Relative Multi-Dimensional Poverty Measurement and Deaton's Distance Function*

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Abstract

In multi-dimensional poverty measurement, Deaton [1979] proposed assessing an individual's contribution to poverty as the fraction of the poverty line bundle to which the individual is indifferent. In this paper, we provide an axiomatic characterization of the set of measures that are concave transformations of Deaton's measure. This result is derived from two key axioms. The first is a transfer principle, which states that a progressive transfer between two poor individuals with homothetic preferences reduces poverty. The second axiom requires that an individual's contribution to poverty remains unchanged when their preferences over the poverty line bundle change, provided their preferences toward their own consumption remain the same.

JEL Classification: D63, I30

Keywords: multi-dimensional poverty measurement, well-being measurement, heterogeneous preferences, distance function.

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1 Introduction

Poverty and inequality are typically measured based on income (or total money value of consumption). It is, however, well acknowledged that income is not always a reliable proxy for well-being, particularly when individuals face different prices, markets are imperfect, certain goods are not marketable, or when individuals rely on subsistence farming and consume self-produced goods. This recognition has led researchers to assess well-being directly from consumption bundles. While some literature has opted to avoid references to well-being entirely – an approach considered appropriate when data do not permit the estimation of preferences [see, e.g., Alkire and Foster, 2011, Alkire et al., 2015] – a long-standing tradition, beginning with Deaton [1979] and others, measures well-being based on consumption and preferences.

In a seminal paper, Deaton [1979] proposed measuring an individual's contribution to poverty as the fraction of the poverty line bundle to which the individual is indifferent. This fraction is referred to as the distance function. The resulting measure of an individual's contribution to poverty – or, equivalently, their well-being – circumvents all the issues associated with using income.

In this paper, we offer an axiomatic characterization of the set of measures that are concave transforms of Deaton's distance function. Our axioms are based on three main ideas. First, an individual's contribution to poverty is easily measured when her preferences are homothetic and, therefore, representable by a homogenous-of-degree-one utility function [a recurrent idea used, for instance, in Blackorby and Donaldson, 1987, Fleurbaey and Tadenuma, 2014, Eden and Freitas, 2023]. Second, allocating resources to a poorer individual decreases poverty, in line with the well-known Pigou-Dalton transfer principle. Third, changes in preferences around the poverty line bundle do not modify the contribution to poverty of one individual whose consumption is strictly below it.

Because the Deaton's measure is consistent with preferences, contribution to poverty and well-being are two sides of the same coin.[1] Our result is therefore connected to Fleurbaey and Maniquet [2017], which offers an axiomatic characterization of a similar measure. This measure equates well-being with the fraction of a reference consumption to which an individual is indifferent. The result of Fleurbaey and Maniquet [2017], contrary to ours, leaves the reference bundle unspecified, so that it could differ from the poverty line bundle. Moreover, the axioms used in Fleurbaey and Maniquet [2017] are based on the lattice structure of the

[1]Throughout the text we will use the two concept interchangeably.

domain of indifference surfaces, whereas we don't exploit this structure here.

Our axiomatic analysis proposes a solution to the well-known problem of introducing inequality aversion within a context of heterogeneous preferences [Fleurbaey and Maniquet, 2011]. Our transfer axiom compares the well-being increases between two situations in which preferences are homothetic and the consumed bundles are proportional to each other. In such a comparison, well-being is required to increase more when an additional vector of resources, proportional to the two considered bundles, is allocated to the worse-off individual.

The above axiom is relatively weak but a mild extension, in which consumption bundles may not be proportional to each other, leads to an impossibility (as demonstrated in Theorem 2). As is typical in the axiomatic analysis of social indices, our proofs rely on a rich domain of preferences. Specifically, the domain must include sufficiently diverse preferences and feature two goods such that some individuals strongly prefer consuming one good, while others strongly prefer consuming the other. In empirical analyses based on parametric demand systems with preference shifters, achieving such diverse preferences may be challenging. However, when preferences are estimated non-parametrically, the domain of admissible preferences easily satisfies our richness requirement.

Our main result (Theorem 3) proposes a way to measure one's contribution to poverty but remains silent on how to aggregate individual contributions. At the risk of stating the obvious, this is a strength, not a weakness of the approach. Indeed a recent literature has studied the possibility of simultaneously building well-being measures and aggregating them [see, e.g., Fleurbaey and Maniquet, 2011, Piacquadio, 2017, Bosmans et al., 2018]. In spite of many possibility results, this literature has made it clear that imposing simplicity requirements on the well-being measure may have drastic consequences on the aggregator, such as imposing an infinite inequality aversion. This suggests to disentangle the two questions. As a result, our measure of contribution to poverty can be used as arguments of a poverty index, for instance an index in the Foster-Green-Thorbecke family [see Foster et al., 1984], or it can be used to measure the distribution of contributions to poverty, in a dominance approach such as in Decancq et al. [2019].[2]

The paper is organized as follows. In Section 2, we introduce the basic notation

[2]The measure we propose takes values in the $[0, 1]$ interval, following the idea that contribution to poverty, or well-being, is relative to the poverty line bundle. There is an alternative tradition, following the pioneer work on inequality of Kolm [1976a,b] and its extension to social welfare functions by Blackorby and Donaldson [1980], that measures well-being, and, therefore, poverty and inequality, in absolute terms. Our measure can be adjusted to this absolute approach as soon as an income level is unambiguously attached to the poverty line bundle. We do not study this question here.

and define the Deaton measure. In Section 3, we develop our axiomatic analysis, discuss the difficulty of imposing transfer principles when the preference domain is sufficiently rich, and derive our main result. In Section 4, we give some concluding comments.

2 Notation and definitions

We assume that there are K divisible and cardinally measurable goods, $K \geq 2$. By a slight abuse of notation, we let K denote both the number and the set of goods. The (closed) consumption set is $\mathcal{X} = \mathbb{R}^K_+$. Individuals have complete, transitive, continuous, convex and monotonic preferences over $\mathcal{X}$. For any two bundles x, $x' \in \mathcal{X}$ we denote weak preference of x over x' as $x\,R\,x'$ (x is at least as good as x'), strict preference of x over x' as xPx' (x is strictly preferred to x'), and indifference as xIx' (x is as good as x').

We also require the following monotonicity property. In words, it says that if strict monotonicity is violated along some axes, then it is violated everywhere along this axis. Observe that this is the case for homothetic preferences, and this is why we impose it for all preferences (see Theorem 3 below). We now define the restriction formally. Vector inequalities are denoted by $\leq, <, \ll$.[3] Our monotonicity requirement is: for all $K' \subseteq K$, if there exists $x, x' \in \mathcal{X}$ such that $x_\ell = x'_\ell = 0$ for all $\ell \in \{1, \ldots, K'\}$ and $x < x'$ and $x\,I\,x'$, then for all $x, x' \in \mathcal{X}$ such that $x_\ell = x'_\ell = 0$ for all $\ell \in \{1, \ldots, K'\}$, $x\,I\,x'$. We denote the set of such preferences by $\mathcal{R}$.

For any $x \in \mathcal{X}$ and $R \in \mathcal{R}$, we denote the lower, upper and indifference (closed) contours as follows $L(x, R) = \{x' \in \mathcal{X} | xRx'\}$, $U(x, R) = \{x' \in \mathcal{X} | x'Rx\}$, $I(x, R) = U(x, R) \cap L(x, R)$.

Our restriction to preferences (and, therefore, the absence of utility functions) underlines the fact that our well-being measures will only depend on ordinal information on individual choices. All information regarding happiness or subjective satisfaction is considered to be normatively irrelevant. An immediate consequence of this starting point is that we avoid the "happy peasants and miserable millionaires" paradox.

We assume that there is a poverty line bundle $y \in \mathbb{R}^K_{++}$. Accordingly, we aim to measure an individual's contribution to poverty, or equivalently, their well-being, as a function of their consumption $x \in \mathcal{X}$, and their preferences $R \in \mathcal{R}$. Let us

[3]That is for $x, x' \in \mathcal{X}$, we write $x \leq x'$ if $x_\ell \leq x'_\ell$ for all $\ell \in \{1, \ldots, K\}$, $x < x'$ if $x \leq x'$ and $x \neq x'$, $x \ll x'$ if $x_\ell < x'_\ell$ for all $\ell \in \{1, \ldots, K\}$.

refer to the pairs (x, R) as situations. Given our focus on poverty, we will only consider situations such that yRx. In other words, we are interested in measuring the well-being of poor individuals, which are individuals that prefer the poverty bundle y to their consumption bundle x [Decancq et al., 2019]. The reader may notice the connection between this and the usual Focus axiom in the poverty literature. Our results, however, do not depend on this restriction; we come back to this in Section 4.

Let us call *well-being measure* a function $W : \mathcal{X} \times \mathcal{R} \to \mathbb{R}$ that satisfies the following three minimal properties.

(i) *Continuity (Cont)*: for all $R \in \mathcal{R}$, $W(\cdot, R)$ is continuous in its first argument;

(ii) *Normalization (N)*: for all $R \in \mathcal{R}$, $W(0^K, R) = 0$ and $W(y, R) = 1$, where 0^K stands for the neutral element of the K-dimensional vector space;

(iii) *Consumption Monotonicity (Mx)*: for all $x, x' \in \mathcal{X}$ and $R \in \mathcal{R}$,

$$W(x, R) > W(x', R) \Leftrightarrow xPx'.$$

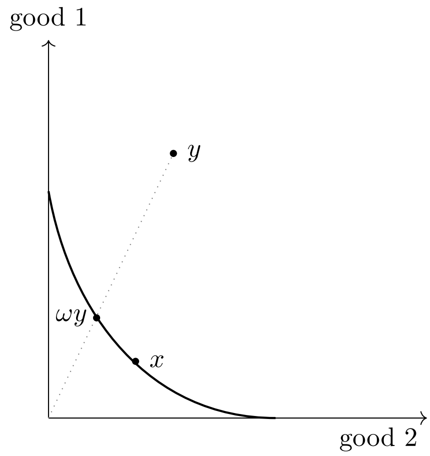
Continuity does not require explanations. Normalization introduces a key feature of multi-dimensional poverty measurement: the contribution to poverty of an individual consuming the poverty bundle is normalized to zero. Since we are defining well-being measures, the axiom normalizes the well-being associated to consuming the poverty line bundle (resp. nothing) to one (resp. zero), independently of individual preferences.

Consumption Monotonicity captures the natural intuition that well-being increases with consumption. Together with Continuity, it implies that $W(\cdot, R)$ is a continuous utility representation of R.

We conclude this section by introducing Deaton's measure, which will turn out to be key in defining the well-being measures characterized in the following section. According to Deaton's distance function [Deaton, 1979], also used by Samuelson [1977] under the name of ray utility, the well-being of an individual with preferences R consuming bundle x, is defined as the fraction of y which this individual finds x indifferent to (see Figure 1 for an illustration).[4]

[4] Observe that when preferences are homothetic, $W_D(\cdot, R)$ is the gauge function with respect to the upper contour set at y.

Figure 1: The Deaton's well-being measure.



Description: Deaton's well-being measure is the fraction of y that is indifferent to x: $W_D(x, R, y) = \omega$

Definition 1. *Deaton's well-being measure is a function* $W_D : \mathcal{X} \times \mathcal{R} \to \mathbb{R}$ *such that, for all* $(x, R) \in \mathcal{X} \times \mathcal{R}$

$$W_D(x, R) = \omega \Leftrightarrow \omega y \in I(x, R).$$

The reader may notice that, if the poverty line bundle y contains at least one entry equal to zero, then W_D fails to satisfy Consumption Monotonicity for weakly monotonic preferences. This is not an issue in this framework since the poverty bundle is assumed strictly positive: $y \in \mathbb{R}^K_{++}$.

The following section develops our axiomatic analysis aimed at characterizing a family of well-being measures for poor individuals. We complete this section with a discussion of the model. We consider that preferences are heterogenous, whereas the poverty line bundle, y, is the same for all. Having the poverty line depend on preferences is a possibility studied, for instance, by Dimri and Maniquet [2020]. We do not follow this route here for two reasons. First, it remains common practice in empirical works to adopt a single poverty line bundle, identical for all. Second, preference heterogeneity may arise from differences in needs, tastes, social norms, and so on. While differences in needs clearly suggest that the poverty line should vary, this seems less compelling when tastes are the only source of heterogeneity. This implies that even in a more comprehensive model – one that distinguishes between needs, tastes, and social norms – it is still necessary to address the case where the poverty line bundle is fixed across individuals.

3 Axiomatic analysis

3.1 Priority to the poorer

Our first main axiom captures the idea that allocating some additional resources to a poorer individual increases well-being more than if the same amount of resources is allocated to a richer individual. This idea echoes the famous Pigou-Dalton transfer principle imposed on inequality averse social aggregators. However, it is well-known that if the consumption-space is multi-dimensional and individual preferences are respected as in our model, a consumption monotonicity axiom like ours or the conventional Pareto axiom clash with Pigou-Dalton types of axioms if one tries to build social welfare function [see Blackorby and Donaldson, 1988, Weymark, 2017, Fleurbaey and Trannoy, 2003] or if one tries to build individual well-being or real income measures [see Van Veelen, 2002, Fleurbaey and Maniquet, 2018]. The following axiom does not lead to an impossibility because we restrict its application to (1) homothetic preferences and (2) bundles that are proportional to each other (see Figure 2 for an illustration).[5]

Let $\mathcal{R}^h \subset \mathcal{R}$ denote the subdomain of homothetic preferences, that is preferences satisfying the property xRx' if and only if $\lambda x R \lambda x'$ for all $x, x' \in \mathcal{X}$, $\lambda > 0$.

Priority to the Poorer on Proportional Bundles - For all $x, x' \in \mathcal{X}$, and $R, R' \in \mathcal{R}^h$, and all $\lambda \in (0, \frac{1}{2}]$, if x is proportional to x', $x \ll x'$ and

$$W(x + \lambda(x' - x), R) < W(x' - \lambda(x' - x), R'),$$

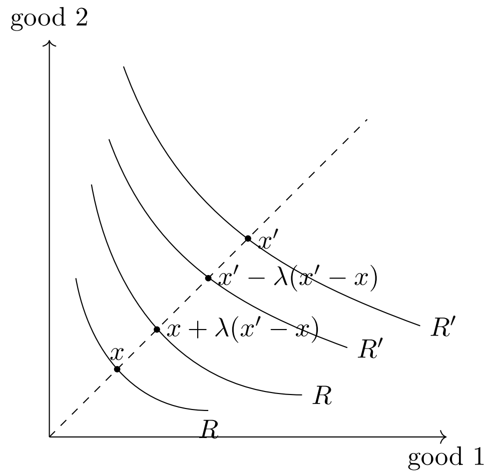
then

$$W(x + \lambda(x' - x), R) - W(x, R) > W(x', R') - W(x' - \lambda(x' - x), R').$$

Observe that $x \ll x'$ does not imply $W(x, R) < W(x', R')$ for any $R, R' \in \mathcal{R}^h$. The axiom is binding only if $W(x + \lambda(x' - x), R) < W(x' - \lambda(x' - x), R')$ which,

[5] In Van Veelen [2002], the respect of individual preferences takes a much weaker shape than the Pareto axiom, as it only requires that real income is different at one point for two different marginal rates of substitution. With our terminology, it can be defined as: there exist $x \in \mathcal{X}$, $R, R' \in \mathcal{R}$ with $mrs(x, R) \neq mrs(x, R')$ such that $w(x, R) \neq W(x, R')$, where $mrs(x, R)$ stands for the marginal rate of substitution vector at x for preferences R. The impossibility emerges when this requirement is combined with a continuity axiom that is weaker than ours and a strengthening of our Priority to the Poorer axiom that is valid for all bundles and all preferences.

Figure 2: Priority to the Poorer on Proportional Bundles



Description: If $W(x + \lambda(x' - x), R) < W(x' - \lambda(x' - x), R')$, then $W(x + \lambda(x' - x), R) - W(x, R) > W(x', R') - W(x' - \lambda(x' - x), R')$.

by Consumption Monotonicity implies $W(x, R) < W(x', R')$ for the given pair $R, R' \in \mathcal{R}^h$. It is therefore a very weak requirement.

This axiom is, nevertheless, sufficient to impose restrictions on how to measure well-being of individuals with heterogeneous preferences. Let $\mathcal{U}^h$ be the set of all homogeneous of degree one utility functions representing homothetic preferences. We can now state our first result.

Theorem 1. *A well-being measure W satisfies Consumption Monotonicity, Normalization, Continuity and Priority to the Poorer on Proportional Bundles if and only if there exists a strictly increasing and strictly concave function $f : \mathbb{R}_+ \to \mathbb{R}_+$ such that $f(0) = 0$ and $f(1) = 1$ and for all $x \in \mathcal{X}$, and $R \in \mathcal{R}^h$,*

$$W(x, R) = f\left(\frac{u(x)}{u(y)}\right)$$

where $u \in \mathcal{U}^h$ represents R.

Proof. (Only if) Let W be defined as in the statement of the theorem. Consumption Monotonicity follows from f being strictly increasing, Normalization follows from $f(0) = 0$ and $f(1) = 1$, and Continuity follows from f being continuous. We need to prove that W satisfies Priority to the Poorer on Proportional Bundles.

Let $x, x' \in \mathcal{X}$, $R, R' \in \mathcal{R}^h$, and $\lambda \in (0, \frac{1}{2}]$, be such that x is proportional to x', $x \ll x'$ and

$$W(x + \lambda(x' - x), R) < W(x' - \lambda(x' - x), R'). \tag{1}$$

Let w and w' be defined by $xIwy$ and $x'I'w'y$, so that $W(x, R) = f(w) < f(w') =$

$W(x', R')$. By homotheticity of the preferences, there exists $\delta > 0$ such that $(x + \lambda(x' - x))I(w + \delta)y$ and $(x' - \lambda(x' - x))I(w' - \delta)y$. By Eq. (1), $\delta \leq \frac{w'-w}{2}$. Therefore, strict concavity of f implies that $f(w+\delta) - f(w) > f(w') - f(w' - \delta)$, that is, $W((w + \delta)y, R) - W(wy, R) > W(w'y, R') - W((w' - \delta)y, R')$, and the desired result follows.

(If) Let W satisfy Consumption Monotonicity, Normalization, Continuity and Priority to the Poorer on Proportional Bundles. Let us choose $R \in \mathcal{R}^h$. Let us define $f_R : \mathbb{R}_+ \to \mathbb{R}_+$ by: for all $\lambda \geq 0$, $f_R(\lambda) = W(\lambda y, R)$. Observe that $\lambda = \frac{u(x)}{u(y)}$ for any $u \in \mathcal{U}^h$ representing R. By Normalization, $f_R(0) = 0$ and $f_R(1) = 1$. By Continuity, f_R is continuous. By Consumption Monotonicity, f_R is strictly increasing. By Priority to the Poorer on Proportional Bundles applied to bundles λy, f_R is strictly concave. Take any other $R' \in \mathcal{R}^h$ and define $f_{R'}$ in the same way. We need to prove that $f_{R'} = f_R$. Assume not. Because $f_{R'}$ and f_R are continuous and strictly increasing, they are invertible. Let g_R and $g_{R'}$ be defined by $g_R(w) = f_R^{-1}(w)$ and $g_{R'}(w) = f_{R'}^{-1}(w)$. Let $h(w) = g_R(w) - g_{R'}(w)$. Because $f_{R'}$ and f_R are continuous and concave, they are absolutely continuous and, therefore, differentiable everywhere but possibly at a finite number of points. Consequently, h is differentiable everywhere but possibly at a finite number of points. Notice that $h(0) = h(1) = 0$ but, since $f_{R'} \neq f_R$, there exists w^* such that either $h(w^*) > 0$ and $h'(w^*) > 0$ or $h(w^*) < 0$ and $h'(w^*) < 0$, where h' denotes the derivative of h. We treat the first case. The second case is treated in a parallel way. Condition $h(w^*) > 0$ implies that $f_R(\lambda) = f_{R'}(\lambda') = w^*$ for some $\lambda > \lambda'$. Condition $h'(w^*) > 0$ implies that for $\delta > 0$ small enough, $f_R(\lambda + \delta) - f_R(\lambda) > f_{R'}(\lambda') - f_{R'}(\lambda' - \delta)$, contradicting Priority to the Poorer on Proportional Bundles. □

According the the previous theorem, when preferences are homothetic, well-being should be measured by a strictly concave transformation of the ratio of utility at consumption to utility at the poverty bundle.

Before exploring well-being measures on the entire preference domain (which includes also non-homothetic preferences) let us come back to Priority to the Poorer on Proportional Bundles. There are multiple reasons why this axiom may look too weak. First, it is restricted to homothetic preferences. Second, it is restricted to bundles that are proportional to each other. Our next axiom is a very light extension of it, because it applies to bundles that are not necessarily proportional to each other (even if the amount of resources that are compared among individuals are proportional to the difference between the bundle of the richer and that of the poorer). We even further weaken the axiom by applying the bundle comparison only among individuals consuming less than the poverty line in all goods.

Priority to the Poorer Below the Poverty Line - For all $x, x' \in \mathcal{X}$, and $R, R' \in \mathcal{R}^h$, and all $\lambda \in (0, \frac{1}{2}]$, if $x, x' \leq y$ and

$$W(x + \lambda(x' - x), R) < W(x' - \lambda(x' - x), R'),$$

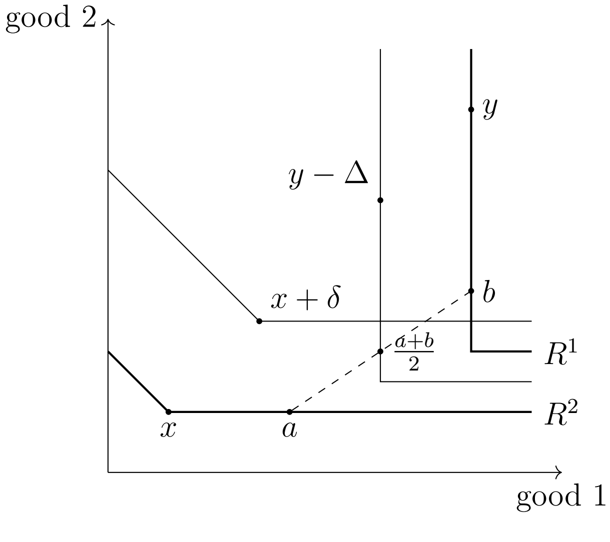
then

$$W(x + \lambda(x' - x), R) - W(x, R) > W(x', R') - W(x' - \lambda(x' - x), R').$$

The following theorem shows that this mild strengthening of Priority to the Poorer on Proportional Bundles leads to an impossibility.

Theorem 2. *No well-being measure W satisfies Consumption Monotonicity, Normalization, Continuity and Priority to the Poorer Below the Poverty Line.*

Figure 3: Illustration of Theorem 2.



Proof. Let $R^1, R^2 \in \mathcal{R}^h$ be defined by the thick indifference curves in Figure 3. By Consumption Monotonicity, Continuity and Normalization, we can find $x \in \mathcal{X}$, $\delta, \Delta \in \mathbb{R}^K_+$, all proportional to y, such that

$$W(x + \delta, R^2) - W(x, R^2) = W(y, R^1) - W(y - \Delta, R^1).$$

Let $a, b \in \mathcal{X}$ be such that: (i) $a_2 = x_2$, (ii) $b_1 = y_1$, (iii) $\frac{a_1+b_1}{2} = y_1 - \Delta_1$, and (iv) $\frac{a_2+b_2}{2} < x_2 + \delta_1$. The construction of a and b is illustrated in Fig. 3. By construction,

- $W(a, R^2) = W(x, R^2)$,
- $W(\frac{a+b}{2}, R^2) < W(x + \delta, R^2)$,
- $W(\frac{a+b}{2}, R^1) = W(y - \Delta, R^1)$,
- $W(b, R^1) = W(y, R^1)$.

As a result,

$$W\left(\frac{a+b}{2}, R^2\right) - W(a, R^2) < W(b, R^1) - W\left(\frac{a+b}{2}, R^1\right),$$

violating Priority to the Poorer Below the Poverty Line. $\square$

The proof of Theorem 2 is developed using piecewise linear preferences. It is not strictly necessary to have them in the domain but what is crucial is to have sufficient heterogeneity among individuals. Even if heterogeneity and piecewise-linearity may be hard to get with parametrical estimations of preferences, they turn out to be the typical result when preferences are estimated non parametrically following the recently developed methods of Decancq and Nys [2021] or De Rock and Moramarco [2023]. The Online Appendix A provides an illustration based on Belgian data.

3.2 Consumption focus

As a result of Theorem 2, we are left with the axioms of Consumption Monotonicity, Normalization, Continuity and Priority to the Poorer on Proportional Bundles, which only tell us how to measure well-being when preferences are homothetic. The next axiom formalizes the idea that, when measuring the well-being of the poor, what matter is the poverty bundle rather than the preferences at this bundle. Formally, it requires that a change in preferences that does not affect the indifference contour at consumption does not alter well-being either.

Consumption Focus - For all $x \in \mathcal{X}$ and $R, R' \in \mathcal{R}$, if $I(x, R) = I(x, R')$, then $W(x, R) = W(x, R')$

This axiom aligns with the idea that the poverty line can be described objectively, as a list of outcomes. This contrasts with a more welfaristic approach, which defines the poverty line in terms of preference satisfaction. The axiomatic analysis in Maniquet and Moramarco [2024] is in line with the welfaristic approach, and the proposed measures account for the entire indifference curve at the poverty bundle. In this paper we focus on the alternative (objective) approach, which also displays desirable properties in the measurement of monetary poverty [see Decerf, 2024, for a comparison of the two approaches].

It is intuitive to expect that this axiom can help us extend the result of Theorem 1 to a larger preference domain. We indeed get the following result, which gives us a complete characterization of well-being measures that works by adopting a concave transform to Deaton's distance function.

Theorem 3. *A well-being measure W satisfies Consumption Monotonicity, Normalization, Continuity, Priority to the Poorer on Proportional Bundles and Consumption Focus if and only if there exists a strictly increasing and strictly concave function $f : \mathbb{R}_+ \to \mathbb{R}_+$ such that $f(0) = 0$ and $f(1) = 1$ and for all $x \in \mathcal{X}$, and $R \in \mathcal{R}$,*

$$W(x, R) = f\left(W_D(x, R)\right). \tag{2}$$

Proof. The proof that W, defined as in the statement of the theorem, satisfies Consumption Focus is simple and omitted here. Let W satisfy Consumption Monotonicity, Normalization, Continuity, Priority to the Poorer on Proportional Bundles and Consumption Focus. By Theorem 1, the claim is proven for all $R \in \mathcal{R}^h$. Let f be the corresponding function. We need to prove that for all $x \in \mathcal{X}$, all $R \in \mathcal{R}$, $W(x, R) = f(\lambda)$ where λ is defined by $xI\lambda y$. Let $R' \in \mathcal{R}^h$ be such that $I(x, R) = I(x, R')$. The existence of R' is guaranteed by the preference domain restriction that we introduced at the beginning of Section 2. Note that R' exists and is unique. By Theorem 1, $W(x, R') = f(\lambda)$. By Consumption Focus, $W(x, R) = W(x, R')$, the desired outcome. □

Following Theorem 3, the well-being of poor individuals should be measured by the Deaton's well-being measure, and transformed by a strictly concave function to ensure priority to the poorer. It is then easy to see how $1 - W(x, R)$ captures the individual's contribution to poverty.

Theorem 3 remains silent on how to aggregate individual contributions to poverty. Following the recent development in the literature [see, e.g., Fleurbaey and Maniquet, 2011, Piacquadio, 2017, Bosmans et al., 2018], the absence of constraints on how to aggregate well-being is an advantage of our proposal.

Indeed, our measures can be used as arguments of a poverty index, for instance an index in the Foster-Green-Thorbecke family [see Foster et al., 1984], or it can be used to measure the distribution of contributions to poverty, in a dominance approach such as in Decancq et al. [2019].

4 Concluding Comments

We have characterized a family of measures that assess an individual's contribution to poverty by the share of the poverty line bundle that the individual considers indifferent to their consumption bundle.

In our axiomatic analysis, the poverty line bundle plays a crucial role. Indeed, our Normalization axiom requires that well-being of all individuals be equal at the poverty line bundle. Theorem 1 shows that it has strong consequences on the way to measure well-being, even if we restrict our attention to homothetic preferences. Moreover, our Consumption Focus axiom requires that information on preferences around the poverty line bundle should be viewed irrelevant. This amounts to requiring that what matters is the poverty line bundle itself and not the satisfaction of preferences it leads to. Of course, preferences still matter, but only around the consumption of the individual.

The well-being measure(s) that we characterize works by applying a concave transform to Deaton's distance function. It, therefore, embodies the idea that giving an amount of goods to a poorer individual increases well-being more. As a result, social evaluation will be consistent with this idea independently of how poverty (or, more generally, social welfare) is defined.

As the careful reader may have noted, none of our results depends on the assumption that well-being is evaluated at a bundle which the concerned individual considers worse than the poverty line bundle. We can, therefore, remove the condition that yRx from our analysis. By doing so, we obtain well-being measures that can be plugged into any social welfare function, not necessarily poverty indices, provided the poverty line bundle remains an argument of social welfare.

As noted in the Introduction, we do not exploit in this paper the fact that indifference surfaces form a lattice.[6] It is customary in measurement theory to study indices by using this lattice structure (for recent works in this vein, see Chambers and Miller [2014a,b], Fleurbaey and Maniquet [2017]). It is worth

[6] A set equipped with a partial order, such as the set of lower contour sets equipped with inclusion, is called a lattice if each pair of elements of this set has a lowest higher element, called the supremum, and a largest lower element, called the infimum.

underlining that that concave transforms to Deaton's distance function all satisfy the following property: if the indifference surface of preferences R through bundle x is the supremum of the indifference curves through x of preferences R' and R'', then $W(x, R) = \max\{W(x, R'), W(x, R'')\}$. This property is a typical derivation from the lattice structure, in line with previous works.

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Online Appendix

Appendix A Preference heterogeneity in practice

The family of well-being measures defined by Eq. (2) satisfies the axiom of Priority to the Poorer on Proportional Bundles. The result of Theorem 2 shows that finding a stronger version of this transfer principle that does not conflict with the other axioms is difficult. This impossibility arises when the preference domain is rich enough to include sufficiently diverse preferences. More precisely, there should exist two goods such that some individuals have stronger preferences for the first good while others are much more interested in the second good. In empirical analysis based on parametric demand systems with preference shifters, it may be difficult to obtain so diverse preferences. Therefore, it is essential to determine whether these preferences are merely theoretical constructs.

Parametric approaches to estimating preferences, such as those based on demand systems [Deaton and Muellbauer, 1980, Banks et al., 1997] or happiness equations [Decancq et al., 2015, 2019], restrict the set of admissible individual preferences. Moreover, when lacking sufficient observations for each individual, these methods often need to assume that similar individuals have the same preferences, therefore capturing only part of the true preference heterogeneity.

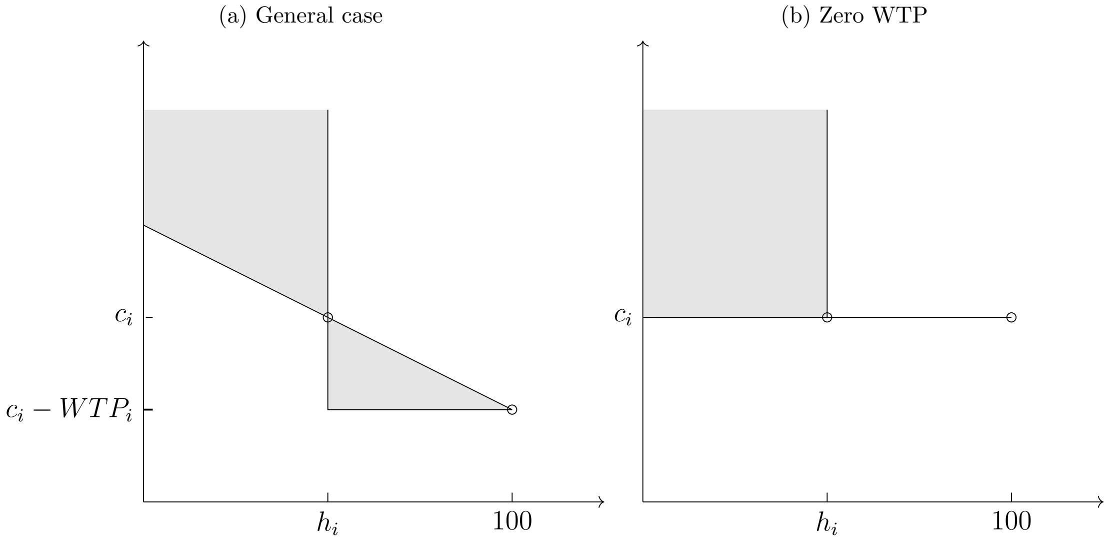
This section demonstrates that if we move beyond parametric estimations of preferences, the domain is rich enough for Theorem 2 to hold. Decancq and Nys [2021] use a non-parametric method (the Adaptive Bisectional Dichotomous Choice) to identify individual indifference curves in the consumption-health space and perform welfare comparisons.[7] Recently, De Rock and Moramarco [2023] implemented a similar approach to analyze multidimensional fairness. Instead of an adaptive survey, they use willingness-to-pay (WTP) information. Compared to Decancq and Nys [2021]'s proposal, WTP allows for less precise identification of indifference curves. However, WTP data are often collected in publicly available surveys. For this reason, we consider De Rock and Moramarco [2023]'s approach for this illustration.

We use the Measuring Equivalent Income (MEqIn) database based on a survey conducted in Belgium in 2016, collecting detailed information on individual consumption, time-use, health, work and other aspects of life for people above 18 years old [see Capéau et al., 2020, for more details].[8] We consider the bi-dimensional outcome space of personal consumption and health score. The former is a measure of the monthly individual private expenditures (e.g. food, clothes, leisure activities, etc.) and a share of the public household expenditures (e.g. utilities, insurances, etc.). The latter is an index from 0 to 100, where 100 represents perfect health, constructed in the survey to summarize individual answers

[7]The idea that individual preferences may not always lead to thin indifference curves is not new in the literature [see, for example, Bernheim and Rangel, 2009]. See also Fleurbaey and Schokkaert [2013], that shows how to derive partial orderings of individuals in these cases.

[8]The MEQIN data are available to all researchers from https://sites.google.com/view/meqin/data.

Figure 4: The set identified Deaton's well-being.



to detailed questions about physical and psychological health.

The sample, after dropping observations with missing values, is composed of 3,034 individuals. We follow De Rock and Moramarco (2022) in using willingness to pay (WTP) for perfect health to set-identify the indifference curves. The idea is that, since the points (h_i, c_i) and $(100, c_i - WTP_i)$ belong to the same indifference curve, they constitute sufficient information to define the subset of $\mathbb{R}^2_{++}$ in which the indifference curve lies (the gray area in Figure 4a).

Whenever $WTP_i = 0$, which is the case for 49.11% of the sample, the set of possible indifference curves takes the shape in Figure 4b.

The reader may observe that the result of Theorem 2 has been proved by considering piecewise linear indifference curves. These preferences are not excluded from our domain, and the fact that almost half of the sample declares zero WTP confirms that some individual indifference curves can be linear on the relevant set of bundles. This strengthens the result of Theorem 2, which is shown to also hold in empirical applications.

Declarations

-Ethical Approval

This paper does not contain any human or animal study.

-Competing interests

The authors have no financial interests related to this paper.

-Authors' contributions

Both authors contributed to this paper.

-Funding

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-Availability of data and materials

The Online Appendix of this paper is based on the MEQIN data, available to all researchers from https://sites.google.com/view/meqin/data.