Density tapering for antenna arrays based on a coordinate transform

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***Abstract*—A differential equation is established to transform an array with constant average density into an array with prescribed density. It is shown that this allows good control over the first few sidelobes. This is especially true when combined with a limited amplitude weighting that compensates for local density variations, with limited effect on the array sensitivity.**

I. Introduction

Non regular arrays are becoming increasingly popular in radar, astronomy and communication applications. This is mainly because such arrangements avoid the occurence of grating lobes, while requiring fewer elements for a given total effective area [1], [2]. Indeed, the effective area of the isolated element is sometimes significantly larger than the element itself, such that it becomes favorable to place the elements at relatively large distances from each other, of the order of one wavelenght or more. A price to pay for this type of arrangement is that, because of mutual coupling, the embedded element pattern is different for each element [3]. This can be corrected to a large extent through proper calibration followed by specific signal-processing techniques and is not the primary topic of this paper.

The main goal of this paper consists of contributing to systematic techniques for shaping the array pattern without significantly altering the array sensitivity. It is clear that strong amplitude tapering leads to lower ratio between effective area and noise, often translated into a G/T (gain over noise temperature) figure. In constrast, space tapering, i.e. adapting the density of elements [1], [2], [4], [5] over the array surface rather than on their amplitudes, may preserve higher values for G/T, at the expense of higher sidelobes at some intermediate angular distance from the main beam [6], [7].

In this paper, we introduce a change of coordinates that allows one to transform an array with a statistically constant density into another array with pre-determined density tapering. This avoids the somewhat arbitrary decomposition of the array surface into a number of annular rings, which entails a piecewise approximation of the tapering function [4], [7]. We will show that this coordinate transform needs to satisfy a specific differential equation. Then, we solve that equation in an approximate way and show how effectively the near-in slidelobes are reduced. Then, we show how a minor change in amplitudes, previously proposed by two of the authors [8], very effectively further improves the sidelobe level, without significantly affecting the G/T figure. Finally, the effect of mutual coupling on such arrays is demonstrated. Section 2 provides the mathematical formulation, while Section 3 shows results for arrays of about 800 elements and Section 4 discusses the results and provides further perspectives.

II. Mathematical formulation

When producing a non regular, space tapered array, it is possible to place elements in an array according to a pre-determined probability density function, proportional to the desired spatial density. However, in order to avoid antenna elements occupying the same physical area, this process may be relatively long and, more importantly, it will correspond to a fundamentally different distribution everytime a new realization is produced. This means that, in an optimization process, it will be difficult to determine whether changes in the sidelobes are due to changes in density tapering or to specific and purely local changes in the density of the elements found in the array realization. This supports the need for a way to change the density tapering while starting from a common density, which would be considered statistically homogeneous (or, at least, produced while considering a constant probability density). This will be done below for arrays that occupy a disk with constant radius $r_\circ$.

Let us denote by $d(r')$ the desired density of the elements along radial coordinate r'. In the initial array, the radial coordinate of the element is given by r, while it is given by $r' = g(r)$ in the final array. The goal hence consists of finding function $g(r)$, given function $d(r')$ and knowing that $g(0) = 0$ and $g(r_\circ) = r_\circ$. The density of elements in a ring of width dr' and centered on radial distance r' is given by the number of elements dN_r in the corresponding ring (radius r and width dr) before transformation, divided by the surface dS of the new ring. We have:

$$\begin{aligned} dN_r &= \frac{N}{\pi\, r_\circ^2}\, 2\pi\, r\, dr && (1) \\ dS &= 2\pi\, r'\, dr' && (2) \end{aligned}$$

Besides, one may write:

$$r'\, dr' = \frac{1}{2}\, d(r'^2) = g(r)\, \frac{dg(r)}{dr}\, dr \qquad (3)$$

From the above equations, one then obtains:

$$\frac{g(r)}{r}\,\frac{dg(r)}{dr} = \frac{N}{\pi\, r_\circ^2}\,\frac{1}{d(r')} \tag{4}$$

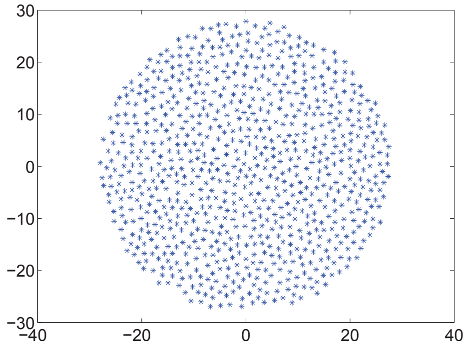

Fig. 1. Array before tapering.

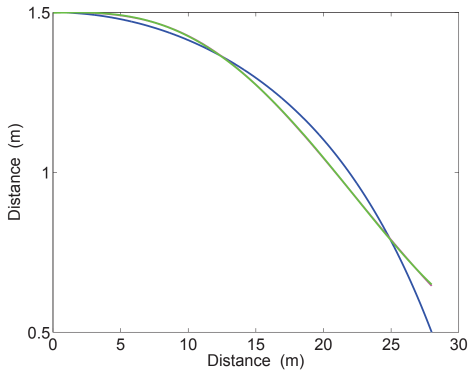

Fig. 2. Objective tapering function, approximation obtained with a fourth-order $g(r)$ and polynomial model of the approximated tapering function.

In the following, a simple tapering law will be considered and equation (4) will be solved in an approximate way by considering a polynomial function $g(r')$ which merely satisfies start end end conditions. The density function is given by

$$d(r') = C\,(1 - \beta\, r') \tag{5}$$

To satisfy $g(0) = 0$, polynom $g(r')$ has no zero-order term; its first-order coefficient is imposed by satisfying (4) at $r' = 0$, while the highest-oder term is connected to lower-order coefficients through the $g(r_\circ) = r_\circ$ condition. In the examples below, a fourth-order will be considered with orders 2 and 3 set to zero.

The imposed space tapering may have an imperfect impact on the sidelobes because of the unavoidably inhomogeneous density of the elements. This effect will be widely compensated through the amplitude modulation proposed in [8], in which the amplitude coefficients are made proportional to the area occupied by each element in the array; that area is estimated through the calculation of Voronoi cells calculated before density tapering. The resulting coefficients, denoted by A_i thus occupy a very limited range, which strongly limits the loss of sensitivity resulting from amplitude modulation. In natural scale, G/T will be reduced through multiplication by the following coefficient:

$$L = \frac{|\sum_i A_i|^2}{N\ \sum_i |A_i|^2} \tag{6}$$

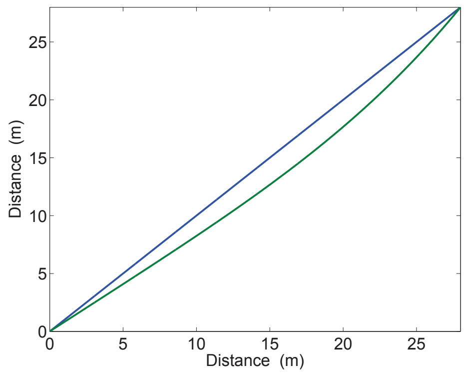

Fig. 3. Transformation function.

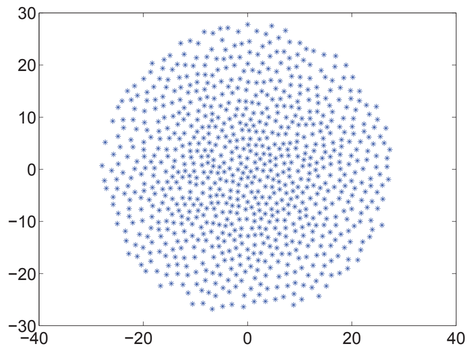

Fig. 4. Array after tapering.

III. Results

Examples will be provided below regarding the array made of 826 elements shown in Fig. 1 (units in meter). In the tapering function (5), β is determined such that the density at the edge corresponds to 1/3 of that at the center (see Fig. 2). A fourth-order $g(r)$ with zero second an third orders is obtained by satisfying the start and end conditions. That function is represented by the curve in Fig. 3 and the corresponding transformed array is shown in Fig. 4.

Considering a 1 meter wavelength, the radiation pattern of the initial array (Fig. 1), is shown in Fig. 5, while the pattern of the transformed array (Fig. 4) is shown in Fig. 6. With the amplitude tapering that corrects for the local density variation (see coefficients in Fig. 7), better sidelobes are obtained, as shown in Fig. 8. Indeed, the first sidelobe has an azimuthally more regular shape and a slightly lower level, while the level of the second and third sidelobes are clearly lower. The standard

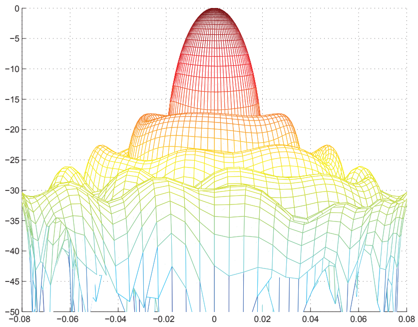

Fig. 5. Inital pattern.

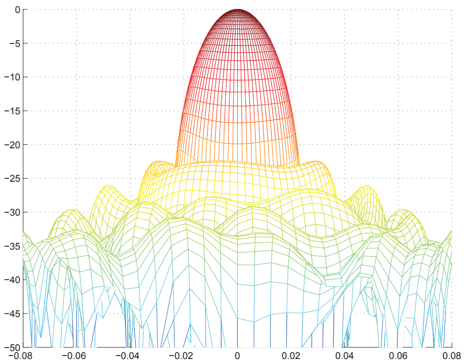

Fig. 6. Pattern with space tapering only.

deviation of the amplitudes A_i is limited to 14 % of their average value. The corresponding loss of sensitivity is of 0.084 dB (G/T multiplied by 0.981).

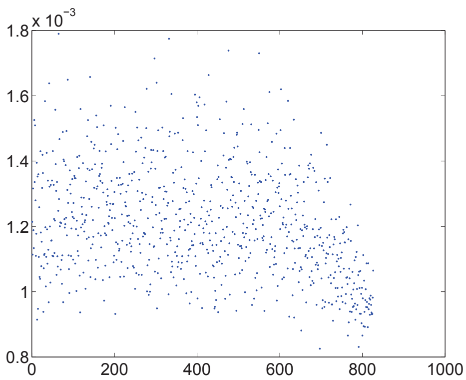

Fig. 7. Correcting amplitude variation.

IV. Conclusion

A diffential equation has been defined for the radial coordinate transform to achieve a pre-determined space tapering in circular antenna arrays. An approximate polynomial solution for the coordinate transform has been used and the effect of this approach on the radiation pattern has been illustrated. It is shown that this method is nicely complemented by a previously published method that corrects for the local density variation through amplitude weighting. Indeed, it allows further reduction of the near-in sidelobes with limited dynamic range for the array weights and hence a very small reduction of the effective area. Further research will include accurate solution of the differential equation that governs space tapering, inclusion of the the effects of mutual coupling and use the initiated theoretical framework to propose rules for the systematic and well controlled combination of amplitude and space tapering.

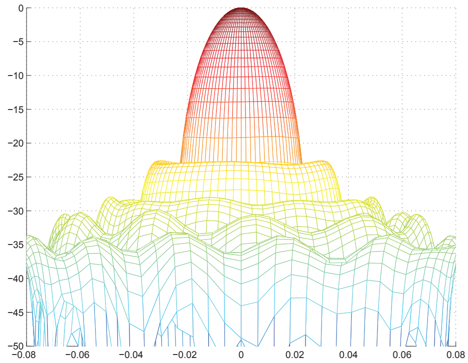

Fig. 8. Pattern with space and amplitude tapering.

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