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and the theory of storage

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Commodities volatility and the theory of storage

Jean-François CARPANTIER¹ and Arnaud DUFAYS²

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Abstract

One implication of the theory of storage states that commodity price volatility should increase when inventories are low. We document this volatility feature by estimating asymmetric volatility models for 16 commodity return series, on the period 1994-2011 and show how to account for this feature in Value-at-Risk forecasting. Our contribution is threefold: (i) This study is the first to investigate systematically the volatility implication of the theory of storage for a large panel of commodity types (agricultural, metals, precious metals and tree crops); (ii) Since inventories are hard to measure and define, especially for high frequency data, we use in the volatility model positive return shocks as a new original proxy for inventories; (iii) We finally develop an original asymmetric version of the spline GARCH model and find that the inventory effect remains robust if we allow the unconditional variance to vary over time.

Keywords: asymmetries, commodities, inventory spline GARCH, VaR.

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1. Introduction

Pricing theories for commodities have not fundamentally changed over the last decades, excepted some adjustments to the theory of storage. This field now raises a renewed interest for three reasons: (i) a significant commodity price boom occurred over the period 2005-2011, commodity indices more than doubled, oil prices were multiplied by three, silver prices by four, etc.; (ii) commodity derivatives occupy a much more important place than before - the Bank of International Settlement thus reports that the amounts outstanding of over-the-counter commodity derivatives (forward and options) exceed 3 trillions dollars as of June 2011; (iii) the pressure on the demand side with the fast development of the BRIC countries, combined with the announced production peak of some commodities raises the questions of the occurrence of potential stock-outs.

Understanding commodity prices dynamics is essential for many agents. First, some countries base their economic development on the production and export of commodities. Such countries are highly exposed to commodity price fluctuations. An efficient macroeconomic policy would require good instruments and models to predict and manage these price fluctuations. In this context, developing appropriate volatility models is especially important. Second, commodities take an ever-growing share in portfolio strategies. Commodity prices are often found to be negatively correlated with equity and bonds, and to provide in the long run returns close to those offered by equity investments. Since commodities are physical assets with specific constraints (such as the non-negativity of inventories), specific volatility models must be developed for commodity prices to optimize their risk management

techniques.

The reference theory for commodity prices and commodity volatility is the theory of storage. This theory explains the difference between spot and futures commodity prices in terms of storage cost, forgone interest and “convenience yield”. One implication of the theory of storage states that the variance of the commodity price increases in time of low inventories and decreases when the inventories are abundantly furnished. One of our objectives is to document this asymmetry and show how to capture it. Since this asymmetry is directly related to the inventories, we label this positive effect of past positive shocks on the variance as the (commodity) “inventory effect”, by analogy with the “leverage effect” commonly found on equities (for which past negative, instead of positive, shocks matter).

Our contribution in this field is threefold. First, this study is the first to investigate systematically the volatility implication of the theory of storage for a large panel of commodity types (agriculturals, metals, precious metals, and tree crops). Other contributions either focus on some specific commodities (metals for Ng and Pirrong (1994)) or focus on the hedging implications of the theory of storage (Gao and Wang (2005) and Lien and Yang (2008)). Second, this study replaces past spot futures spreads, monthly dummies or business cycles by past positive returns as new proxy for the states of inventories, thereby following the suggestion made by Gorton, Hayashi and Rouwenhorst (2007). Indeed, declines in inventories can be signalled by positive price shocks. Third, we propose a technical innovation by developing an asymmetric version of the spline GARCH model initially developed by Engle and Rangel (2008). This new version allows the unconditional variance of

the asymmetric GARCH model to vary over time and permits to check the robustness of our results.

Empirically, we work on a set of 16 commodity prices over the period 1994-2011 on a weekly basis. We first estimate a GARCH model with an asymmetric term capturing the specific impact of past positive shocks, a model called GJR-GARCH and developed by Glosten, Jagannathan and Runkle (1993). We then propose four robustness checks to test alternative frequencies, distributions and specifications. Our results tend to support the theory of storage and in particular the inventory effect. This effect is however not observed for all commodities, and not specific to one type of commodity. It appears that no clear generalizations are possible and that the relevance of asymmetric models has to be examined on a case-by-case basis.

The existence of the commodity inventory effect has some implications for risk management. To illustrate this, we propose in the final section an application where we test whether an appropriate modelling of the commodity prices improves significantly Value-at-Risk (VaR) evaluations. Based on Engle and Manganelli (2004), we find that taking the inventory effect into account improves the VaR estimates for most commodities. A careful investigation of the inventory effect is by consequence relevant before modelling the price of a commodity.

This paper is structured as follows. Section 2 presents the theory of storage and its implications for the volatility. Section 3 describes the data. Section 4 presents the methodology, the GARCH models designed to capture this asymmetry and the results. Robustness checks are proposed in Section 5 and a risk management exercise in Section 6. Finally, Section 7 concludes.

2. Theory of Storage

The theory of storage, in its classical version (Kaldor (1939) and Working (1949)), explains the difference between spot and futures commodity prices in terms of storage cost, forgone interest and “convenience yield”. Convenience yield is for commodities what dividend yield is for equity stocks. It denotes the stream of implicit benefits that an inventory holder receives by having the commodity on hand (i.e. by the ability to respond more flexibly to unexpected excess demand or supply disruptions). More formally,

$$F_{t,T} - S_t = W_{T-t} + r_t S_t - C_t(I_t)$$

where S_t is the spot price of the commodity at date t , $F_{t,T}$ is the price at t of the future contract on the commodity with delivery at time T , W_{T-t} is the global cost of storage between date t and T , r_t the interest rate, $C_t(I_t)$ the convenience yield and I_t the state of the inventories. The difference between $F_{t,T}$ and S_t , also known as the “basis”, can be sometimes positive (the market is said to be in contango) or negative (the market is said to be in backwardation). In case of backwardation, the convenience yield gets larger than the storage costs and forgone interests together.

The variation of the convenience yield is the core of the theory of storage and helps to understand how supply and demand fundamentals affect the basis. When the economy builds up inventories, the potential for stock holders to benefit from the stocks on hand decreases. In other words, the convenience yield decreases. By reduction in inventory build-ups on the contrary, stockholders increase their probability to benefit from a sudden positive spike in spot price. The convenience yield in this case increases. Since supply is less

elastic in the short run than in the long run, spot prices will be more affected than futures prices. As a result, the basis depends positively on the state of inventories.

The modern version of the theory of storage (Deaton and Laroque (1992)) replaces the idea of convenience yield by the probability of occurrence of a stock-out. While in the classical theory, an inventory holder benefits from a convenience yield, the modern version considers that the benefit is related to the implications of a potential stock-out. In both approaches, inventories are the core concept and determine to which extent the cost-and-interest-adjusted basis deviates from zero.

Many empirical papers support the theory of storage. First empirical studies (Brennan (1958) and Telser (1958)) were based on inventory data and found the negative dependence of the cost-and-interest-adjusted basis (i.e. the convenience yield). This approach is however hard to implement due to the difficulty to define an appropriate scope of relevant inventory data (geographically but also due to product differentiations) and due to the difficulty to collect appropriate data on inventories as they are generally not exhaustively known¹. As a consequence, a second wave of applied works relied on various proxies for inventories such as monthly dummies for agriculturals (Fama and French (1987)), business cycles for metals (Fama and French (1988)) or, more recently, price shocks (Gorton et al. (2007)) to highlight the negative dependence of the convenience yield.

Beside these direct tests of the theory of storage, other studies explored

¹Improving the transparency and data availability on physical commodity markets is one of the recommendations made by the G20 in 2011.

some indirect implications of the theory (Ng and Pirrong (1994), Gao and Wang (2005) and Lien and Yang (2008) among others). Ng and Pirrong (1994) list, and test for metals, six implications for the variances and covariance of the spot and futures returns: (1) spot and futures correlation should be positively correlated to the level of the inventory; (2) spot variances should be larger than futures variance; (3) spot and futures variance should be equal when inventories increase; etc. Their results all support the theory of storage. Gao and Wang (2005) propose a unified test incorporating aspects of both direct and indirect approaches of the theory of storage. Lien and Yang (2008) finally test the relevance of including the basis in their variance-covariance specification of spot and futures returns to improve the precision of a dynamic optimal hedge ratio.

The estimation framework changes from one contribution to the other, but they share one feature: they all involve both the spot and futures returns. Baillie and Myers (1991) use a bivariate VAR of spot and futures returns and BEKK-GARCH specification for the variance-covariance equations. Ng and Pirrong (1994) use a bivariate ECM-VAR of spot and futures returns where the correction term is the interest adjusted basis, and include the squared interest-adjusted basis as regressor in the variance equation. Lien and Yang (2008) use the same framework to study optimal futures spot hedge ratios but adjust the variance equation by allowing different effects for negative and positive basis. Gao and Wang (2005) study the interest adjusted basis in a univariate framework with an autoregressive structure in the mean equation and an EGARCH specification in the variance.

We depart from these approaches for two reasons. First, the basis is not

the sole proxy for capturing the state of the inventories. As pointed out by Gorton et al. (2007), the sign of past returns also brings information on the state of the inventories. Positive shocks tend to indicate a deterioration of the stocks. We thus use a “new” (and very simple) proxy which is the sign of the past return, following the suggestion made by Gorton et al. (2007). Using the return sign as a proxy could be particularly relevant and useful for commodities where the futures market is not well developed and the basis not available over a sufficient period (steel for example). Second, our focus is not the relationship between spot and futures returns but the dynamic of the variance of the spot returns. We thus depart from the classical approach related to the theory of storage and focus on spot returns.

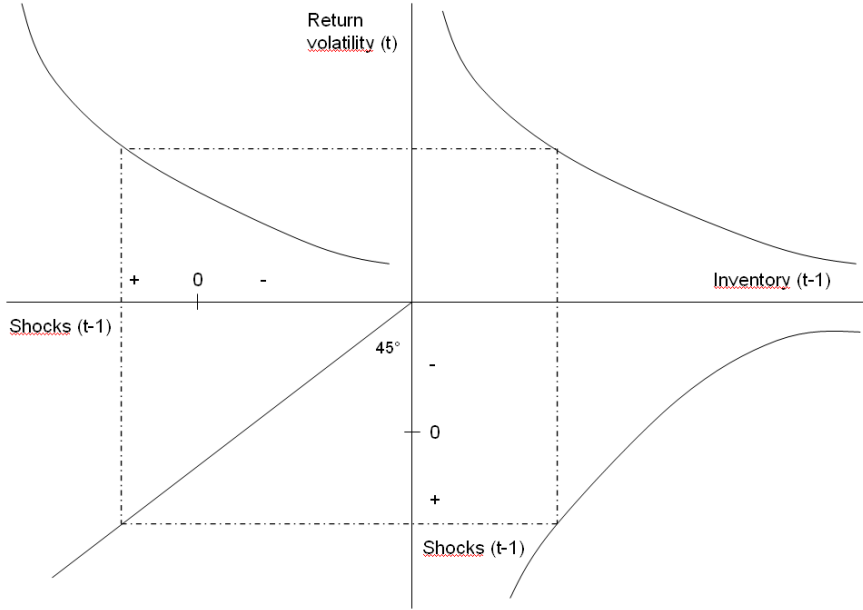


Figure 1: Relation between the return volatility, the inventory and the price shocks

As depicted in Figure 1 the spot return volatility is expected to be nega-

tively related to lagged inventories (upper right corner of the figure). Since those inventories are not observable, we use as proxy the past shocks, which are expected to be on average positive when inventories are low and negative when inventories are high (lower right corner of the figure). We thus estimate whether positive shocks have a positive effect on the variance (upper left corner of the figure).

3. Data

We use a set of 16 commodity spot price series, available in Datastream, which covers the period from 3rd January 1994 to 30th December 2011, 18 years of data on a weekly basis, so 939 observations per series. Our dataset includes five agriculturals (corn, cotton, soybean, sugar and wheat), two tree crops (coffee and rubber), two energies (Brent and gas), four metals (aluminium, copper, nickel and zinc) and three precious metals (gold, palladium and silver). The commodity prices are described in Table 1.

We select these 16 commodity price series on basis of their availability over a sufficiently long period and of their market liquidity. Most price series have no regular pattern, neither in level, nor in log-returns.

Supply and demand factors exert their influence on commodities differently. Agriculturals are affected by seasonality and metals by business cycles. Shocks may also have different implications for different commodities. The impact of a supply shock on agricultural prices has in principle no impact the next year. Tree crops (coffee and rubber) on the contrary may have more lasting effects in case of supply shocks (strong winds or rains detrimental to the trees for example). The cost of storage is another parameter of differentiation for commodities. The relative cost of storage is typically lowest

Table 1: Data description

Series	Description	Type
CORN	Corn No.2 Yellow Cents/Bushel	Agriculturals
COTT	Cotton,1 1/16Str Low -Midl,Memph Cts/lb	Agriculturals
SOYB	Soyabeans, No.1 Yellow Cts/Bushel	Agriculturals
SUGR	Raw Sugar-ISA Daily Price c/lb	Agriculturals
WHEA	Wheat, No.2 Hard (Kansas) Cts/Bushel	Agriculturals
COFF	Coffee-Brazilian (NY) Cents/lb	Tree crops
RUBB	Rubber (MRE) SMR GP FOB Sen/Kg	Tree crops
BRNT	Crude Oil-Brent Current Month FOB USD/BBL	Energy
GAS	Gas Oil-EEC CIF Cargos NWE USD/MT	Energy
ALUM	LME-Aluminium 99.7% Cash USD/MT	Metals
COPP	LME-Copper, Grade A Cash USD/MT	Metals
NICK	LME-Nickel Cash USD/MT	Metals
ZINC	LME-SHG Zinc 99.995% Cash USD/MT	Metals
GOLD	Gold Bullion LBM USD/Troy ounce	Precious
PALD	Palladium USD/Troy Ounce	Precious
SILV	Silver Fix LBM Cash Cents/Troy ounce	Precious

Notes. “LME” holds for London Metal Exchange, “MT” for Metric Tonnes, “SHG” for Special High Grade, “LBM” for London Bullion Market, “lb” for pound-mass, “FOB” for Free on Board, “CIF” for Cost, Insurance and freight included, “NWE” for Northwest Europe, “EEC” for European Economic Community, “BBL” for oil barrel, “SMR” for Standard Malaysian Rubber, “MRE” for Malaysian Rubber Exchange and “GP” for General Purpose.

for precious metals, low for metals, large for agriculturals and very large for animals or perishable products. In addition, though often considered as a unique asset class, commodities are neither clearly correlated (Erb and Harvey (2006)) nor influenced by the same macroeconomic factors (Batten, Ciner and Lucey (2010)). As a consequence, we study the commodities separately in the empirical section and interpret the results by taking into account these different features.

We find that all the logarithmics series possess a unit root and remove the non-stationarity of the logarithmic prices by taking the differences, which are all stationary at 1% according to ADF tests (results available on request). We report in Table 2 the descriptive statistics of the log-returns.

As reported in Table 2, the weekly average log-returns are positive for all series. We see that agriculturals average return, from 0.03% to 0.08%, or on an annual basis from 1.5% to 4.1%, are very low compared to other financial assets (5.5% for the S&P 500 over the same period). Other commodities average returns are larger and closer to stock performances, as documented by Erb and Harvey (2006). Regarding the variance, no clear differences appear according to commodity types. Contrary to Gorton and Rouwenhorst (2006), Gorton et al. (2007), Deaton and Laroque (1992) we find that most of our series exhibit negative skewness (as commonly found for equity stocks). Negative spikes tend to dominate positive spikes in our sample. Without surprise, we also find that all series (especially aluminium, coffee and rubber) are leptokurtic. To take into account the potential effects of the non-zero skewness and excess kurtosis, we reiterate our estimates with a skewed-student distribution (as the one proposed by Lambert and Laurent (2002)) in the

Table 2: Descriptive statistics for commodity log-returns

Variable	Mean	Std. Dev	Min	Max	SK	EK
DLCORN	0.08%	4.29%	-21.1%	22.3%	-0.403	2.670
DLCOTT	0.03%	4.06%	-14.6%	17.4%	0.192	1.353
DLSOYB	0.06%	3.66%	-24.5%	12.3%	-0.798	3.728
DLSUGR	0.08%	4.51%	-21.8%	18.1%	-0.443	2.031
DLWHEA	0.05%	4.09%	-17.5%	22.9%	0.189	2.605
DLCOFF	0.14%	5.65%	-27.5%	48.5%	0.703	7.723
DLRUBB	0.16%	3.08%	-23.1%	12.0%	-1.006	6.775
DLBREN	0.21%	4.93%	-22.0%	15.1%	-0.592	1.193
DLGAS	0.20%	4.61%	-19.8%	15.4%	-0.475	1.190
DLALUM	0.06%	3.00%	-17.8%	9.2%	-0.405	2.133
DLCOPP	0.16%	3.80%	-25.2%	13.5%	-0.844	4.856
DLNICK	0.14%	5.16%	-22.2%	32.0%	0.069	2.571
DLZINC	0.07%	4.08%	-19.8%	15.9%	-0.333	2.511
DLGOLD	0.15%	2.33%	-13.3%	13.1%	-0.161	4.174
DLPALD	0.17%	4.94%	-24.6%	23.4%	-0.171	3.297
DLSILV	0.18%	4.30%	-35.3%	25.6%	-0.932	8.261

Notes. 938 weekly observations. All statistics are computed on commodity log-returns. SK stands for Skewness and EK for Excess Kurtosis.

Robustness section. We use log-returns in the empirical sections.

4. Methodology and Results

GARCH models were initiated by Engle (1982) and Bollerslev (1986) in the eighties to capture the time varying volatility of financial series. The models were extended in the nineties so that we now face a genuine “ARCH-mada” of models (see Terasvirta (2006), Bollerslev (2008) and Bauwens, Hafner and Laurent (2011) for some review in a univariate framework; Bauwens, Laurent and Rombouts (2006) and Silvennoinen and Terasvirta (2008) in the multivariate one). Among the GARCH models, some are developed to cope with the characteristic, originally put forward by Black (1976), that the increase in volatility of equity stocks is usually larger when the returns are negative than when they are positive. The traditional explanation of this so-called “leverage” effect is related to the financial structure of equity stocks. Falling returns give rise to a deterioration of the debt-to-equity ratio, which raises the probability of a default. Some common models capturing this asymmetry are the GJR-GARCH of Glosten et al. (1993) and the EGARCH of Nelson (1991). We use the GJR-GARCH(1,1) model in our empirical sections to capture the commodity inventory effect. Combined with an ARMA conditional mean, the GJR-GARCH model is specified as follows:

$$\begin{aligned}
y_t &= \delta + \phi_1 y_{t-1} + \phi_2 \epsilon_{t-1} + \epsilon_t \\
\epsilon_t &= z_t \sigma_t \\
z_t &\rightarrow i.i.d. \quad D(0, 1) \\
\sigma_t^2 &= \omega + (\alpha + \gamma S_{t-1}^-) \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2
\end{aligned}$$

where y_t is the raw series, ϵ_t is the error term, σ_t^2 is the conditional variance, z_t is the standardized error, identically and independently distributed with mean zero and unit variance, S_{t-i}^- is a dummy variable with value 1 if ϵ_{t-1} is negative and 0 otherwise, γ is the parameter capturing the asymmetric effect in the GJR-GARCH model. A negative γ means that a past positive return has a larger impact on conditional volatility than a past positive return of the same amplitude. If the inventory effect holds, γ should be negative.

We therefore estimate ARMA(1,1)-GJR-GARCH(1,1) models by maximum likelihood in a one-step procedure, based on Gaussian distribution. We report in columns 2 and 3 of Table 3 the value of the asymmetry coefficient and its p-value.

We first note that the γ s, or asymmetry coefficients, of 14 series (out of 16) have a negative sign, as expected. Second, five are significant at 10%.

Regarding agriculturals, sugar is the sole commodity for which an inventory effect is significant. We find significant effects for other agriculturals with alternative specifications, but not in the benchmark case. Fama and French (1987) found, with a different approach, larger effects for agriculturals. For precious metals as well, we find a strong inventory effect, which confirms the results for gold of Engle (2011). These results depart from those of Ng and Pirrong (1994) who found no effect for silver and considered that the absence of effect was due to the lower storage cost of precious metals and the potentially larger buffer such commodities could have. Our results show that the storage cost is not that determinant since the inventory effect affects all types of commodities.

Table 3: GJR-GARCH(1,1) estimates

Frequ.	Weekly		Daily		Weekly	
Distrib.	Gaussian		Gaussian		Sk.-Student	
	γ	p-val	γ	p-val	γ	p-val
CORN	-0.042	0.46	*0.008	0.06	0.019	0.81
COTTON	-0.008	0.77	0.003	0.69	-0.014	0.63
SOYB	-0.032	0.49	*-0.048	0.00	-0.048	0.34
SUGR	*-0.025	0.05	0.002	0.78	-0.011	0.71
WHEAT	-0.009	0.85	*-0.021	0.06	-0.015	0.68
COF	*-0.171	0.02	*-0.054	0.00	*-0.172	0.01
RUBB	-0.076	0.40	-0.008	0.74	*-0.116	0.08
BRENT	0.070	0.24	*0.021	0.05	0.061	0.11
GAS	-0.053	0.11	-0.003	0.80	*-0.041	0.09
ALU	-0.022	0.42	*-0.017	0.09	-0.022	0.42
COPPER	0.036	0.42	0.013	0.30	-0.005	0.87
NICK	-0.016	0.57	-0.003	0.79	-0.010	0.70
ZINC	*-0.078	0.00	*-0.022	0.03	*-0.073	0.00
GOLD	*-0.161	0.02	*-0.056	0.00	*-0.115	0.04
PALD	-0.057	0.19	-0.008	0.70	*-0.099	0.08
SILV	*-0.115	0.10	*-0.034	0.01	*-0.108	0.00
NOBS	938		4694		938	

Notes. The γ coefficients are the asymmetric terms of the GJR-GARCH(1,1) specifications. An asterisk * stands for significance at 10%.

These results globally confirm that there is an inventory effect, but also that this effect is not effective for all series. It appears that no generalizations are possible and that the relevance of asymmetric models has to be examined on a case-by-case basis. It also appears that cost of storage is not as determinant as suggested by Ng and Pirrong (1994) to understand which commodities are most affected. Other parameters should play a role, especially the short term supply elasticity. Huge stocks of gold do not mean that gold holders are willing to sell it directly in case of supply/demand shocks. This view tends to give more support to the classical version of the theory of storage than to the modern one (a probability of stock out is certainly not relevant for gold, since the annual demand oscillates around 4,000 tons compared to above-ground stocks of 158,000 tons).

5. Robustness checks

We first check the robustness of our results to alternative distributions and alternative data frequency. We then estimate a partially non-parametric ARCH model (PNP-ARCH) to detect if our results were induced by the GJR-GARCH parametric form. We finally develop an asymmetric version of the spline GARCH of Engle and Rangel (2008) and check if our results were spuriously induced by a constant conditional variance specification.

5.1. *Alternative frequency*

The benchmark estimates rely on weekly data. Working on weekly data seems an appropriate balance between the objective of reducing the daily noise in the data and the objective of keeping a sufficient sample size. We check in this extension if the results hold for daily data.

The daily estimates are reported in Table 3. We find that now three agriculturals have a significant (at 10%) inventory effect (wheat, corn and soybean). This confirms the results of Benavides-Perales (2010) on corn and wheat, on daily data as well, where they use the cost-adjusted basis as proxy for the inventories. Regarding metals, aluminium and zinc have now a significant (at 10%) inventory effect, as well as gold and silver. The same results were found for gold by Engle (2011) on daily data as well. Ng and Pirrong (1994) also found on their data a confirmation of the theory of storage for their non-precious metals (including copper), but no effect for silver.

5.2. Alternative distribution

Since we found negative skewness for most series and non-zero excess kurtosis for nearly all series, we reiterate our estimates by replacing the Gaussian by a skewed-student distribution (as the one proposed by Lambert and Laurent (2002)). As reported in Table 3, taking the skewness into consideration does not neutralize the asymmetric inventory effect but, on the contrary, even strenghtens the effects (all precious metals have a significant inventory effect under this specification). We found similar results with a skewed-student distribution on daily data. We also note in this case a significant inventory effect for palladium (results non reported).

5.3. Partially non-parametric ARCH model

Engle and Ng (1993) provide a PNP-ARCH for studying the impact of news on the volatility. This approach allows to assess if the inventory effect is spuriously induced by the GJR-GARCH parametric specification. The

PNP-ARCH model relaxes the parametric form assumed on the squared error term lying in the GJR-GARCH model while still preserving a GARCH-type dynamic (therefore “partially” non-parametric). It divides the range of ϵ_{t-1} in m equally spaced intervals that are denoted by the boundary vector $\tau = \{\tau_{-k}, \dots, \tau_{-1}, \tau_0, \tau_1, \dots, \tau_k\}$. The size of the intervals is set by the user and we follow the choice of Engle and Ng (1993) by selecting $\tau_i = i\delta$ (with $k = 2$) where δ is the unconditional variance of the time series. The model is described by the following set of equations :

$$\begin{aligned} y_t &= \delta + \phi_1 y_{t-1} + \phi_2 \epsilon_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_t^2), \\ \sigma_t^2 &= \omega + \beta \sigma_{t-1}^2 + \sum_{i=0}^k \theta_i^- (\epsilon_{t-1} - \tau_{-i})^2 N_i + \sum_{i=0}^k \theta_i^+ (\epsilon_{t-1} - \tau_i)^2 P_i \\ N_i &= 1 \text{ if } \epsilon_{t-1} - \tau_{-i} \leq 0 \text{ (=0 otherwise)} \\ P_i &= 1 \text{ if } \epsilon_{t-1} - \tau_i \geq 0 \text{ (=0 otherwise)} \end{aligned}$$

where ω, β and $\{\theta_i^-, \theta_i^+\}$, ($i = 0, \dots, k$) are constant parameters. The functional form is a quadratic spline with knots specified at τ .

A convenient way to visualize the PNP-ARCH model estimates is to graphically represent the impact of a new shock on the conditional variance, what is known as a News Impact Curve (NIC). It estimates the impact on volatility of a shock given that the model lies in the stationary state.

The NIC of these models are represented in figures 2 and 3. We observe that the NIC of the PNP-ARCH and GJR-GARCH models are very close, exhibiting an asymmetric smile with a sharper slope on the right side for 13 out of 16 series, as expected according to the inventory effect. The inventory effect is thus not an artefact from an inappropriate parametric form. We also report in Table 4 the difference of the first asymmetric coefficients ($\theta_0^- - \theta_0^+$)

and its p-value. We find again that most coefficients are negative (15 out of 16) and five significant, confirming the results reported in Table 3.

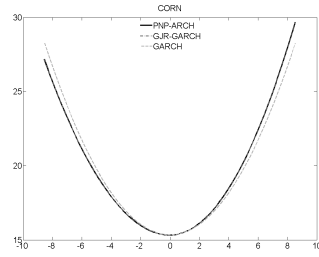
5.4. Spline GJR-GARCH model

Since the return series exhibit different levels of unconditional volatility through time, we rely on the spline GARCH model developed by Engle and Rangel (2008) to allow the unconditional variance to vary over time. The model of Engle and Rangel (2008) consists in spline functions that monitor the unconditional variance of the model instead of fixing it to a constant (i.e. $\frac{\omega}{1-\alpha-0.5\gamma-\beta}$). The interesting feature of the model lies in preserving the short term movements of the volatility. Since the spline GARCH dynamic is symmetrical, we extend its specification to incorporate asymmetric effects such as the inventory effect. We thus reestimate our series with a spline GJR-GARCH model, characterized as follows:

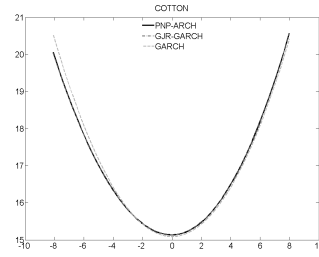
$$\begin{aligned} y_t &= \delta + \phi_1 y_{t-1} + \phi_2 \epsilon_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \tau_t^2 h_t^2), \\ h_t^2 &= (1 - \alpha - 0.5\gamma - \beta) + (\alpha + \gamma S_{t-1}^-)(\epsilon_{t-1}/\tau_{t-1})^2 + \beta h_{t-1}^2 \\ \tau_t^2 &= \omega \exp \left(\lambda_0 t + \sum_{i=1}^k \lambda_i [(t - t_{i-1})_+]^2 \right) \end{aligned}$$

where $(\alpha, \gamma, \beta, \omega, \lambda_0, \dots, \lambda_k)$ are parameters, $(t - t_i)_+ = \max(0, t - t_i)$ and $\{t_0 = 0, t_1, t_2, \dots, t_{k-1}\}$ are time indices (knots) partitioning the sample size T in k equally spaced intervals. The number of knots is selected by the Bayesian Information Criterion.

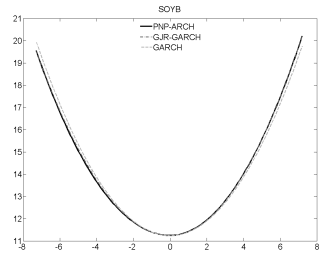
The flexibility of the spline GJR-GARCH model does not rule out the inventory effect as reported in the last columns of Table 4. The signs of all



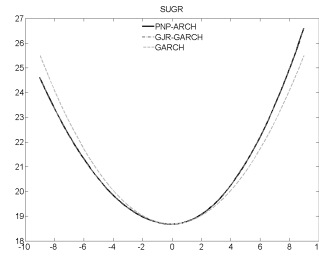
(a) NIC - CORN



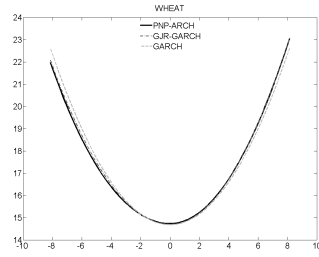
(b) NIC - COTTON



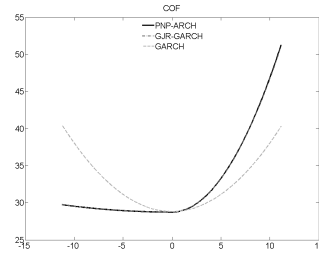
(c) NIC - SOYB



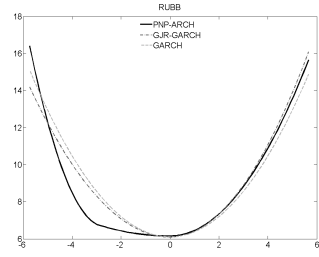
(d) NIC - SUGR



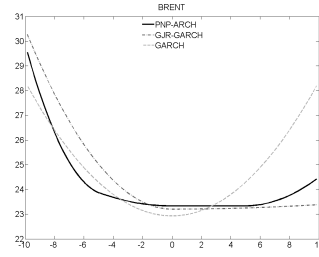
(e) NIC - WHEAT



(f) NIC - COF

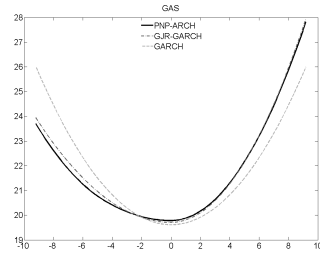


(g) NIC - RUBB

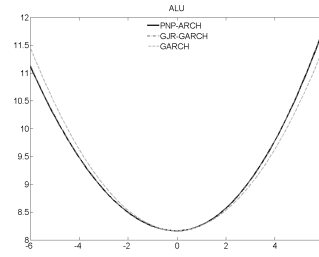


(h) NIC - BRENT

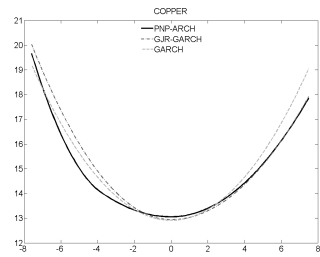
Figure 2: News Impact Curves of weekly commodity percentage returns (1/2)



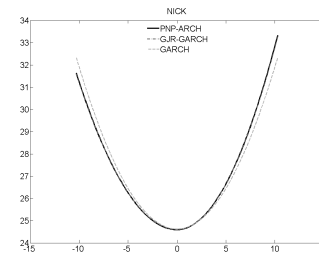
(a) NIC - GAS



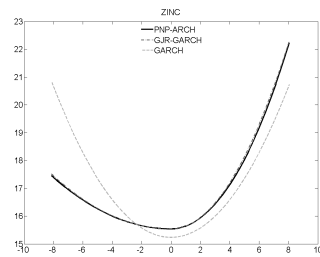
(b) NIC - ALU



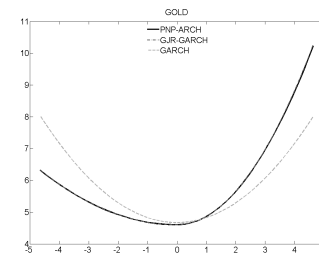
(c) NIC - COPPER



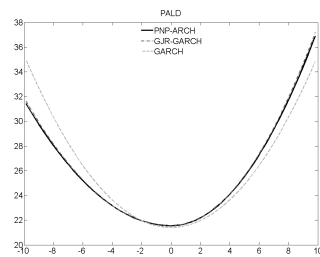
(d) NIC - NICK



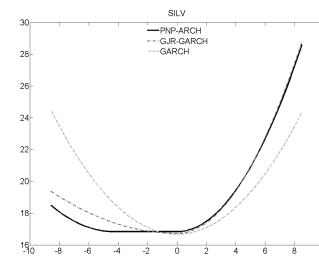
(e) NIC - ZINC



(f) NIC - GOLD



(g) NIC - PALD



(h) NIC - SILV

Figure 3: News Impact Curves of weekly commodity percentage returns (2/2)

Table 4: Robustness checks

Model	PNP-ARCH				spline GJR-GARCH			
Distrib.	Gaussian		Sk-Student		Gaussian		Sk-Student	
	γ	p-val	γ	p-val	" γ "	p-val	" γ "	p-val
CORN	-0.035	0.62	-0.002	0.98	-0.012	0.85	0.050	0.40
COTTON	-0.010	0.87	-0.017	0.77	0.002	0.71	-0.014	0.93
SOYB	-0.022	0.71	-0.050	0.54	0.015	0.78	0.010	0.86
SUGR	-0.025	0.66	-0.013	0.82	-0.026	0.38	0.000	0.95
WHEAT	-0.022	0.79	-0.015	0.84	-0.019	0.94	-0.013	0.84
COF	*-0.173	0.01	*-0.154	0.02	*-0.196	0.00	*-0.180	0.00
RUBB	*-0.227	0.03	*-0.212	0.04	-0.064	0.25	-0.092	0.18
BRENT	0.022	0.73	0.035	0.58	*0.073	0.00	*0.089	0.00
GAS	-0.056	0.28	-0.068	0.22	-0.025	0.46	-0.018	0.68
ALU	-0.019	0.73	-0.020	0.72	-0.024	0.36	-0.025	0.36
COPPER	-0.017	0.79	-0.047	0.35	-0.003	0.78	-0.024	0.52
NICK	-0.016	0.71	-0.010	0.83	-0.013	0.67	0.000	0.99
ZINC	*-0.070	0.04	-0.068	0.11	*-0.083	0.00	*-0.076	0.01
GOLD	*-0.182	0.00	*-0.143	0.04	*-0.173	0.00	*-0.148	0.01
PALD	-0.058	0.28	-0.093	0.37	*-0.108	0.01	*-0.142	0.04
SILV	*-0.162	0.00	*-0.144	0.00	*-0.127	0.00	*-0.106	0.00

Notes. The γ and " γ " coefficients respectively are the asymmetric terms of the spline GJR-GARCH(1,1) model and the sum of the asymmetric negative and positive terms (at value -1 and $+1$) of the PNP-ARCH model. An asterisk * stands for significance at 10%.

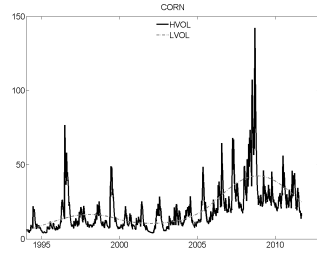
but three asymmetric terms confirm the intuition on the storage theory and, except for the positive estimate of BRENT, all the significant parameters at the 5% level are negative. The conclusions do not change if we reiterate the estimates with a skewed-student distribution instead of the Gaussian. The time-varying unconditional variance of the series are represented in Figures 4 and 5 under the label “LVOL”, together with the conditional variances, labelled “HVOL”. We observe that the inventory effect is robust and does not arise from a change in the level of the variance of the series.

6. Value-at-Risk application

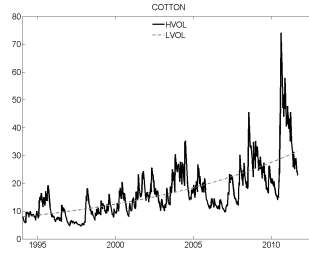
Accounting for the volatility asymmetry documented in the precedent sections can help to improve risk management and VaR in particular.

Trading in commodities, as an alternative investment class to traditional portfolios comprising equity stocks and bonds, has considerably increased in recent years. Gorton and Rouwenhorst (2006) study the properties of commodity futures by building an equally-weighted index of commodity series and find that commodity futures have historically offered the same return and Sharpe ratio as equities. Since return goes hand in hand with risk, we propose in this section an extension to show the relevance of asymmetric models in the context of risk management and investigate whether asymmetric GARCH are valid models for VaR forecasting.

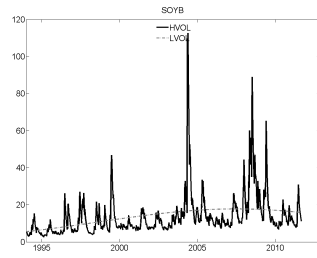
A VaR measure can be defined as a quantitative tool whose goal is to assess the potential loss that can be incurred with a certain level of confidence under normal market conditions by an investor over a given time period and for a given investment. In mathematical terms, the VaR is defined to be the



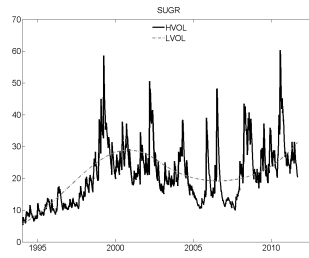
(a) CORN



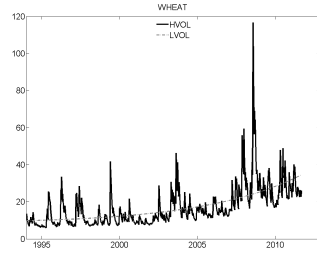
(b) COTTON



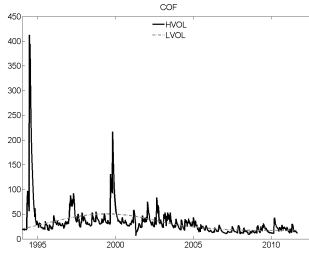
(c) SOYB



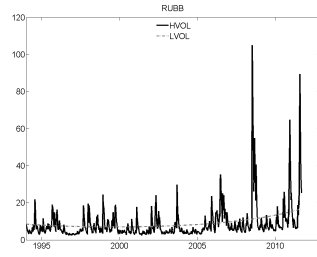
(d) SUGR



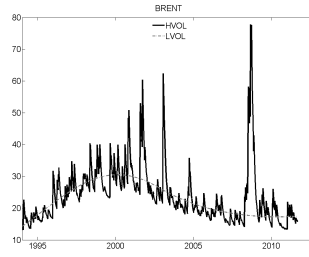
(e) WHEAT



(f) COF

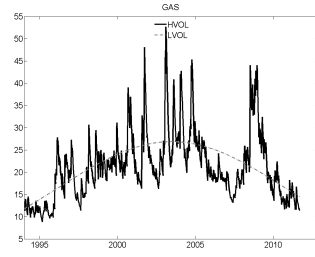


(g) RUBB

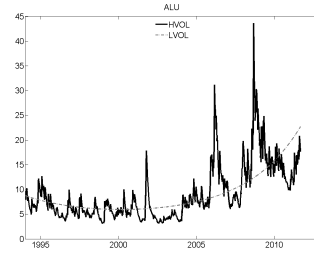


(h) BRENT

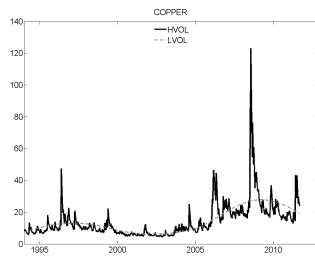
Figure 4: Spline GJR-GARCH unconditional (LVOL) and conditional (HVOL) variances (1/2)



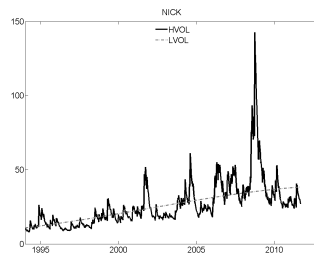
(a) GAS



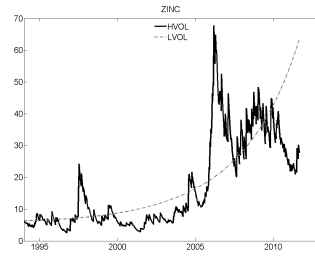
(b) ALU



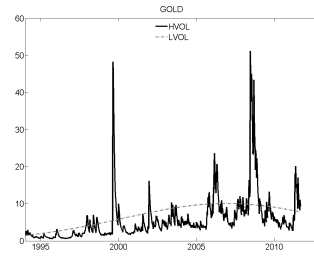
(c) COPPER



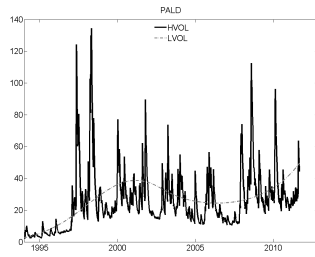
(d) NICK



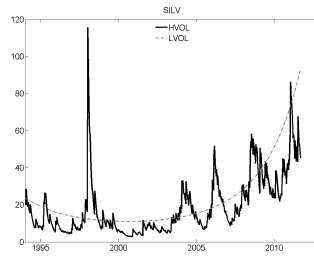
(e) ZINC



(f) GOLD



(g) PALD



(h) SILV

Figure 5: Spline GJR-GARCH unconditional (LVOL) and conditional (HVOL) variances (2/2)

α quantile of the investment's profit and loss distribution

$$VaR_t(\alpha) = F^{-1}(\alpha|\Omega_t)$$

where $F^{-1}(\cdot|\Omega_t)$ refers to the quantile function of profit and losses distributions which varies over time as market conditions change. Modelling the quantile function requires a model that best captures the dynamic of the conditional mean and variance.

Determining the accuracy of a VaR model can be reduced to checking if two properties are satisfied (see Campbell (2005) and Hurlin and Tokpavi (2007) for recent reviews of backtesting procedures). First, the probability of realizing a loss in excess of the reported $VaR_t(\alpha)$ must be precisely equal to $\alpha \times 100\%$ (what is called the unconditional coverage property). Second, the cases where the losses exceed the reported $VaR_t(\alpha)$ must be independently distributed (what is called the independence property).

We report in Table 5 the results of the Dynamic Quantile (DQ) test, developed by Engle and Manganelli (2004), which tests both properties at the same time, by regressing the $Hit_t(\alpha)$ variable (where $Hit_t(\alpha) = 1 - \alpha$ when the loss exceeds the VaR, and $Hit_t(\alpha) = -\alpha$ otherwise) on a constant (equal to 0 if the unconditional coverage property holds) and on its lagged values (non-significant if the independence assumption holds) and by testing the joint significance of the parameters by a Wald test.

The statistics and p-values of the Engle and Manganelli (2004) DQ test are reported in Table 5. We report results for the series where the commodity inventory effect was found to be significant at 10% in the benchmark case (Table 3). Since the commodity inventory effect relates positive shocks to the volatility, we focus on the right side of the return distribution by examining

the VaR for short trading positions only (Giot and Laurent (2003)).

Table 5: Value-at-Risk application: validation of GJR-GARCH(1,1) models by dynamic quantile tests

	γ	DQ test Q95		DQ test Q99	
		stat.	p-val	stat.	p-val
SUGR	-0.025*	11.678	0.07	2.118	0.91
COFF	-0.171*	9.463	0.15	4.317	0.63
ZINC	-0.078*	3.6282	0.73	0.939	0.99
GOLD	-0.161*	3.948	0.68	8.011	0.24
SILV	-0.115*	6.113	0.41	2.702	0.85

Notes. Weekly data. Column 2 “ γ ” reports the γ estimates capturing the effect of positive shocks on the variance, as obtained in the benchmark case (cf Table 3) for the series where the commodity inventory effect is significant at 10%. Columns 3-6 report the Dynamic Quantile test statistics and p-values of Engle and Manganelli (2004) for quantiles 95% and 99%.

We find that the validity of GJR-GARCH(1,1) VaR(95%) and VaR(99%) models cannot be rejected for coffee, zinc, gold and silver. These results confirm those of Giot and Laurent (2003) who compare different GARCH models performances and find that the one allowing asymmetries (APARCH) performs the best². The validity of the GJR-GARCH(1,1) model is rejected for one case (sugar for the DQ test Q95). Bearing in mind this exception, these results overall confirm the relevance of considering an asymmetric volatility model to capture the dynamics of commodity returns in a risk management context.

²They compare their models according to p-values derived from the Kupiec (1995) test. Since the later only tests the unconditional coverage property and not the independence property, we opted in our paper for the Engle and Manganelli (2004) test.

7. Conclusions

The theory of storage implicitly states that commodity prices volatility should increase when the state of the stocks declines. We test empirically the relevance of this statement by using a “new” (and very simple) proxy which is the sign of the past return. Indeed, positive shocks tend to indicate a deterioration of the stocks. We thus test what we call the “inventory effect” by estimating GARCH models with an asymmetric term capturing past positive shocks (GJR-GARCH model), on a set of 16 commodities (agriculturals, tree crops, metals, precious metals) over the period 1994-2011.

As a parallel to the leverage effect requiring asymmetric volatility models for equity, we find that there is also an inventory effect requiring asymmetric volatility models for commodities, that this effect is not detected for all commodities, and not specific to one type of commodity, but robust to a partially non-parametric approach and to alternative distributions and frequencies. We also extend the spline GARCH model into an asymmetric version and find that the inventory effect persists when the unconditional variance is allowed to vary over time. Our results also indicate that the inventory effect is as strong for precious metals as for other commodities, contrary to the belief that commodities with low storage cost (like gold and silver) are less affected by the theory of storage.

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