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ON THE EFFECTS OF PRIVATE CAPITAL FALLING INTO THE PUBLIC DOMAIN

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Abstract. The fact that some private capital eventually slides into the public domain (e.g. taxes on household savings and income channeled to public infrastructures, or R+D investments as patents expire) inefficiently distorts downwards the capital accumulation. This is established for both infinitely-lived agents and overlapping generations setups. I provide next a tax and transfers balanced policy able to decentralize the planner's steady state without resorting to the (impracticable) extension of property rights otherwise needed to address the problem. It consists of (i) subsidizing the rental rate of capital by an amount equal to the depreciation/obsolescence rate of the capital sliding into the public domain, and (ii) taxing households debt issued against future dividends.

1. Introduction

Each generation passes on subsequent generations the results of its achievements, both physical (properties, infrastructures, facilities, ...) and intangible (technology, institutions, organizations, culture, ...),[1] the creation of all of them having required

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[1]Some of them actually fall into both categories like, for instance, cities, metropolitan areas or regions, with their combined nature of public infrastructures and organizations.

at any rate the use of available resources, and hence being capital in a broad sense. Some of this capital is passed on through the trade or bequest of individual property rights (this is notably the case for properties and production facilities, but also for intellectual property rights to the extent they have not expired yet), while some of it eventually just slides into the public domain (certainly organizational schemes and expired intellectual property rights,[2] but also any physical capital infrastructures publicly built out of taxed privately held capital,...).[3] This paper explores the consequences of at least some capital eventually escaping individual property rights.

The model used in this paper to address this issue is deliberately simple, in order to make stand out clearly the mechanism by which the progressive drift of proprietary investments into the public domain necessarily distorts capital accumulation away from the optimal level chosen by a utilitarian planner being able to see the big picture. More specifically, the model is on purpose as aggregate as possible to see *analytically* the essentials of why the gradual drift of capital into the public domain prevents the markets to deliver, under laissez-faire, the optimal level of capital accumulation. On the other hand, as a result of its stripped down character, the model does not seem apt yet for replicating any empirical evidence. That would require a more disaggregated model and is left for later work. The goal, for the moment, is here to highlight the essentials of the inefficiency associated with the drift of capital from private investments into the public domain.

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[2]Property rights over the technologies resulting from R+D investments are temporarily protected by law to allow the investor to get a return from the investment and, hence, supposedly to incentivize growth-enhancing R+D activities. There is currently a heated debate about whether the current patent system implementing those property rights is actually the most adequate for spurring innovation. For instance, Boldrin and Levine (2013) point that the evidence shows no correlation between the number of patents and productivity, and highlight that the rent-seeking nature of *patenting aims rather at preventing further innovation* from competitors, which typically builds on previous innovations. On the other hand, Gould and Gruben (1996) find that intellectual property protection is an important determinant of growth, although this seems to hinge on the openness of the country to international competition, without which it can be detrimental to growth. Also, in a Romer-style endogenous growth model Saint-Paul (2003) argues the crowding out effects of free blueprints on proprietary innovation and, hence, its negative impact on growth and welfare. At any rate, besides the issue of what drives innovation and what incentives are at play, there is the fact that technology is the result of investment, and is hence capital, but one whose property rights are protected only temporarily and thus eventually falls into the public domain.

[3]Strictly speaking, some commonly held physical capital is actually subject to property rights of state institutions at different administrative levels —the *res publicae* and *res universitatis* of Roman law. Notwithstanding, for all purposes, I will consider it to be freely available in the public domain (at least for residents or citizens depending on the case at hand), since the relevant feature characterizing it is the fact of not being subject to *individual* property rights.

The mechanism behind the result is simple enough. Some of the savings lent to firms as capital by households are used to create ways to increase productivity that will eventually become public domain[4] —e.g. taxes on savings or income raised to finance public infrastructures (or even the intangible organizational and legal framework in which the economy operates) and R+D private investments, given the time limits to patents and copyrights, but these are far from being the only examples.[5] As a result of this, firms effectively operate using not only the capital they borrow but also the capital coming from prior private investments that has fallen into the public domain. Since only capital on which property rights can be enforced is remunerated,[6] savers do not take into account the impact that their loans to firms have on the capital in the public domain, and hence on the productivity of factors and on output through this channel.

In a nutshell, by sliding into the public domain, the productivity of the capital doing so is not remunerated to its original investors, but is rather fed into the profits of the firm operating with it for free. Even in an aggregate model, where the representative household is both the lender to the firm —so that he receives the returns to privately held capital— and the owner of the firm —so that he receives the distributed profits too— and therefore receives the entirety of the productivity of the capital used by the firm, whether privately or publicly held, the channel through which this productivity is received matters for the saving decision of the agent. Namely, the productivity of capital in the public domain does not incentivize savings, while that of capital privately held does. This differentiated impact would be even more obvious with heterogeneous agents of which some are lenders and others owners, or all are both but to different extents.

As a consequence, private investments differ from those that a planner able to

[4]Some savings are even deliberately privately invested in capital intended, from the start, to be in the public domain, as it is the case for open source and shareware software. This paper is nonetheless not about such instances of capital deliberately accumulated to be freely available, but rather on the consequences of proprietary capital eventually sliding into the public domain.

[5]A related problem is that of firms' investment in the human capital of their employees. Such investments can be substantial, but they stop being "proprietary" for the firm if the employees quit for another job (since embodied in them). In a sense, such human capital investments by firms, while being proprietary to the employees, have the potentiality of becoming a *de facto* (excludable) public good provided by each firm to the industry.

[6]This is not to say that the productivity of public domain capital is not appropriated by anyone. It is indeed: it accrues firms' profits and eventually feeds into firms' owners wealth as distributed dividends. In effect, with proprietary capital eventually sliding into the public domain even constant returns to scale firms make profits. Nonetheless, even if these profits are distributed to the firms' owners, the latter will still fail to internalize the effect in their saving decision.

take into account the effect of public domain capital would choose. Specifically, in the case of infinitely-lived agents it can be explicitly shown that the market accumulates too little capital. Interestingly enough, in the overlapping generations case this is shown indirectly through the subsidy to capital returns required by the policy decentralizing the planner's allocation. This may underline the different nature of the "sequential" externality at play here (from earlier selves) compared to the "simultaneous" externality exerted by the capital of a firm on the productivity of other firms à la Romer (1986) at any given point in time, see below for details.[7]

In order to address the problem, I provide a policy allowing to steer decentralized choices towards the planner's allocation of choice. Since at the heart of the problem there lays an expiration of property rights, it might seem that a simple all-encompassing extension of property rights would be enough. Nonetheless, since this is clearly impracticable —some of this capital cannot be appropriated (institutions, organizations,...) or is not advisable to be so (because, for instance, of the perverse incentives on innovation of extending indefinitely intellectual property rights pointed by the literature, see Boldrin and Levine (2013))[8]— it is important to devise an implementable policy that avoids running into generating additional inefficiencies. The policy put forward in this paper requires instead (i) to subsidize the rental rate of capital by an amount equal to the depreciation/obsolescence rate of the capital sliding into the public domain, (ii) to tax households debt issued against the future dividends. While the first element of this policy —i.e. the subsidization of capital returns— may be expected (although probably not the exact rate at which this needs to be done), its second element —i.e. taxing households' reliance on credit against future dividends distributed by firms— only makes full sense once it is understood the differentiated impact on the incentives to save of the return to privately held capital and the dividends received from the unremunerated capital in the public domain

In what follows, Section 2 presents the model with two variants for its demographics (infinitely-lived agents and overlapping generations respectively). Section 3, addressing the issue first in the infinitely-lived agents economy, establishes that the

[7]Moreover, overall increasing returns to scale drive the externality in Romer (1986) while the externality analyzed here exists even if returns to scale are constant.

[8]Many considerations about the public interest for patents to expire, to be entirely phased out, or even to invest in freely available technology —for instance, developing open source software as means to invest in network building— are not addressed in this paper (for the case against patents see Boldrin and Levine (2013); for an overview of the economics of open source see Lerner, Josh and Tirole (2005)). While extremely interesting, they address a different point from the one being made here.

market necessarily under-accumulates capital due to the public domain problem. Section 4 addresses then the question for an overlapping generations setup, which provides additional insight on the way the externaliy operates and allows to provide a policy decentralising the planner's choice as a market outcome. This is done through a subsidy on capital returns, and a tax on debt issued against future profits.

2. The model

Consider an economy in which part of savings or capital becomes, after some period of time, non-proprietary and falls into the public domain (e.g. because of being taxed and used to fund public infrastructure and the institutional and legal framework, or the part of investments devoted to R+D that leads to proprietary technology for a limited amount of time only). Specifically, let N_t be the (possibly constant) population at t, and let k_t be the amount of savings lent to firms by the representative household at t, so that the total investment at t used in production at $t+1$ is $N_t k_t$. Without loss of generality, assume that a fraction α of investments becomes public domain after one period, the complement fraction $1-\alpha$ remains proprietary and depreciates at some rate δ. Public domain capital, on the other hand, depreciates or obsolesces at a rate ϕ, so that only a fraction ϕ of it is productive after each period.[9]

Total capital available at any period t is therefore

$$K_t = N_{t-1}k_{t-1} + (1-\alpha)\sum_{i=2}^{+\infty}\delta^{i-1}N_{t-i}k_{t-i} + \alpha\sum_{i=2}^{+\infty}\phi^{i-1}N_{t-i}k_{t-i}$$

Notwithstanding, firms need to remunerate only

(1) the investment $N_{t-1}k_{t-1}$from savings made at $t-1$
(2) and proprietary older capital $(1-\alpha)\delta^{i-1}N_{t-i}k_{t-i}$, for $i = 2, 3, \ldots$ resulting from previous investments

that is to say

$$N_{t-1}k_{t-1} + (1-\alpha)\sum_{i=2}^{+\infty}\delta^{i-1}N_{t-i}k_{t-i}$$

[9]Note that the depreciation rate of proprietary capital δ and public domain capital ϕ need not be the same.

but, crucially, not capital resulting from prior investments that has already slid into the public domain, i.e.

$$\alpha \sum_{i=2}^{+\infty} \phi^{i-1} N_{t-i} k_{t-i}.$$

For the sake of simplifying expressions (and without loss of generality), I will consider below —with no consequence for the main point of the paper— the extreme case in which $\alpha = 1$, so that capital at t

$$K_t = N_{t-1} k_{t-1} + \sum_{i=2}^{+\infty} \phi^{i-1} N_{t-i} k_{t-i}$$

consist of capital that becomes public domain after one period (i.e. the sum in the second term in the right-hand side above).

The production function $F(K_t, N_t)$ is neoclassical, i.e. returns to scale are constant in labor and available capital (proprietary and in public domain), as opposed to the overall increasing returns to scale that are the keystone of the setup considered in Romer (1986).

Finally, as for the demographics, I will consider next both the infinitely-lived agents and the overlapping generations setup. Specifically, a normalized unit of labor is supplied inelastically by each household

(1) when young, if the economy consists of 2-period-lived overlapping generations, and population grows each period by a constant factor n, so that, for all $i = 1, 2, \ldots$

$$N_t = n^i N_{t-i}$$

(2) each period, if agents are infinitely-lived, with a constant population (i.e. $n = 1$) normalized to 1, so that $N_t = 1$, for all t.

3. Market underaccumulation: the infinitely-lived agents economy case

3.1 The planner's allocation for the infinitely-lived agents economy.

A planner is not constrained by property rights, but rather aims at maximizing the welfare of the representative household from a sequence of consumptions c_t while satisfying, at each period t, the feasibility constraint, i.e.

$$\max_{c_t, k_t} \sum_{t=1}^{+\infty} \beta^{t-1} u(c_t)$$
$$c_t + k_t \leq F(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t-i}, 1)$$

where, in each constraint, i.e. for all $t = 1, 2, \ldots$, trivially $k_{t-i} = 0$ for $i > t$, given some initial endowment $k_0 > 0$.

The planner's choice, therefore, must necessarily satisfy, for each $t = 1, 2, \ldots$, and some positive $\lambda_t, \lambda_{t+1}, \ldots$

$$\begin{pmatrix} \beta^{t-1} u'(c_t) \\ 0 \end{pmatrix} = \lambda_t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda_{t+1} \begin{pmatrix} 0 \\ -F_K^{t+1} \end{pmatrix} + \lambda_{t+2} \begin{pmatrix} 0 \\ -F_K^{t+2} \cdot \phi \end{pmatrix} + \ldots$$

where F_K^{t+j} stands for the marginal productivity of capital at $t + j$

$$F_K^{t+j} = F_K(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t+j-i}, 1)$$

from which the next characterisation easily follows.

Proposition 1. *In an infinitely-lived agents economy, a planner's allocation* $\{c_t, k_t\}_{t \in \mathbb{N}}$ *is characterised by*

$$1 = \sum_{j=1}^{+\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} F_K(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t+j-i}, 1) \phi^{j-1}$$

and the feasibility constraint, for all $t = 1, 2, \ldots$, *given some initial endowment* $k_0 > 0$ *—and trivially* $k_{t-i} = 0$ *for* $i > t$.

We will now compare this necessary characterisation of the planner's choice with that of the market allocation next.

3.2 The market allocation for the infinitely-lived agents economy.

A household behaving competitively aims instead at maximising its utility under its budget constraint, as in

$$\max_{c_t, k_t} \sum_{t=1}^{+\infty} \beta^{t-1} u(c_t)$$
$$c_t + k_t \leq w_t + r_t k_{t-1} + \pi_t$$

given the profits π_t,[10] and the factor prices w_t and r_t, determined by their marginal productivities, i.e.

$$w_t = F_L(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t-i}, 1)$$
$$r_t = F_K(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t-i}, 1)$$

where, for all $t = 1, 2, \ldots$, trivially $k_{t-i} = 0$ for $i > t$, given some initial endowment $k_0 > 0$.

The household's choice therefore necessarily satisfies, for all t and some positive λ_t, λ_{t+1},

$$\begin{pmatrix} \beta^{t-1} u'(c_t) \\ 0 \end{pmatrix} = \lambda_t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda_{t+1} \begin{pmatrix} 0 \\ -F_K^{t+1} \end{pmatrix}$$

where F_K^{t+j} stands, as before, for the marginal productivity of capital at $t + j$, and from where the next characterisation easily follows.

Proposition 2. *In an infinitely-lived agents economy in which some private capital eventually falls into the public domain, a market allocation* $\{c_t, k_t\}_{t \in \mathbb{N}}$ *is characterised by*

$$1 = \beta \frac{u'(c_{t+1})}{u'(c_t)} F_K(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t+1-i}, 1)$$

[10] Each firm gets at t an equal share of the positive aggregate profits from the productivity of the capital already in the public domain, $\pi_t = F_K(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t-i}, 1) \sum_{i=2}^{+\infty} \phi^{i-1} k_{t-i}$. Free entry in the industry will drive *per firm* profits down to zero as an unbounded number of firms enter the market, but output and *aggregate* profits remain constant.

and the budget constraint,[11] *for all* $t = 1, 2, \dots,$ *given some initial endowment* $k_0 > 0$ *—and trivially* $k_{t-i} = 0$ *for* $i > t$.

The characterisations provided in Propositions 1 and 2 allow to compare the planner's and market steady state allocations next.

3.3. Planner's vs market steady states in the infinitely-lived agents economy.

When it comes to comparing the steady state allocations of the economy that the market and the planner would deliver, the previous characterisations point to a clear-cut result: the market accumulates less capital at the steady state than the planner, as the next proposition establishes.

Proposition 3. *In an infinitely-lived agents economy in which some private capital eventually falls into the public domain, the market steady state level of capital* $\bar{k}$ *is smaller than the planner's* k^*.

Proof. From the necessary characterisations of propositions 1 and 2 above

$$1 = \sum_{j=1}^{+\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} F_K(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t+j-i}, 1) \phi^{j-1}$$

$$1 = \beta \frac{u'(c_{t+1})}{u'(c_t)} F_K(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t+1-i}, 1)$$

it follows that each steady state —i.e. the planner's k^* and the market $\bar{k}$— is characterised respectively by

$$1 = \beta F_K(\frac{k^*}{1-\phi}, 1) \frac{1}{1-\beta\phi}$$
$$1 = \beta F_K(\frac{\bar{k}}{1-\phi}, 1)$$

[11] Which is equivalent to the feasibility constraint.

from where $k^* > \bar{k}$ follows straightforwardly —since $\beta, \phi \in (0,1)$.[12] $\square$

The impact, in the infinitely-lived agents economy, of private capital sliding into the public domain is therefore very clear: the market accumulates too little capital.

Addressing this same question, in the next section, in an overlapping generations setup instead, will allow us to get additional insight about how to offset within the market the externality from capital sliding into the public domain.

4. Undoing the public domain distortion: the overlapping generations economy case

Because of the need to distinguish variables relating different generations as well as time periods, in the overlapping generations setup, we will use superscript t to identify the t's generation choice variables like, among others, the intertemporal profile of consumption c_0^t, c_1^t, or the amount k^t lent to firms by generation t's representative household for production at $t+1$.

4.1 The planner's problem in the overlapping generations economy.

A utilitarian planner would in this case maximize a discounted sum of the utilities of all households under the feasibility constraint in each period (expressed below in per young terms)

$$\max_{c_0^t, c_1^t, k^t \geq 0} \sum_{t=1}^{+\infty} \eta^{t-1} \big(u(c_0^t) + \beta u(c_1^t) \big)$$

$$c_0^t + \frac{c_1^{t-1}}{n} + k^t \leq F\Big(\frac{1}{\phi} \sum_{i=1}^{+\infty} \Big(\frac{\phi}{n}\Big)^i k^{t-i}, 1\Big), \forall t = 1, 2, \ldots$$

[12] If not all capital slides into the public domain, i.e. $\alpha \in (0,1)$, then the respective steady states are characterised by

$$1 = \beta F_K\Big(\Big[\frac{1-\alpha}{1-\delta} + \frac{\alpha}{1-\phi}\Big]k^*, 1\Big)\Big[\frac{1-\alpha}{1-\beta\delta} + \frac{\alpha}{1-\beta\phi}\Big]$$
$$1 = \beta F_K\Big(\Big[\frac{1-\alpha}{1-\delta} + \frac{\alpha}{1-\phi}\Big]\bar{k}, 1\Big)$$

and the same conclusion follows.

given some initial c_1^0, k^0, a discount factor η for future generations, the households own discounting β of future utility, the population growth factor n, and the depreciation/obsolescence rate ϕ for capital in the public domain.

From the problem above follows the next characterisation of the planner's choice, linking the contribution of savings k^t at any given period t to the marginal productivity of capital at all future $t+j$, for all $j = 1, 2, \dots$ on the one hand, to the marginal rates of intertemporal substitution of consumption for all agents between t and each $t+j$ on the other hand.

Proposition 4. *In an overlapping generations economy, any allocation* $\{c_0^t, c_1^t, k^t\}_{t\in\mathbb{N}}$ *chosen by the planner satisfies*

$$1 = \frac{1}{\phi}\sum_{j=1}^{+\infty}\left[F_K\left(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+j-i}, 1\right)(\phi\beta)^j \prod_{h=0}^{j-1}\frac{u_1(c_0^{t+h}, c_1^{t+h})}{u_0(c_0^{t+h}, c_1^{t+h})}\right]$$

as well as each period feasibility constraint binding.

Proof. The solution to the planner's problem is necessarily characterised, for some $\lambda^{t+i} > 0$ with $i = 0, 1, 2, \dots,$ by

$$\begin{aligned}\begin{pmatrix}\eta^{t-1}u_0(c_0^t, c_1^t)\\ \eta^{t-1}u_1(c_0^t, c_1^t)\\ 0\end{pmatrix} = \lambda^t\begin{pmatrix}1\\0\\1\end{pmatrix} &+ \lambda^{t+1}\begin{pmatrix}0\\ \frac{1}{n}\\ -\frac{1}{\phi}F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1)\frac{\phi}{n}\end{pmatrix}\\ &+ \lambda^{t+2}\begin{pmatrix}0\\0\\ -\frac{1}{\phi}F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+2-i}, 1)(\frac{\phi}{n})^2\end{pmatrix}\\ &+ \dots\end{aligned}$$

for all $t = 1, 2, \dots,$ that is to say, by the following conditions on the marginal rates of substitution

(1) within generations

$$\frac{u_0(c_0^t, c_1^t)}{u_1(c_0^t, c_1^t)} = \frac{\lambda^t}{\lambda^{t+1}}n$$

(2) and across generations

$$\frac{u_0(c_0^t, c_1^t)}{u_0(c_0^{t+i}, c_1^{t+i})} = \eta^i\frac{\lambda^t}{\lambda^{t+i}}$$

and

$$\frac{u_0(c_0^t, c_1^t)}{u_1(c_0^{t+i}, c_1^{t+i})} = \eta^i \frac{\lambda^t}{\lambda^{t+i}} n$$

on top of

$$1 = \frac{1}{\phi} \sum_{j=1}^{+\infty} \left[F_K\left(\frac{1}{\phi} \sum_{i=1}^{+\infty} (\frac{\phi}{n})^i k^{t+j-i}, 1 \right) (\frac{\phi}{n})^j \frac{\lambda^{t+j}}{\lambda^t} \right]$$

and

$$c_0^t + \frac{c_1^{t-1}}{n} + k^t = F(\frac{1}{\phi} \sum_{i=1}^{+\infty} (\frac{\phi}{n})^i k^{t-i}, 1)$$

The necessary condition obtains then from direct substitutions. $\square$

From the previous characterisation can be obtained that of the unique symmetric allocation that a planner treating equally all generations would choose, that is to say the characterisation of the planner's steady state in Proposition 5 next.

Proposition 5. *In an overlapping generations economy, the planner's steady state is characterised by the unique c_0, c_1, k solving*

$$\begin{aligned} \frac{1}{\beta} \frac{u'(c_0)}{u'(c_1)} &= n = F_K(\frac{k}{n-\phi}, 1) + \phi \\ c_0 + \frac{c_1}{n} + k &= F(\frac{k}{n-\phi}, 1) \end{aligned} \qquad \text{(P)}$$

given n, ϕ.

Proof. We will see first that the system above characterizes any steady state, and then we will see that there is a solution to the system and only one.

From Proposition 4 and its proof, a symmetric allocation treating equally all generations —and hence such that $\lambda^t \rho = \lambda^{t+1}$ for all t— is necessarily characterised by

$$\frac{1}{\beta} \frac{u'(c_0)}{u'(c_1)} = n$$

—which implies $\phi\beta\frac{u'(c_1)}{u'(c_0)} < 1$ whenever $n > 1$,[13] so that the series next is convergent—and[14]

$$1 = \frac{1}{\phi} F_K\Big(\frac{k}{n-\phi}, 1\Big) \sum_{j=1}^{+\infty} \Big(\phi\beta\frac{u'(c_1)}{u'(c_0)}\Big)^j$$

i.e. —replacing the series by its value and rearranging terms—

$$\frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} = F_K\Big(\frac{k}{n-\phi}, 1\Big) + \phi$$

since $\phi\beta\frac{u'(c_1)}{u'(c_0)} < 1$.[15]

A planner steady state is clearly locally unique since it is a regular zero of the

[13]Indeed, equivalently $\beta\frac{u'(c_1)}{u'(c_0)} = \frac{1}{n} < 1$, since $n > 1$, from which $\phi\beta\frac{u'(c_1)}{u'(c_0)} < 1$ given that $\phi < 1$ too.

[14]Since $K_t = \sum_{i=1}^{+\infty} \phi^{i-1} k^{t-i} N_{t-i}$ and $N_t = n^i N_{t-i}$, then $\frac{K_t}{N_t} = \frac{1}{\phi}\sum_{i=1}^{+\infty}\left(\frac{\phi}{n}\right)^i k^{t-i}$ which at a steady state becomes $\frac{k}{n-\phi}$.

[15]As a matter of fact, obtaining this characterisation from the one in Proposition 4 hides an argument in the limit, since it requires equal weight for all generations in the planner's problem, i.e. $\eta = 1$, for which the planner's objective is not well defined. Equivalently, a planner weighting equally all generations would solve for the steady state the problem

$$\max_{c_0, c_1, k \geq 0} \; u(c_0) + \beta u(c_1)$$
$$c_0 + \frac{c_1}{n} + k \leq F\Big(\frac{k}{n-\phi}, 1\Big)$$

given ϕ, n. The solution c_0, c_1, k is necessarily characterised, as above, by

$$\begin{pmatrix} u'(c_0) \\ \beta u'(c_1) \\ 0 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ \frac{1}{n} \\ 1 - F_K(\frac{k}{n-\phi}, 1)\frac{1}{n-\phi} \end{pmatrix}$$

for some $\lambda > 0$, and

$$c_0 + \frac{c_1}{n} + k = F\Big(\frac{k}{n-\phi}, 1\Big).$$

The existence of such a solution is guaranteed by the usual differentiably strict quasi-concavity of u (this ensures the continuity of the planner's objective and the strict convexity of its upper contour sets) and the strict concavity of F with respect to capital (this makes the constrained set of the planner to be compact).

left-hand side of the planner's steady state equations

$$
\begin{aligned}
u'(c_0) - n\beta u'(c_1) &= 0 \\
F_k(\frac{k}{n-\phi}, 1) + \phi - n &= 0 \\
c_0 + \frac{c_1}{n} + k - F(\frac{k}{n-\phi}, 1) &= 0
\end{aligned}
$$

In effect,

$$
\begin{vmatrix}
u''(c_0) & -n\beta u''(c_1) & 0 \\
0 & 0 & F_{KK}(\frac{k}{n-\phi},1)\frac{1}{n-\phi} \\
1 & \frac{1}{n} & 1 - F_K(\frac{k}{n-\phi},1)\frac{1}{n-\phi}
\end{vmatrix} =
$$

$$
-F_{KK}(\frac{k}{n-\phi},1)\frac{1}{n-\phi}\left[n\beta u''(c_1) + \frac{1}{n}u''(c_0)\right] < 0.
$$

But it is globally unique too, since if c_0, c_1, k and c_0', c_1', k' were two distinct steady states for the planner, then necessarily $k = k'$ —since the (injective) marginal productivity of capital must match $n - \phi$ for both of them— and $c_0 < c_0'$ would imply $c_1 < c_1'$, which cannot be —since the per young aggregate consumption each period must match the common $F(\frac{k}{n-\phi}, 1) - k$. Therefore $c_0 = c_0'$ and from the feasibility constraint $c_1 = c_1'$ too. □

4.2 Households' problem in the overlapping generations economy.

The representative household born at t can transfer wealth *from its first period into the second* in three ways : (i) lending to firms to get at $t+1$ the rental rate of capital r_{t+1}, (ii) holding real monetary balances,[16] and (iii) taking a stock in the ownership of firms in order to be distributed an equal share d_{t+1} of the aggregate profits π_{t+1} made by firms at $t+1$, as well as an proportional share of the resale value of the firms at $t+1$. Moreover, the household can also transfer wealth *from its second period into the first* by (iv) borrowing from perfectly competitive financial intermediaries operating through the lives of all generations. Thus, let k^t be the

[16] Money is introduced in the model for the benchmark equilibrium to be optimal. In effect, in the Diamond (1965) setup underlying this one, without access to a bubbly asset in which to be able to save there is no hope for the market to implement the planner's choice, independently of whether the additional effect of public domain capital studied here is included or not. The reason is that it is the presence of such an asset which allows the market allocation to replicate the planner's link between the return to capital and the population growth factor.

amount lent by the household to firms, m^t be the household real balances, and s^t be the net saving in assets other than these two, that is to say the *net position* resulting from investing in firm ownership and borrowing from the financial intermediaries. If $s^t > 0$, household t is therefore investing in firm ownership more than it may be borrowing from second period income. If $s^t < 0$ instead, household t is rather borrowing more from second period income than it is investing in firm ownership.[17]

Household t's choice must therefore satisfy the budget constraints

$$\begin{aligned} c_0^t + k^t + s^t + m^t &\leq w_t \\ c_1^t &\leq r_{t+1}k^t + d_{t+1} + s^{t+1}n + \frac{p_t}{p_{t+1}}m^t \end{aligned} \tag{BC}$$

where $d_{t+1} = \frac{\pi_{t+1}}{N_t}$ is the *per* owner distributed profits —given the wage w_t, the rental rate of capital r_{t+1}, the level of prices during his lifetime p_t, p_{t+1}, the profits made by firms when old π_{t+1}, the *per* young net position s^{t+1} of generation $t+1$, and the population growth factor n. Therefore, agent t in principle solves

$$\begin{aligned} \max_{c_0^t, c_1^t, k^t, m^t \geq 0, s^t \in \mathbb{R}} \quad & u(c_0^t) + \beta u(c_1^t) \\ c_0^t + k^t + s^t + m^t &\leq w_t \\ c_1^t \leq r_{t+1}k^t + d_{t+1} &+ s^{t+1}n + \frac{p_t}{p_{t+1}}m^t \end{aligned} \tag{H}$$

but since s^t is not bounded below, this maximization problem for the representative household is not well defined yet. It is necessary to take into account too that as firms necessarily make positive profits *in the aggregate*,[18] households do take a stock in firms ownership. The return from doing so $d^t + s^{t+1}n$ has then to match the return from lending to firms the net position s^t as capital instead, i.e. $r_{t+1}s^t$,[19] that is to say

$$d_{t+1} + s^{t+1}n = r_{t+1}s^t. \tag{NA}$$

[17] For the ease of its interpretation, an $s^t > 0$ can be thought of as the amount payed by each household born at t to the households born at $t-1$ for the firm ownership, i.e. for the right to receive its dividends and the value of its resale to its n children paying each s^{t+1}. The claim on future dividends, and hence the possibility to borrow against them, is what allows to extend the interpretation of s^t to that of a net position that can be negative as well as positive.

[18] Although each firm's profits converge to zero as an unbounded number of firms enter the market. For convenience, it can be thought that only finitely many firms can exist and then they necessarily make individually positive profits. This is nonetheless inessential, aggregate profits are anyway positive and equal to the productivity of the capital in the public domain, even if each of countably many firms makes zero profits.

[19] Or, equivalently, the return from buying the firm $d^t + s^{t+1}n$ must match the return from holding

Households are therefore bound by this condition too as a constraint to be added to their maximization problem, whose Lagrange multiplier delivers the value of the firm.

Note that, necessarily, the first-order conditions with respect to c_0^t, c_1^t, k^t, m^t and s^t imply that the household's choice, at an equilibrium, necessarily satisfies

$$\begin{pmatrix} u'(c_0^t) \\ \beta u'(c_1^t) \\ 0 \\ 0 \\ 0 \end{pmatrix} = \lambda_0^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ \frac{1}{p_t} \\ 1 \end{pmatrix} + \lambda_1^t \begin{pmatrix} x1 \\ -r_{t+1} \\ -\frac{1}{p_{t+1}} \\ 0 \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -r_{t+1} \end{pmatrix}$$

for some $\lambda_0^t, \lambda_1^t > 0$ and $\mu^t \neq 0$, along with the binding budget constraints and condition (NA), or equivalently

$$\begin{aligned} \frac{1}{\beta}\frac{u'(c_0^t)}{u'(c_1^t)} &= \frac{p_t}{p_{t+1}} = r_{t+1} \\ c_0^t + k^t + s^t + m^t &= w_t \\ c_1^t &= r_{t+1}k^t + d_{t+1} + s^{t+1}n + \frac{p_t}{p_{t+1}}m^t \\ d_{t+1} + s^{t+1}n &= r_{t+1}s^t \end{aligned} \tag{HC}$$

where $d_{t+1} = \frac{\pi_{t+1}}{N_t}$. It also follows from the first-order conditions above that the value of the firm for the household is $\mu^t = u'(c_0^t)/r_{t+1} > 0$.

4.3. Firms' problem.

Firms maximize profits at t choosing how much capital to borrow and how much labor to hire —which at equilibrium need be $K^{t-1} = k^{t-1}N_{t-1}$ (i.e. the aggregate of the amount k^{t-1} lent to firms by each of the N_{t-1} households at $t-1$) and N_t respectively— given the rental rate of capital r_t, the wage w_t, and the amount of private capital that has fallen into the public domain ϕK_{t-1}, that is to say

$$\max_{K^{t-1}, N_t} F(K^{t-1} + \phi K_{t-1}, N_t) - r_t K^{t-1} - w_t N_t$$

---

instead the net position s^t in money, i.e. $\frac{p_t}{p_{t+1}} s^t$, that is to say

$$d_{t+1} + s^{t+1}n = \frac{p_t}{p_{t+1}} s^t.$$

The stock of productive capital at t is

$$\begin{aligned} K_t &= K^{t-1} + \phi K_{t-1} \\ &= \sum_{i=1}^{+\infty} \phi^{i-1} k^{t-i} N_{t-i} \\ &= N_{t-1} k^{t-1} + \phi \sum_{i=1}^{+\infty} \phi^{i-1} k^{t-1-i} N_{t-1-i} \end{aligned}$$

whence

$$\frac{K_t}{N_t} = \frac{k^{t-1}}{n} + \frac{1}{n} \sum_{i=1}^{+\infty} (\frac{\phi}{n})^i k^{t-1-i}$$

or, equivalently,

$$\frac{K_t}{N_t} = \frac{1}{\phi} \sum_{i=1}^{+\infty} (\frac{\phi}{n})^i k^{t-i}$$

Note that only K^{t-1} is remunerated by firms, while a fraction ϕ of capital built from previous loans and now in the public domain is not. As a consequence, firms make at t per young profits

$$\frac{\pi_t}{N_t} = F_K(\frac{k^{t-1}}{n} + \frac{1}{n} \sum_{i=1}^{+\infty} (\frac{\phi}{n})^i k^{t-1-i}, 1) \cdot \frac{1}{n} \sum_{i=1}^{+\infty} (\frac{\phi}{n})^i k^{t-1-i}$$

or, equivalently, period t per old profits d_t —i.e. distributed at t to each agent born at $t-1$— are

$$d_t = \frac{\pi_t}{N_{t-1}} = F_K(\frac{k^{t-1}}{n} + \frac{1}{n} \sum_{i=1}^{+\infty} (\frac{\phi}{n})^i k^{t-1-i}, 1) \sum_{i=1}^{+\infty} (\frac{\phi}{n})^i k^{t-1-i} \tag{D}$$

Factor prices are hence determined by

$$\begin{aligned} r_{t+1} &= F_K(K_{t+1}, N_{t+1}) \\ w_t &= F_L(K_t, N_t) \end{aligned} \tag{FP}$$

The optimizing behavior of households in (HC) and firms in (D) and (FP), when compatible, determine a competitive equilibrium of this economy, as stated in the next section.

4.4. Competitive equilibrium of the overlapping generations economy.

A competitive equilibrium of the overlapping generations economy is therefore characterised by the following conditions.

Proposition 6. *In an overlapping generations economy in which some private capital eventually falls into the public domain, a competitive equilibrium is characterised by a consumption profile* c_0^t, c_1^t*, a loan to firms* k^t*, a net position in firms ownership and borrowing from financial intermediaries* s^t*, a real balance* m^t*, and distributed profits* d_{t+1}*, for each agent born in each period* t*, as well as prices* p_t*, for all* t*, such that*

$$\begin{aligned}
\frac{1}{\beta}\frac{u'(c_0^t)}{u'(c_1^t)} &= \frac{p_t}{p_{t+1}} = F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1) \\
c_0^t + k^t + s^t + m^t &= F_L(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}, 1) \\
\frac{c_1^t}{n} = F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1)\frac{k^t}{n} &+ \frac{d_{t+1}}{n} + s^{t+1} + \frac{p_t}{p_{t+1}}\frac{m^t}{n} \\
d_{t+1} + s^{t+1}n &= F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1)s^t \\
d_{t+1} = F_K(\frac{k^t}{n} + \frac{1}{n}\sum_{i=1}^{+\infty}&(\frac{\phi}{n})^i k^{t-i}, 1)\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i} \\
\frac{p_t}{p_{t+1}}\frac{m^t}{m^{t+1}} &= n
\end{aligned} \tag{CE}$$

Proof. The first four lines follow from the household choice in (HC) with the factor prices replaced by the marginal productivities of factors according to the firms' behavior in (FP). The fifth line is the equilibrium profits (D) distributed at each $t+1$.

The sixth line is equivalent to the feasibility of the allocation of resources for this economy, and can be obtained in the usual way adding up the budget constraints of the agents alive at any given period t —of which there are n young agents per old one— and taking into account the homogeneity of degree 1 of the production function, i.e. adding up

$$c_0^t + k^t + s^t + m^t = w_t$$

and

$$\frac{c_1^{t-1}}{n} = r_t \frac{k^{t-1}}{n} + \frac{d_t}{n} + s^t + \frac{p_{t-1}}{p_t} \frac{m^{t-1}}{n}$$

which with (FP) and the feasibility constraint

$$c_0^t + \frac{c_1^{t-1}}{n} + k^t = F(\frac{1}{\phi} \sum_{i=1}^{+\infty} (\frac{\phi}{n})^i k^{t-i}, 1)$$

amounts to

$$\frac{p_t}{p_{t+1}} \frac{m^t}{m^{t+1}} = n$$

at any given t. $\square$

It should be noted, first, that the equilibrium conditions imply that different generations choose, necessarily, a different mix of money and a net position of stocks and borrowing for savings other than loans to firms as the next proposition establishes.

Proposition 7. *In an overlapping generations economy in which some private capital eventually falls into the public domain, there is no competitive equilibrium in which the representative agent monetary savings, m^t, and net position in investing in firm ownership and borrowing, s^t, are constant.*

Proof. In effect, should $m^t = m$ hold for some m and all t, then from the last equation in the system (CE) above

$$\frac{p_t}{p_{t+1}} = n$$

and should $s^t = s$ hold, then the no-arbitrage condition (NA) requires zero profits to be distributed, since

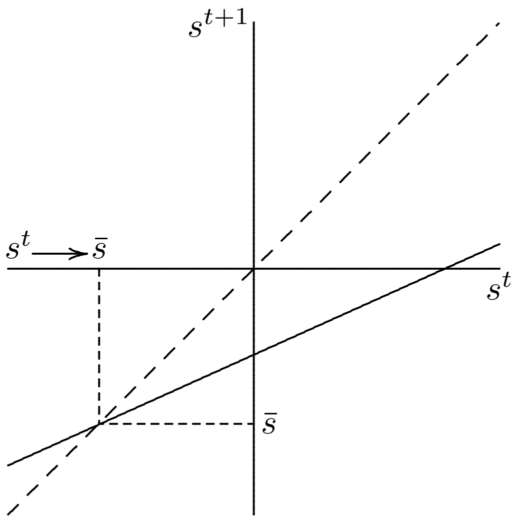
$$d_{t+1} = \Big[F_K(\frac{1}{\phi} \sum_{i=1}^{+\infty} (\frac{\phi}{n})^i k^{t+1-i}, 1) - n\Big] s = 0$$

according to the equilibrium condition $F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}\left(\frac{\phi}{n}\right)^i k^{t+1-i}, 1) = \frac{p_t}{p_{t+1}}$ and $\frac{p_t}{p_{t+1}} = n$ above. However, profits distributed at $t+1$ are positive, since

$$d_{t+1} = F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}, 1)\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i} > 0$$

from which the conclusion follows. $\square$

Therefore a competitive equilibrium steady state will be an allocation where only the consumption profile, the loans to firms and the *total* (but not the composition) of savings in instruments other than loans to firms, $\bar{s}$, will stay constant, as shown in the next proposition. It is shown there too that, in the relevant case in which real balances and the value of the firm do not explode, (i) the share of real balances within $\bar{s}$ converges to zero, so that in the limit the net position of *ownership of the firm and borrowing replaces money* as the bubbly asset in the economy; and (ii) $\bar{s} < 0$, meaning that (in the limit) *each generation pays for the firm* not by saving but *by getting indebted* and repaying when old, the funds of the loan being provided to the lender by the repayment to the financial intermediaries from the previous generation.



Proposition 8. *In the overlapping generations economy in which some private capital eventually falls into the public domain, a competitive equilibrium steady state is characterised by a constant profile of consumptions c_0, c_1 and a constant loan to firms k, for all generations, as well as a constant growth factor of real*

balances m^{t+1}/m^t *satisfying*

$$\frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} = n\frac{m^{t+1}}{m^t} = F_K(\frac{k}{n-\phi},1)$$
$$c_0 + k + \bar{s} = F_L(\frac{k}{n-\phi},1) \tag{M}$$
$$c_0 + \frac{c_1}{n} + k = F(\frac{k}{n-\phi},1)$$

—where $\bar{s} = \frac{d}{r-n}$, *with* $d = r\frac{k}{n-\phi}\phi$ *and* $r = F_K(\frac{k}{n-\phi},1)$*— which determines* c_0, c_1, k, *and* $\frac{m^{t+1}}{m^t}$.

Moreover

$$s^t + m^t = \bar{s}$$

and, if $r < n$, *the household net position in ownership of the firm and borrowing converge to* $\bar{s} < 0$,[20] *while positive real balances converge to zero, i.e.*

$$\lim_{t\to+\infty} s^t = \bar{s} < 0$$
$$\lim_{t\to+\infty} m^t = 0$$

(if $r \geq n$, *savings invested in both real balances and the net position in ownership of the firm and borrowing diverge).*

Proof. From Propositions 6 and 7, a competitive equilibrium steady state is therefore characterised by the conditions next, where consumptions c_0^t, c_1^t, capital savings k^t, and distributed profits d_{t+1} —but not s^t or m^t (nor p_t, *a fortiori*)— stay constant

[20]That is to say, ownership is debt-financed.

at levels c_0, c_1, k, and d,

$$\frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} = \frac{p_t}{p_{t+1}} = F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}\big(\frac{\phi}{n}\big)^i k, 1)$$

$$c_0 + k + s^t + m^t = F_L(\frac{1}{\phi}\sum_{i=1}^{+\infty}\big(\frac{\phi}{n}\big)^i k, 1)$$

$$\frac{c_1}{n} = F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}\big(\frac{\phi}{n}\big)^i k, 1)\frac{k}{n} + \frac{d}{n} + s^{t+1} + \frac{p_t}{p_{t+1}}\frac{m^t}{n}$$

$$d + s^{t+1}n = F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}\big(\frac{\phi}{n}\big)^i k, 1)s^t$$

$$d = F_K(\frac{k}{n} + \frac{1}{n}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k, 1)\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k$$

$$\frac{p_t}{p_{t+1}}\frac{m^t}{m^{t+1}} = n$$

Note that, nonetheless, from the second line above, the aggregate $s^t + m^t$ has necessarily to be a constant, say $\bar{s}$, at a competitive equilibrium steady state, even though s^t and m^t are not.

Furthermore, the steady state conditions imply —adding up the second, third (delayed 1 period), and fifth (divided by n) equations, after having substituted the sixth (delayed 1 period) into the third one— the feasibility of the allocation

$$c_0 + \frac{c_1}{n} + k = F(\frac{k}{n-\phi}, 1)$$

which can replace one of these equations, say the third one. Thus, after substituting the sixth into the equations in the first line and replacing the series by their value, a competitive equilibrium steady state is characterised by

$$\frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} = n\frac{m^{t+1}}{m^t} = F_K(\frac{k}{n-\phi}, 1)$$

$$c_0 + k + \bar{s} = F_L(\frac{k}{n-\phi}, 1)$$

$$c_0 + \frac{c_1}{n} + k = F(\frac{k}{n-\phi}, 1)$$

$$d + s^{t+1}n = F_K(\frac{k}{n-\phi}, 1)s^t$$

whose first three lines are those in (M) as requested. It remains to be checked that the no-abitrage condition in the last line above implies that $\bar{s}$ is the value claimed and that the convergence of s^t to $\bar{s}$ obtains. In effect, from the no-arbitrage condition at the steady state, which can be rewritten as

$$s^{t+1} = \frac{r}{n} s^t - \frac{d}{n}$$

—where $d = r\frac{\phi}{n-\phi}k$ and $r = F_K(\frac{k}{n-\phi}, 1)$ are, respectively, the profits distributed to each agent and the return to capital at a competitive equilibrium steady state— it follows that, whenever $r < n$, the value for s^t converges to

$$\bar{s} = \frac{d}{r-n} < 0.$$

as claimed. □

A few remarks are now in order. Firstly, from conditions (P) in page 12 and (M) in page 20 above, it follows that the planner steady state cannot be decentralized through markets under laissez-faire. In particular, the market leads the agents to consume too early at the steady state, in the sense of choosing a intertemporal marginal rate of substitution smaller than the planner's, as made precise in the next proposition.

Proposition 9. *In the overlapping generations economy in which some private capital eventually falls into the public domain, the planner's steady state cannot be decentralized as a laissez-faire competitive markets outcome. In particular, the market makes the agents choose a profile of consumption whose intertemporal marginal rate of substation is smaller than the planner's.*

Proof. In effect, note first that since $s^t + m^t = \bar{s}$, for $m^t > 0$ it must be that $s^t < \bar{s} < 0$, so that s^t converges to $\bar{s}$ from the left, decreasing in absolute value. Therefore, $m^t = \bar{s} - s^t$ is decreasing, so that $m^t/m^{t+1} > 1$ which, from the equilibrium condition

$$\frac{p_t}{p_{t+1}} \frac{m^t}{m^{t+1}} = n$$

implies $p_t/p_{t+1} < n$.

Now, if $\bar{c}_0, \bar{c}_1$ is the competitive equilibrium steady state profile of consumption, while the profile chosen by the planner is c_0^*, c_1^*, it follows from the respective characterisations in (M) and (P) that

$$\frac{1}{\beta}\frac{u'(\bar{c}_0)}{u'(\bar{c}_1)} = \frac{p_t}{p_{t+1}} < n = \frac{1}{\beta}\frac{u'(c_0^*)}{u'(c_1^*)}$$

as claimed. □

Interestingly enough, it is not immediate in the overlapping generations case (as opposed to the infinitely-lived case) whether the market lends too few or too much capital to firms, compared to what the planner would choose. In effect, since at the competitive equilibrium steady state $p_t/p_{t+1} < n$, it follows from (M) and (P) respectively that

$$F_K(\frac{\bar{k}}{n-\phi}, 1) = \frac{p_t}{p_{t+1}} < n = F_K(\frac{k^*}{n-\phi}, 1) + \phi$$

that is to say

$$F_K(\frac{\bar{k}}{n-\phi}, 1) < n > F_K(\frac{k^*}{n-\phi}, 1)$$

so that the market level of capital $\bar{k}$ could, in principle, be smaller or bigger than the planner's k^*.

Nevertheless, it will follow from the policy decentralising the planner's steady state shown in the next section that savings need to be subsidised, so that —as in the infinitely-lived agents case— the market equilibrium leads to saving too little.

4.5. Market implementation of the planner's steady state through a subsidy on capital returns and a tax on debt.

If households see their returns from loans to firms subsidized by an amount equal to the depreciation/obsolescence rate ϕ and the debt issuance against future profits taxed by a factor $\sigma > 1$, they would face instead

$$\max_{c_0^t, c_1^t, k^t, m^t, s^t} u(c_0^t) + \beta u(c_1^t)$$

$$c_0^t + k^t + s^t + m^t \leq w_t$$

$$c_1^t \leq (r_{t+1} + \phi)k^t + d_{t+1} + \sigma s^{t+1} n + m^t \frac{p_t}{p_{t+1}}$$

$$d_{t+1} + \sigma s^{t+1} n = (r_{t+1} + \phi)s^t$$

given the wage w_t, the rental rate of capital r_{t+1}, the level of prices during his lifetime p_t, p_{t+1}, the profits received as dividends when owner d_{t+1}, the per young repayment of debt when old s_{t+1} and the population growth factor n. As before, the last constrain is the non-arbitrage condition needed to be satisfied for an equilibrium to exist at all.

As a consequence, the household's choice necessarily satisfies

$$\begin{pmatrix} u'(c_0^t) \\ \beta u'(c_1^t) \\ 0 \\ 0 \\ 0 \end{pmatrix} = \lambda_0^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \lambda_1^t \begin{pmatrix} 0 \\ 1 \\ -(r_{t+1}+\phi) \\ -\frac{p_t}{p_{t+1}} \\ 0 \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -(r_{t+1}+\phi) \end{pmatrix}$$

for some $\lambda_0^t, \lambda_1^t > 0$ and $\mu^t \neq 0$, along with the binding budget constraints and the no-arbitrage condition, or equivalently

$$\begin{aligned} \frac{1}{\beta}\frac{u'(c_0^t)}{u'(c_1^t)} &= \frac{p_t}{p_{t+1}} = r_{t+1} + \phi \\ c_0^t + k^t + s^t + m^t &= w_t \\ c_1^t = (r_{t+1}+\phi)k^t &+ \frac{\pi_{t+1}}{N_t} + \sigma s^{t+1} n + \frac{p_t}{p_{t+1}} m^t \\ d_{t+1} + \sigma s^{t+1} n &= (r_{t+1}+\phi)s^t \end{aligned}$$

As before, firms distribute to each old at t dividends

$$d_t = \frac{\pi_t}{N_{t-1}} = \frac{\pi_t}{N_t}\frac{N_t}{N_{t-1}} = F_K\left(\frac{k^{t-1}}{n} + \frac{1}{n}\sum_{i=1}^{+\infty}\left(\frac{\phi}{n}\right)^i k^{t-1-i}, 1\right)\sum_{i=1}^{+\infty}\left(\frac{\phi}{n}\right)^i k^{t-1-i}$$

and factor prices are

$$\begin{aligned} r_{t+1} &= F_K(K_{t+1}, N_{t+1}) \\ w_t &= F_L(K_t, N_t) \end{aligned}$$

The market clearing condition can again be obtained adding up the budget constraints of the agents alive at any given period t, of which there are n young agents per old one, i.e. adding up

$$c_0^t + k^t + s^t + m^t = w_t$$

and

$$\frac{c_1^{t-1}}{n} = (r_t + \phi)\frac{k^{t-1}}{n} + \frac{d_t}{n} + \sigma s^t + \frac{p_{t-1}}{p_t}\frac{m^{t-1}}{n}$$

which after taking into account the feasibility condition amounts to

$$m^t = \phi\frac{k^{t-1}}{n} + (\sigma - 1)s^t + \frac{p_{t-1}}{p_t}\frac{m^{t-1}}{n}$$

at any given t.

A competitive equilibrium is therefore characterised under such a policy by the following conditions.

Proposition 10. *In the overlapping generations economy in which some private capital eventually falls into the public domain, a competitive equilibrium under a policy that (i) subsidises the returns to capital by the depreciation/obsolescence rate ϕ and (ii) taxes debt issued against future profits by a factor $\sigma > 1$, is characterised by a consumption profile c_0^t, c_1^t, a loan to firms k^t, an investment in firms ownership s^t, a real balance m^t, and distributed profits d_{t+1}, for each agent born in each period t, as well as prices p_t, for all t, such that*

$$\frac{1}{\beta}\frac{u'(c_0^t)}{u'(c_1^t)} = \frac{p_t}{p_{t+1}} = F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1) + \phi$$

$$c_0^t + k^t + s^t + m^t = F_L(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}, 1)$$

$$\frac{c_1^t}{n} = \left[F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1) + \phi\right]\frac{k^t}{n} + \frac{d_{t+1}}{n} + \sigma s^{t+1} + \frac{p_t}{p_{t+1}}\frac{m^t}{n}$$

$$d_{t+1} + \sigma s^{t+1} n = \left[F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1) + \phi\right]s^t$$

$$d_{t+1} = F_K(\frac{k^t}{n} + \frac{1}{n}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}, 1)\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}$$

$$m^t = \phi\frac{k^{t-1}}{n} + (\sigma - 1)s^t + \frac{p_{t-1}}{p_t}\frac{m^{t-1}}{n}$$

It is noteworthy that the argument underpinning Proposition 7 does not hold under this policy, so that it does not rule out anymore the possibility of a steady state equilibrium in which s^t and m^t also are constant. As a matter of fact, the equilibrium that decentralises the planner's steady state is indeed such an equilibrium, as the next proposition establishes. This policy finances a subsidy to the return to capital through a lump-sum tax and, interestingly enough, requires too the use of a tax on debt issued against future profits.

Proposition 11. *In the overlapping generations economy in which some private capital eventually falls into the public domain, the planner's steady state is decentralised as a competitive equilibrium steady state by a period-by-period balanced policy subsidising the returns to capital by the depreciation/obsolescence rate ϕ by taxing debt issued on future profits at a rate*

$$\sigma - 1 = -\frac{d}{sn} > 0$$

—given that $s < 0$ for a stable steady state.

Proof. Should there be values for c_0, c_1, k, s, m, d, and p_t/p_{t+1} such that

$$\begin{aligned}
\frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} &= \frac{p_t}{p_{t+1}} = F_K(\frac{k}{n-\phi}, 1) + \phi \\
c_0 + k + s + m &= F_L(\frac{k}{n-\phi}, 1) \\
\frac{c_1}{n} &= \left[F_K(\frac{k}{n-\phi}, 1) + \phi\right]\frac{k}{n} + \frac{d}{n} + \sigma s + \frac{p_t}{p_{t+1}}\frac{m}{n} \\
d + \sigma s n &= \left[F_K(\frac{k}{n-\phi}, 1) + \phi\right] s \\
d &= F_K(\frac{k}{n-\phi}, 1)\frac{k}{n-\phi}\phi \\
m &= \phi\frac{k}{n} + (\sigma - 1)s + \frac{p_{t-1}}{p_t}\frac{m}{n}
\end{aligned} \tag{1}$$

for a given σ, they would characterise a competitive equilibrium steady state under the policy of subsidising returns at a rate ϕ and taxing debt by a factor σ. In particular, the policy that obtains including σ as endogenous variableof the system augmented by the additional equation balancing taxes, subsidies, and transfers

$$\phi k + (\sigma - 1)ns = 0 \tag{2}$$

the system pins down a balanced policy implementing the planner's steady state.

In effect, a solution to (1) and (2) above satisfies $p_t/p_{t+1} = n$, so that

$$\frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} = n = F_K(\frac{k}{n-\phi}, 1) + \phi$$

which is the first line of the planner's system in (P), and the equations in the second, third, fifth and sixth lines imply the feasibility of the allocation. The equation in the fourth line in (1) pins down the necessary σ to be

$$\sigma = 1 - \frac{d}{sn} > 1$$

given that $s < 0$ for a stable steady state, and hence is a tax on debt since it increases the amount that households must repay.

The existence of a solution to (1,2) —and hence of a policy decentralizing the planner's steady state— follows from the existence of the latter. $\square$

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