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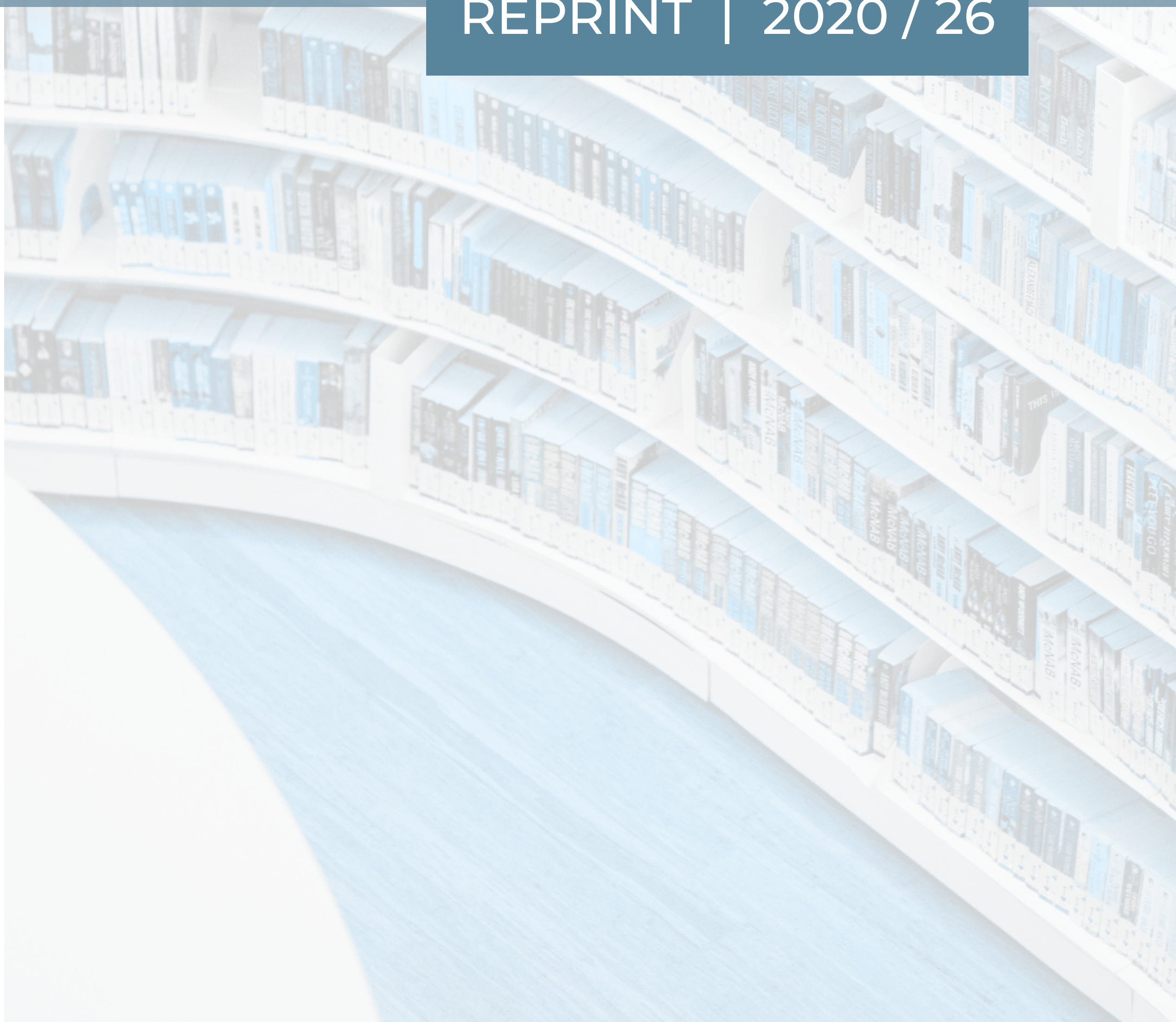

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INVESTING IN YOUR OWN AND PEERS' RISKS: THE SIMPLE ANALYTICS OF P2P INSURANCE

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Abstract

This paper studies a peer-to-peer (P2P) insurance scheme where participants share the first layer of their respective losses while the higher layer is transferred to a (re-)insurer. The conditional mean risk sharing rule proposed by Denuit and Dhaene (2012) appears to be a very convenient way to distribute retained losses among participants, as shown by Denuit (2019a). This paper further considers the case of a P2P insurance scheme when losses are modeled by independent compound Poisson sums with integer-valued severities (resulting from discretization). The amount of contributions paid by participants is determined by splitting it into the price of the stop-loss protection limiting the community's total payout and an appropriate provision for the coverage of the lower layer which is mutualized inside the P2P community. Some extensions are also briefly discussed.

Keywords: conditional expectation, risk pooling, compound Poisson distributions, Panjer recursion.

1 Introduction and motivation

Feeling to belong to a community and to serve societal needs becomes nowadays important for many individuals. In that respect, commercial insurance does not seem to be very appealing. This may be attributed to the progressive de-mutualization of the insurance sector, which has broken the link with the ancestral compensation mechanism consisting in using the contributions of the many to balance the misfortunes of the few. In order to remedy this situation, some insurers have started to promote an offer in line with the collaborative economy by organizing peer-to-peer (P2P) insurance communities, complementing traditional business (see, e.g., Eling and Lehmann, 2018).

P2P insurance thus builds on the fundamental principle of mutuality at the origin of insurance without the equity buffer provided by insurer's capital. Precisely, participants to a P2P insurance scheme agree to pool the first layer of the risks they face, whereas higher losses are still covered by a third party, typically an insurance or reinsurance company. In general, the pool keeps claim amounts up to a given threshold and the part above the threshold is covered by the (re-)insurer with the help of a stop-loss protection limiting the total payout to the sum of the contributions paid by the participants. The main advantage for participants to a P2P insurance scheme is that they can access higher amounts of retention compared to deductibles included in standard insurance covers, thanks to the risk-reducing effect of pooling.

Unclaimed money can be paid back, given to a charity or used to fund a common project to which the community adheres. In the case of a cash-back mechanism, the contract operates as a participating policy. This raises the following question: how to fairly share, in an understandable and transparent way, that part of the premium that has not been used to cover the claims? As everybody is well aware that insurance risks are heterogeneous, equally sharing the total losses among participants may well appear to be unfair. To be successful, P2P insurance schemes require an appropriate risk sharing mechanism recognizing the different distributions of the risks brought to the pool. Intuitively speaking, participants pooling smaller risks should contribute less to the realized total loss. Denuit (2019a) proposed a simple and mathematically correct solution based on the conditional mean risk sharing rule introduced by Denuit and Dhaene (2012). Being based on the concept of mean value, this sharing rule turns out to be quite intuitive as most participants have at least a vague idea about averaging and its risk-reducing effect. Since participants can be informed when they enter the pool about the amount they will have to contribute as a function of the total realized loss, this approach ensures full transparency.

Given the low interest rates and the high volatility inherent to most financial assets, P2P insurance may even be marketed as an attractive investment as long as the size of the pool is large enough so that individual loss experience does not significantly impact on the group's result. Being uncorrelated with financial instruments, accessing insurance risks through P2P communities may offer a nice alternative to classical investment products in terms of diversification. In that respect, some degree of risk classification is important to offer each participant the rate of return he or she deserves.

Let us now discuss the contents of the present contribution. We aim to demonstrate how to

- compute the amount of the initial contribution
- and fairly distribute the surplus among participants to a P2P insurance community,

according to the characteristics of the risks brought to the pool. We work in a frequency-severity setting, decomposing the total loss into the number of insured events (frequency part) and their corresponding costs (severity part). In that setting, compound Poisson distributions are particularly appealing since they naturally arise in the collective approximation to the individual risk model. Also, together with their weak limits, they correspond to the class of infinitely divisible distributions. In this paper, we consider the conditional mean risk allocation principle defined by Denuit and Dhaene (2012). According to this rule, each participant to an insurance pool contributes the conditional expectation of the individual loss brought to the pool, given the total loss experienced by the entire pool. The properties of the conditional mean risk allocation have been studied in Denuit (2019a). See also Denuit (2019b) for related results.

The remainder of this paper is organized as follows. In Section 2, we recall the definitions of the conditional mean risk sharing rule. Section 3 studies this risk allocation rule in the case of independent compound Poisson losses and then applies the conditional mean risk sharing rule to distribute losses among participants to a P2P insurance community. A numerical illustration shows how calculations can be performed. The final Section 4 discusses the result and possible extensions. A dynamic extension including a bonus-malus mechanism is described there, as well as investment opportunities offered in conjunction with the P2P insurance scheme to those participants willing to cover an higher layer.

2 Conditional mean risk sharing rule

Consider n participants to an insurance pool, numbered $i = 1, 2, \ldots, n$. Each of them faces a risk X_i. By risk, we mean a non-negative random variable representing a monetary loss caused by some insurable peril. In the remainder of this paper, we assume that $X_1, X_2, \ldots, X_n$ are independent. This assumption is not restrictive for applications to P2P insurance. Henceforth, we use the notation $S = \sum_{i=1}^n X_i$ for the total risk of the pool. In a risk pooling scheme, each participant contributes ex post an amount $h_i(s)$ where $s = \sum_{i=1}^n x_i$ is the sum of the realizations $x_1, x_2, \ldots, x_n$ of $X_1, X_2, \ldots, X_n$.

In the design of the scheme, it is important that the sharing rule represented by the functions h_i is both intuitively acceptable and transparent. In that respect, the conditional mean risk sharing (or allocation) $h_i^\star$ proposed by Denuit and Dhaene (2012) seems to be particularly attractive. Recall that this allocation is defined as

$$h_i^\star(S) = \mathrm{E}[X_i|S], \;\; i = 1, 2, \ldots, n. \tag{2.1}$$

Clearly, the conditional mean risk sharing (2.1) allocates the full risk S as we obviously have

$$\sum_{i=1}^n h_i^\star(S) = \sum_{i=1}^n \mathrm{E}[X_i|S] = S.$$

In words, participant i must contribute the expected value of the risk X_i he or she brings to the pool, given the total loss S experienced by the entire group.

The key point to explain to participants into a P2P insurance scheme is that the realization of X_i is partly due to chance, and partly reveals the underlying magnitude of the risk brought to the community. In the long run, it is well known that averaging yearly realizations of the risks reveal their mean size according to the law of large numbers. Over one period, (2.1) splits the risk X_i into a random part $X_i - \mathrm{E}[X_i|S]$ shared among participants by virtue of the mutuality (or random solidarity) principle and a structural part $\mathrm{E}[X_i|S]$ to be supported by participant i, individually. Formally, the risk X_i brought by participant i is decomposed into

$$X_i = \underbrace{\mathrm{E}[X_i|S]}_{=\text{structural part}} + \underbrace{X_i - \mathrm{E}[X_i|S]}_{=\text{random departure } \mathcal{E}_i}.$$

Both terms entering this split are uncorrelated and random departures $\mathcal{E}_i$ always sum to 0. In commercial insurance, the variations in S are absorbed by insurer's capital and individual i must contribute a fixed amount

$$\mathrm{E}[X_i] = \mathrm{E}\big[\mathrm{E}[X_i|S]\big] = \mathrm{E}[h_i^\star(S)]$$

to compensate the cost of insured events, supplemented with shareholder's profit for providing the risk-bearing capital. With P2P insurance, each participant must contribute ex post the amount $\mathrm{E}[X_i|S]$ to the pool whereas $\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_n$ are re-allocated among them according to the mutuality principle. The expected contribution is equal to the expected loss, that is, $\mathrm{E}[h_i^\star(S)] = \mathrm{E}[X_i]$, $i = 1, 2, \ldots, n$.

Even if the conditional mean risk sharing rule is not based on participants' preferences, pooling (2.1) appears to be an attractive alternative to conventional, fixed premium insurance as long as the volatility of the contribution $h_i^\star(S)$ is in line with the risk appetite of participant i. In the expected utility setting, every risk-averse decision-maker prefers $h_i^\star(S)$ over the initial risk X_i so that the conditional mean risk sharing rule appears to be beneficial to all participants (as an application of Jensen's inequality). Denuit and Dhaene (2012) established that the conditional mean risk sharing rule is Pareto-optimal for all risk-averse economic agents behaving according to the expected utility paradigm, as long as every function $h_i^\star$ is non-decreasing.

3 Sharing losses within the P2P insurance community

3.1 Conditional mean risk sharing for independent compound Poisson sums

Assume that the loss X_i for participant i to the insurance pool can be represented as

$$X_i = \sum_{k=1}^{N_i} C_{ik} \text{ with } N_i \sim \text{Poisson}(\lambda_i), \quad i = 1, 2, \ldots, \tag{3.1}$$

where the claim severities C_{ik} are positive, all these random variables being independent. Let us assume that the random variables C_{ik} are valued in $\{1, 2, 3, \ldots\}$. This is generally the case in actuarial applications, where claim severities are expressed as multiples of an appropriate monetary unit (after discretization).

The compound Poisson probabilities are readily obtained with the help of Panjer algorithm. Let us denote as C_i a random variable distributed as C_{ik}. Starting from $\mathrm{P}[X_i = 0] = \exp(-\lambda_i)$, Panjer recursive formula allows the actuary to compute the probability mass function of X_i with the help of

$$\mathrm{P}[X_i = k] = \frac{\lambda_i}{k} \sum_{j=1}^{k} j\mathrm{P}[C_i = j]\mathrm{P}[X_i = k - j], \;\; k = 1, 2, \ldots.$$

Coming back to (2.1), we see that $\mathrm{E}[X_i|S = 0] = 0$ and for $s \geq 1$, we have

$$\begin{aligned} \mathrm{E}[X_i|S = s] &= \sum_{k=1}^{s} k\mathrm{P}[X_i = k|S = s] \\ &= \sum_{k=1}^{s} k \frac{\mathrm{P}[X_i = k]\mathrm{P}\left[\sum_{j\neq i} X_j = s - k\right]}{\mathrm{P}[S = s]} \end{aligned}$$

where each probability appearing in the latter expression can be evaluated by Panjer algorithm since X_i, $\sum_{j\neq i} X_j$ and S all obey compound Poisson distributions. Precisely, S is a sum of independent compound Poisson sums and is therefore itself a compound Poisson sum, that is, S is distributed as $\sum_{k=1}^{M} Z_k$ with $M \sim$Poisson$(\lambda_\bullet)$ where $\lambda_\bullet = \sum_{i=1}^{n} \lambda_i$ and $Z_1, Z_2, \ldots$ are independent random variables distributed as Z with

$$\mathrm{P}[Z = k] = \sum_{i=1}^{n} \frac{\lambda_i}{\lambda_\bullet} \mathrm{P}[C_i = k], \;\; k = 0, 1, \ldots,$$

all the random variables being independent. Similarly, $\sum_{j\neq i} X_j = S - X_i$ is distributed as $\sum_{k=1}^{M_i} Z_{ik}$ with $M_i \sim$Poisson$(\lambda_\bullet - \lambda_i)$ and $Z_{i1}, Z_{i2}, \ldots$ are independent random variables distributed as Z_i with

$$\mathrm{P}[Z_i = k] = \sum_{j\neq i} \frac{\lambda_j}{\lambda_\bullet - \lambda_i} \mathrm{P}[C_j = k], \;\; k = 0, 1, \ldots,$$

all the random variables being independent.

3.2 Layering claim severities

P2P insurance splits the risks into several layers, each managed in a specific way (individually by each participant, collectively at the community's level and transferred to an insurer or reinsurer).

A first layer may be retained by participants, individually by introducing a deductible δ_i. The loss pooled is then limited to $(C_{ik} - \delta_i)_+ = \max\{C_{ik} - \delta_i, 0\}$. The amount $\min\{C_{ik}, \delta_i\}$ is retained by participant i. This case is easily accounted for by modifying the Poisson parameter λ_i into $\lambda_i \mathrm{P}[C_i > \delta_i]$ and considering claim severities distributed as $C_i - \delta_i$ given $C_i > \delta_i$ so that we do not discuss this case separately. The introduction of a deductible δ_i of course reduces the contribution paid by participant i.

P2P insurance is also possible when the insured peril may generate large losses, such as in third-party liability insurance, for instance. To avoid that a large claim exhausts the P2P community's capacity, individual losses are capped to a maximum. Precisely, this means that the community is protected by an excess-of-loss cover and retains the first layer $(0, m_i)$ of every claim filed by participant i whereas $(C_{ik} - m_i)_+$ is covered by the (re-)insurer protecting the pool. Calculations are then carried with $\min\{C_{ik}, m_i\}$ instead of C_{ik}. Alternatively, pooling may be restricted to smaller losses and claims with bodily injuries or cases disputed with third parties can be left to the (re-)insurer. In case an excess-of-loss protection with per-claim retention m_i appears to be necessary then participant i must also pay the corresponding price.

To end with, let us mention that calculations are carried on an incurred basis when claim settlement may extend over several years. It is indeed important that participants get detailed accounts of the P2P insurance operations at the end of each period. If some claims remain open at that time, they are ceded to the (re-)insurer covering the higher layer against a price paid the P2P community.

3.3 Participants' contributions

3.3.1 Decomposition according to layers

The P2P insurance community has only limited risk bearing capacity and must thus be protected by a stop-loss arrangement with priority w. The community only retains

$$S_{\mathrm{P2P}} = \min\{S, w\} \text{ where } S = \sum_{i=1}^{n} X_i.$$

Here, the definition of severities in X_i accounts for the presence of deductibles or excess-of-loss protection, if needed. The community thus covers the first layer $(0, w)$ of the total losses S experienced by all participants whereas the upper layer (w, ∞) is transferred to a (re-)insurer. The total amount of contribution π_i paid by participant i is decomposed into

$$\pi_i = \pi_i^{\mathrm{P2P}} + \pi_i^{\mathrm{SL}}$$

supplemented with running expenses (including possible taxes) where

$\pi_i^{\mathbf{P2P}}$ is that part of the total contribution paid by participant i covering the first layer shared within the P2P community, with

$$w = \sum_{i=1}^{n} \pi_i^{\mathrm{P2P}};$$

$\pi_i^{\mathbf{SL}}$ is the contribution to the price of the stop-loss protection, such that

$$\sum_{i=1}^{n} \pi_i^{\mathrm{SL}} = (1 + \theta_{\mathrm{SL}})\mathrm{E}[(S - w)_+]$$

where θ_{SL} is the corresponding loading to obtain the premium charged by the (re-)insurer covering the upper layer (w, ∞).

The value of w can be tuned to ensure that $\mathrm{P}[S < w]$ is high enough so that a surplus $w - S$ can be distributed sufficiently often. The value of w also impacts on both π_i^{P2P} and π_i^{SL} so that it must be selected to make the participation to the P2P insurance scheme affordable and attractive to prospects.

3.3.2 Contribution to the upper layer

The stop-loss premium on S could be shared equally among participants or according to their respective contribution to the upper layer of the losses. Let us discuss the second case. The stop-loss premium corresponding to the upper layer (w, ∞) is decomposed into

$$\begin{aligned}
\mathrm{E}[(S-w)_+] &= \Big(\mathrm{E}[S|S>w] - w\Big)\mathrm{P}[S>w] \\
&= \left(\sum_{i=1}^{n}\mathrm{E}[X_i|S>w] - w\frac{\sum_{i=1}^{n}\mathrm{E}[X_i|S>w]}{\mathrm{E}[S|S>w]}\right)\mathrm{P}[S>w] \\
&= \sum_{i=1}^{n}\mathrm{E}[X_i|S>w]\left(1-\frac{w}{\mathrm{E}[S|S>w]}\right)\mathrm{P}[S>w].
\end{aligned}$$

According to formula (4.2) in Denuit (2019b), we have

$$\mathrm{E}[X_i|S>w] = \mathrm{E}[X_i]\frac{\mathrm{P}[S+T_i>w]}{\mathrm{P}[S>w]}$$

where T_i is a size-biased version of C_i, independent of S, with probability mass function

$$\mathrm{P}[T_i = k] = \frac{k\mathrm{P}[C_i = k]}{\mathrm{E}[C_i]},\quad k \in \{1, 2, \ldots\}.$$

Also,

$$\mathrm{E}[S|S>w] = \frac{\mathrm{E}[S] - \sum_{j=0}^{w} j\mathrm{P}[S=j]}{1-\mathrm{P}[S\leq w]}.$$

Participant i then pays

$$\pi_i^{\mathrm{SL}} = (1+\theta_{\mathrm{SL}})\mathrm{E}[X_i|S>w]\left(1-\frac{w}{\mathrm{E}[S|S>w]}\right)\mathrm{P}[S>w]$$

to cover the higher layer, beyond the P2P community's capacity.

3.3.3 Contribution to the lower layer

Let us now explain how to determine the amount of contribution π_i^{P2P} meant to absorb the pooled part of the losses. Provided the group is large enough, or it satisfies some additional conditions (such as semi-homogeneity, see below), we may expect that $s \mapsto \mathrm{E}[X_i|S=s]$ is non-decreasing. This ensures that the risk sharing is comonotonic and hence Pareto-optimal (Denuit and Dhaene, 2012, Proposition 4.2). Several sufficient conditions for this to hold have been derived in the literature, often involving log-concavity. Henceforth, we make the following assumption:

$$\text{Assumption: } s \mapsto \mathrm{E}[X_i|S=s] \text{ non-decreasing for every } i. \tag{3.2}$$

In practice, this can be checked graphically, displaying the n conditional expectations as a function of s, in case the validity of (3.2) is not supported by theoretical arguments.

The surplus that can be shared among participants or given to a charity is

$$B_{\text{P2P}} = \Big(w - S\Big)_+.$$

If the community decides to distribute B_{P2P} among participants then the benefit B_i paid to participant i is given by

$$B_i = \begin{cases} 0 \text{ if } S \geq w = \sum_{i=1}^n \pi_i^{\text{P2P}} \\ \\ \pi_i^{\text{P2P}} - \mathrm{E}[X_i|S] \text{ if } S < w. \end{cases}$$

However, if $B_i < 0$ for some i then participant i must contribute an additional amount $-B_i$ at the end of the period. This is certainly not desirable and can be avoided by defining π_i^{P2P} as the solution to

$$\pi_i^{\text{P2P}} = \mathrm{E}[X_i|S=w]. \tag{3.3}$$

Then, assumption (3.2) ensures that for any $s \leq w$, the contribution to be paid ex post by participant i is always less than, or equal to π_i^{P2P}, that is,

$$\mathrm{E}[X_i|S=s] \leq \pi_i^{\text{P2P}} = \mathrm{E}[X_i|S=w] \text{ for all } s \leq w$$

so that $B_i \geq 0$ for all i and no additional contribution is required from participant i at the end of the period. Notice that this is in line with the case of survivor funds, or tontines where participants can only loose their initial contribution if they die (Denuit, 2019a, Section 6.2). The threshold w can then be selected so that the total contribution π_i is attractive compared to classical, commercial insurance.

3.4 Particular cases

Let us now review some particular cases where analytical results can be derived.

3.4.1 Homogeneous case

If $X_1, X_2, \ldots, X_n$ are independent and identically distributed then $\pi_i^{\text{P2P}} = \pi$, $w = n\pi$ and

$$\mathrm{E}[X_i|S=n\pi] = \pi \text{ for every } i = 1, 2, \ldots, n.$$

We are then free to select the amount of contribution π so that the retained part w of the losses is acceptable to participants.

3.4.2 Semi-homogeneous case

Assume that heterogeneity is confined to claim frequencies $\lambda_1, \lambda_2, \ldots, \lambda_n$, in the sense that $C_1, C_2, \ldots, C_n$ are identically distributed, and distributed as C, say. Then, we know that

$$\mathrm{E}[X_i|S=s] = \frac{\lambda_i}{\lambda_1 + \ldots + \lambda_n} s. \tag{3.4}$$

See e.g. Denuit (2019a, Section 3.4). Hence, for $s \leq w$, we have

$$\mathrm{E}[X_i|S=s] = \frac{\lambda_i}{\lambda_\bullet} s.$$

The amount of contribution π_i^{P2P} is then defined as

$$\pi_i^{\mathrm{P2P}} = \mathrm{E}[X_i|S=w] = \frac{\lambda_i}{\lambda_\bullet} w = \frac{\lambda_i}{\lambda_\bullet} \sum_{j=1}^{n} \pi_j^{\mathrm{P2P}}.$$

This equality holds true if we select contributions of the form

$$\pi_i^{\mathrm{P2P}} = (1+\rho)\lambda_i \mu \text{ where } \mu = \mathrm{E}[C],$$

with the same retention rate ρ applying to all participants. The retention of the group then amounts to a multiple of the mean loss:

$$w = (1+\rho) \sum_{i=1}^{n} \lambda_i \mu.$$

3.5 Numerical illustration

Consider a P2P insurance community composed of three risk profiles. Each participant brings a compound Poisson loss to the pool, where severities are obtained by discretizing the Beta distribution with mean $a/(a+b)$ and variance $ab/((a+b)^2(a+b+1))$, concentrating the probability mass of each interval $(j/100, (j+1)/100)$ on its upper left endpoint, $j = 0, 1, \ldots, 99$. The corresponding probabilities are then assigned to severity values $10j$, $j = 1, 2, \ldots, 100$. Precisely,

Group 1 (low risks) with $n_1 = 20$ individuals, $\lambda_1 = 5\%$, claim sizes distributed as C_1 obtained by discretizing the Beta distribution with parameters $a = 2$ and $b = 5$.

Group 2 (medium risks) with $n_2 = 10$ individuals, $\lambda_1 = 10\%$, claim sizes distributed as C_2 obtained by discretizing the Beta distribution with parameters $a = 3$ and $b = 5$.

Group 2 (high risks) with $n_3 = 5$ individuals, $\lambda_3 = 20\%$, claim sizes distributed as C_3 obtained by discretizing the the Beta distribution with parameters $a = 4$ and $b = 5$.

The `R` package `actuar` has been used to perform the calculations; see Dutang et al. (2008). Panjer recursion is performed using the function `aggregateDist`.

The probability mass functions of the severities are displayed in Figure 1 for each of the three groups. We can see there that the probability mass progressively shifts towards larger values. Participants in Group 3 bring comparatively higher severities compared to participants in Group 2 who themselves bring comparatively higher severities compared to participants in Group 1. Groups are thus ranked in increasing magnitude of both expected frequencies and severities. Figure 2 displays the probability mass functions of the compound Poisson losses in each group, together with the probability mass function of the grand total S. Notice that the probability mass at zero, respectively equal to $\exp(-\lambda_i)$, $i = 1, 2, 3$ and to $\exp(-\lambda_\bullet)$ are not displayed.

The expected value of S is

$$\mathrm{E}[S] = n_1\lambda_1\mathrm{E}[C_1] + n_2\lambda_2\mathrm{E}[C_2] + n_3\lambda_3\mathrm{E}[C_3] = 1120.16;$$

Assume that the group is willing to cover losses up to $w = 2000$. The functions $s \mapsto \mathrm{E}[X_i|S = s]$ are displayed in Figure 3 for each of the three groups and s up to w. They appear to be non-decreasing so that assumption (3.2) is fulfilled. The relative contributions of each group to the total realized loss $S = s$ are displayed in Figure 4. We can see there

$$\frac{1}{s}\sum_{i\in\text{Group } j}\mathrm{E}[X_i|S = s] \tag{3.5}$$

as a function of s. In relative terms, the contributions of Group 1 and Group 2 decrease as s increases, whereas the contribution of Group 3 increases.

Considering that the maximal amount of loss w that can be supported by the entire group is $w = 2000$ (about twice the mean, as explained above), we obtain the respective contributions

$$\pi_i^{\text{P2P}} = \mathrm{E}[X_i|S = 2000] = \begin{cases} 23.70 \text{ for } i \in \text{Group 1} \\ \\ 67.54 \text{ for } i \in \text{Group 2} \\ \\ 170.11 \text{ for } i \in \text{Group 3} \end{cases}.$$

The probability that there remains some surplus to be shared among participants is

$$\mathrm{P}[S \leq 2000] = 0.8949$$

so that the cash-back mechanism should operate on average 9 years out of 10.

Participants must also pay for the stop-loss cover. Assume that $\theta_{\text{SL}} = 0.5$. Then, we have

$$\mathrm{E}[X_i|S > 2000] = \mathrm{E}[X_i]\frac{1 - \mathrm{P}[S + T_i \leq 2000]}{\mathrm{P}[S \leq 2000]}$$

with

$$\mathrm{P}[S + T_i \leq 2000] = \sum_{k=1}^{100}\frac{10k\mathrm{P}[C_i = 10k]}{\mathrm{E}[C_i]}\mathrm{P}[S \leq 2000 - 10k].$$

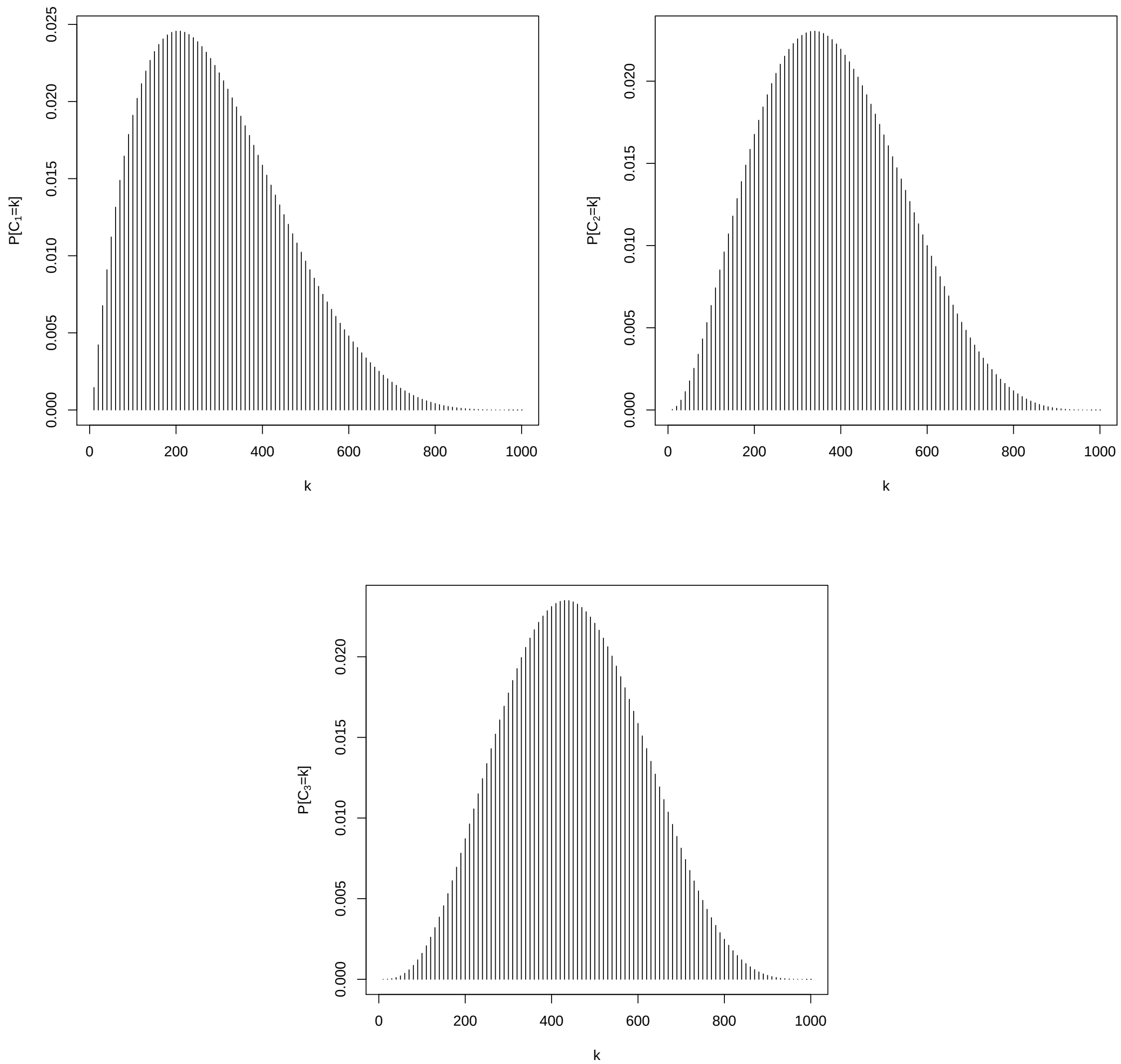

Figure 1: Probability mass functions of the severities C_1 in Group 1 (top left panel), C_2 in Group 2 (top right panel), and C_3 in Group 3 (bottom panel).

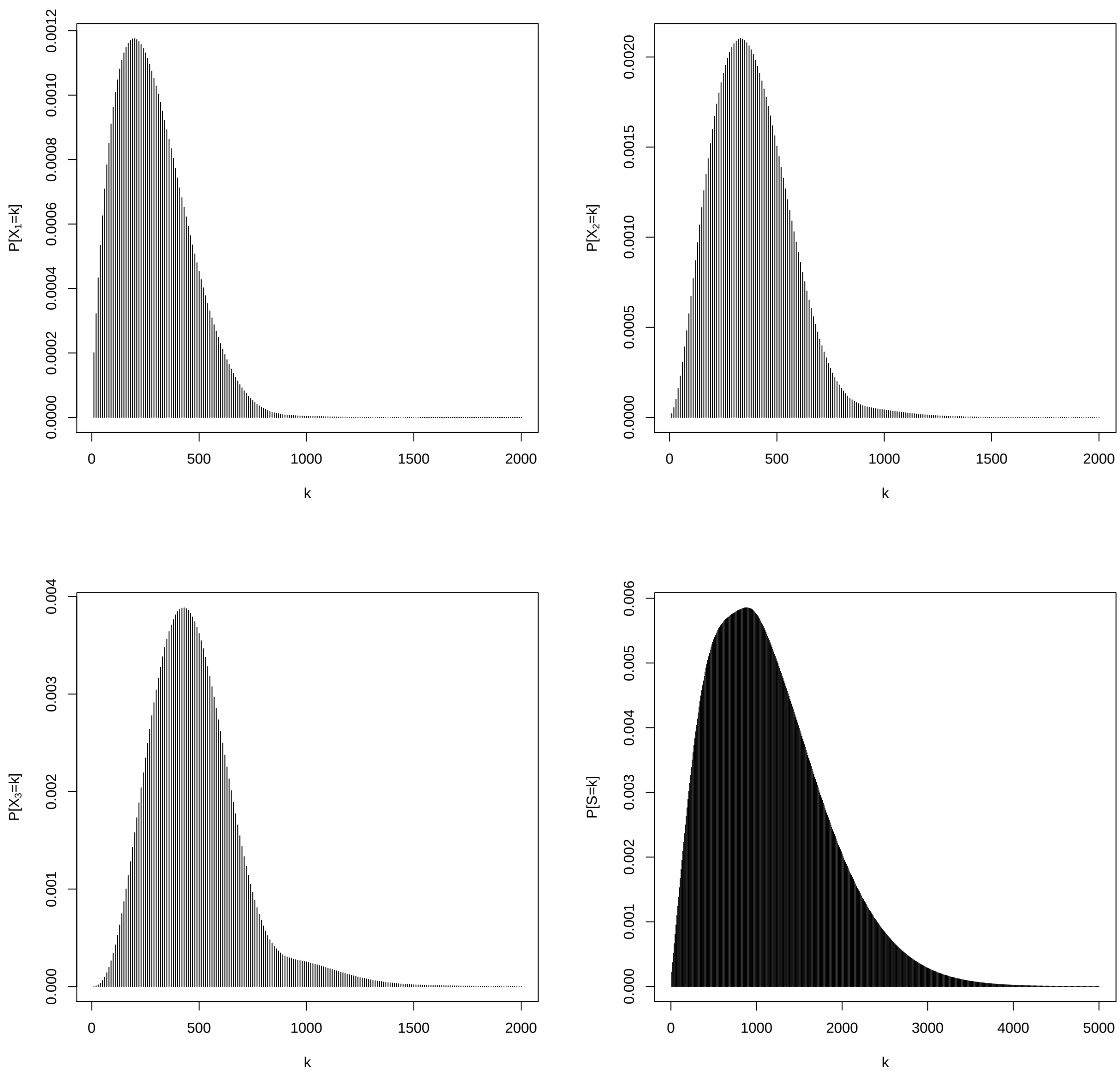

Figure 2: Probability mass functions of $X_i = \sum_{k=1}^{N_i} C_{ik}$ in Group 1 (top left panel), in Group 2 (top right panel), in Group 3 (bottom left panel) and of S (bottom right panel), probability mass at the origin not displayed for graphical reasons.

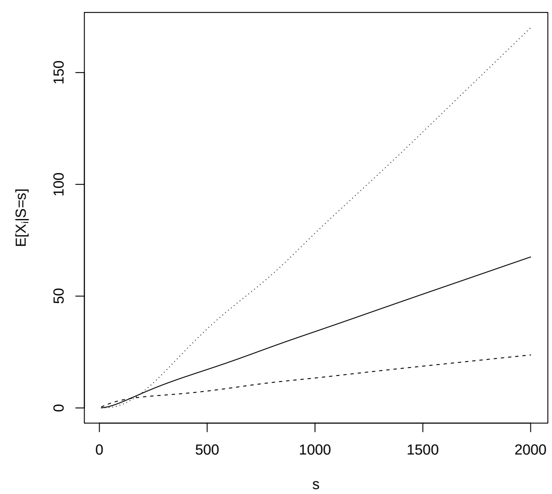

Figure 3: Respective contributions $s \mapsto \mathrm{E}[X_i|S = s]$ in Group 1 (broken line appearing at the bottom), in Group 2 (solid line appearing in the middle), and in Group 3 (dotted line appearing at the top).

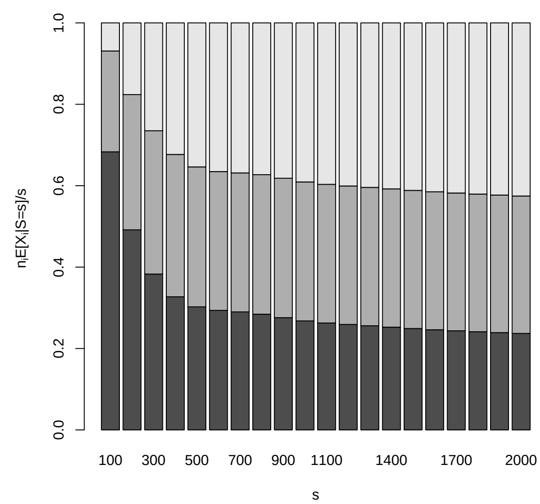

Figure 4: Respective relative contributions (3.5) per group: Group 1 appears at the bottom of each box, Group 2 in the middle, and Group 3 at the top.

This gives

$$\pi_i^{\mathrm{SL}} = 1.5\mathrm{E}[X_i|S > 2000]\left(1 - \frac{2000}{\mathrm{E}[S|S > 2000]}\right)\mathrm{P}[S > 2000] = \begin{cases} 1.27 \text{ for } i \in \text{Group 1} \\ 3.69 \text{ for } i \in \text{Group 2} \\ 9.51 \text{ for } i \in \text{Group 3} \end{cases}.$$

4 Discussion

4.1 Summary

This paper proposes a P2P insurance scheme comprising a stop-loss protection limiting the total amount of losses to the sum of contributions paid by participants. This allows them to pool the first layer of the total losses within the community, while benefiting from the coverage of the upper layer by a (re-)insurer. Participants can thus access to a high, pooled deductible, limiting the risk transfer to the part of the losses that cannot be retained within the pool.

The conditional mean risk sharing rule has been shown to be particularly effective to determine the respective contributions for participants to a P2P insurance scheme, confirming the results obtained by Denuit (2019a). It appears to be intuitively appealing (as it is based on mean value, a familiar concept for most people, at least to some extent) and computationally feasible. Full transparency is achieved as participants can be informed about the cash-back mechanism in function of the realized total loss for the pool when they enter the P2P insurance scheme.

4.2 P2P as an investment opportunity

As explained earlier, the amount of contribution π_i^{P2P} cannot be freely selected by participant i. It must correspond to the highest possible amount of ex post contribution, in order to prevent supplements to be charged to participants at the end of the period. To remain attractive to the majority of participants, the retention level w must be selected so that the total contribution π_i stays moderate compared to the premium corresponding to a commercial insurance policy for the same peril. Hence, some participants might be willing to invest more than π_i^{P2P} to cover the losses experienced by the P2P community. This can be explained by the return perceived as attractive or because they wish to support the initiative beyond their required contribution.

If all participants are willing to contribute over π_i then the community should raise w to a higher level. We must thus consider the case where at least some participants do not wish to contribute more than π_i. A first possibility consists in pooling their losses and to split the intermediate layer fairly among them, according to the conditional mean risk sharing rule, as explained in the preceding section.

In order to benefit from more diversification, it may be relevant to allow those participants willing to contribute more than π_i to invest in all the risks within the P2P community, including those brought by participants who do not wish to pay extra contributions. This means that the latter losses must be shared among those participants who are willing to contribute more than π_i. The design of such a multi-layer scheme, with a lower layer mutualized over the whole community, an intermediary layer covered by participants willing to invest more than π_i and an upper layer ceded to a (re-)insurer, is left for future research.

4.3 Dynamic extension

The conditional mean risk sharing rule can also be justified by actuarial fairness in the long run. Assume that the same rule is applied repeatedly over time among participants in a stable pool. Formally, denote as $\boldsymbol{X}_t = (X_{1t}, X_{2t}, \ldots, X_{nt})$ the experience for year t, with $S_t = \sum_{i=1}^n X_{it}$, and assume that the random vectors $\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots$ are independent and identically distributed. Participant i then pays $\mathrm{E}[X_{it}|S_t]$ in year t and in the long run, the average of $\mathrm{E}[X_{it}|S_t]$ converges to $\mathrm{E}[X_{it}]$ according to the law of large numbers, that is, to the pure premium or fair price for the risk X_{it}.

As explained above, a certain degree of risk classification remains desirable within P2P insurance communities, to prevent adverse selection. Since the available information does not fully reveal the underlying risk, some heterogeneity remains among participants and a mixed Poisson model could replace the homogeneous Poisson one. This means that the Poisson parameter, that is, the expected number of insured event, is allowed to vary beyond the risk factors included in the calculation. Risk classification can then also operate a posteriori, based on credibility corrections. In the Negative Binomial setting for instance, past claim numbers can easily be taken into account to re-valuate next year contributions to the pool. Precisely, the expected number of claims is adapted each year according to the number of claims reported so far. This is achieved by revising the Negative Binomial parameters. The updated values are then used to distribute surpluses the year after, resulting in increased contributions and comparatively less cash-back allocated to participants who reported claims in the past.

References

- Denuit, M. (2019a). Size-biased transform and conditional mean risk sharing, with application to P2P insurance and tontines. ASTIN Bulletin 49, 591-617.

- Denuit, M. (2019b). Size-biased risk measures of compound sums. North American Actuarial Journal, in press.

- Denuit, M., Dhaene, J. (2012). Convex order and comonotonic conditional mean risk sharing. Insurance: Mathematics and Economics 51, 265-270.

- Dutang, Ch., Goulet, V., Pigeon, M. (2008). actuar: An R package for actuarial science. Journal of Statistical Software 25, 1-37.

- Eling, M., Lehmann, M. (2018). The impact of digitalization on the insurance value chain and the insurability of risks. The Geneva Papers 43, 359-396.