

Nondeterministic time dependent mechanics of elementary prestressed and cable stayed concrete bridges models

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ABSTRACT

The time dependent behaviour of two elementary structures, one made of a concrete cantilever beam, suspended at the tip by a pretensioned stay, and the other made of a concrete cantilever beam, post-tensioned through a horizontal cable, has been studied.

After a short recall, which would outline how a suitable stay pretensioning or a suitable cable post tensioning, may balance the deflections due to selfweight only under elastic hypotheses, the effects of creep on the tip vertical displacement and on the tension in stay are studied. The influence on such effects, due to different stay slopes, is discussed.

As well known, the data needed for these analyses involve many uncertain quantities. Thus, in a second part of the paper, through a probabilistic approach, the effects due to large variations of the tension in the cable are studied. On the basis of the achieved results, we can distinguish between two different kinds of structures: those which have a low sensitivity and those which are greatly affected both by creep effects and by uncertainties.

1. INTRODUCTION

The time dependent behaviour of a simple structure, suitable to model the basic mechanics of cable stayed and cantilever bridges, has been studied. Two elementary structures, one made of a concrete cantilever beam, suspended at the tip by a pretensioned stay, and the other, made of a concrete cantilever beam and post-tensioned through a horizontal cable, has been considered.

An introductory deterministic approach recalls how a suitable stay pretensioning or a suitable cable post tensioning, may balance the tip deflection due to selfweight only under elastic hypotheses, or rather in absence of creep and shrinkage.

In presence of delayed behaviours, only the case of a cantilever beam, supported by a vertical stay, whose tension equals the reaction of a rigid support, gives the rigorous result of zero tip

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displacement. When the stay deflects from the vertical, it induces a compressive state, which shortens the beam and reduces the force in the stay. For the case of the prestressed cantilever this balancing effect is only tendentially respected, due to the continuous interaction between losses of prestress and deformations. In any case, the expected attitude of a cable stayed bridge or of cantilever bridge is governed by the antagonist roles of the selfweight and of pretensioning actions in the cables. The final value of the deflections results from the difference of two large and opposite numbers, representing downward and upward deflections.

Now, the conventional assessment of the time dependent behaviour of these types of structures uses tools of analysis based on deterministic criteria and assumes that tensioning forces and all the other factors involved in the structural behaviour are known and certain data. Actually, these data involve many uncertain quantities, such as environmental factors, material characteristics and tension in the cables, thus, in a second part of the paper, the effects due to large variations of the tension in the cable are studied, through a probabilistic approach. For a given set of data, the creep effects has been handled by means of a step by step computation of the stress history, based on a matrix partitioning technique. The effects of the uncertainties have been simulated through a Monte Carlo approach.

With respect to uncertain data concerning the pretensioning/post tensioning force, we can distinguish between two different kinds of structures: those which have a low sensitivity and those which are greatly affected by uncertainties.

2. RECALLS OF VISCOELASTIC STRUCTURAL ANALYSIS

2.1 Stress-strain relationships

In a plane concrete element specimen, for a sustained stress applied at time t_0 , the total strain at time t is

$$\varepsilon_c(t) = \sigma_c(t_0)/E_c(t_0) \cdot [1 + \varphi(t, t_0)] = \sigma_c(t_0) \cdot J(t, t_0) \quad (1)$$

where $\varphi(t, t_0)$ is the dimensionless creep coefficient, function of the age of the concrete t_0 at loading and of its age t when the strain is measured, and $J(t, t_0)$ is the creep function. Several creep functions have been formulated. The most used are those proposed by ACI (ACI Committee 1982), CEB (CEB 1978, CEB 1984, CEB 1999) and Bažant B3 (Bažant et al. 2000). Assuming that the principle of superposition holds, the strain corresponding to a stress history composed of two basic stress histories equals the sum of the strains due to each of these. In general, if the stress intensity varies with time, the total strain of the concrete due to the applied stress is given by:

$$\varepsilon_c(t) = \frac{\sigma_c(t_0)}{E_c(t_0)} \cdot [1 + \varphi(t, t_0)] + \int_{\sigma_c(t_0)}^{\sigma_c(t)} \frac{[1 + \varphi(t, \tau)]}{E_c(\tau)} \cdot d\sigma_c(\tau) + \varepsilon_{sh}(t, t_0) \quad (2)$$

where $\varepsilon_{sh}(t, t_0)$ is the free shrinkage strain and the integral must be understood as a Stieltjes integral. This integral represents the instantaneous strain, plus creep strain due to an increment in concrete stress, gradually introduced during the period t_0 to t .

2.2. Methods for Structural Analysis

For the sake of the structural analyses and referring to Finite Element type solutions, the creep constitutive law has been developed in various forms in order to compile the stiffness or the compliance matrices of the structure. The most frequently used methods refer to:

- Direct approaches, also called algebraic methods, based on simple quadrature rules of the superposition integral, like the Age Adjusted Effective Modulus Method (A.A.E.M.M.) (Trost 1967, Bažant 1972.a, Křitek et al. 1988, Ghali et al., 2002).
- Steps by step solutions in the two versions:
 - those based on the direct integration, with creep and relaxation functions put in matrix form (Bažant 1972.b);
 - those based on the degenerate kernels and Dirichlet series, which require the knowledge of the last step of computation only (Bažant et al. 1973):

In the following, the A.A.E.M. Method will be used for introductory considerations, while the analyses will be carried out through step by step direct integration, based on a matrix partitioning technique, whose details are given in Malerba (Malerba 1986, Malerba 2011).

3. MECHANICS OF AN ELEMENTARY CABLE STAYED BRIDGE MODEL

The expected attitude of a cable stayed bridge is governed by the antagonist roles of the selfweight and of pretensioning actions in the stays. The final value of the deflections results from the difference of two large and opposite numbers, representing downward and upward deflections. In the following the relative influence of these two contributions is examined and compared.

3.1. The role of the stays pretensioning

Through a formal demonstration carried put on a simple model, the role of the stays pretensioning is firstly recalled.

Case A. Cantilever supported at the tip by a non pretensioned stay. With reference to the elementary cable stayed beam shown in Fig. 1.a, we firstly compute the vertical tip displacement when the stay is not pretensioned.

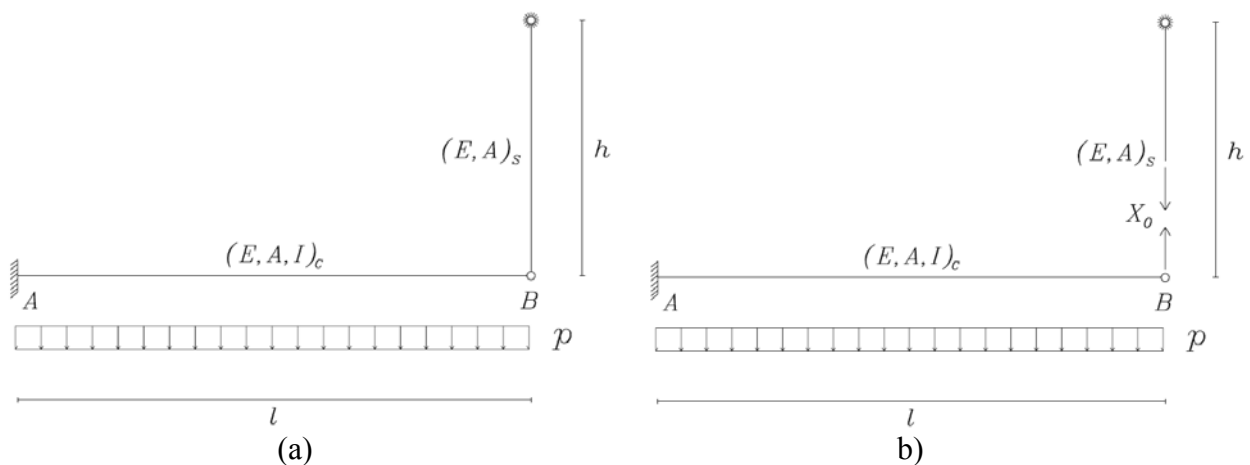


Fig. 1. (a) Elementary cable stayed bridge. (b) Redundant force in the stay at time t_0 .

Assuming as unknown the force $X(t)$ in the stay, through the compatibility equation between the displacements at the tip B, at the loading time t_0 we obtain:

$$X(t_0) = \frac{q \frac{l^4}{8EI}}{\frac{l^3}{3EI} + \frac{h}{EA}} = \frac{u_{10}}{a_{11}^m + a_{11}^s} \quad (3)$$

For a sustained constant load p , the tip deflection of a cantilever beam without any supporting stay, increases with time in affinity to the corresponding concrete creep function. If the tip is hung on a stay, the stay contrasts the downward deflection and, at the same time, keeps increments of the suspending force. The compatibility equation is now written at time t , while the creep effects are dealt with the A.A.E.M. Method:

$$(1 + \chi\varphi) X(t) \cdot a_{11}^m + \varphi(1 - \chi) X(0) \cdot a_{11}^m - (1 + \varphi) \cdot u_{10} = -X(t) \cdot a_{11}^s \quad (4)$$

from which we obtain:

$$X(t) = - \frac{\varphi(1 - \chi) X(0) \cdot a_{11}^m - (1 + \varphi) \cdot u_{10}}{(1 + \chi\varphi) \cdot a_{11}^m + a_{11}^s} \quad (5)$$

It is easy to verify that

$$X(t) > X(t_0) \quad (6)$$

The force in stay increases with time.

Case B. Cantilever supported at the tip by a pretensioned stay. We apply a pretension to the stay. As shown in Fig. 2, the pretensioning is modeled as a uniform temperature decrease.

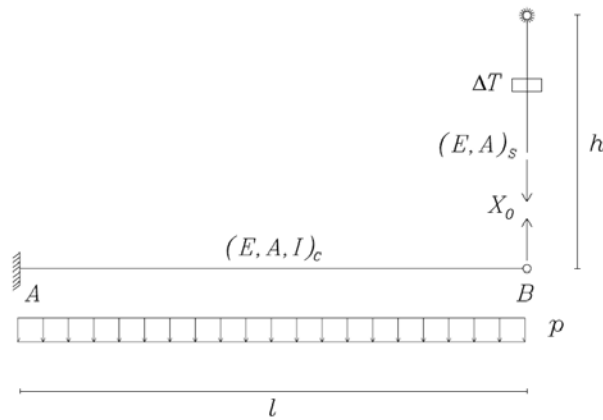


Fig. 2. Stay pretensioning.

Taking into account such new contribution we obtain:

$$X(t) = \frac{\alpha \Delta T h - \varphi(1 - \chi) X(t_0) \cdot a_{11}^m + (1 + \varphi) \cdot u_{10}}{(1 + \chi \varphi) \cdot a_{11}^m + a_{11}^s} \quad (7)$$

where $X(t_0)$ is the force in the stay at time $t = t_0$:

$$X(0) = \frac{\alpha \Delta T h + u_{10}}{a_{11}^c + a_{11}^s} \quad (8)$$

If we impose the condition that the force in the stay be constant in time:

$$X(t) = X(0) \quad (9)$$

After some manipulations, we obtain:

$$\alpha \Delta T h = u_{10} \frac{a_{11}^s}{a_{11}^m} \quad (10)$$

With this value of pretensioning, the force in the stay is constant over time and equals

$$X(t) = X(0) = \frac{u_{10} a_{11}^s / a_{11}^m + u_{10}}{a_{11}^c + a_{11}^s} = \frac{u_{10}}{a_{11}^m} = q \frac{l^4}{8EI} \frac{3EI}{l^3} = \frac{3}{8} ql \quad (11)$$

Such a value equals the rigid support reaction in B. Therefore, by pretensioning the stay with a force equal to that of a rigid support reaction, the force in the stay remains constant over time. It follows that the vertical displacement of the tip B is zero for all times.

3.2. Numerical comparisons

The previous cases have been studied through a step by step F.E. numerical analysis based on the creep and relaxation matrix partitioning technique. The cantilever beam has the double box section and the material characteristics shown in Fig. 3.

Fig. 4 shows the time dependent displacements of the stayed cantilever in the case of non pretensioned (case a) and pretensioned stay (case b). In case (a) the vertical displacement of the tip continues to grow over time, while in case (b) tip B does not move.

For the two cases, Fig. 5 shows the comparisons between normalized displacement functions $s(t)/s(t_0)$ and normalized creep functions $f(t, t_0) \cdot E_c(28)$. The structure as a whole is not homogeneous, but in case (b) the pretensioning of the stay plays the role of a rigid support at tip. As a consequence, the normalized displacement and creep functions coincide and, along the span, the vertical displacements increase with time in affinity to the creep function. In other words, the structural behaviour corresponds to that previewed by the first theorem of viscoelasticity.

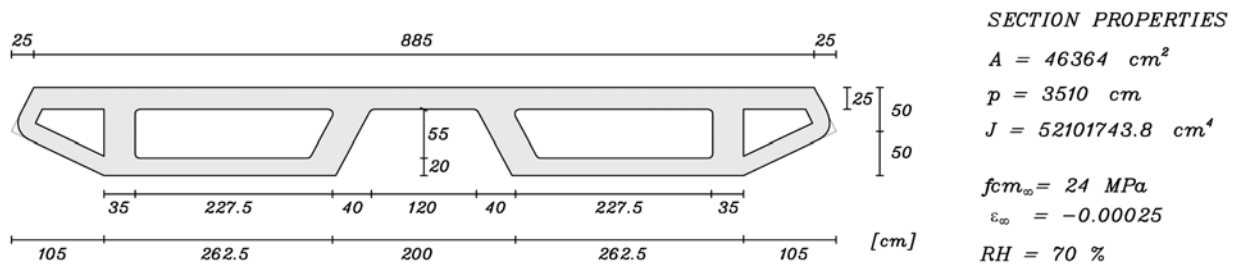


Fig. 3. Cantilever beam section and material properties.

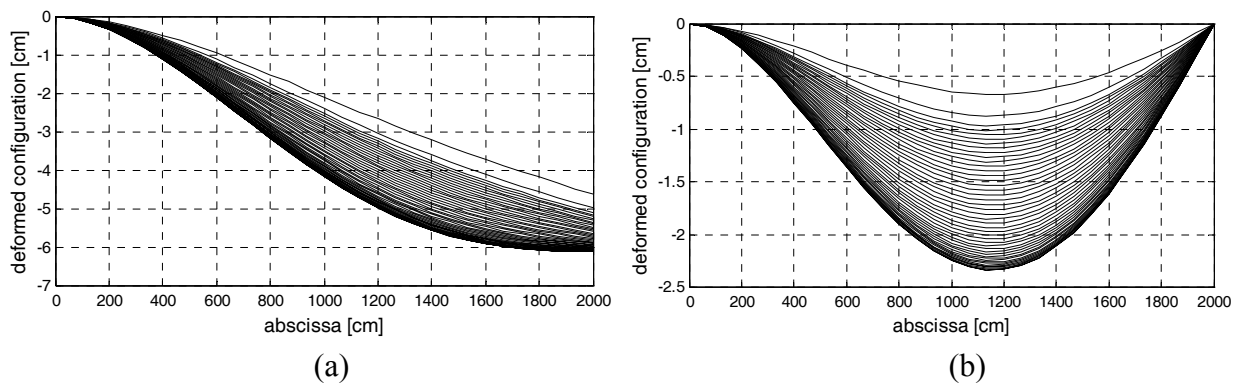


Fig. 4. Time dependent displacements of the stayed cantilever. a) Non pretensioned stay. (b) Stay pretensioned with a force equal to the rigid support reaction.

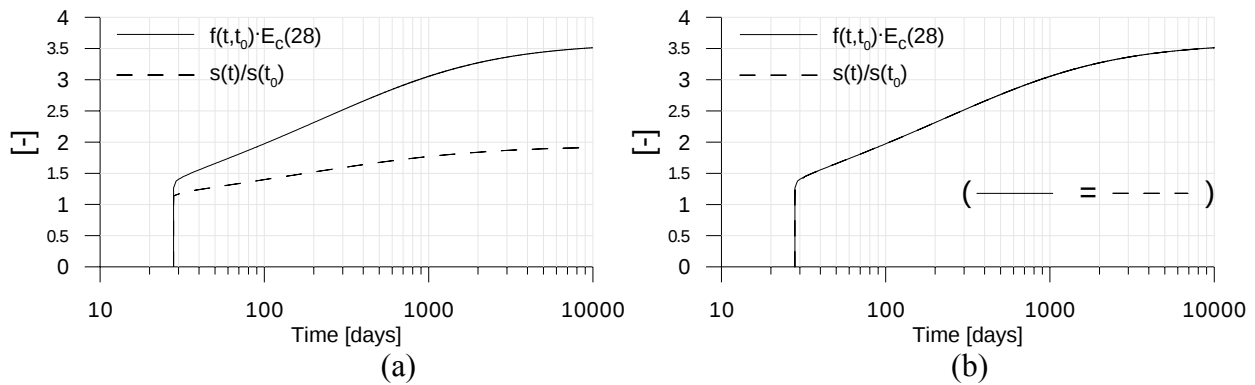


Fig. 5. Comparisons between normalized displacement functions $s(t)/s(t_0)$ and normalized creep functions $f(t, t_0) \cdot E_c(28)$. a) Non pretensioned stay. (b) Stay pretensioned with a force equal to that of the rigid support.

Finally, it may be of interest put in evidence separately what happens in the stay with- and without pretensioning.

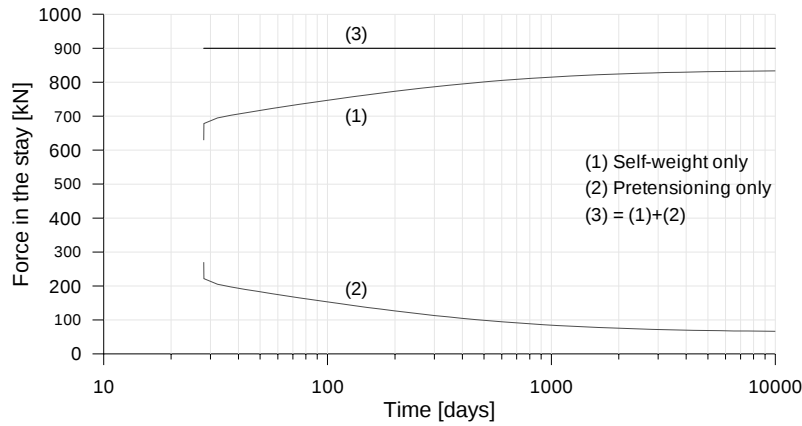


Fig. 6. Time evolution of the force in the stay with- and without pretensioning.

Fig. 6 shows the time evolution of the force in the stay for the following loading conditions:

1. Cantilever beam under the sustained load of the selfweight. Stay not pretensioned.
2. Weightless cantilever beam under the action of the stay pretensioning only.
3. Cantilever beam loaded by selfweight and supported by the pretensioned stay.

Load condition (1) is the same as the case (a). Creep of the concrete beam causes a continuous grow of the force in the stay which suspends the beam at the tip. In load condition (2) creep causes the relaxation of the force in the stay, which decreases with time. Load condition (3) is the sum of the previous ones and is the same as the case (a): the total force in the stay is constant in time and equals the reaction of the rigid support.

$$T = \frac{3}{8}ql = \frac{3}{8} \cdot 120 \frac{kN}{m} \cdot 20 m = 900 kN \quad (12)$$

4. MECHANICS OF AN ELEMENTARY PRESTRESSED CANTILEVER BRIDGE

We consider the elementary cantilever bridge model shown in Fig. 7. The beam is prestressed by single straight cable, having eccentricity e and force N . We want to study its time-dependent behavior.

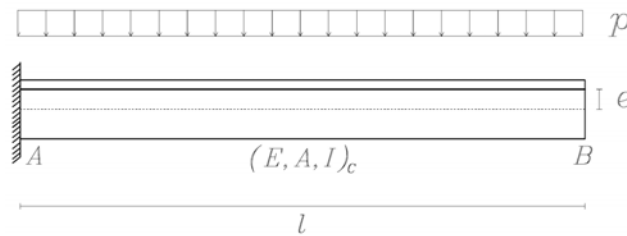


Fig. 7. Elementary prestressed cantilever bridge model. $e = h/6 = 16.67 cm$, $l = 20 m$.

The vertical displacement at the tip B, due to the sustained load p , is:

$$s_B = 1/8 \cdot pl^4 / EI \quad (13)$$

The vertical displacement due to prestressing force N is:

$$s_B = \frac{1}{2} \frac{N \cdot e \cdot l^2}{EI} \quad (14)$$

By equating Eqs. (13) and (14), we obtain that particular value of the prestressing force which, in absence of creep effects, maintains fixed the tip:

$$N = \frac{1}{4} \frac{pl^2}{e} \quad (15)$$

Creep and shrinkage causes delayed deformations and, in particular, an axial shortening of the beam and corresponding losses of prestress.

The problem has been studied through a step by step F.E. numerical analysis based on the creep and relaxation matrix partitioning technique. Also in this case three loading conditions have been considered:

1. Cantilever beam under the sustained load of the selfweight. Cable not prestressed.
2. Weightless cantilever beam under the action of the cable prestressing only.
3. Cantilever beam loaded by selfweight and prestressed.

Fig. 8 shows the vertical displacements along the span. It should be noted that the tip has not zero displacement and, with time, it slightly moves downward (at $t = 10000$ days, $v = -0,237$ cm; displacement/span ratio $\approx 1/8500$),

Finally, it may be of interest put in evidence separately what happens in the stay with- and without pretensioning. Fig. 9 shows the time evolution of the vertical displacements at the tip, for the loading conditions (1) and (2) and for their sum. Curve (3) seems constant with time and thus tendentially confirms the balancing effect between selfweight and prestressing. In truth, as seen before, the tip moves downward, as one can verify through a careful examination of Fig. 10, where the normalized displacement and creep functions are compared. In this case the tip displacement is no longer affine to the creep function.

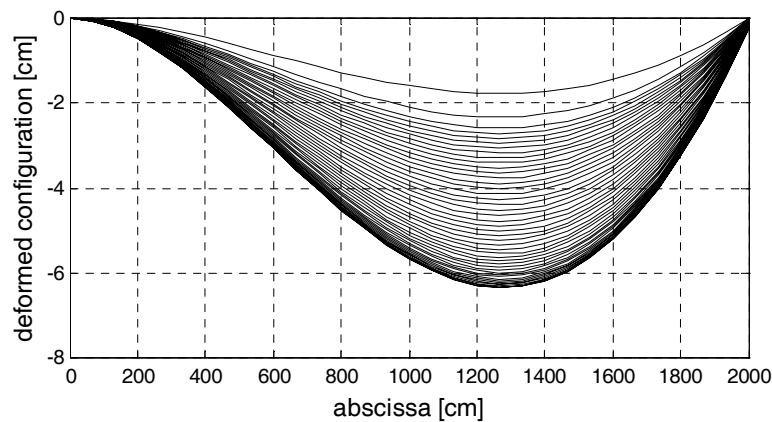


Fig. 8. Time dependent displacements of the prestressed cantilever.

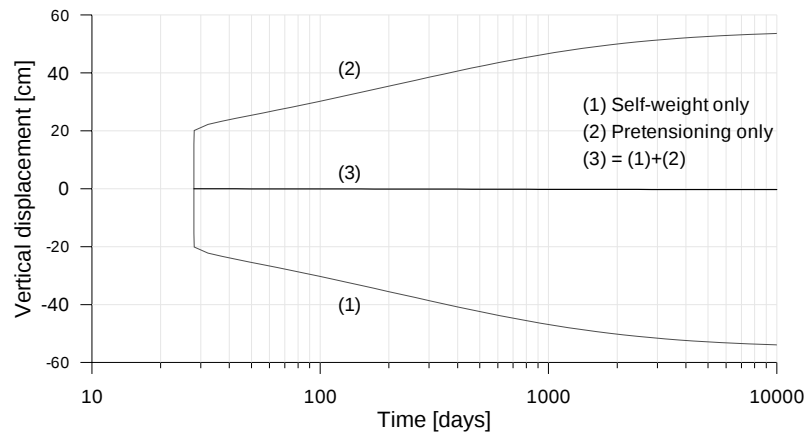


Fig. 9. Time evolution of the tip displacement with- and without prestressing.

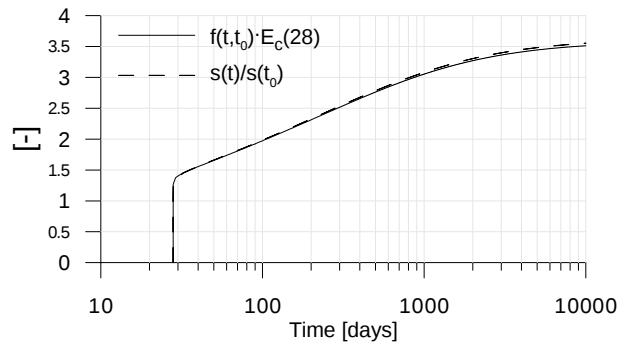


Fig. 10. Comparison between normalized displacement function $s(t)/s(t_0)$ and normalized creep function $f(t, t_0) \cdot E_c(28)$. The two curves do not strictly coincide.

5. FROM CABLE STAYED TO PRESTRESSED CANTILEVER

In the previous sections, with reference to two distinct types of cantilever, one of them cable stayed and the other prestressed, the conditions for zero vertical displacement at the tip have been discussed. It has been shown that, in the case of a cantilever supported by a vertical stay, the vertical displacement at the tip results stable along time, while the tip of a prestressed cantilever moves, due to the prestressing losses caused by the axial shortening of the beam. These concepts are now generalized with reference to a cantilever beam, supported at the tip by a single stay having varying inclinations. We refer to the system shown in Fig. 12. The eight stays slopes listed in Table 1 are considered. The length of the stay is kept constant. The initial cable pretensioning/prestressing force, is that corresponding to zero vertical displacement at time $t = t_0$.

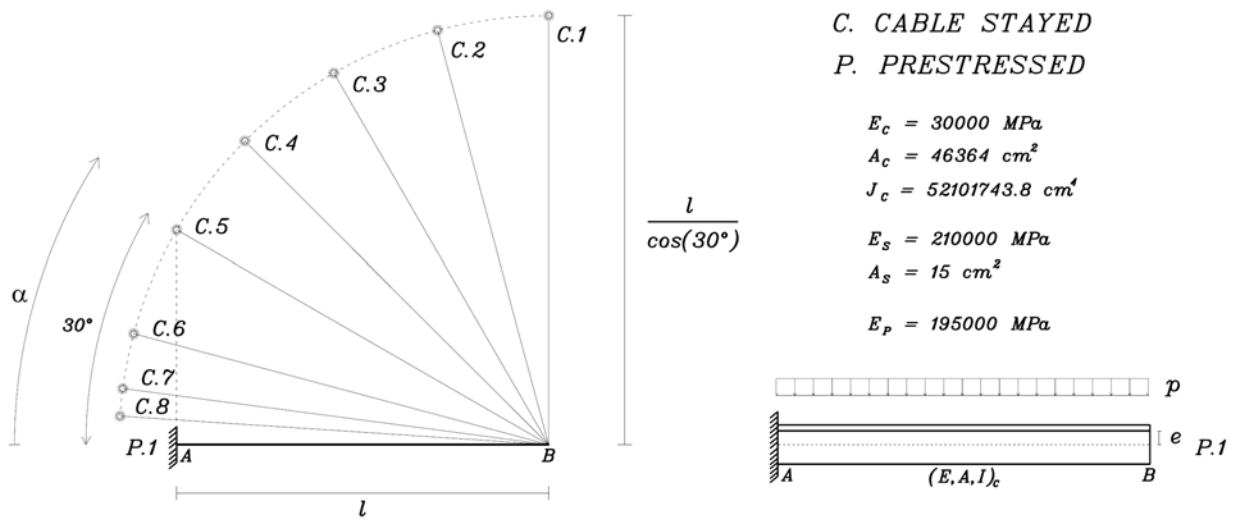


Fig. 11. Cable stayed (C.) and prestressed structures (P.1). $l = 20 \text{ m}$, $p = 120 \text{ kN/m}$.

Table 1. Cases studied. Pretensioning in the stays and prestressing in the cable.

Case	C.1	C.2	C.3	C.4	C.5	C.6	C.7	C.8	P.1
α	90.00	75.00	60.00	45.00	30.00	15.00	7.50	3.75	0.00
T [kN]	900.0	931.0	1039.7	1274.1	1802.7	3483.7	6908.3	13788.0	72000.0

We have seen that, by pretensioning the vertical cable with the force corresponding to the reaction of a rigid support placed at the tip of the beam, the displacement of the node B results zero and that the first theorem of linear viscoelasticity is exactly respected.

Now the slope of the generic stay gives rise to an axial compressive force and the consequent shortening of the beam influence the force in the inclined stays. For cases C.1-C.8 and P1, Fig. 13 shows the ratios between time dependent and initial displacements at tip and a comparison between the displacements evolution and the creep function used in the analyses.

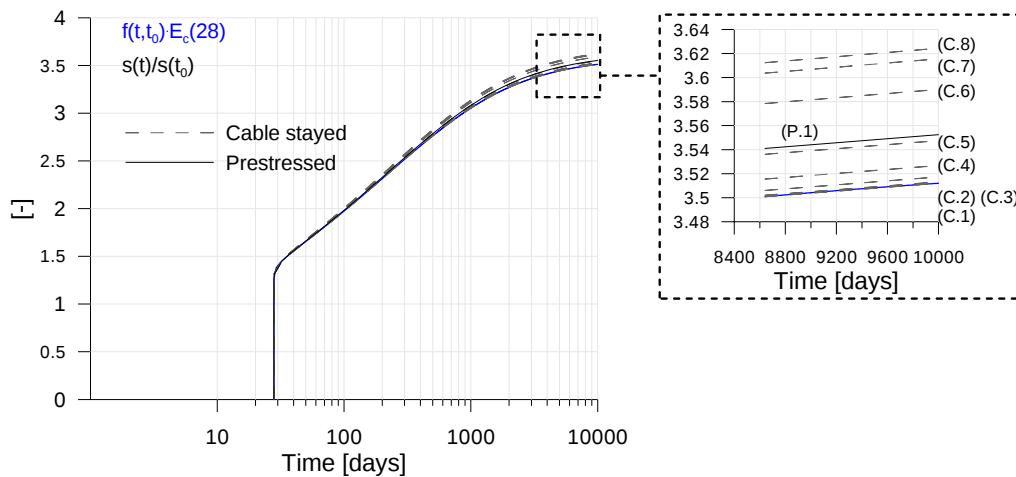


Fig. 12. Ratios between time dependent and initial displacements at tip and comparison between the displacements evolution and the creep function. Enlargement of the final stages of the analysis.

As one can see, for the case C.1, the displacements and creep functions coincide. In the other cases, by tilting the stay and thus compressing the cantilever, these conditions are no longer verified. The tip displacements, even if small (displacement/span ratio $\approx 1/10^4$), grow with the stay slope and with time (Fig. 14).

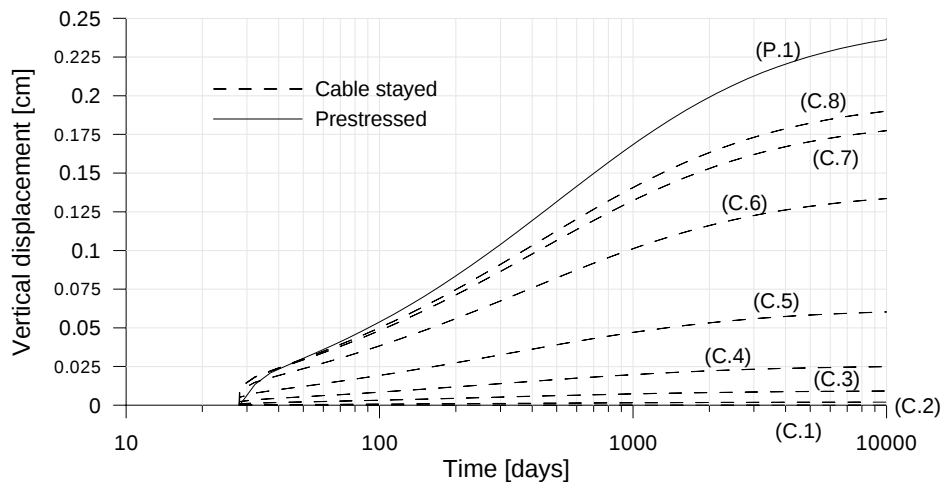


Fig. 13. Vertical displacements of the point B for the considered structures.

Maximum deformation has been computed for the prestressed beam (P.1), whose tip deflection at time $t = 10.000$ days results 2.24 mm.

6. HANDLING WITH UNCERTAINTIES

For the model shown in Fig. 11, prestressing force is considered as an aleatory variable, having normal Gaussian distribution with the mean values μ listed in Table 1 and standard deviations $\sigma = 10\%$. Simulations have been carried out through Montecarlo Method, using 3000 samples. The different influence of the uncertainty on the tension in the cable is highlighted in Fig. 14, showing the probability density function of the tip vertical displacement for the two extreme cases: the cantilever supported by a vertical stay (case C.1) and the cantilever prestressed beam (case P.1). Fig. 14.a, referred at the loading time $t_0 = 28\text{ gg}$, shows for case P.1 an evident greater dispersion in comparison to that of case C.1. At time $t = 10000\text{ gg}$ such a difference is still more stressed. Figs. 18 – 20 give more details as regards these differences and their evolution with time.

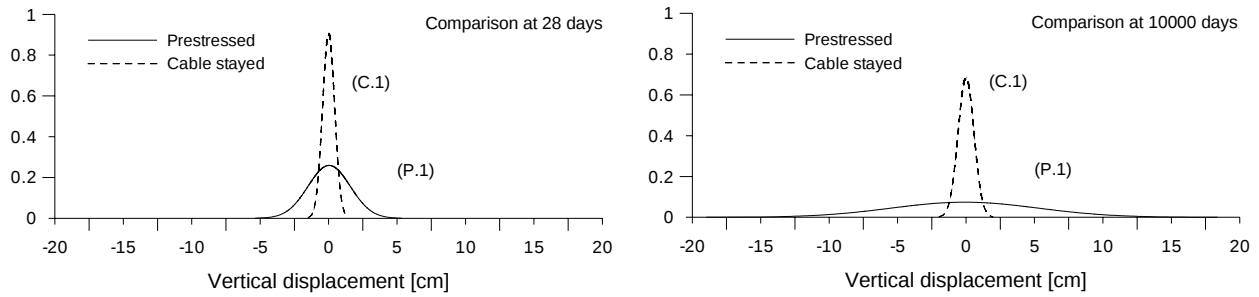


Fig. 14. Comparison of the pdf of the tip vertical displacement at $t_0 = 28\text{ gg}$ and $t = 10000\text{ gg}$.

This result can be put in further evidence by tracing the ratios $\sigma(t)/\sigma(t_0)$ between the standard deviation of the tip displacements at the generic time t and at the loading time $t_0 = 28\text{ gg}$, as shown in Fig. 15.

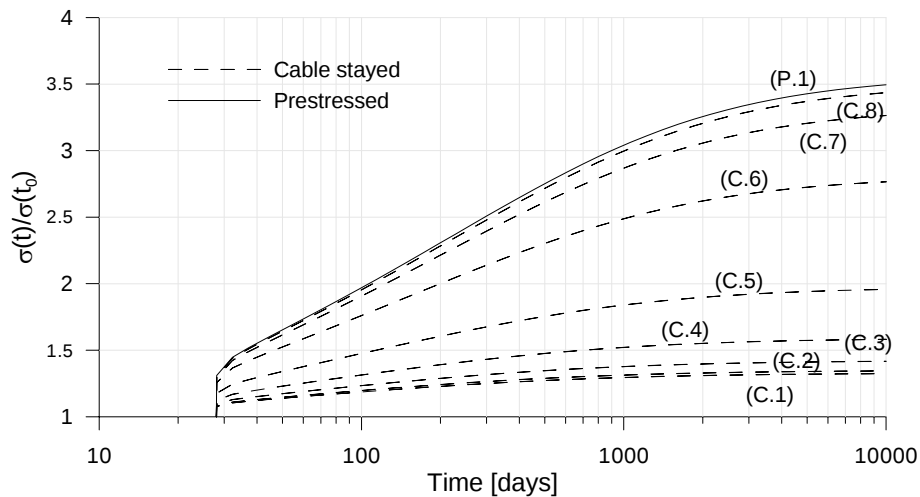


Fig. 15. Evolution with time of the s.d. ratios $\sigma(t)/\sigma(t_0)$ of the tip vertical displacement, for the different slopes of the stay. (C.1, $\alpha = 90^\circ$, C.8, $\alpha = 3.75^\circ$).

Such a ratio grows with time and increases with the slope of the stay with respect to the vertical alignment. The worst case is still represented by the cantilever prestressed beam.

It is of interest, also, outline how evolves with time of the tension in stay for the different slopes examined. Fig. 16 shows the ratios $\sigma(t)/\sigma(t_0)$ between the standard deviation of the force in the stay at time t and at the loading time t_0 . In this case, the dispersion decreases with time and decreases with the slope of the stay.

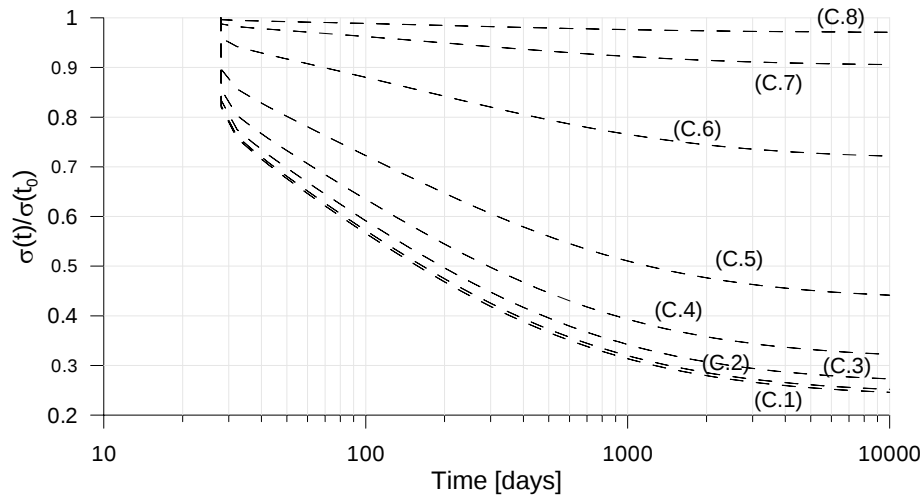
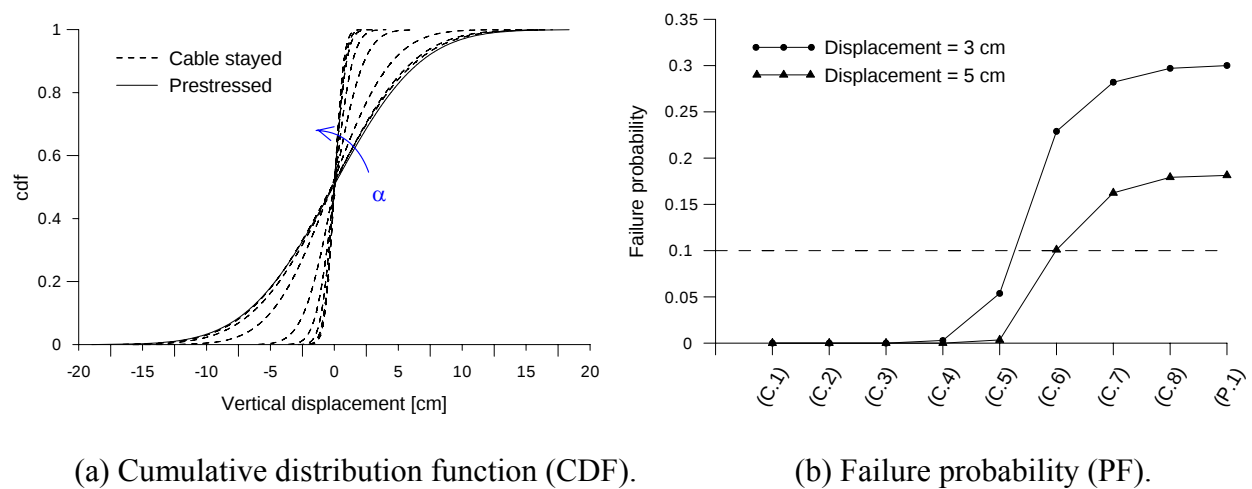


Fig. 16. Evolution with time of the ratios $\sigma(t)/\sigma(t_0)$ between the standard deviation of the force in the stay at time t and at the loading time t_0 . (C.1, $\alpha = 90^\circ$, C.8, $\alpha = 3.75^\circ$).

In particular, it must be outlines how, for equal decreases of the slopes ($\Delta\alpha = 15^\circ$), the corresponding curves C.1 – C.6 are not equally spaced, while for the curves C.7 ($\alpha = 7.50^\circ$) and C.8 ($\alpha = 3.75^\circ$) this ratio $\sigma(t)/\sigma(t_0)$ remains above 0.9.



(a) Cumulative distribution function (CDF).

(b) Failure probability (PF).

Fig. 17. Cumulative density function at $t=10000$ days and failure probability.

Another way to highlight the uncertainty sensitivity in function of the slope of the stay is to trace the cumulative distribution functions (CDF) of the tip vertical displacements for the angles $\alpha = 90^\circ \div 3.75^\circ$, as shown in Fig. 17.a for the measurement time $t = 10000\text{ gg}$.

If we fix two limit states for the tip displacements, for instance LS1 $v_{tip1} = 30\text{mm}$ and LS2 $v_{tip2} = 50\text{mm}$, by assuming as limit probability $PF=0.10$, we show (Fig. 17.b) that LS1 is respected by the configurations C.1 to C.5 ($\alpha = 30^\circ$), while C.6 ($\alpha = 15^\circ$) falls within the wider limit LS2.

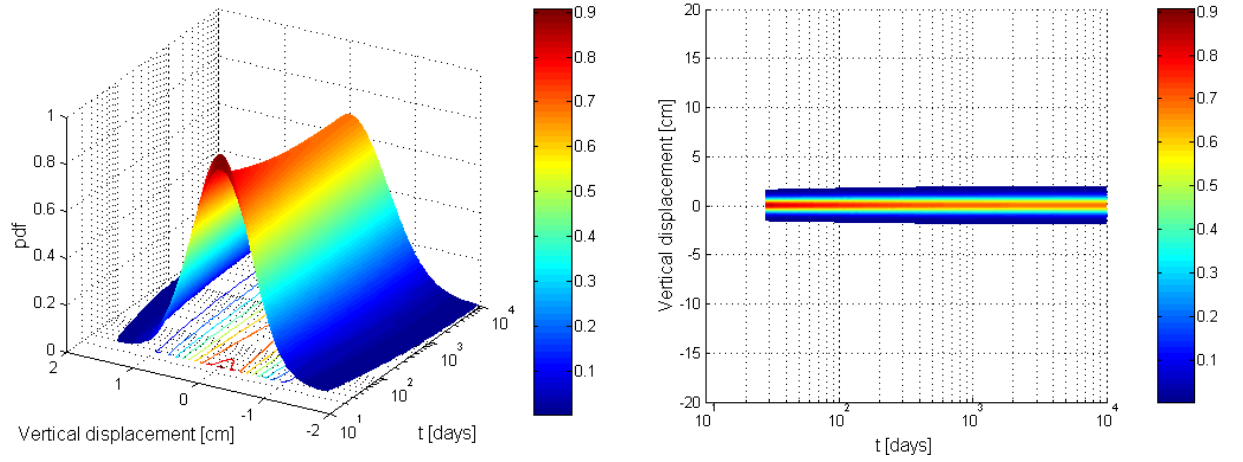


Fig. 18. Case C.1, $\alpha = 90^\circ$. Time evolution of the PDF of the tip vertical displacement.

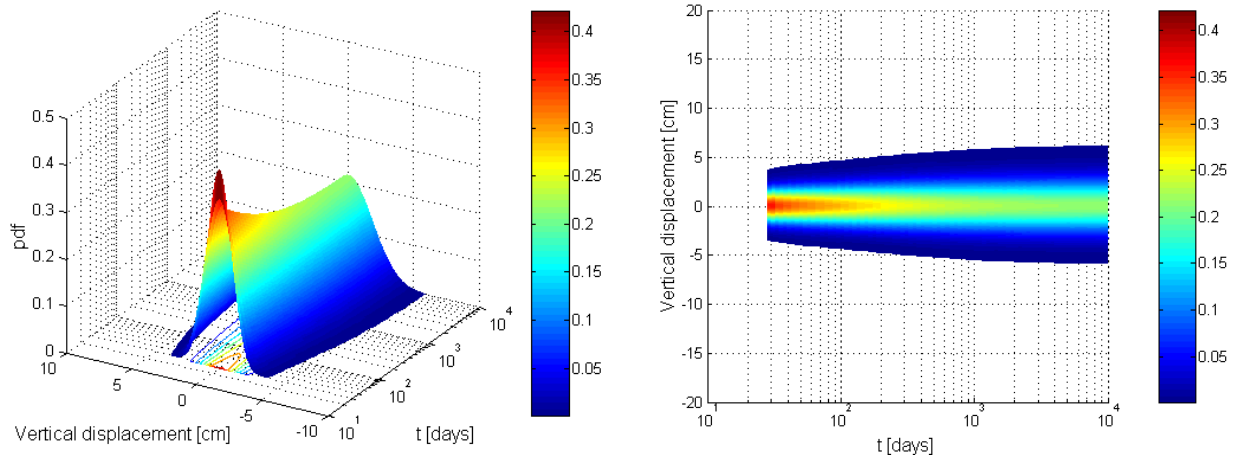


Fig. 19. Case C.5, $\alpha = 30^\circ$. Time evolution of the PDF of the tip vertical displacement.

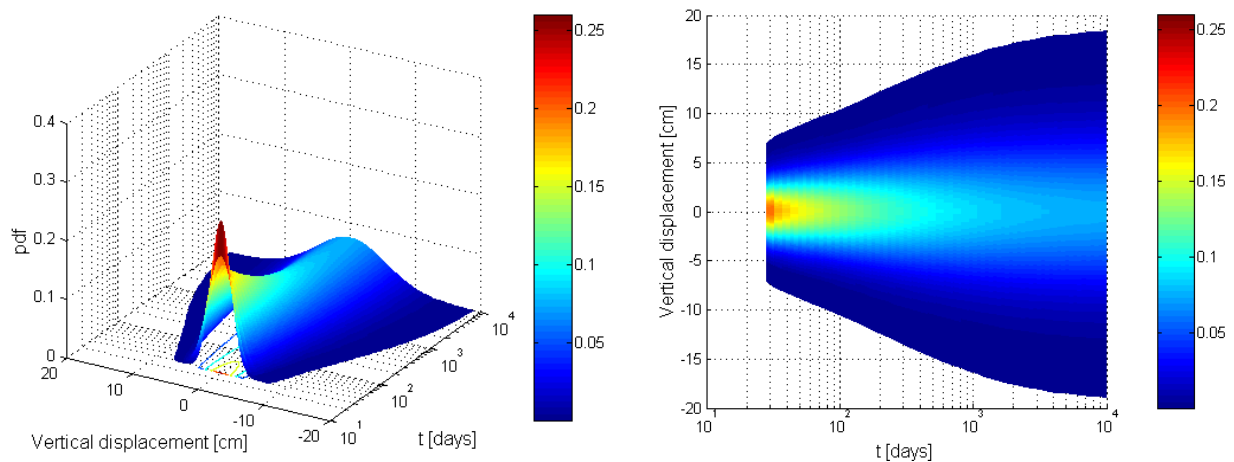


Fig. 20. Case P.1. Time evolution of the PDF of the tip vertical displacement.

CONCLUSIONS

The time dependent behaviour of two elementary structures, one made of a concrete cantilever beam, suspended at the tip by a pretensioned stay, and the other made of a concrete cantilever beam, post-tensioned through a horizontal cable, has been studied.

An introductory deterministic approach recalls how a cantilever beam, supported by a vertical stay, which is tensioned with a force equal to the reaction of a rigid support, rigorously gives zero tip displacement for any time. In the other cases, when the slope deflects from the vertical, the stay induces a compression which shortens the beam and reduces the force in the stay itself. For the case of the prestressed cantilever it is shown how the balancing effect is only tendentially respected, due to the continuous interaction between losses of prestress and deformations.

When we introduce uncertainties in the tension of the cable, we found two different kinds of structures: those which have a low sensitivity and those which are greatly affected by uncertainties.

In the case of a cable stayed bridge, made of a concrete deck, subjected to creep effects and suspended to a pretensioned cable, the role of uncertainties in the pretensioning forces do not influence the time dependent behaviour sensibly. The standard deviation with respect to the mean tension in the cables is relatively small and, most of all, it progressively reduces. The system seems to be self stabilizing over time. In this case the lever arm the stay force is large and equals the length of the beam, while the force in stay is small if it is compared to that involved in the post tensioning case P.1.

On the contrary, in the case of a prestressed cantilever beam, the effects of uncertainties in the pretensioning forces cause a relevant variance of the tip deflections. Both deflections and their variance increase with time. In fact, in this latter case, we have large tensioning forces coupled to very small eccentricities, limited by the fact that the cable must be maintained within the dept of the section. Thus, small variation in the post tensioning force have greter influence in the deformational behaviour of the beam.

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