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RATIONAL EXPECTATIONS
MODELS FOR TWO TYPES OF
LABOUR**

F.C. PALM
G. A. PFANN

Working Paper n° 8916

Département des Sciences Economiques
Collège L.-H. Dupriez
Place Montesquieu, 3 - B-1348 Louvain-la-Neuve (Belgique)

D/1989/3082/16

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Franz C. Palm *

Gerard A. Pfann **

September 1989

ABSTRACT

In this paper we derive and estimate optimal dynamic interrelated labour demand curves for Dutch manufacturing blue and white collar labour. The driving forces of the labour demand model are the predetermined capital stock and the real wage costs of the two types of labour. Interrelation between the demand for two types of labour results only from the presence of serial correlation between the exogenous shocks of the forcing variables. It is found to be optimal for the firm to raise the number of white collar workers employed when increasing its blue collar workforce. On the other hand, a single upswing in white collar employment has a displacement effect on the blue collar workers.

* University of Limburg, Faculty of Economics, P.O. Box 616, 6200 MD
Maastricht, The Netherlands.

** Research Fellow, Département des Sciences Economiques, Louvain-la-Neuve.

We thank David Hendry, Gerard Laman, Steve Nickell, Koos Sneek, an anonymous referee, and participants in seminars at the London School of Economics and Political Science, the University of Limburg, and the Third Congress of the European Economic Society held in Bologna in 1988 for many helpful comments and discussion.

Forthcoming in : Oxford Bulletin of Economics and Statistics.

1. INTRODUCTION

This paper examines interrelated labour demand decisions taken by firms whose expectation formation is assumed to be rational. In line with the approach adopted by Hansen and Sargent (1980,1981) among others, we derive optimal employment decision schemes for blue and white collar workers in closed form solution (CFS). We assume that the driving forces of the labour demand model are the predetermined capital stock, incorporating technological innovations, and the real wage costs of blue and white collar labour. As we assume price-taking behaviour, product demand is not included as a determinant of factor demand.

In table 1 an overview of the main econometric studies which examine interrelated input decisions of production factors is given. Although already in 1974 Nadiri and Rosen pointed out that employment decisions for blue collar workers differ from those of white collar workers, due to differences in hiring and firing costs, only few authors have considered more than one (aggregate) type of labour in their models (see table 1 on this point).

As the overview in table 1 also shows, existing dynamic labour demand models mainly differ in the way expectations are modelled. In models with static expectations, variables that are beyond the direct influence of the firm are assumed to be stationary. At some initial time t the firm chooses the optimal time path for the decision variables by maximizing an intertemporal objective function. The plan remains unchanged through time, regardless of the new information that becomes available and unexpected changes that may take place. The driving forces of the economy are considered to be beyond the scope of this kind of models. Empirical applications are usually based on detrended stationary data.

In nonstatic expectations models, it is assumed that "decisions made by optimizing the objective function with stochastic variables replaced by their conditional expectations are close enough approximations to those which would result if the expected values were maximized" (Morrison, 1986, p.366). Future expectations of exogenous variables are obtained from solutions of ARIMA equations. The nonstationarity of the series of production factors is assumed to be deterministic, and is remedied by incorporating deterministic trends.

In rational expectations intertemporal labour demand models firms are assumed to form expectations which are the conditional expectation given current available and relevant information in order to maximize the expected real present value of profits over a future time horizon. The stochastic process of exogenous variables is needed to form expectations. In each period, expectations and contingency plans are revised in the light of new information.

Kollintzas (1985) determined the unique forward looking solution of the multivariate symmetric linear rational expectations model in terms of its structural parameters. It is rather obvious but does not appear to be widely known that the nondiagonality, in the sequel called interrelation, of the autoregressive part of the employment equations derived from quadratic objective functions does not result from mutual dependence in the production process or in adjustment costs, as was assumed by e.g. Meese (1980) and Hansen and Sargent (1981) for capital and labour. We show that interrelation between blue and white collar labour emerges from serial cross correlation of the disturbances in the firm's economic environment.

The paper is organized as follows. The theoretical interrelation labour demand model is outlined in section 2. Its parameters will be referred to as structural parameters, because they characterize the structure of the decisions that follow from the theoretical model [see also Hendry and Richard (1983) on this point]. In section 3, quarterly data of the Dutch manufacturing sector for the period 1971.I - 1984.IV are used to analyse the closed form solution of the model. The structural parameters are estimated using the method of asymptotic least squares, which is a two step estimation procedure, that is equivalent to maximum likelihood estimation, provided efficient estimates are used for the unrestricted parameters together with an appropriate weighting matrix (Chamberlain (1982), Gouriéroux et al. (1985)). As we expected, it appears to be more costly to adjust the workforce of white collar labour to an unanticipated shift in target level employment than blue collar labour. We compare the model with bivariate dynamic labour demand curves with diagonal autoregressive part. We find that interrelation turns out to be essential to appropriately model the dynamics between blue and white collar employment. Moreover, the moving average representation of the estimated model indicates that more white collar workers are needed when an unforeseen rise in the demand for blue collar workers occurs. A ceteris paribus increase of white collar workers, however, leads to displacement of blue collar workers. In section 4 conclusions are given.

Table 1 : Survey of Interrelated Factor Demand Models.

AGGREGATE LABOUR, CAPITAL (+ MATERIALS, ENERGY)					Variable Inputs	Quasi-fixed Inputs	Expectations
Nadiri & Rosen	1969	Q: 1947.I-62.IV	US	Ma		Blue collar Workers, Hours worked, Capital, Utilization rate	Static
Berndt, Fuss, Waverman	1979	A: 1947-74	US	Ma	Labour, Energy, Materials	Capital	Static
Meese	1980	Q: 1947.I-62.IV	US	Ma		Labour, Capital	Rational
Hansen & Sargent	1981	Theoretical				Labour, Capital	Rational
McIntosh	1982	A: 1950-73	UK	Ag		Labour, Capital	Static
Pindyck & Rotemberg	1983a	A: 1948-71	US	Ma	Energy, Materials	Labour, Capital	Rational
Epstein & Denny	1983	A: 1947-76	US	Ma	Materials	Labour, Capital	Static
Eichenbaum	1984	M: 1959.I-78.12	US	Ma		Hours worked, Inventories	Rational
Epstein & Yatchew	1985	A: 1947-76	US	Ma		Labour, Capital	Nonstatic
Harris	1985	Q: 1968.I-81.IV	UK	E		Hours worked, Capital, Utilization rate	Nonstatic
Kokkelenberg & Bisschoff	1986	Q: 1959.III-77.IV	US	Ma	Labour, Energy	Capital	Nonstatic
Holly & Smith	1986	Q: 1968.III-79.III	UK	Ma		Labour, Capital	Static
Kollintzas & Cassings	1986	Theoretical	US	Ma	Energy, Materials	Labour, Capital	Rational
Morrison	1986	A: 1947-81	US	Ma	Labour, Energy, Materials	Capital	Nonstatic
<u>TWO TYPES OF LABOUR AND CAPITAL</u>							
Nadiri & Rosen	1974	Q: 1947.I-67.IV	US	Ma		Blue collar workers, White collar workers, Hours worked, Capital, Utilization rate, Inventories	Static
Pindyck & Rotemberg	1983b	A: 1947-76	US	Ma	Blue collar workers	White collar workers, Capital	Rational
Kokkelenberg	1984	Q: 1947.I-79.IV	US	Ma	Blue collar workers, Inventories, Hours worked	White collar workers, Capital	Nonstatic
Shapiro	1986	Q:1955.I-80.IV	US	Ma		Blue collar workers, White collar workers, Hours worked, Capital	Rational

A = annual; Q = quarterly; M = monthly; E = engineering; Ma = manufacturing; Ag = aggregate

2. THE MODEL

A firm is assumed to maximize its real present value of profits over an infinite time horizon. It employs blue collar workers (n_1), white collar workers (n_2) and uses capital (k). Decisions to change the capital stock are based on costs which are fixed in the short run, and differ materially from short run decisions to change the input of labour. Consequently, the capital stock is assumed to be predetermined. The adjustment of the decision variables of the firm, blue and white collar labour, is costly. The firm operates under uncertainty. In order to determine the optimal level of the variables n_1 and n_2 , it uses all the relevant information available up to time t . Current real wage costs are part of the employer's information set. Real wage costs are assumed to be given for the firm in the sense that they are not Granger-caused by employment. For empirical evidence on unidirectional causality from real wage costs to employment, we refer to Neftçi (1978) and Sargent (1978) for the U.S., to Harris (1985) for the U.K., and to Palm and Pfann (1987) and section 3 for the Netherlands.

The production technology obeys the usual regularity conditions ($\partial Y / \partial x_i > 0$, $x = (n_1, n_2, k)$; the Hessian matrix is negative definite) and is locally approximated by a quadratic function

$$(2.1) \quad Y_t(n_{1t}, n_{2t}, k_t) = \sum_{i=1}^2 \{ (\alpha_{i1} + s_{it}) n_{it} - \frac{1}{2} \alpha_{i2} n_{it}^2 \} + \alpha_{31} k_t - \frac{1}{2} \alpha_{32} k_t^2 \\ + \alpha_{41} n_{1t} k_t + \alpha_{42} n_{2t} k_t + \alpha_{43} n_{1t} n_{2t}.$$

Y_t denotes the real output per time period, the α_{ij} 's are constant positive parameters. The variables s_{1t} and s_{2t} are exogenous stochastic shocks in the marginal product of labour resulting from changes in the economic environment of the firm.

Rapid adjustment of factor inputs to changes in market conditions can be very costly. Adjustment therefore takes place gradually. In the model, the adjustment costs of labour are assumed to be approximately quadratic. Also as we shall measure n_{it} in terms of numbers of fulltime equivalents, we implicitly assume that the way in which n_{it} is varied (by changing either the number of hours worked per week or the number of persons employed) does not affect the adjustment costs. As capital is predetermined, the adjustment costs of capital are not explicitly included in the adjustment cost function. Although it is unlikely that the adjustment cost function is symmetric, for computational ease we use the following adjustment cost function

$$(2.2) \quad AC_t(\Delta n_{1t}, \Delta n_{2t}) = \sum_{i=1}^2 \frac{1}{2} \beta_i (\Delta n_{it})^2 + \beta_3 \Delta n_{1t} \Delta n_{2t}.$$

It is worth to emphasize here that Nadiri and Rosen (1969) point out that the existence of dynamically interrelated factor input decisions could be explained by implicit trade-off costs that can occur when a firm follows a mixed strategy of holding inventories of production and hoarding some inputs. Therefore β_3 is expected to be negative. With more specific knowledge required, higher screening and hiring costs, and a longer starting period, it seems plausible that the real costs of adjustment for white collar workers will be higher than those for blue collar workers, i.e. $\beta_2 > \beta_1 > 0$.

The variable cost of factor services are assumed to be as follows

$$(2.3) \quad VC_t(n_{1t}, n_{2t}, k_t) = w_{1t}n_{1t} + w_{2t}n_{2t} + c_t k_t,$$

where w_{1t} , w_{2t} and c_t denotes real input prices.

Given the capital stock and investment plans, the rational entrepreneur chooses contingency plans for blue and white collar employment. As a con-

sequence of adjustment costs, at time t , the actual employment level and future employment plans have to be determined simultaneously. In the next period, new information forces the firm to revise its employment plans for the future to determine the optimal employment level at time $t+1$. The firm's objective is to maximize its real present value of profits (PV) at time t , that is

$$(2.4) \quad \text{Max } PV_t(n_{1t}, n_{2t}) = \text{Max } E_t \sum_{j=0}^{\infty} b^j [Y_{t+j} - AC_{t+j} - VC_{t+j}],$$

where b is a real discount factor, $0 < b < 1$, and $E_t(\cdot) = E(\cdot | \Omega_t)$, with Ω_t being the available set of information at time t . First order necessary conditions for maximization of (2.4) with respect to the decision variables n_1 and n_2 consists of the set of Euler equations and a pair of transversality conditions which assure the finiteness of the process.

The Euler equations to this infinite time horizon problem can be expressed as follows

$$(2.5) \quad bB E_t n_{t+1} = A n_t - B n_{t-1} - \Gamma_0 z_t,$$

where $n_t = (n_{1t}, n_{2t})'$, $z_t = (1, s_{1t}, s_{2t}, w_{1t}, w_{2t}, k_t)'$,

$$A = \begin{bmatrix} \alpha_{12} + \beta_1 + b\beta_1 & \beta_3 - \alpha_{43} + b\beta_3 \\ \beta_3 - \alpha_{43} + b\beta_3 & \alpha_{22} + \beta_2 + b\beta_2 \end{bmatrix},$$

$$B = \begin{bmatrix} \beta_1 & \beta_3 \\ \beta_3 & \beta_2 \end{bmatrix},$$

$$\Gamma_0 = \begin{bmatrix} a_1 & I_2 & -I_2 & a_2 \end{bmatrix},$$

with $a_1 = (\alpha_{11}, \alpha_{21})'$, $a_2 = (\alpha_{41}, \alpha_{42})'$ and I_2 the (2x2)-identity matrix.

The planned employment in period $t+1$, $E_t n_{t+1}$, is linearly related to the current and one period lagged employment level and to the current wage

costs and the capital stock. Since $E_t n_{t+1}$ cannot be observed as such it has to be determined within the model. To find a stationary forward looking solution of the Euler linear difference model under rational expectations, that satisfies the transversality conditions we follow Blanchard and Kahn (1980) and transform (2.5) into

$$(2.6) \quad E_t N_t = A N_{t-1} + \Gamma z_t,$$

$$\text{where } N_t = (n_{1t}, n_{2t}, n_{1t+1}, n_{2t+1})'; \text{ and } \Gamma = \begin{bmatrix} 0 \\ \Gamma \end{bmatrix} \text{ and } A = \begin{bmatrix} 0 & I_2 \\ -b^{-1}I_2 & \tilde{A} \end{bmatrix}$$

are (4×2) and (4×4) -matrices respectively with $\Gamma = (bB)^{-1}\Gamma_0$ and $\tilde{A} = (bB)^{-1}A$. The assumption of $\tilde{A}$ being diagonal implies the absence of mutual dependence in the production process ($\alpha_{43} = 0$) and in the adjustment costs ($\beta_3 = 0$). This "diagonal" model, in which the dynamic demand curves are separable, has been analysed by Sargent (1978) and Palm and Pfann (1987) among others. Sargent's model is diagonal for a specific reason, i.e. he distinguishes between straight- and overtime work.

In the sequel of this section, however, we assume $\tilde{A}$ to be a regular (2×2) -matrix. We have the following result.

Proposition 1 : Let A be the regular matrix as defined in (2.6). Let further $\tilde{A}$ be a regular (2×2) -matrix with eigenvalues μ_1 and μ_2 . Then the characteristic polynomial of the matrix A can be written as $(-\lambda^2 + \mu_1\lambda - b^{-1})(-\lambda^2 + \mu_2\lambda - b^{-1})$.

Proof : See Appendix II.

According to proposition 1 the eigenvalues of matrix A can be found analytically as the roots of a product of two quadratic equations.

Proposition 2 : For the quadratic equation $\lambda^2 - \mu\lambda - \delta = 0$ to have one real root λ_a with $|\lambda_a| \leq 1$, and one real root λ_b with $|\lambda_b| > 1$, the following conditions are necessary and sufficient

(i) $\mu^2 + 4\delta \geq 0$

(ii) $(1-\delta)^2 < \mu^2$ or $(1-\delta)^2 = \mu^2$ together with $\delta^2 > 1$.

Proof : Trivial.

If the roots of the characteristics equation of Δ satisfy the conditions of proposition 2, the diagonal (4x4)-matrix Λ of eigenvalues of Δ can be

partitioned as $\Lambda = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix}$, where Λ_1 is a diagonal matrix with eigen-

values λ_1 and λ_2 smaller than or equal to one in absolute value. The diagonal elements of Λ_2 consist of the eigenvalues λ_3 and λ_4 of Δ that lie outside the unit circle. The number of elements of $\text{diag}(\Lambda_2)$ equals the number of stationary forward looking solutions of the difference equation (2.5). When the objective function is quadratic, the number of elements of $\text{diag}(\Lambda_1)$ equals that of $\text{diag}(\Lambda_2)$ (see Hansen and Sargent (1981), Kollintzas (1985)).

The unique stationary forward looking solution of the "diagonal" model yields

$$(2.7) \quad n_t = \Lambda_1 n_{t-1} - \sum_{i=0}^{\infty} \Lambda_2^{-i-1} \Gamma E_t z_{t+i}.$$

Note that $b\Lambda_1 = \Lambda_2^{-1}$. The solution is expressed in terms of the eigenvalues of Δ which are functions of the parameters of the underlying theoretical model.

The difference between the "diagonal" model and the model in (2.5) shows up in the dynamic interrelation of labour demand decisions which is explicitly

modeled in the latter one. If $\mu_1 \neq \mu_2$ the matrix Δ in (2.6) can be transformed such that $\tilde{A}$ is transformed into the diagonal matrix $S^{-1}\tilde{A}S$ with the columns of S being the eigenvectors of $\tilde{A}$. If C is the matrix of the eigenvectors of $S^{-1}\Delta S$ and C is partitioned according to $\tilde{\Delta}$, we have $S^{-1}\Delta S = C\tilde{\Delta}C^{-1}$ or $\Delta = (S\ C)\tilde{\Delta}(S\ C)^{-1}$, where S , C and C^{-1} are (4×4) -matrices decomposed accordingly

$$S = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix}; \quad C = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}; \quad C^{-1} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}.$$

Now A is written in its Jordan canonical form, where $\tilde{A}$ is diagonalized.

The unique stationary forward looking solution of the expected variables in the linear difference equation (2.5) can now be written as follows

$$(2.8) \quad n_t = \Lambda_1 n_{t-1} - (SC_{22})^{-1} \sum_{i=0}^{\infty} \Lambda_2^{-i-1} SC_{22} \Gamma E_t z_{t+i},$$

since $-(SC_{22})^{-1}(SC_{21}) = \Lambda_1$.

Expression (2.8) is the CFS of the labour demand equations for an interrelated factor input decision model under uncertainty. The current employment of blue and white collar workers is linearly related to respectively blue and white collar employment lagged one period, and the expectations of wage costs, capital stock and exogenous shocks in productivity over an infinite time horizon. The parameters of (2.8) are nonlinear functions of the structural parameters of the theoretical model (2.4).

The implications of the diagonal structure of $\tilde{\Lambda}_1$ in (2.8), which is a testable feature of the above model, for identification, stability, and comparative dynamic properties of the model have been thoroughly discussed by Kollintzas (1985). In the present framework it means that interrelation between two production factors through mutual dependence in the production

process and in adjustment costs ($\alpha_{43} \neq 0$, $\beta_3 \neq 0$) does not lead to dynamically interrelated factor demand equations. Therefore, the arguments of Nadiri and Rosen (1969) concerning the dynamic interrelationship between employment decisions do not apply in the present context.

The second of the right hand side term of (2.8) can be expressed as

$$(2.9) \quad -(SC_{22})^{-1} \sum_{i=0}^{\infty} \Lambda_2^{-i-1} SC_{22} \Gamma E_t z_{t+i} =$$

$$\delta_0 + \sum_{i=0}^{\infty} H^{-i-1} (bB)^{-1} E_t s_{t+i}$$

$$- \sum_{i=0}^{\infty} H^{-i-1} (bB)^{-1} E_t w_{t+i}$$

$$+ \sum_{i=0}^{\infty} H^{-i-1} (bB)^{-1} a_2 E_t k_{t+i},$$

where $H^{-i-1} = (SC_{22})^{-1} \Lambda_2^{-i-1} (SC_{22})$, and δ_0 is a (2×1) -vector composed of the unconditional means and deterministic fluctuations of marginal labour productivity.

Now we make assumptions on the stochastic properties of the forcing variables s_t , w_t and k_t . It is probably more realistic to assume that expectations are based on the extrapolative time series representation of the law of motion of the exogenous variables than on specifications that include all available information on relevant determinants of these variables. Therefore, the stochastic components of labour productivity s_t are assumed to be represented by a stationary first order Markov process $E_t s_{t+1} = \tilde{R} s_t$, where $\tilde{R}$ is a (2×2) -matrix. Let Q_s be a (2×2) -matrix, such that $Q_s s_t$ is a stationary AR(1) process with innovation ξ_t^s

$$(2.10) \quad Q_s s_t = R Q_s s_{t-1} + \xi_t^s,$$

where $R = Q_s \tilde{R} Q_s^{-1}$, is a stationary (2×2) -matrix.

Proposition 3 : Let $Q_s = \sum_{i=0}^{\infty} (SC_{22})^{-1} \Lambda_2^{-i-1} SC_{22} (bB)^{-1} \tilde{R}^i$. Then $\text{vec } (Q_s) = [I_2 \otimes [(SC_{22})^{-1} \Lambda_2 SC_{22}] - \tilde{R}^T \otimes I_2]^{-1} \text{vec}((bB)^{-1})$.

Proof : See Appendix II.

The results of proposition 3 enable us to find an explicit solution of $\sum_{i=1}^{\infty} H^{-i-1} (bB)^{-1} E_t s_{t+i}$ in terms of the structural parameters.

In accordance with the literature on dynamic labour demand, real wage costs are assumed to be described by a bivariate autoregressive process. The order of the process has been determined prior to estimation of the model in CFS. A justification of the prior analysis is given in the next section; w_t is described by a bivariate second order autoregression

$$(2.11) \quad w_t = \delta_w + M_1 w_{t-1} + M_2 w_{t-2} + \xi_t^w,$$

where δ_w is a (2×1) -vector of constants, M_1 and M_2 are (2×2) -matrices and ξ_t^w is a (2×1) -vector of innovations of the wage costs. The second r.h.s. term of (2.9) becomes

$$\sum_{i=0}^{\infty} H^{-i-1} (bB)^{-1} E_t w_{t+i} = P_1 w_1 + P_2 w_{t-1}$$

with $P_1 = H^{-1} (bB)^{-1} V$ and $P_2 = H^{-2} (bB)^{-1} V M_2$, where V is a (2×2) -matrix such that $H V M_1 + H^2 V M_2 = I_2$.

Finally, empirical evidence that will be presented in the next section shows that the first difference of the predetermined capital stock can be represented as a random walk with drift

$$(2.12) \quad \Delta k_t = \delta_k + \xi_t^k.$$

Hence, an explicit solution for the third r.h.s. term in (2.9) is

$$(H - I_2)^{-1}(bB)^{-1}a_2k_t.$$

Elimination of the autocorrelation of $Q_s s_t$ is achieved using a Koyck transformation. We substitute the expressions for the expected values of s_t , w_t and k_t into (2.8), assuming that the lagged values of the forcing variables are not correlated with the composite disturbance term ξ_t^n . Applying proposition 3, the reduced form of the labour demand equations becomes

$$(2.13) \quad n_t = \delta + (\hat{A}_1 + R)n_{t-1} - R\hat{A}_1 n_{t-2} + Ck_{t-1} \\ + (P_1 M_1 - RP_1 + P_2)w_{t-1} + (P_1 M_2 - RP_2)w_{t-2} + \xi_t^n,$$

where $C = (I_2 - R)(H - I_2)^{-1}(bB)^{-1}a_2$, $\delta = \delta(\delta_0, \delta_w, \delta_k)$, and $\xi_t^n = \xi_t^n(\xi_t^s, \xi_t^w, \xi_t^k)$.

Labour demand innovations, ξ_t^n , is linearly related to unanticipated changes in labour productivity (ξ_t^s), wage costs (ξ_t^w) and capital (ξ_t^k). From (2.13) it follows that dynamic interrelation in the autoregressive part of the employment equations can only result from the serial cross-correlation in the exogenous productivity shocks. Or, stated more formally, when the objective function is quadratic, interrelation between decisions on the current level of employment of blue and white collar workers occurs if and only if the matrix R of the process for the stochastic components of labour productivity is nondiagonal. If R is a diagonal matrix, both labour demand equations are dynamically interrelated via the capital stock and the wages only. Consequently, the assumption in Meese (1980) of marginal labour productivity being white noise implies that the parameters of interrelation in the reduced form equations can not be identified.

The system of equations (2.11) and (2.13) yields a complete linear multivariate model which can be empirically analysed. The disturbance term $(\xi_t^w, \xi_t^n)'$ is assumed to be normally distributed with mean zero and covariance matrix Σ , which, as a result of the mutual influences of unanticipated exogenous movements of productivity, wage costs, and capital, is assumed to be unrestricted except for symmetry and positive definiteness.

3. AN EMPIRICAL ANALYSIS

In this section we apply the model for interrelated labour demand in CFS to quarterly data of the Dutch manufacturing sector for the period 1971.I to 1984.IV. All series are seasonally unadjusted. The data are obtained from the Central Bureau of Statistics and the Central Planning Bureau. A description of the series and the data are given in Appendix I. Labour demand has been disaggregated into the demand for blue and white collar labour, both expressed in terms of working hours times the number of workers for each category respectively. The effects of possible substitution between hours worked and persons employed has not been taken into account.

Prior to the econometric analysis of labour demand equations, we have tested for Granger causality between wage costs and employment using a linear dynamic model in which the model consisting of (2.11) and (2.13) is nested. Possible causal relations between wage costs and employment were examined separately for each sort of labour by means of a bivariate AR(4)-model, including k_{t-1} , a constant term and seasonal dummy variables in the equations. The chi-square values of leaving four lags of employment out of the wage costs equation were 5.79 and 8.61 (p-values .22 and .075) for blue and white collar labour respectively.

Leaving the four lags of wage costs out of the employment equation yielded chi-square values of 14.23 and 10.66 (p-values .006 and .03) respectively. Consequently, the hypothesis of unidirectional causality from real wage costs to employment is found to be in accordance with the empirical evidence when each type of labour is considered separately. When we consider blue and white collar labour simultaneously we also find stronger evidence in favour of the unidirectional relationship. However, the significance of these tests is not so clear-cut. Chi-square values for joint causality tests with 16 degrees of freedom on a four variate AR(4) model for the four

series, conditional on k_{t-1} , the constant term and the seasonal dummies, amounted to 26.24 (p-value .051) when unidirectional causality from wage costs to employment is tested, and 40.10 (p-value .00) when unidirectional causality of employment to wage costs is analysed. These findings have to be taken with some caution, as later results on the nonstationarity of the underlying variables suggest that none of the statistics reported have classical asymptotic distributions, but rather are functions of Wiener processes and hence have Dickey-Fuller type distributions.

The unidirectional causality assumption enables us to model the dynamic interactions between real wage costs and employment along the lines outlined in the previous section. Moreover, then the wage costs equations can be analysed independently from the employment data. Fuller's τ_t statistic testing for unit roots in w_{1t} and w_{2t} applied to an AR(2) model with constant term yield -1.27 ($\bar{R}^2 = 0.99$) and -2.09 ($\bar{R}^2 = 0.99$), which does not lead to the rejection of the hypothesis of a unit root in both cases.

Bivariate identification of the wage costs series has been carried out by means of a computer package provided by Liu et al. (1986). The results for the first differences of the cost series are presented in Table 2. First differencing yields approximately stationary wage series. Standard errors (s.e.) are given between parentheses. Stationary vector time series can be usefully described by their cross correlation (CCM) (see for further details e.g. Tiao and Box (1981)). A χ^2 -test for the lag length, and the Akaike Information Criteria (AIC) can be used to determine the order of an autoregressive model with constant parameters.

The CCMs of Δw_t cut off after one lag. The χ^2 -statistic and AIC suggest an AR(1) process. Since after one lag the autocorrelations of Δw_{1t} and Δw_{2t} die out and their partial autocorrelations equal zero, we tentatively

Table 2 : Identification of the Wage Costs Series.

I : BIVARIATE IDENTIFICATION OF $\Delta w_t = \Delta(w_{1t}, w_{2t})'$

The Cross Correlation Matrices of Δw_t

lag	1	2	3	4	5	6
CCM	.42* .40*	.13 .14	.04 .18	-.10 .07	-.09 -.16	-.14 -.20
	.41* .53*	.19 .26	.06 .17	-.01 .04	-.05 -.13	-.15 -.24

Criteria for Autoregressive Order Identification

lag	1	2	3	4	5	6
CHI(4)	18.21	1.90	5.94	2.59	3.23	1.85
AIC	-5.887	-5.788	-5.788	-5.711	-5.654	-5.564

II : UNIVARIATE ORDER SELECTION

Autocorrelations of Univariate Series

Δw_{1t} [mean = .143 (.285)]

lag	1	2	3	4	5	6	7	8
AC	.419	.125	.032	.109	-.081	-.143	-.027	-.047
s.e.	(.137)	(.160)	(.162)	(.162)	(.163)	(.164)	(.166)	(.166)

Δw_{2t} [mean = .234 (.460)]

lag	1	2	3	4	5	6	7	8
AC	.528	.252	.162	.017	-.156	-.250	-.254	-.113
s.e.	(.137)	(.171)	(.178)	(.181)	(.181)	(.184)	(.190)	(.196)

Partial Autocorrelations of Univariate Series

Δw_{1t} [s.e. = .137]

lag	1	2	3	4	5	6	7	8
PA	.950	-.057	-.006	-.078	-.062	-.036	-.006	-.030

Δw_{2t} [s.e. = .137]

lag	1	2	3	4	5	6	7	8
PA	.528	.037	.061	-.120	-.172	-.126	-.058	.136

"*" larger than $2n^{-1/2}$, where n denotes the number of observations.

select a bivariate AR(1) process for Δw_t with a diagonal first order autoregressive parameter matrix.

On the basis of the empirical findings, the model (2.11) becomes

$$(2.11)' \quad w_t = (I + M)w_{t-1} - Mw_{t-2} + \xi_t^w,$$

with $M = \text{diag}(m_1, m_2)$.

Fuller's τ_t statistic testing for a unit root in k_t applied to an AR(1) process with constant term yields -0.16 ($\bar{R}^2 = 0.99$). Therefore, we assume that k_t follows a random walk with drift, which leads to the explicit solution of the infinite future expectations of capital as presented in the previous section.

Consequently, the reduced form employment equations fitted to the data are as follows

$$(2.13)' \quad n_t = \delta(\text{CONST}, Q2, Q3, Q4) + (\hat{A}_1 + R)n_{t-1} - R\hat{A}_1 n_{t-2} \\ + Ck_{t-1} + (P_1[I+M] - RP_1 + P_2)w_{t-1} - (P_1M + RP_2)w_{t-2} + \xi_t^w,$$

where R is a nondiagonal (2×2) -matrix and $Q2, Q3, Q4$ denote seasonal dummies for the second, third and fourth quarter, respectively.

The identified structural parameters of the model presented in section 2 are estimated using a two step estimation procedure proposed by Chamberlain (1982) and Gouriéroux et al. (1985). In a first stage, an unrestricted version of the reduced form of the model is estimated that incorporates the restrictions on the order of the system implied by the underlying theory. This model, that will be referred to as model 1, is equivalent to the model of (2.11)' and (2.13)' in which the autoregressive matrices $\hat{A}_1 + R$, $R\hat{A}_1$, $P_1[I+M] - RP_1 + P_2$ and $P_1M + RP_2$ are assumed to be unrestricted nondiagonal (2×2) -matrices, and C is a (2×1) -vector of unrestricted parameters. Parameter estimates of model 1 are obtained by maximum likelihood, and are given in table 3a. In the second stage, the structural parameters are

Table 3a : Maximum Likelihood Estimates of the Unrestricted Model 1 ^{*}.

	Blue Collar Workers	White Collar Workers
Wage Costs	$m_1 = 0.44 \quad (5.48)$	$m_2 = 0.58 \quad (4.25)$
Employment	$n_{1t-1} = 1.27 \quad (8.75)$	$n_{1t-1} = -0.10 \quad (-1.42)$
	$n_{1t-2} = -0.29 \quad (-2.70)$	$n_{1t-2} = 0.16 \quad (3.06)$
	$n_{2t-1} = -1.12 \quad (-3.47)$	$n_{2t-1} = 1.22 \quad (7.69)$
	$n_{2t-2} = 0.99 \quad (2.89)$	$n_{2t-2} = -0.28 \quad (-1.64)$
	$w_{1t-1} = -0.13 \quad (-1.64)$	$w_{1t-1} = -0.06 \quad (-1.50)$
	$w_{1t-2} = 0.07 \quad (0.88)$	$w_{1t-2} = 0.001 \quad (0.37)$
	$w_{2t-1} = -0.09 \quad (-1.50)$	$w_{2t-1} = -0.002 \quad (-0.70)$
	$w_{2t-2} = 0.16 \quad (2.73)$	$w_{2t-2} = 0.07 \quad (2.57)$
	$k_{t-1} = -0.03 \quad (-1.31)$	$k_{t-1} = 0.01 \quad (0.54)$

* Asymptotic t-values are given within parentheses. The estimated coefficients of constants and seasonal dummies are not reported in the table.

Table 3b : Asymptotic Least Squares Structural Parameter Estimates when

$\beta_3 = 0$, $\alpha_{43} = 0$, and $b = 0.98$ *.

	Blue Collar Workers	White Collar Workers
Wage Costs	$m_1 = 0.48 \quad (4.98)$	$m_2 = 0.53 \quad (3.83)$
Technology	$\alpha_{12} = 1.08 \quad (2.61)$ $\alpha_{41} = 0.41 \quad (1.49)$	$\alpha_{22} = 1.17 \quad (2.40)$ $\alpha_{42} = 0.20 \quad (1.12)$
Adjustment Costs	$\beta_1 = 34.31 \quad (5.89)$	$\beta_2 = 85.07 \quad (4.31)$
Serial Correlation R of Productivity Shocks	$\rho_{11} = 0.51 \quad (2.40)$ $\rho_{21} = -0.10 \quad (-0.84)$	$\rho_{12} = -0.81 \quad (-1.59)$ $\rho_{22} = 0.62 \quad (1.92)$

* Asymptotic t-values are given within parentheses.

estimated using the method of Asymptotic Least Squares (ALS) which minimizes the distance between the unrestricted reduced form estimates and the reduced form coefficients expressed as functions of the identified structural parameters. The ALS procedure is asymptotically equivalent to Maximum Likelihood (ML) estimation methods, provided ML estimates of the parameter estimates from the first stage are used together with an optimal weighting matrix and all restrictions between structural parameters and reduced form coefficients have been imposed. In order to obtain the weighting matrix the derivatives of the nonlinear relationships between the parameters estimated in the first stage and the structural parameters are needed.

Since the capital stock is assumed to be predetermined (in the short run), the technology parameters α_{31} and α_{32} in (2.1) are not identified; α_{43} and β_3 are not identified separately either. We impose the prior restrictions $\beta_3 = 0$ and $\alpha_{43} = 0$; the parameter b was chosen a priori to be 0.98 for quarterly data. ALS estimates of α_{12} , α_{22} , α_{41} , α_{42} , β_1 , β_2 , and the elements of the (2×2) -matrices R and M are obtained from the set of implicit relationships relating the 20 parameter estimates of the unrestricted model 1 to the 12 structural parameters of the model in CFS. Estimation results are given in table 3b.

As a byproduct, we computed a WALD statistic testing the appropriateness of the overidentifying restrictions imposed by the underlying theory. We compare the model of the form of (2.11)' and (2.13)' in which all the parameters are functions of the 12 structural parameters with model 1. (We abstract from the parameters of the seasonal dummies which have been included in both models). The test, which is equivalent to the standard LR test on overidentifying restrictions, is asymptotically $\chi^2(8)$ -distributed and yields a value of 10.80 (p-value 0.22).

The estimates of the serial correlation matrix R can be interpreted as measures of the speed of adjustment of productivity of both types of labour to unforeseen productivity shocks. For instance, a positive shock in product demand implying an increase in ξ_t^S is initially absorbed by a large change of blue collar productivity compared with that of white collar labour, in such a way that the discounted effect of both types of labour productivity is positive. This result is a first indication that blue collar labour is more flexibly adjusted than white collar labour. The estimates of the adjustment costs parameters β_1 and β_2 indicate that white collar labour is more quasi-fixed than blue collar labour. Since the hiring procedure will be more expensive, and white collar workers is expected to need more job-specific training, it is plausible that the costs of adjustment (hiring and/or firing) will be much higher for white collar labour than for blue collar labour.

The technology parameters are found to be positive. These findings are in agreement with the assumptions underlying the theoretical model, i.e. technology, adjustment costs and the process for productivity shocks are specified such that the optimality conditions (concavity of production technology, convexity of adjustment costs, both locally approximated using quadratic shapes, and stationarity of productivity shocks) are satisfied.

Using proposition 1 and the structural parameter estimates from table 3 we can compute $\lambda_1 = 0.85$ and $\lambda_2 = 0.90$, and the eigenvalues of $\tilde{A}$, sc. $\mu_1 = (b^{-1} + \lambda_1^2)/\lambda_1 = 2.05$ and $\mu_2 = (b^{-1} + \lambda_2^2)/\lambda_2 = 2.03$. Both the eigenvalues satisfy the necessary and sufficient conditions of proposition 2. Moreover, the estimates of the diagonal elements of Λ_2 , $\lambda_3 = \frac{1}{2}(\mu_1 + \sqrt{\mu_1^2 - 4/b}) = 1.20$ and $\lambda_4 = \frac{1}{2}(\mu_2 + \sqrt{\mu_2^2 - 4/b}) = 1.14$ lie outside the unit circle.

In addition, we compute a LR test comparing model 1 with less complex separate dynamic wage and employment curves. The model with separate demand curves will be referred to as model 2, and is equivalent to the model of (2.11)' and (2.13)' in which the autoregressive matrices $\Lambda_1 + R$, $R\Lambda_1$, $P_1[I+M]-RP_1+P_2$ and P_1M+RP_2 are assumed to be unrestricted diagonal matrices and C is a (2×1) -vector of unrestricted parameters. The LR test is asymptotically $\chi^2(8)$ -distributed and yields a value of 42.20 (p-value 0.00).

The multivariate portmanteau test suggested by Hosking (1980) is a consistent LM statistic for residual autocorrelation. Table 4 contains LM tests for the model (2.11)', (2.13)' for model 1, being the unrestricted version of the model with interrelated employment equations in CFS, and for model 2. The LM test has an asymptotic chi-squared distribution with degrees of freedom that depend on s , the order of autocorrelated lags, the dimension of the model (4), and the number of estimated autoregressive parameters.

Table 4 : Multivariate Lagrange Multiplier Tests on Residual Autocorrelation.

s	Equations (2.11)' (2.13)'		Model 1		Model 2	
	LM	$\chi^2(.95)$	LM	$\chi^2(.95)$	LM	$\chi^2(.95)$
3	40.19	46.19	37.22	43.77	53.93	48.60
4	56.96	65.17	52.45	62.83	66.85	67.51
5	66.64	83.68	61.37	81.38	82.09	85.97
10	119.52	173.00	124.15	170.81	139.46	175.20
15	210.76	259.92	206.62	257.61	241.03	262.07
20	293.02	345.67	280.08	343.53	322.08	347.80

From Table 4 we see that the inclusion of interrelation in labour demand decisions removes the lower order residual autocorrelation. On the whole, we find that the interrelated model is superior to the diagonal model. To summarize, we may conclude that dynamic interrelation between the demand of blue and white collar labour is probably the result of the serial correlation of productivity shocks, and that the inclusion of interrelations between the two types of labour demand has improved the model significantly. The overidentifying restrictions of the rational expectations model, implied by underlying economic theory do seem to be corroborated by the sample evidence.

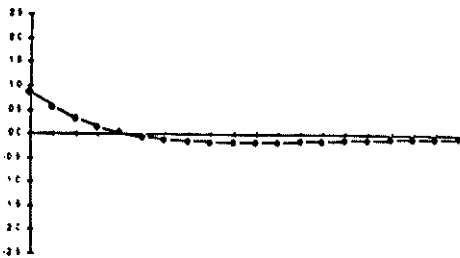
In order to get a better insight into the dynamic properties of the model we compute a moving average representation in terms of orthogonal disturbances of the model in CFS. Figure 1 shows the impulse responses of the demand for blue and white collar labour to a unit innovation of blue and white collar employment respectively. The impulse response traces out the reaction of a firm to uncorrelated shocks in the two demand functions. We note that ξ^n is a linear combination of ξ^s , ξ^w , ξ^k , such that a positive unit innovation in ξ^n can be interpreted as a positive shock in labour productivity or capital, or as a negative shock in real wage costs.

The impulse responses to innovations of labour input differ between the two types. A positive shock in the demand for blue collar workers lead to a rise in the employment of blue as well as white collar labour. The estimated response of n_1 is stronger but shorter, while the estimated response of n_2 is more moderate but lasts longer due to higher adjustment costs. This result could be interpreted as follows. The impact of a shock damps out very quickly with employment returning to its original level. Conversely, if the hiring of white collar labour is costly, the adjustment

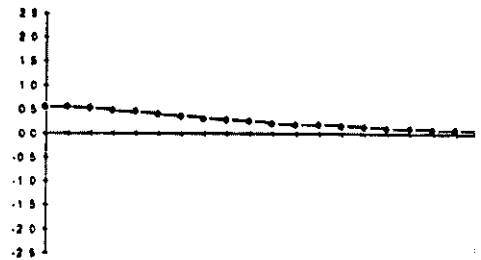
to a shock negatively affects employment of blue collar workers. This leads to the conclusion that on the one hand white collar labour is complementary to blue collar labour. When more workers are hired, more supervisors will be needed. On the other hand, a single upswing in white collar labour demand leads to a displacement effect on the employment of blue collar workers. The firm's work force is upgraded. Blue collar labour can be substituted by white collar labour.

Figure 1 : Impulse response of n_1 and n_2 to innovations in ξ^{n1} and ξ^{n2} .

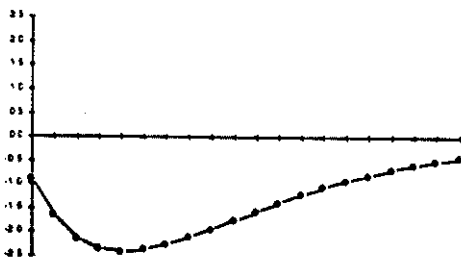
The Impulse response of n_1 to innovations in ξ^{n1}



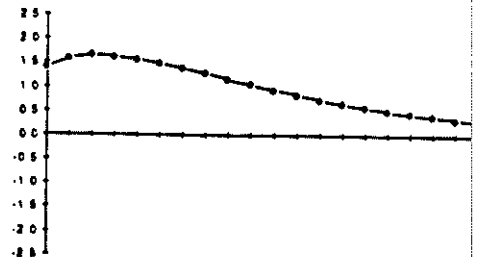
Impulse response of n_2 to innovations in ξ^{n1}



The Impulse response of n_1 to innovations in ξ^{n2}



Impulse response of n_2 to innovations in ξ^{n2}



4. CONCLUSIONS

In this paper we derived and estimated dynamic interrelated labour demand curves for blue and white collar labour, assuming that firms maximize the real present value of profits under rational expectations formation. The driving forces of the labour demand model are the predetermined capital stock, technological innovations, and the real wage costs of blue and white collar labour. We showed that in a linear-quadratic structural model cross dependence of lagged labour input in employment equations does not result from mutual trade-offs in production or in adjustment costs. Interrelation results from the impact of nonzero serial cross-correlation of exogenous shocks on the firm's employment policy outcome, and proves to be essential in appropriately analysing the dynamic structure of the two types of labour in the manufacturing sector in the Netherlands.

The relationships between the parameters of the unrestricted model in closed form solution and the structural parameters of the underlying economic theory have been made explicit. Although these relationships are highly nonlinear and overidentifying, we obtained consistent and asymptotically efficient estimates of the structural parameters of the model.

The econometric analysis of the model, using quarterly time series data for the Dutch manufacturing sector, confirmed an earlier finding by e.g. Nadiri and Rosen (1969) and Pindyck and Rotemberg (1983b) that white collar labour is more quasi-fixed than blue collar labour due to higher adjustment costs. The moving average representation of the estimated model indicated that it is optimal for the firm to raise the number of white collar workers employed when increasing its blue collar workforce. On the other hand, a single upswing in white collar employment has a displacement effect on the blue collar workers.

The present model can easily be extended to include more than two interrelated inputs. For instance, the capital stock can be introduced as a decision variable. Application of such a model is expected to give more insights into the relation between quasi-fixed white collar labour and the capital stock, which is fixed in the short-run. This question has not been addressed in this research.

Matters concerned with the adjustment of labour input and the flexibility of working hours in reaction to unforeseen shocks in the environment of the firm are not considered either. Another interesting subject of analysis is concerned with the stability of the firm's decision rule when expectations are rational. Do all the parameters of this process remain unchanged when a shift in product demand or factor prices occurs which was not anticipated? To put it differently, does the firm interpret an unexpected change in its environment as an innovation or as a structural change in the process generating the exogenous variables? To what extent does the feed-forward model with rational expectations encompass conditional (feedback) alternatives. For more details, we refer to Hendry (1988). The model could be subjected to a more extensive evaluation. A thorough discussion of the evaluation of empirical models can be found in Hendry (1987) and some references therein.

Finally, the assumption of symmetry of the adjustment costs needs to be reconsidered. Most authors agree since long that it is very unlikely that adjustment costs are symmetric, but for computational ease they assume that these costs are quadratic (see e.g. Holt et al. (1960), Nickell (1986)). From interviews with personnel managers of several multinational Dutch corporations, and from simple regression analysis on adjustment costs data from annual reports of Dutch manufacturing companies it has become obvious that adjustment costs are asymmetric (see Pfann and Verspagen (1989)). We are currently investigating possibilities to model asymmetric adjustment costs in a rational expectations context.

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APPENDIX Ia : Sources of the data.

The quarterly series on gross hourly wage costs for blue and white collar workers for the Dutch manufacturing sector (SBI II-III) have been obtained from the Halfjaarlijks Loononderzoek, first published by the CBS in the Sociale Maandstatistiek april 1976, the Driejaarlijks Loonkostenonderzoek, which runs from 1972, and the Kwartaalonderzoek Verdiende Lonen for the period 1964.II - 1985.IV, CBS, Staatsuitgeverij 's-Gravenhage.

The quarterly series on real wage costs for blue and white collar workers have been obtained by deflating the gross wage costs series per category of workers by the quarterly Producentenprijsindexcijfers van de binnenlandse afzet van de Nederlandse Industrie which are unpublished data that we have obtained from CBS, 's-Gravenhage. The real wage costs are expressed in hundreds of guilders per hour worked.

The quarterly series on employment for blue and white collar workers in the Dutch manufacturing sector have been computed from quarterly indices of total employment, published in Centraal Bureau voor de Statistiek (CBS), Algemene Industrie Statistiek, Staatsuitgeverij, 's-Gravenhage, and the ratio of blue and white collar workers in this sector. This last series has been computed as a linear interpolation of ratios obtained from the Loonstruktuur Onderzoek 1972 and the Arbeidskrachtentelling 1975, 1977, 1979, 1981 en 1983, published by CBS, Staatsuitgeverij, 's-Gravenhage.

The quarterly series on average weekly working hours including overtime hours for blue and white collar workers have been obtained from the Halfjaarlijks Loononderzoek, published by CBS. These series are half-yearly data which have been interpolated.

The quarterly series on labour demand are products of the employment series and the series on average weekly working hours. The labour demand series are expressed in tens of millions of average weekly hours.

The quarterly series on capital stock in the Dutch manufacturing sector have been computed from annual data of the capital stock in the Dutch manufacturing sector, and quarterly data of the average capital stock in the Netherlands obtained from the Central Planning Bureau.

The capital stock data are expressed in hundreds of billions of 1983 Dutch guilders.

APPENDIX Ib : The quarterly series.

	W ₁	W ₂	N ₁	N ₂	CAP
1971:1	0.18079	0.24563	0.35066	0.13043	0.66564
1971:2	0.18464	0.24898	0.34598	0.13022	0.68073
1971:3	0.18942	0.25324	0.34471	0.13136	0.69701
1971:4	0.18997	0.27053	0.33555	0.13016	0.70348
1972:1	0.19043	0.28129	0.32864	0.12979	0.71244
1972:2	0.19088	0.29204	0.32427	0.12968	0.72396
1972:3	0.19290	0.29544	0.32297	0.13077	0.73639
1972:4	0.19574	0.29883	0.31793	0.13084	0.74133
1973:1	0.19787	0.30442	0.31324	0.13100	0.74808
1973:2	0.20000	0.30340	0.31150	0.13233	0.75677
1973:3	0.19973	0.29955	0.30976	0.13366	0.76614
1973:4	0.19946	0.30066	0.30583	0.13422	0.76986
1974:1	0.20027	0.29896	0.30193	0.13477	0.77820
1974:2	0.19764	0.29725	0.29585	0.13419	0.78892
1974:3	0.19602	0.30032	0.29531	0.13643	0.80049
1974:4	0.20405	0.30646	0.28844	0.13617	0.80508
1975:1	0.20822	0.31302	0.27859	0.13445	0.81013
1975:2	0.21578	0.32303	0.27456	0.13380	0.81367
1975:3	0.21763	0.32343	0.27138	0.13271	0.81420
1975:4	0.21953	0.32606	0.26875	0.13093	0.82562
1976:1	0.22143	0.32869	0.26633	0.12856	0.82834
1976:2	0.21938	0.32497	0.26574	0.12589	0.83437
1976:3	0.22151	0.32714	0.26787	0.12459	0.83709
1976:4	0.22576	0.33147	0.26516	0.12202	0.84031
1977:1	0.22774	0.33466	0.26263	0.11969	0.84511
1977:2	0.22972	0.33785	0.26352	0.11927	0.84974
1977:3	0.23299	0.34147	0.26409	0.11918	0.85179
1977:4	0.23818	0.35008	0.26122	0.11750	0.85222
1978:1	0.24208	0.35534	0.25651	0.11522	0.85723
1978:2	0.24598	0.36059	0.25646	0.11526	0.86287
1978:3	0.24836	0.36206	0.25642	0.11530	0.86585
1978:4	0.25487	0.37115	0.25359	0.11405	0.87512
1979:1	0.26138	0.38024	0.25355	0.11409	0.87664
1979:2	0.25859	0.37630	0.25071	0.11298	0.88174
1979:3	0.25517	0.36986	0.25350	0.11445	0.88638
1979:4	0.25345	0.36769	0.25036	0.11302	0.89459
1980:1	0.25172	0.36552	0.24834	0.11215	0.90116
1980:2	0.24875	0.36124	0.24772	0.11195	0.90700
1980:3	0.25026	0.36063	0.24883	0.11248	0.91771
1980:4	0.25178	0.36001	0.24511	0.11111	0.91881
1981:1	0.24886	0.35877	0.24084	0.10950	0.92630
1981:2	0.24593	0.35753	0.23913	0.10877	0.93415
1981:3	0.24592	0.35691	0.23827	0.10841	0.93355
1981:4	0.24590	0.35629	0.23431	0.10667	0.94318
1982:1	0.25022	0.36334	0.23068	0.10502	0.96023
1982:2	0.25265	0.36761	0.22868	0.10418	0.96281
1982:3	0.25330	0.37032	0.22697	0.10345	0.96071
1982:4	0.25940	0.37575	0.22165	0.10119	0.97117
1983:1	0.26106	0.38194	0.21721	0.09929	0.98309
1983:2	0.26272	0.37906	0.21549	0.09845	0.98798
1983:3	0.25984	0.37665	0.21294	0.09720	0.99440
1983:4	0.25695	0.37423	0.20962	0.09572	1.00311
1984:1	0.25699	0.37548	0.20798	0.09484	1.01596
1984:2	0.25682	0.37596	0.20768	0.09488	1.02435
1984:3	0.25730	0.37521	0.20793	0.09504	1.03596
1984:4	0.25926	0.37446	0.20710	0.09480	1.04792

APPENDIX II

Proof of Proposition 1 :

Let $S = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix}$, where S is a regular (2×2) -matrix such that $S^{-1}\tilde{A}S$ is

uppertriangular with eigenvalues μ_1 and μ_2 of $\tilde{A}$ on the diagonal.

$$\text{Then } S^{-1} \Delta S = \begin{bmatrix} S^{-1} & 0 \\ 0 & S^{-1} \end{bmatrix} \begin{bmatrix} 0 & S \\ -b^{-1}S & \tilde{A}S \end{bmatrix} = \begin{bmatrix} 0 & I_2 \\ -b^{-1}I_2 & S^{-1}\tilde{A}S \end{bmatrix}.$$

Now, the characteristic polynomial of $S^{-1} \Delta S$ and consequently of Δ equals $(-\lambda^2 + \mu_1\lambda - b^{-1})(-\lambda^2 + \mu_2\lambda - b^{-1})$.

If $\mu_1 \neq \mu_2$ S can be chosen such that $S^{-1}\tilde{A}S$ is a diagonal. Then the columns of S are the eigenvectors of $\tilde{A}$.

Proof of Proposition 3 :

Redefine $H^{-1} = (SC_{22})^{-1} A_2^{-1} SC_{22}$, $\Gamma_1 = (bB)^{-1}$, and $Q_S = \sum_{i=0}^{\infty} H^{-i-1} \Gamma_1 \tilde{R}^i$.

Then $H = (SC_{22})^{-1} A_2 SC_{22}$ and $HQ_S - Q_S \tilde{R} = \Gamma_1 \quad \leftrightarrow$

$$\text{vec}(HQ_S) - \text{vec}(Q_S \tilde{R}) = \text{vec}(\Gamma_1) \quad \leftrightarrow$$

$$(I_2 \otimes H)\text{vec}(Q_S) - (\tilde{R}^T \otimes I_2)\text{vec}(Q_S) = \text{vec}(\Gamma_1) \quad \leftrightarrow$$

$$\text{vec}(Q_S) = [I_2 \otimes H - \tilde{R}^T \otimes I_2]^{-1} \text{vec}(\Gamma_1).$$