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ABSTRACT

Based on the S-matrix approach, we introduce a modified formula for the $\pi^{\pm}$ electromagnetic form factor which describes very well the experimental data in the energy region $2m_{\pi} \leq \sqrt{s} \leq 1.1$ GeV. Using the CVC hypothesis we predict $B(\tau \to \pi\pi\nu) = (24.75 \pm 0.38)\%$, in excellent agreement with recent experiments. Comment: Latex, 10 pages, submitted to Phys. Rev. D

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S-matrix approach to two-pion production in e^+e^- annihilation and τ decay

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Abstract

Based on the S-matrix approach, we introduce a modified formula for the $\pi^{\pm}$ electromagnetic form factor which describes very well the experimental data in the energy region $2m_\pi \leq \sqrt{s} \leq 1.1$ GeV. Using the CVC hypothesis we predict $B(\tau^- \to \pi^-\pi^0\nu_\tau) = (24.75 \pm 0.38)\%$, in excellent agreement with recent experiments.

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I. Introduction.

The processes $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^-\pi^0\nu_\tau$ provide a clean environment for a consistency check of the Conserved Vector Current (CVC) hypothesis [1]. Actually, the measurement of the $\pi^\pm$ electromagnetic form factor in e^+e^- annihilation is used to predict [2] the dominant hadronic decay of the tau lepton, namely $\tau^- \to \pi^-\pi^0\nu_\tau$. The weak pion form factor involved in τ decay is obtained by removing the (model-dependent) I=0 contribution (arising from isospin violation and included *via* $\rho-\omega$ mixing) from the measured pion electromagnetic form factor.

In a previous paper [3] we have applied the S-matrix approach to the $e^+e^- \to \pi^+\pi^-$ data of Ref. [4] and determined the pole parameters of the ρ^0 resonance. In particular, we have fitted the data of Ref. [4] by assuming a constant value for the strength of the $\rho-\omega$ mixing parameter and using different parametrizations to account for the non-resonant background. As a result, the pole position of the scattering amplitude was found [3] to be insensitive to the specific background chosen to fit the experimental data.

The purpose of this Brief Report is two-fold. We first argue that the pole position in $e^+e^- \to \pi^+\pi^-$ is not modified by taking the $\rho-\omega$ mixing parameter as a function of the center-of-mass energy, as already suggested in recent papers [5]. Then we propose a new parametrization for the scattering amplitude of $e^+e^- \to \pi^+\pi^-$, based on the S-matrix approach, which looks very similar to the Breit-Wigner parametrization with an energy-dependent width. This results into an improvement in the quality of the fits (respect to Ref. [3]) while the pole position and $\rho-\omega$ mixing parameters remain unchanged (as it should be). Finally, we make use of CVC to predict the $\tau^- \to \pi^-\pi^0\nu_\tau$ branching ratio, which is found to be in excellent agreement with recent experimental measurements.

II. Energy-dependent $\rho-\omega$ mixing.

We start by giving a simple argument to show that the pole position would not be changed if we choose the $\rho-\omega$ mixing parameter to be $m^2_{\rho\omega}(s) \propto s$ (namely $m^2_{\rho-\omega}(0) = 0$), where $\sqrt{s}$ is the total center-of-mass energy in $e^+e^- \to \pi^+\pi^-$.

Let us consider Eq.(7) of Ref. [3] and replace $y \to y's/s_\omega$ [§], where $s_V = m_V^2 - im_V\Gamma_V$. This yields the following expression for Eq. (7) of Ref. [3]:

$$\begin{aligned} F_\pi(s) &= \frac{A}{s-s_\rho}\left(1+\frac{y's}{s_\omega}\frac{m_\omega^2}{s-s_\omega}\right)+B(s) \\ &= \frac{A'}{s-s_\rho}\left(1+y''\frac{m_\omega^2}{s-s_\omega}\right)+B(s), \end{aligned} \tag{1}$$

where A and $B(s)$ denote the residue at the pole and non-resonant background terms, respectively. The second equality above follows from the approximations:

$$\begin{aligned} A' &\equiv A\left(1+y'\frac{m_\omega^2}{s_\omega}\right) \approx A(1+y'), \\ y'' &\equiv \frac{y'}{1+y'm_\omega^2/s_\omega} \approx \frac{y'}{1+y'} \end{aligned}$$

i.e. by neglecting small imaginary parts of order $y'\Gamma_\omega/m_\omega \approx 10^{-5}$ [3]. Thus, since introducing $m^2_{\rho\omega} \propto s$ is equivalent to a redefinition of the residue at the pole and of the $\rho-\omega$ mixing parameter, we conclude that the pole position

[0§] In the Vector Meson Dominance model, y is related to the usual $\rho-\omega$ mixing strength through $y = m^2_{\rho\omega}f_\rho/(m^2_\rho f_\omega) \simeq -2\times10^{-3}$ [3].

would not be changed if we take a constant or an energy-dependent $\rho - \omega$ mixing parameter.

III. Electromagnetic pion form factor.

Next, we consider a new parametrization for the pion electromagnetic form factor. This parametrization is obtained by modifying the pole term in the following way:

$$s - m_\rho^2 + im_\rho\Gamma_\rho\theta(\tilde{s}) \rightarrow D(s) \equiv [1 - ix(s)\theta(\tilde{s})](s - m_\rho^2 + im_\rho\Gamma_\rho\theta(\tilde{s})), \quad (2)$$

where $\theta(\tilde{s})$ is the step function, with argument $\tilde{s} = s - 4m_\pi^2$.

Observe that if we chose:

$$x(s) = -m_\rho \left(\frac{\Gamma_\rho(s) - \Gamma_\rho}{s - m_\rho^2} \right), \quad (3)$$

then Eq. (2) becomes:

$$D(s) = s - m_\rho^2 + m_\rho\Gamma_\rho x(s)\theta(s - 4m_\pi^2) + im_\rho\Gamma_\rho(s) \quad (4)$$

which, when inserted in (1), looks very similar to a Breit-Wigner with an energy-dependent width, which we will chose to be:

$$\Gamma_\rho(s) = \Gamma_\rho \left(\frac{s - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2} \right)^{3/2} \frac{m_\rho}{\sqrt{s}}\ \theta(s - 4m_\pi^2) \quad (5)$$

with the obvious identification $\Gamma_\rho = \Gamma(m_\rho^2)$.

Using Eq. (2) we are lead to modified expressions for Eqs. (8), (9) and (15) of Ref. [3], namely:

$$F_\pi^{(1)}(s) \ = \ \left(-\frac{am_\rho^2}{D(s)} + b \right) \left(1 + \frac{ym_\omega^2}{s - s_\omega} \right) \quad (6)$$

$$F_\pi^{(2)}(s) = -\frac{am_\rho^2}{D(s)}\left(1+\frac{ym_\omega^2}{s-s_\omega}\right)+b \tag{7}$$

$$F_\pi^{(4)}(s) = -\frac{am_\rho^2}{D(s)}\left(1+\frac{ym_\omega^2}{s-s_\omega}\right)\left[1+b\left(\frac{s-m_\rho^2}{m_\rho^2}\right)\right]^{-1}. \tag{8}$$

Using Eqs. (6-8), we have repeated the fits to the experimental data of Barkov *et al.* [4] in the energy region $2m_\pi \leq \sqrt{s} \leq 1.1$ GeV. As in Ref. [3], the free parameters of the fit are $m_\rho,\ \Gamma_\rho,\ a,\ b$ and y. The results of the best fits are shown in Table 1.

From a straightforward comparison of Table 1 and the corresponding results in Ref. [3] (see particularly, Eqs. (10), (11), (16) and Table I of that reference), we observe that the quality of the fits are very similar. Furthermore, the pole position, namely the numerical values of m_ρ and Γ_ρ, and of the $\rho-\omega$ mixing parameter y, are rather insensitive to the new parametrizations (as it should be). The major effect of the new parametrizations is observed in the numerical values of a (the residue at the pole) and b (which describes the background).

An interesting consequence of the results in Table 1 is an improvement in the value of $F_\pi(0)$, which should equal 1 (the charge of π^+). Indeed, from Eqs. (6-8) and Table 1 we obtain:

$$\begin{aligned}
F_\pi^{(1)}(0) &= a+b \\
&= 0.997 \pm 0.015\ (0.962 \pm 0.020) \\
F_\pi^{(2)}(0) &= a+b \\
&= 0.997 \pm 0.015\ (0.960 \pm 0.017) \\
F_\pi^{(4)}(0) &= \frac{a}{1-b} \\
&= 1.011 \pm 0.010\ (0.987 \pm 0.013)
\end{aligned} \tag{9}$$

where the corresponding values obtained in Ref. [3] are shown in brackets. An evident improvement is observed.

Let us close the discussion on this new parametrization with a short comment: using $F_\pi^{(4)}(s)$ (with imaginary parts and y set to zero) we are able to reproduce very well the data of Ref. [6] in the space-like region -0.253 GeV$^2 \leq s \leq -0.015$ GeV^2.

IV. Prediction for $\tau^- \to \pi^-\pi^0\nu_\tau$.

Finally, using the previous results on the pion electromagnetic form factor, we consider the decay rate for $\tau^- \to \pi^-\pi^0\nu_\tau$. As is well known [2], the CVC hypothesis allows to predict the decay rate for $\tau^- \to (2n\pi)^-\nu_\tau$ in terms of the measured cross section in $e^+e^- \to (2n\pi)^0$. Since for the $\tau^- \to \pi^-\pi^0\nu_\tau$ case the kinematical range extends up to $\sqrt{s} = m_\tau$, let us point out that we have verified that our parametrizations for $F_\pi(s)$ reproduce very well the data of $e^+e^- \to \pi^+\pi^-$ in the energy region from 1.1 GeV to m_τ.

The decay rate for $\tau^- \to \pi^-\pi^0\nu_\tau$ at the lowest order is given by [2]:

$$\Gamma^0(\tau^- \to \pi^-\pi^0\nu_\tau) = \frac{G_F^2|V_{ud}|^2 m_\tau^3}{384\pi^3}\int_{4m_\pi^2}^{m_\tau^2} ds \left(1+\frac{2s}{m_\tau^2}\right)\left(1-\frac{s}{m_\tau^2}\right)^2 \cdot \left(\frac{s-4m_\pi^2}{s}\right)^{3/2} |F_\pi^{I=1}(s)|^2 \quad (10)$$

where V_{ud} is the relevant Cabibbo-Kobayashi-Maskawa mixing angle. In the above expression we have neglected isospin breaking in the pion masses. The form factor $F_\pi^{I=1}(s)$ in the Eq. (10) is obtained from Eqs. (6-8) by removing the I=0 contribution due to $\rho - \omega$ mixing (namely, $y = 0$).

According to Ref. [7], after including the dominant short-distance electroweak radiative corrections the expression for the decay rate becomes:

$$\Gamma(\tau^- \to \pi^-\pi^0\nu_\tau) = \left(1 + \frac{2\alpha}{\pi}\ln\frac{M_Z}{m_\tau}\right)\Gamma^0(\tau^- \to \pi^-\pi^0\nu_\tau). \tag{11}$$

We have not included the effects of long-distance electromagnetic radiative corrections, but we expect that they would not exceed 2.0 % .

In order to predict the branching ratio, we use Eqs. (6)-(8) with $y = 0$, the results of Table 1 and the following values of fundamental parameters (ref. [7, 8]):

$$\begin{aligned} m_\tau &= 1777.1 \pm 0.5 \text{ MeV} \\ G_F &= 1.16639(2) \times 10^{-5} \text{ GeV}^{-2} \\ |V_{ud}| &= 0.9750 \pm 0.0007. \end{aligned}$$

With the above inputs we obtain:

$$B(\tau^- \to \pi^-\pi^0\nu_\tau) = \left(\frac{\tau_{tau}}{2.956 \times 10^{-13} s}\right) \cdot \begin{cases} (24.66 \pm 0.26)\% & \text{from Eq. (6)} \\ (24.62 \pm 0.26)\% & \text{from Eq. (7)} \\ (24.96 \pm 0.32)\% & \text{from Eq. (8)} \end{cases} \tag{12}$$

or, the simple average

$$B(\tau^- \to \pi^-\pi^0\nu_\tau) = (24.75 \pm 0.38)\% \tag{13}$$

which is in excellent agreement with recent experimental measurements and other theoretical calculations (see Table 2). Eq. (13) includes the errors (added in quadrature) coming from the fit to $e^+e^- \to \pi^+\pi^-$ and the 1 % error in the τ lifetime [8]: $\tau_\tau = (295.6 \pm 3.1) \cdot 10^{-15}$ s.

In summary, based on the S-matrix approach we have considered a modified parametrization for the $\pi^{\pm}$ electromagnetic form factor, which describes very well the experimental data of $e^+e^- \to \pi^+\pi^-$ in the energy region from threshold to 1.1 GeV. The pole position of the S-matrix amplitude is not changed by this new parametrization. Using CVC, we have predicted the $\tau^- \to \pi^-\pi^0\nu_\tau$ branching ratio, which is found to be in excellent agreement with experiment.

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TABLE CAPTIONS

1. Best fits to the pion electromagnetic form factor of Ref. [4], using Eqs. (6-8).

2. Summary of recent experimental measurements (Exp.) and theoretical results (Th.) for the $\tau^- \to \pi^- \pi^0 \nu_\tau$ branching ratio. The errors in the first entry arise from use of $e^+e^- \to \pi^+\pi^-$ data, the τ lifetime and radiative correction effects [9], respectively.

Table 1

	m_ρ (MeV)	Γ_ρ (MeV)	a	b	$y(10^{-3})$	$\chi/d.o.f$
$F_\pi^{(1)}$	756.74± 0.82	143.78± 1.16	1.236± 0.008	−0.239± 0.013	−1.91± 0.15	0.998
$F_\pi^{(2)}$	756.58± 0.82	144.05± 1.17	1.237± 0.008	−0.240± 0.013	−1.91± 0.15	1.008
$F_\pi^{(4)}$	757.03± 0.76	141.15± 1.18	1.206± 0.008	−0.193± 0.009	−1.86± 0.15	0.899

Table 2

Reference	$B(\tau^- \to \pi^-\pi^0\nu_\tau)$ (in %)
Th. [9]	$24.58 \pm 0.93 \pm 0.27 \pm 0.50$
Th. [10]	24.60 ± 1.40
Th./Exp. [11]	24.01 ± 0.47
Exp. [8]	25.20 ± 0.40
Exp. [12]	25.36 ± 0.44
Exp. [13]	25.78 ± 0.64