Inverse Approach for the Design of 1-D Sparse-Array Metasurface Antennas

Jonathan Dessy*, Modeste Bodehou†, Christophe Craeye*
*ICTEAM Institute, UCLouvain, Louvain-la-Neuve, Belgium, jonathan.dessy@uclouvain.be
†URPHORAN Research Unit, LETIA/EPAC, Université d'Abomey-Calavi, Abomey-Calavi, Benin

***Abstract*—This paper addresses the design of 1-D sparse-array metasurface (MTS) antennas based on the solution of the electric field integral equation (EFIE). Sparse-array MTSs are regular antenna arrays in which each array cell corresponds to a modulated MTS, with a cell size larger than half wavelength. The overall structure can be described as a periodic modulated surface impedance with periodicity corresponding to the array cell size. In this paper, the planar MTS is modulated along one direction and is invariant along the other direction. The structure infinitely extends along both directions. Given the imposed periodicity, and assuming linear phase shift excitation along the array, the analysis and the design can be restricted to that of a single cell. Harmonic basis functions are proposed for the current and impedance expansion. An inverse solution of the EFIE is formulated to derive the surface impedance from a desired flat-top embedded element pattern (EEP).**

***Index Terms*—Metasurface, antenna arrays, method of moments, inverse problem, surface impedance, flat-top pattern.**

I. INTRODUCTION

Surface-wave (SW)-based metasurface (MTS) antennas can provide a wide range of radiation patterns through a proper transformation of SWs into leaky waves [1], [2]. They consist of a thin grounded dielectric slab on which a MTS layer composed of a variety of sub-wavelength metallic elements is printed [3], [4]. The metallisation can be homogenized and accurately modeled as a surface impedance [5]. This technology can be effectively analyzed by solving the electric field integral equation (EFIE) with the method of moments (MoM) [6], [7], [8].

The design of SW-based MTS antennas consists in finding the appropriate surface impedance to provide desired radiation specifications. Design methods were first based on holographic techniques [3] or Floquet modes theory [9], [10], [11]. Then, more sophisticated techniques directly based on the solution of the EFIE [2], [12] or relying on optimization methods [13], [14] have been developed. Recently, a sparse-array MTS concept has been proposed for continuous beam scanning [15]. The sparse-array MTS is a regular antenna array where each antenna element actually corresponds to a modulated MTS. The latter is designed to provide a desired embedded element pattern (EEP) for the array. The regularity of the array translates into a periodicity of the surface impedance profile. The antenna structure can then be viewed as an infinitely periodic modulated MTS, with periodicity larger than half wavelength.

This paper proposes a new approach for the design of sparse-array MTS antennas. The problem is restricted to two dimensions, i.e. to the plane $y = 0$ considering an infinitely extended structure, periodically modulated along x and completely invariant along y. The design method extends the inversion technique proposed in [12] to the configuration of sparse-array MTS antennas. To that aim, the structure is modeled using a single reference cell with periodic boundary conditions (PBC). An inverse method is formulated as the superposition of a series of periodic problems using the periodic MoM [16], [17], [18] with different PBC [19]. Harmonic basis functions are used for the expansion of the current and surface impedance enabling simple analytical expressions of the resulting system of equations matrix. Numerical results for the design of two symmetric beams as well as a flat-top pattern are shown to illustrate the method.

The remainder of this paper is organized as follows. Section II presents the considered structure as well as the MoM formulation for its analysis. Section III formulates the inverse approach for the design of this type of structures. Numerical results are shown in Section IV. Section V concludes this paper.

II. FORWARD PROBLEM

Let us consider a 1-D sparse-array MTS antenna consisting of a regular antenna array in which each array cell corresponds to a modulated MTS fed at its center. The overall structure is modeled as a sheet impedance Z_s lying on top of a grounded dielectric slab with thickness d_{slab} and relative permittivity ϵ_r. Both the sheet impedance and the dielectric slab extend infinitely along the x and y directions. The sheet impedance is periodically modulated with a period a (of the order of the wavelength) along x, and is invariant along y. The structure is fed using a phased array (along x) of line sources with linear phase shift excitation along the array and a periodicity corresponding to that of the MTS. Each line source is modeled as an infinite wall (along y) of vertical elementary dipoles (VEDs) located in the middle of the substrate (at $z = -d_{slab}/2$, the MTS being located at $z = 0$) [1]. The structure is illustrated in Fig. 1, where the reference cell and the corresponding reference feed are centered at $x = 0$. It consists of a 2-D problem completely invariant along y, with only x-oriented currents on the MTS.

The EEP (response for excitation by a single feed) evaluated at the Floquet wavenumbers $k_x^p = \psi/a + p\,2\pi/a$ can be

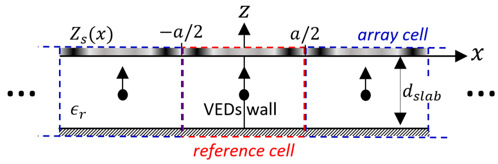


Fig. 1. 1-D sparse-array MTS antenna.

obtained by simulating the periodic structure (response for excitation by the phased array) with an impressed phase-shift $-\psi$ between consecutive feeds [19], [20]. The complete EEP can thus be obtained considering a variety of periodic problems with different PBC. Each periodic problem can be efficiently solved using the periodic MoM. In the latter, the unknown current distribution on the MTS is expanded into a set of basis functions and inserted into the EFIE. Then, the fields are tested using a set of testing functions leading to the following linear system of equations:

$$(\boldsymbol{Z_G} - \boldsymbol{Z_{IBC}})\,\boldsymbol{i} = \boldsymbol{v} \tag{1}$$

where $\boldsymbol{Z_G}$ is the substrate matrix, $\boldsymbol{Z_{IBC}}$ is the impedance boundary conditions (IBC) matrix, $\boldsymbol{v}$ is the excitation vector and $\boldsymbol{i}$ is the vector containing the unknown current expansion coefficients. Details behind this formulation can be found in [20] where it has been validated using Gaussian basis functions. In this paper, harmonic basis functions F_b^n are chosen instead:

$$\boldsymbol{J}(x) = \sum_{n=-N}^{N} i_n\, F_b^n(x)\, \hat{\boldsymbol{x}} \tag{2}$$

with

$$F_b^n(x) = e^{-jk_x^n x} \tag{3}$$

where $k_x^n = \psi/a + n\,2\pi/a$ are the Floquet wavenumbers associated with the impressed phase-shift. For a given Floquet wavenumber, the testing function is taken as the complex conjugate of the corresponding basis function. The periodic sheet impedance is expressed as a weighted sum of harmonic decomposition functions F_d^k with known coefficients c_k:

$$Z_s(x) = jX_0 + \sum_{\substack{k=-K\\k\neq 0}}^{K} c_k\, F_d^k(x) \tag{4}$$

with

$$F_d^k(x) = e^{jk2\pi x/a} \tag{5}$$

where X_0 is the average impedance, and where $c_{-k} = -c_k^*$ (the superscript $*$ stands for the complex conjugate) to ensure a purely imaginary sheet impedance in the absence of loss. This choice of basis and testing functions for current and impedance provides simple expressions for the MoM formulation:

$$Z_G(m,n) = a\,\tilde{G}(k_x^m)\,\delta_{m-n} \tag{6}$$

$$Z_{IBC}(m,n) = jaX_0\,\delta_{m-n} + a\sum_{\substack{k=-K\\k\neq 0}}^{K} c_k\,\delta_{m-n+k} \tag{7}$$

$$v(m) = -\tilde{E}_i(k_x^m) \tag{8}$$

where δ_i is the Kronecker delta function, $\tilde{G}$ is the spatial Fourier transform of the substrate Green's function [21], and $\tilde{E}_i$ is the spectral-domain incident electric field of the excitation [22]. Finally, the electric field can be obtained from the current distribution as

$$\tilde{E}(k_x) = \tilde{G}(k_x)\,\tilde{J}(k_x) + \tilde{E}_i(k_x). \tag{9}$$

A numerical result obtained with the proposed method is shown below. The antenna is operating at a frequency of 24 GHz with a substrate of thickness $d_{slab} = 1.524$ mm and relative permittivity $\epsilon_r = 3.66$. The surface impedance is modulated as

$$Z_s(x) = jX_0\,(1 + M_0 \sin(2\pi x/a)) \tag{10}$$

with $X_0 = -1200\,\Omega$ the average impedance, $M_0 = 1.2$ the modulation depth, and $a = 2\lambda_0/3$, the MTS period where λ_0 is the free-space wavelength [15]. The spectral embedded electric field obtained with harmonic functions (this work) is compared to the one obtained with Gaussian functions (work [20]) in Fig. 2, where $k_0 = 2\pi/\lambda_0$ is the free-space wavenumber. As can be seen, a very good agreement is observed between both curves, which contributes to the validation of the method.

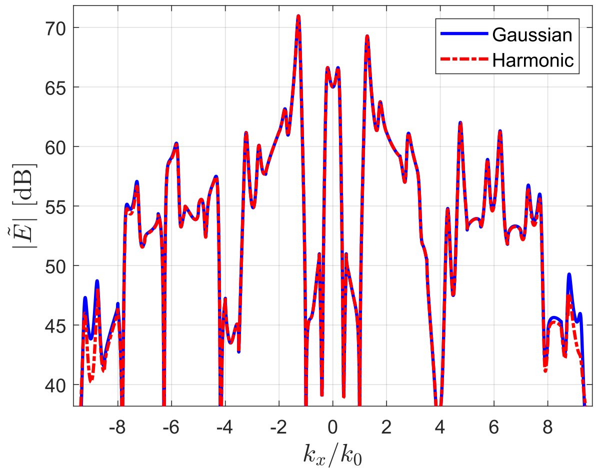


Fig. 2. Spectral embedded electric field obtained with (10) for two types of basis functions: Gaussian and harmonic functions.

III. Inverse Problem

The inverse problem consists of finding an appropriate sheet impedance that will generate a desired EEP. As introduced in [12] for the case of a circular MTS of finite size, the problem can be formulated using the MoM, except that the current is imposed while the sheet impedance has to be determined, i.e. $\boldsymbol{i}$ in (2) is known and $\boldsymbol{c}$ in (4) is the unknown quantity. At this point, it is important to note that the current in the visible region (i.e. for $|k_x| \leq k_0$) can be imposed based on the desired EEP (both in amplitude and in phase) but that the invisible part of the current spectrum (i.e. for $|k_x| > k_0$) remains unknown.

In this work, the invisible part of the current is assumed to be the one generated by the unmodulated sheet impedance (i.e. for $\boldsymbol{c} = \mathbf{0}$), as proposed in [12]. The weight between both parts of the current spectrum can be left as an additional parameter or can be determined based on a power constraint [12]. Thanks to the choice of harmonic basis functions for the current expansion, the vector $\boldsymbol{i}$ simply corresponds to the imposed spectral embedded current sampled at the Floquet wavenumbers. For each periodic problem, an inverse method based on the periodic MoM can be formulated as

$$\boldsymbol{U}\,\boldsymbol{c} = \boldsymbol{w} \tag{11}$$

with

$$\boldsymbol{w} = \boldsymbol{v} - \boldsymbol{Z_G}\,\boldsymbol{i} + jaX_0\boldsymbol{i} \tag{12}$$

and

$$U(m,k) = -a \sum_{n=-N}^{N} i_n\,\delta_{m-n+k}. \tag{13}$$

Similarly to the forward problem, the inverse problem formulation for the considered structure requires inverse periodic formulations for a variety of different impressed phase-shifts. In practice, inverse problems for a number N_ψ of phase-shifts equally spaced in the interval $[0, 2\pi[$ are built using (11) and stacked together in a single larger linear system of equations:

$$[\boldsymbol{U_1}; \boldsymbol{U_2}; ...; \boldsymbol{U_{N_\psi}}]\,\boldsymbol{c} = [\boldsymbol{w_1}; \boldsymbol{w_2}; ...; \boldsymbol{w_{N_\psi}}] \tag{14}$$

where the subscript refers to the impressed phase-shift index. Using Floquet theory, one can choose N_ψ according to the desired spectral sampling Δk_x of the imposed embedded current:

$$\Delta k_x = \frac{\Delta\psi}{a} = \frac{2\pi}{N_\psi a}. \tag{15}$$

To ensure a purely imaginary sheet impedance solution, (14) must be solved while enforcing $c_{-k} = -c_k^*$. This can be done by treating the real and imaginary parts of $\boldsymbol{U}$, $\boldsymbol{c}$, and $\boldsymbol{w}$ separately, which leads to a real linear system of $2N_\psi(2N+1)$ equations for $2K$ unknowns, which can be solved in the least-squares sense.

IV. Numerical results

Two examples of spectral embedded currents are shown below to illustrate the proposed design method: two symmetric beams and a flat-top pattern. The antenna operates at 24 GHz with a period $a = 2\lambda_0$ and a substrate with relative permittivity $\epsilon_r = 3.66$ and thickness $d_{slab} = 1.524$ mm.

A. Two symmetric beams

A spectral embedded current similar to the one provided by an unmodulated MTS, but with two additional peaks at $k_x = \pm 0.1k_0$ is targeted. To that aim, the inverse method presented in Section III is used with an optimization on the value of X_0. It has been found that good results are achieved for a value of $X_0 = -480\,\Omega$. The obtained sheet impedance and currents are shown in Fig. 3. It is striking to note that the impedance is sinusoidal. In fact, those peaks correspond to the first harmonics generated by a sinusoidal modulation [9]. Whereas the MTS period has been fixed, the optimization on the value of X_0 enables to modify the value of the fundamental SW wavenumber such that the first harmonics correspond to the desired beams.

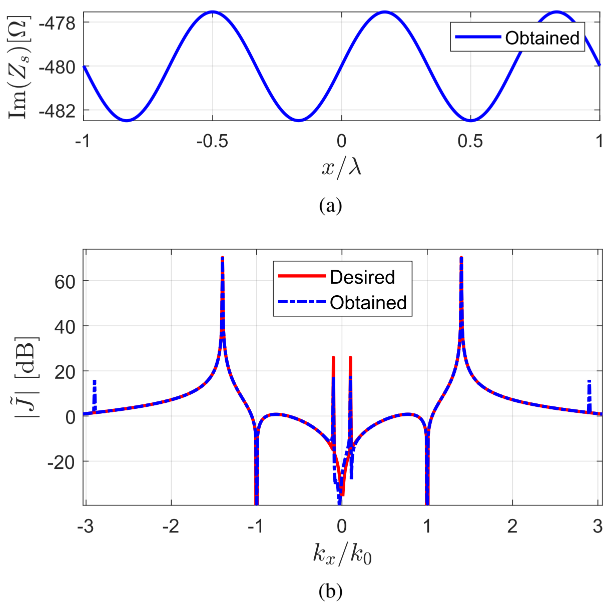
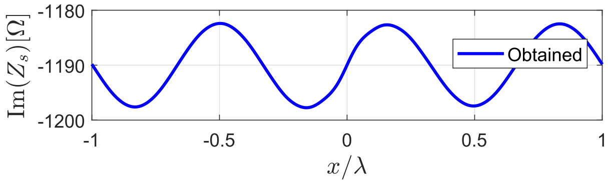


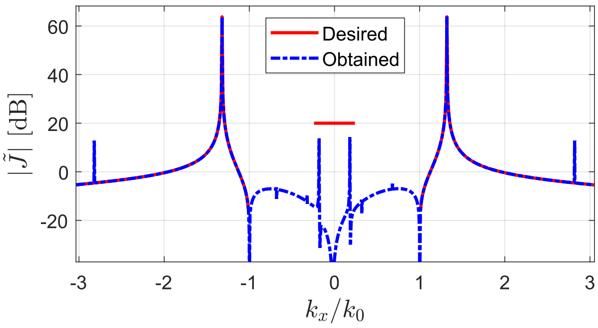
Fig. 3. Obtained (a) sheet impedance and (b) spectral embedded current with $X_0 = -480\,\Omega$ for the desired spectral embedded current with peaks at $k_x = \pm 0.1k_0$.

B. Flat-top pattern

As a second example, a flat-top spectral embedded current in the visible region for $|k_x| \leq 0.25k_0$ is targeted. This type of pattern has already been designed in [15] but using an optimization tool starting from a well-chosen surface impedance (no inverse method). The result using the inverse method proposed in this paper with $X_0 = -1190\,\Omega$ is shown in Fig. 4. Because of the unmodulated current assumption for the invisible part of the current spectrum, the obtained impedance has a weak modulation depth leading to a sharp pattern composed of two peaks located at the edges of the desired rectangular window (corresponding to the first harmonics of the fundamental SW wavenumber). An iterative scheme that updates the invisible part of the current spectrum at each iteration has been implemented but no convincing results were obtained yet. However, in that case, good results can be obtained by increasing the modulation depth of the impedance in a second step. This can be done by multiplying the vector $\boldsymbol{c}$ with an appropriate constant M that can be found through optimization. Fig. 5 shows the results obtained with an optimized value of $M = 195$. It can be observed that a pattern with a similar shape to the one provided in [15] is achieved, but with sharper rectangular edges at the expense of higher side lobes.

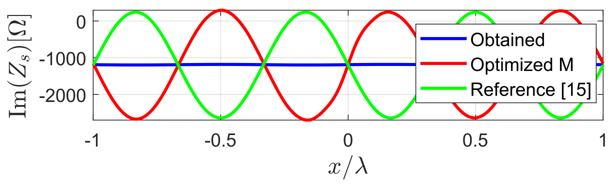


(a)

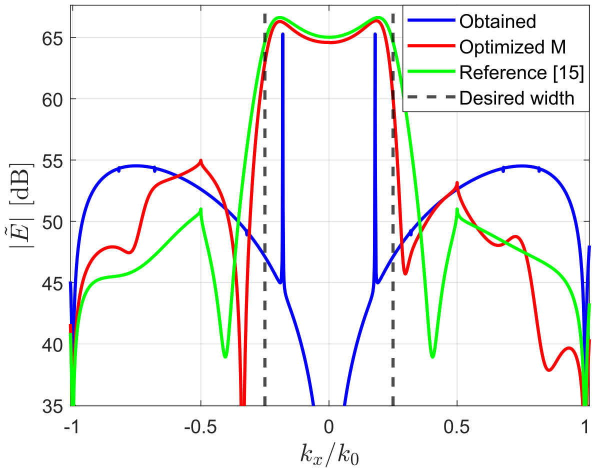


(b)

Fig. 4. Obtained (a) sheet impedance and (b) spectral embedded current with $X_0 = -1190\,\Omega$ for the desired flat-top spectral embedded current.



(a)



(b)

Fig. 5. Optimized (a) sheet impedance and (b) spectral embedded electric field obtained with $X_0 = -1190\,\Omega$ and $M = 195$ (optimized M) for the desired flat-top pattern with a width of $0.5k_0$. It is compared to the initial result (obtained in Fig. 4) and the result from [15].

V. Conclusion

A new approach for the inverse design of 1-D sparse-array MTS antennas has been proposed. It is based on the solution of the periodic MoM using harmonic basis functions. The periodic aspect of the unknown sheet impedance is taken into account by stacking and solving simultaneously a variety of periodic problems with different PBC. Numerical results for the case of two symmetric beams and of a flat-top pattern have been shown. In both cases, convincing results have been obtained with the proposed method combined with an optimization on the values of the average reactance and the modulation depth. The optimization is necessary owing to the assumption on the invisible part of the current spectrum. Further work will focus on an improved model of the invisible current spectrum to enable the direct design of 1-D sparse array MTS antennas that radiate more sophisticated EEPs. Then, the extension of the method for the design of 2-D sparse array MTS antennas will be investigated.

Acknowledgment

Jonathan Dessy's activity is funded under a Research Fellowship of the Fonds de la Recherche Scientifique - FNRS.

References

[1] G. Minatti, F. Caminita, E. Martini, M. Sabbadini, and S. Maci, "Synthesis of modulated-metasurface antennas with amplitude, phase, and polarization control," *IEEE Trans. Antennas Propag.*, vol. 64, no. 9, pp. 3907–3919, Sep. 2016.

[2] M. Bodehou, K. A. Khalifeh, S. N. Jha, and C. Craeye, "Direct numerical inversion methods for the design of surface wave-based metasurface antennas: Fundamentals, realizations, and perspectives," *IEEE Antennas Propag. Mag.*, vol. 64, no. 4, pp. 24–36, Aug. 2022.

[3] B. H. Fong, J. S. Colburn, J. J. Ottusch, J. L. Visher, and D. F. Sievenpiper, "Scalar and tensor holographic artificial impedance surfaces," *IEEE Trans. Antennas Propag.*, vol. 58, no. 10, pp. 3212–3221, Jul. 2010.

[4] G. Minatti, M. Faenzi, E. Martini, F. Caminita, P. De Vita, D. González-Ovejero, M. Sabbadini, and S. Maci, "Modulated metasurface antennas for space: Synthesis, analysis and realizations," *IEEE Trans. Antennas Propag.*, vol. 63, no. 4, pp. 1288–1300, Apr. 2015.

[5] M. A. Francavilla, E. Martini, S. Maci, and G. Vecchi, "On the numerical simulation of metasurfaces with impedance boundary condition integral equations," *IEEE Trans. Antennas Propag.*, vol. 63, no. 5, pp. 2153–2161, May 2015.

[6] R. F. Harrington, "Field computation by moment methods." Piscataway, NJ, USA: Wiley-IEEE Press, 1993.

[7] D. González-Ovejero and S. Maci, "Gaussian ring basis functions for the analysis of modulated metasurface antennas," *IEEE Trans. Antennas Propag.*, vol. 63, no. 9, pp. 3982–3993, Sep. 2015.

[8] M. Bodehou, D. González-Ovejero, C. Craeye, and I. Huynen, "Method of moments simulation of modulated metasurface antennas with a set of orthogonal entire-domain basis functions," *IEEE Trans. Antennas Propag.*, vol. 67, no. 2, pp. 1119–1130, Feb. 2019.

[9] A. Oliner and A. Hessel, "Guided waves on sinusoidally-modulated reactance surfaces," *IRE Trans. Antennas Propag.*, vol. 7, no. 5, pp. 201–208, Dec. 1959.

[10] G. Minatti, F. Caminita, E. Martini, and S. Maci, "Flat optics for leaky-waves on modulated metasurfaces: Adiabatic floquet-wave analysis," *IEEE Trans. Antennas Propag.*, vol. 64, no. 9, pp. 3896–3906, Sep. 2016.

[11] A. M. Patel and A. Grbic, "A printed leaky-wave antenna based on a sinusoidally-modulated reactance surface," *IEEE Trans. Antennas Propag.*, vol. 59, no. 6, pp. 2087–2096, Jun. 2011.

[12] M. Bodehou, C. Craeye, E. Martini, and I. Huynen, "A quasi-direct method for the surface impedance design of modulated metasurface antennas," *IEEE Trans. Antennas Propag.*, vol. 67, no. 1, pp. 24–36, Jan. 2019.

[13] M. Zucchi, F. Vernì, M. Righero, and G. Vecchi, "Current based automated design of realizable metasurface antennas with arbitrary pattern constraints," *IEEE Trans. Antennas Propag.*, vol. 71, no. 6, pp. 4888–4902, Jun. 2023.

[14] H. Lee and D.-H. Kwon, "Printed metasurface leaky wave antennas based on penetrable aperture field synthesis," *IEEE Trans. Antennas Propag.*, vol. 71, no. 6, pp. 4724–4736, Jun. 2023.

[15] J. Dessy, M. Bodehou, J. Cavillot, and C. Craeye, "Sparse-array metasurface for beam scanning," *IEEE Trans. Antennas Propag.*, vol. 72, no. 10, pp. 8088–8093, Oct. 2024.

[16] R. Mittra, C. Chan, and T. Cwik, "Techniques for analyzing frequency selective surfaces-A review," *Proc. IEEE*, vol. 76, no. 12, pp. 1593–1615, Dec. 1988.

[17] L. Trintinalia and H. Ling, "Integral equation modeling of multilayered doubly-periodic lossy structures using periodic boundary condition and a connection scheme," *IEEE Trans. Antennas Propag.*, vol. 52, no. 9, pp. 2253–2261, Sep. 2004.

[18] F. Capolino, C. Craeye, X. Radu, and A. G. Schuchinsky, "Fundamentals of method of moments for artificial materials," in *Theory and Phenomena of Metamaterials*. Boca Raton, FL, USA: CRC Press, 2009, ch. 5, pp. 1–31.

[19] C. Craeye and D. Gonzalez Ovejero, "A review on array mutual coupling analysis," *Radio Sci.*, vol. 46, no. 2, pp. 1–25, Apr. 2011.

[20] J. Dessy, M. Bodehou, and C. Craeye, "Method of moments for the two-dimensional analysis of array-fed metasurface antennas," in *2024 Int. Conf. Electromagn. Adv. Appl. (ICEAA)*, Oct. 2024, pp. 200–204.

[21] N. Das and D. Pozar, "A generalized spectral-domain Green's function for multilayer dielectric substrates with application to multilayer transmission lines," *IEEE Trans. Microw. Theory Techn.*, vol. 35, no. 3, pp. 326–335, Mar. 1987.

[22] K. Michalski and J. Mosig, "Multilayered media Green's functions in integral equation formulations," *IEEE Trans. Antennas Propag.*, vol. 45, no. 3, pp. 508–519, Mar. 1997.