instantaneous. A capacity may limit the size of the shipments. The horizon, NT, is assumed to be infinite ($NT = [1, \infty[$). The objective is to minimize the periodic average transportation cost.

We use the following notations:

- Λ_c: the average rate at which customer c builds its inventory of empty packages.
- Γ: the average rate at which the factory consumes its inventory of empty packages.
- Λ_c^t: the rate at which customer c builds its inventory of empty packages during the interval $[t, t+1]$.
- Γ^t: the rate at which the factory consumes its inventory of empty packages during the interval $[t, t+1]$.
- O_c: the fixed transportation cost incurred for each shipment from customer c to the factory.
- L: the logistic inventory, i.e. the number of empty packages used in the network.
- CAP: the capacity of a truck.
- I_c^t: the instantaneous level of the inventory of empty packages at customer c at time t.
- I_f^t: the instantaneous level of the inventory of empty packages at the factory at time t.

The beginning of the horizon is time 1. The logistic inventory is defined as

$$L = \sum_c I_c^1 + I_f^1$$

5.2.1 *Continuous Operational Problem*

In the continuous version, the shipments may be performed at any time. It means that, if the factory's inventory drops to zero, we can still order a shipment at that moment which would replenish the factory immediately. The rates Λ_c^t and Γ^t might be variable around their average rates. They also might not be known beforehand. It this case, the best estimates for these rates are the average rates Λ_c and Γ. At any time, we can exactly observe the inventory of each location I_c^t. Figure 5.2 illustrates the evolution of the inventory of empty packages at a customer and at the factory.

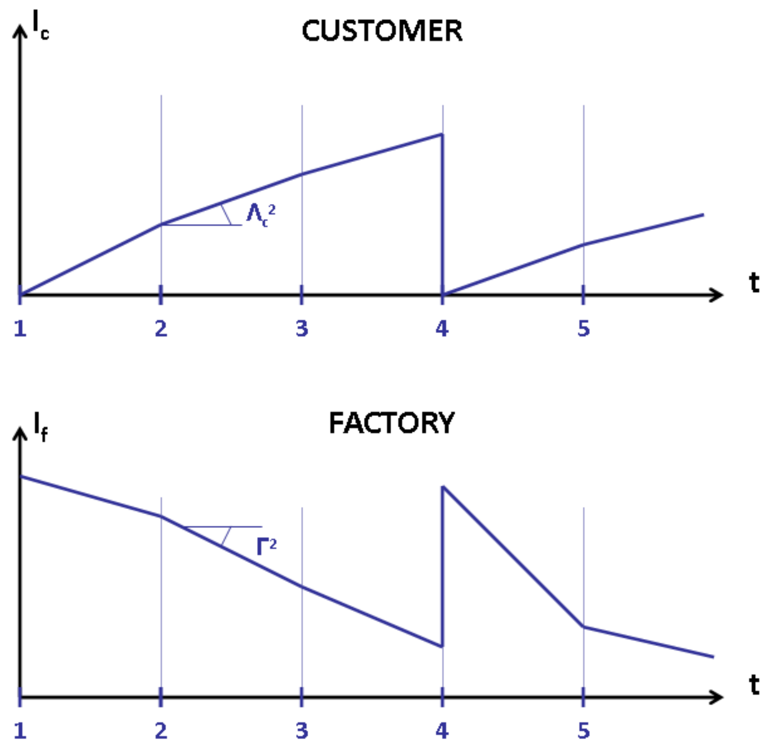

Figure 5.2: Evolution of the inventory level at a customer and at the factory in the continuous version, with a shipment at time 4.

5.2.2 *Discrete Operational Problem*

In the discrete version, shipment decisions are only taken at periodic intervals. The exact inventory level of each location is only observed at the review moments. Let us define

- P: the review periodicity

For example, if $P = 5$, then shipments can only be performed at the following review moments: $\{1, 6, 11, 16, ...\}$. As in the continuous version, the exact rates might be variable around their average, and might be unknown. However, in the discrete version, if the factory gets short of packages at a moment that does not correspond to a periodic review, it would be too late to order a shipment at that time. It follows that, at each review moment, we must ensure to carry out a sufficient number of shipments to cover the need of the factory until the next review. For this reason, we assume that, if we are at a review moment t, the number of empty packages that will be used at the factory from this period until the next review is known exactly. In other words,

$\sum_{v=t}^{t+P} \Gamma^v$ is known exactly[2]. When not otherwise specified, we use $P = 1$ for simplicity.

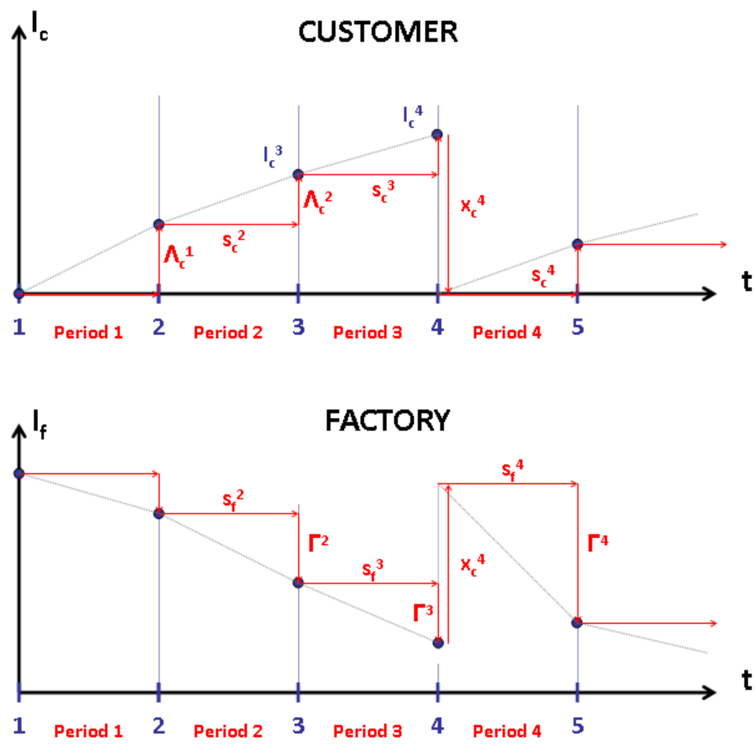

Figure 5.3: Evolution of the inventory level at a customer and at the factory in the discrete version, with a shipment at time 4.

The evolution of the inventories is not impacted by the review policy. However, in the discrete case, the exact inventory position is only observed at the review moments. We use the following additional notations to model the discrete version:

- s_c^t: the number of available empty packages at customer c that is stored from one period to the next.
- s_f^t: the number of available empty packages at the factory after the shipments of period t have just been received.
- x_c^t: the number of empty packages shipped from customer c at the beginning of period t.
- y_c^t: the number of trucks performing shipments from customer c at the beginning of period t.

2 As discussed in Section 1.1, if these rates are not known exactly, we can rely on a safety stock computed in a classical way.

Let us consider the example illustrated on Figure 5.3. At time 3, at the beginning of the third period, we have a certain inventory level I_c^3 at the customer. Because there is no shipment at time 3, it will be the last observed inventory level until the next period. We thus define s_c^3 has the last number of available packages that we observed and that will be stored until the next period. During period 3, some packages are made empty at the customer. At the very beginning of period 4, we observe the new inventory level, which is $I_c^4 = s_c^3 + \Lambda_c^3$ (if $P = 1$). Right after this inventory position is observed, a shipment is performed with all available packages. The size of the shipment is $x_c^4 = I_c^4$. The inventory position is consequently modified, such that the last observed number of available packages is $s_c^4 = I_c^4 - x_c^4$. The use of the notations follows a similar idea at the factory. We have

$$I_c^t = s_c^{t-1} + \Lambda_c^{t-1}$$

$$s_c^t = I_c^t - x_c^t = s_c^{t-1} + \Lambda_c^{t-1} - x_c^t$$

$$I_f^t = s_f^{t-1} - \Gamma^{t-1}$$

$$s_f^t = I_f^t + \sum_c x_c^t = s_f^{t-1} - \Gamma^{t-1} + \sum_c x_c^t$$

We illustrate the discrete version of the problem with an MIP formulation. The discrete horizon $NT = \{1, 2, ..., T\}$ is assumed to be infinite. But, in practice, when solving the problem, we will consider a long finite horizon of which the last period is T.

$$\min_{(s_c^t, x_c^t, y_c^t)} \quad \sum_c \frac{\sum_{t=1}^T y_c^t O_c}{T} \tag{5.1}$$

$$\text{s.t.} \quad s_c^1 = I_c^1 - x_c^1 \qquad \forall c \tag{5.2}$$

$$s_c^t = s_c^{t-1} + \Lambda_c^t - x_c^t \qquad \forall c, t > 1 \tag{5.3}$$

$$s_f^1 = I_f^1 + \sum_c x_c^1 \tag{5.4}$$

$$s_f^t = s_f^{t-1} + \sum_c x_c^t - \Gamma^{t-1} \qquad \forall t > 1 \tag{5.5}$$

$$s_f^t \geq \Gamma^t \qquad \forall t \tag{5.6}$$

$$x_c^t \leq y_c^t CAP \qquad \forall c, t \tag{5.7}$$

$$y_c^t \in \mathbf{Z}_+, \quad x_c^t,\, s_c^t,\, s_f^t \in \mathbf{R}_+ \qquad \forall c, t \tag{5.8}$$

The variables of the model are the number of packages sent by each customer in each period, x_c^t, the number of trucks performing shipments from each customer in each period, y_c^t, and the number of available packages stored from one period t to the next at each customer, s_c^t, and at the factory, s_f^t.

The objective is to minimize the long term periodic transportation cost (5.1) subject to flow conservation constraints of the empty packages in the network (5.2)-(5.5).

The logistic inventory equals the number of empty packages available at the beginning of the horizon. If the flows were balanced in the last period before the horizon, the logistic inventory is equal to the sum of the last level of inventory observed at the customers and at the factory:

$$L = \sum_c I_c^1 + I_f^1 = \sum_c s_c^0 + \Lambda_c^0 + s_f^0 - \Gamma^0$$

This formulation exhibits C subproblems defined by (5.2), (5.3) and (5.7), having a reverse single-item capacitated lot-sizing structure. These subproblems are linked by constraints (5.4) and (5.5). However, the problem appears to be difficult to solve exactly and the lower bounds obtained by linear relaxation are poor. When $\sum_c \Lambda_c^t = \Gamma^t$, the linking constraints can be rewritten as to exhibit a joint inventory bound in each period. We discuss this case in Chapter 6 where we demonstrate that we were able to solve this problem to optimality only on small instances.

The above formulation allows us to illustrate the discrete operational problem. However, using this formulation in practice for the operational problem would present a series of disadvantages.

First, if the rates are variable and unknown, then the formulation has to rely on estimated future flows. If the actual flows do not later correspond to the estimates, there is no guarantee that the solution found remains of good quality or even that it remains feasible. It is likely, in practice, that it would be used on a rolling horizon basis. The solution found would be used only for the few next periods, then the problem would be solved again with updated estimates.

Second, a finite horizon must be used in this formulation. But, in practice, the problem is related to an infinite horizon. If the objective is to minimize the long term average periodic cost, we would have to consider a long horizon, which would make the problem even more difficult to solve.

Third, due to the difficulty of the problem, it is likely that we will not be able to solve it to optimality on real industrial instances (as discussed in Chapter 6). We would then use this formulation heuristically. If we must restrict ourselves to look for good solutions, then we believe that simpler procedures can be sufficient to grant good solutions very quickly.

These observations pushed us to look for some simple heuristics, for example, similar to the Silver-Meal heuristic (Silver and Meal, 1973) used for the determination of optimal shipment sizes. This is the object of Section 5.4.

5.3 LOWER BOUNDS ON THE CASES WITH CONSTANT RATES

In this section, we provide two lower bounds on the optimal cost of the operational problem when the rates (Λ_c^t and Γ^t) are constant over time, and equal to the average rates (Λ_c and Γ). The first lower bound, named *continuous lower bound* (CLB), is valid for both versions of the problem. The second lower bound, named *discrete lower bound* (DLB), is valid for the discrete version of the operational problem.

5.3.1 *Continuous Lower Bound*

The lower bound for the continuous version is actually the solution of the fixed L strategic model presented in Section 2.4. Indeed, we mentioned in Chapter 2 that the perfect coordination assumption represents an ideal scenario. The fixed L strategic model relies on periodic return frequencies. Using the notation presented in Chapter 2, we define the variables

- Q_c: the constant size of the shipments from customer c, ($Q_c \in \mathbf{R}_+$).

We define the problem

$$(FIX - Continuous) \qquad \min_{Q_c} \qquad \frac{\Lambda_c O_c}{Q_c} \tag{5.9}$$

$$s.t. \qquad L = \sum_c \left(\frac{Q_c}{2} + \frac{Q_c}{2}\frac{\Lambda_c}{\Gamma}\right) \leq \bar{L} \tag{5.10}$$

$$Q_c \leq CAP \qquad \forall c \tag{5.11}$$

PROPERTY 5.1 *The cost achieved by any solution of the continuous operational problem without capacities can be achieved by a solution of the problem (FIX-Continuous-U) defined by* (5.9)-(5.10). *The optimal cost solution of the problem (FIX-Continuous-U) is consequently a lower bound on the cost of any solution of the continuous uncapacitated operational problem.*

Proof. Let us consider any solution A of the continuous uncapacitated operational problem and any horizon length T. This solution achieves a transportation cost V^A. This transportation cost is the sum of the transportation costs linked to the shipments of each customer V_c^A, which is directly linked to the number of shipments performed from customer c over the horizon. This number is $n_c = V_c^A / O_c$. Solution A is characterized by a series of shipments from each customer $\{Q_c^{A(1)}, Q_c^{A(2)}, ..., Q_c^{A(n_c)}\}$. We must prove that there exists a feasible solution of (FIX-Continuous-U) that has the same transportation cost.

We prove that the solution B of (FIX-Continuous-U) for which

$$Q_c^B = \frac{\sum_{i=1}^{n_c} Q_c^{A(i)}}{n_c} \quad \forall c \tag{5.12}$$

(1) has the same transportation cost as solution A, and
(2) is feasible with respect to Constraint (5.10).

Proving (1) is straightforward. The same number of shipments is performed from each customer in solutions A and B.

In order to prove (2), we first note that, because the network flows are balanced ($\sum_c \Lambda_c = \Gamma$), the number of empty packages used in the network is constant. It follows that the logistic inventory is exactly equal to the average inventory in the network and, consequently, to the sum of the average inventories of all locations. We will prove that the logistic inventory of solution B is less than or equal to the logistic inventory of solution A. This will imply that if solution A is feasible with respect to (5.10), then solution B is also feasible.

The average inventory at each customer c in solution A is bounded by the following expression

$$\bar{I}_c{}^A \geq \sum_{i=1}^{n_c} \left(\frac{Q_c^{A(i)}}{2} \frac{Q_c^{A(i)}}{\sum_{j=1}^{n_c} Q_c^{A(j)}} \right) \tag{5.13}$$

The inventory is minimized if each shipment $Q_c^{A(i)}$ is sent immediately when it has been built. In this case, each shipment is built during a portion of time equal to the size of this shipment divided by the total number of packages sent over the horizon. This leads to (5.13). This best case average inventory can be simplified and bounded below as follows

$$\bar{I}_c{}^A \geq \frac{\sum_{i=1}^{n_c} (Q_c^{A(i)})^2}{2 \sum_{i=1}^{n_c} Q_c^{A(i)}} \geq \frac{1}{2} \frac{\sum_{i=1}^{n_c} Q_c^{A(i)}}{n_c} = \frac{Q_c^B}{2} = \bar{I}_c{}^B \tag{5.14}$$

where the second inequality derives from a direct application of the Cauchy-Schwarz inequality (Dragomir (2003)).

The average inventory at the factory in solution A can be bounded in a similar way

$$\bar{I_f}^A \geq \sum_c \left(\frac{\Lambda_c}{\Gamma} \frac{\sum_{i=1}^{n_c} (Q_c^{A(i)})^2}{2\sum_{i=1}^{n_c} Q_c^{A(i)}}\right) \geq \sum_c \left(\frac{\Lambda_c}{\Gamma} \frac{1}{2} \frac{\sum_{i=1}^{n_c} Q_c^{A(i)}}{n_c}\right) = \bar{I_f}^B \tag{5.15}$$

It follows that

$$L^B = \sum_c \frac{Q_c}{2} + \sum_c \frac{Q_c}{2} \frac{\Lambda_c}{\Gamma} \leq \sum_c \bar{I_c}^A + \bar{I_f}^A = L^A$$

$\square$

PROPERTY 5.2 *The cost achieved by any solution of the continuous operational problem with transportation capacities can be achieved by a solution of the problem (FIX-Continuous-C) defined by* (5.9)-(5.11). *The optimal cost solution of the problem (FIX-Continuous-C) is consequently a lower bound on the cost of any solution of the continuous capacitated operational problem.*

The proof of Property 5.2 is similar to the proof of Property 5.1. It suffices to note that Q_c^B in (5.12) respects the capacity constraint. This is an immediate consequence of the fact that each $Q_c^{A(i)}$ must be smaller or equal to CAP.

Properties 5.1 and 5.2 provide the continuous lower bound (CLB).

5.3.2 *Discrete Lower Bound*

The discrete lower bound is derived similarly. The difference is that the bound on the average inventory at the factory is adapted to take into account the fact that, at each periodic review, we must ensure that a sufficient inventory of empty packages is present at the factory to sustain its consumption until the next review. We consequently define the problem

$$(FIX - Discrete) \qquad \min_{Q_c} \qquad \frac{\Lambda_c O_c}{Q_c} \tag{5.9}$$

$$s.t. \qquad L = \sum_c \left(\frac{Q_c}{2} + \frac{\Gamma P}{2}\right) \leq \bar{L} \tag{5.16}$$

$$Q_c \leq CAP \qquad \forall c \tag{5.11}$$

PROPERTY 5.3 *The cost achieved by any solution of the discrete operational problem without capacities can be achieved by a solution of the problem (FIX-Discrete-U) defined by* (5.9) *and* (5.16). *The optimal cost solution of the problem (FIX-Discrete-U) is consequently a lower bound on the cost of any solution of the discrete uncapacitated operational problem. Similarly, the optimal cost solution of the problem (FIX-Discrete-C) defined by* (5.9), (5.16) *and* (5.11) *is a lower bound on the cost of any solution of the discrete capacitated operational problem.*

The proof of Property 5.3 is similar to the proof of used for Property 5.1 and 5.2. The bound on the average inventory given by (5.15), can be simply replaced by

$$\bar{I_f}^A \geq \frac{\Gamma P}{2} = \bar{I_f}^B \tag{5.17}$$

Property 5.3 provides the discrete lower bound (DLB).

5.3.3 *Lower Bounds Comparison*

Let us compare the lower bounds in the uncapacitated cases with constant rates. As derived in Section 2.4, the optimal shipment sizes corresponding to the continuous lower bound were given by (2.31):

$$\begin{aligned} Q_c^{*[CLB]} &= \bar{L} \quad \frac{2\sqrt{O_c\Lambda_c}}{\sqrt{1+\frac{\Lambda_c}{\Gamma}}\sum_{d\in NC}\sqrt{O_d\Lambda_d(1+\frac{\Lambda_d}{\Gamma})}} \\ &\approx \bar{L} \quad \frac{2\sqrt{O_c\Lambda_c}}{\sum_{d\in NC}\sqrt{O_d\Lambda_d}} \end{aligned} \tag{5.18}$$

where the approximation $(1 + \Lambda_c/\Gamma) \approx 1$ results from the fact that we consider large granular networks, i.e. networks in which Λ_c is small when compared to Γ. Similarly, we derive

$$Q_c^{*[DLB]} = (\bar{L} - \frac{\Gamma P}{2}) \quad \frac{2\sqrt{O_c\Lambda_c}}{\sum_{d\in NC}\sqrt{O_d\Lambda_d}} \tag{5.19}$$

Equation (5.19) allows us to derive the following corollary.

COROLLARY 5.1 *If the review periodicity evolves for all the actors from P to P', then the logistic inventory has to evolve from L to $(L + \Gamma(P' - P)/2)$ in order to keep the same discrete lower bound on the transportation cost.*

In other words, if it is decided, for example, to lengthen the periodicity of the reviews by one day, then the logistic inventory has to be enlarged by half of the average flow in one day in order to be able to keep the same shipment sizes and, therefore, the same transportation cost. Of course, this is valid if we conjecture that the pictures given by the lower bounds give a good indication of what happens in reality.

The same reasoning can be held for the comparison of a continuous-time and a discrete-time approach. Indeed, if we use the approximation in (5.18), then the comparison with (5.19) leads to the following result.

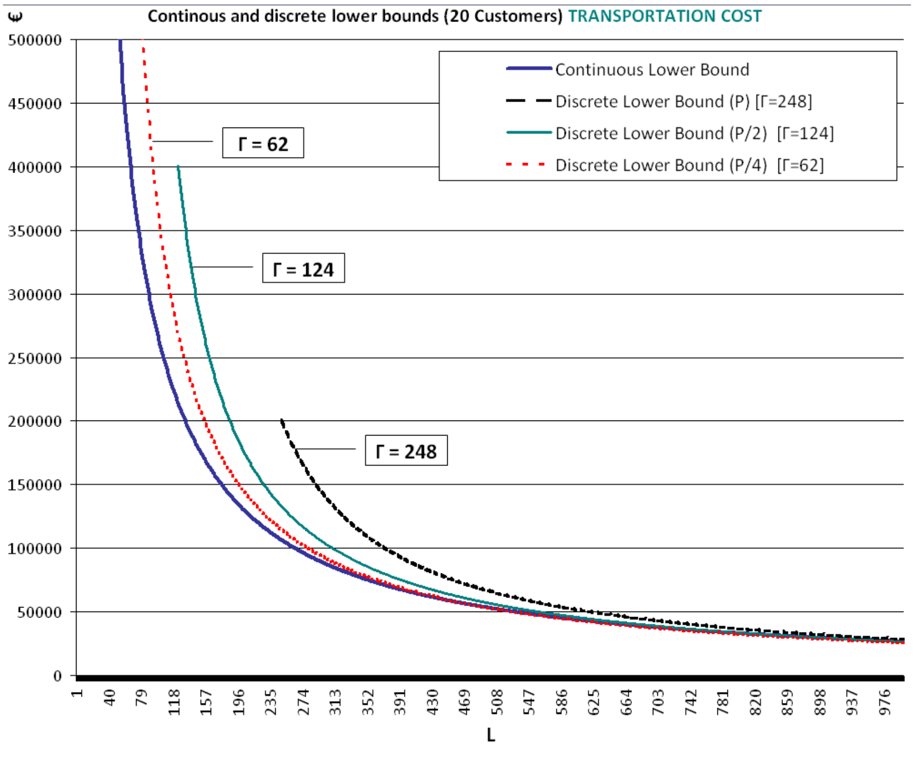

Figure 5.4: Transportation costs for the continuous and discrete bounds for varying P.

COROLLARY 5.2 *If we move from a continuous-time approach to a discrete-time approach with review periodicity P, then an additional inventory of size $\Gamma P/2$ will be needed in order to keep similar lower bounds on the transportation cost.*

Figure 5.4 illustrates these results with the plots of the various lower bounds on the transportation cost for an example with 20 customers and a total flow of 248 units per period. It also follows that, in the discrete version of the problem, there is a minimum logistic inventory (ΓP) under which the problem is infeasible, as the inventory at the factory must be raised to at least ΓP at each review period.

5.4 PORTFOLIO HEURISTIC

In this section, we introduce our heuristic for the operational problem, named portfolio heuristic. It is a myopic heuristic that determines, on the basis of the current inventory levels of empty packages in the network, from which customers return flows are performed when a resupply is needed at the factory. The performance of the heuristic is compared to other classical heuristics, and to the lower bounds derived in Section 5.3.

The world of heuristics is huge. The "impossibility" to find the optimal solution to a hard problem does indeed open the door for creativity. A literature review of all proposed heuristics would bring us back to the work of Knuth (1997) and Aho et al. (1974). Here, however, our goal is simply to develop some expertise with heuristics in order to know whether our problem can be tackled by such simple heuristics or whether more elaborated approaches are needed.

The aim of the heuristic is to coordinate the return flows of the customers over an infinite horizon such as to minimize the average periodic transportation cost in a network where the logistic inventory is given. The heuristic we suggest is myopic. This means that we let the horizon unroll until a decision is needed. When a decision is needed, it is solely based on the information available at that time and it only identifies shipments that are performed immediately. At that time, no decision is taken about future flows.

At the beginning of the horizon, the network is in a specific state. Inventory levels are known for each location. Their sum corresponds to the number of packages used in the network, the logistic inventory. The strategic model of Chapter 2 highlighted that the best case scenario is when the return flows are perfectly coordinated and the shipment sizes are exactly equal to the strategic optimal shipment sizes given by

(5.18) in the continuous version and (5.19) in the discrete version. Our approach is the following:

1. we impose coordination by performing no shipments until they are absolutely required at the factory.
2. when shipments are necessary, we try to choose the "best" customers from which to perform these shipments.

In other words, we impose coordination and try to work with shipments sizes that would bring the transportation cost close to the transportation cost implied by the optimal strategic shipment sizes. Accordingly, we wait until the inventory at the factory drops to zero (in the continuous version) or until, at a review period, the inventory at the factory is not sufficient to cover the needs of the factory until the next review (in the discrete version). At that moment, we select one customer (in the continuous version) or a set of customers (in the discrete version) from which we perform a return flow.

In order to perform this selection, we require:

- a selection criterion (to compare the different customers), and
- a procedure to select a set of shipments that are sufficient (when added to the current inventory of the factory) to meet the requirement of the factory until the next review period (in the discrete version).

5.4.1 *Selection Criteria*

The first question is to know what is the definition of the *best* supplier. A *good* supplier is definitely one that is cheap, in terms of transportation cost, since this is our objective function. There are several practical ways to decline this attribute. Here is a selection.

LARGE SUPPLY FIRST. A supplier with a large inventory is basically a good candidate since we will not need many of them to meet the factory needs. This somehow reduces the transportation costs. A supplier with a *large* inventory I_c is thus a *good* supplier.

SMALL UNIT COST FIRST. Taking both the inventory levels and the transportation costs into account leads to favoring suppliers who have small unit transportation costs. This means that the smaller O_c / I_c is, the better.

SMALL GRADIENT FIRST. The best unit cost method fails however to take into account the non-linearity of the transportation cost function. By choosing customers with a small unit cost, we do not include the fact that this cost could vary over time differently among the customers. For example, it could be better not to select customers with small unit costs because this cost could still be much further reduced at the next period. Since the Lagrangian optimality conditions state that the gradient of the individual cost functions should be equal (see (2.27)), one could qualify the customers with small gradient values $\Lambda_c O_c / I_c^2$ as good candidates.

PORTFOLIO APPROACH. All of the criteria mentioned above aim at comparing the potential candidates for the resupply of the factory. However, none of these approaches take into account the fact that the network can be left in very different states depending on the selected resupply. This is the fundamental principle of the *Portfolio Value* we present now. An inventory I_c at a customer represents a negative charge for the network because it will have to be brought back to the factory at some point. The transportation cost for shipments from customer c is O_c. The inventory I_c should be assigned a part of O_c depending on when we think that the customer will return its inventory. If we think that the nominal shipment size from this customer c is Q_c^*, the negative value of an inventory I_c at a customer c can be defined as $O_c(I_c/Q_c^*)$. Obviously, if the current inventory exceeds the nominal shipment size, we would set $Q_c^* = I_c$.
If we decide to bring back this inventory now, this negative charge is what we save for the future. Since bringing back today the inventory of a customer c costs O_c, the implicit cost of selecting this supplier for immediate shipment is

$$O_c - O_c \frac{I_c}{Q_c^*} = O_c(\frac{Q_c^* - I_c}{Q_c^*})$$

and again, the smaller, the better.

Indeed, each customer inventory can be seen as an asset - a negative one, in our case, since it has to be brought back. The sum of all of these assets is the value of the portfolio. Now, any set of shipments is priced by its corresponding transportation cost but also by the changes it induces in the portfolio value. It follows that the portfolio value is an economic value. Whereas the other selection criteria only allows us to rank the customers, the portfolio value allows for a comparison based on a measurement in €. The expected shipment sizes used for the computation of the portfolio value are the optimal strategic shipment sizes given by (5.18) or (5.19)

One additional selection criteria should be mentioned for the capacitated cases. Indeed, if the inventory level of a customer is equal to or greater than the capacity limit, we know that it will never be possible to return these empty packages at a lower cost. Their portfolio value is equal to zero. When compared with other customers of which the portfolio is zero, but which have not reached the capacity limit, these shipments at full capacity should be favored as there is no chance that the unit transportation cost could further decrease if the inventory grows larger.

5.4.2 *Set Selection*

The second question is how a combination of customers should be selected in order to reach the amount of inventory required by the factory. It only applies in the discrete version of the problem. Again, there are several options.

(i) Rank the customers and select them in sequential order until the factory needs are met.
(ii) Rank the customers and select them in sequential order until the factory needs are met. Then, check if some shipments could be removed while still meeting the requirement.
(iii) Rank the customers, then select the customer with the best value among those that can meet the remaining requirement of the factory. If there is no such customer, just select the overall best one and re-iterate the process.
(iv) Solve a knapsack problem for selecting the best set of customers. The capacity of the knapsack is linked to the factory requirement; the size and the value of each customer are linked to their current inventory and the selection criteria.

In practice, for each of the above criteria, we experimented the various combination rules when applicable. Rule (iii) gave the best results for the first three criteria and the knapsack gave the best results for the portfolio criterion. Note that using a knapsack requires a selection criteria that assign a value to each customer, as does our portfolio value. The other criteria only allow for a ranking of the customers.

5.4.3 *Detailed Algorithm*

Algorithm 2 details the portfolio heuristic used on the continuous version of the operational problem, while Algorithm 3 presents the heuristic applied on the discrete

Algorithm 2 : Portfolio Heuristic for the continuous version

1. Let the horizon unroll until the inventory at the factory drops to zero at time t.

2. At that time, compute the portfolio value (PV) of each customer, using

$$Q_c^* = max[Q_c^{*[CLB]}, I_c^t]$$
$$PV_c^t = O_c(\frac{Q_c^* - I_c^t}{Q_c^*})$$

3. Select a customer and perform the largest shipment possible from this customer (I_c^t or CAP):
if there are customers with $PV_c^t = 0$ **then**
- Select, among the customer whose inventory level has reached the capacity, the customer with the smallest transportation cost O_c.
- If no customer's inventory has reached the capacity limit, select, among the customer with $PV_c^t = 0$, the customer with the smallest gradient $I_c^t O_c / Q_c^{*[CLB]}$.

else
Select the customer with the lowest portfolio value.
end if

4. go back to step 1.

version. Note that, in most iterations, there is no customer with a portfolio value equal to zero, such that the customer selection relies primarily on the portfolio value.

We can immediately observe that the portfolio heuristic aims at implementing the perfect coordination as much as possible. Three arguments support this statement. First, a resupply of the factory is ordered only if needed. Second, it does not request more resupply than needed. Indeed, any additional shipment reduces the knapsack value. Third, it tries to favor shipments from the customers that have an inventory close to or above their optimal shipment size values.

5.4.4 *Numerical Results on Cases with Constant Rates*

We experimented the portfolio heuristic and the other heuristics on networks with 5, 20 and 50 customers respectively. For each network, we considered a large range of possible values for the logistic inventory L. The heuristics were tested without capacity limits on a horizon of 200 periods after an initial warm-up phase. Since the rates are constant, we can make use of the lower bounds we developed to obtain an optimality gap.

Algorithm 3 : Portfolio Heuristic for the discrete version

1. Let the horizon unroll until, at a review period at time t, the inventory of the factory is not sufficient to cover its requirement until the next review.

2. Based on the current inventory of the factory and on the consumption until the next review period, compute the minimum number R of empty packages that must be shipped to the factory at time t.

3.
while $R > 0$ **do**
3.a compute the portfolio value PV_c^t of all customers, using

$$Q_c^* = max[Q_c^{*[DLB]}, I_c^t]$$
$$PV_c^t = O_c(\frac{Q_c^* - I_c^t}{Q_c^*})$$

3.b
if there are customers with $PV_c^t = 0$ **then**
- Select, among the customer whose inventory level has reached the capacity, the customer with the smallest transportation cost O_c.
- If no customer's inventory has reached the capacity limit, select, among the customer with $PV_c^t = 0$, the customer with the smallest gradient $I_c^t O_c / Q_c^{*[CLB]}$.

- Perform the largest shipment possible from this customer, subtract the size of this shipment from R and go back to step 3.

end if
3.c Solve the knapsack problem

$$\begin{aligned} \min_{z_c} \quad & \sum_c z_c PV_c^t \\ s.t. \quad & \sum_c z_c I_c^t \geq R \\ & z_c \in \{0,1\} \quad \forall c \end{aligned}$$

Ship the inventory of the customers selected in the knapsack ($z_c^* = 1$), and set $R = 0$.
end while

For the discrete version of the operational problem, the results are depicted in Figures 5.5 to 5.7. We provide the results achieved on the total cost (transportation and holding of the logistic inventory). In these more than 3000 experiments, the portfolio heuristic always performed better than the other classical heuristics. Table 5.1 gives the minimum, average and maximum relative distance between the cost solution of the portfolio heuristic and of the discrete lower bound. Additionally, we provide the gap achieved at the value of L that minimizes the total cost achieved by the heuristic (at the minimum of the curve that corresponds to the portfolio heuristic), the average gap between the values of L that minimizes the total cost of the discrete lower bound and of the portfolio heuristic solutions, and the gap between the minimum total cost achieved by the portfolio heuristic and the minimum total cost of the DLB.

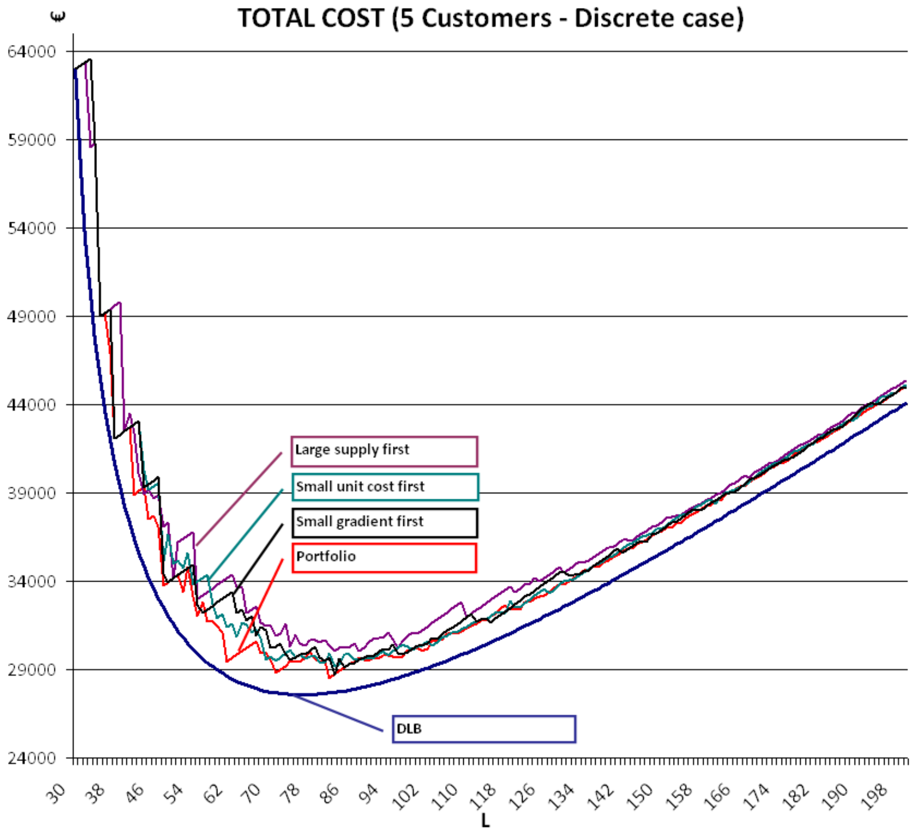

Figure 5.5: Comparison of the heuristic solutions with the discrete lower bound (5C).

Table 5.2 provides the results of the heuristic on the continuous version. In the continuous version, the optimality gaps measured are significantly smaller than in the discrete version. This result is to be expected since the problem is simplified by the fact that we select only one customer at a time.

The portfolio heuristic performs impressively well, not only compared to the other heuristics, but also in an absolute way since the gap becomes very small when the size of the network increases. This can be explained as follows.

In the heuristic, sticking to the perfect coordination assumption implies finding, in each period, customers which are the closest possible to their optimal quantities and, in the discrete version, whose shipments fit as tight as possible the demand of the

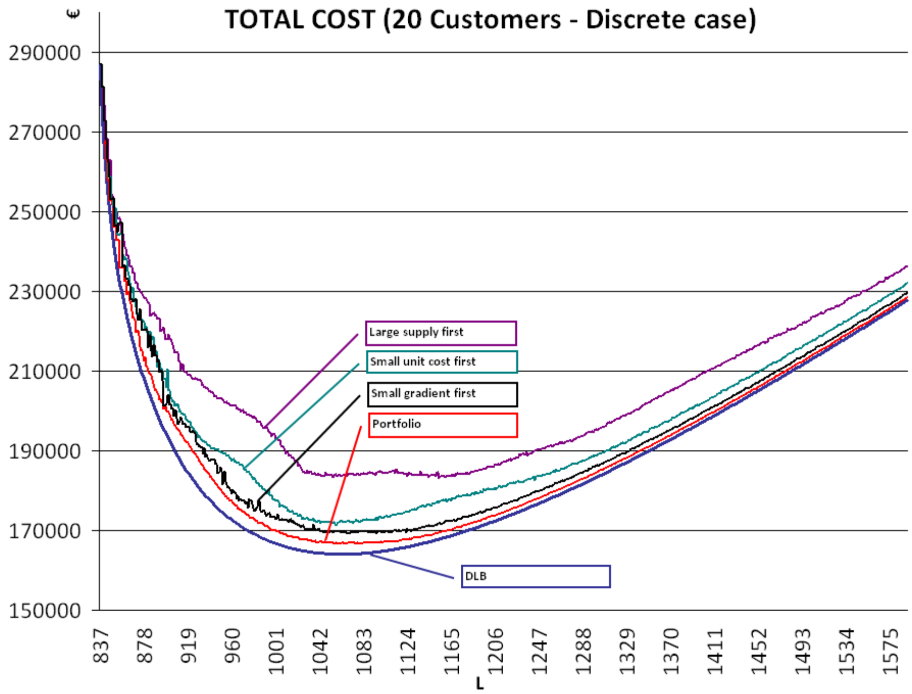

Figure 5.6: Comparison of the heuristic solutions with the discrete lower bound (20C).

Network	5C	20C	50C
Λ Range	3-8	3-30	3-45
Γ	30	245	837
O Range	20-100	20-150	20-150
H	6	6	6
L Range	30-200	245-1000	837-2500
Gap $(HEURSOL - DLB)/DLB$			
Average	5.19%	1.29%	1.08%
Minimum	0%	0%	0%
Maximum	23.59%	4.14%	3.73%
At min cost heuristic	3.06%	1.61%	0.78%
Average btw min costs	6.6%	1.68%	0.86%
Between min costs L	3.4%	1.62%	0.82%

Table 5.1: Results achieved by the portfolio heuristic in the discrete case.

factory for the period. With few customers, the network becomes more rigid, and it is difficult to meet these requirements. In other words, the relatively small number of options will sometimes lead to individual shipments that are far from their nominal optimal sizes and, in the discrete version, to a total resupply that could be far above

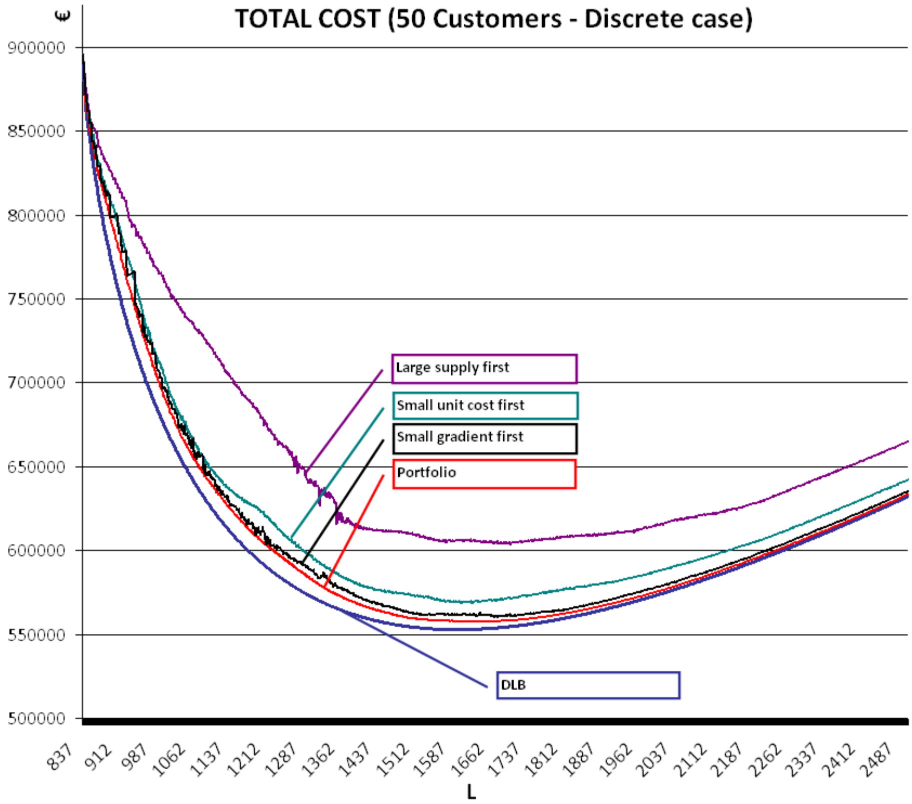

Figure 5.7: Comparison of the heuristic solutions with the discrete lower bound (50C).

Network	5C	20C	50C
Λ Range	3-8	3-30	3-45
Γ	30	245	837
O Range	20-100	20-150	20-150
H	6	6	6
L Range	30-200	245-1000	837-2500
Gap $(HEURSOL - CLB)/CLB$			
Average	0.73%	0.32%	0.14%
Minimum	0%	0.05%	0.05%
Maximum	1.33%	0.74%	0.27%
At optimum L heuristic	0.92%	0.33%	0.12%
Average between L optimum	1.04%	0.4%	0.12%
Between optimum L	0.95%	0.36%	0.12%

Table 5.2: Results achieved by the portfolio heuristic in the continuous case.

what is really needed at the factory.

The importance of the rates of the customers is also a decisive factor in the performance of the heuristic. It is not, however, the size of the Λ_c themselves on which the performance of the heuristic depends. Indeed, it would be easy to verify that if all rates are multiplied by the same factor, we would obtain the same performance. It is rather the absence of very large individual players on which the performance of the heuristic depends. Earlier, we used the term "granular" to refer to the networks for which the approach we use is appropriate. A granular network has customers with individual rates Λ_c that are small when compared to the total flow of the network. Naturally, in order to be considered granular, the network has to contain a certain number of customers. But, our experiments show that this number is quite rapidly reached.

The curves of the solutions obtained with the heuristic exhibit a minimum L value which is slightly larger than the minimum L value of the corresponding lower bound. This is explained by the fact that the lower bound, using the perfect coordination assumption, has an optimistic computation of the logistic inventory used for a set of shipments sizes. In practice, we cannot exactly reach these shipment sizes on average. Consequently, given the convexity of the transportation cost functions, we will find the balance between transportation and holding costs by using a few more empty packages in the network.

Although we tested the portfolio heuristic on cases with capacity limits, we do not provide numerical results as adding capacities actually makes the problem easier. Indeed, when shipments are realized at full capacity, the heuristic uses shipment sizes that are equal to the optimal shipment sizes used in the lower bounds. The performance of the heuristic (in terms of optimality gap) is consequently always better when capacity is added to the problem.

5.5 IMPACT OF VARIABILITY

In this section, we wish to analyze the main impacts of variability. The portfolio heuristic described in Section 5.4 remains identical whether the production rates are constant or variable over time. There are two main effects that we analyze in this section, the *local* and the *global* effects.

First, when considering a single period, the variability of the number of empty packages made available by each customer directly impacts the selection of the shipments to the factory. This selection is based on the optimal strategic shipment sizes. But, when the flows are variable, is the portfolio value based on these optimal shipment sizes still a good criterion? This is the local effect.

Second, when we consider successive periods, the variability will also modify the actual number of empty packages available in the reverse network. Indeed, suppose that for several periods the factory needs are higher than its average, while, at the same time, the releases of the customers are low. Then, more empty packages are consumed then released, and the number of empty packages decreases, making it more difficult to reach shipment sizes close to the optimal sizes used in the lower bound. This is the global effect.

A set of experiments was designed to analyze the impact of these effects. For each customer c and each time period, we drew the number of packages made available from a normal distribution with mean $\mu = \Lambda_c$ and standard deviation $\sigma = CV\Lambda_c$, with CV, the coefficient of variation, ranging from 0 to 1. In order to obtain an integer number of empty packages released in each period, we took the integer part of the result and added the fractional part to the next period. If the result appeared to be negative, it was set to zero. The negative part of the result was deferred to the next period.

5.5.1 *Impact of the Local Effect*

In order to analyze the local effect separately, we have used randomly generated rates Λ_c^t and imposed $\Gamma^t = \sum_c \Lambda_c^t$. The releases are variable, but globally we impose that the network flows be balanced. This allows us to suppress the global effect and to consider the impact of the local effect separately. In Figures 5.8 to 5.10, we compare the solutions achieved by the portfolio heuristic in the discrete version when the coefficient of variation $CV = \sigma/\mu$ is 0 (constant rates), 0.25, 0.5, 0.75 and 1.

The results have been adapted to reflect the cost for an equivalent total flow of units. In the variable case, the total number of units used at the factory is variable. We consequently adapted the total cost obtained in each variable case to correspond to a flow equal to the constant rates case, that is $T\Gamma$, where T is the length of the horizon used for the numerical tests.

Looking at the results, the good news is that it seems that the variability plays a favorable role. In our experiments, we achieve lower total costs when the flows are variable, even lower than the total costs given by the lower bound.

The point here is not that the heuristic performs better, but that the problem renders cheaper solutions. We offer the following intuitive argumentation.

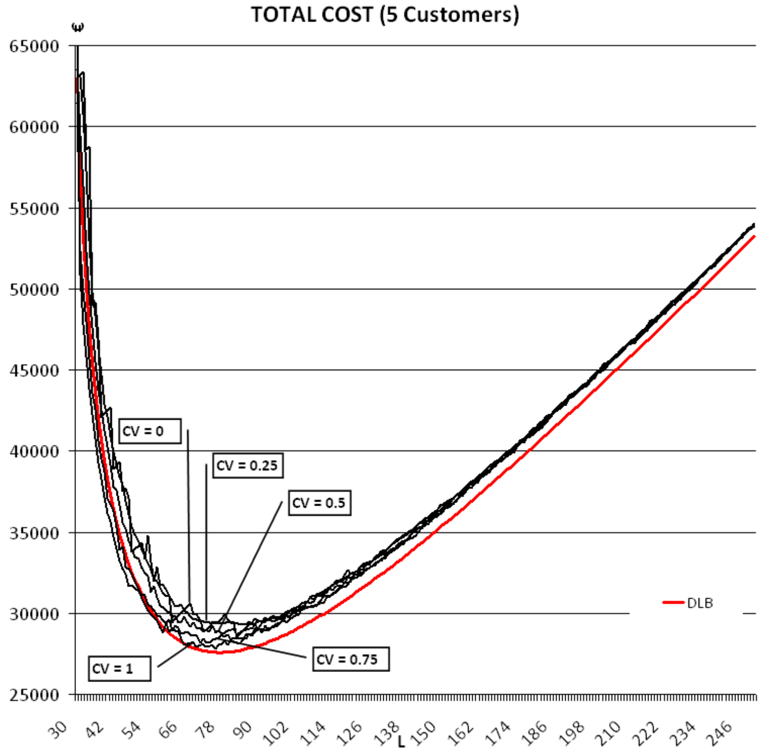

Figure 5.8: Heuristic solutions with zero lead time for different coefficients of variation (5 customers).

Let us consider, for simplicity, the continuous case with C customers having identical parameters and constant factory flows. It would be straightforward to extend this intuition to the general case.

We first consider the case with constant rates. After a long time, the portfolio heuristic has reached a kind of steady-state in which all of the customers have been sequenced. They all send their shipment size when it is their turn. Since everything is deterministic, these shipment sizes will also be identical. Let us denote $Q(constant)$ this shipment size.

Second, let us turn towards the variable case. We will prove that, in this case, the probability that a shipment has a size larger than $Q(constant)$ is more than 0.5. Indeed, when it is time to make a shipment, the portfolio heuristic selects the customer with the largest shipment. Let us assume that we are in period t and it should be the turn of customer d. In a deterministic case, its shipment size would have been equal to $Q(constant)$. However, in the variable-rate case, there is already a 50 percent chance that its shipment size is greater than $Q(constant)$. Let us denote by $\widetilde{Q_d^t}$ the current

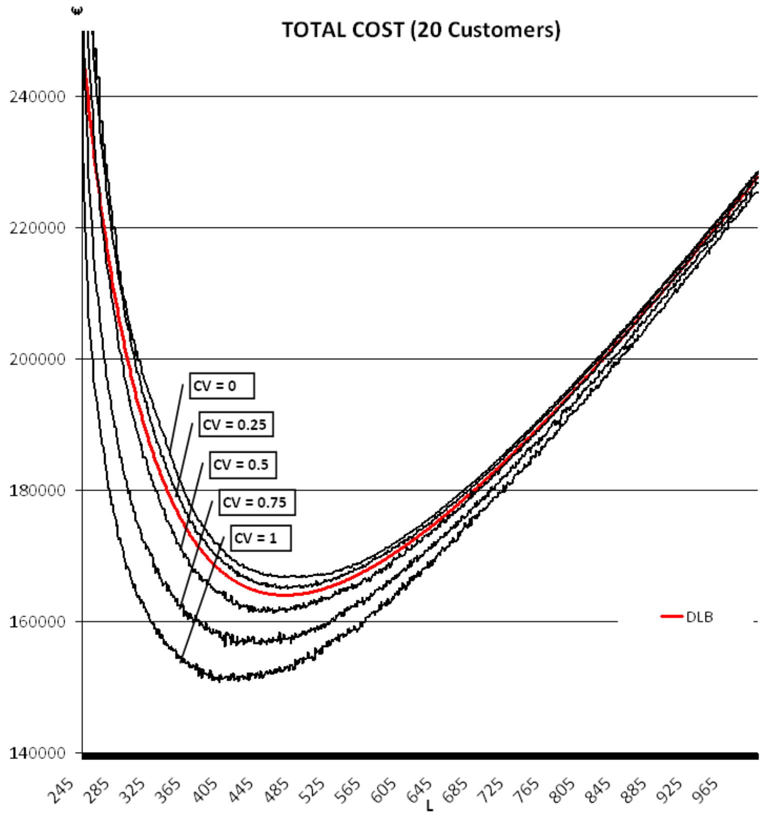

Figure 5.9: Heuristic solutions with zero lead time for different coefficients of variation (20 customers).

shipment size of customer d. In addition to the probability that $\widetilde{Q^t_d} > Q(constant)$, there is also the possibility that the inventory at another customer has exceeded $\widetilde{Q^t_d}$. So, globally, we derive that the shipment sizes with a variable-rate case are larger than the shipment sizes in the corresponding constant-rate case. The total cost achieved by the policy consequently decreases.

In other terms, if we let $\widetilde{Q^t}$ be the random variable that represents the quantity that will be shipped at a time t, the heuristic makes a selection such that

$$\widetilde{Q^t} = \max_{(c \in NC)} [\widetilde{Q^t_c}]$$

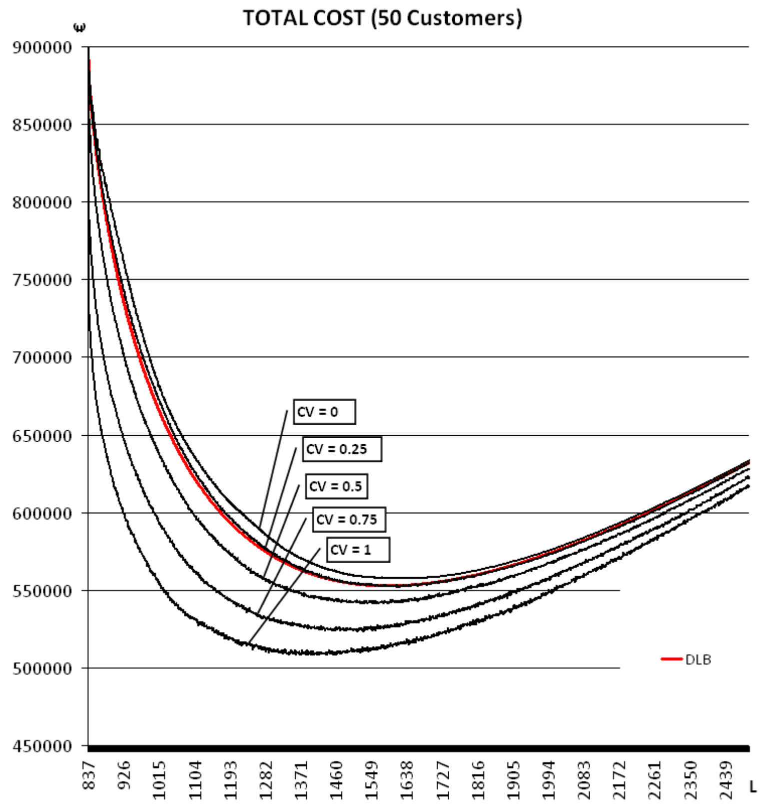

Figure 5.10: Heuristic solutions with zero lead time for different coefficients of variation (50 customers).

This expression shows that $\widetilde{Q^t}$ is an order statistics (Wackerly et al., 2002) for which we can deduce, assuming the independence of the individual random variables,

$$P(\ \widetilde{Q^t} \leq Q(constant)\ ) = P(\ \widetilde{Q^t_d} \leq Q(constant)\ ) \prod_{c \in NC \setminus \{d\}} P(\ \widetilde{Q^t_c} \leq Q(constant)\ )$$

It follows that

$$P(\ \widetilde{Q^t} \geq Q(constant)\ ) > 0.5$$

This beneficial impact is due to the flexibility of the heuristic of selecting the best shipping customer. Without this flexibility, the expected effect of the variability would be a negative cost effect due to the convexity of the transportation cost functions. As a consequence, it will not always be true that the cost solution achieved with variability will be better. Both impacts, the positive impact of the flexibility and the negative impact of the convexity of the cost functions, will act simultaneously. However, we should benefit from the relative flatness of the transportation cost functions around the minimum value and, in large networks, the flexibility will be increased by the number of customers.

In order to illustrate both impacts of the local effect, we ran the following test on the continuous operational problem with 20 customers. We first applied the portfolio heuristic on the case with constant rates (P(constant)). We recorded the sequence in which the customers' inventories were returned. Second, we applied the exact same sequence to the case with variable rates (Fix seq). In this case, the negative impact of the convexity of the cost functions is highlighted in the solution. Third, we applied the portfolio heuristic on the case with variable rates (P(variable)). In this case, the impact of the flexibility is included in the solution. The results are given in the following table for a case with 20 customers, a coefficient of variation equal to 1, and $L = 250$. In this example, the positive impact dominates.

	P(Constant)	Fix seq	P(Variable)
Total cost	154605	155715	147760
# of shipments	2003	2026	1830

5.5.2 *Impact of the Global Effect*

The global effect appears when the number of empty packages available in the network varies over time. This is likely due to the delivery process and the way the goods loaded on the packages are consumed by the customer. Let us, for example, consider that the production level raises for several consecutive periods (because demand for goods is higher). Then, more packages will be used at the factory. More packages will also be received by the customers, and later, more packages will be released by the customers. However, it is likely that it will take some time for the packages to be released. In the meantime, the number of empty packages in the network decreases. As a consequence, smaller shipment sizes will be used, and the total cost increases. Naturally, at some other periods, the network will have more empty packages than its initial number. But, due to the asymmetric cost functions, the negative impact will dominate.

To provide some numerical results, let us define by k the time it takes for the global flow in the network to be balanced. In other words, at some point, there must be a balance between what is used at the factory and what is released at the customers. We thus impose that $\Gamma(t-k) = \sum_c \Lambda_c^t$. The parameter k can be seen as a lead time.

Figure 5.11 displays the heuristic solutions for different values for k with a coefficient of variation equal to 0.75.

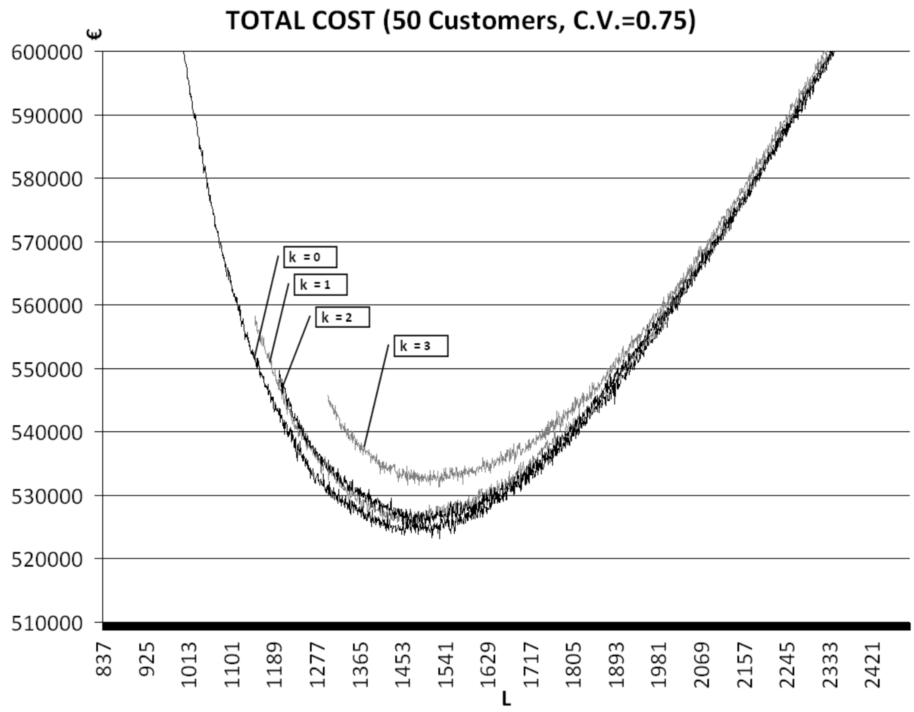

Figure 5.11: Heuristic solutions for the case with 50 customers and $\rho = 0.75$ for different lead times).

It is not our objective to discuss this variability in detail, as it does not give us any indication regarding the robustness of our heuristic. Some policies should be discussed to tackle this unbalance of the flows. For example, it would probably be worthwhile considering whether to allow backlogging some deliveries. In this case, if the network reaches a level of available inventory which is too low, implying huge transportation costs, the factory should postpone some of its requirements.

When considering at the same time the local and the global effect, it is likely, in practice, that the global effect will dominate. Regarding the local effect, it is difficult to assess exactly the performance of the heuristic, as we do not have lower bounds on cases with variable rates. However, the local effect has a positive and a negative impact. Our heuristic proved to be capable of taking advantage of the positive effect due to its flexibility.

5.6 CONCLUSION

In this chapter, we focused on the operational problem of managing the return flows of empty packages on a day-to-day basis. We demonstrated that this problem basically consists in answering the following question: When a factory has to be resupplied with empty packages, from which customer(s) should return shipments be performed?

The operational problem arises in a context in which the strategic design of the reverse network is given. Accordingly, we analyzed the problem on each factory cluster independently. We considered both a continuous and a discrete version of the problem.

The first important contribution achieved in this chapter was to derive lower bounds for both versions of the operational problem when the rates in the network are constant. We demonstrated that these bounds were based on the strategic models of Chapter 2. In other words, the perfect coordination assumption used at the strategic level corresponds to an ideal scenario on which our lower bounds are based. It follows that solving the operational problem additionally provides an answer to the question of the relevance of this assumption at the strategic level.

In this respect, we have shown that, when the network is sufficiently granular, i.e. when each player is small when compared to the whole network, we could achieve cost solutions to the operational problem that are close to the cost given by the lower bounds. This means that the perfect coordination assumption is reasonable when dealing with the strategic questions, like the allocation of the customers, the optimal shipment sizes or the size of the fleet of packages, at least when the network is sufficiently large.

The second contribution was the procedure itself that we presented to solve the operational problem. It is a myopic procedure, named portfolio heuristic, in which we impose a perfect coordination of the flows by not performing any shipment as long as the factory has a sufficient number of empty packages. The main challenge is the choice of the customers from which to perform the shipments when the factory has to be replenished with empty packages. Performing a "good" selection of the customers means that the average shipment sizes that we use are close to the optimal shipment sizes used in the lower bounds. The main achievement was to derive an economic value that allows us to compare the customers based on their inventory levels.

In order to validate the efficiency of the heuristic, it was tested on cases with constant rates, for which lower bounds were derived. The optimality gap observed in these experiments rapidly becomes very small as the network size increases. Besides these experiments, we largely discussed the fact that the good performance of the heuristic on granular network was not very surprising. Actually, even though the combinatorial coordination problem on small networks is very hard, we noted that achieving a good coordination on larger granular networks was relatively natural and easy. The crucial part is that of selecting good sets of customers every time shipments need to be performed. In this respect, we tested several other classical selection criteria, but the portfolio approach outperformed these alternatives.

We also briefly discussed the impact of variability on the solutions achieved. We identified two main effects. First, the local effect relates to our ability to adequately select the customers from which to order shipments when the rates are variable around their average rates, but when the total network flows remains balanced in each period. We demonstrated that this effect has both a positive and a negative impact. We also showed that our portfolio heuristic was able to take advantage of the positive impact due to its flexibility to adapt its selection when actual inventory levels are known. Second, the global effect relates to the fact that the number of empty packages available in the network is modified over time when the flows are variable. This global effect always negatively impacts the cost of the solutions, and is likely to dominate in practice.

Finally, we discussed the impact of the choice of the review periodicity by comparison of the lower bounds. We demonstrated that if, for example, we move from a daily review to a weekly review, then the logistic inventory would have to be larger by approximately half of the difference between the average daily and the average weekly flow in order to maintain the same transportation cost.

6

MIXED INTEGER FORMULATION OF THE OPERATIONAL PROBLEM

6.1 INTRODUCTION

In this chapter, we cover the operational problem over a finite horizon of discrete periods where all future flows are known exactly. In a context where the exact information about future flows is available, considering the problem of generating an optimal plan may be of interest. The problem can be formulated as a mixed integer program (MIP). However, it appears that this MIP does not solve easily. We describe reformulations and valid inequalities that allow us to improve our ability to solve this problem.

More specifically, we first point out the similarity of the polyhedron of our MIP with the one of lot-sizing problems. We demonstrate the equivalence of our problem with a multi-item uncapacitated lot-sizing with joint inventory bounds where the objective function only takes the constant production fixed costs into account. Our problem consequently has Wagner-Whitin non-speculative costs. We focus on improving this lot-sizing formulation. The reason for analyzing the lot-sizing equivalent problem is that lot-sizing mixed integer programs have been much studied since the discrete version of the economic lot size problem has been initiated by the work of Manne (1958) and Wagner and Whitin (1958). The problem has been extensively studied under different variants. We refer to Pochet and Wolsey (2006) for a presentation of the main results achieved in the discrete lot-sizing literature, and to Wolsey (1998) for the integer programming basics used in deriving these results.

The polyhedron of our lot-sizing formulation contains the polyhedra of C uncapacitated lot-sizing problems (LS-U), one for each customer, linked by a joint constraint in each period on the sum of their stock variables. The single-item uncapacitated lot-sizing problem has algorithms in $O(n \log n)$ described by Federgrün and Tzur (1991), Wagelmans et al. (1992) and Aggarwal and Park (1993). With a Wagner-Whitin cost structure, the problem can be solved in $O(n)$. The complete linear description of the convex hull by the (l,S) inequalities is due to Barany et al. (1984). The lot-sizing with inventory bounds is also polynomially solvable. The convex hull description is presented in Atamtürk and Küçükyavuz (2005). With Wagner-Whitin costs, using the tight formulation given in Pochet and Wolsey (1994) and adding the inventory bound constraints, the linear relaxation of the problem produces integer solutions. The complexity of our version of the problem originates in its multi-item structure for which various techniques have been described in Pochet and Wolsey (1991) and Pochet and Wolsey (2006), generally for linking constraints implying the production variables.

6.2 MIXED INTEGER FORMULATION OF THE REVERSE OPERATIONAL PROBLEM

The operational problem over a discrete finite horizon is based on the assumption that all flows (i.e. the number of empty packages made available by each customer and the number of empty packages consumed by the factory in each period) are exactly known at the beginning of the planning horizon. The problem considers C customers and T periods. We use the following notations:

NC: the set of customers,
NT: the set of periods in the horizon,

v_c^t: continuous variables representing the number of empty packages shipped from customer c to the factory in period t,
y_c^t: binary variable indicating if a shipment from customer c to the factory occurs in period t,
z_c^t: continuous variable for the inventory at customer c at the end of period t (after any shipments), if $t = 1$ then z_c^{t-1} denotes the initial inventory at customer c,
zu^t: continuous variable for the inventory at the factory at the end of period t (after receiving shipments), if $t = 1$ then zu^{t-1} denotes the initial inventory at the factory,

O_c: the fixed shipment cost from customer c to the factory,
Λ_c^t: the number of empty packages that customer c adds to its inventory in period t,
$\Gamma^t (= \sum_c \Lambda_c^t)$[1]: the number of empty packages that the factory consumes in period t,
K: the number of empty packages stored from one period to the next[2],

1 The lead time is assumed to be null in order to balance the flows in the network. We refer to Chapter 1 for the discussion about this methodological choice.
2 With respect to the notations used in Section 5.2, we define here $zu^t = s_f^t - \Gamma^t$. Accordingly, the link between the model of Section 5.2 is done by stating that $K = \bar{L} - \Gamma^0$.

M: a large positive value.

The discrete reverse operational problem can be formulated as

$$\begin{aligned}
&min && \sum_{c,t} y_c^t O_c && && (6.1)\\
&s.t. && z_c^{t-1} - v_c^t = -\Lambda_c^t + z_c^t && \forall c, t && (6.2)\\
& && zu^{t-1} + \sum_c v_c^t = \Gamma^t + zu^t && \forall t && (6.3)\\
& && v_c^t \leq y_c^t M && \forall c, t && (6.4)\\
& && \sum_c z_c^t + zu^t = K && \forall t && (6.5)\\
& && y_c^t \in \{0, 1\} && \forall c, t && (6.6)\\
& && z_c^t, v_c^t \geq 0 && \forall c, t && (6.7)\\
& && zu^t \geq 0 && \forall t && (6.8)
\end{aligned}$$

The objective function (6.1) is the sum of the fixed transportation costs for the shipments of all customers over the planning horizon. Constraints (6.2) and (6.3) are flow conservation constraints for each customer and for the factory. Constraint (6.4) links the x variables to the y variables, forcing the y variables to take the value 1 if a shipment occurs. Constraints (6.5) ensure that the total inventory at the end of each period matches the total amount of empty packages used in the reverse network K.

In this formulation, it can be seen that (6.3) is linearly dependent on the other constraints. Using (6.5) and the fact that $\Gamma^t = \sum_c \Lambda_c^t$, Constraints (6.3) may be rewritten as

$$\sum_c z_c^{t-1} - \sum_c v_c^t = -\sum_c \Lambda_c^t + \sum_c z_c^t$$

which is redundant with Constraints (6.2).

Consequently, the problem can equivalently be formulated as

$$\begin{aligned}
min \quad & \sum_{c,t} y_c^t O_c & & \\
s.t. \quad & z_c^{t-1} - v_c^t = -\Lambda_c^t + z_c^t & \forall c,t & \\
& v_c^t \leq y_c^t M & \forall c,t & \\
& \sum_c z_c^t \leq K & \forall t & \quad (6.9) \\
& y_c^t \in \{0,1\} & \forall c,t & \\
& z_c^t, v_c^t \geq 0 & \forall c,t &
\end{aligned}$$

6.3 FORWARD LOT-SIZING FORMULATION OF THE OPERATIONAL PROBLEM

It appears that solving our initial reverse problem is equivalent to solving a symmetric forward problem. As illustrated in Figure 6.1, inverting the direction of the flows as well as the order of the periods of the horizon leads to a problem with forward flows where the solution can immediately be interpreted as the solution of the initial reverse problem. This is done by the following substitutions:

$$\begin{aligned}
& D_c^t = \Lambda_c^{T-t+1} & \forall c,t \\
& s_c^t = -z_c^{T-t} & \forall c,t \\
& x_c^t = -v_c^{T-t+1} & \forall c,t
\end{aligned}$$

We call this problem the forward lot-sizing operational problem (FLS) and the following formulation is denoted P^A:

$$\begin{aligned}
min \quad & \sum_{c,t} y_c^t O_c & & \\
s.t. \quad & s_c^{t-1} + x_c^t = D_c^t + s_c^t & \forall c,t & \quad (6.10) \\
& x_c^t \leq y_c^t M & \forall c,t & \quad (6.11) \\
& \sum_c s_c^t \leq K & \forall t & \quad (6.12) \\
& y_c^t \in \{0,1\} & \forall c,t & \\
& s_c^t, x_c^t \geq 0 & \forall c,t & \quad (6.13)
\end{aligned}$$

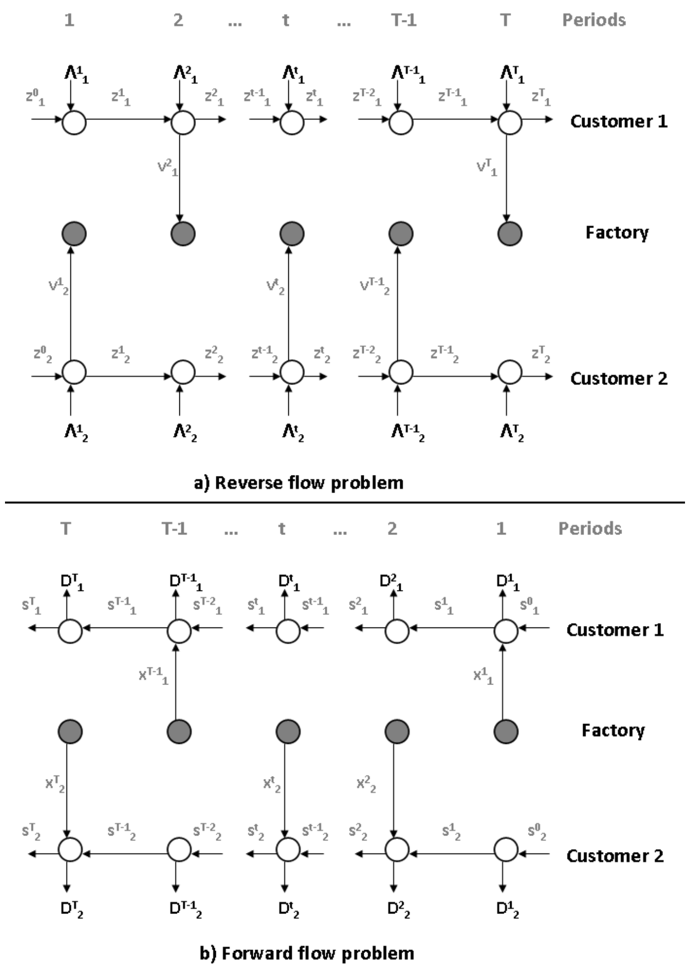

Figure 6.1: Equivalence of the reverse problem (a) and the forward problem (b).

6.4 COMPLEXITY OF THE FORWARD LOT-SIZING FORMULATION

The FLS can be shown to belong to $\mathcal{NPC}$ by observing that the 0-1 knapsack problem is polynomially reductible to the FLS.

Let us consider an instance of the FLS with 2 periods and C customers, an inventory bound equal to K, demands in period 1 which are strictly positive and no initial inventories. In this case, production must occur for each customer in period 1. The minimization of the objective function of FLS is therefore achieved by selecting a set of customers of which the demand in period 2 may be satisfied from production in period 1. The choice of this set of customers is limited by

$$\sum_c D_c^2(1 - y_c^2) \leq K$$

and the objective function may be rewritten as

$$\begin{aligned} & min \sum_c O_c + \sum_c Oc - \sum_c Oc(1 - y_c^2) \\ \Rightarrow \quad & min - \sum_c O_c(1 - y_c^2) \\ \Rightarrow \quad & max \sum_c O_c(1 - y_c^2) \end{aligned}$$

Defining variables $\theta_c^t = (1 - y_c^t)$, the two periods FLS with no initial inventory and positive demands in period 1 is equivalent to

$$\begin{aligned} max \quad & \sum_c O_c \theta_c^t \\ s.t. \quad & \sum_c D_c^2 \theta_c^t \leq K \\ & \theta \in B^C \end{aligned}$$

which is a 0-1 knapsack problem. It follows that any instance of a 0-1 knapsack problem can be polynomially reduced to an instance of the FLS, which proves that FLS $\in \mathcal{NPC}$.

6.5 SOLVING THE FORWARD LOT-SIZING OPERATIONAL PROBLEM

In order to assess the performance of the solver on the formulations that we put forward in this chapter, we rely on two data sets. The first set, referred to as the *small set*, allows us to rapidly solve a small instance to optimality, while the second, named the *medium set*, will generate solutions with a remaining optimality gap.

The small set has 5 customers, a horizon of 10 periods and an inventory bound equal to 20. The data used in the small set are:

$$D = \begin{matrix} 6 & 2 & 9 & 13 & 13 & 14 & 0 & 6 & 13 & 2 \\ 4 & 1 & 0 & 2 & 3 & 0 & 4 & 5 & 8 & 5 \\ 6 & 5 & 14 & 7 & 6 & 5 & 15 & 12 & 15 & 4 \\ 14 & 1 & 11 & 12 & 15 & 7 & 5 & 11 & 5 & 12 \\ 1 & 3 & 1 & 5 & 7 & 8 & 6 & 15 & 1 & 3 \end{matrix}$$

$$O = \begin{matrix} 20 & 40 & 50 & 75 & 100 \end{matrix}$$

The medium set has 15 customers, a horizon of 50 periods and an inventory bound equal to 500. The data used in the medium set was generated arbitrarily. Fixed production costs range from 20 to 150 and demands from 0 to 25. For all tests, the XPRESS solver time was limited to 600 seconds.

In both sets, no initial inventories were included. Initial inventories may easily be removed from the problem by their successive allocation to the demands of the first periods.

In the remaining of this chapter, we set the individual bound in each constraint (5.4) to $min[K + D_c^t, \sum_{u=t}^{T} D_c^u]$.

6.5.1 *Performance on the Initial Formulation*

Using the initial formulation P^A, the following performance on our data sets could be obtained:

Linear relaxation(LR)	Best bound(Bb)	Best solution(Bs)	Gap	Time
Small set				
879.7	1295	1295	0%	0.5s
Medium set				
2658.23	4512.9	7010	35.62%	600s

6.5.2 *Facility Location Reformulation*

Polyhedra of individual uncapacitated lot-sizing for each customer may be identified in the initial formulation. Reformulating each LS-U to obtain a tight formulation allows us to improve the linear relaxation achieved on P^A. We use an extended formulation for the individual LS-U, referred to as the *facility location reformulation* in Pochet and Wolsey (2006). Using other extended formulations or a separation algorithm for the (l, S) inequalities provides equivalent results.

The facility location reformulation, denoted P^B, uses continuous variables x_c^{st} with $t \geq s$ to represent the production of customer c in period s allocated to satisfy the demand of period t. The formulation P^B is

$$\begin{aligned}
min \quad & \sum_{c,t} y_c^t O_c & & \\
s.t. \quad & \sum_{s=1}^{t} x_c^{st} = D_c^t & \forall c, t & \quad (6.14)\\
& x_c^{st} \leq y_c^s D_c^t & \forall c, s, t \geq s & \quad (6.15)\\
& \sum_c \sum_{s=1}^{t} \sum_{u=t+1}^{T} x_c^{su} \leq K & \forall t \in NT \backslash \{T\} & \quad (6.16)\\
& y_c^t \in \{0, 1\} & \forall c, t & \\
& x_c^t \geq 0 & \forall c, t &
\end{aligned}$$

Using formulation P^B, the following results could be obtained:

LR	Bb	Bs	Gap	Time
Small set				
1150.97	1295	1295	0%	1.5s
Medium set				
5798.67	5810.62	6180	5.98%	600s

The facility location reformulation significantly improves the linear relaxation, which allows us to reduce the optimality gap from 35.62% to 5.98%.

6.5.3 *A Class of Simple Valid Inequalities*

In order to derive a class of valid inequalities for P^B, let us consider a subset $R \subseteq C$ of the customers. For each customer i in this subset, and for each period $k \in T$, we must have that the cumulative demand from period k to each period $t_i \geq k$ is covered by the inventory at the end of period $k-1$ and by the production in the interval $[k, t_i]$

$$s_i^{k-1} + M \sum_{u=k}^{t_i} y_i^u \geq D_i^{k,t_i} \quad \forall i \in R, \quad k, t_i \in T : t_i \geq k \tag{6.17}$$

where $D_i^{k,t_i} = \sum_{u=k}^{t_i} D_i^u$.
Summing over the customers in the set R, we obtain

$$\sum_{i \in R} s_i^{k-1} + M \sum_{i \in R} \sum_{u=k}^{t_i} y_i^u \geq \sum_{i \in R} D_i^{k,t_i} \quad \forall k, t_i \in T : t_i \geq k \tag{6.18}$$

Using (6.16), we deduce that the sum of the inventory variables in (6.18) must be less than or equal to K, and consequently

$$M \sum_{i \in R} \sum_{u=k}^{t_i} y_i^u \geq \sum_{i \in R} D_i^{k,t_i} - K \quad \forall k, t_i \in T : t_i \geq k \tag{6.19}$$

From (6.19), we derive the following class κ of valid inequalities for P^B

$$\begin{aligned} &\forall R \subseteq C \quad \forall k, t_i \in T \quad : \quad t_i \geq k, \sum_{i \in R} D_i^{k,t_i} > K \\ &\sum_{i \in R} \sum_{u=k}^{t_i} y_i^u \geq 1 \end{aligned} \tag{6.20}$$

We separated the inequalities of the class κ by solving a 0-1 knapsack problem in each period $t \in T$. The set of potential objects in the knapsack problem is partitioned into C different subsets in which at most one object may be selected. Let us consider a solution $(y, x)^*$ to the linear relaxation of P^B. For the knapsack problem in period t, we denote by V_c the C subsets of objects. Each subset V_c has U_c elements, such that

$$\begin{aligned} &\sum_{u_c=t}^{t+U_c} D_c^{u_c} \leq D_c^t + K \quad , \\ &\sum_{u_c=t}^{t+U_c+1} D_c^{u_c} > D_c^t + K \end{aligned}$$

For each element u_c of the set V_c, we define

$$W_c^{u_c} = D_c^{t(t+u_c)} \quad \forall u_c \in \{1, ..., U_c\} \text{ , the weight of the object}$$
$$P_c^{u_c} = \sum_{v=t}^{t+u_c} (y_c^v)^* \quad \forall u_c \in \{1, ..., U_c\} \text{ , the profit of the object}$$

Each object corresponds to the interval $[t, t + u_c]$ and the separation problem solved at period t strives to find the set of customers, along with an interval of periods for each selected customer, with a cumulated demand greater than K for which the sum of the $(y_c^t)^*$ is minimal. Defining binary variables

$$\tau_c^{u_c} = \begin{cases} 1 \text{ if the object corresponding to customer } c \\ \text{ and interval } [t, t + u_c] \text{ is selected} \\ 0 \text{ otherwise} \end{cases}$$
$$\forall c \in C, u_c \in \{t, ..., U_c\}$$

the separation problem in period t is

$$min \qquad \sum_{c \in C} \sum_{u_c=1}^{U_c} \tau_c^{u_c} P_c^{u_c} \tag{6.21}$$
$$s.t. \qquad \sum_{c \in C} \sum_{u_c=1}^{U_c} \tau_c^{u_c} W_c^{u_c} \geq K + 1 \tag{6.22}$$
$$\sum_{u_c \in V_c} \tau_c^{u_c} \leq 1 \qquad \forall c \in C \tag{6.23}$$

If the optimal objective value is strictly less than 1, a violated inequality in class κ has been found. Otherwise, there is no violated inequality in class κ with starting period in t. The separation algorithm is applied on each period of T.

Since the number of objects in each knapsack is at most CT, each knapsack can be solved in $O(CTK)$, using an adaptation of the classical dynamic recursion for the 0-1 knapsack. Consequently, the separation algorithm can be solved in $O(CT^2K)$.

Applying the separation algorithm on the linear relaxation of P^B until no more inequalities of the class κ could be found, and solving the integer program with the additional constraints added by the separation algorithm, we obtained the following performance on our sets:

LR	Bb	Bs	Gap	nodes	cuts	Sep Time	IP Time
Small set							
1195.59	1295	1295	0%	151	24	0.58s	0.6s
Medium set							
5806.61	5813.19	6210	6.39%	2502	66	710s	600s

Although the linear relaxation of the problem is improved, the optimality gap has slightly deteriorated on the medium set, probably due to the increase in the number of constraints.

6.5.4 *Stock Minimal Solutions*

In order to improve the lower bound we derive on the objective value, we may consider restricting our search to some specific optimal solutions. We observe that there always exist optimal solutions to the FLS which are stock minimal. In other words, considering the initial formulation P^A, there exist optimal solutions for which, in each period t and for each customer c, we have that $s_c^{t-1} y_c^t = 0$. These stock minimal solutions are characterized by the fact that for each customer and in each period t where a production occur, the produced quantity exactly covers the demands of an interval $[t, t+k]$.

Proof. Let us assume an optimal solution $y(x,s,y)^*$ for which there is a customer c and a period t where $(y_c^t)^*(s_c^{t-1})^* > 0$. It follows that $(y_c^t)^* = 1$. Let $k < t$ be the last production period for customer c. Then the solution $(x,s,y)'$ with the following modifications of the solution $(x,s,y)^*$

$$
\begin{aligned}
&(s_c^l)' = (s_c^l)^* - (s_c^{t-1})^* \quad \forall k \leq l \leq t-1 \\
&(x_c^k)' = (x_c^k)^* - (s_c^{t-1})^* \\
&(x_c^t)' = (x_c^t)^* + (s_c^{t-1})^*
\end{aligned}
$$

is an optimal solution. This results from the fact that the solution $(x,s,y)'$ has the same objective value as the solution $(x,s,y)^*$ and that for each period $t \in T$ we have that

$$\sum_c (s_c^t)' \leq \sum_c (s_c^t)^*$$

implying that all constraints remain respected in $(x,s,y)'$. □

Considering only stock minimal solutions and formulation P^B, we immediately deduce that

$$x_c^{tu} = 0 \quad \text{whenever} \quad D_c^{tu} > D_c^t + K$$

Consequently, these variables may be removed from the formulation. Defining for all customers and periods:

- α_c^t as the earliest period of which the production may be used to satisfy the demand in period t and

- β_c^t as the latest period of which the demand may be satisfied by the production of period t

we derive individual inventory bounds for each customer

$$u_c^t = \sum_{v=t}^{\beta_c^t} D_c^v - D_c^t$$

The stock minimal solution also highlight the fact that the inventory bound constraints have the form of knapsack constraints. The inventory at the end of period t is constructed by the demands in periods $u > t$ satisfied by production from periods $s \leq t$. As these demands will be covered exactly, we may reformulate the problem by using new binary variables w_c^{tu} representing the proportion of the demand of period u satisfied by production in period t. Using these binary variables instead of the continuous variables x_c^{tu} will ensure that the knapsack inequalities, that the solver could use for generating cuts automatically, become explicit in the formulation. This leads to the following formulation P^C

$$\begin{aligned}
&\textit{min} && \sum_{c,t} y_c^t O_c && \\
&\textit{s.t.} && \sum_{s=\alpha_c^t}^{t} w_c^{s,t} = 1 && \forall c,t && (6.24)\\
& && w_c^{st} \leq y_c^s && \forall c,s,\beta_c^s \geq t \geq s, D_c^t > 0 && (6.25)\\
& && \sum_c \sum_{s=\alpha_c^t}^{t} \sum_{u=t+1}^{\beta_c^t} D_c^u w_c^{su} \leq K && \forall t \in NT \backslash \{T\} && (6.26)\\
& && y_c^t, w_c^{tu} \in \{0,1\} && \forall c,t,t \leq u \leq \beta_c^t && (6.27)
\end{aligned}$$

However, this formulation does not improve the performance of the solver as it can be seen in the following table:

LR	Bb	Bs	Gap	nodes	cuts	Sep Time	IP Time
Small set							
1195.59	1295	1295	0%	33	19	0.31s	0.6s
Medium set							
5806.61	5813.48	6210	6.39%	783	66	595s	600s

6.5.5 *Options for Future Improvements of the Formulation*

Formulation P^B is the best formulation we could achieve for the FLS. The solution on the small set of the linear relaxation after separation of the valid inequalities of the class κ is displayed in Figure 6.2.

A first option would be to combine the valid inequalities of the class κ to obtain stronger inequalities. Let us consider the instance represented on Figure 6.3. Three valid inequalities of the class κ are represented. In each of these three sets, there must be at least one production period. However, because the intersection of the sets is empty, we conclude that it is not possible to satisfy this requirement of all the three sets with a single production period. Consequently, in the union of the three sets there must be at least 2 production periods. This strengthens the inequalities of the class κ if the total demand in this union is smaller than $2K$, which is the case in our example.

The characterization of stock minimal solutions has created varying individual inventory bounds for each customer. A second option would be to explore how these

Periods	1	2	3	4	5	6	7	8	9	10
Customer 1										
Demand	6	2	9	13	13	14	0	6	13	2
Inventory	1.03	4.37	0	0	0	0	0	4.25	1.65	
Y variable	1	0.49	0.51	1	1	1	0	1	0.69	0.17
Customer 2										
Demand	4	1	0	2	3	0	4	5	8	5
Inventory	1.74	0.74	0.74	2.48	2.08	2.08	2.4	5.88	4.13	
Y variables	1	0	0	0.63	0.37	0	0.48	0.52	0.48	0.17
Customer 3										
Demand	6	5	14	7	6	5	15	12	15	4
Inventory	3.25	4.91	6.34	5.41	3.27	5.19	7.21	2.6	3.31	
Y variables	1	0.35	0.65	0.35	0.35	0.35	0.65	0.4	0.83	0.17
Customer 4										
Demand	14	1	11	12	15	7	5	11	5	12
Inventory	4.86	3.86	7.79	5.26	7.79	8.81	3.81	3.27	4.15	
Y variables	1	0	0.65	0.35	0.65	0.35	0	0.65	0.35	0.65
Customer 5										
Demand	1	3	1	5	7	8	6	15	1	3
Inventory	9.13	6.13	5.13	6.85	6.86	3.92	6.58	4	3	
Y variables	1	0	0	0.37	0.35	0.36	0.35	0.65	0	0

Figure 6.2: Solution of the linear relaxation of P^C.

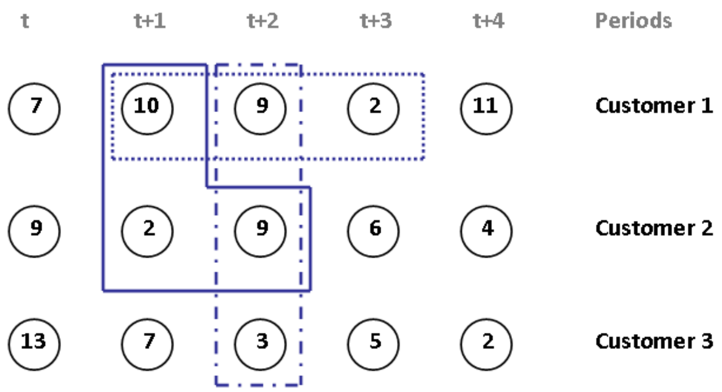

Figure 6.3: Strengthening the valid inequalities of class κ.

individual bounds combine between customers to derive inventory bounds for certain sets of customers. For example, let us consider the bound which may be derived on the sum of the variables s_3^6, s_4^6 and s_5^7. As we can observe in Figure 6.4, the maximal inventory is 39. It is achieved if we supply the demand of customer 3 and 4 in period 7 and the demands of customer 5 in the periods 8 to 10.

Consequently, the bound for these variables is

$$s_3^6 + s_4^6 + s_5^7 \leq 39 \tag{6.28}$$

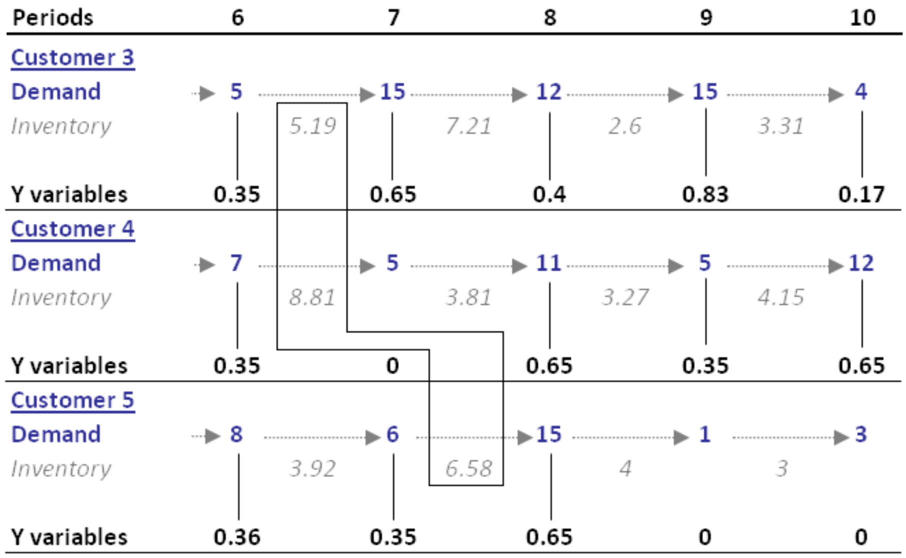

Figure 6.4: Partial solution of the linear relaxation of P^C.

We may further observe that, if no production occurs in period 7 for customer 5, the bound drops to 5. Indeed, if $y_5^7 = 0$ then $s_5^6 = 6$ because period 7 must be supplied from the stock of period 6 which may not, however, cover the demand in period 8. As a consequence, we must have that $s_5^7 = 0$ due to the global inventory bound $K = 20$. For the same reason, we can conclude that $s_3^6 = 0$ and that $s_4^6 \leq 5$. It follows that we may rewrite (5.28) as

$$s_3^6 + s_4^6 + s_5^7 \leq 5 + 34y_5^7 \tag{6.29}$$

which is a violated valid inequality.

From the exploration of the linear relaxation's solution, we can identify a series of such inequalities. For example, the following inequalities are valid:

$$s_3^7 + s_4^6 + s_5^7 \leq 17 + 7y_5^7$$

$$s_4^5 + s_5^4 + s_5^5 + s_3^4 \leq 14 + 21y_4^5$$

$$s_4^5 + s_4^6 + s_5^5 \leq 17 + 8(1 - y_5^6)$$

$$s_3^4 + s_4^4 + s_5^4 \leq 15 + 3(y_2^5 + y_4^4)$$

$$s_3^6 + s_4^6 + s_5^7 \leq 5 + 34y_5^7$$

$$s_3^7 + s_4^6 + s_5^7 \leq 17 + 7(1 - y_5^8)$$

$$s_1^4 + s_4^3 + s_5^1 \leq 17 + 5(1 - y_4^4)$$

Adding this set of inequalities to the formulation improves the linear relaxation from 1195.59 to 1206.63. We could not, however, identify a general description of such inequalities.

6.6 CONCLUSION

In this chapter, we have considered the mixed integer formulation of the discrete operational problem for the return flows of empty packages over a finite horizon and with known future flows. We have first provided a reverse mixed integer formulation for the operational problem. We then have shown the equivalence of this formulation with a multi-item uncapacitated lot-sizing with joint inventory bounds for which the objective function includes only fixed production costs associated with the setup variables. This equivalent problem was named the forward lot-sizing operational problem (FLS). We have shown that FLS belongs to $\mathcal{NPC}$ by proving that all instances of the 0-1 knapsack problem could be reduced polynomially to a 2-period instance of the FLS.

In order to improve our ability to solve the FLS, we derived an extended formulation providing a tight formulation of the polyhedra of each individual uncapacitated lot-sizing. We also derived a simple class of valid inequalities which may be separated in $O(CT^2K)$. On an instance with 15 items and 50 periods, the optimality gap that could be achieved by allowing 600 seconds to solve the mixed integer program is 5.98%. This performance underlines that we were not able to provide a satisfactory formulation nor derive strong valid inequalities that would allow us to rely on a mixed integer program to solve industrial scale instances of the operational problem for the return flows of empty packages.

Future research on the FLS could exploit the structure of stock minimal solutions to derive inventory bounds on specific sets of nodes or the superposition of knapsack-like inequalities in the formulation. Alternative heuristic procedures based on the mixed integer formulation, such as rolling horizons, could also be developed to obtain good solutions to large scale instances. Potentially, it would be worth analyzing the mixed integer formulation with constant flows for each customer and the factory. It is likely that the specific structure offered by constant flows would allow us to derive some additional inequalities. The solutions that would be obtained on this problem with constant flows could be compared with the heuristic solutions obtained in Chapter 4.

7

CONCLUDING CHAPTER

7.1 RESEARCH OBJECTIVES AND METHODOLOGY

7.1.1 *Research Objectives*

This work has presented the scientific and managerial contributions of our research on the management of the return flows of reusable packages. The research was carried out in the context of the TransLogisTIC project financed by the Walloon Region of Belgium. This insertion in an industrial project promoted a strong managerial focus in our research. This industrial focus was reinforced by a collaboration with AGC Flatglass Europe which was carried out during two years.

The objective of the study was to elaborate models to answer the strategic and operational issues faced by the industry in the management of an independent reverse network for reusable package where fixed transportation costs apply. The managerial focus of our study has led to focus on simple and intuitive reasoning in order for the produced knowledge to be easily appropriated by the logistic managers of the industry.

The questions that were raised by the logistic managers of the industry can be classified into four categories.

Fleet of packages. The first category covers the questions about the fleet of reusable packages. It is primarily focused on the size of the fleet and its allocation to potential subsets of the network. It also includes questions about the structure of the fleet when different types of packages are used. Finally, these questions call for a reflexion on the potential sharing of the fleet between customers or factories, or even with the competitors, and on the benefits that could result from the standardization of the fleet. The design and the management of the reverse network only impact a part of the fleet of packages. This part is named the logistic inventory and corresponds to the number of reusable packages that are required to implement the network design decisions.

Network structure. The second category deals with the questions about the structure of the network. It addresses the allocation of the return flows to the production points, the appropriate locations to hold inventories of packages, the transportation policy that should be used or the usefulness of an additional layer of intermediary depots.

Network dynamic. The third category covers the questions about the long term dynamic of the network and the evaluation of its annual cost. It is related to the average transportation cost, the average frequency of the visits at the customers and of the replenishments of the factories and the average inventory held in each location. The logistic managers were particularly interested in the possibility of testing variations on a network's structure and rapidly evaluating their cost impact.

Operational management. The fourth category has to do with the daily management of the flows. It relates to the questions about the periodicity of the planning decisions, the kind of information needed and the timing in which it is needed. It essentially concerns the logic which should support the concrete return decisions at the operational level.

The questions which were specifically addressed in this work are:
1) To which factory(ies) should a customer send its empty packages back?
2) What is the optimal average number of empty packages to hold in the reverse network on the long term?
3) How should we coordinate the flows at the different facilities?
4) What are the optimal return flow frequencies on each link of the network?
5) What are the benefits of pooling the containers between different customers or factories?
6) How should the flows of packages be managed operationally?
7) Where should we locate local accumulation depots?
8) How should the flows be coordinated at the depots?
9) How should we design a network with different types of packages?

7.1.2 *Research Methodology*

The methodology that supported our research relies on a two-level analysis. At a strategic level, we developed models for taking network design decisions whilst, at an operational level, we suggested a procedure for managing the daily return flows in the network. This work presented the fundamental strategic models in Chapter 2. Extensions to the strategic models that exploit the idea of sharing the fixed transportation costs between customers or between different types of packages were presented in Chapters 3 and 4. The operational problem was treated in Chapter 5. Chapter 6 presented a mixed integer formulation of the operational and analyzed ways to improve its formulation.

The strategic models aim at the design of the reverse network. They rely on a series of modeling assumptions that are classical at a strategic level - infinite horizon, constant average flows, continuous timeline - and on an additional assumption about the coordination of the flows, which was referred to as the *perfect coordination assumption*. It states that the return flows will be coordinated at the factories in such a way that each shipment is received at the factory exactly when its inventory drops to zero. In other words, each return flow received is immediately consumed at the factory, providing the largest rotation rate of the empty packages and, consequently, the lowest average inventory at the factory.

In the fundamental strategic models, the design includes the allocation of the customers to the factories and the determination of the optimal return frequencies. Under the assumptions used at the strategic level, the fundamental strategic models are easily solved to optimality. This results from the fact that, in most cases, the problem is separable into two subproblems that can be solved separately in some cases and sequentially in the other cases. In the lot-sizing problem, optimal frequencies are determined on each link of the network. These frequencies have been shown to be independent from the allocation of the customers and represent the optimal shipment sizes that should be applied if the link is selected in the allocation problem. Once the optimal frequencies have been determined, the allocation problem reduces to a classical transportation problem. It follows that, in most cases, we obtain analytical solutions to the problem. Only when the logistic inventory is fixed and when we simultaneously include capacity constraints on transportation do we lose this separability of the two problems. The problem remains easy to solve in a lagrangian fashion, by dualizing the constraint on the available logistic inventory and updating iteratively the associated lagrangian multiplier.

When potential local depots are considered, the additional decisions to be taken are where the depots should be located and how the customers, the depots and the factories should be connected. In this case, the objective function looses its convexity. Our method relies on the fact that the non-convex terms in the objective function originate exclusively from the variables representing the average flows at the depots. The heuristic procedures exploit the fact that, by fixing the values of these variables, we obtain a subproblem which is equivalent in its structure to the fundamental models. This subproblem can be decomposed and solved easily. This observation suggests an iterative procedure in which we first determine an initial estimate of the values of these variables, then solve the corresponding subproblem and, finally, update the estimate according the average flows at the depot implied by the solution of the subproblem.

The strategic extension in which a heterogeneous fleet of containers is analyzed generates continuous convex optimization problems, which are easily solved by any convex optimization method. Additionally, we demonstrate some properties of the problem's structure that enable us to immediately determine from the instance the logistic inventory availability constraints that will be active at optimality. The other availability constraints may be removed from the problem in a first step. In the event that only one availability constraint remains active, the problem is naturally reduced to the fundamental case and can be solved analytically.

At the operational level, we assumed that the design decisions are already taken. We suggested a procedure to take the daily decisions in the management of the return flows. At the operational level, we consequently focused on the coordination of the actual flows in the network. An important aspect of this methodology is the consistency between the two approaches that is granted by the use of the perfect coordination assumption at the strategic level. The heuristic developed for the operational problem largely relies on the solutions of the strategic models. Solving the strategic models by assuming that the operational model will aim at perfect coordination provides a reference for the network's dynamic that we exploit in the operational problem. Indeed, the heuristic will target the frequencies described by the strategic model as they represent the ideal frequencies resulting of perfect coordination. Moreover, the solutions of the strategic model were shown to be lower bounds for the operational problem when rates are constant. Using this reference and these lower bounds, we showed that, although perfect coordination can typically not be reached, the problem of finding very good solutions when the network is large and granular is not too difficult if we can find a good measure to compare the different alternatives with. The portfolio heuristic suggested is based on such a measure, the portfolio value, which represents the future cost associated with the existence of an inventory at a customer. Using this measure allows us to compare the customers adequately when selecting the inventories to be returned. This selection is made in a myopic manner every time shipments are needed.

7.2 OVERALL CONTRIBUTIONS

7.2.1 *Managerial Contributions*

Knowledge about the dominant economic logics

The context in which the research was carried out implied the need to identify the dominant economic logics at stake in the design and the management of the return flows of empty packages. These logics had to be described in a simple and intuitive way in order to be appropriated by the actors of the industry. Four dominant logics were underlined in this study.

The first logic is that of lot-sizing. This logic is based on the impact of the return frequencies on the cost of the system. The essence of the lot-sizing logic is to relate both the fixed transportation and the linear holding costs in the return frequency decision. The objective is to achieve a long term balance between these costs.

The second logic relates to how the allocation of the customers to the factories should be performed. Under the condition that a certain level of substitutability is achieved in the fleet of packages, there will be some flexibility in the choice of the production point(s) to which a customer is allocated. The idea would naturally be to allocate customers in order to reduce the average distances that are traveled in the network. However, flow balance and substitutability rules will have to be taken into account in this allocation process.

The third logic is the sharing of fixed transportation costs. In this study, two extensions were presented which make use of this idea. The first extension illustrated how sharing transportation fixed costs could be done by the local consolidation of customers' inventories. This was modeled by the use of intermediary depots. The benefits of this local consolidation will appear at two levels. First, annual transportation costs will decrease as a consequence of the better truck utilization. Second, annual holding costs will decrease due to the larger rate at which packages accumulate at the depots. These benefits should be compared with the additional costs generated by the implementation of a two-layers network. A second extension analyzed how fixed transportation costs could be shared between the return flows of different types of packages.

The fourth logic is the coordination of the flows. The way the return flows are coordinated will have an important impact on the average inventory of the factories and of the depots. Achieving a "good" coordination will allow the empty packages to be used quickly after they have been returned to a production point. Ideally, each shipment of empty packages should immediately be used at the factories.

Decision supporting models

In addition, models were developed to solve the strategic and operational problems which implement these economic logics. Most importantly, we demonstrate that most models solve easily to optimality. For the most difficult problems, we presented efficient heuristics. The models presented are consequently adapted to the industrial instances' sizes. In Chapter 3, we demonstrated how the strategic model could be tested on the industrial data provided by AGC. The model proved to be successful in treating industrial data and in generating useful lessons on the design of the reverse network. They appeared to be adequate for providing solutions on all design decisions and in testing variations of the retained assumptions and of the various parameters.

Managerial lessons

The models also generate a series of immediate managerial lessons.

Fleet of packages - Strategically, evaluating how many packages should be held in the network is a major issue faced by the logistic managers. The design and the management of the reverse network only impacts a part of the fleet of packages, which we named the logistic inventory. The logistic inventory represents the number of reusable packages that are needed, in addition to the packages used in the production, storage and delivery processes, for implementing the reverse strategy. We demonstrate that finding the adequate size of the logistic inventory was a rather easy problem. Indeed, if we aim at finding the optimal long term fleet size, we apply our fundamental strategic models with the assumption that the logistic inventory is a variable. We showed that the models were particularly easy to solve in this case, as the shipment sizes are determined independently from each other by a simple EOQ-like computation. Capacity constraints are also easily taken into account. If the shipment size calculated by the EOQ-like formula is larger than the transport capacity, it is simply reduced to the capacity limit.

When facing a fleet with different types of packages sharing the same transportation means in a single factory network, we also easily derive the fundamental managerial lessons. To compute the optimal number of empty packages of each type to hold in the network, it is sufficient to compute the optimal shipment size of each customer independently, without taking the existence of different types of packages into account. In other words, we compute the optimal shipment sizes as if there was only one type of packages. Then, summing up adequately these shipment sizes provides the optimal fleet size and structure. Moreover, we proved that, if the investment required to achieve to optimal fleet size cannot be made, this optimal structure of fleet provides the lowest cost for any level of investment. In other terms, the optimal proportions between the number of the packages of each type should always be preserved.

Additionnally, we demonstrate that, in a network that is sufficiently large, granular and in which the flows of the different types of packages are distributed in a relatively uniform manner, if the number of the different types of packages is limited, we can expect to achieve a total cost that is very close to the total cost computed for a standardized fleet. This results requires that the composition of the fleet of packages corresponds to the optimal composition derived in the strategic model for a heterogeneous fleet.

Finally, due the EOQ-structure embedded in our models, the characteristical flatness of the total cost function around the optimal fleet size was observed in our solutions. This is an important factor from a managerial point of view. It provides a relative robustness of the fleet size recommendation against errors in parameters estimation or to variations in the network. It also indicates that the negative impact of an oversized fleet is smaller than the negative impact of an undersized fleet.

Pooling and sharing - A central question in our study related to the benefit of sharing packages between the customers and/or factories. Pooling the fleet of packages requires that the packages are substitutable. In this case, the benefit provided by the sharing of the fleet lies in the fact that factories only have to hold a central inventory of empty packages. This reduces the average inventory at the factories when the return flows are adequately coordinated. As a consequence, less packages are needed in the network.

When comparing the case where each customer uses a specific type of packages with the case where the fleet of packages is completely standardized, we proved that the total benefit of sharing almost amounts to 30% of the total cost (transportation and holding costs) in a large network. If the size of the fleet cannot be adapted, sharing the packages still provides an annual transportation cost reduction which almost reaches

50%. If capacity constraints completely limit the size of the shipments, sharing the fleet generates an annual holding cost reduction that is almost 50%. Intermediary situations where the fleet is only partially standardized will similarily generate sharing benefits. Each specific intermediary situation will have to be evaluated to determine the extent of these benefits.

Sharing the fleet of packages between factories requires the packages to be substitutable between the factories so that packages sent to a customer by a factory may be returned to another factory. In this case, an allocation of the customers to the different factories with respect to their return flows may be considered.

Allocation - Let us assume that the fleet is sufficiently standardized so as to enable to direct the return flows from the customers to any of the factories. In this case, the allocation of the customers to the factories in the forward network must not necessarily be identical to the allocation used in the reverse network. The idea of the re-allocation of the customers in the reverse network is to minimize the average distances that are traveled. Intuitively, we will try to allocate the customers to the closest factory. However, we must ensure that a sufficient number of customers is allocated to each factory to cover its consumption of empty packages. In our models, this proved to be a simple linear transportation problem. The exact benefit of this allocation depends on the allocation in the forward network of each specific instance. It could be that this allocation is already optimal for the reverse network.

An important lesson is that the natural tendency of managers to divide the network into separated clusters is perfectly acceptable in this case. Theoretically, we observed that some customers are allocated to more than one factory in the strategic models. However, we showed that only a few customers could fall in this category. Practically, the allocation model creates almost independent clusters and we will arbitrarily assign these few customers belonging to more than one cluster. Balancing the flows between the clusters could be made by special shipments from time to time.

Depots - The idea of sharing transportation fixed costs between neighboring customers was analyzed through the possibility of establishing local accumulation depots. The benefits generated by the use of depots appear at two levels. First, the number of shipments originating from a region where a depot is located will decrease, since the customers will share the same truck to send their units back. Second, the rate at which the shipment size is built will be larger at a depot than at individual customers. This will reduce the average inventory held locally.

Designing the complete two-layers network is a difficult problem that we tackled with a heuristic. However, we identified the economic factors on which the use of intermediary depots relies. Savings are generated by the use of depots because the shipment sizes used from a depot to a factory will be larger than those from a customer to a factory. Less long-distance shipments will be needed. Additionnally, the shipment size used from a depot to a factory is built faster than the shipment sizes from individual customers. Less packages are thus needed in the network. But, using a depot also implies additional costs. These costs consist of i) the local transportation costs between the customers and the depots, ii) the cost of opening the depots and iii) the handling costs at the depot. Before solving the complete problem, one could have a good idea of the relevance of a network with depots by computing the impact of these factors on certain illustrative local clusters of customers.

Coordination - An important managerial contribution is that the decision process that should support the daily management of the return flows is simple and intuitive. First, if no replenishment of the factory's inventory of empty packages is required, then no return flow should be performed. Second, if return flows must be performed, we must give priority to the most economical customers. The first affirmation is straightforward. Inventories left at the customers' are subject to further increase. Larger inventories imply better truck utilization. The second affirmation is also quite natural. It requires, however, a measure of the economical weight of a customer's inventory. Although the measure we suggested, the portfolio value, is simple, it proved to be very efficient.

This measure is an economical value that may be interpreted as the implicit cost of returning the inventory of a customer. For each customer the portfolio value corresponds to the cost that is incurred with certainty if the return flow is performed (the fixed transportation cost) from which we subtract the implicit cost carried by the inventory (the fixed transportation cost multiplied by the ratio between its actual inventory and its expected shipment size). The expected shipment size was determined at the strategic level by considering the global characteristics of the whole network, i.e. all rates, all costs and the available logistic inventory. Linking an economic value to the decision of returning the inventory of a customer provides the managers with a better way to evaluate the impact of their decisions.

Moreover, the portfolio heuristic is particularly well suited to deal with variable flows. It has the flexibility to adapt to the actual flows that are observed in the network and has the strength to exploit the size of the network. Theoretically, the variation of the flows could either cause the increase or the decrease of the operational cost that can be achieved. The portfolio heuristic proved to be efficient to take advantage of the variations when they impact the network favorably.

Finally, the portfolio heuristic is simple to apply. It requires no elaborated information system and is not likely to monopolize many management efforts. This simplicity is generally expected for the management of secondary activities.

Planning periodicity - The periodicity of the shipment decisions impacts the total cost of the system. Intuitively, if shipments can only be ordered once a week, the inventory level at the factory at the beginning of each week must cover the weekly needs of the factory. This increases the average inventory at the factory. We indicate that the cost effect of the length of the planning periods could be evaluated easily based on a lower bounds comparison. In order for a periodic review policy to preserve the total transportation cost achieved with a continuous policy, a number of additional reusable packages must be held in the network. This number corresponds to half of the average consumption of the factory during a planning period.

7.2.2 *Methodological Contributions*

Perfect coordination assumption

The perfect coordination assumption was the central pillar of our methodology. It is an assumption about how the flow will be coordinated at the operational level. We have highlighted the fact that the coordination problem itself was a hard combinatorial problem. The underlying methodological choice made in our research was to focus on large networks with a high granularity. In these networks, the combinatorial problem itself is untractable. Our ambition was not to tackle the combinatorial problem itself, which was largely studied in the SRMIS literature. Instead, our methodology is based on our ability to approximate the behavior of the solutions of the coordination problem in large granular network. In this sense, the perfect coordination assumption must be seen as a smooth approximation of the coordination.
This perfect coordination assumption was the main strength of our methodology as it appeared
1) to be a very intuitive and natural assumption about the operational management of the return flows (Indeed, in operational management, it would be perfectly natural to delay each return flow until it is needed at the factory, because each inventory left in the network may only grow bigger and reduce the unit transportation cost. More importantly, simply delaying the return flows until they are necessary is sufficient to provide coordination patterns which are close to perfect coordination);
2) to be very reasonable (We were able to show that, in the operational problem, we can easily obtain solutions of which the cost is very close to the cost computed under

the perfect coordination assumption); and
3) to render the fundamental strategic problems easy to solve (Without the perfect coordination assumption, we would have relied on numerical heuristic procedures at the strategic level. With this assumption, we have the opportunity to solve our strategic models exactly and, most of the time, analytically.). This allows us to derive important insights about the problem structure.

It is the perfect coordination that connects the two levels of analysis. At a strategic level, we use this assumption to model the behavior of the flows at the operational level. Because we expounded that this assumption represents a best case scenario for the coordination of the flows at the operational level, the cost of the solutions of the strategic models provide a lower bound for the cost of the operational model in its continuous version with constant rates. The derived lower bound for the discrete version of the operational problem results from an extension of the perfect coordination assumption in the discrete case. Additionally, the solutions of the strategic model embodied a reference of an ideal network. The portfolio heuristic largely relies on this reference, as it aims at reproducing the same economic equilibrium that is depicted by the strategic models.

Fundamental strategic models

Using the perfect coordination assumption, we were able to easily solve our fundamental strategic models to optimality. In many cases, analytical solutions were derived which allowed us to generate the main managerial insights of the study.

We demonstrate that, in most cases, the strategic models decompose into two independent problems: the lot-sizing problem in which the shipment sizes are determined and the allocation problem in which the allocation of the customers to the factories is decided. The independence of the shipment sizes on the actual routing of the flows is an important result. It means that, without knowing the number of packages that a customer will send on average to a specific factory, we are able to determine the shipment size that will be used on this link of the network. This independence means that, since the shipment size is fixed, we can derive a unit transportation cost for the flow that will be allocated to this link of the network in the allocation problem. Somehow, the fixed transportation costs are turned into variable transportation costs. It follows that the allocation problem becomes linear and is solved as an instance of the transportation problem. This decomposition remains applicable when the logistic inventory is fixed or when capacity constraints are considered. However, the two problems become dependent when the logistic inventory is fixed and capacity constraints are included simultaneously. This case was solved in a lagrangian fashion by dualizing the logistic inventory availability constraint into the objective function. The subproblem is similar to the strategic problem with capacity constraints in which the logistic inventory is variable, except that the holding cost was replaced by the lagrangian multiplier. The rapid convergence of the procedure is ensured by the convexity of the problem.

Depot location model

The strategic model in which depots are to be located and allocated to the factories has a non-convex objective function. We presented a formulation of the problem which avoids the use of binary variable, and consequently allows us to obtain a continuous problem. Additionnally, this formulation is close to a linear problem. Indeed, by fixing the values of the depot flow variables, we obtain a linear problem similar in its structure to the fundamental strategic models.

Portfolio heuristic

The portfolio heuristic makes use of the reference embodied in the solution of the strategic model. We established that aiming at the perfect coordination at the operational level can be done by a procedure relying on two principles. First, return flows should only be performed when a replenishment of empty packages is necessary. This first principle ensures that the shipments are rapidly used at the factory. The second principle states that, when return flows must be carried out, the choice of the candidates should rely on the economic impact of bringing back their inventory of empty packages.

In order to compare these economic impacts, we developed a measure: the portfolio value. The portfolio value may be interpreted as the implicit cost of bringing back an inventory. This measure indirectly uses the information about the characteristics of the network because it relies on the optimal shipment sizes determined at the strategic level. Since these optimal shipment sizes represent the shipment sizes that would be used in a perfectly coordinated network, they are considered as the expected shipment sizes.

The fact that the measure represents an economic value, instead of an abstract value, is a requirement to compare the alternative return flows that could be performed.

In the discrete version of the problem, these portfolio values are compared in a 0-1 knapsack problem.

The portfolio heuristic is particularly robust when facing variable flows. This seems natural since variability does not impact the general strategic equilibrium of the network, nor its granularity, which are the factors on which the performance of the heuristic relies. Moreover, we illustrated that variability may have a beneficial impact on the operational cost that may be achieved. Thanks to its flexibility to select the best candidate(s) based on the current inventory situation of the network, the heuristic takes advantage of this beneficial aspect.

7.3 CONCLUDING COMMENTS AND PERSPECTIVES

To conclude this thesis, we discuss several perspectives and a series of future developments that have emerged at the different stages of our research. First, numerous extensions, refinements and complements could be added to the research itself and integrated in the presented models. We restrict the discussion to a number of future developments that, in our opinion, are the most promising to deliver additional knowledge about the research topic. Second, beyond the topic itself, the methodology that we used underlines the type of problems which fall under the scope of our models. We discuss the implications of our methodology and the ambition of our models to provide insights about perspectives for their applicability to contexts other than the management of the return flows of packages.

7.3.1 *Perspectives Emerging from the Research Topic*

First, the models that we presented could be certainly extended in a number of ways. The objective was to tackle the fundamental drivers in the design and the management of the reverse network. Based on these fundamental models, additional configurations, constraints or objectives could be considered. Amongst these extensions, we believe that the following would be of particular interest in gaining more knowledge about our research topic.

Vehicle tours

The logic of consolidating some customers, at the expense of some additional costs (detours, handling,...), was analyzed in this research and an important methodological choice was to use intermediary depots to model the logic. There are, however, other ways of modeling this aggregation. Besides the use of an intermediary layer, a second very common approach is to design tours for the vehicles to visit several customers. This is the second dominant way of modeling local aggregation. Because the impact on the models would be important, we believe that exploring how to model vehicle tours would be an important addition to the knowledge gained in our work.

Operational problem with a heterogeneous fleet

In the case of a heterogeneous fleet of packages, two important comments should be made. First, if the application of the perfect coordination assumption allows us to keep a tractable strategic problem when different types of packages make up the fleet, it implies a stronger simplification of the operational level. Indeed, this assumption states that all the packages of each type of which a shipment consists, are immediately consumed at the factory. The consequence of this assumption is that, every time a shipment is performed from a customer site, all packages available at that customer are loaded in the truck. This is a strong assumption, as it is likely that the shipment will be performed because of a shortage at the factory for one type of packages. The other types shipped at that time will probably not be consumed immediately. It follows that the operational heuristic should include this element when selecting the best candidate. A good candidate should have many packages of the type that is needed at the factory and few of the other types. In other words, the structure of the inventory of the candidate should be compared with the structure of the inventory at the factory. Second, although it would be easy to observe that location-dependent holding costs do not affect the complexity of the models in all the other cases presented, with a heterogeneous fleet, it will force us to reconsider the affirmation that all packages of a customer's inventory are always returned together. Leaving unneeded packages at a customer could be preferable if the holding cost is larger at the factory than at the customer. Integrating these two elements in the portfolio heuristic would require a careful analysis of their impacts on the equilibrium of the system.

Improvement of the portfolio heuristic

In the general case with a standard type of packages, we applied the portfolio heuristic to the cluster of each factory independently. This is a natural way of tackling the operational from a managerial perspective, which is supported by the fact that we showed that the number of customers that could be allocated to more than one factory was limited to $F - 1$. Dealing with these few customers belonging to more than one cluster would merely require a simple adaptation of the heuristic. The results are satisfactory when compared to the global lower bound on the whole network. However, when the flows are variable, we demonstrated that the portfolio heuristic takes advantage of the flexibility of the choice of the best customer after having observed the realization of the random variables representing the availability rates. This selection process implements an order statistics and increases the expectation of the average shipped quantities. In this variable case, increasing the flexibility of the heuristic would reinforce this beneficial effect. Thus, it would be worth considering allowing the factories to select customers which are strategically outside of their cluster. We could, for example, design regions at the intersection of the clusters in which the customers may be selected by more than one factory. This should allow a cluster in an unfavorable position to benefit from customers from another cluster which would happen to be in a favorable position. This would at least offer the same performance as purely fixed clusters, and could be of much interest when the clusters appear to be relatively small.

The idea of the portfolio heuristic is to attempt to mimic a general equilibrium of the ideal network provided by the strategic model. As such, it fundamentally relies on the relevance of the average rates. If we were to be confronted with an environment with arbitrary flows, or unknown average rates, additional information would be necessary regarding forecasted rates in the operational planning horizon. These forecasts could be used to derive a local equilibrium that is constantly updated. An alternative would be to consider the mixed integer formulation of the problem. In Chapter 6, we used this formulation to tackle the combinatorial aspect of the coordination problem. But we believe that MIP based heuristics, such as rolling horizon procedures, could be efficient for finding good solutions when we have forecasts of arbitrary flows.

Implementation in industrial contexts

Finally, interesting lessons would result from the implementation of the models on actual cases. Thanks to the TransLogisTIC project, we had the opportunity to test some of the models presented on the data from our industrial partner and we got to obtain feedback about the relevance of the lessons we presented to their logistic management. Real cases bear many specificities, but some of them are likely to be more widespread. For example, it is to be expected that actual networks will be changing over time with the addition or removal of customers and with varying rates. A complete reevaluation of the network will only be considered periodically. In practice, we would have to derive simple rules for adapting an operating network dynamically. Demand patterns, such as trends or seasonalities, are also very likely to appear in practice. In this case, we should ask ourselves whether to determine different equilibriums or to design a network which will perform well throughout the varying demand patterns.

7.3.2 *Perspectives Emerging from the Research Methodology*

The assumptions and the methodology that we have used outline the ambition and the limitations of our research.

We largely relied on the perfect coordination assumption. This assumption appeared to be essential since a hard combinatorial problem was embedded in the problem we addressed. In a certain way, this perfect coordination assumption can be seen as a smooth approximation of the coordination problem. Using this assumption somehow presupposes that we consider an environment where we can go beyond the combinatorial aspect of the problem. We do not actually aim at solving the combinatorial problem, but instead we strive to approximate its behavior. This immediately draws the boundaries of the problems that we handle. We focus on networks with a high granularity. Small networks or networks with large individual players, in which the combinatorial aspect of the coordination problem dominates are outside the scope of our models. However, we observed that the conditions under which the smooth approximation of the coordination problem is reasonable were quite easily achieved. Once the scope of the models has been clearly underlined by considering the perfect coordination assumption, it seems natural that our portfolio heuristic performs well when the granularity condition is achieved, because it basically tries to implement a smooth solution that remains good beyond the combinatorial aspect of the problem. The robustness to variable flows is also explained by this fit of the heuristic to a smooth environment. Indeed, the fact that the flows become variable does not affect the smoothness of the network, which is the crucial condition for the performance of the heuristic.

Our methodology also fundamentally relies on average rates. Even if various implementation techniques could be used to deal with varying rates, changing networks or demand patterns, the fundamental ambition of the models is not to tackle networks with completely arbitrary flows but to derive a design which is an equilibrium between long term flows and costs. Nevertheless, based on the characterization of this equilibrium, we believe that it would possible to deal with incomplete, uncertain or missing data as long as the network can be smoothly approximated by deriving some simple rules of thumb.

A central decision in the presented models is the choice of the return frequencies, that is the lot-sizing decision. The models are consequently intended for tackling networks in which the lot-sizing logic is not predetermined by the parameters. For example, tight capacity constraints coupled with low holding costs would lead to impose full load transportation from the start. In this case, the problems we analyzed are reduced to more classical problems for which specific methods have been developed. Without depots, our strategic problem would be seen as a classical transportation problem, and with depots it would be reduced to a type of facility location problem.

Exploring whether these methodological principles that define the scope of our models could apply to other contexts is our primary objective for further research. A natural candidate would be distribution management as it has been stressed that models in reverse logistics share most of their characteristics with models in distribution. In this case, the models would consider goods instead of empty packages. Even further, if the characteristics of the forward and reverse networks are sufficiently close, the models could be used for the design of an integrated network for delivering goods and returning empty packages with the same trucks, the same depots and the same logic.

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