Imperfect Competition, Integer Constraints and Industry Dynamics

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Abstract: Amir and Lambson (2003) developed an infinite-horizon, stochastic model of entry and exit by integer numbers of firms facing sunk costs and uncertain market conditions. Here, as examples of the model's usefulness, special cases are applied to the following three issues: (1) the relationship between sunk costs and industry concentration, (2) entry when current profits are negative, and (3) the relationship between entry and the length of the product cycle.

Key Words and Phrases: Entry and Exit, Dynamic Games, Integer Constraints.

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1 Introduction

The traditional industrial organization paradigm explores the interaction between industrial structure, conduct, and performance, with emphasis on the determinants and effects of market power.[1] Wanting to include such pervasive dynamic phenomena as entry and exit in the analysis, economists constructed dynamic models of industrial organization. Almost all of these models are either (1) finite horizon (often two-period) models with an integer number of firms, or (2) infinite horizon models with a continuum of firms. Both have been instructive, but neither is without shortcomings. Finite-horizon integer models suffer from the drawback that the final period behaves like a static (one-period) model, raising questions about their applicability to ongoing industries. Infinite-horizon continuum models, with their infinitessimal price-taking firms, are ill-suited for exploring traditional industrial organization questions regarding market power.

To avoid these shortcomings, Amir and Lambson (2003) developed a stochastic infinite-horizon model where the number of active firms is required to be an integer. Exogenous shocks that are external to the firms—such as demand or factor price changes—generate endogenous entry and exit. The infinite horizon avoids the final period problem and the integer number of firms makes the model suitable for addressing traditional industrial organization questions concerning imperfect competition. The integer constraint—and the resulting inability to satisfy equilibrium zero-profit conditions with equality—has economically relevant implications for the behavior of firms over time. In what follows, we apply special cases of the model to some traditional questions.

Section 2 contains an informal description of the model that is formally described by Amir and Lambson (2003).

Section 3 questions the implicit assumption underlying much of the traditional industrial organization literature that high sunk costs result in high concentration, that is, that high sunk costs reduce the number of firms that are active in an industry. This belief is an

[1]Structure includes the number of firms, the degree of product differentiation, and the height of entry barriers. Conduct refers to pricing and production policies. Performance comprises the normative properties of the conduct. See Scherer and Ross (1990 p. 4).

example of how static thinking can mislead. In a dynamic environment higher sunk costs reduce both entry *and* exit. Thus, although high sunk costs mean that fewer firms enter in good times they also mean that more firms remain active in bad times; hence the effect on the average number of firms over time is ambiguous.

Section 4 considers the question of why new firms often lose money before eventually becoming profitable. This can be explained in a variety of ways that depend on imperfect information. (For example, imperfect capital markets may cause delays in achieving minimum efficient scale or learning-by-doing may be required before a firm's technology becomes profitable.) Our model suggests another possibility that depends critically on the requirement that the number of firms be an integer. The implicit lumpiness of the technology means that the zero-profit entry condition is not exactly satisfied except by chance; thus the expected present value of an entrant typically exceeds its entry cost. In equilibrium, the lure of these economic profits can induce firms to enter sooner than they would like—for example, before growing demand is sufficiently large to allow positive operating profits—in order to hold the place.

Section 5 analyzes the entry decision. When a firm considers whether to enter, the number of periods over which it can recoup its sunk costs matters. In particular, it seems plausible that more firms would enter (or fewer firms exit) given a longer time horizon. Although this is true when firms are infinitessimal price-takers—see Lambson (1992)—it is not true here. Thus, for example, lengthening the product cycle may not result in an increase in the number of initial entrants. Indeed, entry (or exit) in a given period can change non-monotonically as the horizon lengthens.

Section 6 concludes.

2 The Model: An Informal Description

Those who are interested in the technical details of the model are referred to Amir and Lambson (2003). Here an abbreviated, informal description, sufficient for an intuitive understanding of the subsequent examples, is presented.

Consider countably infinitely many identical firms with a countably infinite time horizon.

A *market condition* is a description of all relevant exogenous variables in a given time period. These may include such things as factor prices, demand, entry costs, scrap values, the regulatory environment, and so on. Market conditions follow an exogenous first-order Markov process known to the firms. Finitely many (perhaps zero) firms are initially *active*, that is, capable of producing without paying an entry cost. The other firms are initially *inactive*. At the beginning of each period, firms observe the current market condition, say m, and then simultaneously decide whether to be active. A strategy for a firm dictates whether to be active in the current period as a function of the current market condition and which firms were active in the previous period. A firm's payoff is the present expected value of paying ξ_m each time it changes from being inactive to active (enters), receiving $\chi_m < \xi_m$ each time it changes from being active to inactive (exits), and earning $\pi_m(y)$ in periods that it is one of y active firms facing the market condition m. The function π_m is a reduced-form profit function; it is assumed to be non-increasing in y.[2] Firms discount their profit streams with the discount factor $\delta \in (0, 1)$.

Define subgame perfect Nash equilibrium in the obvious way. A subgame perfect Nash equilibrium can be characterized by a list of integer pairs, $\{(N_m, X_m)\}$, one for each market condition. If there were y active firms in the previous period and if the market condition is m in the current period then the number of active firms in the current period, y', is

$$y' = \min\{X_m, \max\{N_m, y\}\}.$$

In words, if there are fewer than N_m firms coming into the period then there will be entry up to N_m firms, if there are more than X_m firms coming into the period then there will be exit down to X_m firms, and otherwise there will be no change in the number of firms. Mathematically, N_m and X_m are, respectively, the largest integers that result in an expected present value weakly exceeding entry costs and scrap values, respectively, for the marginal firm given subsequent equilibrium behavior.

Amir and Lambson (2003) established more general results by allowing market conditions to be driven by very general—not necessarily Markov—stochastic processes. Truncated ver-

[2] General conditions for $\pi_m(y)$ to be nonincreasing in Cournot models are derived in Amir and Lambson (2000).

sions of the model were solved using backward induction in the sense that equilibrium integer pairs were constructed for each finite history. Limiting arguments established equilibrium integer pairs in the infinite horizon model. The additional structure imposed here allows a rather straightforward (if sometimes tedious) algebraic derivation of equilibrium integer pairs and a concrete characterization of the stochastic equilibrium path. These derivations are relegated to the appendix.

3 Concentration and Entry Barriers

It is natural—but, as this section demonstrates, incorrect—to believe that higher sunk costs necessarily decrease the number of firms in an industry. Put another way, it is natural to believe that higher sunk costs increase an industry's concentration, typically defined as the combined market share of the (usually four) largest firms. Partly because of this belief, much effort has gone into debating the empirical importance of entry barriers such as sunk costs.[3] Little of this literature has challenged the assertion that entry barriers and concentration are positively correlated.[4] This section challenges that assertion, showing by example that it depends critically on static reasoning. In a dynamic context, higher entry barriers indeed reduce the number of entrants when times are good but they also reduce the number of firms that exit when times are bad. The net effect on the average number of firms over time is ambiguous.

Suppose there are two market conditions: market condition H exhibits high demand and market condition L exhibits low demand. Market conditions follow the i.i.d. stochastic process characterized by the probabilities ρ_H and ρ_L of the respective market conditions. In market condition $m \in \{H, L\}$ inverse demand is $P = a_m - Q$. Production is costless, so

[3] A lot of this debate has centered around a perceived positive correlation between concentration and profit rates across industries. See, for example, Weiss (1974), Demsetz (1974), Dewey (1976), La Manna (1986) and Lambson (1987).

[4] An exception is Bernheim (1984) who, in a finite model of sequential entry, made the point that when future entry deterrence is more costly, current potential entrants may be less likely to enter. Bentolila and Bertola (1990) and Lambson (1992) point out that in dynamic competitive models higher sunk costs reduce both entry in good times and exit in bad times and that either effect may dominate on average.

Cournot operating profit is $\pi_m(y) = \left(\frac{a_m}{y+1}\right)^2$. Assume that both market conditions exhibit the same entry cost ξ and the same scrap value χ. The discount factor is $\delta \in (0, 1)$.

Let $\{N_L, X_L\}$ and $\{N_H, X_H\}$ be the ordered integer pairs associated with the respective market conditions. Hysteresis effects (see Dixit (1989)) imply $N_L \leq X_L$ and $N_H \leq X_H$. Naturally, high demand conditions support more firms than low demand market conditions, so $N_L \leq N_H$ and $X_L \leq X_H$. These restrictions together imply that either (1) $N_L \leq N_H \leq X_L \leq X_H$ or (2) $N_L \leq X_L < N_H \leq X_H$. In both cases N_L and X_H are observed only finitely many times (with probability one) and do not affect long run averages. So the relevant comparison is between N_H and X_L.

If $N_H \leq X_L$ then after finite time (with probability one) there will be no further entry or exit. The number of active firms depends on initial conditions. If initially there are fewer than N_H firms then the first time market condition H is realized there is entry up to N_H. Similarly, if initially there are more than X_L firms then the first time market condition L is realized there is exit down to X_L. Finally, if initially there are between N_H and X_L firms (inclusive) then there is never any entry or exit.

If $X_L < N_H$ then the long run average number of firms does not depend on initial conditions. There is entry up to N_H firms when demand rises and exit down to X_L firms when demand falls.

For the purposes of the example, impose the following values:

$$a_H = 2, a_L = 1, \rho_H = \rho_L = .5, \delta = .9, \ \chi = 1.35.$$

To calculate N_H, let $v_H(y)$ be the value of an active firm (gross of the entry cost) when demand is high. Now $v_H(y) = \max\{v_H^s(y), v_H^x(y)\}$, where $v_H^s(y)$ is the firm's value if it (and the other active firms) stay active when demand falls and $v_H^x(y)$ is the firm's value if it intends to exit when demand falls. The following table is derived in the appendix:

Table 3.1:

y	$v_H^s(y)$	$v_H^x(y)$	$v_H(y)$
3	1.65625	$1.55\overline{90}$	1.65625
4	1.06	$1.39\overline{54}$	$1.39\overline{54}$
5	$0.736\overline{11}$	$1.30\overline{65}$	$1.30\overline{65}$

As reflected in the last column, if $\xi \in (1.39\overline{54}, 1.65625]$ then $v_H(3) > \xi > v_H(4)$ so $N_H = 3$. If $\xi \in (1.30\overline{65}, 1.39\overline{54}]$ then $v_H(4) > \xi > v_H(5)$ so $N_H = 4$.

To calculate X_L, let $v_L^n(y)$ be the value of an active firm in the low-demand market condition given that it will be one of $\max\{y, n\}$ active firms in the high market condition. The following table is derived in the appendix:

Table 3.2:

y	$v_L^3(y)$	$v_L^4(y)$
1	2.50	2.095
2	$1.736\overline{11}$	$1.33\overline{11}$
3	1.46875	1.06375
4	0.94	0.94

If $\xi \in (1.39\overline{54}, 1.65625]$ recall that Table 3.1 implies $N_H = 3$ and note that Table 3.2 implies

$$v_L^3(3) = 1.46875 > 1.35 = \chi > 0.94 = v_L^3(4).$$

So $X_L = 3$. Now $N_L = X_L = 3$ implies that the number of firms is eventually constant at 3.

By contrast, if $\xi \in (1.30\overline{65}, 1.39\overline{54}]$ recall that Table 3.1 implies $N_H = 4$ and note that Table 3.2 implies

$$v_L^4(1) = 2.095 > 1.35 = \chi > 1.33\overline{11} = v_L^4(2).$$

So $X_L = 1$. Now $N_H = 4$ and $X_L = 1$ together imply that high demand periods exhibit 4 firms and low demand periods exhibit 1 firm. Since the two demand conditions are equally likely, the long run average number of firms is 2.5, compared to 3 firms when $\xi \in (1.39\overline{54}, 1.65625]$.

This example establishes that the long run average number of firms may be lower with lower entry barriers, contrary to the conventional wisdom. Intuitively, although lower entry costs induce more entry when times are good, they also induce more exit when times are bad; hence the effect on the long-run average number of active firms is ambiguous.

4 Preemptive Entry

Firms entering an industry often lose money for some time before becoming profitable. This can be explained in a variety of ways that depend on imperfect information. (For example, imperfect capital markets may cause delays in achieving minimum efficient scale or learning-by-doing may be required before a firm's technology becomes profitable.) This section contains another possibility that depends critically on lumpy sunk costs, that is, on the requirement that the number of firms be an integer. The lumpiness of the technology means that, even allowing for entry, economic profits can be made; if the constraint that the number of firms must be an integer is binding—as will be the case except by chance—then the expected present value of entrants exceeds their entry costs. In equilibrium, the lure of these economic profits can induce firms to enter sooner than they would like in order to hold the place.[5]

Suppose there are three market conditions: H, M, and L (meant to suggest high, medium, and low demand, respectively). Market conditions follow an i.i.d. stochastic process characterized by the probabilities ρ_H, ρ_M, and ρ_L, respectively. In market condition $m \in \{H, M, L\}$ inverse demand is $P = a_m - Q$. Production requires a fixed cost of ϕ, but no marginal cost, so Cournot operating profit is $\pi_m(y) = \left(\frac{a_m}{y+1}\right)^2 - \phi$. Assume that all market conditions have the same entry cost ξ and the same scrap value χ. The discount factor is $\delta \in (0, 1)$.

To construct the example, impose the following values:

$$\rho_H = .6, \rho_M = .3, \rho_L = .1, a_H = 15, a_M = 8, a_L = 3, \xi = 1, \chi = 0, \phi = 2, \delta = .9$$

Under these assumptions, equilibrium satisfies $N_L \leq X_L \leq N_M \leq X_M \leq N_H \leq X_H$. The extreme values N_L and X_H are observed only finitely many times. The other values are realized infinitely often. These turn out to be $X_L = 2$, $N_M = 5$, $X_M = 6$, and $N_H = 8$. (See the appendix.) Thus, after finitely many periods, equilibrium exhibits $N_H = 8$ firms in market condition H, $X_M = 6$ firms in market condition M if the previous distinct market condition was H, $N_M = 5$ firms in market condition M if the previous distinct market

[5] This explanation is related to the rent dissipation literature. See, for example, Fudenberg and Tirole (1985, 1987).

condition was L, and $X_L = 2$ in market condition L. The history-dependence of the number of active firms in market condition M reflects *hysteresis* caused by the sunk costs.

To see that this example exhibits entry with negative current profits infinitely often, note that $\pi_M(5) = -\frac{2}{9}$; thus whenever the market condition changes from L to M, three firms enter even though current profits are negative. They do so hoping to earn $\pi_H(8) = \frac{7}{9}$ often enough in the future to cover the entry cost before they are forced to exit when the market condition returns to L.

In this example, equilibrium is clearly inefficient, even subject to the constraint that active firms must behave as Cournot (not perfect) competitors. To see this, consider what happens when the market condition changes from L to M. With five active firms (as equilibrium requires), standard calculations establish that current consumer surplus is approximately 22.22 while current industry profit is approximately -1.11; thus current social surplus is approximately 21.11. If, by contrast, only four firms were active, current consumer surplus would be 20.48 while current industry profit would be 2.24, resulting in a current social surplus of 22.72. It follows that a social planner could increase the expected present value of social surplus relative to the equilibrium level by making the single change of allowing only four active firms (rather than five) when the market condition M follows the market condition L. Not only would it increase current social surplus in those cases, it would also save on entry costs. Entry is excessive when demand increases from low demand to medium demand.

Note that the fifth firm would have an incentive to delay entry until H is realized, if it could do so without fear of another firm taking its place; by doing so it could avoid some losses while delaying payment of the entry cost. Equilibrium does not allow such delay, however, because of the incentive for other firms to enter instead: firms must enter to hold the place before it is socially optimal to do so.

The insight that unregulated entry can be excessive entry when competition is imperfect is not new. It was formally established in a two-stage model by Mankiw and Whinston (1986). The result was generalized by Amir and Lambson (2003).

5 Entry, Exit, and Horizon Length

A longer time horizon gives entering firms (or, similarly, firms that refrain from exiting) more time to recoup entry costs (or, similarly, foregone scrap values); this effect tends to increase the current number of firms. However, a longer time horizon may also increase the future number of active firms, potentially making it more difficult to recoup entry costs or foregone scrap values; this effect tends to reduce the current number of active firms.

In dynamic models with infinitesimal price-taking firms there is no ambiguity: the first effect dominates. The intuition is not hard to grasp. In continuum models zero-profit conditions can be exactly satisfied, so a firm's equilibrium present value is maximized (and equal to the entry cost) when there is entry. Hence future increased entry due to a longer horizon implies that future profit streams are maximally attractive, and thus cannot reduce the attractiveness of current activity. It follows that the number of firms in any given period is (weakly) increasing in the time horizon. Lambson (1992) used this monotonicity to show that an infinite-horizon equilibrium can be constructed as the limit of finite-horizon equilibria when there is a continuum of infinitesimal price-taking firms.

When the number of firms must be an integer, by contrast, either effect may dominate, because zero-profit conditions need not be satisfied with equality. Since entry doesn't imply maximal present values, future increased entry due to a longer horizon may (but need not) reduce the attractiveness of future profit streams, make current activity less attractive, and thus reduce the current number of firms. This ambiguity suggests that the number of active firms in a given time period need not be monotonic in the horizon. The remainder of this section contains an example of the non-monotonicity phenomenon. Thus monotonicity arguments that are useful for proving existence in the analogous model with a continuum of firms fail in the present circumstance.[6]

Identify market conditions with periods, that is, say that market condition t occurs in period t with probability one. Suppose inverse demand in period t is $P_t = a_t - Q_t$ and that there are no production costs, so Cournot operating profit in period t is $\left(\frac{a_t}{y+1}\right)^2$. Assume

[6] This difficulty was noted without example by Amir and Lambson (2003) where an existence proof is presented that does not depend on monotonicity.

there is no scrap value—that is, $\chi_t = 0$ for all t—and let the entry cost be $\xi_t = 1$ in every period. The discount factor is $\delta = \frac{1}{2}$. Suppose the a_t values are given by $a_1 = 1$, $a_2 = 4 - \varepsilon$, and $a_t = 6$ for $t \geq 3$. Let $\left\{N_t^T, X_t^T\right\}$ be the equilibrium integer pair in period t if the horizon length is $T \geq t$. Now $X_t^T = \infty$ for all T and all $t \leq T$—that is, there is never any exit—because there is no scrap value and Cournot profits are always strictly positive.

First consider the truncated model where firms expect the market to last for one period. Then N_1^1 is the largest integer satisfying $\left(\frac{1}{N_1^1+1}\right)^2 \geq 1$, that is,

$$N_1^1 = 0.$$

In the one-period truncation there is no entry if there are initially no firms.

Next consider the two-period truncation, that is, let $T = 2$. Applying backward induction, N_2^2 is the largest integer satisfying $\left(\frac{4-\varepsilon}{N_2^2+1}\right)^2 \geq 1$. Thus for small $\varepsilon > 0$, $N_2^2 = 2$. Then N_1^2 is the largest integer satisfying $\left(\frac{1}{N_1^2+1}\right)^2 + \frac{1}{2}\left(\frac{4-\varepsilon}{\max(N_1^2,2)+1}\right)^2 \geq 1$. So for small $\varepsilon > 0$, $N_1^2 = 1$. In summary,

$$N_1^2 = 1;\ N_2^2 = 2.$$

In the two-period truncation one firm enters in period 1 and then an additional firm enters in period 2.

Finally, consider the three-period truncation, that is, let $T = 3$. Apply backward induction. Assuming (as turns out to be the case) that there is entry in each period, N_3^3 is the largest integer satisfying $\left(\frac{6}{N_3^3+1}\right)^2 \geq 1$. Thus $N_3^3 = 5$. Next, N_2^3 is the largest integer satisfying $\left(\frac{4-\varepsilon}{N_2^3+1}\right)^2 + \frac{1}{2}\left(\frac{6}{\max(N_2^3,5)+1}\right)^2 \geq 1$. Thus, for small $\varepsilon > 0$, $N_2^3 = 4$. Finally, N_1^3 is the largest integer satisfying $\left(\frac{1}{N_1^3+1}\right)^2 + \frac{1}{2}\left(\frac{4-\varepsilon}{\max(N_1^3,4)+1}\right)^2 + \frac{1}{4}\left(\frac{6}{\max(N_1^3,5)+1}\right)^2 \geq 1$. Thus $N_1^3 = 0$. In summary,

$$N_1^3 = 0;\ N_2^3 = 4;\ N_3^3 = 5.$$

In the three-period truncation, no firms enter in period 1, four firms enter in period 2, and an additional firm enters in period 3.

Notice the non-monotonicity of N_1^T: $N_1^1 = 0$, $N_1^2 = 1$, and $N_1^3 = 0$. The intuition for why this arises is as follows. In the one-period truncation no firm enters; demand doesn't allow even one firm to recoup the entry cost so quickly. In the two-period truncation, demand is higher in the second period, thus allowing two firms each to recoup much more than the entry cost by producing only in period 2. These super-normal profits occur because of the integer constraint: if a third firm entered it couldn't recoup its entry cost in period 2 alone. The lure of super-normal profits in period 2 (even discounted) suffices to lure one firm to enter in period 1. In the three-period truncation there is higher demand still in period 3, sufficient for five firms to more than recoup their entry costs just in period 3. Once again, the lure of super-normal profits in period 3 induces additional entry in period 2, but the example is constructed so that the super-normal profits are dissipated in period 2: the integer constraint does not bind very tightly. Thus there are no second-period super-normal profits to lure entrants in period 1.

6 Concluding Remarks

This paper has applied a general, stochastic, infinite-horizon model of entry and exit with imperfect competition. It should be applicable to many other traditional industrial organization questions, incorporating as it does an explicitly dynamic framework and requiring an integer number of firms. Some examples of applications were presented in the paper, where it was shown that high sunk costs can actually reduce average industry concentration over time, that the lumpiness of the technology that results in imperfect competition can explain entry when current profits are negative, and that the amount of entry may not be positively correlated with the length of the product cycle.

Appendix

Section 3 Derivations

It follows from standard recursive methods that if there are always y_H firms active in high demand conditions and y_L firms active in low demand conditions, then the value of an active firm that never exits, denoted $w_H(y_H, y_L)$ when demand is high and $w_L(y_H, y_L)$ when demand is low, satisfies

$$w_H(y_H, y_L) = \pi_H(y_H) + \delta[\rho_H w_H(y_H, y_L) + \rho_L w_L(y_H, y_L)]$$

and

$$w_L(y_H, y_L) = \pi_L(y_L) + \delta[\rho_H w_H(y_H, y_L) + \rho_L w_L(y_H, y_L)].$$

Solving these two equations for w_H and w_L yields

$$w_H(y_H, y_L) = \frac{(1-\delta\rho_L)\pi_H(y_H) + (\delta\rho_L)\pi_L(y_L)}{(1-\delta)}$$

and

$$w_L(y_H, y_L) = \frac{(1-\delta\rho_H)\pi_L(y_L) + (\delta\rho_H)\pi_H(y_H)}{(1-\delta)}.$$

Then

$$v_H^s(y) = w_H(y, y)$$

Substituting the imposed values generates the column of Table 3.1 labelled $v_H^s(y)$. A firm that is active when demand is high but intends to exit when demand is low has value $v_H^x(y)$ satisfying $v_H^x(y) = \pi_H(y) + \delta[\rho_H v_H^x(y) + (1-\rho_H)\chi]$. Therefore,

$$\nu_H^x(y) = \frac{\pi_H(y) + \delta\rho_L\chi}{1-\delta\rho_H}.$$

Substituting the imposed parameter values into the previous two equations generates Table 3.1. Finally, note that

$$v_L^n(y) = w_L(\max\{y, n\}, y\}.$$

Substituting the imposed parameter values generates Table 3.2.

Section 4 Derivations

There are four kinds of market condition changes to consider: changes to H (from either M or L), changes to M from H, changes to M from L, and changes to L (from either H or M). The derivations of the first two changes will be sketched. The others are similar and are omitted. In what follows, ν_{mn} is the expected present value of an active firm in market condition m that intends to exit the next time the market condition exhibits lower demand than market condition n.

When the market condition changes to H, if there are eight firms, then the expected present value of a firm that intends to exit when demand falls satisfies $\nu_{HH} = \pi_H(8) + \delta\rho_H\nu_{HH}$; so $\nu_{HH} = \pi_H(8)/(1-\delta\rho_H) \simeq 1.69 > \xi$. If, however, that marginal firm were one of nine, its expected present value would be only $\pi_H(9)/(1-\delta\rho_H) \simeq .54 < \xi$. Thus $N_H = 8$.

When the market condition changes from H to M, provoking exit, a remaining firm that intends to exit when L is next realized (and understanding that there will be entry up to eight firms if H is realized), has an expected present value v_{MM} satisfying $\nu_{MM} = \pi_M(6)+\delta[\rho_M\nu_{MM}+\rho_H\nu_{HM}]$ where ν_{HM} satisfies $\nu_{HM} = \pi_H(8)+\delta[\rho_M\nu_{MM}+\rho_H\nu_{HM}]$. Solve these two equations to write $\nu_{MM} = [(1-\delta\rho_H)\pi_M(6)+\delta\rho_H\pi_H(8)]/(1-\delta\rho_M-\delta\rho_H) \simeq .53 > \chi$. By contrast, if a seventh firm remained active with the intention of exiting the first time market condition L is realized, its expected present value would only be $\nu_{MM} = [(1-\delta\rho_H)\pi_M(7) + \delta\rho_H\pi_H(8)]/(1-\delta\rho_M-\delta\rho_H) \simeq -.21 < \chi$. Thus $X_M = 6$.

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