a Geometric-based Method for the Perceptive Design of Hyperstatic Rigid Frames

Thesis registered in 2018

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How to design hyperstatic rigid frames insightfully with graphic statics ?

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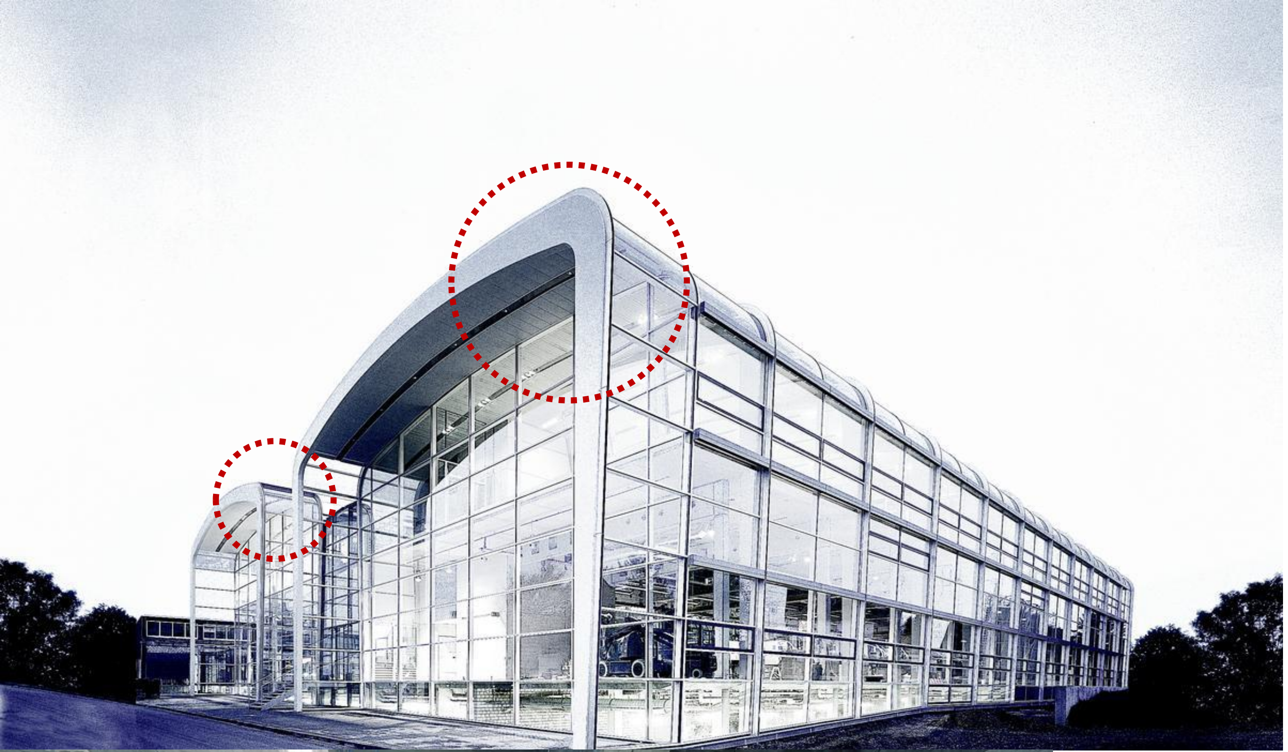
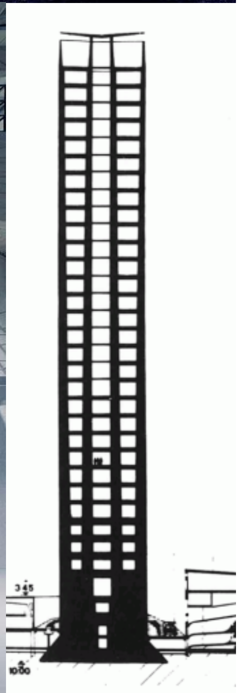


GIRA Production Hall (Ingenhoven Architects, source:[GIRA Giersiepen GmbH & Co. KG](#))



Pirelli Tower ([source](#):BMS)

Key feature:
Rigid/moment-resisting connections and members

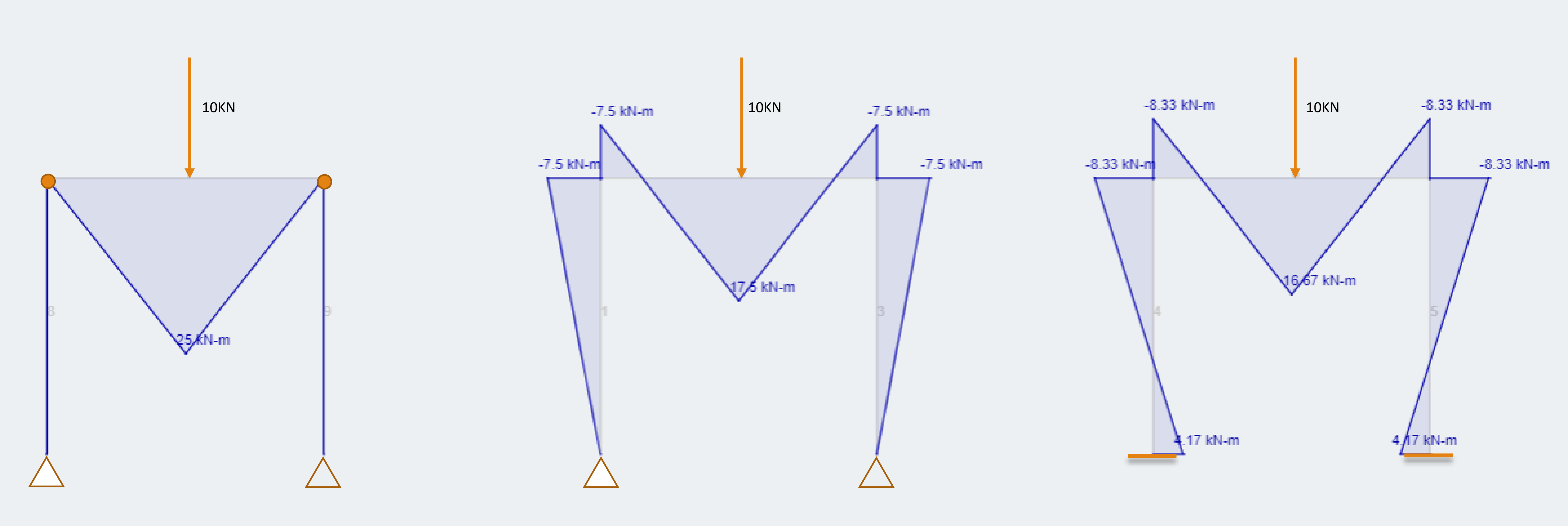
statically indeterminacy/
hyperstatic

The structural relevance of statically indeterminacy

- Structural efficiency through hyperstability
- Structural redundancy

The structural relevance of rigid frames

- Geometrical relevance
- Lateral resistance capacity
- Constructability



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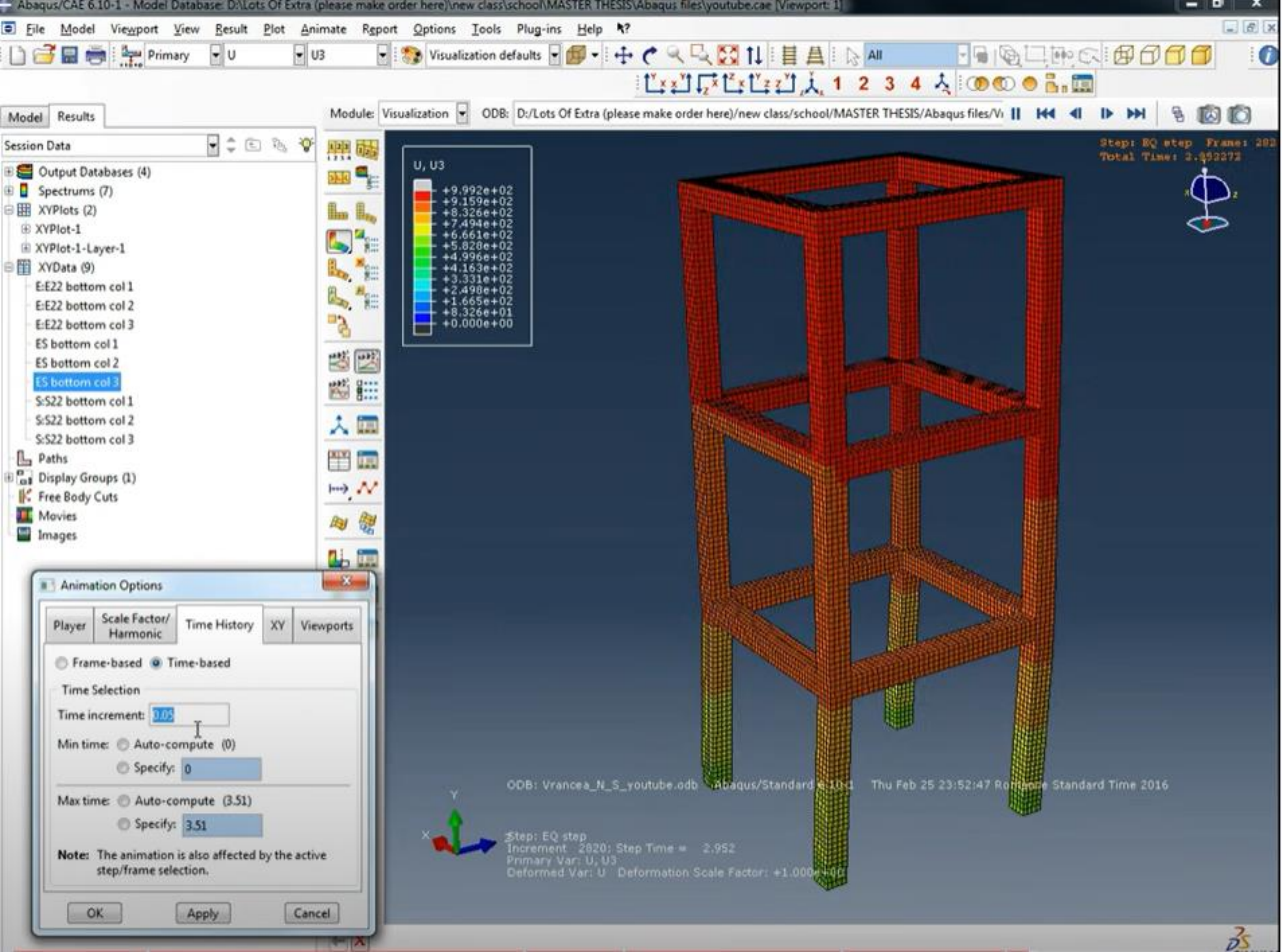


The current design practices

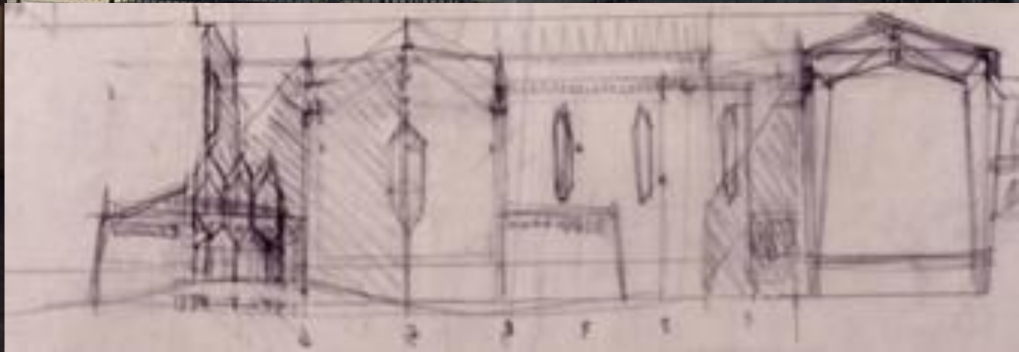
- Not insightful into the relation between form and forces.
- One-way approach: results are also not interactive
- Demanding of mathematics knowledge

- “Black-box” tools
- Trial-and-error approach
- Architects are deterred from in-depth form-finding of frames

- Often materially-inefficient
- Aesthetic value is often unexploited

FEM analysis (source:youtube)

- Materially-efficient: varying sections according to bending moment
- Aesthetic value: an integrated element for architectural richness and elegance



Church of Saint Francis of Assisi 'al Fopponino'
Milano, Italy. 1964(completed)
Architect: Gio Ponti
Engineer: Antonio Fornaroli(to be confirmed)
source: Hidden Architecture

How to design hyperstatic rigid frames insightfully with graphic statics ?

Insight into the relation between form and forces

-44] EXAMPLES OF FRAME- AND STRESS- DIAGRAMS. 157

metrical to the triangle 1, 17, 18; the rectangle 8, 45, 43, 42 to the rectangle 2, 17, 19, 20: and so on.

All the upper bars are in compression, and all the lower ones are in tension.

The diagonals and contra-diagonals are all in compression; finally two of the verticals **23, 39** are in tension, and all the rest in compression.

44. Figure 9*a** represents one half of a locomotive shed. The external forces are the weights **1, 2, 3, 4, 5** applied at

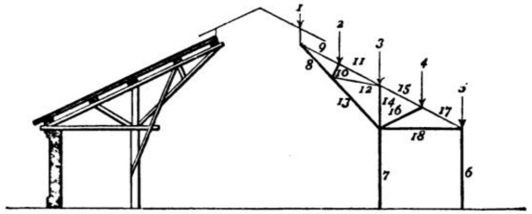


Fig. 9*a*.

the upper joints of the frame, and the reactions **6** and **7** of the wall and column. Again, all the external forces are parallel, and consequently the polygon of forces reduces, in diagram *b*, to one straight line. The force **6** (taken in the opposite sense to that in which it really acts) is equal to a certain part of the weight **5**; by adding the difference to the other weights we get the magnitude of the force **7**.

In the diagram *b* the direction of the lines 8 and 13 coincide; the first is a part of the second. Here then we have for the polygon corresponding to the joint (**8**, **10**, **12**, **13**) one of those degenerate forms about which we spoke in Art. 33; the polygon is in fact a quadrilateral 8, 10, 12, 13, having three of its vertices (13, 8), (8, 10), (12, 13) in one straight line.

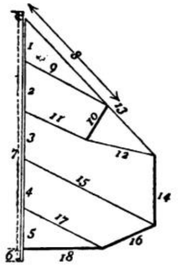
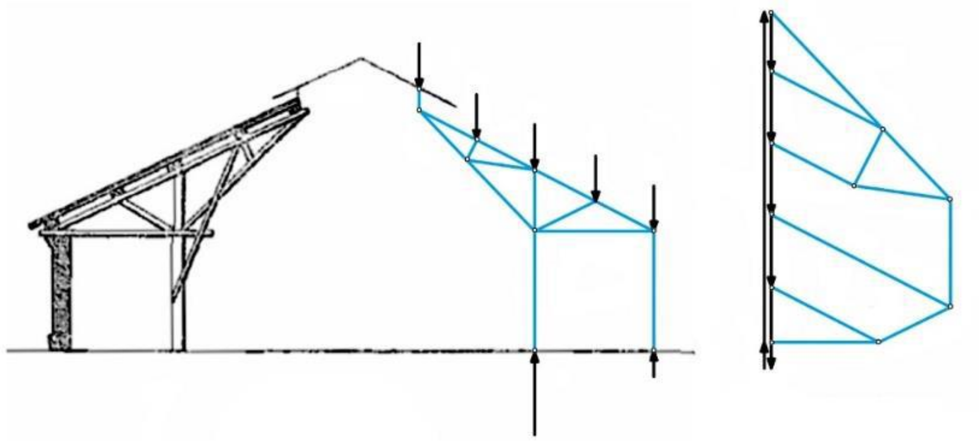
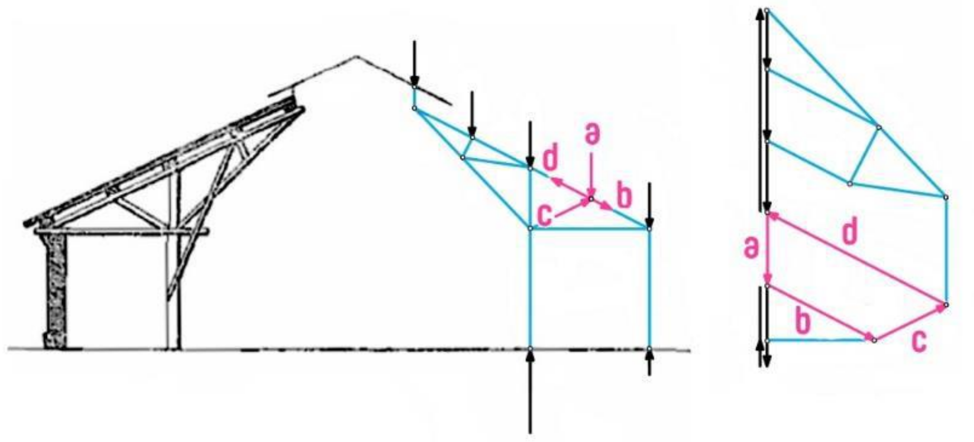


Fig. 9*b*.

The polygon 5, 17, 18, 6, corresponding to the point where

* This example is taken from Pl. xix. of the atlas of *Graphische Statik* of CULMANN, 1st edition. As previously stated, the two diagrams are not rigorously reciprocal.

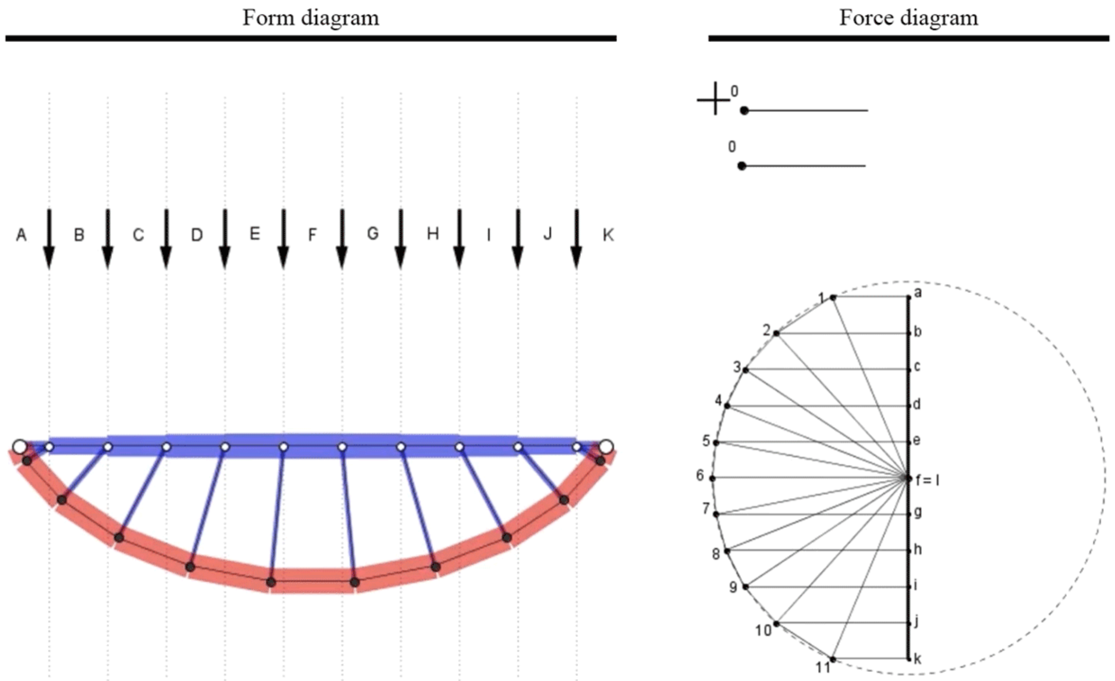




Source: Cremona's Le figure reciproche nella statica grafica

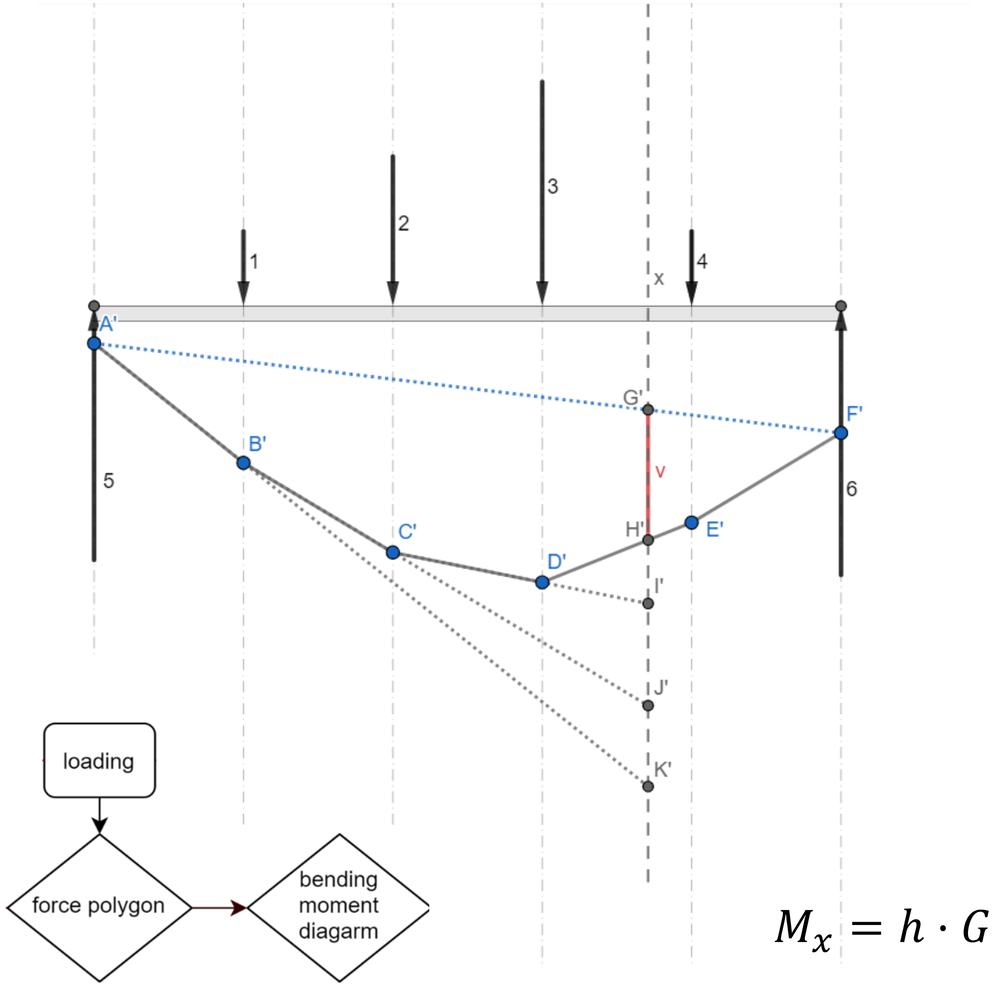
Advantages:

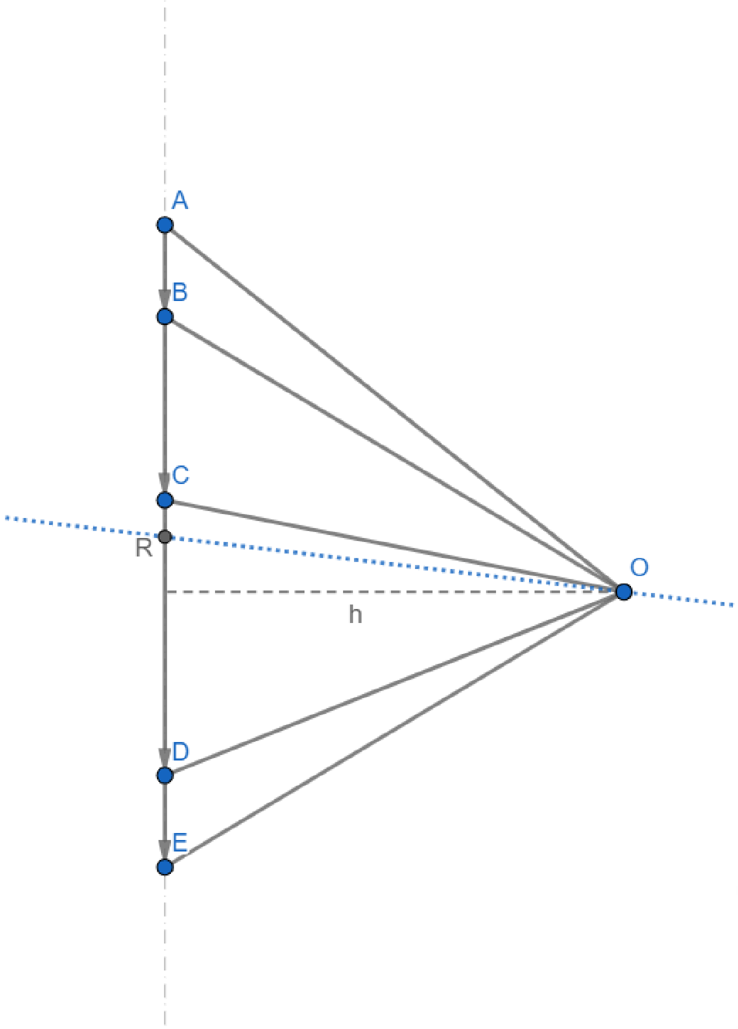
- insightful into the relation between form and forces.
- Two-way approach
- Less demanding of mathematics knowledge



Source: Block's reasearh group

The analysis of the bending moment of a beam with graphic statics





$$M_x = h \cdot G'K' - h \cdot J'K' - h \cdot I'J' - h \cdot H'I' = h \cdot G'H'$$

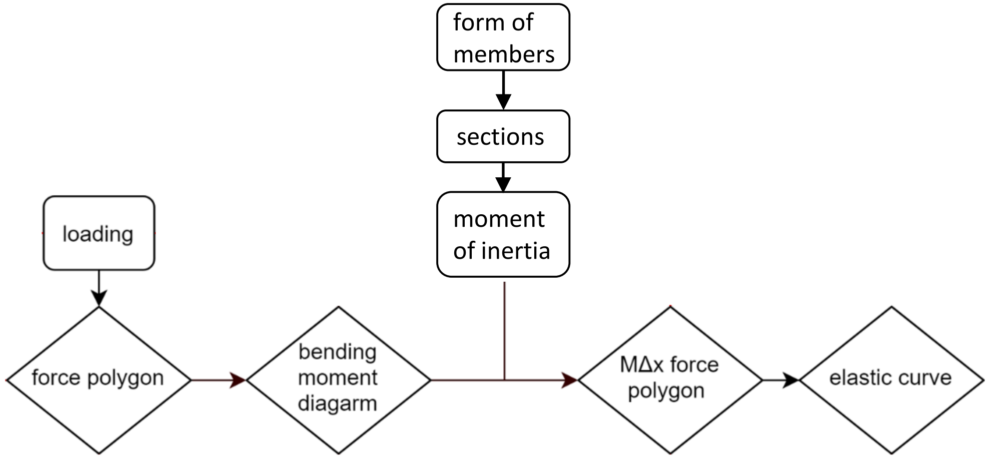
First funicular polygon/bending moment diagram

force polygon

Beams with varying **moment of inertia**

a) the pole distance for each segment in proportional to its corresponding inertia and equals $\frac{EI}{n \cdot H}$,

b) Each ray intersects the previous one or its extension on the pole.



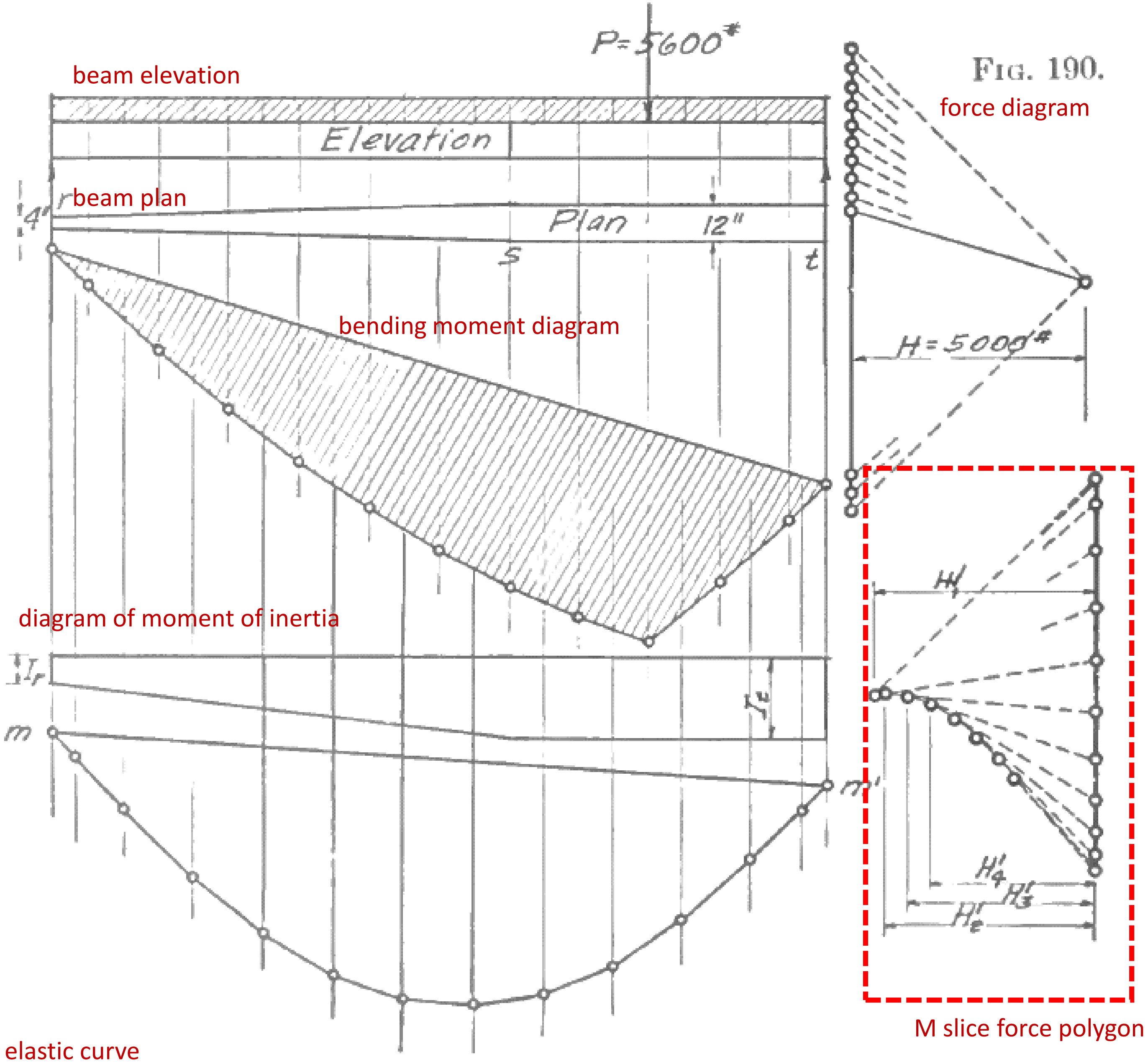
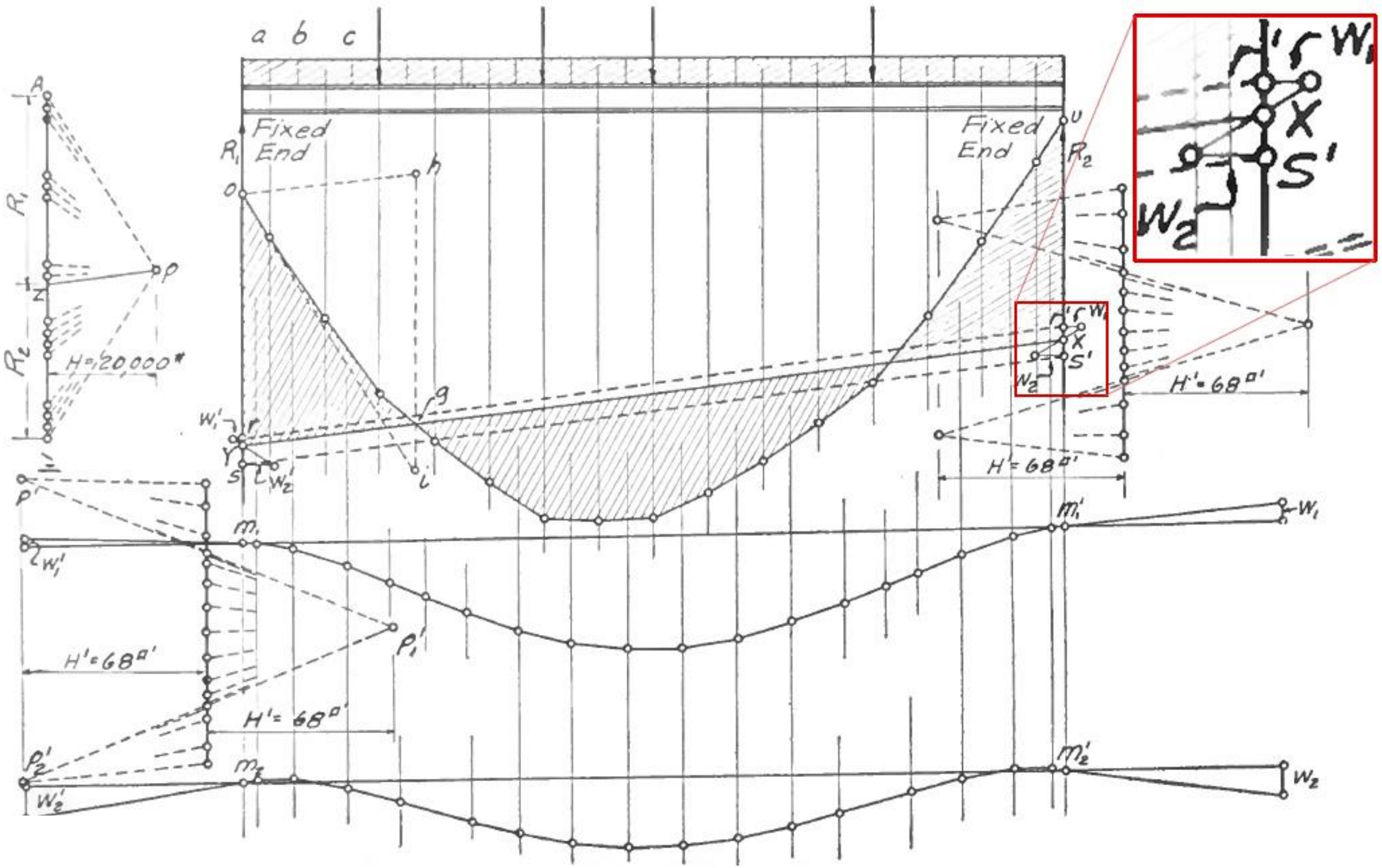
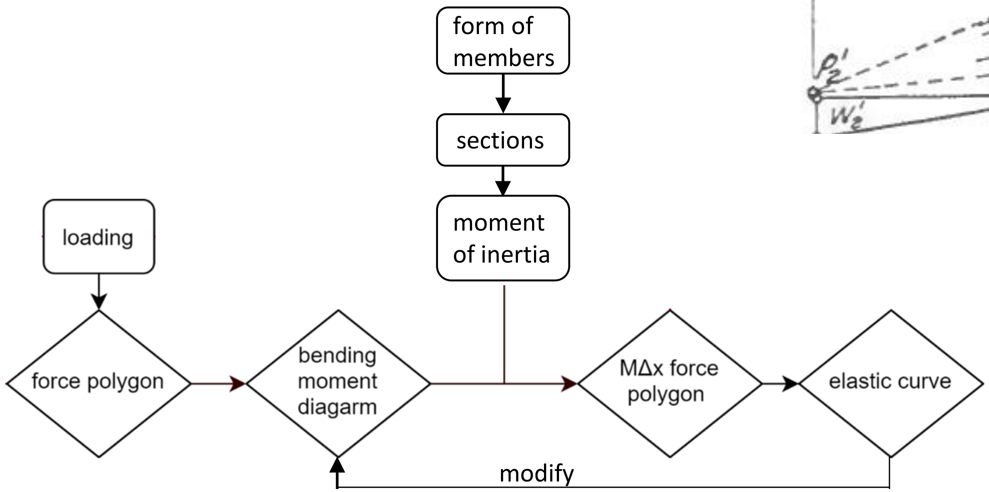


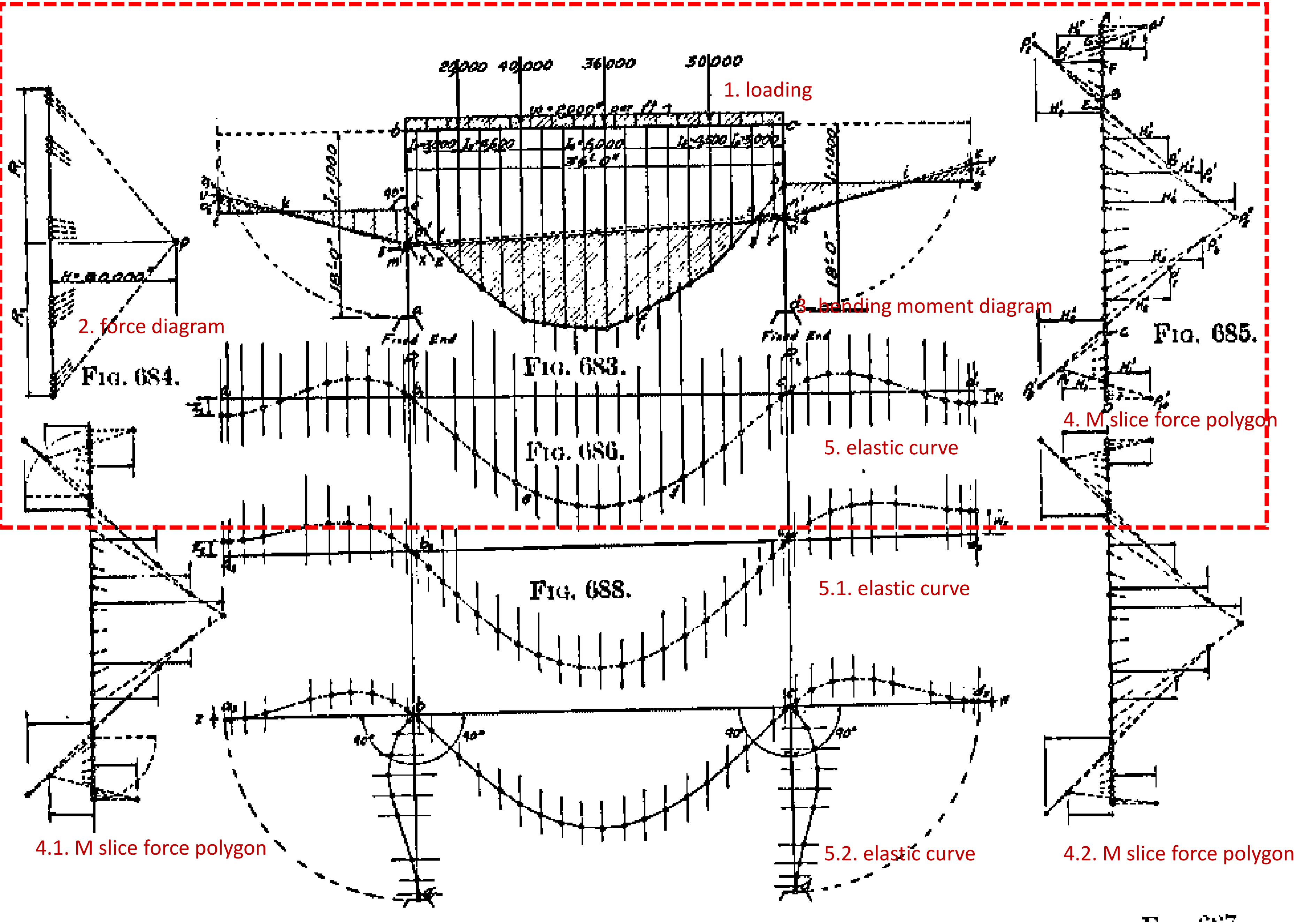
FIG. 191. FIG. 193. (Wolfe 1921)



Fixed End
Fixed End
H=20,000#
H'=68□'



form of members
sections
moment of inertia
loading
force polygon
bending moment diagarm
MΔx force polygon
elastic curve
modify



Key techniques for hyperstatic rigid frames:

1. Frames are rotated into continuous beams
2. Using trial bending moment diagrams for hyperstaticity
3. Using elastic curves to check and adjust

Limitations:

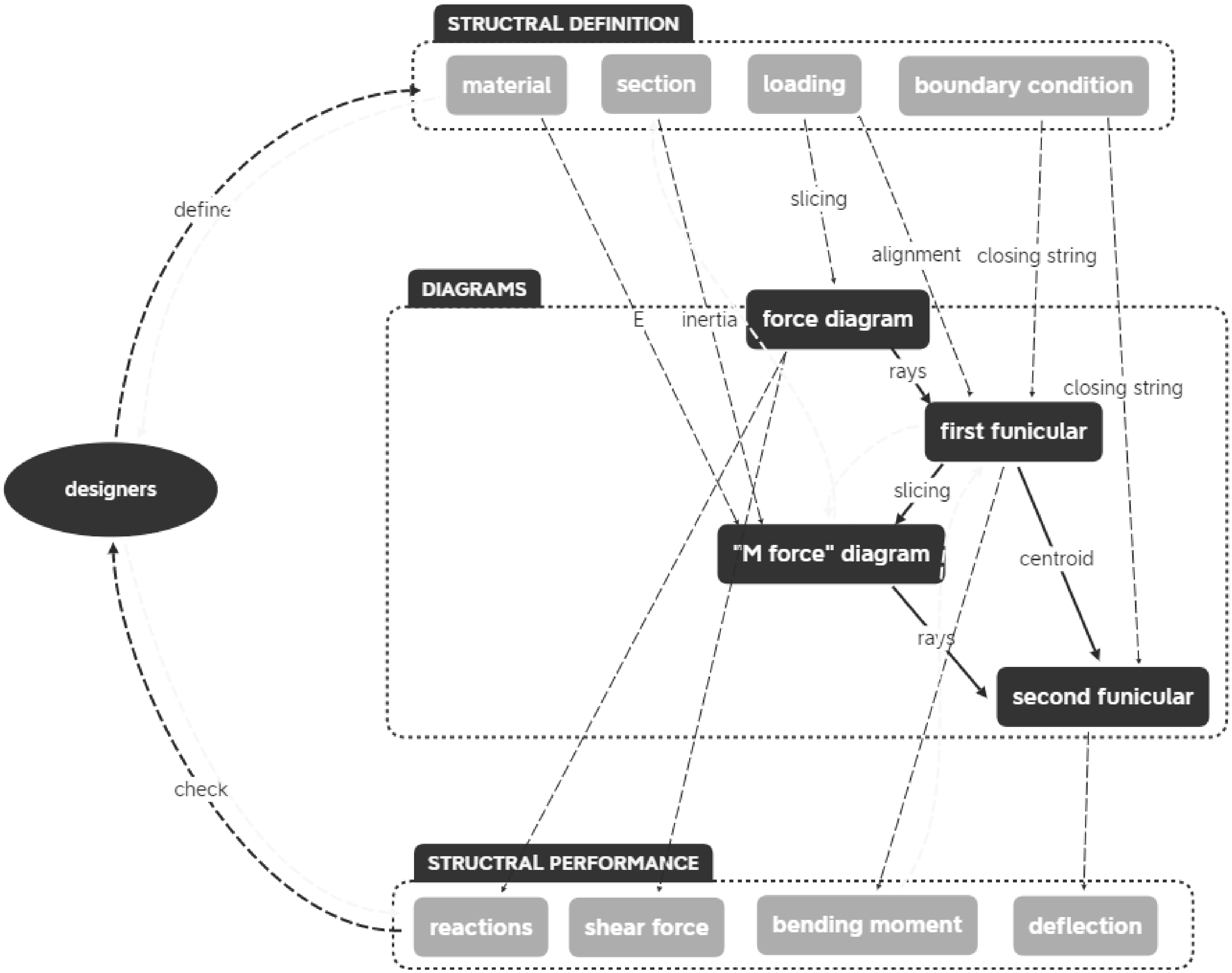
1. limited to orthogonal portal frames under vertical loading
2. Slow and cumbersome with hand-drawing
3. imprecise for frames with static indeterminacy more than one degree

(Wolfe 1921)

Intuitive and insightful,

But still only a analyzing method,

And an incomplete one…



How to design hyperstatic rigid frames insightfully with graphic statics ?

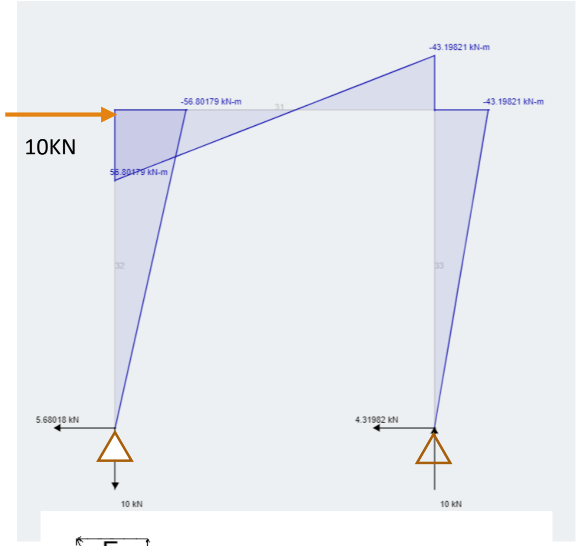
LIMITATIONS:

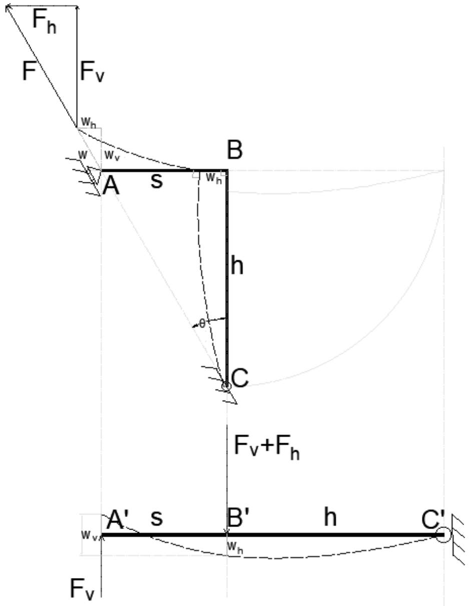
1. Limited to orthogonal portal frames under vertical loading
2. Slow and cumbersome with hand-drawing
3. Difficult to modify

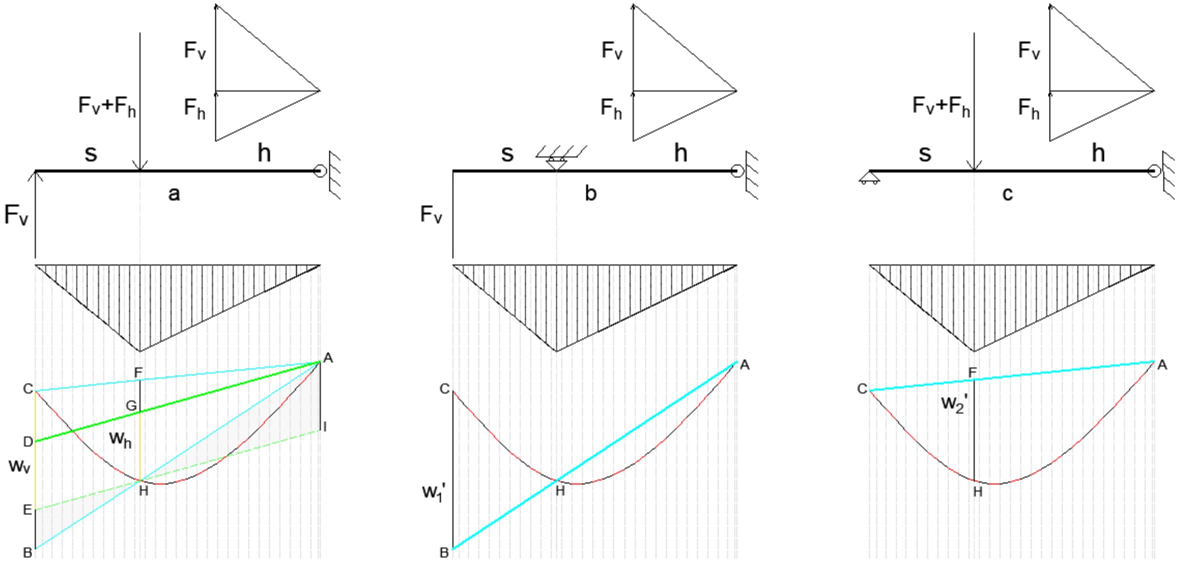
DEVELOPMENT TO BECOME A DESIGN TOOL

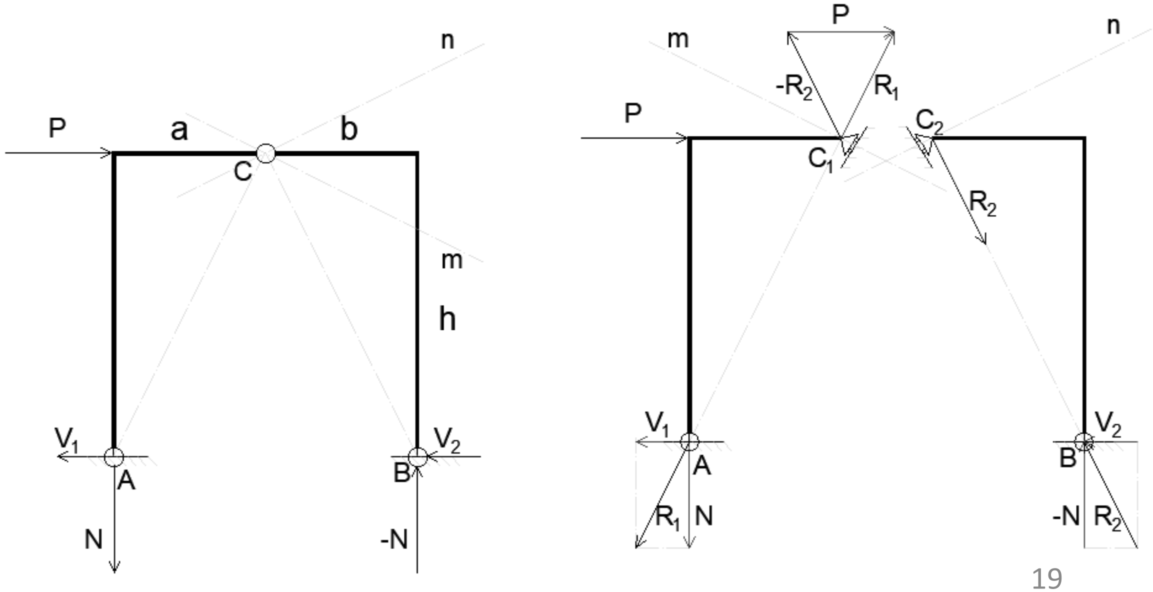
1. Portal frames under horizontal loading
2. The portal frames with oblique members
3. Multi-story and multi-span frames
4. Computerization and parameterization

1. Portal frames under horizontal loading

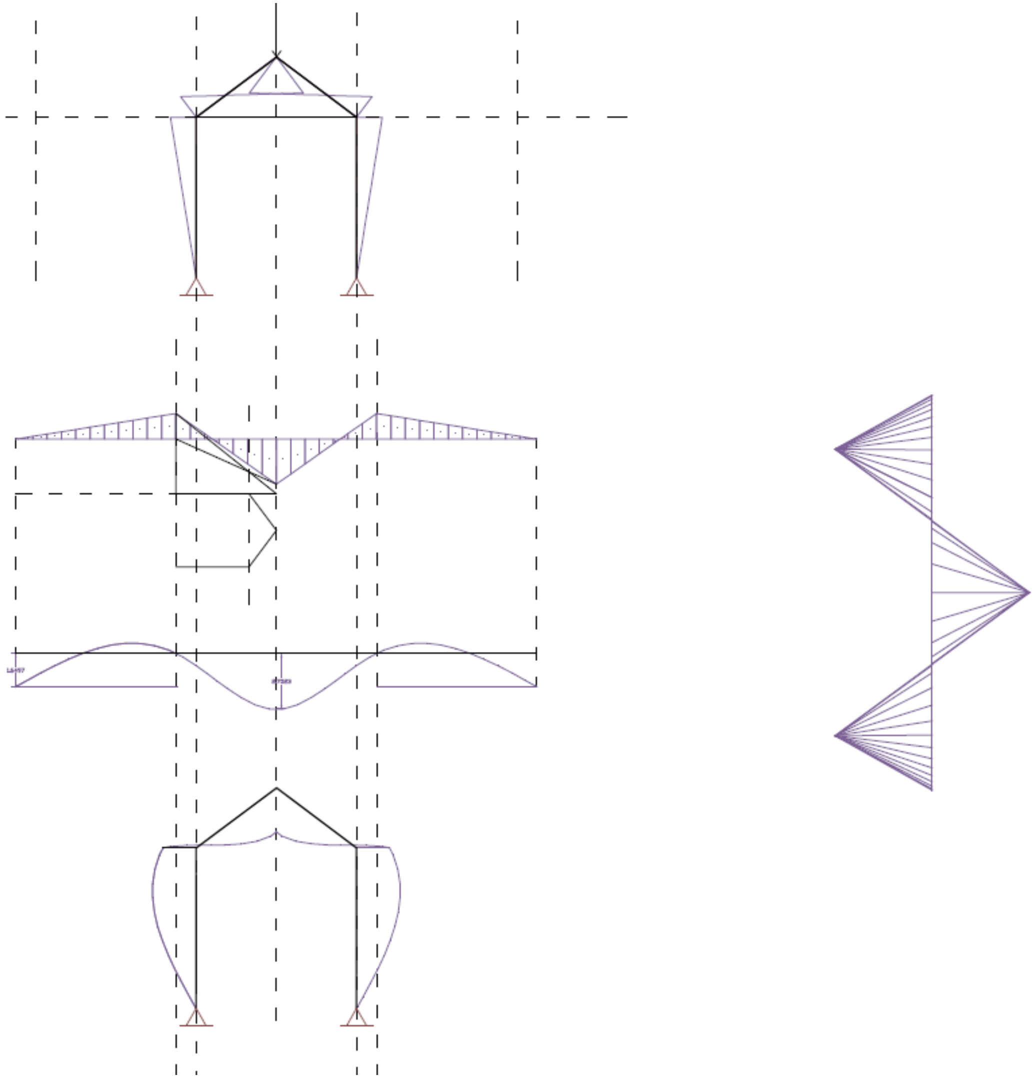


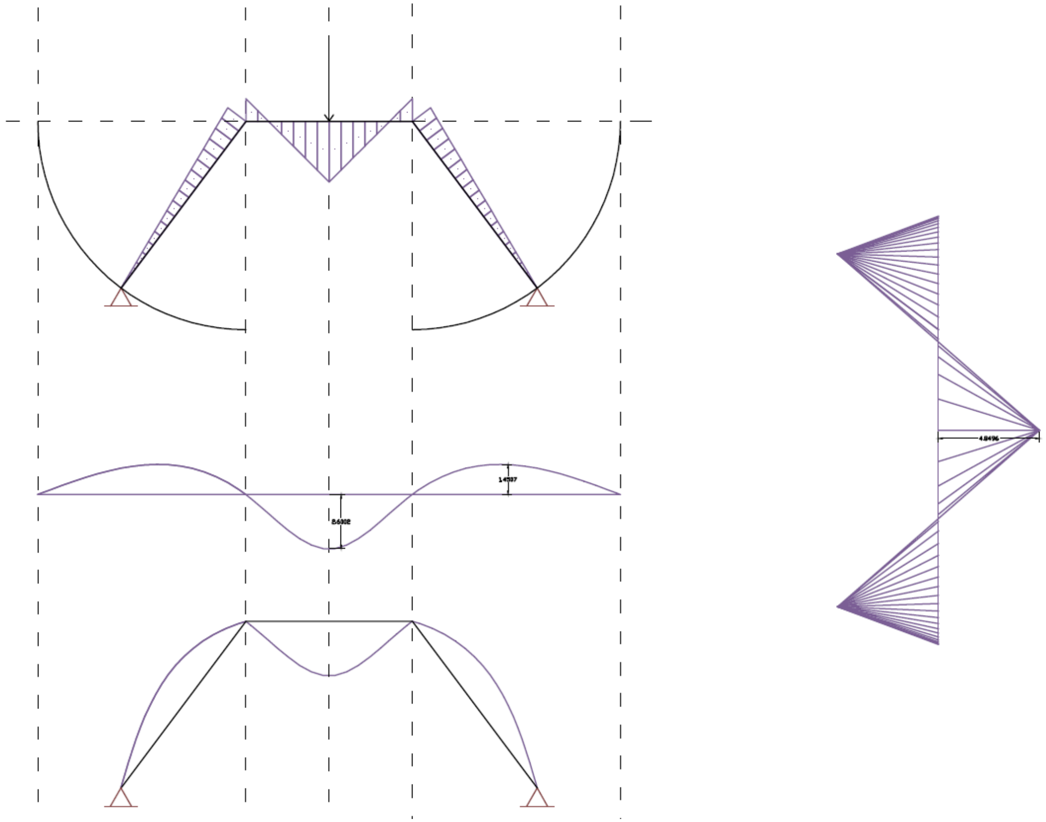




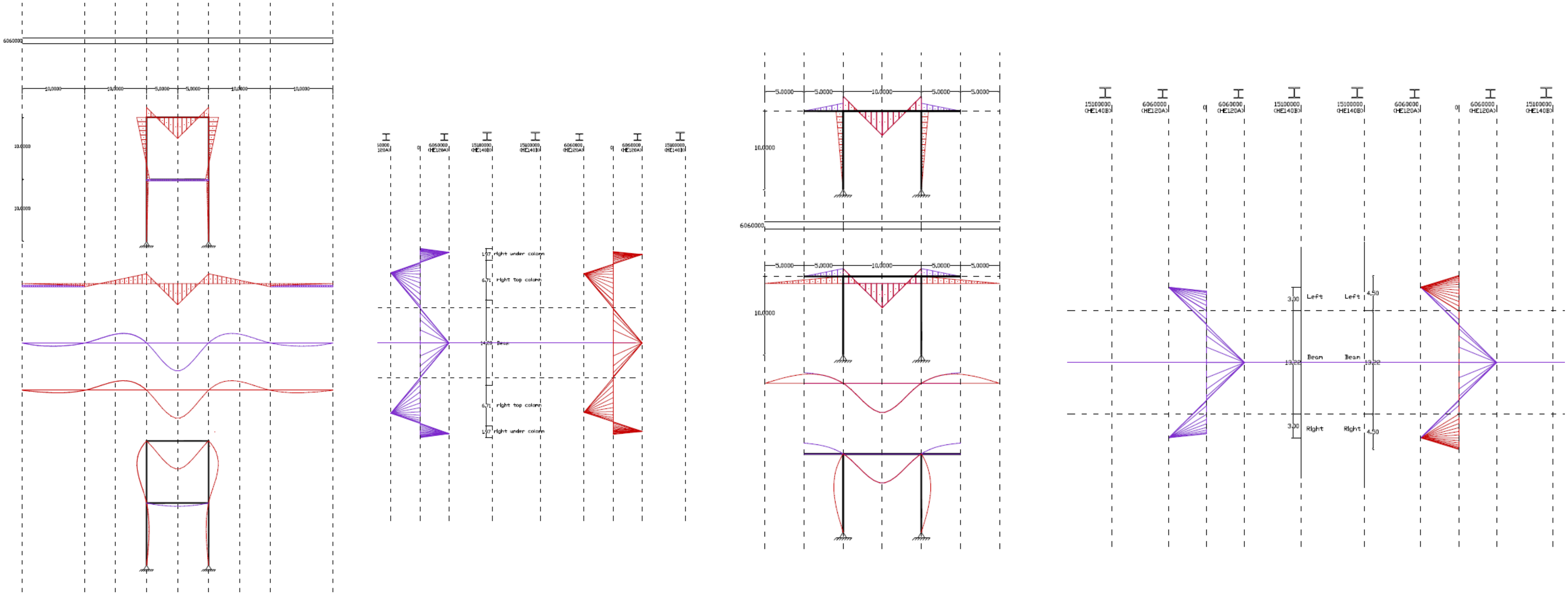


2. Portal frames with oblique members

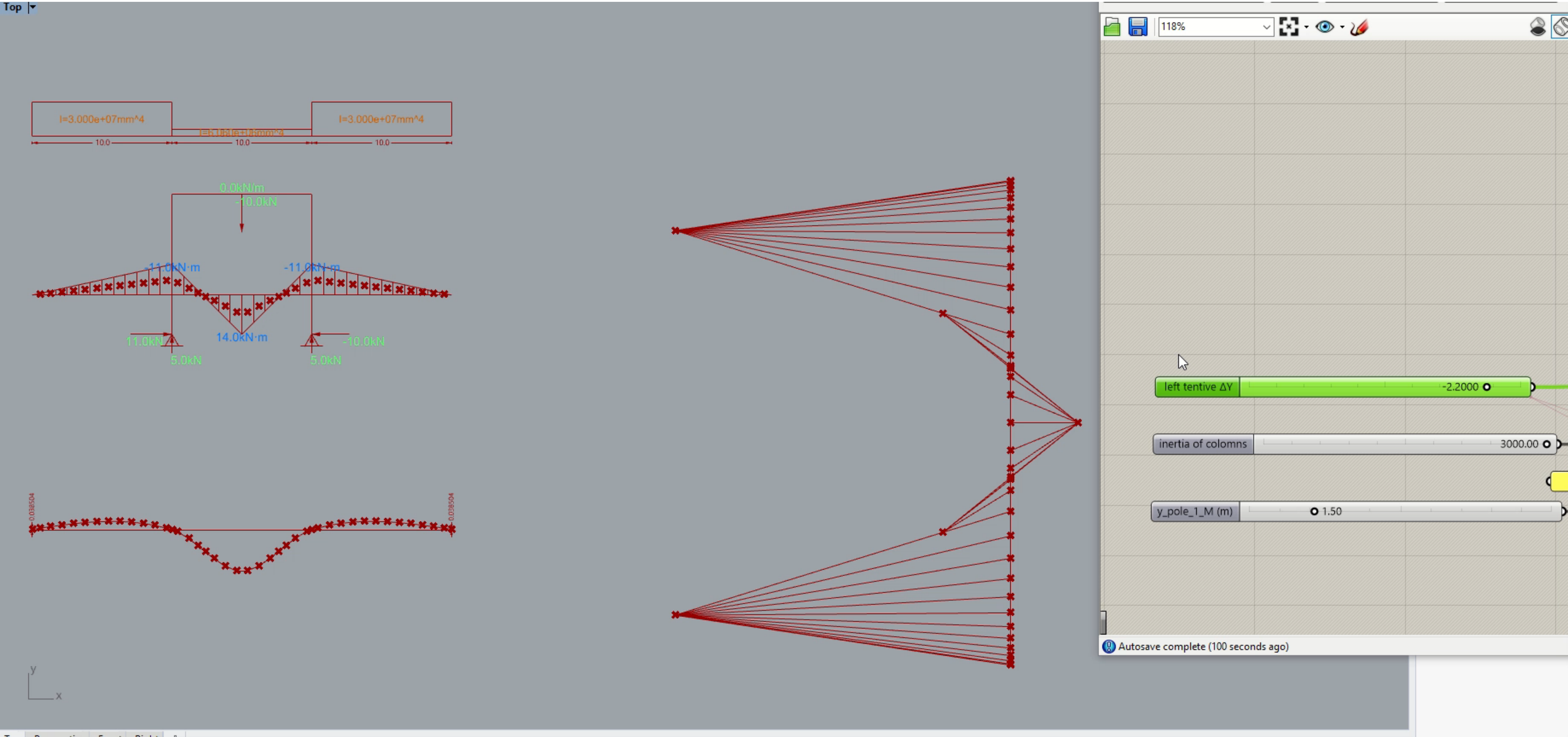


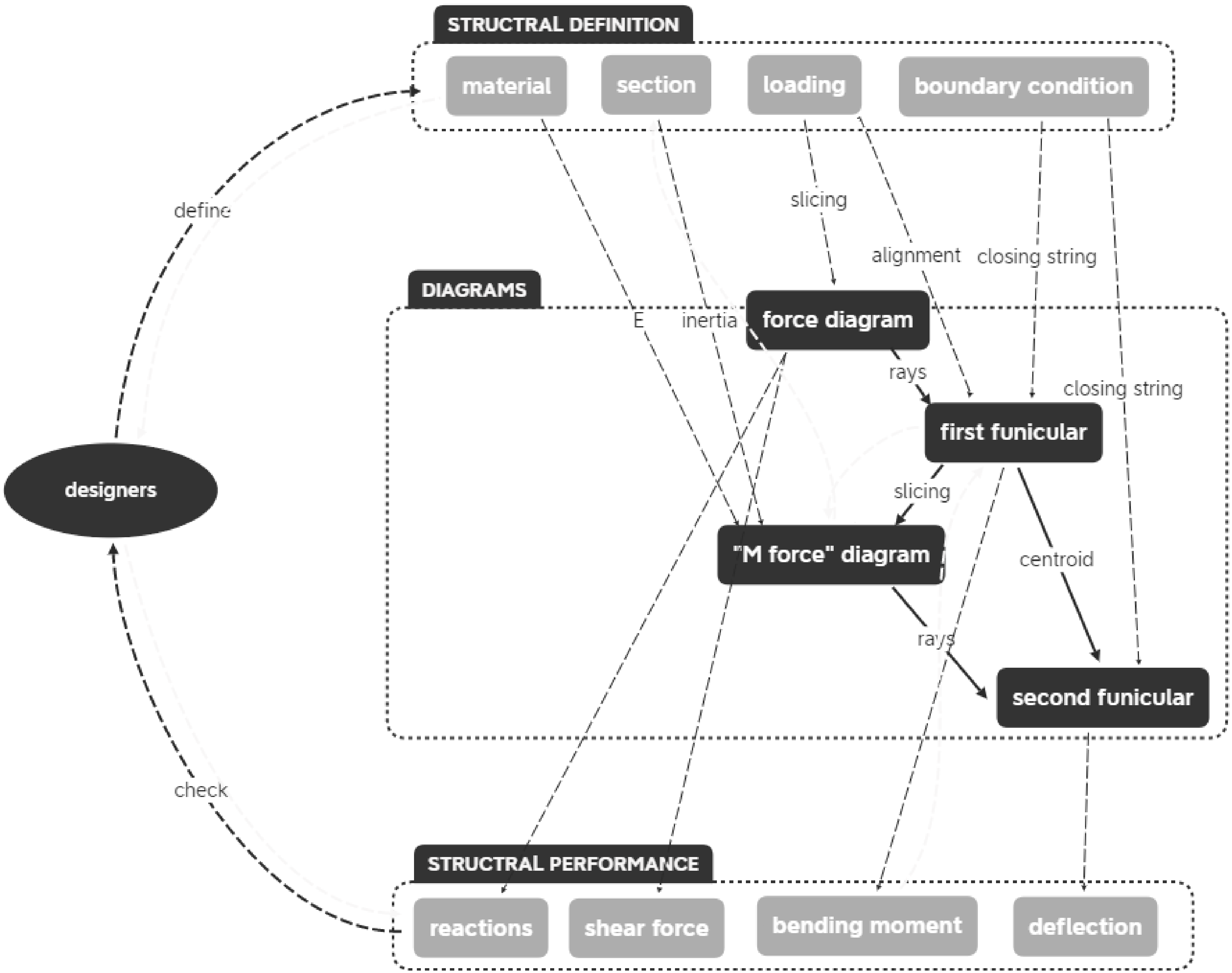


3. Multi-story and multi-span frames



4. Computerization and parameterization




STRUCTRAL DEFINITION
material
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loading
boundary condition
define
slicing
alignment
closing string
DIAGRAMS
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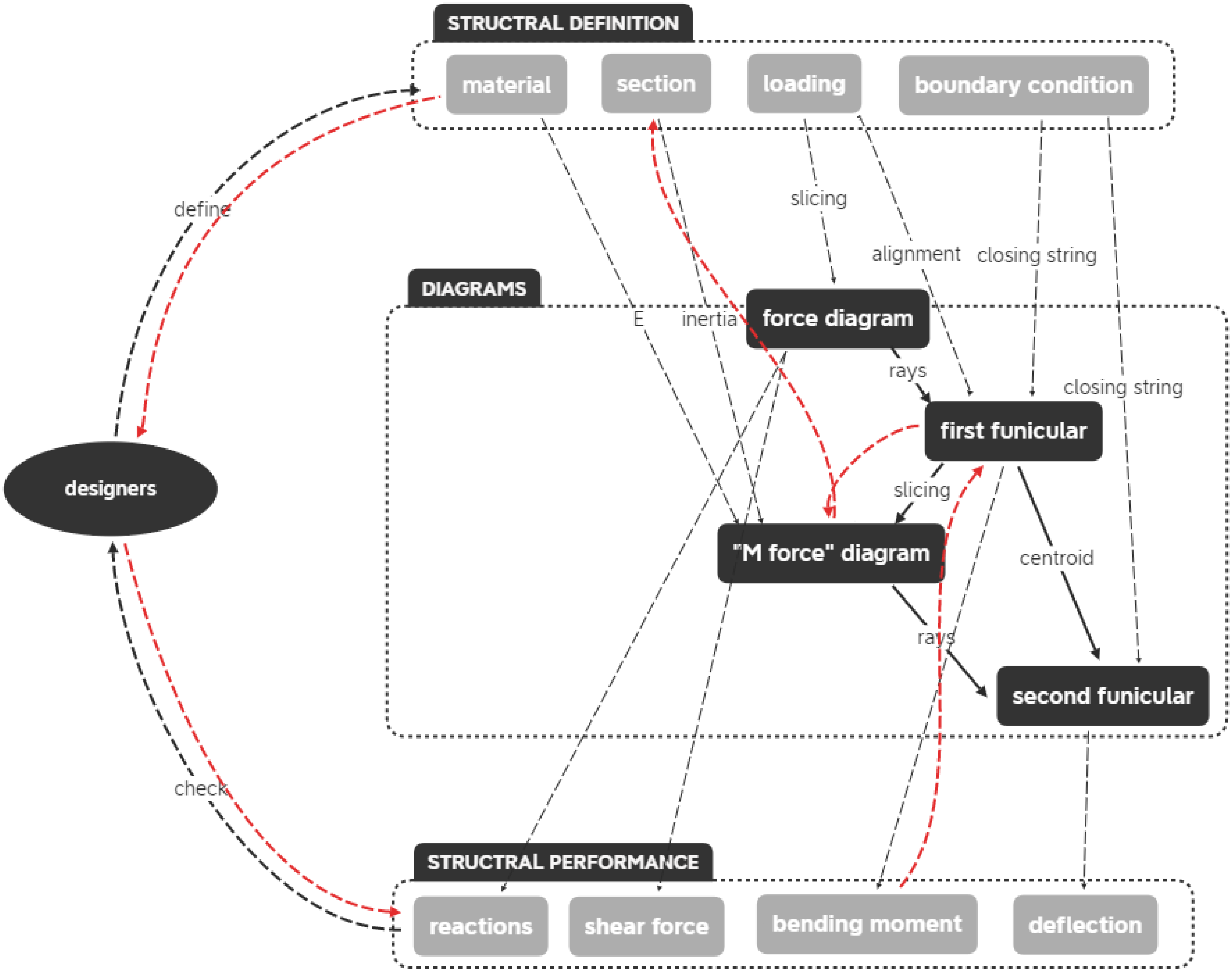
inertia
force diagram
rays
closing string
first funicular
designers
slicing
"M force" diagram
centroid
rays
second funicular
check
STRUCTRAL PERFORMANCE
reactions
shear force
bending moment
deflection



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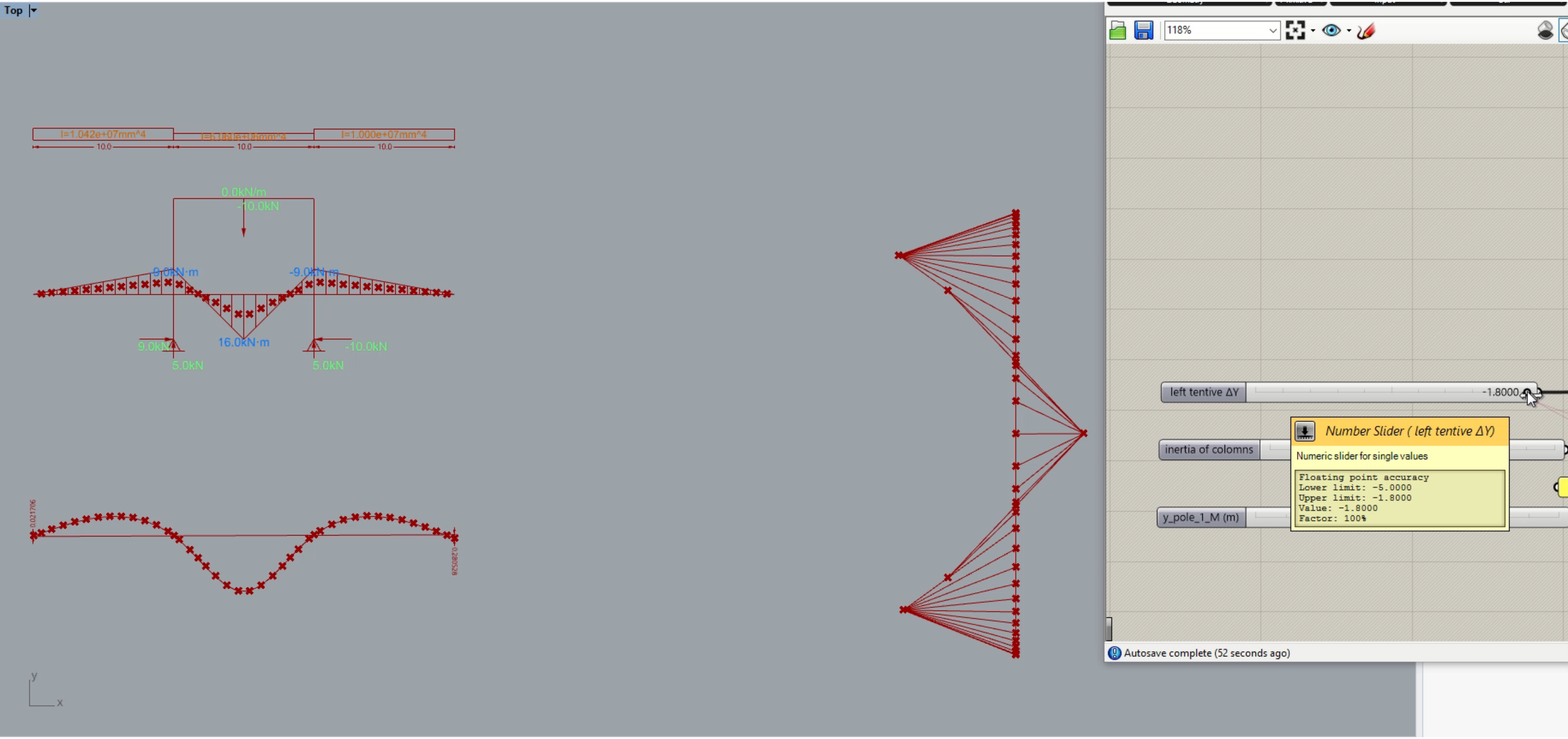
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I=1.000e+07mm^4
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10.0
0.0kN/m
-10.0kN
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5.0kN
5.0kN
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-1.8000
Number Slider (left tentive ΔY)
Numeric slider for single values
Floating point accuracy
Lower limit: -5.0000
Upper limit: -1.8000
Value: -1.8000
Factor: 100%
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y_pole_1_M (m)
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TO BE CONTINUED…

5. DISCUSSION

- **Advantages**
- Insightful
- Chronology-free
- Less demanding for mathematics

- **Limitation**
- The relation between form(member section) and moment of inertia is still to be worked on.
- Difficulty with high degree of indeterminacy
- Interactivity is still limited

THANK YOU !