

Extending the EU Savings Directive*

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Abstract

This paper is motivated by the current debate concerning the possibility to extend the EU Directive on Savings Income Taxation. With the help of a two-country multi-asset game-theoretic model, we assess how this measure could reshape the system of cross-border taxation. Then we analyze the consequences of its limited scope of application. For these purposes, we study the functioning of four taxation systems: a pure source-based system of withholding taxation; the model implied by the OECD Convention on Income and Capital; the scheme of coordinated withholding taxation negotiated by Austria, Belgium, and Luxembourg as a transitory measure before their full adoption of the Directive; and the residence-based regime finally adopted in 2003 which characterizes the EU Directive on Savings Income Taxation. Through analytical comparison we show that (i) the OECD system gives countries strong incentives to undertake non-collaborative policies; (ii) a system of coordinated withholding taxation cannot be adopted as a general rule in a structure like the EU where, in tax matters, decisions have to be taken at unanimity; (iii) the EU Directive offers an innovative design for the game, but its limited field of application encourages countries to issue financial products escaping the Directive; (iv) the extension of the scope of the EU Directive would permit to achieve a non-distorting scheme of international taxation allowing countries to tax individuals on their European-wide income.

Keywords: International Saving, Taxation of Savings Income, European Integration.

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1 Introduction

In an open economy, savings invested abroad can in principle be taxed by the country where they generate income and most often where that income is paid out, called the source country, or by the country whose the investor is a resident, named the residence country, or even by both. For a long time, saving income arising from a foreign source was actually subject only to a withholding tax levied at source; indeed no exchange of information was organized between the source and the residence country and each country was a tax heaven for its neighbour. The Model Tax Convention on Income and Capital proposed by the Organization for Economic Co-operation and Development (OECD) was a first attempt to allow the residence country to effectively tax worldwide income of its resident, in accordance with the Haig-Simmons view of horizontal equity in tax matters (Haig, 1921; Simmons, 1938). The OECD Convention allowed the source country to levy a withholding tax, whilst the country of residence of the taxpayer was left free to subject or not the income to its own tax system, under provision of a credit for the withholding tax levied abroad. A consequence of this system was that the residence country needed to get information from the source country in order to be able to apply the crediting mechanism. Residence country legal authorities were hence entitled to request such exchange, but the information could be passed from the source country administration only if it could be legally obtained by this latter.

The absence of a well performing international tax scheme for capital income was rather unimportant as long as savings abroad remained contained. Within the European Union (EU), the increasing capital mobility resulting from the accomplishment of the Single Market in the late 1980's turned a marginal phenomenon into a larger one. The willingness to preserve undistorted capital movements across EU Member States, as well as the possibility of making savings income contributing to government revenues, led the governments of the EU Member States to find a way to avoid tax competition among countries. Two avenues were therefore suggested: either introducing a system of coordinated withholding taxes levied at source or systematically exchanging information among Member States. The first attempt to harmonize the taxation of savings income was led by the EU Commissioner Christiane Scrivener. The proposal aimed at coordinating the withholding taxes levied by the EU Member States on interest income paid out to residents of other EU Member States. The adoption of this proposal failed due to its rejection by a group of countries including Luxembourg and the United Kingdom, as in the EU unanimity is required in tax matters. The second attempt was directed by Mario Monti, at the time Commissioner for Internal Market, Financial Services and Financial Integration, Customs, and Taxation. Monti's suggested reform asked each Member State to decide on one of the following two options: either a withholding tax levied at source, like in the Scrivener proposal, or a systematic exchange of information across EU internal borders. Even the adoption of that proposal, however, failed. Finally, an agreement was reached in 2000, during the EU

Council held in Feira, Portugal, whose text served as basis for the EU Directive on Savings Income Taxation (Cattoir, 2006).

The EU Directive on Savings Income Taxation currently at work is based on an automatic system of exchange of information, but tentatively allows Austria and Luxembourg to apply the alternative system of coordinated withholding taxation, which is also in use in a series of countries not belonging to the European Union, namely Switzerland, Liechtenstein, Monaco, Andorra, and San Marino.¹ The main concern with the EU Directive comes from the limitation of its scope to savings income in the form of interest payments and to some interest-based financial products. This leaves away from the field of application income from innovative financial products, insurance contracts, and dividends, which are close to debt claims. Today the extension of the EU Savings Directive to a larger class of financial assets is at stake and its assessment is one of the aims of this work.

In this paper we assess various tax designs for cross-border savings by adopting a formalization which is as close as possible to actual and potential institutional features. We contribute to the literature in finding out which of four institutional frameworks best fits the EU needs in that matter. Namely we consider a pure source based scenario where no information at all is exchanged, the OECD tax model, a system of coordinated withholding tax like the one currently at work between Austria or Luxembourg and the other EU Member States, and the residence based system prescribed in the Directive. In each of these frameworks, Member States decide simultaneously and non-cooperatively on the tax rates to apply within their borders and choose unanimously the rules governing the general scenario of the game. We show that a non distorting system of taxation allowing countries to tax individuals on their EU-wide income could be achieved only through an extension of the scope of the Directive.

After a brief review of the literature in section 2, in sections 3 to 5 we introduce a two step Nash game and solve that game backwards for the first three tax settings introduced above. In section 6 we adapt the model to assess the institutional framework underpinning the EU Savings Directive. Section 7 concludes.

2 Review of the literature

The literature on cross-border savings taxation has generally tried to assess whether countries strategically prefer a system of information exchange over a withholding tax system, for which reasons they can prefer one above the other, and what are the determinants of their choice. In their seminal papers Bacchetta and Espinosa (1995, 2000) examine the incentives leading a government to share

¹Belgium decided to stop applying the withholding tax and to start exchanging information starting from January 1st, 2010.

information about foreign investment with another government. They reach the conclusion that the informational behaviour of governments is a strategic variable that should be taken into account when designing the optimal international tax system. In addition, they show that the incentives to transmit information depend on the precise institutional setup of the international tax system. In fact, they prove that, under some features of the tax system, there is no room at all for information sharing, while such exchange may exist and be used for strategic purposes under other institutional arrangements. Similarly, Eggert and Kolmar (2002, 2004) point out that, in a context of optimal taxation, the integration of international capital markets may lead governments to establish sub-optimal tax rates and, hence, to under-provide public goods. By contrast, cooperation among governments leads to another equilibrium solution, characterized by an efficient level of public goods provision as well as by complete and voluntary information exchange between national tax authorities.

The most recent theoretical literature on the subject is inspired by the series of proposals formulated since the late 1980s by the EU Commission and culminated into the adoption of the EU Council Directive on Savings. Huizinga and Nielsen (2003), for example, analyze the minimum withholding tax proposal made by the EU Commissioner Christiane Scrivener in 1989. They set up a three-country model in which a typical EU country has to deal with an inside and an outside tax heaven. They investigate what happens to the welfare of each country when the inside tax heaven is forced to raise its withholding tax. In the same way, Keen and Ligthart (2006a, 2006b) and Gérard (2004) assess the impact on the social welfare of the innovative mechanisms of revenue sharing and shift from a source-based to a residence-based criterion of taxation, which the EU Commission implemented when the idea of a minimum withholding tax faced the opposition of some EU Member States. In particular, these authors find that automatic transfers of withholding tax revenues from the source to the residence country have no effect on the equilibrium tax rates. By contrast, rules allocating some of the proceeds from information exchange to the source country have adverse strategic effects on total revenues, even though such arrangements may help large and high tax countries to convince small and low tax countries to provide information voluntarily. Most importantly, they demonstrate that any kind of withholding tax regime is Pareto dominated by a system of residence-based taxation implying full exchange of information.

The adoption of the Savings Directive by the EU Council in 2003 gave also birth to some empirical literature. Huizinga and Nicodème (2004) investigate the determinants of international deposits, providing evidence that foreign deposits do react to taxation. Also, Ligthart and Voget (2008) analyze the key factors leading governments to exchange information and find that sharing decisions are taken on a reciprocal basis in most cases. Further, Johannesen (2010) and Hemmelgarn and Nicodème (2010) show that the Directive did not lead EU residents to manage major shifts in their international allocation of savings, even if the authors themselves advice to interpret this result with caution given

the small quantity of data available.

The present work develops Gérard (2004) and Keen and Ligthart (2006a, 2006b) and brings two contributions to the literature. First, agents are here considered investors, who choose how to allocate their wealth between national and foreign assets. They are assumed to be able to hold both kinds of financial products simultaneously. In Gérard (2004), Keen and Ligthart (2006a, 2006b), and Bacchetta (1995, 2000), agents only match the after-tax revenues of a domestic asset with those of a foreign asset discounted by a factor representing the higher difficulty in finding information beyond national frontiers. From this comparison, authors calculate the distance at which agents are indifferent between investing locally or abroad, with the consequence that agents located closer to the border invest their whole wealth abroad while agents living more inside the country buy only national assets. In this model, on the contrary, the allocation of savings is made on the basis of the after tax net return of investors' portfolio. Second, agents can hold two kinds of foreign assets at the same time. In section 6, indeed, agents can choose between a financial product regulated by the EU Directive and one falling out of its rules. This allows us to tackle the shortcomings due to the limited coverage of the EU Directive.

3 The Model, Baseline Scenario

In this model we focus on two countries called *home* and *foreign*. We assume that there is a single kind of financial asset providing investors from both countries with a fixed rate of interest r . A distinction is made based on the country where the interests are paid out, that is the source country or the paying agent country. Therefore, a subscript h will indicate that interests are paid through a financial institution in country *home*, while a subscript f will refer to income paid out in country *foreign*.² In addition we assume that in each country there exists a government and that each country is populated by homogeneous economic agents maximizing their utility function. In what follows therefore we indicate with a superscript h the variables referring to the representative agent of population living in country *home* and with a superscript f those belonging to the representative agent of country *foreign*.

The model takes the form of a two-step Nash game, solved backwards. In step 1, governments fix their respective tax rates on savings income in a non-cooperative way. In step 2, investors select their portfolio, that is they decide on the distribution of their savings among the two locations and thus on the way they will have their interests paid out. We apply this game under four possible main scenarios. In the first one, the baseline scenario, no information is exchanged between the two governments and each of them levies taxes on interests paid out in its territory, possibly at a different rate for resident and

² r_h may differ from r_f for various reasons including differences in terms of risk either on exchange rates or country specific.

non-resident investors. In the second scenario, the OECD scenario, interests are again taxed at source by the paying country, which can still discriminate between local and foreign investors. However, unlike in the baseline framework, the residence country now may also tax interests obtained abroad by its residents, provided it grants them a credit for the withholding tax levied in the partner country and get the pertinent information, even though not always in the desired quality. The third scenario refers to the EU Directive exception for Austria and Luxembourg, also adopted in agreements between the EU and some non-EU countries like Switzerland and Liechtenstein. In this scenario there is no systematic exchange of information between countries but part of the revenue from the withholding tax is shared with the residence country of the investor, without disclosing her identity. In the fourth scenario, which corresponds the EU Savings Directive, governments refrain from levying withholding taxes, but fully exchange information so that savings income can be taxed by investors' country of residence.

3.1 The investors

As already pointed out, in the baseline scenario governments can choose different tax rates for local and foreign investors, because foreign interests are taxed at source by the host country and no information at all is exchanged between the residence and the host country. In such a framework, the representative investor of each country has a positive and real wealth w , with $w \in \mathbb{R}_+$, and selects the share of her wealth that she invests in both jurisdictions so to maximize her investment income.

Since in this model agents can only invest in riskless financial products with a fixed interest rate and since agents keep their assets until maturity, they only care about maximizing their return, disregarding the degree of risk aversion. Thus agents' utility maximization problem boils down to maximizing their net portfolio return. Given an arbitrary initial allocation of agents' savings, the representative investor of country *home* decides how to invest her wealth solving the following maximization problem

$$\begin{aligned} \text{Max}_{a_h^h, a_f^h} \quad & U^h = w^h \sum_{k=h,f} z_k^h a_k^h - \frac{v}{2} \sum_{k=h,f} \left(a_k^h - a_{0,h}^h \right)^2, \\ \text{s.t.} \quad & \sum_{k=h,f} a_k^h = 1 \quad \text{and} \quad \sum_{k=h,f} a_{0,k}^h = 1, \end{aligned} \tag{1}$$

where $a_{0,k}^h \in [0, 1]$ and $a_k^h \in [0, 1]$ are respectively the initial and final fraction of savings invested in country k with $k \in \{h, f\}$, $v \in \mathbb{R}_+$ is the adjustment cost due to a change in the initial allocation of the agent's savings, and $z_k^h \in \mathbb{R}_+$ is

the after-tax interest rate.³ This latter can be written as

$$z_h^h = (1 - t_h^h) r_h, \quad z_f^h = (1 - t_f^h) r_f, \quad (2)$$

where $r_k \in \mathbb{R}_+$, again with $k \in \{h, f\}$, stands for the before-tax interest rate paid out in country k , while $t_k^h \in (0, 1)$ indicates the tax rate on the interest paid out in country k to a resident of *home*.

Using the first order conditions, we find that

$$a_h^{h*} = a_{0,h}^h + \frac{z_h^h - z_f^h}{2} \frac{w^h}{v}, \quad a_f^{h*} = a_{0,f}^h + \frac{z_f^h - z_h^h}{2} \frac{w^h}{v}. \quad (3)$$

The representative agent of country *foreign* acts similarly to the investor of country *home* due to the symmetry of the two countries. Throughout the paper, we will hence report the results for country *foreign* only when necessary.

In each country the portfolio allocation of the representative agent depends on the initial share of wealth invested at home, the ratio between adjustment costs and agent's wealth, and the differential between the after-tax net returns on the asset in the two countries.

To better illustrate the relationship between agents' portfolios and tax rates, throughout the paper we will also write the results for a simpler case with entirely domestic initial savings ($a_{0,h}^h = a_{0,f}^f = 1$) and equal interest rate in both countries ($r_h = r_f = r$). Then,

$$a_h^{h*} = 1 - \frac{t_h^h - t_f^h}{2} \frac{w^h}{v} r, \quad a_f^{h*} = \frac{t_h^h - t_f^h}{2} \frac{w^h}{v} r. \quad (4)$$

Proposition I: *If no exchange of information takes place between jurisdictions, then the optimal portfolio allocation of representative agents corresponds to the initial share of wealth invested in each country, adjusted for a function of possible country differences in returns net of taxation.*

3.2 The governments

In the second step of this game, each government maximizes its revenue and chooses the tax rates under its control, knowing the best response functions of investors. In this baseline scenario, where no information is exchanged between the two jurisdictions at stake, government of country *home* decides on the tax rate it charges on the domestic income of its residents, t_h^h , and on the withholding

³The utility function U has been checked to be concave and twice continuously differentiable with respect to its arguments. We have controlled for the validity of this result in all the sections of the paper. First and second conditions have been equally verified throughout the whole development of the model presented in this work.

tax it levies on interest payments paid out in its territory to residents of the other country, t_h^f . The objective function of government h is hence equal to⁴

$$\begin{aligned} \text{Max}_{t_h^h, t_h^f} W^H &= t_h^h a_h^{h*} r_h w^h N^h + t_h^f a_h^{f*} r_h w^f N^f, \\ \text{s.t. } t_h^h, t_h^f &\leq 1, \end{aligned} \quad (5)$$

with N^h and N^f being respectively the population of country *home* and *foreign*.⁵

Solving the maximization programs of both governments, we obtain the best responses for the tax rates charged by *home*⁶

$$\begin{aligned} t_h^{h*} &= \frac{1}{r_h} \left[a_{0,h}^h \frac{v}{w^h} + \frac{r_h - r_f(1 - t_f^h)}{2} \right], \\ t_h^{f*} &= \frac{1}{r_h} \left[a_{0,h}^f \frac{v}{w^f} + \frac{r_h - r_f(1 - t_f^f)}{2} \right]. \end{aligned} \quad (6)$$

The tax rates charged result to be influenced by four factors. First, they are a positive function of the importance of adjustment costs v with respect to agent's wealth. The higher the cost that agents face to move their savings, the higher can be the tax rate charged by a country. Second, they are also positive functions of the initial investment. The higher the initial share of investors' wealth invested in a country, the higher the tax rate set by the government of that country, reflecting the lower need of this latter to attract further savings from the partner jurisdiction. Third, the rates increase with the difference between the interest paid out in the two countries. Fourth, tax rates raise when the complementary tax rate paid by agents domestically or abroad increase.

If we introduce the assumption of the simple case above, the optimal tax rates respectively become

$$t_h^{h*} = \frac{4}{3r} \frac{v}{w^h}, \quad t_h^{f*} = \frac{t_f^*}{2}. \quad (7)$$

It thus turns out that tax rates charged to resident investors are twice as high as those charged to non-resident investors in both countries, showing that,

⁴As for the agents' utility function, we have checked that government's revenue function is concave, with the concavity given by the presence of the functions giving portfolio allocations. Moreover we have also verified that the function is twice continuously differentiable with respect to its arguments as well as controlled for the validity of this result in all the sections of the paper. First and second conditions have been also examined for any of the described scenarios.

⁵In this paper, the two governments are assumed to behave in a Leviathan way. For a justification of such assumption, see Keen and Ligthart (2006a, 2006b).

⁶In this paper we only consider interior solutions and focus on realistic values of the tax rates.

in a context of pure source-based withholding taxation, countries do try to attract foreign capitals through “beggar-thy-neighbour” policies.

Proposition II: *If no exchange of information takes place between jurisdictions, then withholding tax rates charged to foreign investors are half than those charged to resident investors, when initial savings are entirely domestic and interest rates are equal in the two countries.*

4 The OECD Model

According to the OECD Model Tax Convention on Income and Capital, the source country can levy a withholding tax on savings income, whilst the country of residence of the taxpayer can subject the same income to its own tax system with a credit for the withholding tax levied abroad.

4.1 The investors

In such a scenario, the representative agent of country *home* faces the same utility maximization problem as in (1). The return coming from the share of her savings invested abroad, however, will now also depend on the tax rate applied on her domestic income, if this latter rate is higher than the withholding tax charged in country *foreign*, and on the quality of the information regarding her cross-border savings, which is provided by the banks or authorities of country *foreign*, or even by the investor herself, to the authorities of *home*. This is why, as we have already noted above, the OECD system may function properly only if the residence country receives due information from the source country. Following Keen and Ligthart (2006a, 2006b) we introduce the parameter p^f for qualifying the quality of the information obtained by country *home* from country *foreign* and, similarly, the parameter p^h to indicate the information obtained by country *foreign* from country *home*. Consequently, p^h and $p^f \in (0, 1)$, with upper and lower boundaries excluded because representing none and perfect exchange of information respectively.⁷

We model the dependence by the quality of information by modifying the definitions of the after-tax real net returns. For instance, for the representative resident of country *home* these are now defined as

$$z_h^h = (1 - t_h^h) r_h, \quad z_f^h = \left[1 - t_f^h - p^f (t_h^h - t_f^h) \mathbf{1}_{\{t_h^h > t_f^h\}} \right] r_f. \quad (8)$$

where $\mathbf{1}_{\{t_h^h > t_f^h\}}$ is an indicator function taking value 1 for values of t_h^h higher

⁷Notice that we do not consider the case in which the investor herself reveals her savings invested abroad to the authorities of her country of residence. In absence of exchange of information between the two jurisdictions or auditing of foreign banks by the residence country, in fact, it has been shown that the investor has little incentive to report her income. See Gérard and Hadrhi (1994).

than t_f^h and 0 otherwise.⁸ In what follows this function will be omitted for not making the notation heavier.

The allocation of savings for the representative investor of country *home* continues to follow the formula found in (3). In the simple case explained above, however, the two shares become equal to

$$a_h^{h*} = 1 - (1 - p^f) \frac{t_h^h - t_f^h}{2} \frac{w^h}{v} r, \quad a_f^{h*} = (1 - p^f) \frac{t_h^h - t_f^h}{2} \frac{w^h}{v} r. \quad (9)$$

The share of savings invested in the country of residence continues to decrease when the domestic tax rate increases and to increase when the foreign withholding tax rate goes up. Nonetheless the outflow of savings toward the partner jurisdiction is cushioned by the importance of the exchange of information, as the share of savings invested abroad decreases when the quality of the information transmitted improves. Eventually, when that quality is at its top value, $p = 1$, the investor becomes insensitive to the difference between the domestic tax rate and the foreign withholding tax and keeps unchanged the initial distribution of her savings in order to avoid unprofitable, costly portfolio allocations.⁹

Proposition III: *If exchange of information does take place between jurisdictions, then the share of representative agent's portfolio invested in the domestic asset is a positive function of the quality of the information exchanged between jurisdictions. By contrast, the foreign share decreases with any rise in the quality of the information exchanged.*

4.2 The governments

The objective function of each of the two governments now integrates a new source of tax revenue, i.e. that arising from their capacity to tax foreign income of residents thanks to the cross border exchange of information. As a result, the government of country *home* faces the following maximization program

$$\begin{aligned} \text{Max}_{t_h^h, t_h^f} W^H &= [t_h^h a_h^{h*} r_h + p^f (t_h^h - t_f^h) a_f^{h*} r_f] w^h N^h + t_h^f a_h^{f*} r_h w^f N^f, \\ \text{s.t. } &t_h^h, t_h^f \leq 1. \end{aligned} \quad (10)$$

⁸Indeed, the credit granted to taxpayers in their country of residence for the withholding tax paid abroad is limited to the excess of the domestic tax liabilities on the income at stake. Therefore no country will charge a withholding tax larger than the upper limit of the creditable amount.

⁹Taking the derivative of domestic investments in each country with respect to the domestic and the withholding tax rate, in fact, we obtain respectively $\frac{da_h^h}{dt_h^h} \leq 0$ and $\frac{da_h^h}{dt_f^h} \geq 0$.

The best response function for the tax rates charged by *home* are given by

$$\begin{aligned} t_h^{h*} &= \frac{a_{0,h}^h r_h + p^f (a_{0,f}^h r_f)}{(r_h - p^f r_f)^2} \frac{v}{w^h} + \frac{(r_h - r_f) + r_f (1 - 2p^f) t_f^h}{2(r_h - p^f r_f)}, \\ t_h^{f*} &= \frac{a_{0,h}^f}{r_h (1 - p^h)} \frac{v}{w^f} + \frac{r_h (1 - p^h t_f^f) - r_f (1 - t_f^f)}{2r_h (1 - p^h)}. \end{aligned} \quad (11)$$

In the simplified case, the equilibrium solutions for these rates become

$$t_h^{h*} = \frac{4}{r(3 - 5p^f + 2p^{f2})} \frac{v}{w^h}, \quad t_h^{f*} = \frac{t_f^{f*}}{2}. \quad (12)$$

As in the baseline scenario, the tax rate charged to resident investors is twice as high as that charged to non-resident investors. In addition, tax rates are positive functions of the quality of the information received from the partner country. Indeed, an increase in the quality of information exchange reduces the risk of tax motivated outflows of savings and thus allows governments to increase their respective domestic tax rate. Similarly, it incentivizes governments to augment their withholding tax on income paid out to non-residents, since this is credited on tax liabilities in the country of residence of the investors.¹⁰

Proposition IV: *If exchange of information does take place between jurisdictions, then tax rates are positive function of the quality of the information exchanged between jurisdictions. Withholding tax rates charged to foreign investors continue to be half than those charged to resident investors, as in the case of no exchange of information between countries.*

5 The Coordinated Withholding Tax Model

Under this scenario countries *home* and *foreign* transfer each other part of their respective revenues coming from cross-border savings taxation.

5.1 The investors

Nothing changes for the agents decision with respect to what was said in the baseline model. Agents continue to solve the maximization problem explained in (1) and invest their savings according to (3).

¹⁰The sufficient condition for having the existence of an equilibrium and reaction functions as strategic complements requires that the derivative of each tax rate with respect to its complementary tax rate is positive and minor than one. We have verified the existence of this condition also for the OECD scenario. In fact the derivative of domestic tax rates with respect to the quality of information is positive and less than one for the whole field of definition of this latter, that is $0 < \frac{dt_h^h}{dp^f} < 1$ for $p^f \in (0, 1)$ and $\frac{dt_h^h}{dp^f} = 0$ for $p^f = \frac{5}{4}$.

5.2 The governments

On the contrary, the maximization problem for the government of country *home* can be modified as follows

$$\begin{aligned} \max_{t_h^h, t_h^f} W^H &= (t_h^h a_h^{h*} r_h + x t_h^f a_f^{h*} r_f) w^h N^h + (1-x) t_h^f a_h^{f*} r_h w^f N^f, \\ \text{s.t. } &t_h^h, t_h^f \leq 1. \end{aligned} \quad (13)$$

where $x \in [0, 1]$ is the percentage of the withholding tax that the two countries have exogenously agreed ex-ante to pass each other.¹¹ The best response functions for tax rates charged by country *home* are now equal to

$$\begin{aligned} t_h^{h*} &= \frac{a_{0,h}^h}{r_h} \frac{v}{w^h} + \frac{r_h - r_f \left[1 - (1+x) t_f^h \right]}{2r_h}, \\ t_h^{f*} &= \frac{a_{0,h}^f}{r_h} \frac{v}{w^f} + \frac{r_h - r_f \left(1 - t_f^f \right)}{2r_h}. \end{aligned} \quad (14)$$

In the simplified scenario, we have the following equilibrium tax rates

$$t_h^{h*} = \frac{4}{r(3-x)} \frac{v}{w^h}, \quad t_h^{f*} = \frac{t_f^{f*}}{2}. \quad (15)$$

The tax rates levied by the two countries correspond to those found in (7), apart for the presence of the sharing factor x . In particular, for $x = 0$, the expressions found here coincide with those of the baseline scenario. The closer to 1 the x , the higher are the domestic tax rates levied by each country.¹²

Proposition V: *In a framework of coordinated withholding taxation, domestic tax rates are higher than those levied in case of no exchange of information due to the sharing agreement. Withholding tax rates charged to foreign investors, though, keep following the case of no exchange of information between countries.*

6 The EU Directive

The 2003 EU Council Directive on Savings Income, which is currently applied by all EU Member States apart Austria and Luxembourg, has shifted the power to tax cross-border savings from the source to the paying agent country. This framework constitutes the antithetic scenario to what we have presented as baseline model. Indeed, in this system of taxation each country can tax its citizens on the basis of their European-wide income.

¹¹See Keen and Lighthart (2006a, 2006b) for a more detailed explanation of the system.

¹²This is because $\frac{\partial t_h^{h*}}{\partial x} > 0$ and $\frac{\partial t_h^{f*}}{\partial x} > 0$.

6.1 The investors

The representative agent of country *home* continues to maximize the same utility function as in the baseline model (1), but her after-tax real net return on the two assets is now given by

$$z_h^h = (1 - t_h^h) r_h, \quad z_f^h = (1 - t_h^h) r_f. \quad (16)$$

The return on foreign savings income is subject to the tax rate applied to domestic revenues by country *home* and the same applies in country *foreign*.

The allocation of savings is similar to (3). Nevertheless, in the simplified setting, the two after-tax real net returns become equal and the final portfolio allocation always reflect the initial choice of agents.

6.2 The governments

The maximization problem for the government becomes

$$\begin{aligned} \text{Max}_{t_h^h} W^H &= (t_h^h a_h^{h*} r_h + t_h^h a_f^{h*} r_f) w^h N^h, \\ \text{s.t. } t_h^h &\leq 1. \end{aligned} \quad (17)$$

Each country has just one tax rate to choose. Using the first order condition, it is easy to derive that for country *home*, its optimal value is given by

$$t_h^{h*} = \frac{1}{2} + \frac{(a_{0,h}^h r_h + a_{0,f}^h r_f)}{(r_h - r_f)^2} \frac{v}{w^h}. \quad (18)$$

Tax rate equilibrium values are obtained straight from the first order conditions, implying that the two countries fix independently their respective tax rate. This rules out the possibility of setting strategically withholding tax rates, in order to attract foreign assets as in the three previous scenarios. Taxation is, hence, neutral for the allocation of savings.

6.3 The limited field of application of the EU Directive

The EU Directive applies only to savings income in the form of interest payments and to some interest-based financial products. The limited scope of the Directive may incentivize agents to hold products falling out of its range of application like insurance contracts. In what follows, we alternatively assume that these latter can be emitted by only one of the two countries or by both of them.

We firstly consider that only in country *foreign* there exist a financial product falling out of the range of application of the Directive. Agents' income coming from holding this asset is taxed exclusively by the country where interests are paid out, i.e. country *foreign*. In this case, the representative agent of

country *home* maximizes the following utility function

$$\begin{aligned} \text{Max}_{a_h^h, a_f^h, a_{f'}^h} \quad U^h &= w^h \sum_{l=h, f, f'} z_l^h a_l^h - \frac{v}{2} \sum_{l=h, f, f'} \left(a_l^h - a_{0,l}^h \right)^2, \\ \text{s.t.} \quad \sum_{l=h, f, f'} a_l^h &= 1 \quad \text{and} \quad \sum_{l=h, f, f'} a_{0,l}^h = 1. \end{aligned} \quad (19)$$

where f' is the insurance product emitted only in country *foreign*. The after-tax interest rate on f' perceived by the resident of country *home* can now be written as

$$z_{f'}^h = (1 - t_{f'}^j) r_{f'}, \quad (20)$$

where $t_{f'}^j$ is the tax rate at which the income coming from the detention of an insurance product is taxed by country *foreign* disregarding the nationality of the asset owner, i.e. $j \in \{h, f\}$.

As in (3), the solution to the agent's maximization problem is given by

$$a_{l \in k}^{h*} = a_{0,l \in k}^h - \frac{\sum_{i \in k \neq l} z_i^h - (K-1) z_{l \in k}^h}{K} \frac{w^h}{v}, \quad (21)$$

where i and l are different elements of the $k \in \{h, f, f'\}$, and K is the total number of elements contained in k .¹³

Whilst the government of country *home* continues to solve the maximization problem stated in (17), the government of country *foreign* now maximizes

$$\begin{aligned} \text{Max}_{t_f^f, t_{f'}^j} \quad W^F &= t_f^f (a_f^{f*} r_f + a_h^{f*} r_h) w^f N^f + t_{f'}^j (a_{f'}^{h*} w^h N^h + a_{f'}^{f*} w^f N^f) r_{f'}, \\ \text{s.t.} \quad t_f^f, t_{f'}^j &\leq 1. \end{aligned} \quad (22)$$

The contemporaneous solution of the two maximization problems gives the relative best response function for the three taxation rates

$$t_h^{h*} = \phi(t_{f'}^{j*}), \quad t_f^{f*} = \varphi(t_{f'}^{j*}), \quad t_{f'}^{j*} = \chi(t_f^{f*}, t_h^{h*}). \quad (23)$$

Tax rates become interdependent and taxation is not more neutral to agents' decisions. Under the usual simplifying assumptions, the equilibrium tax rates are equal to

$$\begin{aligned} m_h^{hH*} &= \frac{1}{r} \left(\frac{2w^h N^h + w^f N^f}{2w^h N^h} \right) \frac{v}{w^h}, \\ m_f^{fF*} &= \frac{1}{r} \left[\frac{3}{4} + \frac{w^f}{w^h} \left(\frac{w^h N^h + 2w^f N^f}{2w^h N^h} \right) \right] \frac{v}{w^f}, \quad m_{f'}^{jF*} = \frac{1}{r} \left(\frac{w^h N^h + 2w^f N^f}{2w^h N^h} \right) \frac{v}{w^h}. \end{aligned} \quad (24)$$

Optimal tax rates are even functions of the relative size of the two countries in terms of population and wealth. In the response functions each country

¹³This expression is nothing else than a general version of (3). This shorter notation is easier to use when $K \geq 3$.

weights more favourably its population and wealth. Having two tax rates to establish, country *foreign* can set its domestic tax rate at a higher level than country *home* and use the withholding tax rate to attract foreign savings. Country *home*, instead, has to align its decision to the withholding tax rate chosen by the other jurisdiction.

If also in country *home* there is a financial product falling out of the range of application of the Directive (h'), then agents' income coming from holding h and f is taxed exclusively by the country of residence of the taxpayer, while income from h' and f' is taxed only by the country where interests are paid out.

In country *home*, the agent's maximization problem and its solution are similar to (19) and (21), with k now enlarged to comprehend even h' . The government of this country fixes its respective tax rates according to the following new maximization problem

$$\begin{aligned} \text{Max}_{t_h^h, t_{h'}^j} W^H &= t_h^h (a_h^{h*} r_h + a_f^{h*} r_f) w^h N^h + t_{h'}^j (a_{h'}^{h*} w^h N^h + a_{h'}^{f*} w^f N^f) r_{h'}, \\ \text{s.t. } t_h^h, t_{h'}^j &\leq 1. \end{aligned} \quad (25)$$

while the government of country *foreign* continues to behave as in the previous section.

The functions of best response for the government of country *home* are

$$\begin{aligned} t_h^{h*} &= \phi' \left(t_{f'}^{j*}, t_{h'}^{j*} \right), & t_{h'}^{j*} &= \psi' \left(t_h^{h*}, t_f^{f*}, t_{f'}^{j*} \right), \\ t_f^{f*} &= \varphi' \left(t_{f'}^{j*}, t_{h'}^{j*} \right), & t_{f'}^{j*} &= \chi' \left(t_h^{h*}, t_f^{f*}, t_{h'}^{j*} \right). \end{aligned} \quad (26)$$

As in the previous case, tax rates are interdependent and dependent on the relative size of the two countries. The only difference relies in the fact that now also country *home* has two assets. The two countries, hence, return to be symmetric. Annex I provides the details.

6.4 Extending the EU Savings Directive

Extending the scope of the Directive would be a way to restore the efficiency of taxation and avoid competition among governments. Removing the limits of its application, the principle of residence would be valid for all kinds of assets paying interest rates.

Nothing would change in the maximization problem faced by the investors apart the after-tax interest rates, which for agents living in *home* would become

$$\begin{aligned} z_h^h &= (1 - t_h^h) r_h, & z_f^h &= (1 - t_h^h) r_f, \\ z_{h'}^h &= (1 - t_h^h) r_{h'}, & z_{f'}^h &= (1 - t_h^h) r_{f'}. \end{aligned} \quad (27)$$

Governments, instead, would maximize

$$\begin{aligned} \text{Max}_{t_h^h} W^H &= t_h^h (r_h a_h^{h*} + r_f a_f^{h*} + r_{h'} a_{h'}^{h*} + r_{f'} a_{f'}^{h*}) w^h N^h, \\ \text{s.t. } t_h^h &\leq 1. \end{aligned} \quad (28)$$

and set independently the two optimal tax rates.

$$t_h^{h*} = \frac{1}{2} + \frac{K}{2} \frac{\sum_{i \in k} a_{0,i}^h r_i}{\sum_{l \in k} (r_l - r_{m \neq l, \in k})^2} \frac{v}{w^h} \quad \forall (r_l - r_{m \neq l, \in k})^2 \neq (r_{m \neq l, \in k} - r_l)^2$$

where i , l , and m are different elements of the $k \in \{h, f, f', h'\}$, while K is always the total number of elements contained in k . This last expression constitutes the extension of (18) to the case of multiple assets.

Summarising what found in this section before to conclude, we can say that in a framework of perfect exchange of information for all assets, the allocation of assets depends only on the initial allocation given by agents' preferences and return differentials. Any asset mobility due to taxation disappears. In case of exchange of information limited to only some assets, the tax rates levied on financial products for which information exchange hold are direct functions of the tax rates levied on products escaping any exchange of information. Any change in these latter rates has consequences for the tax rates on products for which information is exchanged. This “pass-through” effect means that governments can compete and distort the allocation of assets by choosing the tax rates levied on products not subject to information exchange. As a result, only a full exchange of information on all products avoid mobility of assets due to tax reasons and thus guarantee a “Pareto-optimal ” allocation.

Proposition VI: *Only a system of full exchange of information for all mobile assets guarantees neutrality of taxation.*

7 Conclusions

In this study, with the help of a two-step two-country two-asset game theoretic model solved backwards, we show what system of international taxation is strategically chosen by governments of an highly integrated area like the EU, what the prevailing tax rates in four taxation designs are, and how they influence the optimal decision of investors. By using the hypothesis that the two countries playing the game may both issue financial products falling within the range of application of the rules of the EU Directive, while only one of the two may also issue a financial product escaping such rules, we are able to analyze what the drawbacks of the limited scope of the EU Savings Directive are and how its extension would overcome these shortcomings.

Through analytical comparisons and a numerical exercise, we show that (i) the OECD system gives strong incentives to jurisdictions to undertake non-collaborative policies; (ii) a system of coordinated withholding taxation cannot

be adopted as a general rule in a structure like the EU where, in tax matters, decisions have to be taken at unanimity; (iii) the EU Directive offers an innovative design for the game, even though it incentivizes countries to issue financial products escaping from the application of the EU provisions; (iv) the extension of the scope of the EU directive would permit to achieve a non-distorting scheme of international taxation allowing countries to tax individuals on their European-wide income.

Directions for further research may include the comparison between the different proposals of extension of that Directive and competing systems, like US Qualified Intermediary and FATCA, as well as further studies for generalizing the approach we adopt in this paper including more variables in agents' and governments' objective functions.

References

- [1] Bacchetta, P. and M. P. Espinosa (1995), "Information Sharing and Tax Competition Among Governments", *Journal of International Economics*, 39, pp. 103-121.
- [2] Bacchetta, P. and M. P. Espinosa (2000), "Exchange-of-Information Clauses in International Tax Treaties", *International Tax and Public Finance*, 7, 275-293.
- [3] Cattoir, P. (2006), "A History of the Tax Package: The Principles and Issues Underlying the Community Approach," *Taxation Papers 10*, Directorate General Taxation and Customs Union, European Commission, revised Dec 2006.
- [4] Council of the European Union (2003), "Council Directive on Taxation of Savings Income in the Form of Interest Payments", *Council Directive 2003/48/EC*, 3 June 2003.
- [5] European Commission (1989), "Tax Measures to be Adopted by the Community in Connection with the Liberalization of Capital Movements. Proposal for a Council Directive on a Common System of Withholding Tax on Interest Income", *Communication from the Commission to the Council*, COM(89) 60 final.
- [6] European Commission (2008), "Proposal for a Council Directive Amending Directive 2003/48/EC on Taxation of Savings Income in the Form of Interest Payments", *Communication from the Commission to the Council*, COM(2008) 727 final.
- [7] Eggert, W. and M. Kolmar (2002), "Residence-Based Capital Taxation in a Small Open Economy: Why Information is Voluntarily Exchanged and Why it is Not", *International Tax and Public Finance*, 9, pp. 465-482.
- [8] Eggert, W. and M. Kolmar (2004), "The Taxation of Financial Capital Under Asymmetric Information and the Tax Competition Paradox", *Scandinavian Journal of Economics*, 106, pp. 83-105.
- [9] Gérard, M. (2004), "Combining Dutch Presumptive Capital Income Tax and US Qualified Intermediaries to Set Forth a New System of International Savings Taxation", *CESifo, Munich*, WP 1340.
- [10] Gérard, M. and M. Hadhri (1994), "The European Tax Game and Welfare", in W. Pommerhehne (ed.) *Public Finance and Irregular Activities*, *Public Finance/Finances Publiques*, 49, pp. 211-231.
- [11] Haig, R. M. (1921), "The Concept of Income Economic and Legal Aspects". *The Federal Income Tax*. New York: Columbia University Press. pp. 128.

- [12] Hemmelgarn, T. and G. Nicodème (2010), "Tax Co-ordination in Europe: Assessing the First Years of the EU-Savings Taxation Directive", Centre Emile Bernheim Working Paper 10/006, Université Libre de Bruxelles - Solvay Brussels School of Economics and Management, January.
- [13] Huizinga, H. and S. B. Nielsen (2003), "Withholding Taxes or Information Exchange: The Taxation of International Interest Flows", *Journal of Public Economics*, 87, pp. 39-72.
- [14] Huizinga, H. and G. Nicodème (2004), "Are International Deposits Tax-Driven", *Journal of Public Economics*, 88(6), pp. 1093-1118.
- [15] Keen, M. and J. E. Ligthart (2006a), "Information Sharing and International Taxation: A Primer", *International Tax and Public Finance*, 13(1), pp. 81-110.
- [16] Keen, M. and J. E. Ligthart (2006b), "Incentives and Information Exchange in International Taxation", *International Tax and Public Finance*, 13(2), pp. 163-180.
- [17] Johannesen, N. (2010), "Tax Evasion and Foreign Bank Deposits. Evidence from a Natural Experiment", University of Copenhagen, Economic Policy Research Unit, Working Paper 2010-05.
- [18] Ligthart, J. and J. Voget (2008), "The Determinants of Tax Information Sharing: A Panel Data Analysis", Tilburg University Working Paper.
- [19] Simons, H. (1938), "Personal Income Taxation: the Definition of Income as a Problem of Fiscal Policy". Chicago: University of Chicago Press. p. 49.

Annex I.

A. One country emitting new financial products

In this case, the solution to the agent's maximization problem is given by

$$\begin{aligned} a_h^{h*} &= a_{0,h}^h - \frac{\left[(r_f - r_h) (1 - m_h^{hH}) + r_{f'} (1 - m_{f'}^{jF}) - r_h (1 - m_h^{hH}) \right] w^h}{3} \frac{w^h}{v}, \\ a_f^{h*} &= a_{0,f}^h - \frac{\left[(r_h - r_f) (1 - m_h^{hH}) + r_{f'} (1 - m_{f'}^{jF}) - r_f (1 - m_h^{hH}) \right] w^h}{3} \frac{w^h}{v}, \\ a_{f'}^{h*} &= a_{0,f'}^h - \frac{\left[(r_h + r_f) (1 - m_h^{hH}) - 2r_{f'} (1 - m_{f'}^{jF}) \right] w^h}{3} \frac{w^h}{v}, \end{aligned}$$

that is

$$a_{l \in k}^{h*} = a_{0,l \in k}^h - \frac{\sum_{i \in k \neq l} z_i^h - (K-1) z_{l \in k}^h}{K} \frac{w^h}{v}. \quad A)$$

The best response function for the three taxation rates are

$$\begin{aligned} m_h^{hH*} &= \frac{3 \left(a_{0,h}^h r_h + a_{0,f}^h r_f \right)}{4 \left[r_h^2 - r_f (r_f - r_h) \right]} \frac{v}{w^h} + \frac{(r_h - r_f)^2 + (r_h + r_f) \left[(r_h + r_f) - r_{f'} (1 - m_{f'}^{jF}) \right]}{4 \left[r_h^2 - r_f (r_f - r_h) \right]} \\ m_f^{fF*} &= \frac{3 \left(a_{0,h}^f r_h + a_{0,f}^f r_f \right)}{4 \left[r_f^2 - r_h (r_h - r_f) \right]} \frac{v}{w^f} + \frac{(r_f - r_h)^2 + (r_h + r_f) \left[(r_h + r_f) - r_{f'} (1 - m_{f'}^{jF}) \right]}{4 \left[r_f^2 - r_h (r_h - r_f) \right]} \\ m_{f'}^{jF*} &= \frac{3v \left(a_{0,f'}^h w^h N^h + a_{0,f'}^f w^f N^f \right)}{4r_{f'} (w^{h2} N^h + w^{f2} N^f)} + \\ &\quad + \frac{\left[2r_{f'} - (r_h + r_f) (1 - 2m_{f'}^{jF}) \right] w^{f2} N^f + [2r_{f'} - (r_h + r_f) (1 - m_h^{hH})] w^{h2} N^h}{4r_{f'} (w^{h2} N^h + w^{f2} N^f)} \end{aligned}$$

B. Two countries emitting new financial products

If also in country *home* there is a financial product falling out of the range of application of the Directive, then the representative agent of country *home* maximizes

$$\begin{aligned} \text{Max}_{a_h^h, a_f^h, a_{h'}^h, a_{f'}^h} \quad U^h &= w^h \sum_{l=h,f,h',f'} z_l^h a_l^h - \frac{v}{2} \sum_{l=h,f,h',f'} \left(a_l^h - a_{0,l}^h \right)^2 \\ \text{s.t.} \quad \sum_{l=h,f,h',f'} a_l^h &= 1 \quad \text{and} \quad \sum_{l=h,f,h',f'} a_{0,l}^h = 1 \end{aligned}$$

The after-tax interest rate on asset $l = h, f, h', f'$ perceived by the resident of country *home* can now be written as

$$\begin{aligned} z_h^h &= (1 - m_h^{hH}) r_h, & z_f^h &= (1 - m_h^{hH}) r_f, \\ z_{h'}^h &= (1 - m_{h'}^{jH}) r_{h'}, & z_{f'}^h &= (1 - m_{f'}^{jF}) r_{f'}. \end{aligned}$$

In country *home*, the solution to the agent's maximization problem is given by (A). The functions of best response for the government of country *home* are

$$\begin{aligned} m_h^{hH*} &= \frac{2(a_{0,h}^h r_h + a_{0,f}^h r_f)}{[(r_h - r_f)^2 + 2(r_h^2 + r_f^2)]} \frac{v}{w^h} + \frac{3r_h^2 - r_h [2r_f + r_{f'} (1 - m_{f'}^{jF}) + r_{h'} (1 - 2m_{h'}^{jH})]}{2[(r_h - r_f)^2 + 2(r_h^2 + r_f^2)]} + \\ &+ \frac{r_f [3r_f + r_{f'} (1 - m_{f'}^{jF}) + r_{h'} (1 - 2m_{h'}^{jH})]}{2[(r_h - r_f)^2 + 2(r_h^2 + r_f^2)]}, \\ m_{h'}^{jH*} &= \frac{4v(a_{0,h'}^h w^h N^h + a_{0,h'}^f w^f N^f)}{6r_{h'} (w^{h2} N^h + w^{f2} N^f)} + \frac{[3r_{h'} - (r_h + r_f) (1 - m_f^{jF}) - r_{f'} (1 - m_{f'}^{jF})] w^{f2} N^f}{6r_{h'} (w^{h2} N^h + w^{f2} N^f)} \\ &+ \frac{[3r_{h'} - (r_h + r_f) (1 - 2m_h^{hH}) - r_{f'} (1 - m_{f'}^{jF})] w^{h2} N^h}{6r_{h'} (w^{h2} N^h + w^{f2} N^f)}, \end{aligned}$$

while those for the government of country *foreign* are

$$\begin{aligned} m_f^{fF*} &= \frac{2(a_{0,h}^f r_h + a_{0,f}^f r_f)}{[(r_h - r_f)^2 + 2(r_h^2 + r_f^2)]} \frac{v}{w^f} + \frac{3r_h^2 - r_h [2r_f + r_{f'} (1 - 2m_{f'}^{jF}) + r_{h'} (1 - m_{h'}^{jH})]}{2[(r_h - r_f)^2 + 2(r_h^2 + r_f^2)]} + \\ &+ \frac{r_f [3r_f + r_{f'} (1 - 2m_{f'}^{jF}) + r_{h'} (1 - m_{h'}^{jH})]}{2[(r_h - r_f)^2 + 2(r_h^2 + r_f^2)]}, \\ m_{f'}^{jF*} &= \frac{4v(a_{0,f'}^h w^h N^h + a_{0,f'}^f w^f N^f)}{6r_{f'} (w^{h2} N^h + w^{f2} N^f)} + \frac{[3r_{f'} - (r_h + r_f) (1 - 2m_f^{jF}) - r_{h'} (1 - m_{h'}^{jH})] w^{f2} N^f}{6r_{f'} (w^{h2} N^h + w^{f2} N^f)} \\ &+ \frac{[3r_{f'} - (r_h + r_f) (1 - m_h^{hH}) - r_{h'} (1 - m_{h'}^{jH})] w^{h2} N^h}{6r_{f'} (w^{h2} N^h + w^{f2} N^f)}. \end{aligned}$$

The optimal tax rates, under the usual simplifying conditions, are equal to

$$\begin{aligned} m_h^{hH*} &= \frac{(w^{h2} N^h + w^{f2} N^f) [70w^{h2} N^h + w^f N^f (29w^h + 36w^f)]}{9r (8w^{h4} N^{h2} + 17w^{h2} N^h w^{f2} N^f + 8w^{f4} N^{f2})} \frac{v}{w^h}, \\ m_{h'}^{jH*} &= \frac{2v [26w^{h3} N^{h2} + 16w^{f3} N^{f2} + w^h N^h w^f N^f (19w^h + 20w^f)]}{9r (8w^{h4} N^{h2} + 17w^{h2} N^h w^{f2} N^f + 8w^{f4} N^{f2})}, \\ m_f^{fF*} &= \frac{(w^{h2} N^h + w^{f2} N^f) [70w^{f2} N^f + w^h N^h (29w^f + 36w^h)]}{9r (8w^{h4} N^{h2} + 17w^{h2} N^h w^{f2} N^f + 8w^{f4} N^{f2})} \frac{v}{w^f}, \\ m_{f'}^{jF*} &= \frac{2v [26w^{f3} N^{f2} + 16w^{h3} N^{h2} + w^f N^f w^h N^h (19w^f + 20w^h)]}{9r (8w^{h4} N^{h2} + 17w^{h2} N^h w^{f2} N^f + 8w^{f4} N^{f2})}. \end{aligned}$$