

On the location and ‘lock-in’ of cities: geography vs. transportation technology*

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Abstract

We investigate where cities are located in a spatial economy and why they tend to get ‘locked-in’ at particular sites. Building on Fujita and Krugman (1995) we show that geography and/or transportation technology must exhibit some ‘non-smoothness’ for cities to possibly become ‘locked-in’ in location space.

Our results establish that no asymmetric monocentric equilibrium can be generically sustained when space is homogenous and transportation technologies are ‘smooth’, whereas it can in the presence of transportation hubs and/or concave transport cost functions. This suggests that cities are drawn to transportation hubs during the early stages of economic development, whereas they can be sustained almost everywhere during later stages.

Keywords: transport hubs; transport costs; economic geography; location theory; smoothness.

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1 Introduction

The question of *why* cities form in a spatial economy has attracted a lot of attention in recent years and many microeconomic mechanisms underlying the formation of cities have been uncovered in the literature (see, e.g., Fujita and Thisse, 2002; Duranton and Puga, 2004). The question as to *where* cities are located is, on the contrary, often regarded as belonging more to the realm of geography proper and has, thus, received less attention. This can be clearly seen from the fact that most models of spatial economics voluntarily abstract from geographical issues by treating cities either as ‘floating islands’ (see, e.g., Henderson, 1974, 1988), or by focusing on a setting involving two symmetric regions only (see, e.g., Krugman, 1991a; Ottaviano *et al.*, 2002). While fundamental to our understanding of the workings of a spatial economy under many respects, such a modelling strategy has the drawbacks that space is shapeless and that the location of cities is both largely undetermined and unexplained. Even when space has some shape that is explicitly accounted for, it is our contention that the literature has focused so far on the *relative location of cities*, leaving the question of the *absolute location* largely untouched. Fujita *et al.* (1999a,b) develop, e.g., a framework in which new cities form in response to population growth. They characterize the distance separating the emerging cities from the existing ones, yet absolute location is not analyzed since the location of the first city is exogenously given.

As an additional by-product of such modelling choices, models of economic geography usually exhibit *multiple equilibria*, which reduces their predictive power when it comes to analyzing the location of cities in the real world.¹ To deal with this indeterminacy, ‘new’ economic geography (henceforth, NEG) heavily draws on the concept of *historical accident*, i.e., the fact that some process of agglomeration once started in a particular location, yet could equally well have started somewhere else (Krugman, 1991b; Arthur, 1994). Stated differently, even if cities must appear in response to some fundamental economic, social, and political principles, such a view suggests that the present landscape is largely the outcome of some ‘process of random historical accidents’, thus implying again that NEG has not much to say about absolute location.²

The objective of the present paper is two-fold. First, it analyzes some

¹Note that this critique has been recurrently addressed to fields of economic theory in which increasing returns and indivisibilities play an important role (Arthur, 1994).

²This may also explain why geographers and historians usually have at best ‘mixed’ feelings when it comes to NEG (see, e.g., Ottaviano and Thisse, 2004; Scott, 2004).

of the various ways in which geography and transportation technologies interact in determining city location, by merging insights from NEG, location theory, and transportation economics. We argue that once we take into account not only changes in the level of transport costs but also *changes in the functional form itself*, more can be said about city location, indeterminacy, and ‘lock-in’ from a historical point of view. This also clearly shows that *NEG needs to pay more attention to the modelling of transportation itself*, a point that has been neglected until now because of the dominance of the ‘iceberg’-approach. Second, our paper offers a methodological contribution by showing how some apparently innocuous technical assumptions largely condition the equilibrium results. In particular, we highlight the crucial role of smoothness properties in models of economic geography, an issue that remains largely unexplored until now.

In order to investigate more closely how geographical features and transportation technologies interact in shaping the space-economy, we follow Krugman (1993a,b) and integrate more location theoretic aspects into models of NEG. More precisely, we build on the framework developed by Fujita and Krugman (1995), as subsequently extended by Fujita and Mori (1997) and by Fujita *et al.* (1999a,b), in which *location space is continuous and one-dimensional*. Although this framework, in which there is some ‘true’ location space where shape becomes important, allows for the analysis of absolute city location, the question as to where a city is established has, to the best of our knowledge, not been investigated until now. This is because the bulk of the literature has only focused on *symmetric settings* in which the city is a priori located at the geographical center of the economy. Although such an assumption is useful for reducing the analytical complexity of the model, it turns out to be quite restrictive because it rules out *boundary phenomena* that play an important role in making geographical configurations *asymmetric* in the real world. Imagine, e.g., a von-Thünen-like economy with a growing population, in which the area of cultivated agricultural land expands around the city. When there is enough arable land on both sides, new farmers will locate symmetrically in order to minimize transport costs. Yet, such a symmetric expansion may be impossible in the presence of geographical features like mountains, rivers, and oceans. In that case, any further expansion of the agricultural area will make the city location asymmetric, a feature that is largely observed in the real world.³ The question of how much asymmetry can be sustained before the equilibrium breaks down

³Indeed, most major cities are located on either coasts or rivers, so that location space cannot be considered as being a priori symmetric.

becomes then of major interest.

In order to investigate the impacts of such asymmetries, we drop the assumption of symmetry in this paper. This allows us to analyze under which conditions on geography and transportation technologies *asymmetric monocentric equilibria* (henceforth, AME) can be sustained, i.e., we would like to know the absolute location of the city within the agricultural area and whether there is a trade-off between city-size and centrality, as there is in Weberian location theory (see, e.g., Weber, 1909; Witzgall, 1964; Love *et al.*, 1988). To this end, we investigate the sustainability of the monocentric equilibrium under smooth and non-smooth transport cost specifications and in the presence (resp. absence) of natural transportation hubs.

Our analysis highlights two key results. First, *the sustainability of an AME depends crucially on the functional form of transport costs*. More precisely, we show that an asymmetric location can never be sustained as a monocentric equilibrium when the transport cost function is smooth, whereas it can, within some parameter ranges, when it is non-smooth. This suggests that some form of long-haul economies in transportation are required for an AME to possibly arise in the absence of geographical features. An undesirable by-product of this result is that the so-called ‘lock-in’ of cities vanishes when both transport costs and geography are smooth, thus showing that statements like “the very presence [of cities] generates the lock-in effect in the location space, from which individual agents find it difficult to escape, and to which new agents tend to be attracted” (Fujita and Mori, 1996, p. 96) possibly require some additional qualification. In particular, the location of the city may change with the shape of the agricultural area when transport costs are smooth, so that it ‘floats around’ as the size of the hinterland increases asymmetrically. Second, we show that *cities located at transportation hubs can in general be sustained as an AME, no matter whether transport costs are smooth or not*. This suggests that transportation hubs provide another form of long-haul economies and are very likely locations for cities to form in the presence of mobile profit-maximizing firms. It is of interest to note that this result is reminiscent of the *Hakimi Theorem* (Hakimi, 1964), where it is shown that the nodes of a transportation network always contain an optimal location for cost-minimizing firms. Stated differently, hubs attract firms and cities thus tend to form there (see, e.g., Fujita and Mori, 1996), *independently of the properties of the transportation technology itself*.

The remainder of this paper is organized as follows. In Section 2, we present a heuristic model based on Krugman (1996) and Fujita *et al.* (1999a, ch. 9) that allows to boost intuition and to expose clearly the principal

mechanisms underlying our results. The approach is a location theoretic one, namely partial equilibrium with transport cost minimization. We also provide a tentative historical interpretation that suggests how changes in transportation technologies may impact on the location patterns of cities. In sections 3 and 4, we then cast this heuristic model into a full-fledged spatial economics framework building on Fujita and Krugman (1995) and Fujita *et al.* (1999a) and show that all results established previously carry through to this more complete specification. The approach is one of ‘new’ economic geography, namely spatial general equilibrium with profit maximization. Some conclusions are presented in Section 5.

2 A heuristic model of firm location

In this section, we develop a heuristic model that allows to present our key results in a simplified setting. This basic framework is subsequently extended in sections 3 and 4.

2.1 The linear economy

We build on the heuristic location model developed by Krugman (1996) and Fujita *et al.* (1999a, ch. 9) in order to gain some first insights into the lock-in of agglomerations and the role of transportation hubs. Consider a linear spatial economy, stretching out on the real line between 0 and 1. Let $\mu \in [0, 1]$ stand for the industrial population share and let $1 - \mu$ stand for the agricultural population share. We assume that the μ industrial workers are clustered into a single agglomeration, which is located at $r \in (0, 1)$, whereas the $1 - \mu$ agricultural workers are *evenly spread* over the whole segment (see Figure 1 for an illustration).⁴

Insert Figure 1 about here

We further assume that each worker consumes inelastically one unit of some given good. Unit transport costs for this good depend on a *cost function* γ , which depends itself on some measure of distance between locations. We define a cost function as follows:

Definition 1 (cost function) *A function $\gamma : \mathbb{R} \rightarrow \mathbb{R}^+$ is said to be a cost function if it satisfies the following properties: (i) $\gamma(x) = \gamma(-x)$ (symmetry);*

⁴Note that such a problem can be interpreted in terms of a Fermat-Weber location problem with a density of demand points (see, e.g., Carrizosa *et al.*, 1998).

(ii) $\gamma(x) = 0$ if and only if $x = 0$ (non-degenerate); and (iii) $\gamma(x) > \gamma(y)$ for all $x > y \geq 0$ (strictly increasing).

In what follows, we further assume that γ is a continuous function, that it is differentiable everywhere, except eventually at $x = 0$, and that it is locally Lipschitz in the vicinity of $x = 0$. Note that we could replace (ii), without loss of generality, by $\gamma(0) = F$, which captures the existence of fixed set-up costs in transportation technologies.⁵ Note also that when f is differentiable everywhere, (iii) can be replaced by $f'(x) > 0$ for all $x > 0$. It is readily verified that $f'(0) = 0$ must then hold from property (i).

In what follows, we assume that the per unit shipping cost between locations t and s is simply given by $\gamma(t - s)$. To keep our analysis general, we consider cost functions that can be smooth (i.e. differentiable) or non-smooth (i.e. non-differentiable) at the origin.

Consider a firm that seeks to establish a single production facility in the linear economy. Production costs and demands are independent of location so that the firm's problem consists in finding the transport cost minimizing site for delivery to the consumers. Given our assumption of a uniform spatial distribution of demand, total transport costs for a firm operating at s are given by

$$c(s) \equiv \mu\gamma(r - s) + (1 - \mu) \int_0^1 \gamma(t - s)dt. \quad (1)$$

The question we are interested in is the following: *under which conditions is the agglomeration in r a transport cost minimizing location, i.e. under which conditions does the firm choose to establish its plant at r ?* Stated differently, we would like to know when the optimal location coincides with the already existing agglomeration, i.e. when are firms' location choices "sluggish" in the sense that location occurs at a few potential sites only?

Let us start with a useful lemma.

Lemma 2 *The function*

$$g(s) = \int_0^1 \gamma(t - s)dt \quad (2)$$

is similar to an integral-convolution and thus inherits the same properties. In particular, it is differentiable and its derivative is given by

$$g'(s) = \gamma(s) - \gamma(1 - s).$$

⁵Alonso (1975) has suggested the following transportation technology: $\gamma(0) = 0$ and $\lim_{x \rightarrow 0+} \gamma(x) = F > 0$, where F captures the cost of loading and unloading the shipped goods. Such a function fails to be continuous and has no directional derivatives at $x = 0$. Albeit interesting, we disregard such cases by assuming that γ is continuous.

Proof. Note that (2) is similar to the integral-convolution of γ with the constant function 1. Use the change of variable $u = t - s$ to rewrite expression (2) as follows:

$$g(s) = \int_{-s}^{1-s} \gamma(u) du.$$

Hence g is differentiable (see, e.g., Theorem 12 in Appendix C) and

$$g'(s) = \gamma(-s) - \gamma(1-s) = \gamma(s) - \gamma(1-s),$$

where the last inequality results from the symmetry of γ . ■

It is of interest to note that g is differentiable even when the cost function γ is not, provided that γ is a continuous function.⁶ In what follows, we assume that γ is an arbitrary cost function (which may be smooth or not). As shown in Appendix A, the generalized derivative in the sense of Clarke (1975, 1983) of the total cost function c at $s = r$ is given by:

$$\partial c(r) = \left[(1-\mu)[\gamma(r) - \gamma(1-r)] - \mu \lim_{\substack{s \rightarrow r \\ s < r}} \gamma'(r-s), \right. \\ \left. (1-\mu)[\gamma(r) - \gamma(1-r)] - \mu \lim_{\substack{s \rightarrow r \\ s > r}} \gamma'(r-s) \right].$$

The first order necessary condition for r to be a minimizer of c is $0 \in \partial c(r)$ (see, e.g., Clarke, 1983, Proposition 2.3.2, p. 38). This yields the following optimality condition:

$$\lim_{\substack{s \rightarrow r \\ s > r}} \gamma'(r-s) \leq \frac{1-\mu}{\mu} [\gamma(r) - \gamma(1-r)] \leq \lim_{\substack{s \rightarrow r \\ s < r}} \gamma'(r-s) \quad (3)$$

because, by monotonicity of γ , the leftmost term is non-positive, whereas the rightmost term is non-negative.

Condition (3) allows us to establish the following result:

Proposition 3 *When γ is a continuously differentiable cost function, then $r \in \arg \min_s c(s)$ if and only if either $\mu = 1$ or $r = 1/2$.*

⁶The function g also remains differentiable if we assume that the population is distributed according to some continuous density p . In this case,

$$g(s) = \int_0^1 p(t) \gamma(t-s) dt,$$

which has an even stronger ‘integral-convolution flavor’.

Proof. When γ is continuously differentiable, we have

$$\lim_{\substack{s \rightarrow r \\ s > r}} \gamma'(r - s) = \lim_{\substack{s \rightarrow r \\ s < r}} \gamma'(r - s) = \gamma'(0) = 0.$$

By consequence, the optimality condition (3) reduces to

$$\frac{1 - \mu}{\mu} [\gamma(r) - \gamma(1 - r)] = 0. \quad (4)$$

Given that γ is strictly increasing on $\mathbb{R}^+$, (4) can only be met when $\mu = 1$ or when $\gamma(r) = \gamma(1 - r)$, i.e. when $r = 1/2$, which proves our claim. ■

Note that neither of the two cases covered by Proposition 3 is of great economic interest. In the first one, the whole population is clustered into the agglomeration, which must hence be the sole transport cost minimizing location, whereas in the second one, the geographical center is a minimizer due to locational symmetry and the assumption of a uniform distribution of the agricultural demand. The interesting and quite disturbing fact is that *the city is never a transport cost minimizer* when the cost function is differentiable. Stated differently, firms' location behavior is not sluggish in the sense that firms are no longer attracted by the existing agglomeration.

It is also of interest to note that, as shown by Proposition 3, a wide range of usual cost functions, like the logistic and the Cauchy functions, lead to non-sluggish location behavior of firms. Paradoxically, these functions are often used because they are differentiable and, therefore, analytically 'easier' to handle. Yet, as shown by Proposition 3, such apparent simplicity may come at the cost of some theoretical handicap.

When γ is a non-differentiable cost function, r may or may not be a transport cost minimizing location, *depending crucially on the trade-off between weight ('attraction') and centrality ('accessibility')*. This result may be summarized as follows.

Proposition 4 *When γ is a non-differentiable cost function, then (i) for any given value of $r \in (0, 1)$, there exists a value of $\mu \in [0, 1]$ such that $r \in \arg \min_s c(s)$; and (ii) for any given value of $\mu \in (0, 1)$, there exists a value of $r \in [0, 1]$ such that $r \in \arg \min_s c(s)$.*

Proof. When γ is non-differentiable at 0, it must be that

$$\lim_{\substack{s \rightarrow r \\ s > r}} \gamma'(r - s) < 0 \quad \text{and} \quad \lim_{\substack{s \rightarrow r \\ s < r}} \gamma'(r - s) > 0. \quad (5)$$

For any given and fixed value of r , we clearly have

$$\lim_{\mu \rightarrow 1} \frac{1 - \mu}{\mu} [\gamma(r) - \gamma(1 - r)] = 0, \quad (6)$$

whereas, because γ is continuous, for any given value of μ we have

$$\lim_{r \rightarrow 1/2} \frac{1-\mu}{\mu} [\gamma(r) - \gamma(1-r)] = 0. \quad (7)$$

Combining (6) and (7) with (5) shows that the necessary conditions (3) can therefore always be met when γ is non-differentiable, provided that μ is sufficiently close to 1 and r is sufficiently close to 1/2. ■

Proposition 4 captures of course the well-known trade-off between a more central location (*‘accessibility’*) and economic weight (*‘attraction’*), which is crucial to location theory and economic geography (see, e.g., Weibull, 1976, for an axiomatic approach to this trade-off). It further shows that *any off-center location may be a minimizer provided it is sufficiently large*. Clearly, such a result can be viewed as some natural extension of Witzgall’s (1964) *Majority Theorem*.

The optimality condition (3) captures a phenomenon we will refer to as the *lock-in of agglomeration* (Fujita and Thisse, 2002, p. 360, call it an ‘urban shadow’). Roughly speaking, this reflects the fact that the city located in r may be a transport cost minimizer for a large range of parameter values. When the firms are agglomerated at r , they have no incentive to deviate because r minimizes transport costs for shipping to the consumers, whereas a new firm has always an incentive to locate where the already existing firms are established.

It is of interest to take a look at the example in which the cost function is given by the absolute value, since this special case has been widely used in the literature (see Krugman, 1996; Fujita *et al.*, 1999a, ch. 9).

Example 5 Assume that γ is given by the absolute value. In this case,

$$\lim_{\substack{s \rightarrow r \\ s > r}} \gamma'(r-s) = -1 \quad \text{and} \quad \lim_{\substack{s \rightarrow r \\ s < r}} \gamma'(r-s) = 1$$

so that, by condition (3), r is a cost minimizing location if and only if

$$-1 \leq \frac{1-\mu}{\mu} (2r-1) \leq 1 \quad (8)$$

holds. Stated differently, for all sets of parameter values satisfying (8), the firm will choose to establish its plant at the city located in r . Clearly, firm’s location behavior is sluggish in the sense that its optimal location is of zero measure for a non-zero measure set of parameter values. Rewrite the optimality condition (8) as follows

$$\frac{1-2\mu}{2(1-\mu)} \leq r \leq \frac{1}{2(1-\mu)} \quad (9)$$

to see that r is always a transport cost minimizing location when $\mu \geq 1/2$. This illustrates in a very neat way the Majority Theorem (Witzgall, 1964), which states that a location hosting more than one-half of the total demand is always a transport cost minimizer.⁷

2.2 Transportation hubs

In order to boost intuition for our main results, we first extend the basic model presented in the previous subsection. Consider again a linear economy on the interval $[0, 1]$, but assume now that there are $n \geq 1$ identical branches stretching out from a hub located at $b \in (0, 1)$. Figure 2 depicts such an economy when $n = 3$.

Insert Figure 2 about here

In what follows, we show that *the transportation hub always creates a kink in total transport costs so that b is usually an optimal location for firms, even when the cost function γ is differentiable*. This result confirms the robustness of the observation made by Fujita and Mori (1996, p. 97) that “transport nodes induce sharp kinks in market potential curves, which tends to generate new cities there”. Hence, a transportation hub naturally generates a lock-in for firms and economic activities, which largely explains why agglomeration takes place in locations that offer good access to markets and consumers. Geographers, economists and historians have repeatedly argued that a city usually develops at the junction of two or more transport routes of the same kind, or at the junction of two or more transport routes of different kinds. Cooley (1894) already emphasized that “population and material resources tend to accumulate wherever there is a *break in transport lines*” (Bairoch, 1985, p. 143 of the English translation, our emphasis), thus showing that the role of non-differentiability in explaining the attractive power of hubs has been implicitly recognized since long.

Let γ be an arbitrary cost function. For analytical simplicity we assume, without loss of generality, that there is no longer an initial agglomeration.⁸

⁷Strictly speaking, Witzgall’s theorem has only been proven for the special case in which there is a finite number of demand points and in which distance is measured by a norm. Yet, similar results seem to hold for a vast variety of spatial problems involving transportation costs and profit maximizing firms. In models of *home market effects*, one can show that the so-called ‘dominant market effect’ is strongly related to the Majority Theorem when a special metric is used to measure distance (see Behrens *et al.*, 2004).

⁸Our results carry of course through to the case where there is some initial agglomeration of mass $\mu > 0$ in the economy. Yet, in order to state our point as concisely as possible, we assume that $\mu = 0$ in what follows.

Total transport costs of a firm established at s are then given by

$$\begin{aligned} c_1(s) &= \int_0^1 \gamma(t-s)dt + n \int_b^1 \gamma(t-s)dt & \text{if } s \in [0, b] \\ c_2(s) &= \int_0^1 \gamma(t-s)dt + n \int_b^1 \gamma(t+s-2b)dt & \text{if } s \in [b, 1]. \end{aligned}$$

It is of interest to note that, as shown by Lemma 2, both c_1 and c_2 are differentiable functions of s . Further, given that the cost function γ is strictly increasing, both c_1 and c_2 are convex. When $b < s$, we have $t-s < t-2b+s$ so that

$$\int_b^1 \gamma(t-s)dt < \int_b^1 \gamma(t+s-2b)dt$$

and hence $c_1 < c_2$. The reverse inequality holds when $s < b$, so that we can rewrite the total transport cost c in a more compact way as the pointwise supremum of two convex functions

$$c(s) \equiv \sup \{ c_1(s), c_2(s) \}, \quad (10)$$

which remains convex. Yet, as is well-known, such a function is usually non-differentiable at all x satisfying $c_1(x) = c_2(x)$ (see, e.g., Clarke, 1983; Hiriart-Urruty and Lemaréchal, 1993). This clearly shows that *even when the cost function is smooth*, non-differentiability continues to play a fundamental role in the presence of a hub. Yet, this non-differentiability is of ‘first nature’ (i.e. due to exogenously given features of the landscape), and no longer of ‘second nature’ (i.e. due to the transportation technology itself). Hence, the non-differentiability is directly related to some spatial heterogeneity, since the hub creates a “break in transport lines”, thus providing a very convenient location for (industrial) agglomeration.⁹

It is readily verified that $c_1(b) = c_2(b)$, which shows that the total transport cost function usually fails to be differentiable at (and, by convexity, only at) the hub. Given the results established in the previous subsection, this suggests that the hub will very likely be a transport cost minimizing location. Using Lemma 2, the generalized derivative at the hub is given by the convex hull of the derivatives of c_1 and c_2 (see, e.g., Clarke, 1983, p. 47):

$$\partial c(b) = \left[\gamma(b) - (n+1)\gamma(1-b), \gamma(b) + (n-1)\gamma(1-b) \right].$$

⁹The “break in transport lines” becomes even more obvious when one takes into consideration the fact that the non-differentiability of the total cost c corresponds to a *discontinuity* in its first derivative.

As before, the first order necessary and sufficient condition for a local minimum at the hub is given by $0 \in \partial c(b)$. This leads to the following result.

Proposition 6 *Assume that γ is a differentiable cost function. Then, for any given $n \geq 1$, all hub locations b satisfying*

$$-(n+1) \leq \frac{-\gamma(b)}{\gamma(1-b)} \leq n-1$$

are minimizers of total transport costs.

Proof. We can rewrite the first order conditions $0 \in \partial c(b)$ as follows:

$$-(n+1) \leq \frac{-\gamma(b)}{\gamma(1-b)} \leq n-1 \quad (11)$$

When $n = 0$, $\gamma(b) = \gamma(1-b)$ must hold and we hence fall back on the case covered in Section 2, where only the central location $b = 1/2$ yields optimality. When $n > 0$, $-(n+1) < -1$ and $n-1 > 0$ so that $b = 1/2$ is always a solution to the problem. Given the continuity of $-\gamma(b)/\gamma(1-b)$, we may hence conclude that there exists a whole range $\mathcal{S}$ of values of b (centered on $b = 1/2$) such that the hub is an optimal solution to the minimization problem of the firm. ■

Condition (11) shows that firms' location behavior becomes once again sluggish in the presence of a hub and it highlights two important results. First, if the number n of branches is large, the hub is likely to be a cost minimizer, no matter its location b . Second, if the hub is centrally located, i.e. b is close to $1/2$, the hub is also likely to be an optimal location, even when the number of branches n is quite small. Therefore, condition (11) *embodies the trade-off between attraction and accessibility* already mentioned previously. This is reminiscent of Barioch (1985, p. 143 of the English translation), who argues that this trade-off is crucial in explaining whether a hub offers an interesting site: "Naturally, the importance of waterways derives not only from their own intrinsic properties, but also from the economic value of the hinterlands they serve: an excellent natural harbor in a desert region without resources will remain rural".

Let us summarize the foregoing developments as follows.

Proposition 7 *The existence of natural transportation hubs creates non-differentiabilities in firms' transport cost functions. Since minima of such a function are likely to be located at cusps, hubs offer privileged locations for transport cost minimizing economic agents, no matter whether cost functions are intrinsically smooth or not.*

Note that a similar result holds for the case of a transport cost minimizing firm locating in a network, when demands are distributed on the nodes only (Hakimi, 1964). Indeed, it is shown in this setting that if the transport cost function is arc-wise concave, the vertices of such a network always contain a transport cost minimizing solution. Our results show that once demands are not only distributed on the nodes of the network but are spread uniformly over the arcs, optimality at a node may also hold for convex costs. Yet, even in the case of a concave cost function, optimality of the node is no longer guaranteed since it now involves its *relative position in the network*. Thus, “geography matters again”. More work is called for here in our opinion.

2.3 First vs second nature: a historical perspective

The distinction between strictly convex and concave cost functions has, in our opinion, both a theoretical and a historical interest. From a theoretical point of view, strictly convex cost functions are often used in spatial competition models (especially, the function $c(x) = x^2$). Thus, we believe it is of interest to investigate what their impact is on equilibria in models of NEG, and to compare this with the results derived with concave functions.

From a historical point of view, the distinction between convex and concave cost functions may be related to the evolution of transportation technologies. All modern transportation technologies do exhibit very significant scale economies in volume and economies of long haul in distance (see, e.g., Boyer, 1997; Hummels, 1999). Yet, this seems to have not always been the case. As argued by Armstrong (1989), even as late as back in the 18th century, the lack of scale economies due to restricted carrying capacities still made most means of land-transportation, based on either animal or human energy, unsuited for the shipment of bulkier loads. Thus, it is fair to say that significant scale and long-haul economies in (land) transportation are a relatively recent phenomenon that dates back to the Industrial Revolution and the rapid rise of the railroads in the 19th century.

Economies of long-haul in transportation manifest themselves through cost functions that are *concave* with respect to distance. Clearly, such cost functions fail to be differentiable at the origin. Hence, at least some (strict) convexity around $x = 0$ is required for γ to possibly be differentiable. Yet, when γ is convex, transport costs can be reduced by trans-shipment through intermediate points, which seems to be characteristic of traditional transportation technologies. It is, e.g., well known that horse-drawn transportation required the frequent change of horses due to their rapid exhaustion, which amounts to some kind of trans-shipment through

supply points. To sum it up, we believe that modern transportation technologies are mostly characterized by important fixed costs, concavity with respect to distance and, therefore, non-smoothness; whereas more traditional animal-based transportation technologies are characterized by small fixed costs, strict convexity with respect to distance and smoothness. Hence, the distinction between smooth and non-smooth cost functions captures, in our opinion, most of the differences between present and past transportation technologies.

Given such a distinction between modern and traditional transportation technologies, our results suggest the following historical evolution. In the past, when transportation technologies did not exhibit economies of long-haul, cities were established at (or somehow drawn to) natural transportation hubs, because such hubs provided some advantage in transportation and the necessary lock-in for cities to be sustained there. Hence, in Cronon's (1991) terms, *first nature* largely explained location patterns during these early stages of economic development. As argued in subsections 2.1 and 2.2, the number of possible locations compatible with monocentric equilibria was thus quite small in such a context, which therefore suggests that the location of cities was not much subject to historical accidents. Stated differently, *agglomerations either emerged at hubs or were, in the long-run, drawn to them*. As transportation technologies subsequently improved during later stages of economic development, increasing returns and economies of long-haul became internal to the technologies themselves. Thus, *second nature* became (and still is) a major determinant of city location. Because transportation advantage can now a priori be provided everywhere in the space-economy via technology, historical accidents play an important role in explaining the observed spatial pattern.

To sum it up, although the current structure of the city system is only one of the potential possibilities that could be sustained in the presence of modern transportation technologies, it has been somehow "selected" in the past when first nature was more important than second nature. This suggests that, given the historical evolution of transportation technologies, the possible outcomes could not have been radically different from the currently observed ones, which provides one possible explanation for the persistence of urban structures throughout most of history.¹⁰

¹⁰This agrees with the historical fact that the geographical structure of Europe's actual city system was already largely determined during the Middle-Ages and has only changed little since then (Bairoch, 1985; Hohenberg and Lees, 1985). It is also consistent with the observation by Fujita and Mori (1996) that many of today's cities are located at natural transportation hubs, although these hubs no longer play a significant role in explaining

3 The generalized market potential function

In this section, we show that the results derived in the heuristic location model carry over to the more complex ‘new’ economic geography model by Fujita and Krugman (1995). This suggests that our results are quite robust with respect to different modelling choices and apply to both location models with transport cost minimization, and to economic geography models with profit maximization. In what follows, we extend the monocentric spatial setting developed by Fujita and Krugman (1995) and refined by Fujita *et al.* (1999a,b) by allowing for an arbitrary cost function and an arbitrary location of the center. To shorten the paper, we do not present the model in detail and refer the reader to Fujita *et al.* (1999a, ch. 9) for all further developments.

We consider a linear monocentric economy, whose unique center is located at $r \in [-\ell, \ell]$, where ℓ stands for the agricultural fringe distance. We assume, for analytical simplicity, that transport costs for both agricultural (A-) and manufacturing (M-) goods are of the ‘negative exponential iceberg’ type. The cost of shipping goods between locations is now measured by an arbitrary cost function γ . When one unit of A-good (resp. of M-good) is shipped from location r to location s , only a fraction

$$e^{-\tau^A \gamma(r-s)} \leq 1, \quad \tau^A > 0, \quad \text{respectively} \quad e^{-\tau^M \gamma(r-s)} \leq 1, \quad \tau^M > 0$$

arrives at its destination, whereas the rest ‘melts away en route’. Using the same normalizations as Fujita *et al.* (1999a), the derivation of the market potential function proceeds then in exactly the same way. We just need to replace the absolute value with the cost function γ , and keep track of the fact that the location of the center is no longer given by 0 but by r .

Although all derivations are as in Fujita *et al.* (1999a), one issue deserves some more discussion. Indeed, the equalization of agricultural real wages in all locations requires that real wages be equalized at the fringe. This does usually imply that the center must be located in the middle of the agricultural area $[-\ell, \ell]$, i.e. $r = (-\ell + \ell)/2 = 0$. Assume that $r \neq 0$ and re-center the agricultural area on the origin by letting $a = r - \ell$ and $b = \ell - r$. It is then straightforward to show that the agricultural real wages in the new fringe locations a and b are given by

$$\omega^A(a) = \frac{(Gp^A)^{-\mu}}{c^A} e^{-\mu(\tau^A + \tau^M)\gamma(r-a)}$$

their economic performance.

and

$$\omega^A(b) = \frac{(Gp^A)^{-\mu}}{c^A} e^{-\mu(\tau^A + \tau^M)\gamma(r-b)}$$

respectively, so that, by monotonicity of the cost function, $\omega^A(a) = \omega^A(b)$ if and only if $r = (a + b)/2 = 0$, which contradicts our initial assumption. Hence, the fringe wages will differ when the agglomeration is not located in the dead center of the agricultural area. This is because “land in the neighbourhood of a town gives a greater rent than land equally fertile in a distant part of the country. Though it may cost no more labour to cultivate the one than the other, it must always cost more to bring the produce of the distant land to market” (Smith, 1776, Book I, Ch. 11: I.11.12). When the center is not located in the middle of the agricultural area, farmers on the ‘long side’ of the economy find it profitable to relocate to the ‘short side’ in order to decrease transport costs and, therefore, increase real wages. Yet, *such a relocation may be impossible because of some natural obstacles like mountains, rivers or oceans*. Hence, an asymmetric agricultural area may form because there is more arable land on one side of the center than on the other (think, e.g., of cities located on the coast). A similar idea is used by Mori (1997) in the case of megalopolitanization between regionalizing economies. He argues that a strictly positive land rent at the fringe reflects *boundary conditions*, i.e. a saturated constraint. It is of interest to note that the land rent differential at the fringe of the economy is a *scarcity rent*, whereas land rent differentials elsewhere just reflect differences in transport costs for shipping the agricultural output to the center (*differential rent*).

In what follows, we assume that *location space is bounded to the right of the center*, i.e. there exists some $\ell_{\max}$ such that agriculture is impossible for all $\ell > \ell_{\max}$. Because the agricultural fringe distance ℓ is strictly increasing with population size (see, e.g., Fujita and Krugman, 1995) this constraint will become binding when the population gets sufficiently large. In that case, any further increase in population will make the location of the center asymmetric (i.e. $r \neq 0$). Note that such a setting seems to be the rule and not the exception in the real world. Although space may be regarded as close to homogenous over vast areas, the unboundedness hypothesis is too strong an assumption. Most major cities are located close to rivers, oceans and sometimes also mountain ranges.¹¹

¹¹Fujita *et al.* (1999b) illustrate their model with the help of a historical setting in which the hypothesis of symmetry is questionable. The authors consider the 19th century westward expansion of the US city system, starting from the old north-eastern colonies. Even if location space seems to have been ‘unbounded’ to the west, the unboundedness to the east is more questionable. As shown in this section, when the cost function is smooth,

Following Fujita *et al.* (1999a), the *generalized market potential function* is defined as follows:

$$\Omega(s) = \left[\frac{\omega^M(s)}{\omega^A(s)} \right]^\sigma = \left[\frac{w^M(s)}{w^A(s)} \right]^\sigma,$$

which can be rewritten as

$$\Omega(s) = \left[\mu e^{-(\sigma-1)\tau^M \gamma(r-s)} + (1-\mu)\psi(s) \right] e^{\sigma[(1-\mu)\tau^A - \mu\tau^M]\gamma(r-s)} \quad (12)$$

where

$$\psi(s) \equiv \frac{\int_{-\ell}^{\ell} e^{[(\sigma-1)\tau^M - \tau^A]\gamma(t-r) - (\sigma-1)\tau^M \gamma(s-t)} dt}{\int_{-\ell}^{\ell} e^{-\tau^A \gamma(t-r)} dt}. \quad (13)$$

Note that ψ is quite similar to an integral convolution. In Appendix B, we show that it is a differentiable function whenever γ is differentiable or when γ is given by the absolute value. Letting $r = 0$ and $\gamma = |\cdot|$, we fall of course back on the ‘classical’ market potential function, as derived by Fujita *et al.* (1999a). We begin by investigating some properties of Ω in order to check for the feasibility of an AME. Rewrite (12) as follows:

$$\Omega(s) = \left[\mu a(s) + (1-\mu)\psi(s) \right] c(s)$$

where

$$a(s) = e^{-(\sigma-1)\tau^M \gamma(r-s)} \quad \text{and} \quad c(s) = e^{\sigma[(1-\mu)\tau^A - \mu\tau^M]\gamma(r-s)}.$$

We assume that γ is an arbitrary cost function, which may be smooth or not. As shown in Appendix C, the generalized derivative of Ω in the sense of Clarke (1975, 1983) at $s = r$ is given by:

$$\partial\Omega(r) = \text{co} \left\{ (1-\mu)\psi'(r) + k \lim_{s \rightarrow r, s > r} \gamma'(r-s), (1-\mu)\psi'(r) + k \lim_{s \rightarrow r, s < r} \gamma'(r-s) \right\},$$

where $k = \sigma[(1-\mu)\tau^A - \mu(1+\rho)\tau^M]$ is a bundle of parameters, and where co stands for the convex hull.¹²

the results may then vastly differ in the presence of asymmetries.

¹²Expressing this convex hull directly as an interval, as in the previous section, is generally impossible, because we have no a priori information on the sign of k .

3.1 The non-differentiable case

Consider the traditional case, as developed by Fujita and Krugman (1995) and Fujita *et al.* (1999a), where $r = 0$ and $\gamma = |\cdot|$. In this case

$$\lim_{\substack{s \rightarrow r \\ s > r}} \gamma'(r - s) = -1 \quad \text{and} \quad \lim_{\substack{s \rightarrow r \\ s < r}} \gamma'(r - s) = 1.$$

Further, ψ is differentiable and $\psi'(0) = 0$ when $r = 0$ (see Appendix C). Hence, the necessary conditions for a local extremum at $r = 0$ are given by

$$0 \in \text{co} \left\{ -\sigma [(1 - \mu)\tau^A - \mu(1 + \rho)\tau^M], \sigma [(1 - \mu)\tau^A - \mu(1 + \rho)\tau^M] \right\}, \quad (14)$$

which always holds. In order for this extremum to be a local maximum,

$$(1 - \mu)\tau^A - \mu(1 + \rho)\tau^M \leq 0 \quad \Rightarrow \quad \frac{1 - \mu}{\mu(1 + \rho)} \leq \frac{\tau^M}{\tau^A} \quad (15)$$

must hold (see, e.g., Fujita *et al.*, 1999a; Fujita and Thisse, 2002, p. 360). Note that when τ^A is sufficiently large, (15) never holds, no matter the population size N . In that case, no city develops in the economy, which is in accord with the historical fact that sufficient agricultural surplus was required for non-agricultural agglomerations to develop (see, e.g., Bairoch, 1985). Because the existence of a city is obviously a prerequisite in order to investigate its location, we assume in what follows that (15) always holds.

As can be seen from (14), the slope of the market potential function (or, more precisely, the extent of the cusp) at 0 does not depend on population size as given by either N or ℓ . This is due to the fact that $\psi'(0) = 0$, which no longer holds when the agglomeration is asymmetrically located. Note that an asymmetric location may remain compatible with a monocentric equilibrium, provided the agglomeration is not too far off-center.

Proposition 8 *For each given value of r , there exists an expenditure share $\tilde{\mu} < 1$ such that r is a monocentric equilibrium for all $\mu \geq \tilde{\mu}$. This property does neither depend on the cost function γ , nor on the population size N .*

Proof. Assume that $r \neq 0$. Using the necessary condition $0 \in \partial\Omega(r)$, the center remains a local maximum provided that

$$(1 - \mu)\tau^A - \mu(1 + \rho)\tau^M \leq \frac{(1 - \mu)\psi'(r)}{\sigma} \leq -(1 - \mu)\tau^A + \mu(1 + \rho)\tau^M. \quad (16)$$

Taking the limit of (16) when $\mu \rightarrow 1$, we readily have

$$-(1 + \rho)\tau^M \leq 0 \leq (1 + \rho)\tau^M,$$

which shows that every location r can be sustained as a monocentric equilibrium. The result then follows by continuity of (16). ■

Therefore, any off-center location may be sustained as a monocentric equilibrium, provided the share of revenue spent on M-goods is sufficiently large. One should note that the parameter μ indirectly stands for the relative share of population agglomerated at r . The agglomeration effect is stronger the closer μ is to 1, the closer ρ is to 0 and the higher τ^M , although the last two factors play only a lesser role when compared to μ . Note also that our result, establishing the existence of a continuum of locations compatible with an AME, is reminiscent of that by Berliant and Kung (2003, p. 13), who have shown that the “the equilibrium set is generically a continuum of rather high dimension”.¹³

Insert Figure 3 about here

Figure 3 depicts the case in which an AME arises. As can be seen, despite its asymmetric location the agglomeration creates a cusp in the market potential function, which locks-in its position and makes it a stable equilibrium. As we show next, this result unfortunately only holds when $\mu = 1$ in the case where γ is differentiable, whereas, as shown by Proposition 8, it also holds for some $\mu < 1$ when γ is non-differentiable.

3.2 The differentiable case

Let us start with the symmetric case, i.e., $r = 0$. Assume that γ is a differentiable cost function. In this case, $\gamma'(0) = 0$ and Ω is differentiable, so that

$$\partial\Omega(0) = \{0\}, \tag{17}$$

since $\psi'(0) = 0$ when $r = 0$ (see Appendix B). Condition (17) shows that, provided (15) is met, the agglomeration in $r = 0$ is a local maximum of the

¹³More precisely, Berliant and Kung (2003) show that if the cost function is smooth, the equilibrium set when there are $K \geq 1$ cities is a $\mathcal{C}^\infty$ -manifold of dimension $K - 1$ for almost all regular parametrizations of the model. In the monocentric case, where $K = 1$, this implies that there is at most one equilibrium when the cost function is smooth. As shown in this paper, there is a continuum of equilibria when the cost function is non-smooth, even when $K = 1$. Stated differently, non-differentiability increases the indeterminacy of the model. These results are reminiscent of classical issues involving smoothness as discussed by, e.g., Debreu (1970).

market potential function. Hence, the monocentric configuration remains sustainable when the population size N is small enough.¹⁴

Yet, just as in the Section 2, no asymmetric configuration can be sustained as a monocentric equilibrium when the cost function is differentiable, as shown by the following proposition.

Proposition 9 *Assume that γ is differentiable. When $\mu < 1$, $r = 0$ is the only possible monocentric equilibrium.*

Proof. When γ is a differentiable function, whereas $r \neq 0$, the derivative of Ω at r is given by

$$\Omega'(r) = (1 - \mu)\psi'(r).$$

A necessary condition for r to be a local maximum is $\Omega'(r) = 0$, which holds if and only if $(1 - \mu)\psi'(r) = 0$. This implies that either $\mu = 1$ or $\psi'(r) = 0$ must be met. As shown in Appendix C, $\psi'(r) \neq 0$ when $r \neq 0$. Hence, when $\mu < 1$, an off-center location is incompatible with a monocentric equilibrium, no matter the value of N . ■

Note that our result is highly reminiscent of, yet slightly different from, the one derived by Berliant and Kung (2003, p. 14). Indeed, these authors show that when the cost function is $\mathcal{C}^\infty$, there is no asymmetric spatial equilibrium. Yet, because they assume that there are no boundaries on the agricultural area, so that $R(-\ell) = R(\ell) = 0$ holds, their result cannot be straightforwardly extrapolated to our setting.

Insert Figure 4 about here

Figure 4 depicts an example in which $r \neq 0$ whereas γ is differentiable. As can be seen, the market potential actually increases when one moves from $r = 0.4$ to the center, which shows that some firms have an incentive to relocate to the ‘long side’ of the economy. It is of interest to note that Proposition 9 is almost identical to Proposition 3, which shows that the results of the location model carry through to the more complex general equilibrium specification used in this section.

As shown above, an off-center location r is never a monocentric equilibrium when γ is differentiable, whereas it may be one when γ is non-differentiable. In the latter case, there is a crucial trade-off between centrality (i.e. r) and attraction (i.e. μ). In the limit, when μ becomes sufficiently

¹⁴Yet, this equilibrium is kind of ‘unstable’, depending crucially on the analytical form of the cost function γ . Indeed, locations sufficiently close to the center in $r = 0$ are now nearly as attractive as the center itself, which shows that the lock-in effect and the urban shadow may be very weak in this case.

large, any off-center location may be sustained with a non-differentiable cost function γ , whereas such a result does not hold when γ is smooth. As shown in the next section, the existence of a transportation hub significantly changes these findings in the differentiable case, thus providing a result analogous to the one derived in the location model of Subsection 2.2.

4 Transport hubs and the lock-in of cities

In what follows, we assume that γ is differentiable. We still consider the same setting as developed in Section 3, but we assume now that there is a hub at $b \geq 0$. Just as Fujita *et al.* (1999a, p. 227), we view our hub as “a metaphorical representation for any sort of transportation hub, whether created by the crossing of transportation routes or even by the availability of a port through which goods may be shipped to and from distant regions”. There are n identical branches bifurcating at the hub and extending to a distance of ℓ each (see Figure 2 for an illustration with $n = 3$ branches).

Almost all analytical expressions derived in the monocentric setting without a hub still apply. The only changes concern the expressions of $w^M(s)$ and of the agricultural surplus S^A . Let

$$\delta = \begin{cases} 0 & \text{if } \ell \leq b \\ 1 & \text{if } \ell > b \end{cases}$$

be an indicator of the presence of a hub, and let

$$d(t, s) = \begin{cases} \gamma(t - s) & \text{if } t \leq b \\ \gamma(t + s - 2b) & \text{if } t > b. \end{cases} \quad (18)$$

The manufacturing wage in location s can then be expressed as

$$w^M(s) = \left[Y(s)e^{-(\sigma-1)\tau^M\gamma(s-r)}G(s)^{\sigma-1} + \int_{-\ell}^{\ell} Y(t)e^{-(\sigma-1)\tau^M\gamma(r-t)}G(t)^{\sigma-1}dt + n\delta \int_b^{\ell} Y(t)e^{-(\sigma-1)\tau^M d(s,t)}G(t)^{\sigma-1}dt \right]^{\frac{1}{\sigma}},$$

whereas the agricultural surplus at the city is now given by

$$S^A = \mu \left[\int_{-\ell}^{\ell} e^{-\tau^A\gamma(r-t)}dt + n\delta \int_b^{\ell} e^{-\tau^A\gamma(r-t)}dt \right], \quad (19)$$

which is independent of the firm location s . Thus, the generalized market potential function with transportation hub is given as follows:

$$\begin{aligned}\Omega_H(s) &= e^{\sigma[(1-\mu)\tau^A - \mu\tau^M]\gamma(r-s)} \left[\mu e^{-(\sigma-1)\tau^M\gamma(r-s)} \right. \\ &\quad + \frac{\mu(1-\mu)}{S^A} \int_{-\ell}^{\ell} e^{[(\sigma-1)\tau^M - \tau^A]\gamma(t-r) - (\sigma-1)\tau^M\gamma(s-t)} dt \\ &\quad \left. + \frac{\mu(1-\mu)}{S^A} n\delta \int_b^{\ell} e^{[(\sigma-1)\tau^M - \tau^A]\gamma(r-t) - (\sigma-1)\tau^M\gamma(s+t-2b)} dt \right]. \quad (20)\end{aligned}$$

As can be seen from (20), when there is no hub (i.e. $\delta = 0$), Ω_H boils down to Ω . Rewrite the generalized market potential function with hub as follows:

$$\Omega_H(s) = \left[\mu e^{-(\sigma-1)\tau^M\gamma(r-s)} + \frac{\mu(1-\mu)}{S^A} \left[\tilde{\psi}(s) + n\delta \overline{\psi}(s) \right] \right] e^{\sigma[(1-\mu)\tau^A - \mu\tau^M]\gamma(r-s)} \quad (21)$$

where, by definition (18) of d ,

$$\begin{aligned}\tilde{\psi}(s) &= \int_{-\ell}^{\ell} e^{[(\sigma-1)\tau^M - \tau^A]\gamma(t-r) - (\sigma-1)\tau^M\gamma(s-t)} dt. \\ \overline{\psi}(s) &= \begin{cases} \int_{-\ell}^{\ell} e^{[(\sigma-1)\tau^M - \tau^A]\gamma(r-t) - (\sigma-1)\tau^M\gamma(s-t)} dt & \text{if } s \leq b \\ \int_b^{\ell} e^{[(\sigma-1)\tau^M - \tau^A]\gamma(r-t) - (\sigma-1)\tau^M\gamma(s+t-2b)} dt & \text{if } s > b. \end{cases}\end{aligned}$$

Note that the expression of $\overline{\psi}$ changes at $s = b$ which shows that $\overline{\psi}$ and, therefore, Ω_H will usually be non-differentiable at this point. Yet, both ‘parts’ of Ω_H are smooth functions. As shown in Appendix D, the generalized derivative of Ω_H in the sense of Clarke (1975, 1983) at b , conditional upon $r = b$, is given as follows:

$$\partial\Omega_H \Big|_{r=b} (b) = \text{co} \left\{ k[\tilde{\psi}'(b) - n\delta \overline{\psi}'_1(b)], k[\tilde{\psi}'(b) + n\delta \overline{\psi}'_1(b)] \right\}, \quad (22)$$

where

$$k = -\frac{\mu(1-\mu)(\sigma-1)\tau^M}{S^A} < 0$$

is a bundle of parameters and where

$$\overline{\psi}'_1(b) = -(\sigma-1)\tau^M \int_b^{\ell} \gamma'(b-t) e^{-\tau^A\gamma(b-t)} dt. \quad (23)$$

Note that for Ω_H to be differentiable at b when $r = b$ requires that (23) be equal to zero, which is impossible on the domain of integration $[b, \ell]$ by the very definition of γ . This shows that *there is always a cusp at the hub even when γ is a smooth function*.¹⁵ Figure 5 illustrates this situation.

Insert Figure 5 about here

The first order necessary condition for a local maximum at the hub is given by $0 \in \partial\Omega_H(b)$. Combining this with the sufficient condition for a local maximum, the hub may be a monocentric equilibrium provided that

$$-n\delta\bar{\psi}'_1(b) < \tilde{\psi}'(b) < n\delta\bar{\psi}'_1(b)$$

holds. When $b < \ell$, and hence $\delta = 1$, this can be summarized by the unique condition

$$\left| \frac{\int_{-\ell}^{\ell} \gamma'(b-t)e^{-\tau^A \gamma(b-t)} dt}{\int_b^{\ell} \gamma'(t-b)e^{-\tau^A \gamma(b-t)} dt} \right| < n, \quad (24)$$

since $\gamma'(-x) = -\gamma'(x)$. This leads to the following proposition.

Proposition 10 *A city located at a hub may be an AME, independently of the functional form of γ , provided that (i) the population is not too large; (ii) the hub is not too off-centrally located; and (iii) the hub has sufficient attraction (i.e., n is large).*

Proof. The proof follows from (24). First, for any given ℓ , the denominator goes to zero as $b \rightarrow \ell$, whereas the numerator has a finite limit. Hence, the hub cannot be too off-centrally located since this would violate the equilibrium conditions for any fixed number n of branches. Conversely, consider that b is fixed. Then one can always choose n sufficiently large so that (24) holds. For n and ℓ not too large and given, the numerator goes to zero as $b \rightarrow 0$. This is due to the continuity of the numerator and the fact that it is equal to zero if b is zero (which can straightforwardly be verified by recalling that $\gamma'(-x) = -\gamma'(x)$). Hence, when ℓ is not too large, an AME can be sustained at the hub. ■

¹⁵This non-differentiability does of course not automatically imply that the monocentric configuration is an equilibrium at the hub. Yet, it shows that an off-center agglomeration at the hub may possibly be an AME even when γ is differentiable, which is never the case when there is no hub.

Proposition 10 shows that, *even if the transport cost function is differentiable, an AME can be sustained, provided the city is located at a transportation hub*. Condition (24) highlights that there is a trade-off between centrality of the hub (as given by b), attraction of the hub (as given by n) and size of the population (as given by ℓ). Note finally that, since the left-hand side of expression (24) is strictly positive, the off-center agglomeration is not sustainable as a spatial equilibrium when there is no hub and when γ is differentiable. In this case, we fall back on the result of Proposition 9.

5 Conclusions

When taken together, propositions 6, 7 and 10 may provide a rationale for why cities are mostly located at transportation hubs. Our results suggest indeed that in the early stages of economic development, characterized by convex small-scale transportation technologies without economies of long-haul, agglomerations can only find some lock-in when established at transportation hubs. Being then locked-in, these agglomerations stay there, which largely explains why we do not observe ‘floating cities’ in a spatial economy.

When the cost functions are non-smooth, as is e.g. the case with modern increasing returns transportation technologies, a city can become locked-in almost anywhere in the space-economy, so that asymmetric monocentric equilibria can be sustained. This suggests that the degree of indeterminacy of city location rises with the evolution of transportation technologies, whereas it was most often driven by ‘first nature’ in the past. Finally, this also suggests that transportation technology can *create new hubs*; since the location of transportation activities is itself endogenously determined, we may conclude that the location of hubs is nowadays driven by ‘second nature’, whereas it was exclusively driven by ‘first nature’ in the past (see Cronon, 1991). To the best of our knowledge, this conceptual and analytical similarity between economies of long-haul in transportation technologies and transportation hubs has been overlooked in the literature until now.

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Appendix A

Let K_f stand for the set of points at which the total cost function c fails to be differentiable. The generalized derivative at $x \in K_f$ is then best computed with the help of the following characterization theorem due to Clarke (1983). Let $\partial f(x)$ stand for the set of generalized derivatives of f at x , whereas f' refers to a standard derivative.¹⁶

Theorem 11 (Clarke, 1983, p. 63) *Let f be Lipschitz near x , and suppose S is any set of Lebesgue measure 0 in $\mathbb{R}^n$. Let K_f be the set of points at which f fails to be differentiable. Then*

$$\partial f(x) = \text{co}\{\lim \nabla f(x_i) : x_i \rightarrow x, x_i \notin S, x_i \notin K_f\}, \quad (25)$$

where co stands for the convex hull.

Theorem 11 roughly states that we may evaluate the function in the vicinity of the kink $x \in K_f$ (where it is differentiable), and then compute the generalized derivative as the convex hull of all limit points of sequences such that the gradient converges for $x_i \rightarrow x$.

Because we assume that the cost function γ is at most non-differentiable at 0, the function c has a single kink located at $s = r$. We can then compute the generalized derivative of c at $s = r$ with the help of Theorem 11. Note that, because γ is a strictly positive, increasing and continuous function, g is convex and, therefore, c is convex too. This implies that c is locally Lipschitz in the vicinity of r . From Lemma 2, we know the derivative of g , whereas the derivative of γ for $s \neq r$ is given by $\gamma'(s - r)$. Taking the limits and the convex hull then yields the desired result.

¹⁶We do not go into more technical details in this paper. Let us just mention that all functions involved are sufficiently regular for the Clarke derivative to work like some 'natural extension' of the directional derivative. For further details see Clarke (1975, 1983). Note also that, because all functions involved in Section 2 are convex, we could resort to subdifferential calculus (see, e.g., Hiriart-Urruty and Lemaréchal, 1993).

Appendix B

In this appendix, we show that ψ is a differentiable function of s and that $\psi'(r) = 0$ if and only if $r = 0$. The following theorem, known as the *Leibnitz Integral Rule* (see, e.g., Kaplan, 1992, pp. 256-258) will be useful.

Theorem 12 (Leibnitz Integral Rule) *Let f be a continuous function of the couple (t, s) and assume that $\partial f / \partial s$ exists and is continuous with respect to (t, s) . Given two differentiable functions u and v of s , the function*

$$\varphi(s) = \int_{u(s)}^{v(s)} f(t, s) dt$$

is differentiable with respect to s and its derivative is given by

$$\varphi'(s) = \int_{u(s)}^{v(s)} \frac{\partial f}{\partial s}(t, s) dt + f[v(s), s]v'(s) - f[u(s), s]u'(s).$$

Dropping the denominator of ψ (which is constant with respect to s , and hence has no impact), we can focus on

$$\psi(s) \equiv \int_{-\ell}^{\ell} e^{[(\sigma-1)\tau^M - \tau^A]\gamma(t-r) - (\sigma-1)\tau^M\gamma(s-t)} dt = \int_{-\ell}^{\ell} e^{a_1\gamma(t-r) - a_2\gamma(s-t)} dt.$$

Case 1. Assume first that γ is differentiable. In this case, ψ is obviously differentiable and we hence need only to show that its derivative at r is zero if and only if $r = 0$. Straightforward application of Theorem 12 yields

$$\begin{aligned} \frac{\partial \psi}{\partial s}(r) &= -a_2 \int_{-\ell}^{\ell} \gamma'(r-t) e^{a_1\gamma(t-r) - a_2\gamma(r-t)} dt \\ &= -a_2 \int_{-\ell}^{2r-\ell} \gamma'(r-t) e^{a_1\gamma(t-r) - a_2\gamma(r-t)} dt \\ &\quad + a_2 \int_{2r-\ell}^r \gamma'(r-t) e^{a_1\gamma(t-r) - a_2\gamma(r-t)} dt \\ &\quad + a_2 \int_r^{\ell} \gamma'(r-t) e^{a_1\gamma(t-r) - a_2\gamma(r-t)} dt \end{aligned}$$

By symmetry with respect to r and because $\gamma'(-x) = -\gamma'(x)$, the two last terms cancel out. Hence, the derivative of ψ at r is zero if and only if

$$\int_{-\ell}^{2r-\ell} \gamma'(r-t) e^{a_1\gamma(t-r) - a_2\gamma(r-t)} dt = 0.$$

Since $2r - \ell < r$, the function under the integral is strictly positive. Hence, the derivative can only be equal to zero when $r = 0$.

Case 2. When γ is non-differentiable, things become more involved. In what follows, we focus on the special case in which $\gamma(\cdot) = |\cdot|$. We need to distinguish the two cases $s < r$ and $s > r$. Within each of these cases, three sub-cases may arise. Let us develop the case $s < r$ only (the second case is strictly analogous):

(i) when $t < s < r$, the function under the integral of ψ is given by $e^{-a_1(t-r)-a_2(s-t)}$, and its derivative with respect to s is $-a_2 e^{-a_1(t-r)-a_2(s-t)}$.

(ii) when $s < t < r$, the function under the integral of ψ is given by $e^{-a_1(t-r)+a_2(s-t)}$, and its derivative with respect to s is $a_2 e^{-a_1(t-r)+a_2(s-t)}$.

(iii) when $s < r < t$, the function under the integral of ψ is given by $e^{a_1(t-r)+a_2(s-t)}$, and its derivative with respect to s is $a_2 e^{a_1(t-r)+a_2(s-t)}$.

Because f is differentiable on the different domains of integration, we can apply Theorem 12. Yet, we must note that the integration bounds now depend on the variable s . We hence get

$$\begin{aligned} \left. \frac{\partial \psi}{\partial s} \right|_{s < r} (s) &= -a_2 \int_{-\ell}^s e^{-a_1(t-r)-a_2(s-t)} dt + e^{-a_1(s-r)} \\ &\quad + a_2 \int_s^r e^{-a_1(t-r)+a_2(s-t)} dt - e^{-a_1(s-r)} \\ &\quad + a_2 \int_r^\ell e^{a_1(t-r)+a_2(s-t)} dt \end{aligned}$$

which simplifies to

$$\begin{aligned} \left. \frac{\partial \psi}{\partial s} \right|_{s < r} (s) &= -a_2 \int_{-\ell}^s e^{-a_1(t-r)-a_2(s-t)} dt + a_2 \int_s^r e^{-a_1(t-r)+a_2(s-t)} dt \\ &\quad + a_2 \int_r^\ell e^{a_1(t-r)+a_2(s-t)} dt \end{aligned}$$

because the two exponential terms cancel out. A similar calculation shows that

$$\begin{aligned} \left. \frac{\partial \psi}{\partial s} \right|_{s > r} (s) &= -a_2 \int_{-\ell}^r e^{-a_1(t-r)-a_2(s-t)} dt + a_2 \int_r^s e^{a_1(t-r)-a_2(s-t)} dt \\ &\quad + a_2 \int_s^\ell e^{a_1(t-r)+a_2(s-t)} dt. \end{aligned}$$

The differentiability of ψ then follows from

$$\lim_{s \rightarrow r} \left. \frac{\partial \psi}{\partial s} \right|_{s < r} (s) = \lim_{s \rightarrow r} \left. \frac{\partial \psi}{\partial s} \right|_{s > r} (s). \quad (26)$$

Finally, we can apply the same technique as in case 1 to show that $\psi'(r) = 0$ if and only $r = 0$. It suffices to note that in this case

$$\int_{-\ell}^r e^{(-a_1+a_2)(t-r)} dt = \int_r^{\ell} e^{-(a_1+a_2)(t-r)} dt \quad (27)$$

must hold, which is impossible if $r \neq 0$.

Appendix C

The generalized market potential function Ω is neither differentiable nor sub-differentiable in the sense of convex analysis. Yet, we can use generalized derivatives in the sense of Clarke (1975, 1983) in order to investigate its properties. This allows us to avoid the use of directional derivatives of composite functions, which cuts short some technical developments and eases the burden of notation.

By assumption, γ is at most non-differentiable at 0 so that Ω has a single kink located at $s = r$. Because Ω is locally Lipschitz in the vicinity of r , we may apply Theorem 11 to compute the generalized derivative at $s = r$. We thus have

$$\Omega'(s) \big|_{s \neq r} = \mu \left[a'(s)c(s) + a(s)c'(s) \right] + (1 - \mu) \left[\psi(s)c'(s) + \psi'(s)c(s) \right],$$

where

$$a'(s) \big|_{s \neq r} = -(\sigma - 1)\tau^M \gamma'(r - s) e^{(\sigma-1)\tau^M \gamma(r-s)}$$

and

$$c'(s) \big|_{s \neq r} = -\sigma[(1 - \mu)\tau^A - \mu\tau^M] \gamma'(r - s) e^{\sigma[(1-\mu)\tau^A - \mu\tau^M] \gamma(r-s)}.$$

Since a , c and γ are continuous functions, and since $\gamma(0) = 0$, we have

$$\lim_{\substack{s \rightarrow r \\ s > r}} a'(s) \big|_{s \neq r} = -\sigma\rho\tau^M \lim_{\substack{s \rightarrow r \\ s > r}} \gamma'(r - s)$$

and

$$\lim_{\substack{s \rightarrow r \\ s > r}} c'(s) \big|_{s \neq r} = -\sigma[(1 - \mu)\tau^A - \mu\tau^M] \lim_{\substack{s \rightarrow r \\ s > r}} \gamma'(r - s),$$

where $\rho \equiv (\sigma - 1)/\sigma$ is used for notational convenience. Symmetric expressions hold when $s < r$.

Because $\lim_{s \rightarrow r} a(r) = \lim_{s \rightarrow r} \psi(r) = \lim_{s \rightarrow r} c(r) = 1$, and because ψ is continuously differentiable, we finally get

$$\begin{aligned} \lim_{\substack{s \rightarrow r \\ s > r}} \Omega'(r) &= \sigma \left[(1 - \mu)\tau^A - \mu(1 + \rho)\tau^M \right] \lim_{\substack{s \rightarrow r \\ s > r}} \gamma'(r - s) + (1 - \mu)\psi'(r) \\ &= k \lim_{\substack{s \rightarrow r \\ s > r}} \gamma'(r - s) + (1 - \mu)\psi'(r). \end{aligned} \quad (28)$$

A similar calculation shows that

$$\begin{aligned} \lim_{\substack{s \rightarrow r \\ s < r}} \Omega'(r) &= \sigma \left[(1 - \mu)\tau^A - \mu(1 + \rho)\tau^M \right] \lim_{\substack{s \rightarrow r \\ s < r}} \gamma'(r - s) + (1 - \mu)\psi'(r) \\ &= k \lim_{\substack{s \rightarrow r \\ s < r}} \gamma'(r - s) + (1 - \mu)\psi'(r). \end{aligned} \quad (29)$$

Because γ is locally Lipschitz in the vicinity of 0, the limits are finite so that (28) and (29) imply that

$$\partial\Omega(r) = \text{co} \left\{ k \lim_{\substack{s \rightarrow r \\ s < r}} \gamma'(r - s) + (1 - \mu)\psi'(r), k \lim_{\substack{s \rightarrow r \\ s > r}} \gamma'(r - s) + (1 - \mu)\psi'(r) \right\}.$$

Appendix D

We may proceed as in appendices A and C to calculate the derivative by using Theorem 11. Calculating the derivative of Ω_H for $s \neq b$, we have

$$\begin{aligned} \Omega'_H(s) &= \left\{ \mu(\sigma - 1)\tau^M \gamma'(r - s) e^{-(\sigma-1)\tau^M \gamma(r-s)} + \frac{(1 - \mu)}{TC^A} \left[\tilde{\psi}'(s) \right. \right. \\ &\quad \left. \left. + n\delta \overline{\psi}'(s) \right] \right\} e^{\sigma[(1-\mu)\tau^A - \mu\tau^M]\gamma(r-s)} - \left[\sigma((1 - \mu)\tau^A - \mu\tau^M) \gamma'(r - s) \right] \\ &\quad \times \left\{ \mu e^{-(\sigma-1)\tau^M \gamma(r-s)} + \frac{\mu(1 - \mu)}{S^A} \left[\tilde{\psi}(s) + n\delta \overline{\psi}(s) \right] \right\} e^{\sigma[(1-\mu)\tau^A - \mu\tau^M]\gamma(r-s)} \end{aligned}$$

because $\overline{\psi}$ is differentiable for all $s \neq b$. We are interested in evaluating this derivative at $r = b$ in order to show that the hub creates some cusp, independently of the mathematical properties of the cost function γ . *Assume hence in what follows that $r = b$* and let us examine the different components of the derivative of Ω_H .¹⁷ First, using the results of Appendix B, we have

$$\tilde{\psi}'(s) = -(\sigma - 1)\tau^M \int_{-\ell}^{\ell} \gamma'(s - t) e^{[(\sigma-1)\tau^M - \tau^A]\gamma(t-b) - (\sigma-1)\tau^M \gamma(s-t)} dt$$

¹⁷In order to alleviate notations, we omit to explicitly recall $|_{r=b}$ for all expressions.

and thus

$$\tilde{\psi}'(b) = -(\sigma - 1)\tau^M \int_{-\ell}^{\ell} \gamma'(b-t)e^{-\tau^A \gamma(b-t)} dt$$

since γ is symmetric. Denote by $\bar{\psi}_1$ the ‘left’ part of $\bar{\psi}$ (associated with $s < b$) and by $\bar{\psi}_2$ the ‘right’ part of $\bar{\psi}$ (associated with $s > b$). Then we have

$$\bar{\psi}'_1(s) = -(\sigma - 1)\tau^M \int_b^{\ell} \gamma'(s-t)e^{[(\sigma-1)\tau^M - \tau^A]\gamma(t-b) - (\sigma-1)\tau^M \gamma(s-t)} dt$$

which, by continuity, yields

$$\lim_{\substack{s \rightarrow b \\ s < b}} \bar{\psi}'_1(s) = -(\sigma - 1)\tau^M \int_b^{\ell} \gamma'(b-t)e^{-\tau^A \gamma(b-t)} dt \quad (30)$$

since γ is symmetric. Since $\gamma'(-x) = -\gamma'(x)$, we have

$$\lim_{\substack{s \rightarrow b \\ s > b}} \bar{\psi}'_2(s) = (\sigma - 1)\tau^M \int_b^{\ell} \gamma'(b-t)e^{-\tau^A \gamma(b-t)} dt = -\lim_{\substack{s \rightarrow b \\ s < b}} \bar{\psi}'_1(s). \quad (31)$$

Note that expressions (30) and (31) are exactly the opposite of each other, which implies that there will be a cusp.

Assembling the different parts, the generalized derivative of Ω_H at b , conditional upon the fact that $r = b$, is given by

$$\partial\Omega_H \Big|_{r=b} (b) = \text{co} \left\{ k[\tilde{\psi}'(b) - n\delta\bar{\psi}'_1(b)] , k[\tilde{\psi}'(b) + n\delta\bar{\psi}'_1(b)] \right\},$$

which establishes the result.



Figure 1: Location in the linear economy

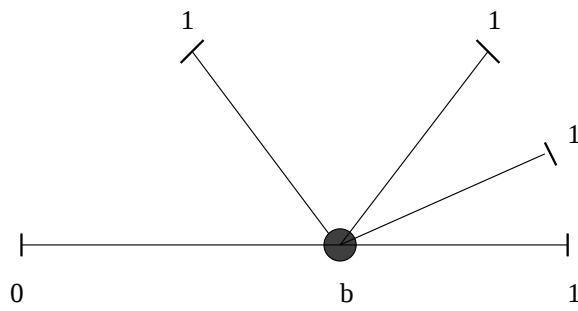


Figure 2: Location in a transport network

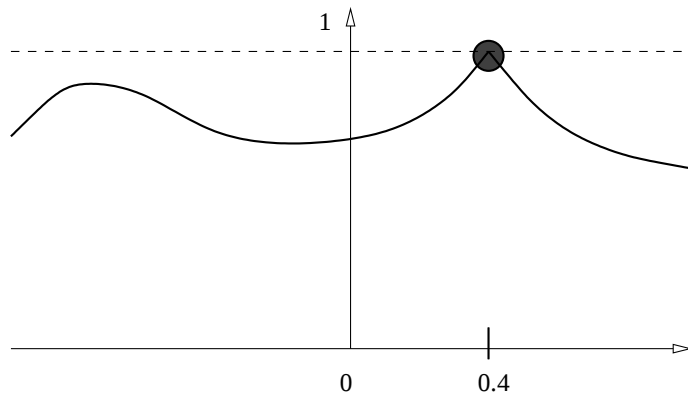


Figure 3: The non-differential case: existence of AME

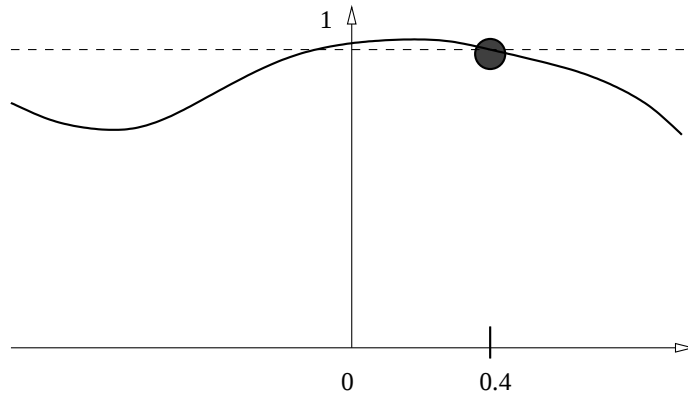


Figure 4: The differentiable case: AME does not exist

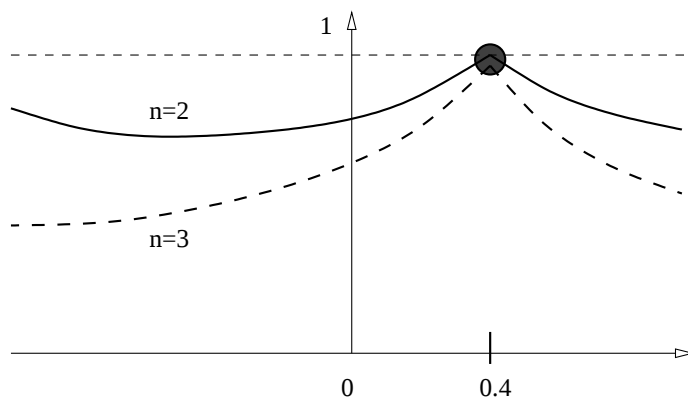


Figure 5: The differentiable case: an AME at the hub