Bargaining and Delay in Patent Licensing

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Abstract

We consider a model of licensing of a non-drastic innovation in which the patent holder (an outside innovator) negotiates either up-front fixed fees or per-unit royalties with two firms producing horizontally differentiated brands and competing à la Cournot. We investigate how licensing schemes (fixed fee or per-unit royalty) and the number of licenses sold (exclusive licensing or complete technology diffusion) affect price agreements and delays in reaching an agreement. We show that, under complete information, the patent holder prefers to license by means of up-front fixed fees whatever the degree of product differentiation, the innovation size and the level of bargaining power. Once there is private information about the relative bargaining power of the parties, the patent holder may prefer licensing by means of per-unit royalties even if market competition is strong. Moreover, the delay in reaching an agreement is greater whenever the patent holder chooses to negotiate up-front fixed fees instead of per-unit royalties.

Key words: Patent licensing, fixed fee, royalty, bargaining, private information.

JEL classification: C78; D21; D43; D45; L13.

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1 Introduction

A license on a patent protected technology consists of a contract for which its legal holder gives the right to exploit the technology to a third party (licensee) in exchange for some up-front fixed fee or royalties. Patent licensing is a profitable practice for the innovator to diffuse the innovation. The innovator can be either an outside innovator or one of the incumbent producers in the industry. The theoretical literature on licensing of cost-reducing innovations has mainly considered outside innovators who have full bargaining power. That is, innovators are able to impose some up-front fixed fee or per-unit royalty. In industries where firms produce homogeneous goods and compete à la Cournot, licensing by means of per-unit royalties turns to be inferior to licensing by means of posting an up-front fixed fee or auctioning licenses for an outside innovator, regardless of the industry size and the innovation size (see Kamien and Tauman, 1986; Katz and Shapiro, 1986; Kamien, Oren and Tauman, 1992).[1]

However, there is some evidence that the relative bargaining power of a patent holder and a licensee greatly influences the price of patents in licensing negotiations.[2] Sakakibara (2010) has empirically examined the determinants of the price of patent licensing using data about 661 patent licensing contracts in Japan which took place between 1998 and 2003. She has found that factors affecting the profitability of patents and the bargaining power of the patent holder are good predictors of per-unit royalties, while proxies for the reservation price of patent holders are less important for the determination of per-unit royalties. In addition, she has found that the fit of the per-unit royalty regression is always better than that of the fixed fee regression, suggesting that the per-unit royalty represents patent licensing price better than fixed fee payment.[3]

[1] Sen (2005a) has shown that if the number of licenses can take only integer values, then for an outside innovator in a Cournot oligopoly (with at least three firms), royalty licensing could be superior to both fixed fee and auction. Sen and Tauman (2007) have analyzed optimal combinations of up-front fees and per-unit royalties for cost-reducing innovations for both outside and incumbent innovators.

[2] Based on survey data, Caves, Crookwell and Killing (1983) have found that on average only 40 percent of the rent from licensed technology is captured by a patent holder.

[3] Empirical studies have shown the wide prevalence of per-unit royalties in practice. For instance, Rostoker (1984) and Macho-Stadler, Martinez-Giralt and Perez-Castrillo (1996) have found that licensing by means of up-front fixed fees is less frequently used than choosing per-unit royalties or combinations of up-front fees and royalties. Vishwasrao (2007) has considered a data set of all the foreign technology licensing agreements entered into by manufacturing firms in India between

In the present paper we consider a model of licensing of a non-drastic innovation in which the patent holder (an outside innovator) negotiates either up-front fixed fees or per-unit royalties with two firms producing horizontally differentiated brands and competing à la Cournot. The main feature of our model is that both the patent holder and the firms may have private information. To describe the bargaining process, we adopt Rubinstein's (1982) alternating-offer bargaining model with two-sided incomplete information, which allows for the occurrence of delays in equilibrium.

We find that, under complete information, the patent holder prefers to license by means of up-front fixed fees rather than per-unit royalties whatever the degree of product differentiation, the innovation size and the level of bargaining power. However, once there is private information about the relative bargaining power of the parties, the patent holder may prefer licensing by means of per-unit royalties even if market competition is strong. In addition, per-unit royalties may be the optimal choice even if the bargaining with two-sided incomplete information is close to one with complete information.[4]

We also obtain that the maximum delay in reaching an agreement is greater whenever the patent holder chooses to negotiate up-front fixed fees instead of per-unit royalties and remains finite even when the period length between two offers shrinks to zero. When brands are substitutable, royalty settlements create spillover effects (by altering the firms' relative competitive positions in the product market) that have implications for the outcome of negotiations. Spillover effects increase with the degree of market competition and drive the parties to concede more rapidly. In case of per-unit royalties, the number of licenses sold has an ambiguous effect on

1989 and 1993. Industry, firm, and contract characteristics are used to explain differences between the forms of payment in licensing contracts. She has found that licensing contracts are more likely to use royalties when sales are relatively high, while increased volatility of sales and greater profitability favor fixed fee contracts.

[4]The prevalence of royalties over fixed fees in practice has been explained in the theoretical literature by Bertrand competition (Muto, 1993), spatial competition (Caballero-Sanz, Moner-Colonques and Sempere-Monerris, 2002; Poddar and Sinha, 2004), variation in the quality of innovation (Rockett, 1990), incumbent innovator (Shapiro, 1985; Wang 1998, Kamien and Tauman, 2002, Li and Yanagawa, 2011), or asymmetry of information (Gallini and Wright, 1990; Macho-Stadler and Perez-Castrillo, 1991; Beggs, 1992; Sen, 2005b). Recently Amir, Encaoua and Lefouili (2011) have provided another rationale for the prevalence of royalties over fixed fees, that is weak patents. They have shown that a high probability of invalidation increases the attractiveness of per-unit royalties because the per-unit royalty may allow the patent holder to deter litigation.

the maximum real delay time in reaching an agreement. When the patent holder chooses complete technology diffusion, firms have incentives to concede more rapidly because of increased spillover effects. However, complete technology diffusion raises the potential payoffs for the patent holder, and in expanding the payoff set, also increases the scope for delay (longer negotiations may be needed for screening the private information). In case of up-front fixed fees, the rents to be divided do not depend on the negotiated fees. Hence, the maximum real delay time in reaching an agreement in case of up-front fixed fees is greater than the one in case of per-unit royalties.

Thus, the relative bargaining power of a patent holder and a licensee provides a rationale for the wide prevalence of royalties in practice. Royalty could dominate fixed fee even if parties have almost complete information, independently of the intensity of the market competition. In addition, the likelihood of having more inefficient outcomes in case of licensing by means of up-front fixed fees may explain why licensing by means of per-unit royalties is commonly used in practice.

Our paper is linked to a recent strand of the literature that applies a cooperative game theoretic approach to patent licensing. Starting from the observation that real world licensing agreements are the result of negotiations, this literature studies patent licensing as a bargaining outcome. Kishimoto and Muto (2012) have compared fixed fee versus royalty in the presence of an incumbent innovator.[5] They have considered an homogeneous Cournot duopoly in which the patent-holding firm negotiates with its rival firm the price of licensing a cost-reducing innovation. They have shown that royalty licensing is more profitable than fixed fee licensing for both firms as long as the innovation is non-drastic. Although, in the non-cooperative approach, an incumbent innovator prefers royalty versus fixed fee licensing (Wang, 1998), Kishimoto and Muto (2012) have shown that, in the Nash bargaining solution, both the innovating firm and the rival firm strictly prefer royalty, so that royalty would prevail also in the case where a licensing policy is decided through negotiation between the two firms. Moreover, royalty licensing is shown to be preferred from the social welfare point of view so that licensing through negotiation is socially better than the take-it-or-leave-it licensing manner. In our paper, we depart from Kishimoto and Muto (2012) in that we compare fixed fee versus royalty licensing for an outside innovator in an horizontally differentiated Cournot duopoly. Also, we introduce incomplete information in the bargaining process.

[5] Other contributions are Tauman and Watanabe (2007) and Watanabe and Muto (2008).

Our analysis is also linked to the literature of patent licensing under asymmetric information. This literature has considered other sources of asymmetry that may explain the prevalence of royalties over fixed fees in practice. In Gallini and Wright (1990), the value of the innovation is private information to the innovator and the innovator is the only party who can make offers. In Macho-Stadler and Perez-Castrillo (1991), the value of the innovation is private information to the buyer (a monopolist) and the innovator is the only party who can make offers. In Beggs (1992), the value of the innovation is private information to the buyer (a monopolist) and the monopolist is the only party who can make offers. In Sen (2005b), the marginal cost of production is private information to the buyer (a monopolist) and the innovator is the only party who can make offers. So, in all these models either the innovator has full bargaining power or the firm has full bargaining power. In this paper, we provide a model of licensing where both the innovator (patent holder) and the firms (duopolists) have private information and can make offers and counter-offers. In addition, our model can explain the number of days between the date of application of the licensed patents and the date of the licensing contracts as reported in Sakakibara (2010) as well as the effects of market competition on price agreements and delays in reaching an agreement.[6]

The paper is organized as follows. In Section 2 we present the model. In Section 3 we describe and solve the up-front fixed fee (per-unit royalty) bargaining games for the case of complete information. In Section 4 we analyze the up-front fixed fee (per-unit royalty) bargaining games with private information and we derive the maximum delay in reaching an agreement. In Section 5 we conclude.

2 Model

We consider a duopolistic industry. Each firm is producing one brand of a differentiated product. Let firm i produce brand i in quantity q_i. There is no entry or threat of entry, and both firms are quantity setters (Cournot competition). The inverse demand function for brand i of the differentiated product is given by $p_i(q_i, q_j) = a - q_i - \gamma q_j$, $i \neq j$. The parameter $\gamma \in [0, 1]$ represents the degree of

[6]Recently, Gordanier and Miao (2011) have analyzed a multi-period model with potential future innovations and some irreversibility of technology transfer where an innovator jointly decides the payment scheme and the length of the contract. They have found that it may be optimal to license the innovation for less than the full length of the patent and that royalty contracts can be more profitable than fixed-fee licensing even in the absence of private information.

substitutability between both brands. The higher the γ, the higher is the degree of substitutability between i and j. When $\gamma = 0$, each firm becomes a monopolist; when $\gamma = 1$, both brands are perfect substitutes. With the old technology, both firms produce with the identical constant marginal cost c where $0 < c < a$. An outside innovator (the patent holder) has been granted a patent for a non-drastic innovation that reduces the marginal cost from c to $c - \varepsilon$. A non-drastic innovation is such that the non-purchasing firm would produce a positive quantity at equilibrium. Without loss of generality we set $(a - c) = 1$; $\varepsilon \in (0, 1)$ is the innovation size. The patent holder decides to license the new technology to one or both firms but cannot enter the market of the final good directly.[7]

The strategic interaction between the patent holder and the duopolists is modelled as a three-stage game. In the first stage, the patent holder decides how many licenses to sell (complete technology diffusion or exclusive licensing) and the licensing schemes (a non-negative up-front fixed fee or a non-negative per-unit linear royalty). In the second stage, the patent holder and the duopolists bargain either over a fixed fee or a per-unit royalty. In the third stage, the Cournot competition takes place. The model is solved backwards. Let Π_i ($\overline{\Pi}_i$) be firm i's Nash equilibrium profits when both firms produce with the old (new) technology. Let $\overline{\Pi}_i$ ($\underline{\Pi}_i$) be firm i's Nash equilibrium profits when firm i produces with the new (old) technology while firm j produces with the old (new) technology, $j \neq i$. In case of non-negative up-front fixed fees, we have

$$\begin{aligned}
\Pi_i^F &= \left[\frac{1}{2+\gamma}\right]^2, \text{ and } q_i = \frac{1}{2+\gamma};\\
\overline{\Pi}_i^F &= \left[\frac{(2-\gamma)+2\varepsilon}{(2-\gamma)(2+\gamma)}\right]^2 - F_i, \text{ and } q_i = \frac{(2-\gamma)+2\varepsilon}{(2-\gamma)(2+\gamma)};\\
\underline{\Pi}_i^F &= \left[\frac{(2-\gamma)-\gamma\varepsilon}{(2-\gamma)(2+\gamma)}\right]^2, \text{ and } q_i = \frac{(2-\gamma)-\gamma\varepsilon}{(2-\gamma)(2+\gamma)};\\
\overline{\underline{\Pi}}^F &= \left[\frac{1+\varepsilon}{2+\gamma}\right]^2 - F_i, \text{ and } q_i = \frac{1+\varepsilon}{2+\gamma};
\end{aligned}$$

where F_i is the up-front fixed fee which is negotiated between the patent holder and

[7] A drastic innovation would be licensed to a single firm as, by definition, the innovating firm would become a monopolist.

firm i. In case of non-negative per-unit royalties, we have

$$\begin{aligned}
\Pi_i^R &= \left[\frac{1}{2+\gamma}\right]^2, \text{ and } q_i = \frac{1}{2+\gamma};\\
\overline{\Pi}_i^R &= \left[\frac{(2-\gamma)+2(\varepsilon-R_i)}{(2-\gamma)(2+\gamma)}\right]^2, \text{ and } q_i = \frac{(2-\gamma)+2(\varepsilon-R_i)}{(2-\gamma)(2+\gamma)};\\
\underline{\Pi}_i^R &= \left[\frac{(2-\gamma)-\gamma(\varepsilon-R_j)}{(2-\gamma)(2+\gamma)}\right]^2, \text{ and } q_i = \frac{(2-\gamma)-\gamma(\varepsilon-R_j)}{(2-\gamma)(2+\gamma)};\\
\underline{\overline{\Pi}}^R &= \left[\frac{(2-\gamma)(1+\varepsilon)-2R_i+\gamma R_j}{(2-\gamma)(2+\gamma)}\right]^2, \text{ and } q_i = \frac{(2-\gamma)(1+\varepsilon)-2R_i+\gamma R_j}{(2-\gamma)(2+\gamma)};
\end{aligned}$$

where R_i is the per-unit royalty which is negotiated between the patent holder and firm i.

3 Bargaining with complete information

3.1 Fixed fee licensing game

We denote by $F(k)$ the fixed fee licensing game where the patent holder decides to sell k licenses in the first stage.

We first consider the fixed fee licensing game with exclusive licensing, $F(1)$, where the patent holder and firm i negotiate over the up-front fixed fee F_i. As in Sen and Tauman (2007), decisions about quantities to be produced occur only when a price for the new technology has been settled.[8] The negotiation proceeds as in Rubinstein's (1982) alternating-offer bargaining model. The patent holder and firm i make alternate fixed fee offers, with firm i making offers in odd-numbered periods and the patent holder making offers in even-numbered periods. The length of each period is Δ. The negotiation starts in period 0 and ends when one of the negotiators accepts an offer. Firm j will continue producing with the old technology.[9] The patent holder and firm i are assumed to be impatient. The patent holder and firm i

[8] It is implicitly assumed that in the meantime firms continue producing using the old technology and that the length of the negotiation is short compared to the duration of the contract. Macho-Stadler, Martinez-Giralt and Perez-Castrillo (1996) have reported that more than half of the contracts with royalties have a duration of at least five years; contracts based on fixed fees tend to have a shorter duration.

[9] In case no agreement is reached between the patent holder and firm i, the patent holder cannot try to reach an agreement with firm j. The results we obtain are qualitatively robust to this assumption, except that it makes more likely that the patent holder will choose complete technology diffusion (see Sempere-Monerris and Vannetelbosch, 2001).

have time preferences with constant discount rates $r_p > 0$ and $r_f > 0$, respectively. To capture the notion that the time it takes to come to terms is small relative to the life of the patent, we assume that the time between periods is very small. This allows a study of the limiting situations in which the bargaining procedure is essentially symmetric and the potential costs of delaying agreement by one period can be regarded as negligible. As the interval between offers and counteroffers shortens and shrinks to zero, the alternating-offer model has a unique limiting subgame perfect equilibrium (SPE), which approximates the Nash bargaining solution to the bargaining problem (see Binmore, Rubinstein and Wolinsky, 1986).[10] Thus the predicted fixed fee is given by

$$F_i^{\mathrm{SPE}}(1) = \arg\max [F_i]^{\alpha} \cdot \left[\left[\frac{(2-\gamma)+2\varepsilon}{(2-\gamma)(2+\gamma)} \right]^2 - F_i \right]^{1-\alpha}$$

where $\alpha \in (0,1)$ is the patent holder bargaining power which is equal to $r_f/(r_p+r_f)$ and the status quo payoffs are zero, subject to firm i's outside option of not buying the license ($\overline{\Pi}_i^F \geq \Pi_i^F$ if and only if $4\varepsilon(2+\varepsilon-\gamma)(2-\gamma)^{-2}(2+\gamma)^{-2} \geq F_i$). Then, the equilibrium fixed fee is equal to

$$F_i^{\mathrm{SPE}}(1) = \frac{\alpha(2(1+\varepsilon)-\gamma)^2}{(2-\gamma)^2(2+\gamma)^2} \text{ if } \alpha \leq \alpha_1^F \tag{1}$$

and to $4\varepsilon(2+\varepsilon-\gamma)(2-\gamma)^{-2}(2+\gamma)^{-2}$ if $\alpha > \alpha_1^F$, where the upper limit on the patent holder bargaining power that guarantees that firm i prefers the Nash bargaining solution to the outside option is given by $\alpha_1^F \equiv 4\varepsilon(2+\varepsilon-\gamma)(2(1+\varepsilon)-\gamma)^{-2}$. The patent holder's equilibrium revenue, $V^{F^*}(1)$, is equal to $F_i^{\mathrm{SPE}}(1)$. Firm i's equilibrium profits are equal to

$$\Pi_i^{F^*}(1) = \frac{(1-\alpha)(2(1+\varepsilon)-\gamma)^2}{(2-\gamma)^2(2+\gamma)^2} \text{ if } \alpha \leq \alpha_1^F \tag{2}$$

and to $(4(1-\gamma)+\gamma^2)(2-\gamma)^{-2}(2+\gamma)^{-2}$ if $\alpha > \alpha_1^F$.

We now consider the fixed fee licensing game with complete technology diffusion, $F(2)$, where the patent holder and firm 1 negotiate over the up-front fixed fee F_1

[10] The Rubinstein's alternating-offer bargaining model provides a useful guide for the interpretation and identification of the status quo or disagreement point in static models. The interpretation of the status quo is no loss no gains as compared with the players' positions during the negotiation. So, it is not the outside options of the bargaining parties that are defined to be the best alternatives that players can obtain if they withdraw from the bargaining process. The presence of outside options just places restrictions on the solution. See Binmore, Rubinstein and Wolinsky (1986).

while the patent holder and firm 2 negotiate over the up-front fixed fee F_2. The negotiations occur simultaneously and firms are unaware of any proposals made in related negotiations. The predicted fixed fees are given by

$$F_i^{\text{SPE}}(2) = \arg\max [F_i]^{\alpha} \cdot \left[\left[\frac{1+\varepsilon}{2+\gamma} \right]^2 - F_i \right]^{1-\alpha},$$

$i = 1, 2$, $i \neq j$, where the status quo payoffs are zero, subject to firm i's outside option of not buying the license ($\overline{\Pi}_i^F \geq \underline{\Pi}_i^F$). Notice that $\overline{\Pi}_i^F \geq \underline{\Pi}_i^F$ if and only if $4\varepsilon \left(2 + \varepsilon \left(1 - \gamma\right) - \gamma\right) \left(2 - \gamma\right)^{-2} \left(2 + \gamma\right)^{-2} \geq F_i$. Then, the equilibrium fixed fees are equal to

$$F_i^{\text{SPE}}(2) = \frac{\alpha \left(1 + \varepsilon\right)^2}{\left(2 + \gamma\right)^2} \text{ if } \alpha \leq \alpha_2^F$$

and to $4\varepsilon \left(2 + \varepsilon \left(1 - \gamma\right) - \gamma\right) \left(2 - \gamma\right)^{-2} \left(2 + \gamma\right)^{-2}$ if $\alpha > \alpha_2^F$, where the upper limit on α that guarantees that firm i prefers the Nash bargaining solution to the outside option is given by $\alpha_2^F \equiv 4\varepsilon \left(2 + \varepsilon \left(1 - \gamma\right) - \gamma\right) \left(1 + \varepsilon\right)^{-2} \left(2 - \gamma\right)^{-2}$; with $\alpha_2^F > \alpha_1^F$. The patent holder's equilibrium revenue is equal to

$$V^{F^*}(2) = \frac{2\alpha \left(1 + \varepsilon\right)^2}{\left(2 + \gamma\right)^2} \text{ if } \alpha \leq \alpha_2^F \tag{3}$$

and to $8\varepsilon \left(2 + \varepsilon \left(1 - \gamma\right) - \gamma\right) \left(2 - \gamma\right)^{-2} \left(2 + \gamma\right)^{-2}$ if $\alpha > \alpha_2^F$. Firm i's equilibrium profits are equal to

$$\Pi_i^{F^*}(2) = \frac{\left(1 - \alpha\right) \left(1 + \varepsilon\right)^2}{\left(2 + \gamma\right)^2} \text{ if } \alpha \leq \alpha_2^F \tag{4}$$

and to $\left(4 \left(1 - \gamma \left(1 + \varepsilon\right)\right) + \gamma^2 \left(1 + 2\varepsilon + \varepsilon^2\right)\right) \left(2 - \gamma\right)^{-2} \left(2 + \gamma\right)^{-2}$ if $\alpha > \alpha_2^F$. Comparing $V^{F^*}(1)$ with $V^{F^*}(2)$ we obtain the following lemma. The proof of this lemma, as well as the other proofs, may be found in the appendix. Define

$$\varepsilon^F(\gamma) \equiv \frac{2 \left(1 - \gamma\right) \left(2 - \gamma\right) + \gamma \left(2 - \gamma\right) \sqrt{2}}{2 \left(4\gamma - \gamma^2 - 2\right)} \tag{5}$$

which is a decreasing function of γ. We have that[11]

$$\varepsilon \leq \varepsilon^F(\gamma) \text{ if and only if } \gamma \leq \frac{\left(2 - \sqrt{2}\right) \left(\varepsilon + 1\right)}{\varepsilon + 1 - \frac{1}{2}\sqrt{2}} \equiv \gamma^F(\varepsilon).$$

[11] Notice that $\varepsilon^F(1) = 0.707$ and $\varepsilon^F(\gamma) < 1$ if and only if $\gamma > 0.91$.

Lemma 1. *Consider the fixed fee licensing game. The patent holder prefers complete technology diffusion rather than exclusive licensing except if the innovation size is big and the market competition is strong (namely if* $\varepsilon > \varepsilon^F(\gamma)$*, possible only for* $\gamma > 0.91$*, the patent holder prefers exclusive licensing).*

Notice that the patent holder's preferences towards technology diffusion are independent of the bargaining power. The patent holder appropriates a share α of the profits, so that the higher the profits of the firms are the higher the revenue of the patent holder is. When market competition is mild (γ small), both firms make high profits so that the patent holder prefers to sell two licenses. In contrast, when market competition is strong and the innovation is almost drastic, the patent holder prefers exclusive licensing because the innovating firm will have a large cost advantage which allows the innovating firm to conquer most of the market. Exclusive licensing is more likely as brands become closer substitutes (in fact in this case monopoly profits are larger than the sum of duopoly profits).

3.2 Royalty licensing game

We denote by $R(k)$ the per-unit royalty licensing game where the patent holder decides to sell k licenses in the first stage.

We first consider the per-unit royalty licensing game with exclusive licensing, $R(1)$, where the patent holder and firm i negotiate over the per-unit royalty R_i. The negotiation still proceeds as in Rubinstein's (1982) alternating-offer bargaining model. Thus the predicted royalty is given by

$$R_i^{\text{SPE}}(1) = \arg\max \left[\frac{(2-\gamma)+2(\varepsilon-R_i)}{(2-\gamma)(2+\gamma)}R_i\right]^{\alpha} \cdot \left[\frac{(2-\gamma)+2(\varepsilon-R_i)}{(2-\gamma)(2+\gamma)}\right]^{2(1-\alpha)}$$

where $\alpha \in (0,1)$ is the patent holder bargaining power which is equal to $r_f/(r_p+r_f)$ and the status quo payoffs are zero, subject to firm i's outside option of not buying the license ($\overline{\Pi}_i^R \geq \Pi_i^R$ if and only if $\varepsilon \geq R_i$). Then, the equilibrium royalty is equal to

$$R_i^{\text{SPE}}(1) = \frac{\alpha(2(1+\varepsilon)-\gamma)}{4}$$

if $\alpha \leq \alpha^R$ and to ε if $\alpha > \alpha^R$, where $\alpha^R \equiv 4\varepsilon/(2(1+\varepsilon)-\gamma)$ is the upper limit on the patent holder bargaining power that guarantees that firm i prefers the Nash bargaining solution to the outside option. This upper limit is increasing in γ and ε. If the patent holder is very powerful ($\alpha > \alpha^R$), then the equilibrium per-unit royalty,

$R_i^{\text{SPE}}(1)$, is settled equal to ε. The patent holder's equilibrium revenue, $V^{R^*}(1)$, is equal to

$$V^{R^*}(1) = \frac{\alpha\,(2-\alpha)\,(2\,(1+\varepsilon)-\gamma)^2}{8\,(2-\gamma)\,(2+\gamma)} \text{ if } \alpha \leq \alpha^R \tag{6}$$

and to $\varepsilon/(2+\gamma)$ if $\alpha > \alpha^R$. Firm i's equilibrium profits are equal to

$$\Pi_i^{R^*}(1) = \left(\frac{(2-\alpha)\,(2\,(1+\varepsilon)-\gamma)}{2\,(2-\gamma)\,(2+\gamma)}\right)^2 \text{ if } \alpha \leq \alpha^R \tag{7}$$

and to $1/(2+\gamma)^2$ if $\alpha > \alpha^R$. Firm j's equilibrium profits are equal to

$$\Pi_j^{R^*}(1) = \left(\frac{\alpha\gamma\,(2\,(1+\varepsilon)-\gamma)+4\,(2-\gamma\,(1+\varepsilon))}{4\,(2-\gamma)\,(2+\gamma)}\right)^2 \text{ if } \alpha \leq \alpha^R$$

and to $1/(2+\gamma)^2$ if $\alpha > \alpha^R$.

We now consider the per-unit royalty licensing game with complete technology diffusion, $R(2)$, where the patent holder and firm 1 negotiate over the per-unit royalty R_1 while the patent holder and firm 2 negotiate over the per-unit royalty R_2. The predicted per-unit royalties are given by

$$R_i^{\text{SPE}}(2) = \arg\max \left[\frac{(2-\gamma)\,(1+\varepsilon)-2R_i+\gamma R_j}{(2-\gamma)\,(2+\gamma)}R_i\right]^{\alpha} \cdot \left[\frac{(2-\gamma)\,(1+\varepsilon)-2R_i+\gamma R_j}{(2-\gamma)\,(2+\gamma)}\right]^{2(1-\alpha)},$$

$i = 1, 2$, $i \neq j$, where the status quo payoffs are zero, subject to firm i's outside option of not buying the license ($\overline{\Pi}_i^R \geq \underline{\Pi}_i^R$ if and only if $\varepsilon \geq R_i$). Then, the equilibrium royalties are equal to

$$R_i^{\text{SPE}}(2) = \frac{\alpha\,(2-\gamma)\,(1+\varepsilon)}{(4-\alpha\gamma)} \text{ if } \alpha \leq \alpha^R$$

and to ε if $\alpha > \alpha^R$, $i = 1, 2$. The patent holder's equilibrium revenue, $V^{R^*}(2)$, is equal to

$$V^{R^*}(2) = \frac{4\alpha\,(2-\alpha)\,(2-\gamma)\,(\varepsilon+1)^2}{(\alpha\gamma-4)^2\,(\gamma+2)} \text{ if } \alpha \leq \alpha^R \tag{8}$$

and to $2\varepsilon/(2+\gamma)$ if $\alpha > \alpha^R$. Firms' equilibrium profits are equal to

$$\Pi_i^{R^*}(2) = \left(\frac{8\,(1+\varepsilon)-4\,((1+\varepsilon)\,\alpha+\gamma)+\alpha\gamma^2}{(4-\alpha\gamma)\,(2-\gamma)\,(2+\gamma)}\right)^2 \text{ if } \alpha \leq \alpha^R \tag{9}$$

and to $1/(2+\gamma)^2$ if $\alpha > \alpha^R$. As long as the patent holder is not too powerful ($\alpha \leq \alpha^R$), the equilibrium per-unit royalty is increasing in α and ε but decreasing

in γ. Notice that $R_i^{\text{SPE}}(1) \geq R_i^{\text{SPE}}(2)$. Comparing $V^{R^*}(1)$ with $V^{R^*}(2)$ we obtain the following lemma. Define

$$\alpha_1 \equiv \frac{4\left(2\varepsilon-\gamma+2\right)^2+4\sqrt{2}\left(\varepsilon+1\right)\left(2-\gamma\right)\left(2\varepsilon-\gamma+2\right)}{\gamma\left(2\varepsilon-\gamma+2\right)^2}$$

with $\alpha_1 > 0$ if and only if $\gamma > (2\left(\varepsilon+1\right)\left(2\varepsilon+1\right)-2\varepsilon\left(\varepsilon+1\right)\sqrt{2})/(4\varepsilon+2\varepsilon^2+1) = \gamma^F\left(\varepsilon\right)$,[12] and $\alpha_1 < \alpha^R$.

Lemma 2. *Consider the royalty licensing game. The patent holder prefers complete technology diffusion rather than exclusive licensing except if the patent holder's bargaining power is low, the market competition is strong and the innovation size is big (namely if $\alpha \leq \alpha_1$, possible only for $\gamma > \gamma^F\left(\varepsilon\right)$ and $\varepsilon > 0.707$, the patent holder prefers exclusive licensing).*

When the patent holder is very powerful ($\alpha > \alpha^R$), the negotiation leads to a royalty equal to the innovation size, independently of the degree of product differentiation (γ). Hence, the patent holder prefers to sell licenses to both firms.

In contrast, when the bargaining power of the patent holder is weak ($\alpha < \alpha^R$), the equilibrium royalty rate depends on α, on the toughness of the market competition (γ) as well as on the innovation size (ε). In particular it increases with α and ε, and it decreases with γ. In this case as long as the competition in the market is sufficiently mild ($\gamma < \gamma^F$, always for small enough innovations, i.e. $\varepsilon < 0.707$) both firms can sustain a sufficiently high per-unit royalty such that the patent holder prefers to sell the license to both firms. In contrast, for ε and γ high the patent holder prefers exclusive licensing when $\alpha < \alpha_1$. In particular, the patent holder can negotiate a higher price under exclusive licensing because the innovating firm, thanks to the cost advantage which is increasing with the competition, makes higher profits and can afford a higher marginal cost. The more competitive the market and the larger the innovation size, the more likely exclusive licensing will be chosen (as α_1 is increasing in γ and ε).

3.3 Fixed fee versus royalty

Which mode of licensing a non-drastic innovation does the patent holder prefer under complete information? Remember that $\varepsilon^F(\gamma)$ (Expression 5) is the cut-off value on ε such that, in case of licensing by means of fixed fees, the patent holder

[12] Notice that $\gamma^F\left(\varepsilon\right) < 1$ if and only if $\varepsilon > 0.707$.

prefers exclusive licensing to complete technology diffusion if and only if $\varepsilon > \varepsilon^F(\gamma)$. This cut-off value, $\varepsilon^F(\gamma)$, decreases with γ.

Proposition 1. *In case of bargaining with complete information, the patent holder always prefers licensing by means of fixed fees to licensing by means of per-unit royalties, $\forall \gamma, \varepsilon, \alpha$. In addition, the patent holder prefers to choose complete technology diffusion if and only if $\varepsilon < \varepsilon^F(\gamma)$.*

The patent holder always prefers to negotiate fixed fees whatever the degree of product market competition, the innovation size and its bargaining power.[13] The intuition is as follows. A per-unit royalty rate represents an additional marginal cost of production for the firms (so it implies a distortion); hence, decreasing their profits. The patent holder prefers to bargain a fixed fee rather than a per-unit royalty because the fixed fee allows the patent holder to appropriate a higher share of the profits. Moreover, technology diffusion depends on the innovation size as well as the degree of product differentiation: as goods become closer substitutes, it is more likely that exclusive licensing occurs (as $\varepsilon^F(\gamma)$ decreases with γ).

Proposition 1 is in line with previous literature on optimal licensing of an external patentee under Cournot competition (see Kamien (1992), Kamien, Oren and Tauman (1992) and Kamien and Tauman (2002)).[14] Kishimoto and Muto (2012) have compared fixed fee versus royalty in an homogeneous Cournot duopoly in the presence of an incumbent innovator. They have shown that royalty licensing is more profitable than fixed fee licensing for both firms as long as the innovation is non-drastic. In our paper, we show that the patent holder always prefers to negotiate fixed fees whatever the degree of product market competition, the innovation size and its bargaining power in the presence of an outside innovator. The relevant assumption in order to derive the different results is not the degree of product market competition nor the patent holder's bargaining power but the kind of innovator (an incumbent versus an outside innovator). Namely, the outside innovator has no distortion incentives as its aim is to maximize the licensees' profits in order to extract these profits up to the outside options via the fixed fees. In contrast, the incumbent innovator finds it profitable to distort the rival's production cost through a positive

[13] Notice that the patent holder has the same bargaining power, α, whatever the licensing scheme (fixed fee or per-unit royalty). Hence, the comparison between the patent holder revenues $V^{R^*}(1)$ and $V^{F^*}(2)$ is done for the same given bargaining power α.

[14] In case of Bertrand competition, Muto (1993) has shown that licensing by means of royalties is more profitable when the innovation size is small and the patent holder has full bargaining power. Indeed, higher prices can compensate for higher royalty costs under Bertrand competition.

per-unit royalty in order to get a competitive advantage. However, as it is shown in the next section, the outside innovator may also prefer licensing by means of per-unit royalties once we introduce incomplete information in the bargaining process.

4 Bargaining with private information

4.1 Fixed fee or royalty

Both the asymmetric Nash bargaining solution and Rubinstein's model predict efficient outcomes of the bargaining process (in particular, agreement is reached immediately). This is not true if we introduce incomplete information into the bargaining. In this case, the early rounds of negotiation are used for information transmission between the two negotiators. We now suppose that negotiators have private information. Neither negotiator knows the impatience (or discount rate) of the other party. It is common knowledge that the firm's discount rate is included in the set $[r_f^{\mathrm{P}}, r_f^{\mathrm{I}}]$, where $0 < r_f^{\mathrm{P}} \leq r_f^{\mathrm{I}}$, and that the patent holder's discount rate is included in the set $[r_p^{\mathrm{P}}, r_p^{\mathrm{I}}]$, where $0 < r_p^{\mathrm{P}} \leq r_p^{\mathrm{I}}$. The superscripts "I" and "P" identify the most impatient and most patient types, respectively. The types are independently drawn from the set $[r_i^{\mathrm{P}}, r_i^{\mathrm{I}}]$ according to the probability distribution p_i, for $i = p, f$. We allow for general distributions over discount rates. This uncertainty implies bounds on the patent holder's bargaining power which are denoted by $\underline{\alpha} = r_f^{\mathrm{P}}/(r_p^{\mathrm{I}} + r_f^{\mathrm{P}})$ and $\overline{\alpha} = r_f^{\mathrm{I}}/(r_p^{\mathrm{P}} + r_f^{\mathrm{I}})$. We assume that the upper bound on the patent holder bargaining power is below some critical level, $\overline{\alpha} < \alpha_1^F$. This assumption guarantees that firms do not take their outside options when bargaining occurs in the presence of private information.

Watson (1998) has characterized the set of perfect Bayesian equilibrium (PBE) payoffs that may arise in Rubinstein's alternating-offer bargaining game and constructed bounds (which are met) on the agreements that may be made. The bounds and the PBE payoffs set are determined by the range of incomplete information and are easy to compute because they correspond to the SPE payoffs of two bargaining games with complete information. These two games are defined by matching one player's most impatient type with the opponent's most patient type. As Watson (1998) stated, "each player will be no worse than he would be in equilibrium if it were common knowledge that he were his least patient type and the opponent were his most patient type. Furthermore, each player will be no better than he would be in equilibrium with the roles reversed". Since we allow for general distributions

over types, multiplicity of PBE is not an exception. There are PBE in which the outcome is close to the upper bound, and there are PBE in which the outcome is close to the lower bound.

In complete information, the patent holder always prefers to license by means of fixed fees for any given bargaining power. But, once the patent holder and the firms have private information, this complete information result does not necessarily hold because of the multiplicity of PBE. PBE close to the upper bound in case of non-exclusive licensing and fixed fees are preferred by the patent holder to PBE close to the upper bound in case of exclusive licensing and royalties. Similarly, PBE close to the lower bound in case of non-exclusive licensing and fixed fees are preferred by the patent holder to PBE close to the lower bound in case of exclusive licensing and royalties. However, the patent holder prefers PBE close to the upper bound in case of exclusive licensing and royalties to PBE close to the lower bound in case of non-exclusive licensing and fixed fees once there is enough uncertainty about the bargaining power. When incomplete information vanishes (in that $\underline{\alpha}$ and $\overline{\alpha}$ converge), we recover the results under complete information.

Suppose that firms are producing homogeneous brands, $\gamma = 1$. Suppose that the patent holder chooses to negotiate a fixed fee. In case of non-exclusive licensing, the patent holder and the firms may reach a fixed fee agreement close to the lower bound, i.e. the outcome of the complete information game when it is known that the patent holder bargaining power is $\underline{\alpha}$. For instance, the firm may decide to update much more optimistically its beliefs about the patent holder bargaining power (putting probability one on the patent holder's weakest type) in the case the patent holder deviates from the equilibrium path. The new beliefs lead to a continuation game in which the patent holder's prospects have diminished, which deters deviation in the first place and supports the equilibrium close to the lower bound.[15] Suppose now that the patent holder chooses to negotiate a per-unit royalty and to sell only one license. Choosing an alternative mode of licensing modifies the bargaining environment and can induce the bargaining process to switch to an equilibrium close to the upper bound, i.e. the outcome of the complete information game when it is known that the patent holder bargaining power is $\overline{\alpha}$. Since the upper bound on the patent holder's revenue under exclusive licensing by means of a per-unit royalty is greater than the lower bound on the patent holder's revenue under non-exclusive

[15] Perfect Bayesian equilibrium allows great latitude for such revision of beliefs, because it occurs off the equilibrium path.

licensing by means of fixed fees, the patent holder may prefer exclusive licensing by means of a per-unit royalty to non-exclusive licensing by means of fixed fees.[16]

4.2 Maximum delay in reaching an agreement

Inefficient outcomes are possible, even as the period length shrinks to zero. The bargaining game may involve delay, but not perpetual disagreement, in equilibrium.[17] In fact, delay is positively related to the distance between the discount rates of the most and least patient types of the players. If the range of types is reduced, then this leads to a smaller range of possible payoffs and less delay. Delay can occur even when the game is close to one of complete information (as the type distributions converge to point mass distributions). We propose to analyze the maximum delay time in reaching an agreement. Only on average is this measure a good proxy for actual delay.[18] In the appendix we compute the maximum delay in equilibrium which shows that an agreement is reached in finite time and that delay time equals zero as incomplete information vanishes (in that r_i^{P} and r_i^{I} converge).

In case of licensing by means of fixed fees, the maximum real delay time in reaching an agreement is given by

$$D(F) = \min\left\{D^p(F), D^f(F)\right\} \tag{10}$$

where

$$D^p(F) = -\frac{1}{r_p^{\mathrm{P}}} \cdot \log\left[\frac{r_f^{\mathrm{P}}}{r_f^{\mathrm{I}}} \cdot \frac{r_p^{\mathrm{P}} + r_f^{\mathrm{I}}}{r_p^{\mathrm{I}} + r_f^{\mathrm{P}}}\right] \tag{11}$$

is the maximum real time the patent holder would spend negotiating, and

$$D^f(F) = -\frac{1}{r_f^{\mathrm{P}}} \cdot \log\left[\frac{r_p^{\mathrm{P}}}{r_p^{\mathrm{I}}} \cdot \frac{r_f^{\mathrm{P}} + r_p^{\mathrm{I}}}{r_f^{\mathrm{I}} + r_p^{\mathrm{P}}}\right] \tag{12}$$

[16] The patent holder may even prefer exclusive licensing by means of a per-unit royalty in case of bargaining with almost complete information. Suppose there are three possible types r_i^{P}, r_i^*, r_i^{I} where $r_i^{\mathrm{P}} < r_i^* < r_i^{\mathrm{I}}$ and the distribution over these types $(r_i^{\mathrm{P}}, r_i^*, r_i^{\mathrm{I}})$ is $(\beta, 1-2\beta, \beta)$. Watson (1998) has shown that, as β converges to zero, PBE outcomes do not converge to a single outcome, despite that the distribution over types converges to a point mass distribution. The limiting set of equilibrium payoffs is defined by each player's greatest possible discount rate and the limiting discount rates; the players' lowest possible discount rates do not play a role.

[17] Watson (1998) has constructed equilibria with delay in which the types of each player behave identically (no information is revealed in equilibrium), players use pure strategies, and players make non-serious offers until some appointed date.

[18] It is not uncommon in the literature on bargaining to analyze the maximum delay before reaching an agreement. See, for instance, Cramton (1992) and Cai (2003).

is the maximum real time the firm would spend negotiating. In fact, $D^p(F)$ is the maximum real time the patent holder would spend negotiating if it were of the most patient type. Similarly, $D^f(F)$ is the maximum real time the firm would spend negotiating if it were of the most patient type. So, $D^p(T)$ and $D^f(T)$ are the upper bounds on the maximum time the patent holder of type r_p and the firm of type r_f would spend negotiating. This maximum time decreases with type r_p (r_f). So, the more patient a player is the greater the delay that may be observed. Since $D^p(T)$ and $D^f(T)$ are positive, finite numbers, the maximum real delay in reaching an agreement in case of licensing by means of fixed fees is finite and converges to zero as r_i^{I} and r_i^{P} become close.

In case of exclusive licensing by means of a per-unit royalty, the maximum real delay time in reaching an agreement is given by

$$D(R(1)) = \min\left\{D^p(R(1)), D^f(R(1))\right\} \tag{13}$$

where

$$D^p(R(1)) = -\frac{1}{r_p^{\mathrm{P}}} \cdot \log\left[\frac{r_f^{\mathrm{P}}}{r_f^{\mathrm{I}}} \cdot \frac{r_f^{\mathrm{P}} + 2r_p^{\mathrm{I}}}{r_f^{\mathrm{I}} + 2r_p^{\mathrm{P}}} \cdot \left(\frac{r_p^{\mathrm{P}} + r_f^{\mathrm{I}}}{r_p^{\mathrm{I}} + r_f^{\mathrm{P}}}\right)^2\right] \tag{14}$$

is the maximum real time the patent holder would spend negotiating, and

$$D^f(R(1)) = -\frac{1}{r_f^{\mathrm{P}}} \cdot \log\left[\left(\frac{2r_p^{\mathrm{P}} + r_f^{\mathrm{I}}}{2r_p^{\mathrm{I}} + r_f^{\mathrm{P}}}\right)^2 \cdot \left(\frac{r_f^{\mathrm{P}} + r_p^{\mathrm{I}}}{r_f^{\mathrm{I}} + r_p^{\mathrm{P}}}\right)^2\right] \tag{15}$$

is the maximum real time the firm would spend negotiating. Since $D^p(R(1))$ and $D^f(R(1))$ are positive, finite numbers, the maximum real delay in reaching an agreement in case of exclusive licensing by means of a per-unit royalty is finite and converges to zero as r_i^{I} and r_i^{P} become close.

In case of complete technology diffusion by means of per-unit royalties, the maximum real delay time in reaching an agreement is given by

$$D(R(2)) = \min\left\{D^p(R(2)), D^f(R(2))\right\} \tag{16}$$

where

$$D^p(R(2)) = -\frac{1}{r_p^{\mathrm{P}}} \cdot \log\left[\frac{r_f^{\mathrm{P}}}{r_f^{\mathrm{I}}} \cdot \frac{r_f^{\mathrm{P}} + 2r_p^{\mathrm{I}}}{r_f^{\mathrm{I}} + 2r_p^{\mathrm{P}}} \cdot \left(\frac{4r_p^{\mathrm{P}} + (4-\gamma)\, r_f^{\mathrm{I}}}{4r_p^{\mathrm{I}} + (4-\gamma)\, r_f^{\mathrm{P}}}\right)^2\right] \tag{17}$$

is the maximum real time the patent holder would spend negotiating, and

$$D^f(R(2)) = -\frac{1}{r_f^{\mathrm{P}}} \cdot \log \left[\left(\frac{2r_p^{\mathrm{P}} + r_f^{\mathrm{I}}}{2r_p^{\mathrm{I}} + r_f^{\mathrm{P}}} \right)^2 \cdot \left(\frac{(4-\gamma)\, r_f^{\mathrm{P}} + 4r_p^{\mathrm{I}}}{(4-\gamma)\, r_f^{\mathrm{I}} + 4r_p^{\mathrm{P}}} \right)^2 \right] \tag{18}$$

is the maximum real time the firm would spend negotiating. Since $D^p(R(2))$ and $D^f(R(2))$ are positive, finite numbers, the maximum real delay in reaching an agreement in case of complete technology diffusion by means of per-unit royalties is finite and converges to zero as r_i^{I} and r_i^{P} become close. We have that

$$\frac{\partial D^p(R(2))}{\partial \gamma} > 0 \text{ and } \frac{\partial D^f(R(2))}{\partial \gamma} < 0.$$

Proposition 2. *In case of complete technology diffusion by means of per-unit royalties, a decrease in product differentiation (as γ increases) increases the maximum real time the patent holder would spend negotiating but decreases the maximum real time the firm would spend negotiating.*

When brands are substitutable, i.e., $\gamma > 0$, royalty settlements create spillover effects (by altering the firms' relative competitive positions in the product market) that have implications for the outcome of negotiations. Spillover effects are decreasing with the degree of product differentiation. Spillover effects create incentives for firms to concede (the tougher the competition in the market the higher the value of the innovation for competing firms) but incentives for the patent holder to make less concessions in order to obtain greater royalties. Hence, it is ambiguous whether $D(R\,(2))$ increases or decreases with γ.

Comparing (12) with (15) and (15) with (18) we have that $D^f(F) > D^f(R(1)) \geq D^f(R(2))$. Comparing (11) with (17) and (14) with (17) we have that $D^p(F) > D^p(R(2)) \geq D^p(R(1))$. The maximum real time the firm would spend negotiating in case of licensing by means of royalties is decreasing with the number of licenses sold, but the maximum real time the patent holder would spend negotiating is increasing with the number of licenses sold. Thus, in case of per-unit royalties, the number of licenses sold has an ambiguous effect on the maximum real delay time in reaching an agreement.[19] When the patent holder chooses complete technology diffusion, firms have incentives to concede more rapidly because of increased spillover effects. However, complete technology diffusion raises the potential payoffs for the patent

[19] Spillover effects are clearly absent for $\gamma = 0$, that is when goods are perfectly differentiated, which coincides, in case of two firms, with two local monopolies. In this case: $D^f(R(1)) = D^f(R(2))$, $D^p(R(1)) = D^p(R(2))$ and in turn $D\,(R\,(1)) = D\,(R\,(2))$.

holder, and in expanding the payoff set, also increases the scope for delay (longer negotiations may be needed for screening the private information). In case of fixed fees, the rents to be divided do not depend on the negotiated fees. For instance, anticipating that an agreement will be reached in the negotiation with firm j, the negotiation between the patent holder and firm i has no incidence on the payoff set of the negotiation with firm j, and vice versa. Hence, the delay in reaching an agreement does not depend on the number of licenses sold but only depends on the private information. In addition, the maximum real delay time in reaching an agreement in case of fixed fees is longer than the maximum real delay time in case of per-unit royalties. The reason is the absence of spillovers.

Then, we obtain the following proposition.

Proposition 3. *The maximum real delay time in reaching an agreement is longer when the patent holder and the firm negotiate over fixed fees. That is,* $D(F) > D(R(2))$ *and* $D(F) > D(R(1))$.

Suppose that $r_p^{\mathrm{P}} = r_f^{\mathrm{P}}$ and $r_p^{\mathrm{I}} = r_f^{\mathrm{I}}$. Then, comparing (17) with (18) we have that, in case of licensing by means of royalties, the maximum real delay time in reaching an agreement is decreasing with the number of licenses sold.

Corollary 1. *Suppose that* $r_p^P = r_f^P$ *and* $r_p^I = r_f^I$. *Then,* $D(F) > D(R(1)) \geq D(R(2))$.

Thus, licensing by means of fixed fees instead of royalties would increase the maximum real delay time in reaching an agreement. So, the likelihood of having more inefficient outcomes in case of licensing by means of fixed fees may explain why licensing by means of per-unit royalties is commonly used in practice. We now provide an example of the maximum delay. In this example, let $r_f^{\mathrm{P}} = r_p^{\mathrm{P}} = r^{\mathrm{P}}$, $r_f^{\mathrm{I}} = r_p^{\mathrm{I}} = r^{\mathrm{I}}$, $r^{\mathrm{I}} = 0.36 - r^{\mathrm{P}}$ with $r^{\mathrm{P}} \in [0.04, 0.18]$. Table 1 gives the integer part of the maximum delay for the different modes of licensing and for different values of the parameter γ. We can interpret r_i as the annual discount rate and the numbers in Table 1 as the maximum number of days needed to reach an agreement. Indeed, the integer part of the maximum delays for $\Delta = 1/365$ are exactly the numbers in Table 1. The data seem consistent with the number of days between the date of application of the licensed patents and the date of the licensing contracts as reported in Sakakibara (2010). We observe that (i) the real delay time in reaching an agreement is not negligible: many bargaining rounds may be needed in equilibrium before an agreement is reached; (ii) D^p and D^f are increasing with the amount of

private information $\left|r_i^{\mathrm{P}} - r_i^{\mathrm{I}}\right|$; (iii) $D(R(2))$ is increasing with the degree of product differentiation (γ); (iv) $D(R(k))$ is decreasing with the number of licenses sold (k).

	$F(k)$		$R(1)$		$R(2);\gamma=\frac{1}{2}$		$R(2);\gamma=\frac{3}{4}$		$R(2);\gamma=1$	
r^{P}	D^p	D^f	D^p	D^f	D^p	D^f	D^p	D^f	D^p	D^f
0.18	0	0	0	0	0	0	0	0	0	0
0.17	0	0	0	0	0	0	0	0	0	0
0.16	1	1	0	0	1	0	1	0	1	0
0.15	2	2	1	1	1	1	1	1	2	0
0.14	3	3	2	2	2	1	2	1	3	1
0.13	4	4	2	2	3	2	3	1	4	1
0.12	5	5	3	3	4	2	5	2	5	2
0.11	7	7	5	4	6	3	6	3	7	2
0.10	9	9	6	5	7	4	8	4	9	3
0.09	12	12	8	7	9	5	10	5	11	4
0.08	15	15	10	9	12	7	13	6	14	5
0.07	20	20	14	11	16	9	18	8	19	6
0.06	26	26	19	15	22	12	23	10	25	8
0.05	36	36	26	19	30	15	32	13	34	11
0.04	51	51	38	26	43	21	46	18	49	15

Table 1: Maximum delay in reaching an agreement

5 Conclusion

We have considered a model of licensing of a non-drastic innovation in which the patent holder (an outside innovator) negotiates either up-front fixed fees or per-unit royalties with two firms producing horizontally differentiated brands and competing à la Cournot. We have studied how licensing schemes (fixed fee or per-unit royalty) and the number of licenses sold (exclusive licensing or complete technology diffusion) affect price agreements and delays in reaching an agreement. We have obtained that, under complete information, the patent holder prefers to license by means of up-front fixed fees. However, once there is private information about the relative bargaining power of the parties, the patent holder may prefer licensing by means of per-unit royalties even if market competition is strong. In addition, the delay in reaching an agreement is greater whenever the patent holder chooses to negotiate up-front fixed

fees instead of per-unit royalties. Thus, the relative bargaining power of a patent holder and a licensee provides another rationale for the wide prevalence of royalties in practice.

Sen and Tauman (2007) have shown that the optimal licensing mode for a drastic innovation is a fixed fee when the patent holder has all bargaining power as the patent holder earns monopoly profit. This result extends to any level of bargaining power.[20]

Some extensions may be worthwhile. A first one is to extend the bargaining model to allow parties to negotiate over the choice of a licensing scheme. A second one is to consider more general licensing schemes like two-part tariffs.

Appendix

A Proofs

Proof of Lemma 1.

- **Case 1:** $\alpha < \alpha_1^F < \alpha_2^F$. We have that $V^F(2) > V^F(1)$ for $\gamma < 0.585$. For $\gamma > 0.585$, we have

$$V^F(2) > V^F(1) \text{ if and only if } \varepsilon < \frac{2(1-\gamma)(2-\gamma)+\gamma(2-\gamma)\sqrt{2}}{2(4\gamma-\gamma^2-2)} \equiv \varepsilon^F(\gamma).$$

  Notice that $\varepsilon^F(\gamma)$ is a decreasing function of γ. In particular, $\varepsilon^F(1) = 0.707$ and $\varepsilon^F(\gamma) < 1$ if and only if $\gamma > 0.91$. Therefore, only for high values of γ and ε, we have $V^F(1) > V^F(2)$. Notice also that

$$\varepsilon < \varepsilon^F(\gamma) \text{ if and only if } \gamma < \frac{2(2-\sqrt{2})(\varepsilon+1)}{2\varepsilon+2-\sqrt{2}} \equiv \gamma^F(\varepsilon).$$

- **Case 2:** $\alpha_1^F \leq \alpha \leq \alpha_2^F$. We have that

$$V^F(2) - V^F(1) = \frac{2\alpha(1+\varepsilon)^2}{(2+\gamma)^2} - \frac{4(2-\gamma+\varepsilon)\varepsilon}{(2+\gamma)^2(2-\gamma)^2} > 0$$

  if and only if

$$\alpha > \frac{2(2-\gamma+\varepsilon)\varepsilon}{(1+\varepsilon)^2(2-\gamma)^2} \equiv \alpha^T < \alpha_2^F.$$

[20] In case of a drastic innovation, the patent holder profit under a fixed fee is $\alpha(\varepsilon+1)^2/4$ whereas the profit under a per-unit royalty is $(2-\alpha)(\varepsilon+1)^2\alpha/8$. Thus, the patent holder always prefers a fixed fee.

Notice that $\alpha^T < \alpha_1^F$ if and only if $\varepsilon < \varepsilon^F(\gamma)$. Thus, if $\varepsilon < \varepsilon^F(\gamma)$ then $V^F(1) < V^F(2)$; otherwise, $V^F(1) < V^F(2)$ for $\alpha > \alpha^T$ and $V^F(1) > V^F(2)$ for $\alpha < \alpha^T$.

- **Case 3:** $\alpha > \alpha_2^F$. We have that

$$V^F(2) - V^F(1) = \frac{4(2-\gamma+\varepsilon-2\varepsilon\gamma)\varepsilon}{(2+\gamma)^2(2-\gamma)^2} > 0.$$

Proof of Lemma 2.

- **Case 1:** $\alpha > \alpha^R$. We have that $V^R(2) - V^R(1) = \varepsilon/(\gamma+2) > 0$.

- **Case 2:** $\alpha \leq \alpha^R$.
We have that $V^R(2) - V^R(1) = 4\alpha(2-\alpha)(2-\gamma)(\varepsilon+1)^2/((\alpha\gamma-4)^2(\gamma+2))$ $-\alpha(2-\alpha)(2(1+\varepsilon)-\gamma)^2/(8(2-\gamma)(2+\gamma)) > 0$ if and only if $\alpha^2\gamma^2(2\varepsilon-\gamma$ $+2)^2 - \alpha 8\gamma(2\varepsilon-\gamma+2)^2 - 16(8\varepsilon-4\gamma-12\gamma\varepsilon+\gamma^2+4\varepsilon^2-8\gamma\varepsilon^2+4\gamma^2\varepsilon+2\gamma^2\varepsilon^2$ $+4) < 0$. That is, $V^R(2) - V^R(1) > 0$ if and only if $\alpha > (4(2\varepsilon-\gamma+2)^2 +$ $4\sqrt{2}(2-\gamma)(\varepsilon+1)(2\varepsilon-\gamma+2))/(\gamma(2\varepsilon-\gamma+2)^2) \equiv \alpha_1$. Notice that $\alpha_1 > 0$ if and only if $\gamma > \gamma^F(\varepsilon)$, and $\gamma^F(\varepsilon) < 1$ if and only if $\varepsilon > 0.707$. Notice also that α_1 is increasing in ε and γ. If $\varepsilon < 0.707$ then $\gamma^F(\varepsilon) > 1$ and $\alpha_1 < 0$ and so $V^R(2) - V^R(1) > 0$. If $\varepsilon > 0.707$ then $\gamma^F(\varepsilon) < 1$ and so $\alpha_1 > 0$ if and only if $\gamma > \gamma^F(\varepsilon)$ and so $V^{R^*}(2) - V^{R^*}(1) > 0$ if and only if $\alpha > \alpha_1$.

Proof of Proposition 1. We give the proof for the case of homogeneous brands ($\gamma = 1$) and the case of monopoly ($\gamma = 0$). The general proof involves tedious comparisons and is available from the authors upon request. We have the following ranking for the α thresholds:

$$\alpha_1 < \alpha_1^F \leq \alpha_2^F < \alpha^R.$$

- **Homogeneous brands case.** Let $\gamma = 1$.

  - Consider first the region $\gamma < \gamma^F(\varepsilon)$ (always true for $\varepsilon < 0.707$). We know from Lemma 1 that $V^F(2) > V^F(1)$ and from Lemma 2 that $V^R(2) > V^R(1)$. For $\alpha \in (0, \alpha_2^F)$, we have that $V^R(2) - V^F(2) =$ $-(2/9)((\varepsilon+1)^2(\alpha^2-2\alpha+4)\alpha)/(\alpha-4)^2 < 0$.
  Notice that $\alpha \in (\alpha_2^F, \alpha^R)$ if and only if $\varepsilon \in (\varepsilon_\alpha(1) = \alpha/2(2-\alpha), \varepsilon_1(1) =$ $(2-\alpha-2\sqrt{1-\alpha})/\alpha)$. For $\alpha \in (\alpha_2^F, \alpha^R)$, we have that $V^R(2) - V^F(2) =$

$-4\left(3\alpha\varepsilon^2\left(\alpha-2\right)+\varepsilon\left(8\alpha^2-28\alpha+32\right)+3\alpha^2-6\alpha\right)/9\left(\alpha-4\right)^2>0$ if and only if $\varepsilon<\left(14\alpha-4\alpha^2-16+\left(\alpha-4\right)\sqrt{\left(7\alpha^2-20\alpha+16\right)}\right)/\left(-6\alpha+3\alpha^2\right)<\varepsilon_\alpha\left(1\right)$, so that $V^R\left(2\right)-V^F\left(2\right)<0$. For $\alpha\in\left(\alpha^R,1\right)$, we have that $V^R\left(2\right)-V^F\left(2\right)=-2\varepsilon/9<0$.

- Consider next the region $\gamma>\gamma^F\left(\varepsilon\right)$ (possible only for $\varepsilon>0.707$). We know from Lemma 1 that $V^F\left(2\right)<V^F\left(1\right)$ and from Lemma 2 that $V^R\left(2\right)>V^R\left(1\right)$ if and only if $\alpha>\alpha_1$. For $\alpha\in\left(0,\alpha_1\right)$, we have that $V^R\left(1\right)-V^F\left(1\right)=-(1/72)\alpha\left(2\varepsilon+1\right)^2\left(3\alpha+2\right)<0$. For $\alpha\in\left(\alpha_1,\alpha_1^F\right)$, we have that $V^R\left(2\right)-V^F\left(1\right)=-\alpha(4\varepsilon^2(\alpha^2-5\alpha+10)+4\varepsilon(\alpha^2-2\alpha+4)+(4\alpha+\alpha^2-8))/9(\alpha-4)^2>0$ if and only if $\varepsilon<\left(-2\left(\alpha^2-2\alpha+4\right)+2(\alpha-4)\sqrt{3\left(2-\alpha\right)}\right)/4\left(\alpha^2-5\alpha+10\right)<0.707$. Then $V^R\left(2\right)-V^F\left(1\right)<0$. For $\alpha\in\left(\alpha_1^F,\alpha^R\right)$, we have that $V^R\left(2\right)-V^F\left(1\right)=-4\left(\varepsilon+1\right)\left(3\alpha\left(\alpha-2\right)+\varepsilon\left(4\alpha^2-14\alpha+16\right)\right)/9\left(\alpha-4\right)^2>0$ if and only if $\varepsilon<3\alpha\left(2-\alpha\right)/(4\alpha^2-14\alpha+16)<0.707$. Then $V^R\left(2\right)-V^F\left(1\right)<0$. For $\alpha\in\left(\alpha^R,1\right)$, we have that $V^R\left(2\right)-V^F\left(1\right)=\left(-2\varepsilon/9\right)\left(2\varepsilon-1\right)>0$ if and only if $\varepsilon<1/2<0.707$. Then $V^R\left(2\right)-V^F\left(1\right)<0$.

- **Monopoly case.** Let $\gamma=0$, so that we are in the parameter region $\gamma<\gamma^F\left(\varepsilon\right)$.

  For $\alpha\in\left(0,\alpha_1^F=\alpha_2^F=\alpha^F\right)$, we have that $V_R^M-V_F^M=((2-\alpha)(\varepsilon+1)^2\alpha)/8-\alpha(\varepsilon+1)^2/4=(-1/8)\alpha^2(\varepsilon+1)^2<0$.

  Notice that $\alpha\in\left(\alpha^F,\alpha^R\right)$ if and only if $\varepsilon\in\left(\varepsilon_\alpha\left(0\right),\varepsilon_1\left(0\right)\right)$, with $\varepsilon_\alpha\left(0\right)=\alpha/\left(2-\alpha\right)$ and $\varepsilon_1\left(0\right)=\left(2\alpha+\sqrt{4-4\alpha}-2\right)/2\left(1-\alpha\right)$. For $\alpha\in\left(\alpha^F,\alpha^R\right)$, we have that $V_R^M-V_F^M=((2-\alpha)(\varepsilon+1)^2\alpha)/8-\varepsilon(\varepsilon+2)/4=-(\varepsilon^2(\alpha^2-2\alpha+2)+\varepsilon(2\alpha^2-4\alpha+4)+\alpha^2-2\alpha)/8>0$ if and only if $\varepsilon<(2\alpha-\alpha^2+\sqrt{2}\sqrt{-2\alpha+\alpha^2+2}-2)/(-2\alpha+\alpha^2+2)<\varepsilon_\alpha\left(0\right)$. Then $V_R^M-V_F^M<0$.

  For $\alpha\in\left(\alpha^R,1\right)$, we have that $V_R^M-V_F^M=\varepsilon/2-\varepsilon\left(\varepsilon+2\right)/4=-\varepsilon^2/4<0$.

B Maximum delay

B.1 Fixed fee licensing game

We first consider the fixed fee licensing game with exclusive licensing, $F(1)$, where the patent holder and firm i negotiate over the up-front fixed fee F_i. The negotiation proceeds as in Rubinstein's (1982) alternating-offer bargaining model. The patent

holder and firm i make alternate fixed fee offers, with firm i making offers in odd-numbered periods and the patent holder making offers in even-numbered periods. The negotiation starts in period 0 and ends when one of the negotiators accepts an offer. The patent holder and firm i are assumed to be impatient. The patent holder and firm i have time preferences with constant discount factors $\delta_p \in (0,1)$ and $\delta_f \in (0,1)$, respectively. For any fixed fee bargaining which leads to an agreement F_i at period n, $\delta_f^n \cdot \overline{\Pi}_i^F(F_i)$ and $\delta_p^n \cdot V^F(F_i)$ are, respectively, firm i's payoff and the patent holder's payoff. For any fixed fee bargaining which leads to perpetual disagreement, disagreement payoffs are set to zero. As in Binmore, Rubinstein and Wolinsky (1986), the SPE fixed fee outcome is such that

$$\begin{cases} \overline{\Pi}_i^F(F_{ip}) = \delta_f \cdot \overline{\Pi}_i^F(F_{if}) \\ V^F(F_{if}) = \delta_p \cdot V^F(F_{ip}), \end{cases}$$

where F_{ip} is the SPE fixed fee outcome if the patent holder makes the first offer, F_{if} is the SPE fixed fee outcome if the firm makes the first offer, and subject to firm i's outside option of not buying the license ($\overline{\Pi}_i^F \geq \Pi_i^F$). Notice that $\overline{\Pi}_i^F \geq \Pi_i^F$ if and only if $4\varepsilon(2+\varepsilon-\gamma)(2-\gamma)^{-2}(2+\gamma)^{-2} \geq F_i$. Since the patent holder makes the first offer, the unique SPE fixed fee is given by

$$F_i^{\text{SPE}}(1,\delta_p,\delta_f) = \min\left\{\frac{1-\delta_f}{1-\delta_f\delta_p}\left[\frac{(2-\gamma)+2\varepsilon}{(2-\gamma)(2+\gamma)}\right]^2, \frac{4(2+\varepsilon-\gamma)\varepsilon}{(2-\gamma)^2(2+\gamma)^2}\right\},$$

from which we get the SPE profits and the SPE payoff of the patent holder,

$$\begin{aligned} V^{F^*}(1,\delta_p,\delta_f) &= \min\left\{\frac{1-\delta_f}{1-\delta_f\delta_p}\left[\frac{(2-\gamma)+2\varepsilon}{(2-\gamma)(2+\gamma)}\right]^2, \frac{4(2+\varepsilon-\gamma)\varepsilon}{(2-\gamma)^2(2+\gamma)^2}\right\}, \\ \Pi_i^{F^*}(1,\delta_p,\delta_f) &= \max\left\{\frac{\delta_f(1-\delta_p)}{1-\delta_f\delta_p}\left[\frac{(2-\gamma)+2\varepsilon}{(2-\gamma)(2+\gamma)}\right]^2, \Pi_i^F\right\}. \end{aligned}$$

We now consider the fixed fee licensing game with complete technology diffusion, $F(2)$, where the patent holder and firm 1 negotiate over the up-front fixed fee F_1 while the patent holder and firm 2 negotiate over the up-front fixed fee F_2. The SPE fixed fee outcome is such that

$$\begin{cases} \underline{\Pi}_i^F(F_{ip}) = \delta_f \cdot \underline{\Pi}_i^F(F_{if}) \\ V^F(F_{if}) = \delta_p \cdot V^F(F_{ip}), \end{cases}$$

where F_{ip} is the SPE fixed fee outcome if the patent holder makes the first offer, F_{if} is the SPE fixed fee outcome if the firm makes the first offer, and subject to firm

i's outside option of not buying the license ($\overline{\Pi}_i^F \geq \underline{\Pi}_i^F$). Notice that $\overline{\Pi}_i^F \geq \underline{\Pi}_i^F$ if and only if $4\varepsilon\left(2+\varepsilon\left(1-\gamma\right)-\gamma\right)\left(2-\gamma\right)^{-2}\left(2+\gamma\right)^{-2} \geq F_i$. Since the patent holder makes the first offer, the unique symmetric SPE fixed fee is given by

$$F_i^{\text{SPE}}(2,\delta_p,\delta_f) = \min\left\{\frac{1-\delta_f}{1-\delta_f\delta_p}\left[\frac{1+\varepsilon}{2+\gamma}\right]^2, \frac{4\left(2+\varepsilon\left(1-\gamma\right)-\gamma\right)\varepsilon}{\left(2-\gamma\right)^2\left(2+\gamma\right)^2}\right\},$$

from which we get the SPE profits and the SPE payoff of the patent holder,

$$\begin{aligned} V^{F^*}(2,\delta_p,\delta_f) &= \min\left\{\frac{2(1-\delta_f)}{1-\delta_f\delta_p}\left[\frac{1+\varepsilon}{2+\gamma}\right]^2, \frac{8\left(2+\varepsilon\left(1-\gamma\right)-\gamma\right)\varepsilon}{\left(2-\gamma\right)^2\left(2+\gamma\right)^2}\right\}, \\ \Pi_i^{F^*}(2,\delta_p,\delta_f) &= \max\left\{\frac{\delta_f(1-\delta_p)}{1-\delta_f\delta_p}\left[\frac{1+\varepsilon}{2+\gamma}\right]^2, \frac{4\left(1-\gamma\left(1+\varepsilon\right)\right)+\gamma^2\left(1+2\varepsilon+\varepsilon^2\right)}{\left(2-\gamma\right)^2\left(2+\gamma\right)^2}\right\}, \end{aligned}$$

for $i=1,2$.

Suppose now that the players have private information. They are uncertain about each others' discount factors. It is assumed that player i's discount factor is included in the set $[\delta_i^{\text{I}},\delta_i^{\text{P}}]$, where $0<\delta_i^{\text{I}}\leq\delta_i^{\text{P}}<1$, and $(1-\delta_f^{\text{I}})/(1-\delta_f^{\text{I}}\delta_p^{\text{P}}) \leq 4\varepsilon\left(2+\varepsilon-\gamma\right)\left(2\left(1+\varepsilon\right)-\gamma\right)^{-2}$ which guarantees that each firm will never take the outside option at equilibrium. Since we allow for general probability distributions over discount factors, multiplicity of PBE is not an exception. From Watson (1998), we have that for any PBE, the payoff of the patent holder belongs to $[V^{F^*}(k,\delta_p^{\text{I}},\delta_f^{\text{P}}), V^{F^*}(k,\delta_p^{\text{P}},\delta_f^{\text{I}})]$ and the payoff of the firm belongs to $[\Pi_i^{F^*}(k,\delta_p^{\text{P}},\delta_f^{\text{I}}), \Pi_i^{F^*}(k,\delta_p^{\text{I}},\delta_f^{\text{P}})]$. In case of exclusive licensing ($k=1$), the maximum number of bargaining periods the patent holder would spend negotiating, $I\left(m^p(F(1))\right)$, is given by

$$V^{F^*}(1,\delta_p^{\text{I}},\delta_f^{\text{P}}) = \left(\delta_p^{\text{P}}\right)^{m^p(F(1))}\cdot V^{F^*}(1,\delta_p^{\text{P}},\delta_f^{\text{I}}),$$

from which we obtain

$$m^p(F(1)) = \frac{1}{\log\left(\delta_p^{\text{P}}\right)}\cdot\log\left[\frac{1-\delta_f^{\text{P}}}{1-\delta_f^{\text{I}}}\cdot\frac{1-\delta_f^{\text{I}}\delta_p^{\text{P}}}{1-\delta_f^{\text{P}}\delta_p^{\text{I}}}\right].$$

Notice that $I\left(m^p(F(1))\right)$ is simply the integer part of $m^p(F(1))$. It is customary to express the players' discount factors in terms of discount rates, r_p and r_f, and the length of the bargaining period, Δ, according to the formula $\delta_i=\exp\left(-r_i\Delta\right)$, for $i=p,f$. With this interpretation, player i's type is identified with the discount rate r_i, where $r_i\in[r_i^{\text{P}},r_i^{\text{I}}]$. We thus have that $\delta_i^{\text{I}}=\exp(-r_i^{\text{I}}\Delta)$ and $\delta_i^{\text{P}}=\exp(-r_i^{\text{P}}\Delta)$, for $i=p,f$. Note that $r_i^{\text{I}}\geq r_i^{\text{P}}$ since greater patience implies a lower discount rate. As Δ approaches zero, we have (using l'Hopital's rule): $(1-\delta_f^{\text{P}})/(1-\delta_f^{\text{I}})$ converges to

$(r_f^{\mathrm{P}})/(r_f^{\mathrm{I}})$; $(1-\delta_f^{\mathrm{I}}\delta_p^{\mathrm{P}})/(1-\delta_f^{\mathrm{P}}\delta_p^{\mathrm{I}})$ converges to $(r_f^{\mathrm{I}}+r_p^{\mathrm{P}})/(r_f^{\mathrm{P}}+r_p^{\mathrm{I}})$; and $\Delta/\log(\delta_p^{\mathrm{P}})$ converges to $(-1/r_p^{\mathrm{P}})$. These facts imply that

$$D^p(F(1)) = \lim_{\Delta\to 0}\left(m^p(F(1))\cdot\Delta\right) = -\frac{1}{r_p^{\mathrm{P}}}\cdot\log\left[\frac{r_f^{\mathrm{P}}}{r_f^{\mathrm{I}}}\cdot\frac{r_p^{\mathrm{P}}+r_f^{\mathrm{I}}}{r_p^{\mathrm{I}}+r_f^{\mathrm{P}}}\right],$$

which is a positive, finite number. Notice that $D^p(F(1))$ converges to zero as r_i^{P} and r_i^{I} become close, for $i=p,f$. The maximum number of bargaining periods firm i would spend negotiating, $I(m^f(F(1)))$, is given by

$$\Pi_i^{F^*}(1,\delta_p^{\mathrm{P}},\delta_f^{\mathrm{I}}) = (\delta_f^{\mathrm{P}})^{m^f(F(1))}\cdot\Pi_i^{F^*}(1,\delta_p^{\mathrm{I}},\delta_f^{\mathrm{P}}),$$

from which we obtain

$$m^f(F(1)) = \frac{1}{\log(\delta_f^{\mathrm{P}})}\cdot\log\left[\frac{\Pi_i^{F^*}(1,\delta_p^{\mathrm{P}},\delta_f^{\mathrm{I}})}{\Pi_i^{F^*}(1,\delta_p^{\mathrm{I}},\delta_f^{\mathrm{P}})}\right],$$

and as Δ approaches zero,

$$D^f(F(1)) = \lim_{\Delta\to 0}\left(m^f(F(1))\cdot\Delta\right) = -\frac{1}{r_f^{\mathrm{P}}}\cdot\log\left[\frac{r_p^{\mathrm{P}}}{r_p^{\mathrm{I}}}\cdot\frac{r_f^{\mathrm{P}}+r_p^{\mathrm{I}}}{r_f^{\mathrm{I}}+r_p^{\mathrm{P}}}\right],$$

which is a positive, finite number. The maximum real delay time before reaching an agreement is given by

$$D(F(1)) = \min\left\{D^p(F(1)), D^f(F(1))\right\}.$$

In case of complete technology diffusion ($k=2$), the maximum number of bargaining periods the patent holder would spend negotiating, $I\left(m^p(F(2))\right)$, is given by

$$V^{F^*}(2,\delta_p^{\mathrm{I}},\delta_f^{\mathrm{P}}) = \left(\delta_p^{\mathrm{P}}\right)^{m^p(F(2))}\cdot V^{F^*}(2,\delta_p^{\mathrm{P}},\delta_f^{\mathrm{I}}),$$

from which we obtain, as Δ approaches zero,

$$D^p(F(2)) = \lim_{\Delta\to 0}\left(m^p(F(2))\cdot\Delta\right) = -\frac{1}{r_p^{\mathrm{P}}}\cdot\log\left[\frac{r_f^{\mathrm{P}}}{r_f^{\mathrm{I}}}\cdot\frac{r_p^{\mathrm{P}}+r_f^{\mathrm{I}}}{r_p^{\mathrm{I}}+r_f^{\mathrm{P}}}\right],$$

which is a positive, finite number. The maximum number of bargaining periods firm i would spend negotiating, $I(m^f(F(2)))$, is given by

$$\Pi_i^{F^*}(2,\delta_p^{\mathrm{P}},\delta_f^{\mathrm{I}}) = (\delta_f^{\mathrm{P}})^{m^f(F(2))}\cdot\Pi_i^{F^*}(2,\delta_p^{\mathrm{I}},\delta_f^{\mathrm{P}}),$$

from which we obtain, as Δ approaches zero,

$$D^f(F(2)) = \lim_{\Delta \to 0} \left(m^f(F(2)) \cdot \Delta \right) = -\frac{1}{r_f^{\mathrm{P}}} \cdot \log \left[\frac{r_p^{\mathrm{P}}}{r_p^{\mathrm{I}}} \cdot \frac{r_f^{\mathrm{P}} + r_p^{\mathrm{I}}}{r_f^{\mathrm{I}} + r_p^{\mathrm{P}}} \right],$$

which is a positive, finite number. The maximum real delay time before reaching an agreement is given by

$$D(F(2)) = \min \left\{ D^p(F(2)), D^f(F(2)) \right\},$$

and we have that $D^p(F(1)) = D^p(F(2))$, $D^f(F(1)) = D^f(F(2))$, $D(F(1)) = D(F(2))$.

B.2 Royalty licensing game

We first consider the per-unit royalty licensing game with exclusive licensing, $R(1)$, where the patent holder and firm i negotiate over the per-unit royalty R_i. The negotiation still proceeds as in Rubinstein's (1982) alternating-offer bargaining model. Thus the SPE per-unit royalty is such that

$$\begin{cases} \overline{\Pi}_i^R(R_{ip}) = \delta_f \cdot \overline{\Pi}_i^R(R_{if}) \\ V^R(R_{if}) = \delta_p \cdot V^R(R_{ip}), \end{cases}$$

where R_{ip} is the SPE royalty if the patent holder makes the first offer, R_{if} is the SPE royalty if the firm makes the first offer, and subject to firm i's outside option of not buying the license ($\overline{\Pi}_i^R \geq \Pi_i^R$ if and only if $\varepsilon \geq R_i$). Since the patent holder makes the first offer, the unique SPE royalty is given by

$$R_i^{\mathrm{SPE}}(1, \delta_p, \delta_f) = \min \left\{ \frac{\left(1 - \sqrt{\delta_f}\right)\left(2(1+\varepsilon) - \gamma\right)}{2\left(1 - \delta_f \delta_p\right)}, \varepsilon \right\},$$

from which we get the SPE profits and the SPE payoff of the patent holder,

$$\begin{aligned} V^{R^*}(1, \delta_p, \delta_f) &= \min \left\{ \frac{\left(1 - \sqrt{\delta_f}\right)\sqrt{\delta_f}\left(1 - \sqrt{\delta_f}\delta_P\right)(2+\gamma)\left(2(1+\varepsilon) - \gamma\right)^2}{2\left(1 - \delta_f \delta_p\right)^2 (2 - \gamma)}, \frac{\varepsilon}{2+\gamma} \right\}, \\ \Pi_i^{R^*}(1, \delta_p, \delta_f) &= \max \left\{ \frac{\delta_f \left(1 - \sqrt{\delta_f}\delta_P\right)^2 (2+\gamma)^2 \left(2(1+\varepsilon) - \gamma\right)^2}{\left(1 - \delta_f \delta_p\right)^2 (2-\gamma)^2}, \left(\frac{1}{2+\gamma}\right)^2 \right\}. \end{aligned}$$

We now consider the per-unit royalty licensing game with complete technology diffusion, $R(2)$, where the patent holder and firm 1 negotiate over the per-unit royalty R_1 while the patent holder and firm 2 negotiate over the per-unit royalty R_2. The SPE per-unit royalties are such that

$$\begin{cases} \underline{\Pi}_i^R(R_{ip}, R_j) = \delta_f \cdot \underline{\Pi}_i^R(R_{if}, R_j) \\ V^F(R_{if}, R_j) = \delta_p \cdot V^F(R_{ip}, R_j), \end{cases}$$

where R_{ip} is the SPE royalty if the patent holder makes the first offer, R_{if} is the SPE royalty outcome if the firm makes the first offer, and subject to firm i's outside option of not buying the license ($\overline{\Pi}_i^R \geq \underline{\Pi}_i^R$ if and only if $\varepsilon \geq R_i$). Since the patent holder makes the first offer, the unique symmetric SPE royalty is given by

$$R_i^{\text{SPE}}(2, \delta_p, \delta_f) = \min\left\{ \frac{\left(1-\sqrt{\delta_f}\right)(1+\varepsilon)(2-\gamma)}{2\left(1-\delta_f\delta_p\right) - \left(1-\sqrt{\delta_f}\right)\gamma}, \varepsilon \right\},\ i = 1, 2,$$

from which we get the SPE profits and the SPE payoff of the patent holder,

$$\begin{aligned} V^{R^*}(2, \delta_p, \delta_f) &= \min\left\{ \frac{4\sqrt{\delta_f}\left(1-\sqrt{\delta_f}\right)\left(1-\sqrt{\delta_f}\delta_P\right)(1+\varepsilon)^2(2-\gamma)}{(2+\gamma)\left(2\left(1-\delta_f\delta_p\right) - \left(1-\sqrt{\delta_f}\right)\gamma\right)\left(2-2\delta_f\delta_p-\gamma+\gamma\sqrt{\delta_f}\right)}, \frac{2\varepsilon}{2+\gamma} \right\}, \\ \Pi_i^{R^*}(2, \delta_p, \delta_f) &= \max\left\{ \frac{4\delta_f\left(1-\sqrt{\delta_f}\delta_P\right)(1+\varepsilon)^2}{(2+\gamma)^2\left(2-2\delta_f\delta_p-\gamma+\gamma\sqrt{\delta_f}\right)^2}, \left(\frac{1}{2+\gamma}\right)^2 \right\},\ i = 1, 2. \end{aligned}$$

Consider now the case where players have private information. It is assumed that player i's discount factor is included in the set $[\delta_i^{\text{I}}, \delta_i^{\text{P}}]$, where $0 < \delta_i^{\text{I}} \leq \delta_i^{\text{P}} < 1$, and $(1-\delta_f^{\text{I}})/(1-\delta_f^{\text{I}}\delta_p^{\text{P}}) \leq 4/\left(2\left(1+\varepsilon\right)-\gamma\right)$ which guarantees that each firm will never take the outside option at equilibrium. In case of exclusive licensing ($k = 1$), the maximum number of bargaining periods the patent holder would spend negotiating, $I\left(m^p(R(1))\right)$, is given by

$$V^{R^*}(1, \delta_p^{\text{I}}, \delta_f^{\text{P}}) = \left(\delta_p^{\text{P}}\right)^{m^p(R(1))} \cdot V^{R^*}(1, \delta_p^{\text{P}}, \delta_f^{\text{I}}),$$

from which we obtain

$$m^p(R(1)) = \frac{1}{\log\left(\delta_p^{\text{P}}\right)} \cdot \log\left[\frac{1-\sqrt{\delta_f^{\text{P}}}}{1-\sqrt{\delta_f^{\text{I}}}} \cdot \frac{\sqrt{\delta_f^{\text{P}}}}{\sqrt{\delta_{\text{f}}^{\text{I}}}} \cdot \frac{1-\sqrt{\delta_f^{\text{P}}}\delta_p^{\text{I}}}{1-\sqrt{\delta_f^{\text{I}}}\delta_P^{\text{P}}} \cdot \left(\frac{1-\delta_f^{\text{I}}\delta_p^{\text{P}}}{1-\delta_f^{\text{P}}\delta_p^{\text{I}}}\right)^2 \right].$$

Notice that $I\left(m^p(R(1))\right)$ is simply the integer part of $m^p(R(1))$. We can express the players' discount factors in terms of discount rates, r_p and r_f, and the length of the bargaining period, Δ, according to the formula $\delta_i = \exp\left(-r_i\Delta\right)$. Then, player i's type is r_i, where $r_i \in [r_i^{\text{P}}, r_i^{\text{I}}]$; $\delta_i^{\text{I}} = \exp(-r_i^{\text{I}}\Delta)$ and $\delta_i^{\text{P}} = \exp(-r_i^{\text{P}}\Delta)$. As Δ approaches zero, we obtain that

$$D^p(R(1)) = \lim_{\Delta\to 0}\left(m^p(R(1)) \cdot \Delta\right) = -\frac{1}{r_p^{\text{P}}} \cdot \log\left[\frac{r_f^{\text{P}}}{r_f^{\text{I}}} \cdot \left(\frac{r_p^{\text{P}}+r_f^{\text{I}}}{r_p^{\text{I}}+r_f^{\text{P}}}\right)^2 \cdot \frac{2r_p^{\text{I}}+r_f^{\text{P}}}{2r_p^{\text{P}}+r_f^{\text{I}}} \right],$$

which is a positive, finite number. The maximum number of bargaining periods firm i would spend negotiating, $I(m^f(R(1)))$, is given by

$$\Pi_i^{R^*}(1, \delta_p^{\mathrm{P}}, \delta_f^{\mathrm{I}}) = (\delta_f^{\mathrm{P}})^{m^f(R(1))} \cdot \Pi_i^{R^*}(1, \delta_p^{\mathrm{I}}, \delta_f^{\mathrm{P}}),$$

from which we obtain

$$m^f(R(1)) = \frac{1}{\log(\delta_f^{\mathrm{P}})} \cdot \log \left[\frac{\delta_f^{\mathrm{I}}}{\delta_f^{\mathrm{P}}} \cdot \left(\frac{1 - \sqrt{\delta_f^{\mathrm{I}} \delta_p^{\mathrm{P}}}}{1 - \sqrt{\delta_f^{\mathrm{P}} \delta_p^{\mathrm{I}}}} \right)^2 \cdot \left(\frac{1 - \delta_f^{\mathrm{P}} \delta_p^{\mathrm{I}}}{1 - \delta_f^{\mathrm{I}} \delta_p^{\mathrm{P}}} \right)^2 \right],$$

and as Δ approaches zero,

$$D^f(R(1)) = \lim_{\Delta \to 0} \left(m^f(R(1)) \cdot \Delta \right) = -\frac{1}{r_f^{\mathrm{P}}} \cdot \log \left[\left(\frac{r_p^{\mathrm{I}} + r_f^{\mathrm{P}}}{r_p^{\mathrm{P}} + r_f^{\mathrm{I}}} \right)^2 \cdot \left(\frac{2r_p^{\mathrm{P}} + r_f^{\mathrm{I}}}{2r_p^{\mathrm{I}} + r_f^{\mathrm{P}}} \right)^2 \right],$$

which is a positive, finite number. The maximum real delay time before reaching an agreement is given by

$$D(R(1)) = \min \left\{ D^p(R(1)), D^f(R(1)) \right\}.$$

In case of complete technology diffusion ($k = 2$), the maximum number of bargaining periods the patent holder would spend negotiating, $I\left(m^p(R(2))\right)$, is given by

$$V^{R^*}(2, \delta_p^{\mathrm{I}}, \delta_f^{\mathrm{P}}) = \left(\delta_p^{\mathrm{P}}\right)^{m^p(R(2))} \cdot V^{R^*}(2, \delta_p^{\mathrm{P}}, \delta_f^{\mathrm{I}}),$$

from which we obtain

$$m^p(R(2)) = \frac{1}{\log\left(\delta_p^{\mathrm{P}}\right)} \cdot \log \left[\frac{1 - \sqrt{\delta_f^{\mathrm{P}}}}{1 - \sqrt{\delta_f^{\mathrm{I}}}} \cdot \frac{\sqrt{\delta_f^{\mathrm{P}}}}{\sqrt{\delta_f^{\mathrm{I}}}} \cdot \frac{1 - \sqrt{\delta_f^{\mathrm{P}}} \delta_P^{\mathrm{I}}}{1 - \sqrt{\delta_f^{\mathrm{I}}} \delta_P^{\mathrm{P}}} \cdot \left(\frac{2(1 - \delta_f^{\mathrm{I}} \delta_p^{\mathrm{P}}) - \gamma \left(1 - \sqrt{\delta_f^{\mathrm{I}}}\right)}{2(1 - \delta_f^{\mathrm{P}} \delta_p^{\mathrm{I}}) - \gamma \left(1 - \sqrt{\delta_f^{\mathrm{P}}}\right)} \right)^2 \right].$$

and as Δ approaches zero,

$$D^p(R(2)) = \lim_{\Delta \to 0} \left(m^p(R(2)) \cdot \Delta \right) = -\frac{1}{r_p^{\mathrm{P}}} \cdot \log \left[\frac{r_f^{\mathrm{P}}}{r_f^{\mathrm{I}}} \cdot \frac{2r_p^{\mathrm{I}} + r_f^{\mathrm{P}}}{2r_p^{\mathrm{P}} + r_f^{\mathrm{I}}} \cdot \left(\frac{4r_p^{\mathrm{P}} + (4 - \gamma) r_f^{\mathrm{I}}}{4r_p^{\mathrm{I}} + (4 - \gamma) r_f^{\mathrm{P}}} \right)^2 \right],$$

which is a positive, finite number. The maximum number of bargaining periods firm i would spend negotiating, $I(m^f(R(2)))$, is given by

$$\Pi_i^{R^*}(2, \delta_p^{\mathrm{P}}, \delta_f^{\mathrm{I}}) = (\delta_f^{\mathrm{P}})^{m^f(R(2))} \cdot \Pi_i^{R^*}(2, \delta_p^{\mathrm{I}}, \delta_f^{\mathrm{P}}),$$

from which we obtain

$$m^f(R(2)) = \frac{1}{\log(\delta_f^{\mathrm{P}})} \cdot \log\left[\frac{\delta_f^{\mathrm{I}}}{\delta_f^{\mathrm{P}}} \cdot \left(\frac{1-\sqrt{\delta_f^{\mathrm{I}}}\delta_p^{\mathrm{P}}}{1-\sqrt{\delta_f^{\mathrm{P}}}\delta_p^{\mathrm{I}}}\right)^2 \cdot \left(\frac{2(1-\delta_f^{\mathrm{P}}\delta_p^{\mathrm{I}}) - \gamma\left(1-\sqrt{\delta_f^{\mathrm{P}}}\right)}{2(1-\delta_f^{\mathrm{I}}\delta_p^{\mathrm{P}}) - \gamma\left(1-\sqrt{\delta_f^{\mathrm{I}}}\right)}\right)^2\right],$$

and as Δ approaches zero,

$$D^f(R(2)) = \lim_{\Delta\to 0}\left(m^f(R(2))\cdot\Delta\right) = -\frac{1}{r_f^{\mathrm{P}}} \cdot \log\left[\left(\frac{2r_p^{\mathrm{P}} + r_f^{\mathrm{I}}}{2r_p^{\mathrm{I}} + r_f^{\mathrm{P}}}\right)^2 \cdot \left(\frac{r_f^{\mathrm{P}} + r_p^{\mathrm{I}}}{r_f^{\mathrm{I}} + r_p^{\mathrm{P}}}\right)^2\right],$$

which is a positive, finite number. The maximum real delay time before reaching an agreement is given by

$$D(R(2)) = \min\left\{D^p(R(2)), D^f(R(2))\right\}.$$

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