

DYNAMICS MODELLING OF LARGE SUSPENDED PARALLEL CABLE-DRIVEN ROBOTS

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Abstract. *This paper proposes a multibody approach to model and simulate the dynamics of large cable robot, considering hefty cables. Each cable is discretized into finite rigid segments interconnected by ball joints including small stiffness and damping effects. This finite-segment model also involves the development of a simple winding model to transfer the mass of the first translating segment onto the rotating drum. The choice of the required number of segments to use is a key issue that is also discussed in this work. This discussion is based on the comparison of ten models involving one to ten segments per cable regarding their static equilibria, their behavior when the platform is elevated and their eigen frequencies. Finally, an application is proposed to simulate the control of the chosen model. An existing quasi-static analytical model is also used for comparison and to devise the control strategy.*

1 INTRODUCTION

Parallel cable-driven robots mainly consist of a mobile platform connected in parallel to a base by cables (see Fig. 1). Each cable is wound and unwound by actuated winches which are generally fixed to the base. The winches control the position and orientation of the platform by modifying the length of their respective cable. Since the overall mass and inertia of the moving parts are reduced, cable robots exhibit interesting properties compared to rigid-link robots. According to their size and practical realization, they may be less expensive and easier to build, transport and reconfigure.



Figure 1: Fatronik/LIRMM prototype of a 3-meter-side 6-cable robot.

Another useful property of cable robots is their *scalability*. Indeed, cables with small to very large lengths are easily stored on drums permitting the realization of parallel cable-driven robots with a workspace of global dimensions ranging from a few ten or so centimeters to hundred of meters. In this research project, we are interested in very large cable robots (e.g. 20 m), possibly carrying heavy to very heavy loads (e.g. 100 kg). In practice, such manipulators may be used in aircraft or shipbuilding industries to improve safety and cut the costs of some operations such as welding or painting over huge distances.

The goal of this work is to simulate the dynamics of large cable robots with a robust and accurate model in order to improve mechanical design and control. Since heavy loads are involved, the mass of the cables must be taken into account in the model as well as their flexibility and extensibility. This is particularly true when the platform mass is small compared to the cable mass, e.g. when the manipulator is unloaded. Cable dynamics has been studied for various applications such as large telescope antenna [3], marine vessel towing [2], flexible marine riser [4] or towed-aerial-cable sensors [7]. In the context of multibody dynamics with large deformations, a lumped-mass model is usually chosen for its computational efficiency [4] and the ability to simulate the pay-out/reel-in process [2, 7]. Similar to the lumped-mass model, a finite-segment model is proposed in this work to take into account bending and sagging effects. A variable-cable-length model is then devised to simulate the pay-out/reel-in process. This model can be refined to consider the extension and possibly the torsion of the cables.

The paper is organized as follows. In Section 2, the model of hefty cables is studied from different points of view. A multibody approach with finite segments is developed in Section 2.1. An analytical model taken from [5] is described for quasi-static behaviors in Section 2.2. In Section 2.3 a simple model is devised to consider the winding process of the finite-segment cables around the drums. After this general modelling, a discussion is proposed in Section 3

to choose the number of segments required to discretize a cable. Three situations are therefore analyzed in Sections 3.1 to 3.3: a comparison of the static equilibria, a simple simulation to lift up the platform and a modal analysis. Once the number of segments is determined, the application of the finite-segment model is illustrated in Section 4 to design a controller based on the quasi-static model of Section 2.2.

2 HEFTY CABLE MODELLING

This section will present first two approaches to model hefty cables: a multibody approach for the dynamics and an analytical model for quasi-static behaviors. Next, a simplified model of the winding process is proposed.

2.1 Multibody dynamic model using finite segments

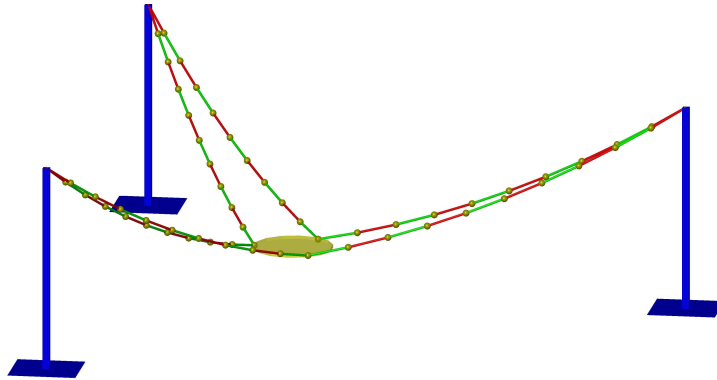


Figure 2: Multibody model a large 20-meter-side 6-cable robot using 10 segments per cable.

As depicted in Fig. 2, a suspended cable-driven robot consists of m ($m \geq 6$) cables to manipulate the six degrees of freedom of the platform. Each cable is discretized into n rigid segments ($m = 6$ and $n = 10$ in Fig. 2). These segments are interconnected by ball joints including small bending stiffness (e.g. 0.01 Nm) and small damping coefficients (e.g. 0.01 Nms) as shown in Fig. 3. Note that most lumped-mass models neglect these bending terms. The value of these coefficients will be determined experimentally. The ball joints connecting the cables to the base and to the platform are assumed to be ideal (e.g. no friction, no backlash). A prismatic joints is inserted between the first segment of each cable and the corresponding base ball joint to pull or slacken the cables. Future work will include additional prismatic joints between the segments to consider the extensibility of the cables.

The full multibody model of the robot is generated thanks to the Robotran[©] software, which provides the equations of motion in a compact symbolic form using recursive algorithms [6]. This has been done for a variable number of segments composing the six cables. The total number of degrees of freedom is substantial, varying from 42 for one segment per cable to 204 for 10 segments. A discussion about the number of segments required for an sufficiently accurate model is developed in Section 3.

2.2 Analytical quasi-static model

In this Section, a simplified analytical model of a cable robot with hefty cables is described for a quasi-static behavior. All the results are summarized from [5]. It does not consider the

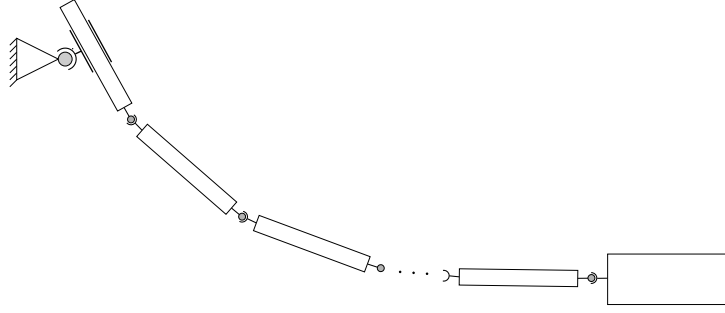


Figure 3: Finite-segment model of a hefty cable.

dynamics of the cables but the dynamics of the platform itself can be taken into account as an external wrench (forces and torques) applied to the system.

When the mass of the cables is significant compared to the mass of the load, the cables are not taut in general but are sagging between both attachment points. In that case, the cable profile is determined by the cable length, the position of the attachment points as well as the tension in these points. Computing the inverse kinematics of the robot is thus not straightforward since cable lengths and applied forces are completely coupled [1]. However, under some assumptions, the authors of [5] have proposed the following simplified quasi-static model in which tensions and lengths are decoupled.

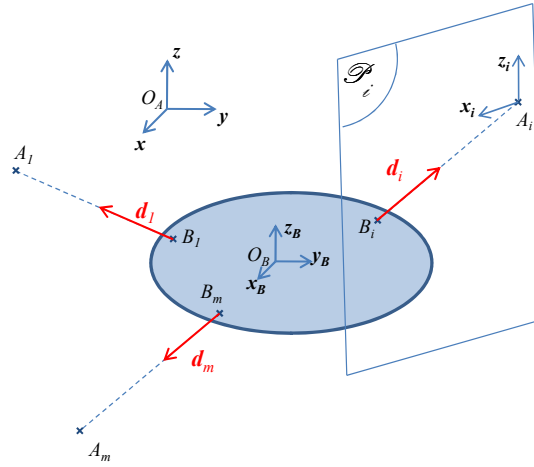


Figure 4: Platform and notations.

Let us consider the cable robot of Fig. 4 whose platform is connected to the base by m cables. These cables are assumed inextensible and are attached to the base in A_i and to the platform in B_i . They are sagging in their respective vertical plane $\mathcal{P}_i$ for their dynamics is disregarded. According to these conditions, the static equilibrium of the platform of a parallel robot driven by m cables on which an external wrench $\mathbf{f}_e$ is applied, is satisfied when:

$$\mathbf{W}_x \boldsymbol{\tau}_x = \mathbf{f}_e - \mathbf{f}_{cab}, \quad (1)$$

where:

- $\mathbf{W}_x$ is a 6 by m matrix defined as:

$$\mathbf{W}_x = \begin{bmatrix} \mathbf{q}_1 & \mathbf{q}_2 & \cdots & \mathbf{q}_m \\ \mathbf{Q}\mathbf{b}_1 \times \mathbf{q}_1 & \mathbf{Q}\mathbf{b}_2 \times \mathbf{q}_2 & \cdots & \mathbf{Q}\mathbf{b}_m \times \mathbf{q}_m \end{bmatrix}, \quad (2)$$

with $\mathbf{q}_i = [\cos \gamma_i \quad \sin \gamma_i \quad \tan \beta_{0i}]^T$, γ_i corresponding to the angle between the unit vectors $\mathbf{x}_i$ and $\mathbf{x}$, and β_{0i} being the angle between $\mathbf{x}_i$ and the chord $A_i B_i$;

- $\boldsymbol{\tau}_x$ is a vector containing the horizontal components $\tau_{l_{ix}}$ of the tension in the cables in B_i :

$$\boldsymbol{\tau}_x = [\tau_{l_{1x}} \quad \tau_{l_{2x}} \quad \cdots \quad \tau_{l_{mx}}]^T \quad (3)$$

- $\mathbf{f}_{cab}$ is a wrench vector accounting for the sum of cable weight contributions:

$$\mathbf{f}_{cab} = \sum_{i=1}^m \begin{bmatrix} \mathbf{f}_i \\ \mathbf{Q}\mathbf{b}_i \times \mathbf{f}_i \end{bmatrix}, \quad (4)$$

with $\mathbf{f}_i = [0 \quad 0 \quad \rho_0 g L_i / 2]^T$.

In the previous definitions, matrix $\mathbf{Q}$ is the rotation matrix from the inertial frame $\mathcal{R}_A$ to frame $\mathcal{R}_B$ attached to the platform and L_i is the distance of the chord $A_i B_i$. From Eq. 1, tension components τ_x can be obtained as:

$$\boldsymbol{\tau}_x = \mathbf{W}_x^{-1} (\mathbf{f}_e - \mathbf{f}_{cab}). \quad (5)$$

It is shown in [1] that this model is valid only if the ratios r_i are smaller than one, i.e.:

$$r_i = \frac{\rho_0 g L_i}{\tau_{l_{ix}}} \leq 1 \quad \forall i = 1, \dots, m, \quad (6)$$

where ρ_0 is the linear density of the cables and g is the norm of the gravity vector. From this validity ratio r_i , the actual lengths l_i of each cable are now computed as corrections with respect to L_i :

$$l_i = L_i \left(1 + \frac{r_i^2}{24} - \frac{r_i^4}{640} + \dots \right). \quad (7)$$

Finally, the norm τ_{0i} of the tension of the cables in A_i can be obtained with the following expression:

$$\tau_{0i} = \sqrt{\tau_{l_{ix}}^2 + \left(\frac{\rho_0 g L_i}{2} - \tau_{l_{ix}} \tan \beta_{0i} \right)^2}. \quad (8)$$

This will be useful to estimate the needed actuator torques (see Section 4).

2.3 Winding process model

To simulate the winding process with finite-segment cables, we propose to model it as a mass transfer between the cable and the rotating drum as illustrated in Fig. 5. Indeed, when the first rigid segment of the cable is pulled or slackened, part of it must be respectively wound or unwound onto the drum (see gray parts in Fig. 5). Therefore, the mass of the first rigid segment increases (resp. decreases) and the mass of the spool decreases (resp. increases) proportionally

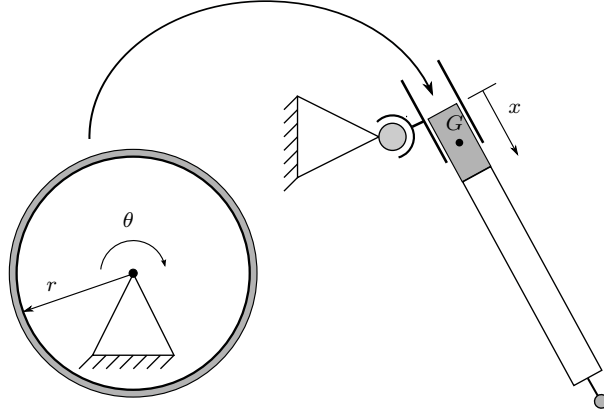


Figure 5: Mass transfert from the drum to the first cable segment while unwinding the cable.

to the displacement x of the cable w.r.t. the supporting ball joint. The transferred mass Δm can thus be written as:

$$\Delta m = \rho_0 x, \quad (9)$$

where ρ_0 is the linear density of the cable.

Practically, a variable punctual mass is added in G to model the variation of the first segment and a variable hoop is added on the circumference of the drum to model the winding/unwinding of the first segment. More precisely, the mass is transferred between both additional bodies. If $\mathbf{R}$ denotes the position vector of G of the additional mass on the first segment, its equation of motion is:

$$\frac{d}{dt} (\Delta m \dot{\mathbf{R}}) = \Delta m \mathbf{g} + \mathbf{F}_c \quad (10)$$

$$\Leftrightarrow \rho_0 \dot{x} \dot{\mathbf{R}} + \rho_0 x \ddot{\mathbf{R}} = \rho_0 x \mathbf{g} + \mathbf{F}_c, \quad (11)$$

where $\mathbf{g}$ is the gravity vector and $\mathbf{F}_c$ is the constraint force providing that the translation of G is half the translation of the first segment. On the other hand, due to the mass transfer, the mass of the additional arc on the drum becomes $-\Delta m$, i.e. a moment of inertia of $-\Delta m r^2$ about the axis of rotation. Assuming no slipping of the cable on the spool, the rotation θ of the drum is kinematically constrained by the position x of the prismatic joint actuating the first segment. This constraint equation is:

$$\dot{x} = r \dot{\theta}. \quad (12)$$

Following this kinematic constraint, the rotation angle of the hoop is half the rotation angle of the drum, since the position of G is half the position of the first segment in translation. The scalar equation of rotation of the hoop becomes:

$$\frac{d}{dt} \left(-\Delta m r^2 \frac{\dot{\theta}}{2} \right) = T_c \quad (13)$$

$$\Leftrightarrow -\rho_0 \dot{x} r^2 \frac{\dot{\theta}}{2} - \rho_0 x r^2 \ddot{\theta} = T_c, \quad (14)$$

where T_c is the constraint torque required to rotate the hoop at half the velocity angle of the drum.

For a horizontal displacement of the first segment, it can be checked by virtual works that constraint efforts $\mathbf{F}_c$ and T_c yield opposite contributions, meaning that these are internal forces.

Indeed, virtual displacements δx of the segment and $\delta\theta$ of the drum lead to the following virtual works:

$$F_c \delta x = \rho_0 \frac{\dot{x}^2}{2} \delta x + \rho_0 x \frac{\ddot{x}}{2} \delta x \quad (15)$$

$$T_c \delta\theta = -\rho_0 \dot{x} r^2 \frac{\dot{\theta}}{2} \delta\theta - \rho_0 x r^2 \frac{\ddot{\theta}}{2} \delta\theta, \quad (16)$$

which are exactly opposite when considering constraint (12).

The effects of this winding model will be analyzed in the application of Section 4.

It should be pointed out that if the cable length increased beyond the length of the first segment, a new segment should be inserted in the model. Conversely, the reduction of the cable length should involve the withdrawal of a segment. Since the topology of the multibody system is thereby modified, a new system of equations of motion should be considered. The transition would have to be carefully performed with appropriate initial conditions for the integration of the new differential equations. However, this eventuality has not been considered in this work but is a future perspective of this research.

3 HOW MANY SEGMENTS FOR THE DYNAMIC MODEL OF THE CABLES ?

To analyze the effect of the number n of segments, 10 models have been built with one to 10 segments in each cable. Except the size of the segments, all these models share the same geometric features: the positions of the drawing points A_i of cables on the base and the positions of the attachment points B_i on the platform. They are reported in Table 1.

	x (m)	y (m)	z (m)
A_1	11.55	0.00	6.00
A_2	11.55	0.00	6.00
A_3	-5.77	10.00	6.00
A_4	-5.77	10.00	6.00
A_5	-5.77	-10.00	6.00
A_6	-5.77	-10.00	6.00
B_1	0.58	-1.00	0.00
B_2	0.58	1.00	0.00
B_3	0.58	1.00	0.00
B_4	-1.15	0.00	0.00
B_5	-1.15	0.00	0.00
B_6	0.58	-1.00	0.00

Table 1: Positions of points A_i in inertial frame $\mathcal{R}_A$ and B_i in body-fixed frame $\mathcal{R}_B$ (See Fig. 4).

Since experimental data are not yet available, stiffness k_s and damping D_s parameters of the ball joints between the segments are chosen at a small value depending on the number of segments: $k_s = \frac{0.01 \text{ Nm}}{n}$ and $D_s = \frac{0.01 \text{ Nms}}{n}$.

In the following, the 10 models are compared on the basis of three criteria: the static equilibrium height of the platform, a simple dynamic simulation and a modal analysis. This will enable us to choose the required number of segments.

3.1 Static equilibrium

For each of the 10 models, the static equilibrium is computed with 12-meter constant lengths of the 6 cables. The equilibrium height of the platform is then compared to the value obtained with the analytical model of Section 2.2. The results are illustrated in Fig. 6. It is observed that the equilibrium height increases with the number of segments. From 6 to 10 segments, the value does not change significantly and remains different from the analytical value drawn with a dotted line. Let us notice that the one-segment model does not yield an equilibrium but rather the closed-loop configuration of a rigid mechanism.

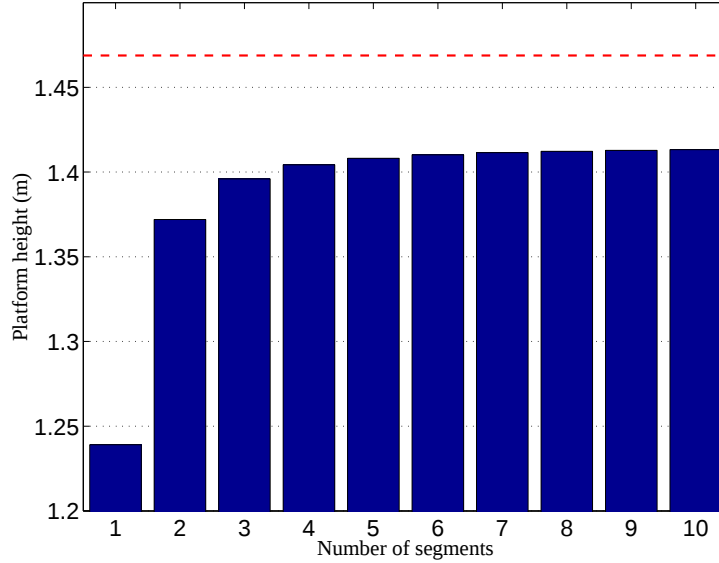


Figure 6: Equilibrium height of the platform for 12-meter cables w.r.t. the number of segments in each cable. The dotted line denotes the equilibrium obtained with the analytical model of Section 2.2.

3.2 A first simulation

Starting from the equilibrium configurations obtained in the previous Section, a first simulation is carried out. The input torques actuating the winches are the same for each model. They are computed from the inverse dynamics of the one-segment model so that the platform is elevated one meter in two seconds from the initial equilibrium pose. To simulate a critical case, the mass of the unloaded platform (10 kg) is lower than the mass of the six cables ($6 \times 3.6 \text{ kg} = 21.6 \text{ kg}$).

The comparison of the 10 models is illustrated in Fig. 7. The one-segment model exactly follows the prescribed trajectory. A discrepancy of more than 20 cm between the one-segment model and the 10-segment one can be observed, showing the need of modelling the cable bending for control purposes (see Section 4). As in the previous Section, the behaviors of the models are very similar from 6 to 10 segments. From 2 to 10 seconds of simulation, oscillations occur because of the small damping factors and the relatively small platform mass with respect to the cable mass. Finally, it should be noticed that the curve of the 2-segment simulation does not exhibit higher-order eigen frequencies than the 10-segment simulation. This last effect is analysed in the next Section.

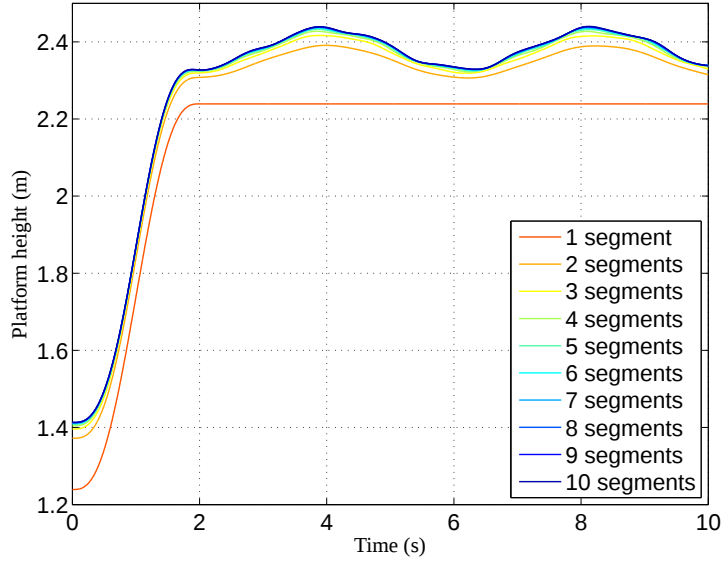


Figure 7: Simulation from the equilibrium height with 12-meter cables, applying the input torques to raise the platform one meter higher. The one-segment model exactly follows the prescribed trajectory.

3.3 Modal analysis

In order to give a more accurate description of the 10 models, a modal analysis is performed about the equilibrium configurations obtained in Section 3.1. Part of the resulting eigen frequencies is reported in Table 2. It obviously shows an increasing number of modes with the number of segments. Depending on the application and the type of simulation, a cut-off frequency is chosen and thereby a number of segments. In our case of large cable robot, motions are relatively slow but the effects of oscillating cables are important when controlling the pose of the platform. This will be illustrated in the next Section.

mode number	2 seg. (Hz)	3 seg. (Hz)	4 seg. (Hz)	5 seg. (Hz)	6 seg. (Hz)	7 seg. (Hz)	8 seg. (Hz)	9 seg. (Hz)	10 seg. (Hz)
7	1.05	1.03	1.01	1.00	0.99	0.99	0.99	0.99	0.99
11	-	1.72	1.65	1.60	1.57	1.56	1.55	1.54	1.53
15	-	-	2.60	2.53	2.45	2.40	2.36	2.33	2.31
18	-	-	-	3.43	3.35	3.25	3.17	3.11	3.06
23	-	-	-	-	4.29	4.26	4.16	4.06	3.99
26	-	-	-	-	-	5.12	5.12	5.02	4.90
30	-	-	-	-	-	-	5.95	6.00	5.90
33	-	-	-	-	-	-	-	6.77	6.85
38	-	-	-	-	-	-	-	-	7.59

Table 2: Partial eigen frequencies of the six-cable robot w.r.t. the number of segments in each cable.

In the following, considering the current application needs, a model with 6 cables is chosen (see Fig. 8). It provides a good trade-off between accuracy in terms of the three studied criteria and the model complexity (e.g. CPU time of simulation).

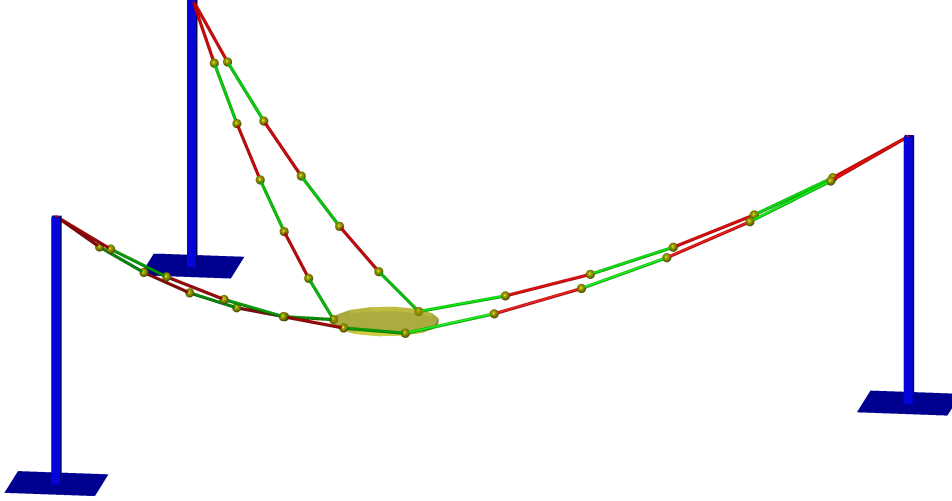


Figure 8: Multibody model of the large 6-cable robot using 6 segments per cable with dimensions defined in Table 1.

4 APPLICATION TO CONTROL DESIGN

It is proposed to design a simple controller for pick-and-place applications. The basic idea is to use the results of the simplified quasi-static model developed in Section 2.2 as a feedforward component with a PID controller to adjust the output by feedback. This strategy is illustrated in Fig. 9. In this Figure, the different variables are:

- x_d : the desired pose of the platform;
- Q_d : the desired torques computed with the analytical model;
- α_d : the angles of the winches also computed with the analytical model;
- α_m : the same angles measured on the robot model;
- Q : the input torques applied to the six winches.

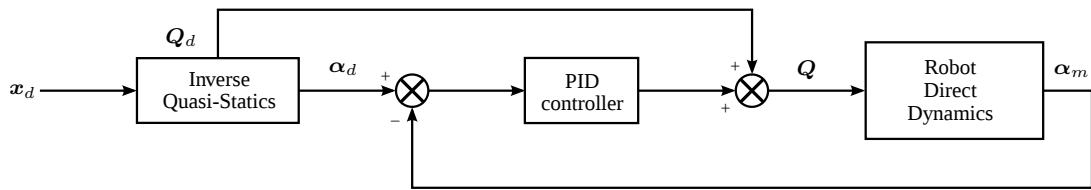


Figure 9: Control diagram.

Given the platform pose x_d , the linear density of the cables and the positions of the attachment points, the actual length l_i of the cables can be computed thanks to Eqs. 2 to 7. Moreover, the norm τ_{0i} of the cable tension at the drawing points can be obtained with Eq. 8. Then, the outputs of the inverse quasi-statics block finally result from:

$$Q_{di} = \tau_{0i} R_w \quad (17)$$

$$\alpha_{di} = \frac{l_i}{R_w}, \quad (18)$$

where R_w is the radius of the winches and $i = 1, \dots, 6$. The gains of the PID controller are: 3 (proportional), 1 (integral) and 0.5 (derivative). These are probably not optimal but tuning the best controller is not the goal here. The following example is only illustrative of the importance of an accurate model to devise the control strategy.

In our example, the task of the robot is to displace the unloaded platform of 10 kg from point $[0 \ 0 \ 1.4102]^T$ to point $[0 \ 1.5 \ 1.4102]^T$ with an average velocity of 0.5 m/s. In details, the motion planning is divided into three parts:

1. vertical motion from $[0 \ 0 \ 1.4102]^T$ to $[0 \ 0 \ 2.4102]^T$;
2. horizontal motion from $[0 \ 0 \ 2.4102]^T$ to $[0 \ 1.5 \ 2.4102]^T$;
3. vertical motion from $[0 \ 1.5 \ 2.4102]^T$ to $[0 \ 1.5 \ 1.4102]^T$.

Each parts of the planning is determined by a five-order polynomial in position to ensure zero velocities and accelerations at both start and end points. The total trajectory is achieved in 7 seconds. The vertical and horizontal motions are plotted in Fig. 4. The corresponding actuator torque of the first cable is plotted in Fig. 11.

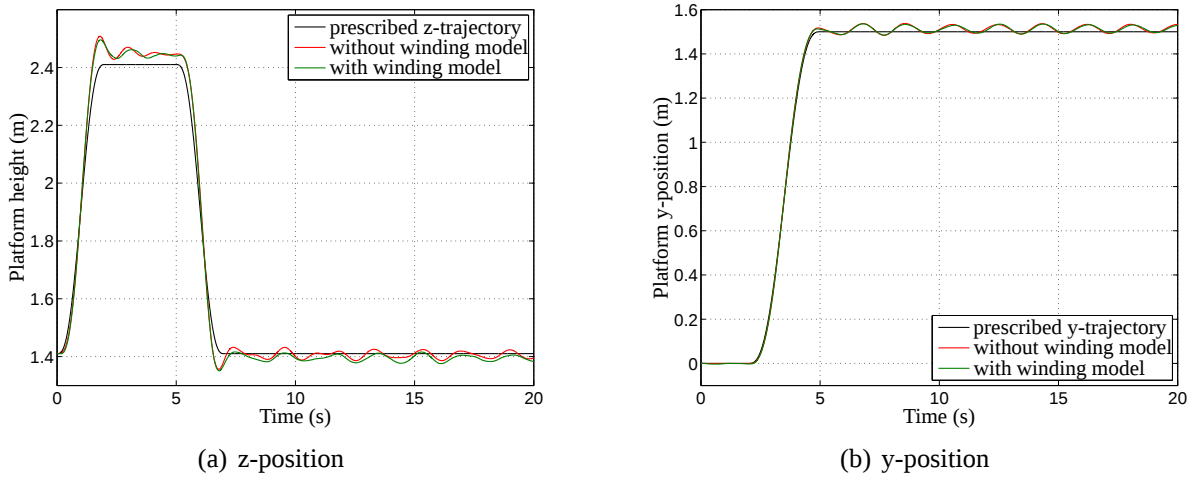


Figure 10: Evolution of the platform position.

In general, the results show the difficulty to control the pose of the unloaded platform because of the bending oscillations in the cables. This disturbance is particularly observed at rest after the displacement of the platform to the end point. These oscillations will restrict the dynamics of this pick-and-place motion. It is also interesting to observe the effects of the winding model developed in Section 2.3. In terms of position of the platform, these effects are relatively low. However, it affects more the behavior of the actuator torques as shown in Fig. 11.

5 CONCLUSIONS

In conclusion, the modelling of large cable-driven robots has been performed in a multibody approach. Each hefty cable has been discretized into finite rigid segments interconnected by ball joints. This has lead to large models with more than 100 degrees of freedom. Such models have been generated thanks to the Robotran[©] software which provides the equations of motion in a symbolic compact form. To refine the multibody model, the winding process of the cable onto the rotating drum has also been modelled. A simple variable mass transfer between the

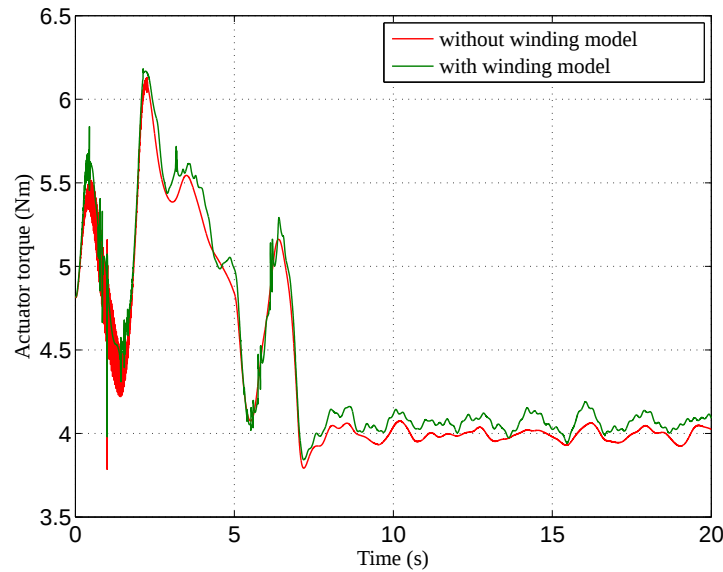


Figure 11: Evolution of the first actuator torque.

cable and the drum is considered. Beside this multibody model, an analytical quasi-static model is also summarized from [5] to compare the results and to develop the control strategy.

Following that multibody modelling, the issue about the required number of segments in each cable has been discussed. Ten models with one to ten segments per cable have been compared in three situations: the obtained static equilibrium for given-length cables, the behavior of the model when elevating the platform as well as their eigen frequencies. The number of segments has been chosen as a trade-off between these three results and the model complexity. Finally, as application, the chosen model has been simulated to design a controller based on the aforementioned quasi-static model. It has shown the importance of the model to highlight the disturbances due to the oscillations of the cables.

In terms of prospects, this model can still be refined considering the extensibility of the cable and adjusting the stiffness and damping parameters according to experimental data. For larger motion, the model should be carefully extended to insert or withdraw segments in the cables. The proposed control strategy is only illustrative and should also be improved before using it with the future prototype.

6 ACKNOWLEDGEMENTS

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