UNIVERSITE CATHOLIQUE DE LOUVAIN
Faculté de Psychologie et Sciences de l'Education

When [5] looks like [6]

A deficit of the number magnitude representation in developmental dyscalculia: Behavioural and brain-imaging investigation

by

Christophe Mussolin

<u>Jury</u>: Profs. Marie-Pascale Noël and Anne De Volder, UCL, Promoters
Prof. Jacques Grégoire, UCL, President
Prof. Mauro Pesenti, UCL
Prof. Manuela Piazza, Università di Trento
Prof. Xavier Seron, UCL

Thesis submitted in partial fulfilment of the requirements for the degree of
"Docteur en Psychologie, orientation Neurosciences"

2009

A la mémoire de ma maman

Acknowledgements

La réalisation d'une thèse n'est pas qu'un travail personnel, mais le fruit de collaborations, de relations professionnelles et amicales. Beaucoup de personnes ont contribué à cette thèse, et je voudrais les remercier.

Mes premiers remerciements vont à mes deux promotrices. Marie-Pascale, je pourrais te remercier pour ta disponibilité, pour ton aide précieuse, pour ton écoute attentive et tes encouragements lors des moments difficiles. Mais ce n'est pas le plus important. Tu m'as vraiment donné le goût de la recherche. Si j'ai progressé durant ces années de doctorat, c'est grâce à toi. Tu as su me comprendre et trouver les mots et les attitudes adéquats pour me faire avancer. A ton contact, j'ai repoussé mes limites et n'ai jamais baissé les bras même si j'en ai eu souvent l'envie. Bien sûr notre cheminement mutuel n'a pas été qu'un long fleuve tranquille, mais je suis convaincu que c'est en affrontant les difficultés que l'on progresse. Tu as été présente depuis le début de mon parcours de jeune chercheur, et j'espère que tu continueras à me suivre pour un petit bout de chemin.

Anne, cela a vraiment été un plaisir de travailler avec toi. Ta bonne humeur, ton enthousiasme, ta patience, ta gentillesse ont été autant de pierres pour mener à bien cet édifice. Je n'oublierai pas de sitôt nos longs après-midi à Woluwé, à croiser les doigts pour qu'aucun problème technique ou autres ne viennent perturber l'enregistrement des données fMRI. Je garde en mémoire également ton aide précieuse lors de l'écriture des articles scientifiques. En un mot, je n'aurais pas pu choisir meilleure co-promotrice que toi.

Mauro, peu de gens le savent mais ce n'est pas le désir de travailler avec Xavier Seron, ou l'envie de présenter une réunion devant le groupe calcul, ou encore le plaisir de pouvoir contredire Stanislas Dehaene lors d'une conférence internationale qui m'ont amené à faire de la recherche à l'UCL. Sache que mon seul objectif était de jouer avec toi dans l'équipe des Black Rats. Depuis, j'ai parcouru un peu de chemin et j'ai pu apprécier tes qualités de chercheur et d'homme. Tu m'as toujours apporté ton aide quand j'en avais besoin, et tu l'as fait avec beaucoup de sérieux et d'implication. Pour ces différentes raisons, tu resteras à jamais Cap'tain Mo.

Xavier, je me souviens de notre première confrontation lorsque, jeune étudiant, j'ai dû un peu me battre avec toi pour changer de thème de mémoire. Tes réticences initiales se sont muées en félicitations lors de ma défense. Cette situation illustre bien ta personnalité. Tu pousses les autres à te prouver qu'ils ont raison, mais tu sais aussi reconnaître tes torts (de temps en temps). Tu es un sage, et par là je ne veux pas parler de ton grand âge, mais de ta capacité à mener à bien une discussion, à en ressortir l'essentiel. Tu es et resteras un exemple pour le jeune chercheur que je suis.

Jacques, dans ce monde cruel et dur de la recherche, qu'il est bon de trouver des personnes comme vous qui savez mettre en évidence le positif et soutenir les initiatives. Je garde en mémoire notre souper dans un petit restaurant français à Berlin au cours duquel j'ai appris à mieux vous connaître.

My sincere gratitude goes also to Professor Manuella Piazza for his acceptance to be member of the jury, and for undertaking the journey from Rovereto to Louvain-la-Neuve just to listen to me.

Merci aussi …

…à Cécile Grandin et Laurent Hermoye pour leur aide précieuse et leur patience durant les longues journées passées en neuroimagerie à Woluwé

…à mes étudiantes Nathalie, Julie, Patricia qui ont largement contribué à cette thèse

…à Yolande, pour sa bonne humeur et sa présence stimulante dans notre unité

…à Pierre Feyereisen, Marie-Anne Schelstraete, et Michel Hupet pour leur soutien

…à Nathalie Lefèvre pour ses nombreux conseils statistiques

…aux directeurs des écoles de Namur et de Bruxelles qui ont accepté de participer à nos différents projets

…et surtout aux enfants et à leurs parents, sans qui cette thèse n'aurait pas eu lieu

Et puis la recherche, c’est aussi une histoire d’amitié…

Merci à Coralie pour nos discussions professionnelles et personnelles durant les longs trajets quotidiens entre Louvain-la-Neuve et Namur

Merci à Anne, Sandrine M., et plus récemment Mandy pour les nombreux repas à la Galoute durant lesquels nous avons partagé tant de discussions

Merci aux Black Rats Denis, Pierre, Nico, Salva, Val, Mike, Dam, Gaets, Xavier, Gilles, David, les Mathieux, ainsi que le team PSOR, pour m’avoir permis de marquer tant de buts et surtout de passer des rares moments de détente

Merci à tous les autres membres des unités CODE et NESC : Auriane, Charlotte, Christine, Valérie, Marie, Matthieu, Véro, Arnaud, Fred, Thomas, Adé, Béné, Caro, Corentin, Peggy, Bruno, René, Raymond, et Agnesa avec qui j’ai partagé cinq ans de ma vie

Un petit clin d’œil particulier à me voisins de bureau Pierre et Sue qui ont réussi à ma transmettre un peu de leur énergie à travers la cloison

Enfin, si je ne devais retenir qu’une seule chose durant ces années de recherche, ce sont les moments passés en dehors de l’université avec mes amis Sandrine C., Maria, Angels, Marina, Fab, Sam, et Virginie. En particulier, l’école d’été en Sicile ainsi que les très nombreuses sorties « arrosées ». Mais surtout, je garderai en mémoire cette soirée spéciale passée chez Fabrizio.

Merci à toi Laetitia pour m’avoir soutenu et supporté pendant ces derniers mois. Je n’y serais certainement pas arrivé sans toi

Table of contents

List of abbreviations

ANOVA	ANalysis Of VAriance
BOLD	Blood Oxygen Level Dependent
DD	Developmental Dyscalculia
ERPs	Event-Related Potentials
fMRI	functional Magnetic Resonance Imaging
FWE	Family-Wise Error
GLM	General Linear Model
IPS	IntraParietal Sulcus
IQ	Intellectual Quotient
KRT	Kortrijk Rekenen Test
MD	Mathematical Disabilities
MNI	Montreal Neurological Institute
PET	Positron Emission Tomography
PoorAF	Poor Arithmetic Facts abilities
ROI	Region Of Interest
RTs	Response Times
SNARC	Spatial-Numerical Association of Response Codes
SOA	Stimulus Onset Asynchrony
SPM 2	Statistical Parametric Mapping, version 2
TMS	Transcranial Magnetic Stimulation
VCFS	Velo-Cardio-Facial Syndrome

GENERAL INTRODUCTION

The manipulation of numbers is a fundamental ability involved in a wide range of daily activities, like estimation of quantities, managing money, telling the time, or planning scientific experiments. Numbers can express different numerical properties such as the cardinality of a set, the order of elements in a series, and even a nominal assignment. The numeral "8", for instance, refers to twice four, a quantity larger than seven, the place before the ninth, the number of chapters in this thesis, or the shirt number of football player Steven Gerrard.

In recent decades, research has begun to address the origins of the human mind's competence for numerical processing. Using recent methods of cognitive neuroscience, one can now ask which internal representations are used to manipulate numbers mentally, when and how they develop, and which brain areas are involved. Different models have been developed to account for number processing in adults. McCloskey (1992) proposed a functional architecture including number comprehension, calculation, and production systems. His model assumes that these subsystems communicate through a common semantic quantity code represented as composites of powers of 10. For instance, the meaning of the number 128 is represented by $<1> \times [10^2] + <2> \times [10^1] + <8> \times [10^0]$. Although this model has several advantages, it implies an exact representation of number only. On the contrary, in the triple-code model, Dehaene (1992) postulated that numbers are internally represented as roughly analog magnitudes along a kind of a left-to-right oriented *mental number line*. This semantic representation is used to understand and manipulate both symbolic (e.g., the number word "two" or the Arabic numeral "2") and non-symbolic quantities (e.g., set of objects or pattern of dots), and can serve as a foundation of more elaborate mathematical competencies like calculation. Furthermore, it has been proposed that this representation is implemented in the parietal cortex of both hemispheres.

The learning of mathematics covers a variety of skills, such as comparing quantities, counting the number of items in a set, dealing with the numerical systems (i.e., writing and reading numbers), performing simple and complex calculations, or solving word problems. Typically, a majority of children are able to master these abilities, but an appreciable percentage of them does not and are then referenced as having developmental dyscalculia.

It is clear that not being able to count efficiently, to understand the meaning of numbers, or to calculate as other children do, rapidly becomes a handicap during the development, not only at school but also in society in general, in the same way as not being able to read is. Despite the growing interest observed over these last few years, research on developmental dyscalculia or more general mathematical disability is actually much less advanced than research on dyslexia. It could be due to the complexity of the mathematics field. Several hypotheses have been proposed to account for this learning deficit, but the origin(s) of developmental dyscalculia remain(s) unclear. Research first focused on the role of the auxiliary cognitive functions not directly related to number processing. In particular, the different components of the working memory were extensively measured in children with mathematical difficulties. Other theories are based on potential weak spatial abilities, low speed of processing, or difficulties in retrieving information from long-term memory. More recently, Butterworth (1999b) proposes that humans are born with a capacity specialised for recognising and mentally manipulating numerosities. Unlike the previous hypotheses, he argues for “a highly selective and specific deficit of a very basic capacity for understanding numbers, which leads to a range of difficulties in learning about number and arithmetic”(p. 455). In the same vein, Dehaene (1997) speaks about the “number sense” as the ability to represent and manipulate number magnitude nonverbally that could be impaired in developmental dyscalculia.

The Present Thesis

The aim of this thesis is to try to bring more light on the question of the origin of developmental dyscalculia. In particular, we investigate the hypothesis of a deficit of the number magnitude representation in dyscalculic children. To this end, both behavioural and neuroimaging experiments have been conducted to compare performances between children with mathematical disabilities (MD) and age-matched controls. The present thesis falls within Butterworth’s proposal favouring a deficit of specialised numerical circuits in dyscalculia. However, the concept of “number module” as well as the “number sense” proposed by Dehaene (1997) is very broad and covers a wide range of processes ranging from basic to more sophisticated numerical abilities. Our lines of research are more precise by postulating a specific impairment of the number magnitude representation. It should be noted that we do not consider the semantic system for numbers as the only source of problems leading to dyscalculia. The mathematical domain is complex and requires the engagement of both specific and non-specific cognitive processes. It is thus more likely that various causes can induce some form of mathematical difficulties, and that different populations of MD children co-exist depending on the kind of impairment. Our claim is that a majority of dyscalculic children could suffer from a core deficit in "number sense" which leads to a poor understanding of numbers.

The idea that we follow in this thesis is not to consider the number magnitude representation as unusable in dyscalculia. Even in the most severe cases, dyscalculic children can select the larger of two sets of objects, or can judge that 8 is more than 5. However, they are slower and more error-prone relative to children of the same age. Sometimes, they resort to immature strategies to compensate for their inability. Our statement is that the number representation could be less precise in dyscalculia than in typical development. This imprecision would lead to difficulties in activating analog magnitudes, particularly for adjacent numerosities, resulting in poor performances in children with MD during number processing. The title "When [5] looks like [6]" refers to this hypothesis.

In this thesis, we have focused on four main questions with respect to number development. First, we wanted to test whether or not difficulties encountered by mathematically disabled children are specific to the numerical domain. Children with mathematical disabilities manifesting poor calculation abilities were compared to control children of the same age during various tasks of retrieving information from long-term memory (Experiment 1). The second aim was to examine the integrity of the number magnitude representation in dyscalculia. We analysed potential differences in the slope of the numerical distance effect, which reflects the nature of analog magnitudes, in children with or without mathematical disabilities when they had to select the larger of two quantities presented in different formats (Experiment 2). These experiments are presented in Chapters 4 and 5. The two other questions are dedicated to the analysis of brain areas involved in number development using functional magnetic resonance imaging (fMRI). Our third objective was to examine age-related changes in frontal and parietal regions between children and adults during number comparison (Experiment 3). Finally, we conducted a second neuroimaging study to explore the potential neural correlates of dyscalculia.

Cerebral activity of both children with pure dyscalculia and control children was analysed during a numerical and a non-numerical comparison tasks (Experiment 4). We also investigated whether or not these differences in brain activation were specific to number processing. These experiments are reported in Chapters 6 and 7. These studies are exposed under the form of papers published or submitted in scientific journals. The results of these studies will be then discussed and integrated in a general discussion.

We would like to call the reader's attention to three points. First, our data were collected on either children who were mainly characterised by weak arithmetic fact abilities, or on children who were diagnosed by a neuropediatric department as having developmental dyscalculia, confirmed by a battery of numerical reasoning and calculation tests. In our own experiments, these children were referenced as MD (or PoorAF) and DD children, respectively. Considering that research on dyscalculia suffers from a high heterogeneity in the selection criteria of participants, we used the term MD children in all other studies reported in this thesis. The second point is that, although the semantic system of numbers is rich and not restricted to cardinality, we focused on the number magnitude representation which supports early numerical knowledge and provides a foundation for the development of mathematics. Finally, various terms like analog magnitude, approximate, or preverbal system will be used to refer to this representation.

Scope and structure of the thesis

The present thesis is divided in eight parts. The introductory chapter provides a description of the typical development of numerical abilities. We first review the characteristics of the approximate and exact systems of number representations in adults. Then, we examine how the number magnitude representation evolves during the development, and its impact on the acquisition of the exact representation of numbers. In Chapter 2, we explore the brain imaging evidence in favour of a parietal contribution to number processing. We present the criteria proposed by Dehaene and colleagues (2003) for a specific role of the intraparietal sulcus in number representation and manipulation, and confront them to the most recent brain imaging results. Then, we propose a synthesis of the studies investigating possible age-related changes dedicated to number processing. Chapter

3 investigates developmental dyscalculia as a learning deficit affecting school-aged children. We review the definition, prevalence, manifestations, but also the extent to which the previous studies have contributed to the comprehension of mathematical disabilities. The various hypotheses which postulate difficulties with general cognitive systems (working memory, visuospatial attention, language, speed of processing) in mathematically disabled children will also be discussed. This part ends with the hypothesis of a specific deficit of the number system in developmental dyscalculia. We address the original idea proposed by Butterworth (1999) in greater details, and examine the behavioural and neuroimaging results exploring a possible impairment of the number magnitude representation in dyscalculic children. This hypothesis is further described in the following of this thesis. In Chapter 4, we report an original experiment that aims at testing the specificity of the difficulties encountered in dyscalculia. Chapter 5 presents a second study that is more directly dedicated to the possible deficit of number magnitude representation in children with MD. The two subsequent chapters investigate the neural correlates of number processing in children with and without MD. In Chapter 6, we compare brain activation in healthy children and adults during a number comparison task, and analyse to what extent the age-related changes are specific to numbers. In Chapter 7, we explore the differences in brain activation between a group of pure dyscalculic children and age-matched control children when they compare the magnitudes of Arabic numerals, or the colours of non-numerical symbols. Finally, Chapter 8 provides a general discussion on the potential contribution of our findings to the literature, and offers perspectives for further investigation on developmental dyscalculia.

CHAPTER 1

The development of number representations

Human adults are able to quantify precisely the number of persons in a small group, but can only estimate the number of persons in a crowd. It has been suggested that these abilities reflect two distinct systems, one for processing large numerosities in an approximate way, and another for supporting exact enumeration and arithmetic. While the former is supposed to be biologically determined and shared with other species, the latter is culturally-dependent. Chapter 1 explores the development of these two systems for number representation. First, we review the different characteristics of the approximate and exact number representations in adults. In the second part of this chapter, we investigate how the number magnitude representation develops throughout the lifespan, and interacts with the exact number representation. The hypothesis of an early preverbal numerical knowledge in infants is considered and confronted with recent data. Then, we trace the acquisition of an exact number representation in children through the use of verbal and finger counting, and through the learning of Arabic numbers and arithmetic.

1. Two different systems of number representation in adults

A large body of evidence indicates that human adults process number by means of two different systems (e.g., Dehaene & Cohen, 1991). One is analog, language-independent, and approximate; it is common to a variety of species (Dehaene, Dehaene-Lambertz, & Cohen, 1998; Gallistel & Gelman, 1992). The other is discrete, language-specific, exact, and culturally-derived (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Hurford, 1987). In the subsequent sections, we first describe the nature of the approximate number system in greater details as well as the various psychophysical effects related to this representation. Additional supports for nonverbal numerical abilities in cultures with only few counting words and in animals are reported. Then, we review dissociations between exact and approximate number representations based on brain-damaged patients and bilinguals performances. Finally, the triple-code model (Dehaene, 1992) is briefly presented as an interesting hypothesis on the anatomo-functional processes involved in these two systems.

1.1. A system for approximate number representation

A language-independent representation of number magnitude, akin to a mental "number line" is used for quantity manipulation, comprehension, and approximation (Dehaene & Cohen, 1995). The main feature of this internal representation is that it does not exactly reflect the reality but that it is noisy and obeys Weber-Fechner law (Fig. 1-1). This implies that the discriminability of two magnitudes is a function of their ratio (for a review, see Gallistel & Gelman, 2005). It is more difficult to discriminate close than distant magnitudes ("distance effect"), and to discriminate large than small magnitudes ("size effect"). Accordingly, human adults compare sets of dots and sequences of tones when the items are too numerous and too briefly presented for verbal counting, with an accuracy that is dependent upon the ratio of the two sets (Barth, Kanwisher, & Spelke, 2003; van Oeffelen & Vos, 1982). Moreover, when adults produce verbal or nonverbal estimates of the sizes of sets without counting, the variability in their estimation is proportional to numerosity (Cordes & Gelman, 2005; Whalen, Gelman, & Gallistel, 1999). The ratio-dependent performances showed little or even no cost in comparing numerosities either across- and within-modality (visual and auditory), or across- and within-format (sequential or simultaneous) presentation (Barth et al., 2003). These findings support the view that approximation is based on an abstract representation of numerosity.

The signature of the Weber-Fechner law is also found during the processing of exact symbolic quantities through various psychophysical effects (for a discussion, see Dehaene, 1997). Moyer and Landauer (1967) were the first to demonstrate the existence of an analog number representation in humans, even if this suggestion was considered by other researchers in the past (e.g., Galton, 1880; Piaget, 1952). They found, in Arabic numeral comparisons, that response times (RTs) and error rates decreased as the numerical difference between the numerals to be compared increased. For example, it is easier to decide that 8 is larger than 2, than that 8 is larger than 7. The classical interpretation of this *numerical distance effect* supposes that numbers are automatically converted into an internal representation like analog magnitudes that are in turn compared with each other (but see Verguts & Fias, 2004, for a different view). The distance effect has also been reproduced in the comparison of written words (Foltz, Poltrock, & Potts, 1984; Tzeng & Wang, 1983), and dot patterns (Buckley & Gillman, 1974; van Oeffelen & Vos, 1982). Even during two-digit Arabic number comparisons, latencies decreased monotonically as the distance between the target and the standard[1] increased, and showed no discontinuities between decades (Dehaene, Dupoux, & Mehler, 1990). This suggests that two-digit numbers are represented holistically on the same approximate number line than single-digit numbers (Brysbaert, 1995) although there are some pieces of evidence that they can also be processed compositionally as separate magnitudes for the decade- and unit-digits (e.g., Nuerk, Weger, & Willmes, 2001).

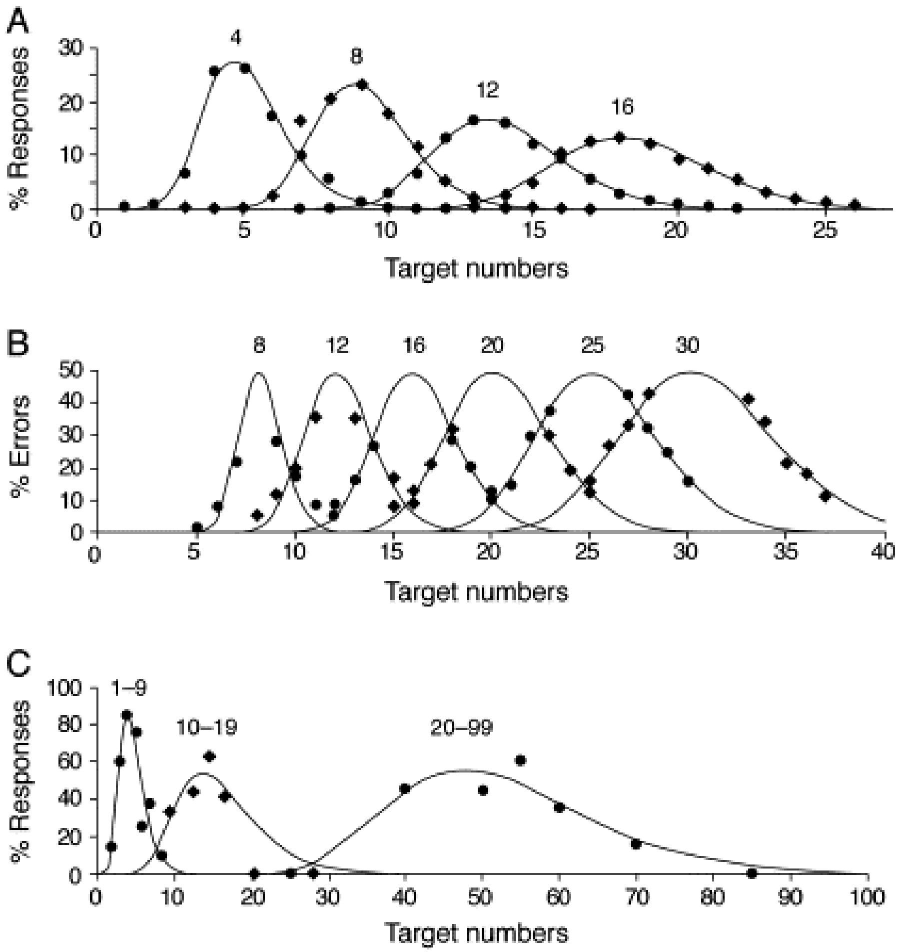

Figure 1-1. Performance in number processing becomes increasingly imprecise as the numbers involved get larger, resulting in an increasing dispersion of responses: (A) in rats comparing the number of lever presses to a fixed standard of 4, 8, 12 or 16; (B) in human adults comparing the numerosity of a dot pattern to a fixed standard of 8, 12, 16, 20, 25 or 30; (C) in a patient who failed in exact calculation having to verify simple addition problems (reproduced from Dehaene et al., 1998).

[1] The standard is a reference number fixed once and for all before the experiment. The participant must keep the standard in mind and compare it to each target.

Beyond the numerical distance, other effects are supposed to depend on the number magnitude representation. The *size effect* leads to higher latencies in comparing large than small numbers (Moyer & Landauer, 1967). For example, it is more difficult to tell which of the numerals 8 or 9 is the larger than to decide between 2 and 3, although they are equally distant numerically. This effect is generally interpreted as stronger compression (Dehaene, 1992, 2003) or a higher variability (Gallistel & Gelman, 1992) for larger numbers (but see Verguts, Fias, & Stevens, 2005 for a different view).

The so-called *SNARC effect* (i.e., Spatial-Numerical Association of Response Codes) is supposed to reflect the spatial orientation of the number representation (Dehaene, Bossini, & Giraux, 1993). It consists of an association between the relative number size and the response side, such that larger numbers are associated with right side and smaller numbers with left side (for a review, see Fias & Fischer, 2005). This effect was initially demonstrated in parity judgments when adult participants responded faster to small numbers with the left-hand, and to large numbers with the right-hand (Dehaene et al., 1993; Fias, 2001; Fias, Brysbaert, Geypens, & d'Ydewalle, 1996). The SNARC was also observed during numerical tasks requiring phoneme monitoring (Fias et al., 1996) or orientation detection (Lammertyn, Fias, & Lauwereyns, 2002). Moreover, the central presentation of small numbers speeded subsequent detection of peripheral stimuli in the left visual field, while the presentation of larger numbers speeded detection in the right visual field (Fischer, Castel, Dodd, & Pratt, 2003).

Finally, the *size congruity effect* (e.g., Besner & Coltheart, 1979; Ganor-Stern, Tzelgov, & Ellenbogen, 2007) was extensively investigated in adults using a Stroop-like paradigm in which participants have to select the physically larger of two Arabic numbers varying along both numerical and physical dimensions. A decrease in RTs (facilitation component) is observed when the decision on physical size is congruent with that on numerical size (e.g., 2 4), and an increase in RTs (interference component) when the decision on physical and numerical dimensions is incongruent (e.g., 2 4).

Two main hypotheses have been proposed regarding the nature of the analog magnitude representation. Gallistel and Gelman (1992) proposed that internal number magnitudes are linearly represented as a series of equally spaced distributions with increasing spread (Fig. 1-2A). Close numbers show overlapping distributions of activation and are thus more difficult to discriminate ("distance effect"). This model assumes scalar variability in the mapping from sensory input to this linear representation: the variability increases with number magnitudes leading to more noisy representation for larger numbers ("size effect"). In other words, the ratio between the variability of estimations and the magnitudes to be estimated remains constant across a range of quantities (Gallistel & Gelman, 1992; Whalen et al., 1999). As an alternative to scalar variability, Dehaene (1992) postulated that numbers are represented with a constant variability on a scale that is more compressed for larger numbers (Fig. 1-2B). This logarithmic model implies a less accurate representation and a worse discriminability of large numbers relative to small numbers ("size effect"), and of close numbers relative to far numbers ("distance effect"). At a behavioural level, psychological predictions of the linear and logarithmic models are essentially equivalent (Dehaene, 2003). In both models, larger numerosities are represented by distributions that overlap increasingly with nearby magnitudes. This variability decreases the discrimination power between numerosities. Yet, recent data indicate that performances on number-space mapping in Amazonian indigene culture (Dehaene, Izard, Spelke, & Pica, 2008), and single-cell recording in primates during a matching numerosity task (Nieder & Miller, 2003) were better plotted onto a logarithmic scale than a linear scale.

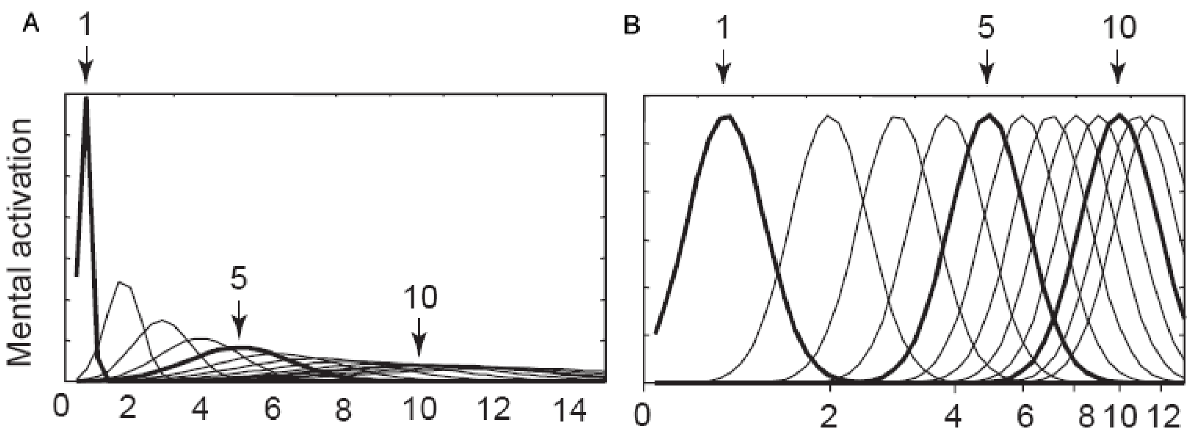

Figure 1-2. Two models of the number magnitude representation: (A) linear model with scalar variability; (B) logarithmic model with fixed variability (reproduced from Feigenson et al., 2004).

The approximate number representation seems to be biologically determined. Several data support the view that this system appears early in the development. As we will see in the subsequent section, performances of human infants during number discrimination and comparison tasks show the signature of the analog magnitude representation in the first year of life, before the acquisition of number words (Brannon, 2002; Brannon, Wusthoff, Gallistel, & Gibbon, 2001; Wynn, Bloom, & Chiang, 2002; Xu & Spelke, 2000). When 2- to 7-year-old children estimate the numerosities of briefly presented groups of objects, their performances are consistent with analog models of magnitude representation: errors increase as the ratio of compared numerosities approaches 1 (Huntley-Fenner, 2001; Huntley-Fenner & Cannon, 2000; Starkey and Cooper, 1995), exactly as in adults. Moreover, verbal counting abilities of the participants do not predict performance on the numerical discrimination task. Further support is provided by the study of numerical competence in situations in which the language of numbers is either absent or reduced. The comprehension of numbers was recently assessed in Mundurukù, an Amazonian indigene culture with little access to education (Pica, Lemer, Izard, & Dehaene, 2004). Although the Mundurukù lack words for numbers beyond 5, they are able to compare and add large approximate numbers that are far beyond their naming range. When they are asked to compare two sets of dots, controlled for various non-numerical variables, their performances are far above chance level and decrease as the ratio between quantities, in the same way than French controls. Thus, the Mundurukù clearly can represent large numbers and understand the concept of relative magnitude.

The nonverbal representation that is supposed to underlie the human "number sense" (Dehaene, 1997) probably has a long evolutionary history. The ability to discriminate numerical quantity might have a key role for the survival, such as discerning the number of approaching predators (McComb, Packer, & Pusey, 1994). Not only non-human primates (Brannon & Terrace, 1998; Brannon & Terrace, 2000; Hauser & Carey, 2003; Hauser, Carey, & Hauser, 2000; Hauser, Dehaene, Dehaene-Lambertz, & Patalano, 2002), but also other animal species (dolphins: Kilian, Yaman, von Fersen, & Gunturkun, 2003; rats: Meck & Church, 1983; salamanders: Uller, Jaeger, Guidry, & Martin, 2003; pigeons: Xia, Emmerton, Siemann, & Delius, 2001) can discriminate sets of objects that differ only in numerosity (for a review, see Gallistel & Gelman, 2005). More importantly, animals' performances in discrimination and comparison tasks are subjected to the same psychophysical effects as reported in humans. Both distance and size effects have been observed in various species (Gallistel & Gelman, 1992). Although most experiments required extensive training, numerically relevant behaviour has also been observed in the wild (Hauser, MacNeilage, & Ware, 1996) or in situations in which number appears to be extracted spontaneously by animals (Hauser & Carey, 2003; Hauser et al., 2002; McComb et al., 1994; Meck & Church, 1983; Uller et al., 2003). In addition, cross-modal generalisation in numerosity discrimination has been observed (Church & Meck, 1984). Rats initially trained to press a lever on the left in response to two flashes or two sounds, and a lever on the right in response to four flashes or four sounds, spontaneously pressed the right lever when presented with a combination of two sounds and two flashes. This suggests an extraction of numerosity based on the abstract total number of events, rather than on modality-specific representations. Finally, the violation-of-expectation paradigm, initially used with infants (Wynn, 1992), has been replicated with untrained monkeys tested in the wild (Hauser et al., 1996). This suggests that animals have a sense of basic arithmetical relationships as they could also perform simple problems as 1+1 = 2 but not 1, and 2-1 = 1 but not 2.

1.2. A system for precise number representation

The approximate system is not the only source of numerical information. Language enables humans to develop a precise system to represent the numbers of individual objects, and to support exact calculation and higher mathematics. Lesion data provides evidence for a distinction between a language-dependent system for exact arithmetic and a language-independent system for tasks requiring an approximate representation of numerical quantity. Patients with cortical or subcortical lesions in left frontal areas suffered from language deficits which strongly affected exact calculation such as multiplication, but succeeded relatively well on tasks based on magnitude representation such as subtraction, approximation, subitizing[2], or estimation (Cohen & Dehaene, 2000; Dagenbach & McCloskey, 1992; Dehaene & Cohen, 1997; Lampl, Eshel, Gilad, & Sarova-Pinhas, 1994; Lee, 2000; McNeil & Warrington, 1994; Pesenti, Seron, & Van der Linden, 1994; van Harskamp &

[2] The subitizing is the ability to enumerate a small group of four or fewer objects fast and accurately without counting.

Cipolotti, 2001). The reverse pattern was observed in patients with left parietal lesion or atrophy. They were able to retrieve the exact product of a multiplication, but exhibited difficulties in solving subtraction problems, or in comparing two numbers (Dehaene & Cohen, 1997; Delazer & Benke, 1997; Lemer, Dehaene, Spelke, & Cohen, 2003; van Harskamp & Cipolotti, 2001; van Harskamp, Rudge, & Cipolotti, 2002).

Studies with bilinguals support the existence of a discrete system of numbers, differing from the number magnitude representation. Russian-English bilinguals were taught exact or approximate additions in one of their two languages (Dehaene et al., 1999). When tested on trained exact addition problems, participants performed faster in the teaching language than in the untrained language, irrespective of whether training was in English or Russian. This suggests that exact problems are stored in a language-specific format and show a cost due to the required language-switching. By contrast, performance for approximate problems was equivalent in the two languages, providing evidence for storage in a language-independent form.

Another study examined more directly the shift in brain activation related to language manipulation (Venkatraman, Soon Chun, Chee, & Ansari, 2006). English-Chinese bilinguals were trained on an approximate estimation task or on a base-7 addition task in one of their two languages. After training, all participants were tested in both tasks and in both languages during the fMRI scanning. In the exact addition task, a language switching effect was found bilaterally in the frontal regions, as well as in the left inferior and superior parietal lobules extending to the angular gyrus. Language switching effects for the approximate task were observed in the left prefrontal cortex, and bilaterally in the posterior parietal areas. These differences in brain activation were interpreted as a greater emphasis on visuospatial and quantity manipulation during approximation, and a language-based retrieval of stored facts during exact calculation. However, the areas activated in the estimation task could also reflect a cost to translate the problem to the trained language in order to retrieve the response.

Unlike the approximate representation which exists even in population with reduced numbers lexicon, the exact system is culturally-derived (Pica et al., 2004). In the Mundurukù indigene culture, all numerals, except for the words for one and two, are used in relation to a range of approximate quantities rather than to a precise number as in Western societies.

Although the Mundurukù did not differ qualitatively from French controls in approximating quantities, they failed in exact arithmetic with numbers larger than four or five when they had to predict the outcome of a subtraction of dots from an initial set comprising one to eight items. Once again, these findings imply a distinction between a nonverbal system for number approximation and a language-based counting system for exact number and arithmetic.

Relative to other species, humans are the only ones to possess a sophisticated language, which can serve as a foundation of more elaborate mathematical reasoning. A few experiments indicated that nonhuman primates (Boysen & Capaldi, 1993; Matsuzawa, 1985; Washburn & Rumbaugh, 1991) could be taught to match the symbols 1–9 with the appropriate number of objects. For instance, chimpanzees trained with Arabic numerals can identify two symbols such as 2 and 3 and point to their sum 5 amongst other Arabic numerals (Boysen & Capaldi, 1993). However, these performances are exceptional, show considerable intra- and inter-species variability, require years of training and are never found in the wild (Dehaene et al., 1998). Thus, there is no evidence of an exact system for numbers within the normal behavioural repertoire of animals.

1.3. The triple-code model

Several models have tried to describe human abilities in the numerical domain. Originally, they have been proposed to account for performances of brain-damaged patients in calculation and transcoding (e.g., Deloche & Seron, 1987; McCloskey, Caramazza, & Basili, 1985), and thus mainly focused on an exact representation of numbers. Amongst the various theories, the triple-code model, originally proposed by Dehaene (1992; Dehaene & Cohen, 1995) presents at least two major advantages. First, the cognitive processes described in this model are not only functionally but also anatomically specified. Second, this proposal accounts for both the approximate and the precise systems of number representation. In particular, the triple-code model assumes three representation systems differently recruited depending on the task (Fig. 1-3). The *quantity system* refers to the core semantic representation of numbers, in the form of analog magnitudes, supposed to be located in areas around the intraparietal sulcus of both hemispheres (see Chapter 2). This system is used during number comparison, approximate calculation, and subtraction. The *visual* *system*, localised in the left and right occipito-temporal areas, is implicated in the processing of Arabic numbers. The *verbal system* is used to represent numbers as orthographical, phonological and syntactical entities, and is thought to support exact calculation, especially the retrieval of automatized arithmetic facts from long-term memory as during multiplication. The localisation of this representation in the classical frontal language areas, as initially postulated by the model, has been criticised (Pesenti, Thioux, Seron, & De Volder, 2000; Zago et al., 2001), and has been later assumed to depend on the angular gyrus.

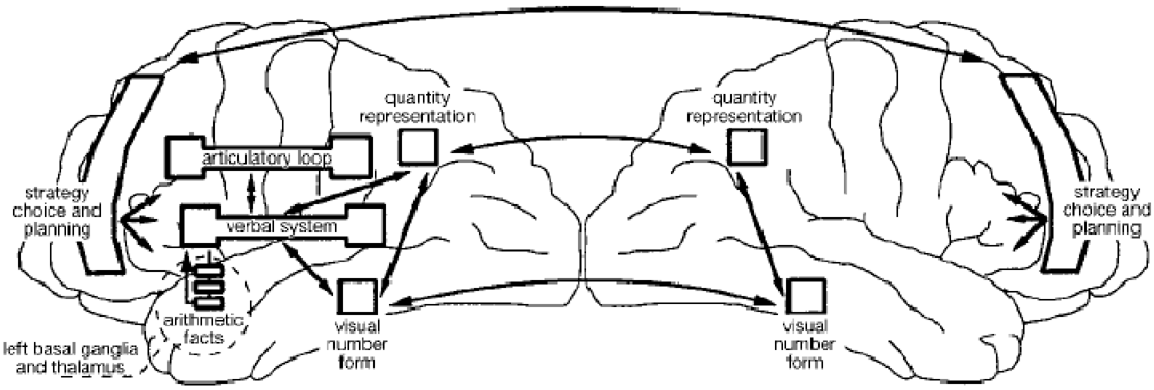

Figure 1-3. Schematic anatomical and functional depiction of the triple-code model (adapted from Dehaene & Cohen, 1995).

In summary, data converge to prove the existence of two distinct systems of number representations in adults. They use the approximate system depending on the number magnitude representation when comparing or estimating symbolic and non-symbolic numerical quantities. This system most probably exists in indigene cultures without a well-formed lexicon of counting words, and in animals even in a crude form. Moreover, adults have an exact system to precisely count the number of individual objects and to support exact calculation. This system does not appear in indigene cultures or animals as it depends on language. However, one important question concerns the development of these number systems. In particular, we investigate how the number magnitude representation evolves throughout the lifespan, and interacts with the exact number system. These issues are addressed in the subsequent sections in reference to (1) the possible preverbal numerical knowledge in infants; (2) the development of an exact number system in children; (3) the learning of Arabic numbers; and (4) the acquisition of arithmetical skills.

2. Early numerical knowledge in infants

The last 30 years have provided a number of experimental studies demonstrating numerical discrimination and elementary operation abilities in preverbal infants. Most researchers agree to postulate the existence of preverbal representations of numerosity early in the development. However, some authors have proposed that infants' performance on such discrimination and transformation tasks is based on continuous perceptual variables of the displays rather than the number of elements (Clearfield & Mix, 1999; Feigenson, Carey, & Spelke, 2002) and would be better explained by visual cognitive processes that are not specifically numerical (Mix, Huttenlocher, & Levine, 2002).

Discrimination of visual numerosity was first investigated in young infants using the *habituation-recovery paradigm* of looking time (Starkey & Cooper, 1980). Babies see repeated slides with a fixed number of items until their looking time decreases, indicating habituation. Then, a slide with a different number of items is presented. An increase in looking time to the novel number, indicating dishabituation, suggests that infants discriminate the numerosities. There is some evidence that infants can discriminate between small displays varying in numerosity across different modalities during the first year of life. Studies using the habituation-dishabituation paradigm reported successful discriminations of 1 versus 2 and 2 versus 3 items at many ages including newborns, for visually presented dot patterns (Antell & Keating, 1983; Starkey & Cooper, 1980; Wynn & Bloom, 1999), pictures of objects varying in shape, size and position (Starkey, Spelke, & Gelman, 1983; Strauss & Curtis, 1981), for three-dimensional objects occlusion (Feigenson, Carey, & Spelke, 2002), for moving shapes undergoing occlusion (Loosbroek & Smitsman, 1990), and for successive jumps of a puppet (Wynn, 1995, 1996). The ability of infants' to discriminate numerosities is not limited to visual sets of objects, but was also observed with auditory stimuli (Lipton & Spelke, 2001; but see Mix, Huttenlocher, & Levine, 1997). For instance, 4-day-old newborns are surprised, as measured by an increase in sucking rate, when they hear a three-syllable utterance after habituating to a variety of two syllable utterances, and vice versa (Bijeljac-Babic, Bertoncini, & Mehler, 1993). Performances in discrimination improve with age, as 6- to 12-month-old infants extend this capacity to comparisons of 3 versus 4 photographs or drawings of sets of objects (Starkey, Spelke, & Gelman, 1990; Strauss & Curtis, 1981), and sometimes until 4 versus 5 moving shapes (van Loosbroek & Smitsman, 1990). Moreover, infants' abilities are not restricted to small numerosities. Six-month-old infants are sensitive to the difference between 8 versus 16 and between 16 versus 32 (Xu & Spelke, 2000), but fail to discriminate between 16 and 24 (Xu, Spelke, & Goddard, 2005). This result has also been replicated with sequential sets of sounds (Lipton & Spelke, 2003). Therefore, the discrimination of large numerosities seems to be ratio-dependent as infants can discriminate numerosities separated by a 1:2 ratio, but not when a 2:3 ratio is used. As previously discussed, these performances reflect the nature of the number magnitude representation which shows an increasing overlap between analog magnitudes with nearby numerosities. Furthermore, there is also some evidence for cross-modal numerosity matching in 6- to 8-month-old infants. When infants hear either two or three drumbeats, and have to chose between a slide with two visual objects or at another with three objects, they spend more time looking at the slide whose numerosity matches the number of sounds that they hear (Starkey et al., 1983). Similarly, 6-month-old infants, habituated to associate dropping objects and the sounds produced, look longer at a display when the number of sounds do not match the number of objects dropped (Kobayashi, Hiraki, & Hasegawa, 2005).

The second source of empirical support for early numerical competence comes from *violation-of-expectation transformation* studies. This paradigm assesses whether or not infants are able to anticipate the result of simple addition and subtraction problems. Five-month-old infants see two successive toys moving behind a screen, and one object is added (or removed) surreptitiously. They look longer at the unexpected outcome of three objects than at the expected outcome of two (or one) objects, suggesting that they have represented numerically appropriate expectations analogous to the arithmetic operations 1+1=2 and 2-1=1 (Wynn, 1992). Similar results are obtained when objects are placed on a rotating tray so that their location behind the screen is unpredictable (Koechlin, Dehaene, & Mehler, 1997), and when the identity of the objects is changed surreptitiously in some of the trials before the screen is dropped (Simon, Hespos, & Rochat, 1995). Ten-month-old infants succeed in larger problems including 2+1=3 but not 2 and 3-1=2 but not 3 (Baillargeon, Miller, & Constantine, 1994; Wynn, 1995). Together, these findings provide further evidence that infants represent in an abstract fashion the number of objects, independent of their identity and location.

2.1. The accumulator model

The habituation and transformation studies indicate that infants can discriminate the numerosity of small sets of objects, but also of larger sets when the ratio between them is sufficiently important. However, one important question concerns the nature of human infants' numerical knowledge. The accumulator model, originally proposed to explain both the timing and counting competencies in animals (Meck & Church, 1983), was later used by Gallistel and Gelman (1992) and Wynn (1995) to account for early sensitivity to number. According to this model (Fig. 1-4), objective quantities (e.g., time, number, distance) are represented subjectively as continuous magnitudes. For each discrete numerosity counted, a roughly equal amount of energy enters at a constant rate in a mental accumulator. The total amount is directly proportional to the numerosity. However, because the amount of energy entering the accumulator slightly varies from trial to trial, the generated analog magnitudes are only approximate, and follow the characteristic signature of Weber-Fechner law. This variability in the representation accounts for the numerical distance and number size effects or the ratio effect, which combines the two others.

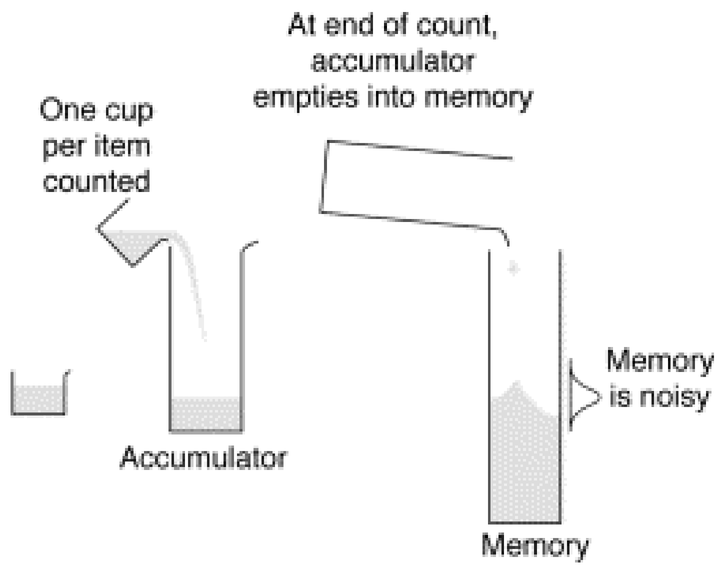

Figure 1-4. Schematic depiction of the accumulator model (reproduced from Gallistel & Gelman, 2000).

2.2. An alternative view

Most of the above studies have been criticised for failing to control adequately for non-numerical perceptual variables that tend to co-vary with numerosity, such as contour length of the display (i.e. the sum of the perceptual contours of the items in the display), total surface area, brightness, or density of the items. These experiments may not preclude that infants responded to some continuous quantitative dimensions, instead of number itself. The question is whether or not infants respond to the numerosities of small sets of objects, when controlled strictly for these variables (e.g., Feigenson, Spelke & Carey, 1998). For instance, 10- and 12-month-old infants were able to choose the larger of two sets of crackers contrasting 1 versus 2 and 2 versus 3, but they based their judgments on the total surface area or volume rather than number (Feigenson, Carey, & Hauser, 2002). In the same line, it was found that 6- to 8-month-old infants dishabituated to the novel contour length but not to the novel number (Clearfield & Mix, 1999). In another experiment, infants were habituated to small sets of three-dimensional objects, and then presented with novel items (Feigenson, Carey, & Spelke, 2002). When number and extent were unconfounded, infants looked longer at the display that was novel in total surface area than at the display that was novel in number. Moreover, when the novel item was controlled for continuous extent, infants did not demonstrate sensitivity to number, either in the habituation or the transformation paradigm. However, two other studies reported that infants responded more significantly to changes in number than in continuous extend (Brannon & Gautier, 2003; Leslie, Glanville, & Lerner, 2003).

This sensitivity to perceptual attributes led some researchers to propose that infants' performance on such tasks was better explained by appealing to cognitive capacities that were not specifically numerical (Mix et al., 2002). The object-file model, which firstly described how humans track discrete objects in their visual environment (Kahneman, Treisman, & Gibbs, 1992), was proposed as an alternative representational system for small sets discrimination in both infants and adults. This model postulates that the elements of a quantified set are represented by mental tokens ("object-files") which are in turn stored in short-term memory to be compared (e.g., Carey, 2001; Uller, Carey, Huntley-Fenner, & Klatt, 1999). In this view, infants' performance in numerical discrimination and transformation tasks would reflect the operation of specialised visual mechanisms to represent and track individual objects within a scene, rather than sensitivity to numerosity. Even if object files are discrete and can be counted, they can be represented in parallel, but not as a cardinal quantity. The fact that this object-file system is limited to represent up to three or four objects (Pylyshyn, 1989; Scholl & Pylyshyn, 1999; Trick & Pylyshyn, 1994) could explain why infants are able to discriminate two objects from three objects (Feigenson, Carey, & Spelke, 2002; Xu, 2003) or three from four objects (Leslie, Xu, Tremoulet, & Scholl, 1998), but not four from six because these numerosities exceed the capacity of the object-file system.

Nonetheless, the object-file model cannot easily account for infants' ability to discriminate non-visual elements, such as two from three actions (Wynn, 1996), tones (Lipton & Spelke, 2001; Starkey et al., 1990), or speech syllables (Bijeljac-Babic et al., 1993), or to show cross-modal discrimination (Starkey et al., 1983). Moreover, infants can also discriminate between arrays containing more than four elements, given large enough comparison ratios (Lipton & Spelke, 2003; Xu & Spelke, 2000). Finally, there are a number of experiments in which infants’ performances fail to be explained by quantification of continuous perceptual variables alone (e.g., Wynn & Bloom, 1999; Xu & Spelke, 2000). In a recent study (Wynn, Bloom, & Chiang, 2002), 5-month-old infants were presented with two or four collections, each composed of three objects moving independently from each other so that the overall configuration and contour of the collection varied continually. Infants habituated to two collections would look longer at four collections during test, while those habituated to four collections would look longer at two collections during test.

3. The development of an exact number system in children

Beside the early preverbal knowledge for approximate numerical quantities, another system for precise quantities appears in children. In this section, we investigate the development of exact number representations. The first manifestation is represented by the verbal counting consisting in the one-to-one mapping between each element of the set and a specific counting number word. Moreover, fingers are used by children to support verbal counting and enable to represent objects as iconic numerosities. We discuss whether or not the approximate system serves as a foundation of counting skills. The different theories are described as well as the recent studies examining the role of counting in numerosity estimation. In a next point, we explore how children learn Arabic numbers and how they map these visual symbols to precise values. Hence, counting abilities and Arabic number knowledge provide support for more sophisticated mathematical competencies. We briefly review the various strategies used by children to solve simple arithmetic problems, and trace the shift from immature to more efficient strategies during normal development.

3.1. The acquisition of counting

The verbal counting implies that the child does master the sequence of number words, and does understand the inherent principles associated with the counting. The acquisition of the sequence of number words is a long-lasting process that starts around the age of 2 and takes the child up to about the age of 6. Although most 3-year-old children can recite the ordered list of number words at least up to 6, it takes them several months to learn the meanings of these words (Wynn, 1990). English children acquire the meaning of the word ‘‘one’’ about 6 months before the meaning of ‘‘two’’ and about 9 months before the meaning of ‘‘three’’ (Wynn, 1992). Once they have mastered the numerals up to about “four”, however, they appear to grasp the logic of the system, and understand that each word in their count list refers to a unique cardinal value that is one higher than the value picked out by the prior word. Then, numbers up to “twenty” are mastered around the age of 6 (Gelman & Gallistel, 1978). A relatively similar development was reported in Belgian children (Noël, 2001).

Fuson (1988) proposes to distinguish two phases in the development of the number word sequence. In the acquisition phase, the child realises that there are special words that refer to numbers, and learns to use them in particular contexts (e.g., counting). At this stage, the child is able to correctly and consistently produce the beginning part of the sequence only (stable conventional portion). The elaboration phase requires to progressively differentiate the words in the sequences, and includes five levels: at the *string level*, number words are undifferentiated and form a whole unidirectional structure that can only be recited starting from 1 (“one-two-three-four-…”); at the *unbreakable chain level*, the sequence words are distinguishable, but they are still closely connected and the sequence can only be produced starting from the beginning (“one, two, three…”); at the *breakable chain level*, the child is able to recite the sequence from arbitrary entry points (“five, six, seven…”); at the *numerable chain level*, the number words are distinct units which can be counted and matched to a set of items (the child is able to solve 3+4 by counting three steps from 4, such as “four… five, six, seven”); and finally at the *bidirectional chain level*, there is a flexible use of the counting sequence in the forward and backward directions (“seven, six, five…”).

Gelman and Gallistel (1978) argue that objects enumeration is governed by five implicit counting principles. The *one-to-one correspondence principle* refers to the matching between exactly one counting word and each object in the empirical set. The child understands that objects must be counted once and only once. The *stable order principle* implies that the counting words are consistently used in a fixed order. The *order irrelevance principle* refers to the fact that the elements can be counted in any order, without affecting the cardinality of the set. The *abstraction principle* indicates that anything can be counted, independent of the nature of the elements. Finally, the *cardinality principle* reflects the fact that the last word in the count represents the cardinality of the set.

Children often use their fingers, parallel to verbal counting, to represent the number of objects in a set. Finger-counting strategies most probably underlie our understanding of the base-10 system (Butterworth, 1999a). Unlike number words, individual fingers do not represent a specific numerosity, except numerosity one. They provide an iconic representation of numerosity as each finger raised figures an object (Fayol & Seron, 2005). Fingers refer to concrete numerosities which most probably support the learning of the conventional sequence of counting words (Fuson, Richards, & Briars, 1982). It is likely that finger movements play a crucial role in the emergence of the one-to-one correspondence principle, particularly by tagging and partitioning the items to be counted (Gallistel & Gelman, 1992). Indeed, pointing to the objects is present prior verbal counting (Alibali & DiRusso, 1999; Graham, 1999) and could help the child to assign a concrete value to the number words. Moreover, as fingers occupy fixed positions on the hand, they constitute a material which is always available to count objects in a sequential and stable order. It is thus plausible that the sequence of finger manipulation may underline the stable-order principle. Later on, fingers progressively become flexible numerical tools to achieve more sophisticated numerical abilities such as basic arithmetical operations. In line with this view, it has been recently demonstrated that finger-counting strategies influence the way that numerical information is mentally represented and processed by adults (Di Luca, Granà, Semenza, Seron, & Pesenti, 2006).

The question of how the exact number system resulting from verbal and finger counting develops in reference to the preverbal approximate system is still debated. Preverbal numerical knowledge may serve as a foundation upon which verbal counting is built (Dehaene, 1997; Wynn, 1992). Despite their approximate character, the analog magnitudes might help the child to have a basic conception of counting, especially the ordering of the numerical sequence. Gelman and Gallistel (1978) offer a slightly different view by postulating that the counting principles are innate and guide the development of verbal enumeration (i.e., “principles before skills”). Conversely, Fuson (1988; Fuson et al., 1982) emphasises that children learn to count by rote and discover the regularities of the conventional sequence of counting words through practice only (i.e., “skills before principles”). In the same vein, Wiese (2003) suggests that finger counting most likely results from a habit that evolves naturally given the fixed position of fingers on the hand. Others propose that children initially have no understanding of the logic of the natural number system, and that verbal counting procedures are based on a distinct preverbal mechanism for exact representations (Mix et al., 1997) which develops independently of the nonverbal mechanisms (Carey, 2001; Spelke, 2000).

Some studies have examined whether preschool children map words for large numbers onto approximate number representation before or after the acquisition of skilled verbal counting. Participants of 3 and 4 years had to verbally estimate the number of elements in sets without counting (Le Corre & Carey, 2007). They have been categorised on the basis of the number words for which they have learned exact numerical meanings. Children who hadn't acquire the counting principles mapped "one" to "four" onto core representations of small sets, but failed to apply larger numerals to larger sets when they were presented with sets beyond their counting range. Moreover, only half of the children who mastered the counting procedures had mapped numerals beyond "four" onto magnitudes. This mapping was succeeded around 4 years, about 6 months to one year later than the average age at which children first acquired the counting principles (Le Corre, Van de Walle, Brannon, & Carey, 2006; Wynn, 1990, 1992). The second main result is that, although the variability of skilled counters' estimates of large sets (6-10) was scalar, estimates of small sets (1–4) did not show the scalar variability in any of the groups of children. This suggests that children did not rely on analog magnitudes alone to estimate small set sizes.

It was also investigated whether children who had not yet mastered the principles of verbal counting had an innate concept of cardinality (Condry & Spelke, 2008). Three-year-old children, who could count up to "ten" but who failed to comprehend the exact meaning of numbers beyond "four", were tested on their ability to map unknown number words to arrays of objects. When children were told that one of two arrays varying in 2:1 ratio contains a certain number of objects (e.g., four sheep), they did understand that a different number word designated the other set (e.g., eight sheep). By contrast, they were at random to apply non-numerical adjectives (e.g., happy or some sheep) to both labelled and unlabelled arrays. When they were presented with the same arrays without label, they were at chance to point out to the set corresponding to an unknown number word while they succeeded well on known number words. In another experimental situation, children were told through short verbal stories that they possessed a different number of objects than the experimenter, and had to decide which of the two related number words was the larger. They were able to select the correct unknown number word only when the other corresponded to a known number word. For instance, they succeeded on 2 versus 7 objects, but not on 5 versus 10 objects. They did understand that an unknown number word referred to a greater numerosity than the known one. Furthermore, children judged reliably that a different known number word should designate a set after addition or subtraction of objects, but that the same number word continued to refer to a set after rearrangement. However, they were at chance on similar manipulations with unknown number words. These results were interpreted as children who had learned the principles of the first counting words failed to consider that the other words in their counting range referred to cardinal values.

In another experiment (Lipton & Spelke, 2005), 5-year-old children classified as skilled or unskilled counters (based on their ability to count up to "hundred"), and adults had to assign number words to large sets of objects without counting in three different tasks. First, children who could not count reliably beyond "sixty" failed to estimate, to match, or to understand the logic of estimation for number words larger than sixty. By contrast, adults and children who could count to "hundred" performed well these tasks for all number words. Second, children who failed to count reliably to "hundred" succeeded at the estimation and comprehension tasks when tested with number words within their counting range (from "twenty" to "sixty") but failed for larger number words (from "eighty" to "one hundred and twenty"). These results indicate that preschool children are able to map symbolic and non-symbolic representations for number words within their counting range, but not for number words beyond that range. However, this does not mean that unskilled counters know nothing about the meanings of number words beyond their counting range.

A following study investigated what children understood about large-number words before they could count to large numbers (Lipton & Spelke, 2006). Five-year-old children were presented with a number word that correctly labelled a large set of items beyond their counting range. They understood that the cardinality of the set changed when items were removed from or added to the set, but remained the same when items were simply rearranged. They also knew that a large-number word continued to apply to a set if one item was removed from the set, and replaced by a different item.

In summary, what emerges from current findings is that verbal counting plays a crucial role in mapping symbolic numbers onto the non-symbolic representation. Children are able to verbally estimate large sets of objects only when number words fall within their counting range (Lipton & Spelke, 2005). Moreover, their performances show scalar variability which reflects the nature of the number magnitude representation (Le Corre & Carey, 2007). It seems also that 5-year-old children possess some understanding of cardinality even for numbers beyond their counting range (Lipton & Spelke, 2006). They understand that the same number word is applied to a set of objects after rearrangement, but not after adding or subtracting objects. Although 3-year-old children do not show this pattern, they correctly judge that a number word, for which they have no exact meaning, refers to greater numerosities than known number words. This suggests that they grasp the logic of the counting sequence, even if it is not clear whether their decision is based on ordinal (the number that comes after) or cardinal (the number that is larger) properties (Condry & Spelke, 2008).

Some recent findings cast more doubt on the role of language in the understanding of cardinality. Butterworth and colleagues (Butterworth, Reeve, Reynolds, & Lloyd, 2008) proposed that language is useful but not necessary to develop an exact representation of numbers. They showed that Australian children aged from 4 to 7 years, whose languages (i.e., Warlpiri and Anindilyakwa) lack a counting vocabulary, possess the same numerical concepts than English-speaking indigenous Australian children. There was no language effect on performances when children had to recall from memory the number of counters manipulated by the experimenter, to judge the correctness of cross-modal numerosities matching, and to reproduce nonverbal simple addition problems. Accuracy decreased in a linear way from 1 to 9 objects for all groups. These data favour an innate capacity to represent exact numerosities which is used by children to learn counting sequence. To reconcile these apparent conflicting results, it may be that young children possess some form of exact cardinality embodied in the number magnitude representation, but that the mapping between these pre-existing concepts and the number words sequence requires mastering counting principles.

3.2. The learning of Arabic numbers

As seen in the previous section, children acquire the meaning of number words and counting procedures around the age of 4 or 5 years. In occidental cultures, they do also master the system of Arabic numbers which plays an important role in their capacity to adapt to future life. Although the acquisition of Arabic numbers has been much less investigated than counting, different stages can be distinguished through which the child learns to recognise and manipulate them. Young children are confronted very early with these symbols printed on various objects, like toys, phone, remote control, etc. The first step is to differentiate 0-to-9 Arabic numerals from other non-numerical symbols. Of particular interest are the results of young Belgian children who had to decide whether or not visually presented symbols corresponded to numerals (Noël, 2001). Around the age of 3 years, performances are at chance. Children of 4-years 6-months old adequately accept 90 percents of Arabic symbols. Nevertheless, they tend to accept letters or other non-relevant symbols as Arabic numerals too. Five-year-old children identify almost all numerals, but they are able to find the correct number word related to each of them in 70 percents of the cases only. These children understand that Arabic symbols constitute a specific numbers system, and begin to map numerals and the semantic value they convey. They learn that each of these visual symbols refers to a precise quantity (i.e., the cardinality). The use of counting words which are mastered earlier helps the child to perform this mapping. Indeed, the assignment of a cardinal value from Arabic numerals is first achieved through verbal representations of number.

Children are able to compare the magnitude of Arabic numerals from the age of 6. In a pioneer experiment, Sekuler and Mierkiewicz (1977) investigated the performances of American children from kindergarten, first-, fourth-, and seventh-grade as well as adult students in comparing pairs of Arabic numerals. The authors observed a slowdown in latencies as the numerical distance between numerals of each pair decreased. Moreover, this numerical distance effect was steeper for younger participants than for older ones. Duncan and McFarland (1980) reported similar results even in the absence of explicit number magnitude processing. Children from kindergarten, third-, and fifth-grade had to judge whether or not two Arabic numerals were physically identical. Although numerical distance influenced RTs on different pairs in all groups, this effect was more important in young children. Together, this indicates that children's ability to activate a quantity conveyed by Arabic numerals is similar to adults' one (Moyer & Landauer, 1967), and depends on the approximate system. Furthermore, the fact that the slope of numerical distance decreased with age might reflect a progressive reining of the acuity with which number magnitudes are discriminated. In line with this view, children could automatically access the magnitude of Arabic numerals at the age of 7-8 only (Girelli, Lucangeli, & Butterworth, 2000; Rubinsten, Henik, Berger, & Shahar-Shalev, 2002), about one to two years later than the age at which they were able to compare the magnitude of these numerals. Similarly, children exhibited the SNARC effect as early as 7 years when they compared the magnitude of Arabic numerals, but at the age of 9 only when they performed parity or detection tasks in which the number magnitude was less relevant (Berch, Foley, Hill, & Ryan, 1999; van Galen & Reitsma, 2008). As experiences with numerals increase, children extend these abilities to larger Arabic numbers. They learn to recognise, to read, and to write them so that the mapping between these numerical symbols and the magnitude representation becomes automatic as reflected by a size congruity effect in children of 8-9 years when processing two-digit Arabic numbers (Mussolin & Noël, 2007, 2008).

3.3. The acquisition of arithmetical skills

Both counting skills and symbolic knowledge enable the learning of more elaborate numerical competencies. The ability to rapidly and reliably retrieve the result of simple calculation is one of the most fundamental activities mastered during school years. From the age of 4 or 5 years, children have a mixture of strategies to solve single-digit additions (Siegler & Shrager, 1984b). For instance, when confronted with the problem 5+4, children initially add both operands starting from 1 by using a combination of finger counting and verbal counting. A more sophisticated strategy involves stating the larger value operand and then counting a number of times equal to the value o the smaller operand, such as 5… 6, 7, 8, 9. These two procedures refer to *counting all* and *counting on* (Baroody, 1987; Fuson, 1982), or to *sum* and *min* procedures, respectively (Ashcraft, 1982; Groen & Parkman, 1972). Children learn by themselves and during schooling to apply the most efficient procedure resulting in a gradual shift from the frequent use of counting all to counting on (Baroody, 1984; Siegler & Jenkins, 1989). With practice, children progressively abort counting strategies and store basic arithmetic facts in long-term memory. These representations support *direct retrieval* through which children quickly produce the answer without overt signs of counting, such as stating "/nain/" (i.e., nine) when presented with 5+4. When a direct retrieval is not possible, children sometimes apply *decomposition* which implies solving the problem based on the retrieval of a partial result. Tie problems that are memorised very early (e.g., Ashcraft, 1982) could serve as a basis for solving more complex problems. For example, the problem 5+4 might be solved by adding 1 to the answer of the retrieved fact 4+4, "/eyt/" (i.e., eight). Both transversal (Houlihan & Ginsburg, 1981; Siegler, 1987; Svenson, Hedenborg, & Lingman, 1976) and longitudinal (Carpenter & Moser, 1984; Svenson & Sjöberg, 1983) studies converge to show a developmental progression from an initial reliance on counting-all to an increasing use of retrieval and counting-on procedures during elementary school.

Table 1-1. Strategies used by children in solving arithmetical problems

Operations	Strategies	Examples
Addition	counting all or sum	5+4 → "1, 2, 3… 9"
(e.g., Ashcraft, 1982)	counting on or min	5+4 → "5… 6, 7, 8, 9"
	direct retrieval	5+4 → "it is 9"
Multiplication	repeated additions	3×5 → 3+3+3+3+3
(e.g., Siegler, 1988b)	count by one of the multiplicands	3×5 → "5, 10, 15"
	reconstruct the answer from a known product	3×4=12 → "3×5 is 3 more"
	direct retrieval	3×5 → "it is 15"
Subtraction	counting down	9-5 → "9…8, 7, 6, 5, 4"
(e.g., Siegler, 1989)	counting up	9-5 → "5… 6, 7, 8, 9, so it is 4"
	retrieve a complementary addition fact	5+4=9 → "so 9-5 is 4"
	direct retrieval	9-5 → "it is 4"
Division	retrieve a complementary multiplication fact	12÷4 → "?×4=12, so it is 3"
(Hittmair-Delazer et al., 1994)	add the divisor until to reach the answer	12÷4 → 4+4+4
	direct retrieval	12÷4 → "It is 3"

Beside addition problems, the other arithmetical operations are also solved by various strategies (see Table 1-1), and show a typical shift from counting-based to more efficient retrieval-based strategies in memory (e.g., Siegler, 1988b). However, the storage format of these problems is still discussed. While McCloskey et al. (1985) suggested an abstract representation, others proposed that the nature of the arithmetic facts varies according to the operation (i.e., a verbal storage for multiplication and simple addition, Dehaene & Cohen, 1995, 1997), or even to individual preference (Noël & Seron, 1993, 1995). Beyond this debate, a main characteristic of this network is the so-called *problem-size effect* (Ashcraft, 1992). The ability to retrieve the answer of arithmetic facts varies with the size of the operands. Large problems (e.g., 9×7) induce slower RTS and higher error rates than small problems (e.g., 2×3). The problem-size effect has been found in addition (Ashcraft & Battaglia, 1978), subtraction (Geary, Frensch, & Wiley, 1993), multiplication (Campbell, 1987), and division (Robinson et al., 2006) problems. It is generally interpreted as reflecting differences in associative strength within the arithmetical network, even if other hypotheses postulated differences in problem familiarity (Campbell & Tarling, 1996; Hamann & Ashcraft, 1986). Siegler and Shrager (1984a) proposed that the retrieval of a given answer depends on the associate strength of this answer and the problem relative to the associate strength of other (correct or not) responses to the same problem. For instance, the problem 3×5 was related to a peaked distribution with a high probability of retrieving the answer 15, while a flat distribution was observed for the problem 6×9 (Siegler, 1988b). Furthermore, the retrieval-based strategies are moderated by a *confidence criterion* that represents an internal threshold through which the child judges his or her confidence in the exactness of the retrieved response (Siegler, 1988a). This individual factor influences performances on simple calculation in the same way as the inherent characteristics of arithmetical network.

The acquisition of arithmetical abilities is a long-lasting process which requires an exact manipulation of numbers. One crucial question is whether these skills develop independently of the preverbal approximate system, or whether they are built on early numerical knowledge. As noted before, even preverbal infants are able to judge the relevance of outcome on basic arithmetical problems like 1+1=2 but not 3 (Wynn, 1992). More recently, several studies using computer-animated tasks have investigated whether preschool children without any formal instruction on mathematics grasp the logic of simple calculation (Barth et al., 2006; Barth, La Mont, Lipton, & Spelke, 2005). In these experiments, children were presented with a dots pattern which was occluded, and then a second number of dots moved behind the screen to join the first one. In a final step, the screen disappeared to reveal a third quantity which differed from the correct sum by a small or a large ratio/distance, and the participants had to compare it to the expected sum. A similar procedure was adapted to subtraction problems. The results indicated that 5 and 6 years children could perform above chance on both approximate addition and subtraction of non-symbolic visual sets although they were at chance on the same problems when presented symbolically. Their accuracy decreased as the ratio of the compared quantities approached 1, reflecting the approximate number representation. Moreover, children's performance did not depend on perceptual factor (Barth et al., 2006) or strategies based on exact calculation (Gilmore, McCarthy, & Spelke, 2007). Together, these data are taken as evidence that the preverbal number representation serves as a basis for formal mathematics.

Further supports for this hypothesis are provided by recent investigations on relationships between individual performances on basic numerical tasks, like number comparisons or numerosity estimations, and more elaborate mathematical competencies. For instance, it was found that, amongst various cognitive measures, the speed of numeral comparisons was a reliable predictor of individual differences on mathematical scores of 7- to 10-year-old children (Durand, Hulme, Larkin, & Snowling, 2005). These results were obtained when performances were controlled for all other variables including measures of processing speed, favouring a specific role of the speed of numeral comparisons.

Another study tested the extent to which the numerical distance effect of 6- to 8-year-old children, measured during symbolic and non-symbolic number comparisons, correlated with scores on an arithmetical test (Holloway & Ansari, in press). There was a significant negative correlation between calculation scores and the size of the symbolic distance effect, indicating that large distance effect was associated with lower performances on simple arithmetic. The non-symbolic distance effect was correlated neither with mathematics achievement, nor with the symbolic distance effect. Similar analyses conducted by groups showed that the relationship between mathematics achievement and the symbolic distance effect mainly influenced 6-year-old children and decreased by the age of 8. The fact that mathematical skills were specifically linked to symbolic, but not to non-symbolic, distance effect was interpreted as individual variability in the connection between symbols and magnitudes rather than in the representation of number magnitudes *per se*. However, it should be noted that such associations could simply occur as the result of a common mechanism to identify Arabic numbers or to map these symbols onto a magnitude representation during number comparisons and the arithmetical task.

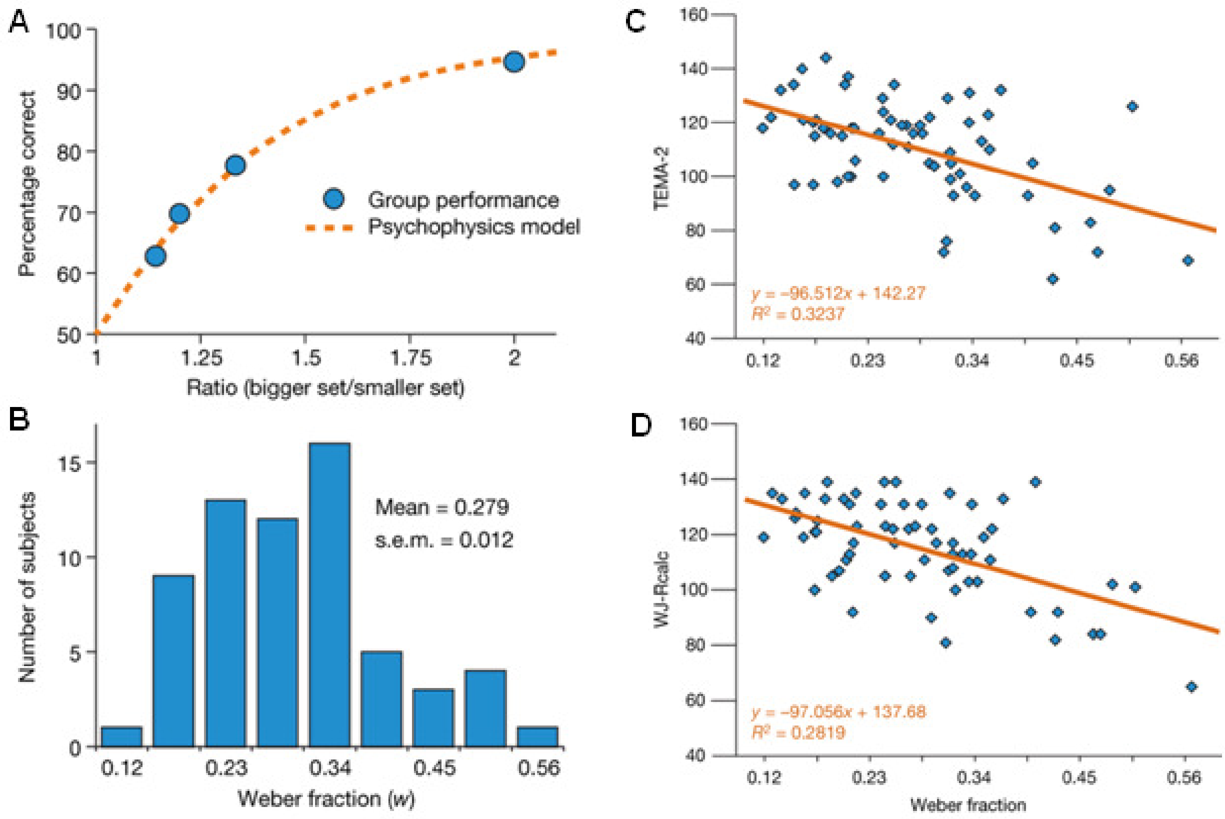

Figure 1-5. (A) Group performance and modelled best-fit for all trials in the numerical discrimination task performed on dot patterns; (B) Individual differences in Weber fraction as a marker of number sense acuity; (C-D) Relationship between individual symbolic mathematical achievement score on the test of early mathematical ability, second edition (TEMA-2) or on the Woodcock–Johnson revised calculation subtest (WJ-Rcalc) and the acuity of number sense (adapted from Halberda, Mazzocco, & Feigenson, 2008).

Finally, possible individual differences in the "number sense" acuity were investigate using a non-verbal number discrimination task, and were related to mathematical achievement (Halberda et al., 2008). Fourteen-year-old children had to estimate the larger of two intermixed dots patterns presented in different colours. The ratio between the two sets varied from 1:2 to 7:8 in order to determine the level of acuity (i.e., ratio-limited discrimination) for each participant. Results showed that there was a high variability across children in the Weber fraction ranging from 0.12 to 0.56 (see Fig. 1-5). In other words, some children were able to discriminate numerical ratio as fine as 7:8 whereas others could discriminate 2:3 ratio only. Moreover, differences in acuity measured at ninth grade retroactively correlated with scores on mathematical achievement tests from kindergarten to sixth grade, even controlled for speed of processing and general intelligence. This indicates that difficulty in estimating fine differences between non-symbolic displays at 14 years was closely related to symbolic mathematical achievement from as early as 5 years old. Nevertheless, these findings do not demonstrate a causal role of "number sense" acuity for mathematical performance, as one cannot exclude the possibility that children with high-ability in mathematics develop a more precise magnitude representation throughout the lifespan.

4. Conclusions

Human beings are able to estimate the cardinal value of non-symbolic displays, without verbal counting. As accuracy performance is ratio-limited and shows the signature of Weber-Fechner law, it has been postulated to depend on an approximate representation of numbers. These preverbal abilities have been observed in educated adults (Whalen et al., 1999), preschool children (Lipton & Spelke, 2005), adults in an indigene Amazonian group with only small lexicon of number words (Pica et al., 2004), and even in various non-human animal species (for a review, see Dehaene et al., 1998). Furthermore, it seems that a sensitivity to numerosity exists early in the development, at least in a crude form. Infants can discriminate between numerosities presented as visual dots patterns (Starkey & Cooper, 1980), sequences of sounds (Starkey et al., 1990), or successive actions (Wynn, 1995), suggesting that they possess a multimodal and potentially abstract representation of numerosities. However, it is clear that continuous amounts like surface area and density also influence infants' discrimination (Feigenson, Carey, & Spelke, 2002). In some conditions, object-files could be used to rapidly track discrete elements in a visual scene, rather than number itself. Nevertheless, perceptual mechanisms can not account for all performances, and thereby do not rule out the existence of an early "number sense". More particularly, infants are able to discriminate between numerosities controlled for non-numerical dimensions that were too large to be explained by an object-files system. In addition, more responses for number have been reported even when controlled for perceptual dimensions (Brannon & Gautier, 2003; Leslie et al., 2003).

Another system for exact quantities appears later on the development. Between the age of 2 and 6, children acquire the counting principles through which they assign a cardinal value to each of number words. Contrarily to early preverbal abilities, the verbal counting enables the development of a precise representation for numbers. Fingers are used to support verbal counting, and therefore probably contribute to the acquisition of this system. Children of Western societies are also confronted with Arabic numerals very early on, but an accurate understanding of these symbols is achieved at the age of 5-6 only. Both counting abilities and symbolic knowledge enable the acquisition of more elaborate skills. School-aged children learn to solve simple calculation by first resorting to counting procedures. They progressively abort these strategies in favour of a direct retrieval of the answer in memory.

Of particular interest is whether or not the exact system for numbers is built on preverbal approximate representations. Although this question is still open, there is convincing evidence that many tasks dealing explicitly with exact symbolic numerosities automatically activate the non-symbolic number system. Performances on comparison of Arabic numbers are influenced by the nature of the approximate representation in both adults (e.g., Dehaene et al., 1990) and preschool children (Temple & Posner, 1998). Even early abilities in arithmetic have been found in children without any formal instruction in symbolic mathematics (e.g., Gilmore et al., 2007). Once again, the ability to estimate the result of additions or subtractions on non-symbolic patterns was dependent on the approximate system. With regard to verbal counting, it appears that children can map number words in their counting range onto approximate representation, but not number words outside this range. However, it is not clear whether an exact meaning of cardinality is needed to achieve this mapping, or whether the understanding of the logic of counting sequence is sufficient.