ADMM-based Subcarrier Allocation and Power Control for Uplink OFDMA Channels

Zijian Wang†, Yuting Mou*, and Luc Vandendorpe†
†Institute of Information and Communication Technologies, Electronics and Applied Mathematics
Université catholique de Louvain, Louvain-la-Neuve, Belgium
* Center for Operations Research and Econometrics, Université catholique de Louvain, Louvain-la-Neuve, Belgium
Emails: {zijian.wang; yuting.mou; luc.vandendorpe}@uclouvain.be

***Abstract*—In this paper, the problem of subcarrier allocation and power control for uplink orthogonal frequency division multiple access (OFDMA) channels is studied. Each user may have a different strategy of power control due to his/her own limits of battery, quality-of-service (QoS) requirement, and the amount of information to transmit. The problem is first formulated as a minimization problem, and the alternating direction method of multipliers (ADMM) is then introduced to solve the problem in a distributed manner. Necessary conditions of each subproblem are derived to obtain a deeper understanding in the structure of the solutions. Numerical results show that the algorithm converges fast and obtains a near-optimal solution within only a few iterations.**

I. Introduction

Nowadays, heterogeneity characterizes a variety of network aspects: user condition, devices, services, etc. On another hand, energy consumption is a concern, leading users to target either power minimization, energy efficiency (EE) maximization, or rate maximization depending on the desired information rate and the available power budget [1]. Out of these criteria, EE maximization is the most complex one, and power minimization and rate maximization can be viewed as special cases [2].

Uplink orthogonal frequency division multiple access (OFDMA) channels aiming at maximizing individual EE were studied in papers [3]–[9]. The transmit power of each user was optimized by formulating a game theoretic problem in paper [3]. Since the problem was challenging, it was assumed that only one subcarrier is allocated to one user. Centralized algorithms were studied in papers [4]–[9]. Subcarriers were iteratively allocated to the user with the maximum EE increase in paper [4]–[6], or to the user with the maximum increase of throughput in paper [7]. For a max-min problem, subcarriers were iteratively allocated to the user with the minimum EE in paper [8]. Subcarriers were iteratively or simultaneously allocated to the user with the maximum value of the first order derivative of the Lagrangian function in papers [9].

Contrary to the centralized algorithms, distributed algorithms are more advantageous in terms of decoupling variables, parallel computing, and flexibility, etc. The alternating direction method of multipliers (ADMM) has been shown to be superior to solve distributed problems [10] and was exploited for resource allocation in wireless networks in papers [11]–[15]. Beamformings were optimized to maximize the sum EE in papers [11], [12]. In papers [13] and [14], power allocation was respectively optimized with a wireless power transfer setup and a simultaneous wireless information and power transfer (SWIPT) setup. Power allocation was optimized by separating constraints into several groups using ADMM in paper [15].

In this paper, multi-user OFDMA uplink channels are studied, where each user operates in one of the three transmission modes: power minimization, EE maximization, and rate maximization. To the best of our knowledge, no work has considered this model in the literature. Related considerations can however be found in [16]–[18]. Paper [16] considered a multi-relay-aided cluster downlink and optimized user selection and power allocation with the help of a multi-objective (power and throughput) framework. Paper [17] considered the optimization of power and channel allocation in a two-tier OFDMA downlink, also with the help of the multi-objective (two objectives out of network power, EE, and spectral efficiency (SE)) framework. In paper [18], a game theoretic approach was proposed to study a network with heterogeneous users' objectives.

In the current paper, we propose an ADMM-based subcarrier allocation algorithm. The advantages of ADMM are parallel optimization with low complexity and fast convergence, and keeping the distributed objective functions confidential and flexible. Our analysis and numerical results will show these properties.

II. System model and problem formulation

Let us consider an uplink OFDMA system where M users communicate with one access point (AP), over a wideband channel which is divided into K subcarriers. All devices are assumed to have a single antenna. $\rho_{k,m} \in \{0,1\}$ is the indicator of subcarrier allocation where $\rho_{k,m} = 1$ (resp. $= 0$) implies that subcarrier k is (resp. is not) allocated to user m. Each subcarrier is allocated to one user, therefore there exists no interference among users. $h_{k,m} = \hat{h}_{k,m}/(\sigma_k\sqrt{\tau_m})$ is the normalized channel between user m and the AP at subcarrier k, where $\hat{h}_{k,m}$ is the Rayleigh fading channel including the link budget, τ_m is the SNR gap depending on the modulation and coding scheme, and σ_k^2 is the power of noise of subcarrier k at the receiver.

Denoting $q_{k,m}$ as the power allocated to the symbol to be transmitted by user m at subcarrier k, the transmit power and the information rate of user m in the uplink channel are respectively

$$P_m = \sum_{k=1}^{K} \rho_{k,m} q_{k,m} \tag{1}$$

$$R_m = \sum_{k=1}^{K} \rho_{k,m} \log_2 \left(1 + |h_{k,m}|^2 q_{k,m}\right). \tag{2}$$

The users are divided into three groups depending on their individual requirements. The users in group $\mathcal{M}_1$ aim at minimizing their individual transmit power, the users in group $\mathcal{M}_2$ aim at maximizing their individual EE, and the users in group $\mathcal{M}_3$ aim at maximizing their individual information rate. The optimization concerning all users can be formulated as a minimization problem which is expressed as:

$$\begin{aligned}\min_{\{q_{k,m}\},\{\rho_{k,m}\}} \quad & \sum_{m\in\mathcal{M}_1} w_m(\phi_m P_m + Q_m) \\ & - \sum_{m\in\mathcal{M}_2} w_m \frac{R_m}{\phi_m P_m + Q_m} - \sum_{m\in\mathcal{M}_3} w_m R_m \end{aligned} \tag{3}$$

$$s.t. \quad P_m \le \bar{P}_m, \quad 1 \le m \le M \tag{4}$$

$$R_m \ge \bar{R}_m, \quad 1 \le m \le M \tag{5}$$

$$\sum_{m=1}^{M} \rho_{k,m} = 1, \quad 1 \le k \le K, \tag{6}$$

where w_m is the weight of user m, ϕ_m is the inverse of the efficiency of amplifier of user m, $\bar{P}_m$ is the transmit power limit, $\bar{R}_m$ is quality-of-service (QoS) in terms of the minimal rate and Q_m is the circuit power of user m.

III. Proposed ADMM-based method

In this section, we solve the minimization problem formulated in the previous section. Two kinds of ADMM-based methods are introduced.

A. Problem reformulation

We propose to use ADMM methods to solve the distributed problem. Because the convergence of ADMM is guaranteed only for convex problems [10], [19], we first reformulate the problem to a convex one.

To this end, we rewrite the expressions and relax the subcarrier allocation constraint. Observe that (1) and (2) are not jointly convex w.r.t. $q_{k,m}$ and $\rho_{k,m}$. To solve this, let us introduce a new vector $\mathbf{p} \in \mathbb{R}_{\ge 0}^{MK\times 1}$, whose $((m-1)K+k)$-th element is $p_{k,m} = \rho_{k,m} q_{k,m}$. Then, we have

$$P_m = \sum_{k=1}^{K} p_{k,m} \tag{7}$$

$$R_m = \sum_{k=1}^{K} \rho_{k,m} \log_2 \left(1 + |h_{k,m}|^2 \frac{p_{k,m}}{\rho_{k,m}}\right), \tag{8}$$

where (7) is affine and (8) is concave w.r.t. $\mathbf{p}$ and $\boldsymbol{\rho}$ [20]. Denote the objective function in (3) as $f(\mathbf{p}, \boldsymbol{\rho})$, where the $((m-1)K+k)$-th element of $\boldsymbol{\rho}$ is $\rho_{k,m}$. Obviously, $f(\mathbf{p}, \boldsymbol{\rho})$ is jointly quasi-convex w.r.t. $\mathbf{p}$ and $\boldsymbol{\rho}$. The constraint of subcarrier allocation can be converted to be convex by introducing time-sharing such that $\rho_{k,m} \in \{0,1\}$ becomes $\rho_{k,m} \in [0,1]$. In this way, the problem becomes convex.

Please note that we will consider both $\rho_{k,m} \in \{0,1\}$ and $\rho_{k,m} \in [0,1]$ in the following since $\rho_{k,m} \in \{0,1\}$ produces the accurate solution and $\rho_{k,m} \in [0,1]$ obtains a lower bound of the objective function. The problem with $\rho_{k,m} \in \{0,1\}$ can be solved in a way quite similar to the problem with $\rho_{k,m} \in [0,1]$, although the former one is not guaranteed to converge theoretically.

B. An ADMM-based method

By introducing a new variable $\mathbf{z}$ in ADMM, we reformulate the problem as

$$\min_{\mathbf{p},\boldsymbol{\rho},\mathbf{z}} \quad f(\mathbf{p}, \boldsymbol{\rho}) + s_i(\mathbf{z}) \tag{9}$$

$$s.t. \quad (4)(5), \tag{10}$$

$$\boldsymbol{\rho} = \mathbf{z}, \tag{11}$$

where $i = 1$ or 2 depending on whether the $\rho_{k,m}$ should be discrete or continuous, and $s_i(\mathbf{z}) = \{0, +\infty\}$ indicates whether $\mathbf{z}$ belongs or not to set $\mathcal{C}_i$, with

$$\mathcal{C}_1 = \left\{ z_{k,m} \in \{0,1\} \mid \sum_{m=1}^{M} z_{k,m} = 1, \quad 1 \le k \le K \right\} \tag{12}$$

$$\mathcal{C}_2 = \left\{ z_{k,m} \in [0,1] \mid \sum_{m=1}^{M} z_{k,m} = 1, \quad 1 \le k \le K \right\}. \tag{13}$$

The augmented Lagrangian (using the scaled dual variable) is [10]

$$L_\theta(\mathbf{p}, \boldsymbol{\rho}, \mathbf{z}, \mathbf{u}) = f(\mathbf{p}, \boldsymbol{\rho}) + s_i(\mathbf{z}) + \frac{\theta}{2}\|\boldsymbol{\rho} - \mathbf{z} + \mathbf{u}\|_2^2, \tag{14}$$

so the scaled form of ADMM for this problem is [10]

$$\mathbf{p}^{j+1}, \boldsymbol{\rho}^{j+1} := \operatorname*{argmin}_{\mathbf{p},\boldsymbol{\rho}\in(4),(5)} \left(f(\mathbf{p}, \boldsymbol{\rho}) + \frac{\theta}{2}\|\boldsymbol{\rho} - \mathbf{z}^j + \mathbf{u}^j\|_2^2 \right) \tag{15}$$

$$\mathbf{z}^{j+1} := \operatorname*{argmin}_{\mathbf{z}\in\mathcal{C}_i} \|\boldsymbol{\rho}^{j+1} - \mathbf{z} + \mathbf{u}^j\|_2^2 \tag{16}$$

$$\mathbf{u}^{j+1} := \mathbf{u}^j + \boldsymbol{\rho}^{j+1} - \mathbf{z}^{j+1}. \tag{17}$$

Please note that $\boldsymbol{\rho}$ in (15) can be any non-negative real number. The constraints (6), and $\rho_{k,m} \in \{0,1\}$ or $\rho_{k,m} \in [0,1]$ are however transferred to $\mathbf{z}$ as indicated by (16).

In the following, we will consider both $\mathcal{C}_1$ and $\mathcal{C}_2$ to update $\mathbf{z}$, and refer to them respectively as 'discrete $\mathbf{z}$' and 'continuous $\mathbf{z}$'. Obviously, the optimum with continuous $\mathbf{z}$ can be viewed as a lower bound of that of discrete $\mathbf{z}$. This will be validated by the numerical results. Keep in mind that as the algorithms iterate, $\boldsymbol{\rho}$ approaches to $\mathbf{z}$. The solution is the global optimum if $\boldsymbol{\rho}$ equals to $\mathbf{z}$ [10].

Let us introduce a group indicator $\delta_m(\mathcal{M})$ for each user m where $\delta_m(\mathcal{M}) = 1$ if user m belongs to group $\mathcal{M}$ and

$\delta_m(\mathcal{M}) = 0$ if user m does not belong to group $\mathcal{M}$. Then we have $f(\mathbf{p}, \boldsymbol{\rho}) = \sum_{m=1}^{M} f_m$, where

$$f_m = w_m \left(\delta_m(\mathcal{M}_1)(\phi_m P_m + Q_m) \right. \\ \left. -\delta_m(\mathcal{M}_2) \frac{R_m}{(\phi_m P_m + Q_m)} - \delta_m(\mathcal{M}_3) R_m \right). \quad (18)$$

Therefore, in the $j+1$-th iteration in (15), the user m

$$\mathbf{p}_{(m)}^{j+1}, \boldsymbol{\rho}_{(m)}^{j+1} := \underset{\mathbf{p}_{(m)}, \boldsymbol{\rho}_{(m)}}{\operatorname{argmin}} \left(f_m + \frac{\theta}{2} \|\boldsymbol{\rho}_{(m)} - \mathbf{z}_{(m)}^j + \mathbf{u}_{(m)}^j\|_2^2 \right), \quad (19)$$

where $\mathbf{p}_{(m)}$ (resp. $\boldsymbol{\rho}_{(m)}$, $\mathbf{z}_{(m)}$, $\mathbf{u}_{(m)}$) is the m-th K entries of $\mathbf{p}$ (resp. $\boldsymbol{\rho}$, $\mathbf{z}$, $\mathbf{u}$).

One can clearly observe that each user m will minimize his/her own local function f_m without informing other users or the AP any parameters or configurations inside f_m. This keeps local data confidential and provides flexibility of changing the transmission modes.

IV. Solutions of Subproblems

In this section, the solution of each subproblem is derived. In the user subproblem, we assume that the problem is always feasible. Feasibility discussions are beyond the scope of this paper.

A. Subproblem 1

If $\delta_m(\mathcal{M}_1) = 1$, the problem becomes

$$\min_{\mathbf{p}_{(m)}, \boldsymbol{\rho}_{(m)}} \quad \sum_{k=1}^{K} w_m \phi_m p_{k,m} + \frac{\theta}{2} \|\boldsymbol{\rho}_{(m)} - \mathbf{z}_{(m)}^j + \mathbf{u}_{(m)}^j\|_2^2 \quad (20)$$

$$s.t. \quad \sum_{k=1}^{K} p_{k,m} \leq \bar{P}_m \quad (21)$$

$$\sum_{k=1}^{K} \rho_{k,m} \log_2 \left(1 + |h_{k,m}|^2 p_{k,m}/\rho_{k,m}\right) \geq \bar{R}_m, \quad (22)$$

which is a convex problem. Under the assumption that the problem is feasible, the constraint (21) is redundant. The Lagrangian is written as

$$L(p_{k,m}, \rho_{k,m}, \lambda_1) = w_m \phi_m \sum_{k=1}^{K} p_{k,m} \\ + \frac{\theta}{2} \sum_{k=1}^{K} \left(\rho_{k,m} - z_{k,m}^j + u_{k,m}^j\right)^2 \\ + \lambda_1 \left(\bar{R}_m - \sum_{k=1}^{K} \rho_{k,m} \log_2 \left(1 + |h_{k,m}|^2 p_{k,m}/\rho_{k,m}\right) \right). \quad (23)$$

The KKT conditions are:

$$q_{k,m} = \left(\frac{\lambda_1}{w_m \phi_m \cdot \ln 2} - \frac{1}{|h_{k,m}|^2} \right)^+, \quad (24)$$

$$\rho_{k,m} - z_{k,m}^j + u_{k,m}^j = \frac{\lambda_1}{\theta \ln 2} A\left(|h_{k,m}|^2 q_{k,m}\right), \quad (25)$$

where x^+ is defined as $\max\{x, 0\}$ and $A(x) = \ln(1+x) - \frac{x}{1+x}$. The solution can be found by finding λ_1 such that (22) is satisfied with equality.

B. Subproblem 2

If $\delta_m(\mathcal{M}_2) = 1$, the problem becomes

$$\min_{\mathbf{p}_{(m)}, \boldsymbol{\rho}_{(m)}} \quad - w_m \frac{\sum_{k=1}^{K} \rho_{k,m} \log_2 \left(1 + |h_{k,m}|^2 p_{k,m}/\rho_{k,m}\right)}{\phi_m \sum_{k=1}^{K} p_{k,m} + Q_m} \\ + \frac{\theta}{2} \|\boldsymbol{\rho}_{(m)} - \mathbf{z}_{(m)}^j + \mathbf{u}_{(m)}^j\|_2^2 \quad (26)$$

$$s.t. \quad (21)(22), \quad (27)$$

which is jointly quasi-convex w.r.t. $\boldsymbol{\rho}_{(m)}$ and $\mathbf{p}_{(m)}$. The Lagrangian is written as

$$L(p_{k,m}^{\mathrm{U}}, \rho_{k,m}, \lambda_1, \lambda_2) \\ = -w_m \frac{\sum_{k=1}^{K} \rho_{k,m} \log_2 \left(1 + |h_{k,m}|^2 p_{k,m}/\rho_{k,m}\right)}{\phi_m \sum_{k=1}^{K} p_{k,m} + Q_m} \\ + \frac{\theta}{2} \sum_{k=1}^{K} \left(\rho_{k,m} - z_{k,m}^j + u_{k,m}^j\right)^2 + \lambda_2 \left(\sum_{k=1}^{K} p_{k,m} - \bar{P}_m \right) \\ + \lambda_1 \left(\bar{R}_m - \sum_{k=1}^{K} \rho_{k,m} \log_2 \left(1 + |h_{k,m}|^2 p_{k,m}/\rho_{k,m}\right) \right). \quad (28)$$

One can observe that the KKT conditions would be quite complex and do not provide a clear insight. For simplicity, let us assume that both (21) and (22) are satisfied with inequality, that is, $\lambda_1 = \lambda_2 = 0$. It is a quite reasonable assumption because the solution will be the same as in subproblem 1 or in subproblem 3 (see later) otherwise.

Then the KKT conditions are respectively

$$q_{k,m} = \left(\frac{\phi_m \sum_{k=1}^{K} p_{k,m} + Q_m}{\phi_m \sum_{k=1}^{K} \rho_{k,m} \ln\left(1 + |h_{k,m}|^2 q_{k,m}\right)} - \frac{1}{|h_{k,m}|^2} \right)^+ \quad (29)$$

and

$$\rho_{k,m} - z_{k,m}^j + u_{k,m}^j = \frac{w_m A\left(|h_{k,m}|^2 q_{k,m}\right)}{\theta \ln 2 \left(\phi_m \sum_{k=1}^{K} p_{k,m} + Q_m\right)}. \quad (30)$$

$\rho_{k,m}$ and $p_{k,m}$ can be calculated by (30) for given $q_{k,m}$. Thus, the solution can be found by updating the water level in (29).

C. Subproblem 3

If $\delta_m(\mathcal{M}_3) = 1$, the problem becomes

$$\min_{\mathbf{p}_{(m)}, \boldsymbol{\rho}_{(m)}} \quad - w_m \sum_{k=1}^{K} \rho_{k,m} \log_2 \left(1 + |h_{k,m}|^2 p_{k,m}/\rho_{k,m}\right) \\ + \frac{\theta}{2} \|\boldsymbol{\rho}_{(m)} - \mathbf{z}_{(m)}^j + \mathbf{u}_{(m)}^j\|_2^2 \quad (31)$$

$$s.t. \quad (21)(22), \quad (32)$$

TABLE I: Comparison of exchange information

	Distributed ADMM	Centralized ADMM
Power control	autonomous	non-autonomous
Transmission mode	confidential	public
CSI at users	necessary	unnecessary
Q_m	confidential	public
ϕ_m	confidential	public
τ_m	confidential	public
$\bar{P}_m$	confidential	public
$\bar{R}_m$	confidential	public
$\boldsymbol{\rho}^j$	exchange	no exchange
$\mathbf{u}^j - \mathbf{z}^j$	exchange	no exchange

which is a convex problem. Under the assumption that the problem is feasible, (22) is redundant. Therefore, the Lagrangian is written as

$$
\begin{aligned}
&L(p_{k,m}, \rho_{k,m}, \lambda_2) \\
&= -w_m \sum_{k=1}^{K} \rho_{k,m} \log_2 \left(1 + |h_{k,m}|^2 p_{k,m}/\rho_{k,m}\right) \\
&+ \frac{\theta}{2} \sum_{k=1}^{K} \left(\rho_{k,m} - z^j_{k,m} + u^j_{k,m}\right)^2 + \lambda_2 \left(\sum_{k=1}^{K} p_{k,m} - \bar{P}_m\right).
\end{aligned}
\tag{33}
$$

The KKT conditions are:

$$q_{k,m} = \left(\frac{w_m}{\lambda_2 \cdot \ln 2} - \frac{1}{|h_{k,m}|^2}\right)^+ \tag{34}$$

$$\rho_{k,m} - z^j_{k,m} + u^j_{k,m} = \frac{w_m}{\theta \ln 2} A\left(|h_{k,m}|^2 q_{k,m}\right). \tag{35}$$

Similarly as in subproblem 1, the solution can be found by finding λ_2 such that (21) is satisfied with equality.

D. Solution of continuous z

From the KKT condition of $z_{k,m}$ in (16), it is straightforward to obtain

$$z_{k,m} = \left(\rho_{k,m} + u_{k,m} - \lambda_k\right)^+, \tag{36}$$

where λ_k is the Lagrangian multiplier for the k-th subcarrier constraint. $z_{k,m}$ can be found by finding λ_k such that $\sum_{m=1}^{M} z_{k,m} = 1$.

E. Solution of discrete z

Since for each k, there is only a unique m such that $z_{k,m} = 1$, we get $z_{k,m^\star(k)} = 1$ where

$$m^\star(k) = \arg\max_m \left(\rho_{k,m} + u_{k,m}\right). \tag{37}$$

Therefore, updating $\mathbf{z}$ for discrete $\mathbf{z}$ has less complexity than for continuous $\mathbf{z}$.

V. Practical considerations and algorithms

In this section, first the choice of w_m and θ is discussed, then the algorithms are briefly explained.

Because the objective function of each kind of transmission mode (f_m) has various magnitudes, w_m should be first designed to make each objective function having approximately the same magnitude, so that all f_m are efficiently optimized. Then, each value of w_m can be modified within a reasonable region to emphasize on some f_m if a larger w_m is set. The value of θ should be designed such that the term $\frac{\theta}{2}\|\boldsymbol{\rho}_{(m)} - \mathbf{z}^j_{(m)} + \mathbf{u}^j_{(m)}\|_2^2$ is in the same magnitude or a slightly smaller magnitude with f_m [10].

As analyzed in the last section, the algorithms consist of an outer algorithm which updates $\mathbf{z}$ and $\mathbf{u}$ and M inner algorithms which optimize $\mathbf{p}$ and $\boldsymbol{\rho}$. Each inner algorithm being convex or quasi-convex, a bisection method or a subgradient method can be used to update the Lagrangian multipliers in the derived KKT conditions above to obtain the global optimum for each inner problem. The convergence condition of the outer algorithm is to make the normalized primal residual smaller than a threshold, where the normalized primal residual is defined as

$$\frac{\|\boldsymbol{\rho}^j - \mathbf{z}^j\|^2}{\max\{\|\boldsymbol{\rho}^j\|^2, \|\mathbf{z}^j\|^2\}}. \tag{38}$$

Moreover, the setup and the impact of w_m, θ, and the threshold will be illustrated in the numerical results in the next section.

Please note that the ADMM-based methods are logically 'distributed', where the main advantage is to optimize the M subproblems in parallel, which reduces to operation time of these M subproblems by a factor of M. The proposed algorithms in this paper can be either implemented at the AP in a centralized manner, or at users in a distributed manner. In Table I, we compare these two schemes. To summarize, the centralized one does not need to feedback channel state information (CSI) to users (indicated as 'unnecessary' in the table) and does not exchange $\boldsymbol{\rho}^j$ and $\mathbf{u}^j - \mathbf{z}^j$ between users and the AP. The distributed one provides more flexibility and confidentiality on the transmission modes, modulation and coding schemes, power controls, and personal parameters, etc.

VI. Numerical results

In this section we show the numerical results. Parameters are set as follows: $K = 64$, $P_c = 100$mW, $\phi_m = 3$, $\tau_m = 1$, $\bar{P}_m = 100$mW, and $\bar{R}_m = 5$bits/s/Hz. The SNR of the channel of each subcarrier is 30dB and the channel is Rayleigh fading. $\delta_c = 0.1$ unless otherwise specified. We assume there is one user in each group, which is sufficient to illustrate the performances of different transmission modes. Therefore, $\mathcal{M}_1 = \{1\}$, $\mathcal{M}_2 = \{2\}$, $\mathcal{M}_3 = \{3\}$. $w_1 = 100, w_2 = 300, w_3 = 1$ is used to make the values of individual objective functions belong to the same ranges.

We begin the investigation of convergence by plotting the primal residual and the dual residual respectively in Fig. 1 and Fig. 2, which are key criteria in ADMM. As is known in the literature, the value of the obtained objective function is smaller and the convergence is slower if θ becomes smaller [10]. As continuous $\mathbf{z}$ guarantees convergence and discrete $\mathbf{z}$ does not guarantee convergence mathematically, both cases are plotted to illustrate their convergence. In Fig. 1, the normalized primal residual defined as in (38) for discrete $\mathbf{z}$ converges with a similar speed with the continuous $\mathbf{z}$, except some unstable vibrations before the convergence. In Fig. 2, averaged

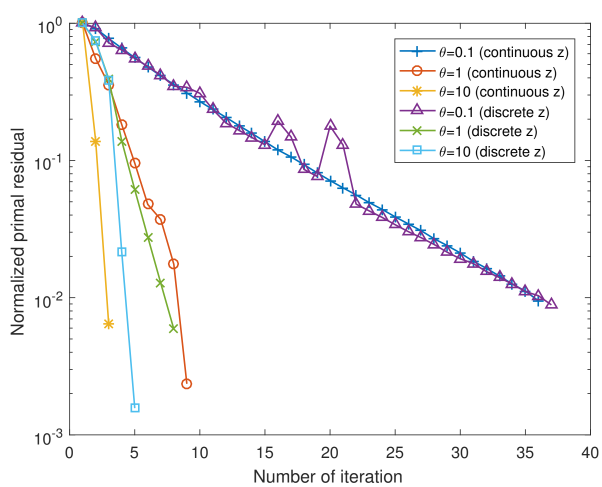

Fig. 1: Convergence performance of primal residual.

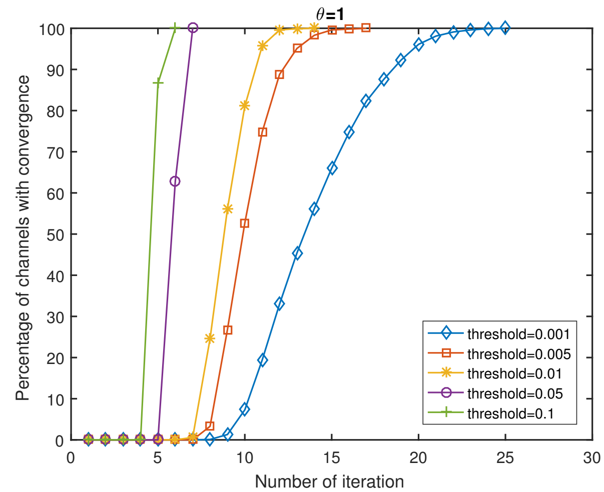

Fig. 3: The impact of convergence condition.

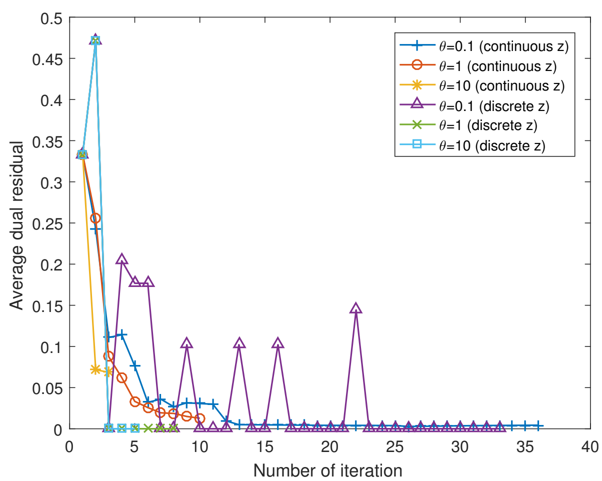

Fig. 2: Convergence performance of dual residual.

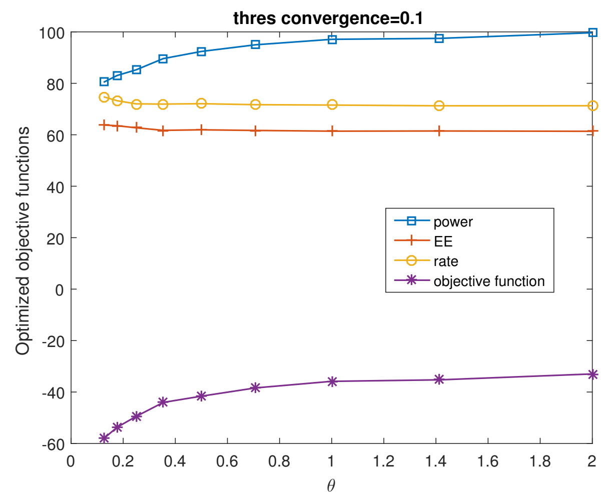

Fig. 4: Impact of θ on the objective function.

dual residual, $\|\mathbf{z}^{j+1} - \mathbf{z}^{j}\|^2/(KM)$, is plotted to study the convergence of each subcarrier indicator. Because the values are zero at some points, we do not plot it in logarithm as in Fig. 1. Again, discrete $\mathbf{z}$ converges with a speed similar to the case of continuous $\mathbf{z}$, except some unstable oscillations before the convergence.

In Fig. 3, we investigate the performance of convergence by calculating the cumulative distribution function (CDF) versus the number of iterations. A smaller value of 'threshold' implies a stricter condition of convergence. The curves are plotted by accumulating 1000 channel realizations. Continuous $\mathbf{z}$ is considered in this figure. The figure illustrates that algorithm converges within 5 iterations for 'threshold'$= 0.1$ and within 25 iterations for 'threshold'$= 0.001$ which is already quite precise.

In Fig. 4, we compare the performance for various values of θ. As stated in the explanation for Fig. 1 and Fig. 2, the value of the obtained objective function is larger and the convergence is faster if θ becomes larger. The observation in Fig. 4 is consistent with this property. In addition, the change of values of individual objective functions is not large, meaning that the considered optimization is robust in terms of θ, which benefits the practical design.

In Fig. 5, we investigate the impact of individual weights on the optimized individual objective functions. Seven cases of weights are considered where the corresponding values of weights are shown in Table II. Case 1 is a benchmark for all the other cases. In Case 2 and Case 3, w_2 decreases and increases respectively, which means the importance of EE maximization of user 2 is reduced and enhanced respectively. It is observed that the EE of user 2 is reduced in Case 2 and increased in Case 3, which is consistent with the changes of w_2. Similar investigations can be found in Case 4 and Case 5 for power minimization of user 1, and in Case 6 and Case 7 for rate maximization of user 3. Therefore, by changing individual weights, the network can modify the importance of each user.

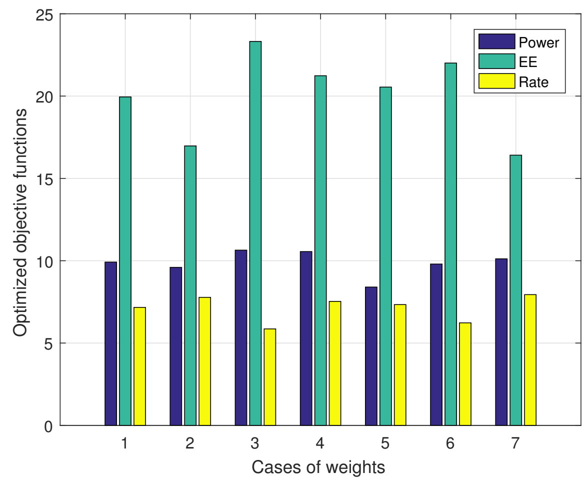

Fig. 5: Impact of weights on the objective function.

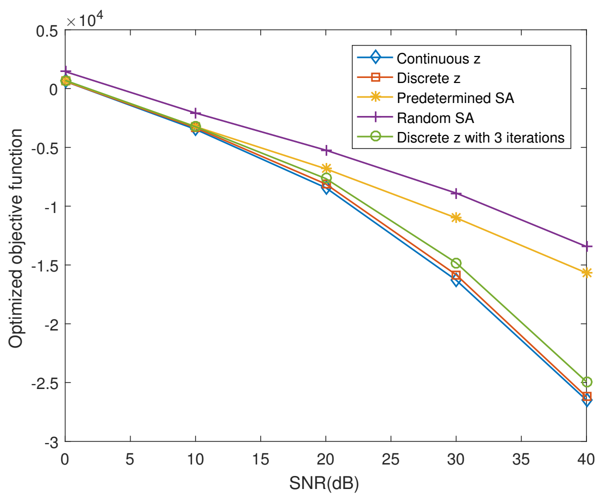

Fig. 6: Comparison of objective functions for various optimization schemes.

In Fig. 6, the obtained values of objective functions for various optimization schemes are compared. Note that because the optimization is a minimization problem, a smaller value means a better solution. It is observed that the continuous $\mathbf{z}$ obtains the best solution, whose value of objective function can be viewed as a lower bound to that obtained with the discrete $\mathbf{z}$. The performance with discrete $\mathbf{z}$ with only a few iterations (for example 3 in the figure) is quite comparable with the optimal one. The performance of the predetermined subcarrier allocation, where each subcarrier is allocated to the user with the maximum channel gain, is similar with the optimal one for low SNR but quite inferior to the optimal one for high SNR. This is because EE and rate for user 2 and 3 converge to zero for low SNR and the power allocation is dominated by strong channels for user 1 due to water-filling. Random subcarrier allocation is the benchmark of the worst case.

TABLE II: Values of weights in Fig. 5.

Cases of weights	1	2	3	4	5	6	7
w_1	100	100	100	10	1000	100	100
w_2	300	30	3000	300	300	300	300
w_3	1	1	1	1	1	0.1	10

VII. Conclusion

The problem of subcarrier allocation and power control for uplink OFDMA channels has been studied. Individual transmission modes, including power minimization, EE maximization, and rate maximization, are considered. An ADMM-based distributed algorithm is proposed to solve the subcarrier allocation and power allocation problems. The proposed algorithm provides high flexibility and confidentiality and also low complexity. Necessary conditions of each subproblem based on KKT conditions are derived. The impact of weights of each subproblem and the square error term is also illustrated. Numerical results show that the algorithm converges fast and obtains the near-optimal solution within only a few iterations.

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References

[1] E. Bjornson, E. A. Jorswieck, M. Debbah and B. Ottersten, "Multiobjective Signal Processing Optimization: The way to balance conflicting metrics in 5G systems," *IEEE Signal Process. Magazine*, vol. 31, no. 6, pp. 14-23, Nov. 2014.

[2] C. Isheden, Z. Chong, E. Jorswieck, and G. Fettweis, "Framework for link-level energy efficiency optimization with informed transmitter," *IEEE Trans. Wireless Commun.*, vol. 11, no. 8, pp. 2946-2957, Aug. 2012.

[3] S. Buzzi, G. Colavolpe, D. Saturnino and A. Zappone, "Potential Games for Energy-Efficient Power Control and Subcarrier Allocation in Uplink Multicell OFDMA Systems," in *IEEE J. Sel. Topics Signal Process.*, vol. 6, no. 2, pp. 89-103, April 2012.

[4] H. Ye, G. Lim, L. J. Cimini and Z. Tan, "Energy-Efficient Resource Allocation in Uplink OFDMA Systems under QoS Constraints," *MILCOM 2013 - 2013 IEEE Military Communications Conference*, San Diego, CA, 2013, pp. 424-428.

[5] H. Ye, G. Lim, L. J. Cimini and Z. Tan, "Energy-Efficient Scheduling and Resource Allocation in Uplink OFDMA Systems," in *IEEE Commun. Lett.*, vol. 19, no. 3, pp. 439-442, March 2015.

[6] C. Y. Ho and C. Huang, "Non-Cooperative Multi-Cell Resource Allocation and Modulation Adaptation for Maximizing Energy Efficiency in Uplink OFDMA Cellular Networks," in *IEEE Wireless Commun. Lett.*, vol. 1, no. 5, pp. 420-423, October 2012.

[7] Z. Zheng, L. Dan, Y. Xiao, G. Wu and S. Hu, "Low-Complexity Energy-Efficient Resource Allocation for Uplink OFDMA Systems," *2013 IEEE 78th Vehicular Technology Conference (VTC Fall)*, Las Vegas, NV, 2013, pp. 1-5.

[8] C. Xiong, G. Y. Li, S. Zhang, Y. Chen and S. Xu, "Energy-Efficient Resource Allocation in OFDMA Networks," in *IEEE Trans. Commun.*, vol. 60, no. 12, pp. 3767-3778, December 2012.

[9] Á. Ricieri Castro e Souza, J. Roberto de Almeida Amazonas and T. Abrão, "Power and Subcarrier Allocation Strategies for Energy-Efficient Uplink OFDMA Systems," in *IEEE J. Sel. Areas Commun.*, vol. 34, no. 12, pp. 3142-3156, Dec. 2016.

[10] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Foundations and Trends in Machine Learning*, vol. 3, no. 1, pp. 1-122, 2010.

[11] F. Han, S. Zhao, L. Zhang, K. Yang and J. Shen, "Decentralized Beamforming for Weighted Sum Energy Efficiency Maximization in MIMO Systems," *GLOBECOM 2017 - 2017 IEEE Global Communications Conference*, Singapore, 2017, pp. 1-6.

[12] O. Tervo, L. Tran and M. Juntti, "Decentralized coordinated beamforming for weighted sum energy efficiency maximization in multi-cell MISO downlink," *2015 IEEE Global Conference on Signal and Information Processing (GlobalSIP)*, Orlando, FL, 2015, pp. 1387-1391.

[13] Z. Chang, Z. Wang, X. Guo, Z. Han and T. Ristaniemi, "Energy efficient and distributed resource allocation for wireless powered OFDMA multi-cell networks," *2017 15th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt)*, Paris, 2017, pp. 1-6.

[14] X. Hu, B. Li, K. Huang, Z. Fei and K. Wong, "Secrecy Energy Efficiency in Wireless Powered Heterogeneous Networks: A Distributed ADMM Approach," *IEEE Access*, vol. 6, pp. 20609-20624, 2018.

[15] X. Li, X. Ge, X. Wang, J. Cheng and V. C. M. Leung, "Energy Efficiency Optimization: Joint Antenna-Subcarrier-Power Allocation in OFDM-DASs," *IEEE Trans. Wireless Commun.*, vol. 15, no. 11, pp. 7470-7483, Nov. 2016.

[16] R. Devarajan, S. C. Jha, U. Phuyal and V. K. Bhargava, "Energy-Aware Resource Allocation for Cooperative Cellular Network Using Multi-Objective Optimization Approach," *IEEE Trans. Wireless Commun.*, vol. 11, no. 5, pp. 1797-1807, May 2012.

[17] M. Masoudi, H. Zaefarani, A. Mohammadi and C. Cavdar, "Energy and Spectrum Efficient Resource Allocation in Two-Tier Networks: A Multiobjective Approach," *2017 IEEE Wireless Communications and Networking Conference (WCNC)*, San Francisco, CA, 2017, pp. 1-6.

[18] I. Stupia, L. Sanguinetti, G. Bacci and L. Vandendorpe, "Power Control in Networks With Heterogeneous Users: A Quasi-Variational Inequality Approach," *IEEE Trans. Signal Process.*, vol. 63, no. 21, pp. 5691-5705, Nov.1, 2015.

[19] R. Takapoui, N. Moehle, S. Boyd, and A. Bemporad, "A simple effective heuristic for embedded mixed-integer quadratic programming," *arXiv preprint arXiv:1509.08416*, 2015.

[20] S. Boyd and L. Vandenberghe, "Convex Optimization," *Cambridge University Press*, 2004.