AN ALTERNATIVE THEORY OF HEDGING AND SPECULATION

by

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ABSTRACT

This paper develops an alternative non-maximizing approach to individual speculator decision-making. The economic agent is seen to determine his market position by equating his estimate of the probability of loss with his willingness to take risk expressed as a function of market commitments. This alternative approach is applied to speculative buying and selling in commodity futures, and to carrying charge and selective hedgers, although it may be applicable to other speculative activities. Two specific forms of expectations function are used, and comparison is made between the non-maximizing approach and two expected utility models regarding changes in current price, expectations, etc.

Theoretical and empirical work on individual decision-making in the context of organized commodity markets has mostly assumed that individual economic agents pursue, or behave as if they pursue, the aim of maximum expected utility (Stein 1961, 1964 based on Tobin 1958; L.L. Johnson 1967; Ward and Fletcher 1971; Rutledge 1972) or maximum expected net income (Telser 1958, 1965; Brennan 1958). The assumption of maximum expected utility is also used in the theory of asset pricing (see for example Fama 1971).

For the reader who believes that individuals do not maximize expected utility because of inadequate information, excessive adjustment time, inability to distinguish local from global maxima, utility functions which do not permit maximization, or for some other reason, and for the reader who finds the aim of maximum expected utility introspectively unsatisfactory, this paper presents an alternative non-maximizing approach. Whether this theory yields predictions different from those of the expected utility theory, is of course one of the questions of this paper.

The main determinants of an individual speculator's or hedger-speculator's market position are his expectations of the relevant variable relating to the commodity[1] in which he is speculating, his attitude toward risk and his decision rule. This paper presents an alternative model first for the case of an individual speculator, who may be either risk averse or risk-loving, and secondly for the case of a discretionary hedger who combines both hedging and speculative elements, and who may be either a carrying-charge or selective hedger

1. The main variable relevant here is of course the price, but expectations of quality and of delivery location may also be relevant.

in the sense of Working (1953). It will be shown that conventional results, as regards the nature of the trader's supply and demand functions and with respect to certain parameter changes, can be obtained by a route alternative to that of utility maximization. Moreover, it will be seen in passing that market stability will be attained with this alternative approach under conditions which are economically meaningful. Finally, a comparison will be made between the predictions of this model and those of two important expected utility models.

For the benefit of the non-specialist reader in the area of forward commodity markets, we begin with a brief review of the literature on the role of speculation, on expectations and on attitude toward risk in this context.

1 : SPECULATION AND DISCRETIONARY HEDGING IN FUTURES MARKETS: ROLE, EXPECTATIONS AND ATTITUDE TO RISK

1.1 Role of Speculation

In the Keynes (1930)-Hicks (1936) model speculators were seen as a group which took up the balance of hedgers' positions in the futures market: they were net long when hedgers were net short (the model was not developed for the reverse case). Their role was one of risk - bearing rather than forecasting, although the expectation on which the market position was based could be said to be a forecast (Cf. Rockwell, 1967). They were not however, involved in making forecasts of short-term price changes. In Kaldor's development (1939, 1961) of the Keynes-Hicks model the scope of speculation was extended to include actuals, and long speculators in futures and actuals shared the same expected spot price and marginal risk premium. In these treatments hedgers were routine hedgers seeking risk reduction only. Speculators in the Peston-Yamey (1960) model of market prices performed much the same functions as in Kaldor's model: their 'merchants' were long speculators in actuals, while long speculators in futures performed a risk-bearing function. The authors went on to admit discretionary hedgers (their 'mixed traders') and a group speculating on the long side of both spot and futures markets (their 'mixed speculators').

Houthakker (1957) estimated returns to speculators who were assumed to perform a risk-bearing function, for three U.S.A. markets, while Rockwell's tests with wider data enabled him to show that the bulk of *large* speculators' profits was derived from forecasting while small speculators performed a risk-bearing function.

In the Holbrook Working analysis hedging and speculative elements are combined, and in pursuit of an uncertain profit the trader's market position became a function of his expectations of the price spread. Provision was also made for stocks to be under-or over-hedged (selective hedging) where expectations of the price level are formed. Discretionary hedging thus became part of the firm's program of inventory control, product pricing and input purchasing (and risk reduction *per se* became a lesser motive in hedging.) In the discretionary hedging models of Stein and L.L. Johnson the hedger-speculator holds expectations of changes in both spot and futures prices. By maximizing expected utility, itself a function of risk and expected return, the optimum proportion of stock to be hedged was defined. In Johnson's case over-hedging and long positions in futures were also possible, while Ward and Fletcher extended the analysis to cover long hedging. In these presentations the pursuit of an uncertain profit by discretionary hedgers was part of the inventory and input program while at the same time determining the extent of that program.

The Brennan (1958)-Telser (1958) storage analysis is a development of the actuals speculation side of Kaldor's model, with merchants holding unhedged stocks in support of their expectations of the spot price. This stockholding is equivalent to a supply of storage, the marginal risk premium being one of the costs of storage, and deduction of the marginal convenience yield gives net marginal carrying cost. This supply of storage function is used for single equation estimation of the risk premium and convenience yield as a compound residual.

Rutledge (1972) used the portfolio model approach to estimate hedgers demand for futures contracts in the Chicago soybeans complex, and found *inter alia* that hedgers' demand for futures is elastic with respect to price spread expectations but inelastic with respect to price expectations. Dusak (1973) used the capital asset pricing model of Sharpe (1964) and Lintner (1965) to estimate that returns to long speculative positions in three U.S.A. commodities for the period 1952-67 were near-zero, and that relative risk for these commodities (taking into account the risk on wealth in general) was also near-zero, even though risk in the sense of price variability was positive.

1.2: Expectations and Attitude to Risk

An expectation in the sense of De Morgan is the view of a person, or group of persons, of a coming event or any other matter on which absolute knowledge does not exist (quoted in Georgescu-Roegen, 1958).

Relevant to the study of expectations are first the person or group by whom it is held, second the evidence known to that economic agent, and third the agent's prediction of the coming event, Ozga (1965). Expectations may be uniform or heterogeneous as between individuals; this may arise partly because different individuals may observe different evidence from given facts, or may make different predictions from given evidence.

Little is known about the precision with which commodity traders form price expectations, although some evidence is presented in Goss (1972). Precise representation of expectations in futures market literature has usually taken the form of an assumed or implied subjective probability distribution, either by reference to an expected value with the deduction of a risk premium from expected return, (e.g. Kaldor, Brennan) or by using the mean as a measure of expected return and the variance as a measure of risk, (e.g. Stein, L.L Johnson). In some cases uniformity (or near uniformity) of individuals' expectations is required for market equilibrium, (Kaldor, Brennan, Telser, 1958).

While progress has been made with 'direct measurement' (questionnaire) of consumer attitudes, investment intentions and business expectations in recent years (e.g. see Klein 1972), little is known of the factors which cause commodity traders with speculative and discretionary hedging positions to revise their price expectations. One theoretical explanation of the revision of expectations is embodied in the *adaptive* expectations hypothesis, developed by Nerlove (1958) and others, which postulates that current expectations are revised in proportion to the error in expectations in the preceding period. Since this model implies that the current price expectation is a weighted average of past actual prices, it has the convenient feature from the empirical viewpoint that an expected price may be estimated from objective data. This approach has the disadvantage however, that if current expectations are correct no revision will be made for next period even if new information comes to hand.

Mills' (1962) concept of *implicit* expectations does not exclude any relevant information, and his method leads to the estimate of an expected price which is implicit in the sense that it is consistent with the observed value of the variable representing the speculative commodity (e.g. supply of storage). This approach, which joins together the decision-making and expectations formation functions, has its rationale in areas of operation where economic agents predict actual values well.

Muth (1961) argues that a *rational* expectation, whatever may be the process by which it is formed, will incorporate all available information, so that it will not be possible to improve the predictive accuracy of an expectation by including lagged actual values along with the expectation. Expectations which are rational in this sense will have prediction errors which are not correlated with the expectation (see also Hirsch and Lovell, 1969).

In the area of futures markets Brennan and Peck (1976) have employed the adaptive expectations hypothesis, and Cox (1976) employed the Muth approach to obtain estimates of the degree of price dependence which support the view that futures trading increased the information available to traders, and that this information was fully reflected in market prices.[2]

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2. This implies that price changes in futures markets do not exhibit significant dependence, and this has been supported by the results of studies testing efficient market hypotheses using futures markets data; see especially Larson (1960), Stevenson and Bear (1970) and Praetz (1975).

'Risk' in this context means risk of loss from a particular market situation, and the individual's attitude toward risk is expressed in his market decision, given his expectations and current prices. Normally the prospect of increased returns is accompanied by increased risk, so that for a risk-averse trader the question of trade-off arises.

In the literature on futures trading a subjective estimate of the risk involved in various market positions has usually been derived from an individual trader's distribution of expected prices. While a trader may, in theory, be risk-averse, risk-neutral or risk-loving, this literature has almost invariably assumed risk-aversion. This has been expressed in two main ways: first by the requirement of a marginal risk premium in an expected revenue function (e.g. Kaldor, Brennan, Telser), and second by an increasing MRS of risk for expected return on expected utility contours in portfolio-type models (e.g. Stein, Johnson 1967, Ward and Fletcher). This latter approach corresponds to the assumption of falling marginal utility of income in a von Neumann-Morgenstern hypothesis.

Peck (1975) argued that relevant risk should not be measured by the variance of returns, but that once an economic agent has made a price forecast, risk may be better measured by the unexplained variation in returns. Dusak (1973) defines risk for a futures market trader in relative terms, as the contribution the futures market asset makes to the variability of return of a diversified portfolio containing assets representing wealth in general.

There is evidence however, to indicate that futures prices may belong to the family of stable Paretian distributions, which have infinite variances, so that the variance of expected or actual returns is not a suitable measure of risk (see Fama, 1971). In this paper risk is measured by the probability of loss as derived from the trader's subjective distribution of expected prices. Also relevant to the trader's decision-making is his willingness to take risk specified as a function of the trader's market commitments.

2 : A NON-MAXIMIZING MODEL OF SPECULATION

In this model, based on the general specification in Goss (1972), the key variables are the individual trader's price expectations and his willingness to take risk. The model is presented first for the cases of long and short speculation in futures, applied briefly to the cases of increasing risk, the Working "carrying charge" hedger and the selective hedger, and finally a comparison is made between the responses of this model to changes in current prices, expected prices, etc. and those of a simple model of expected utility maximization.

The individual trader is assumed to have a subjective probability distribution of expected futures prices given by $f(S)$ where S is a random variable with mean $S^* = \int S\, f(S) dS$ and variance $\sigma^2 = \int S^2 f(S) dS - S^{2*}$. In this paper two alternative forms of $f(S)$ are assumed, each being selected because of its possession of desired properties and in the interests of simplicity of analysis. In the first of these alternatives, form A, it is assumed that the individual trader forms price expectations which can be expressed by the subjective density function

$$f(S) = \frac{1}{\pi(1+s^2)} \qquad (1A)$$

giving a cumulative frequency function

$$F(s) \quad = \quad \frac{1}{\pi} \text{ artan a} + \tfrac{1}{2} \qquad \text{(2A)}$$

where the current futures price is measured in terms of deviations from the mean expected value: $s = P - S^*$ where P_o is the current futures price. This form has some desired properties (asymptotic to 0 and 1 in the form of (2A), precise to work with in subsequent analysis) and has mean equal to zero, and variance equal to $\frac{1}{\pi}$ (s - artan s).

In the second alternative employed here, form B, it is assumed that the individual trader forms expectations which may be represented, in cumulative frequency form, by a type of logistic function:

$$F(s) \quad = \quad \frac{1}{1 + e^{-s}} \qquad \text{(2B)}$$

which implies a subjective density function

$$f(s) \quad = \quad \frac{1}{2 + e^{s} + e^{-s}}$$

$$= \quad \frac{1}{2 + 2 \cosh s} \qquad \text{(1B)}$$

where the current futures price is again defined in terms of deviations from the mean expected value. The logistic, which has been used for estimation purposes in economics (Griliches (1957)) is again, as expressed in (2B) asymptotic to 0 and 1 and gives rise to precise forms in analysis.

The individual trader's willingness to take risk is defined by

$$R = R(x) \tag{3}$$

where x = market position measured by number of units held

and $0 < R(x) < 1$.

Risk aversion is defined by

$$\frac{\partial R(x)}{\partial x} < 0 \tag{4}$$

and $$R(x) < 0.5 \tag{5}$$

For simplicity it is assumed that $R(x)$ is linear of the form

$$R = a + bx \qquad \text{where} \quad a > 0 \quad b < 0 \tag{6}$$

although this assumption about the restriction on b will be reversed in the section dealing with Increasing Risk.

There is some evidence that in practice the form of $R(x)$ for an individual trader is determined by his psychological make-up and his past history of profits and losses (Goss (1972)). It is assumed here that the individual is willing to take a smaller (total) risk of loss as his total commitment increases, and that he is not willing to take a market position for which he estimates the probability of loss to exceed 0.5.

Therefore when the current futures price $P < S^*$ the individual trader is a buyer, and his estimate of the risk of loss *in the current price situation is*

$$\alpha = \int_{-\infty}^{P} f(S)\,dS \tag{7}$$

Similarly when $P > S^*$ the individual trader is a seller, and his estimate of the risk of loss *in the current price situation* is

$$\alpha = \int_P^\infty (S)dS \tag{7A}$$

Individual equilibrium is defined by

$$\alpha = R(x) \tag{8}$$

That is, it is assumed that the individual determines his market position by equating his willingness to take risk as defined in equations (3-5) with his estimate of the probability of loss in the current price situation, (7-7A).

For a smaller market position than that defined in (8) the individual would be willing to take a greater risk than that involved in the current price situation: hence an extension of holdings is indicated. Similarly for a larger market position than that in (8) he is willing to take less risk than the one he believes he is currently taking: a contraction of holdings is indicated.

2.1: Individual's Supply and Demand Functions

The question is then whether functions can be derived to describe the change in the individual trader's market position in response to a change in the current price of the contract traded, that is, whether supply and demand functions for futures contracts for the individual can be derived.

Form A Expectations

Suppose that the individuals price expectations can be represented by form A above, so that the relevant density function is given by (1A). If the individual expects the futures price to rise so that $s = P - S^* < 0$ he will be long in futures and the buyer's risk will be given by (2A).

Individual equilibrium requires that

$$\frac{1}{\pi} \text{artan } s + \tfrac{1}{2} = R(x)$$
$$= a + bx$$

so that

$$s = \tan \pi[(a-\tfrac{1}{2}) + bx] \tag{9}$$

which is the equation to the individual's demand curve, and has period $\frac{1}{b}$.

If the individual expects the price to fall so that $s > 0$ he will be short in futures and the seller's risk will be given by

$$-\frac{1}{\pi} \text{artan } s + \tfrac{1}{2} .$$

Assuming the individual has the same $R(x)$ function for long and short positions the equation to his supply function for futures contracts will be

$$s = -\tan \pi[(a-\tfrac{1}{2}) + bx] . \tag{10}$$

2.10 *Change in Current Price*

The effect of a change in the current futures price on the individual's demand for futures is given by $\frac{\partial x}{\partial P}$ from (9)

$$\frac{\partial x}{\partial P} = \frac{1}{b\pi(1+s^2)} < 0 \qquad \text{since} \qquad b < 0 \tag{11}$$

Hence a rise in the current price *cet.par.* will lead to a contraction of the individual's demand for futures.

If the individual is short in futures the response of his market position to a change in the current price is given by $\frac{\partial x}{\partial P}$ obtained from (10) and is the negative of (11). Hence a rise in the current futures price *cet. par.* will cause an extension of the individual's supply of futures contracts.

2.11 *Change in Price Expectations*

Suppose the individual revises his mean price expectation S^*. The effect of such a change *cet.par.* upon his demand for and supply of futures contracts is found by obtaining the differential $\frac{\partial x}{\partial S^*}$ from (9) and (10) respectively. From (9)

$$\frac{\partial x}{\partial S^*} = \frac{-1}{b\pi(1+s^2)} > 0 \tag{12}$$

so that an upward revision of price expectations would cause an upward shift in the individual's demand for futures. From (10) we see that $\frac{\partial x}{\partial S^*}$ is the negative of (12) so that an upward revision of price expectations will lead to a downward shift in the individual's supply function.

2.12 *Change in Variance (of Price Expectations)*

Although the variance of the distribution of expected prices is not a decision variable of the individual in this model, for purposes of comparison we observe that an increase in variance *cet.par.* results in an increase in buyer's risk for $s < 0$. We would therefore expect a decrease in supply and in demand for a risk averse individual.

This is easily shown as follows. With variance $\sigma_s^2 = \frac{1}{\pi}$ (s-artan s) (s-artan s) and $s > 0$ (seller), $\frac{\partial \sigma^2}{\partial s} > 0$ for $s^2 > 0$ (i.e. $s \neq 0$); hence there is a rise in the trader's supply price. For $s < 0$ (buyer) $\frac{\partial \sigma^2}{\partial s} < 0$ for $s^2 > 0$ (i.e. $s \neq 0$); the result is a fall in the trader's demand price. With risk aversion as defined in (6) the result is a decrease in supply and in demand.

2.2: Form B Expectations

If we suppose that the individual's price expectations are represented by form B above, and if $s < 0$ the trader will be long in futures, the buyer's risk being given by (2B). Individual equilibrium requires

$$\frac{1}{1 + e^{-s}} = R(x) = a + bx$$

The equation to the individual's demand function is therefore:

$$s = \log \left[\frac{a + bx}{1 - (a+bx)} \right] \tag{13}$$

which is a decreasing convex function of x. The effect of a price change *cet.par.* on the number of units held by the buyer is given by $\frac{\partial x}{\partial p}$ from (13).

$$\frac{\partial x}{\partial p} = \frac{e^{-s}}{b(1+e^{-s})^2} < 0 \qquad \text{since} \quad b < 0 \ , \ s < 0 \qquad 14)$$

Hence a rise in the current futures price will cause a contraction of the trader's demand for futures.

The effect of a revision in the individual's mean expected price *cet.par.* is given by $\frac{\partial x}{\partial S^*}$ from (13) and is the negative of (14). Hence an increase in the trader's mean expectation *cet.par.* will lead to an increase in demand for futures contracts.

If $s > 0$ so that the individual is short in futures, the seller's risk is given by $F(s) = \frac{1}{1 + e^{s}}$.

Individual equilibrium requires:

$$\frac{1}{1 + e^{s}} = R(x) = a + bx.$$

The equation to the trader's supply of futures function is therefore

$$s = \log \left[\frac{1 - (a+bx)}{a + bx} \right] \qquad (15)$$

which is an increasing concave function of x .

An increase in the current futures price *cet.par.* will cause an extension of supply because from (15)

$$\frac{\partial x}{\partial P} = \frac{-e^{s}}{b(1+e^{s})^{2}} > 0 \qquad (16)$$

On the other hand, an upward revision in mean expectation *cet.par.* will cause a decrease in supply of futures by the individual, because $\frac{\partial x}{\partial S^*}$ is the negative of (16).

2.3: Carrying Charge Hedger

A carrying charge hedger, following Working (1953/1962) is defined as a trader who is simultaneously long in actuals and short in futures, in pursuit of a gain from an expected change in the price spread, that is from an expectation of a decline in the forward premium or an increase in the spot premium. Working believed that the change in the price spread which could reasonably be expected was indicated by the current value of the price spread. Hence the carrying charge hedger used the price spread as a guide to inventory control, and the volume of carrying charge hedging was seen as an increasing function of the forward premium.

In this model it is assumed that the carrying charge hedger forms expectations of the price spread only, and deals in futures and actuals in a one-for-one ratio. It is further assumed that the trader is risk averse in the senses defined above.

Since the individual is a short hedger, we shall concentrate on the seller's risk given by (assuming form A expectations)

$$F(m) = \frac{-1}{\pi} \arctan m + \tfrac{1}{2}$$

where $m = B - M^* > 0$ and $B = P - A$ where A is the current actuals price, B is the current price spread, and M^* is the individual's expectation of M, the prospective price spread.

Individual equilibrium requires

$$\frac{-1}{\pi}\text{ artan } m + \tfrac{1}{2} = a + by \tag{17}$$

where y is the trader's market position comprising spot and futures contracts in a 1:1 ratio.

Hence $$m = -\tan \pi[(a-\tfrac{1}{2}) + by] \tag{18}$$

which we shall call the carrying charge hedger's supply of storage function.

From (17) we see that $\frac{\partial y}{\partial B} > 0$ so that the volume of carrying charge hedging is an increasing function of the forward premium. Again, the trader can use the current price spread as a guide to inventory control. Since $\frac{\partial^2 y}{\partial m^2} < 0$ the supply of storage function, existing in the first quadrant only, will be concave up. With $\frac{\partial y}{\partial M^*} < 0$ an upward revision in the expected value of the price spread will lead to a decrease in the supply of storage.

2.4: Selective Hedger

A selective hedger not only forms expectations of the price spread, but also takes a view about the absolute magnitudes of actuals and futures prices, and accordingly is prepared to under or over-hedge his actuals holdings. In this paper it is not assumed that the trader forms precise expectations about the levels of actuals and futures prices, but that in addition to expectation of the price spread, he takes a view as to which price is likely to change more. The proportion of actuals to futures is given by

$$\beta = g(q) \qquad g' < 0 \qquad (19)$$
$$0 < q < 1$$

where β = ratio of actuals to futures

q = subjective probability that the futures price will change more than the actuals price.

Individual equilibrium will be given by

$$\frac{-1}{\pi} \arctan m + \tfrac{1}{2} = a + bx,$$

where x = short position in futures for the individual.

The trader's supply of futures function is given by

$$m = -\tan \pi \left[(a-\tfrac{1}{2}) + bx\right]$$

The quantity of actuals held (long) will be given by (19).

2.5: Increasing Risk Trader

A risk-loving trader is defined here as one who is willing to take a greater risk of loss, as given by $R(x)$, as his investment increases, although it is not assumed that he is actually willing to take a market commitment where he believes that the odds are against him.

It is therefore assumed that $0 < R(x) < 0.5$ and $\frac{\partial R(x)}{\partial x} > 0$.

We assume again that the trader's willingness to take risk function is linear as in (6), but that a, b > 0.

It is also assumed that the trader is not interested in dealing in the market until some minimum level of risk is involved.

If $s < 0$ the trader will again be long in futures, and this equilibrium market position is given by (using form A expectations)

$$\frac{1}{\pi} \text{artan } s + \tfrac{1}{2} = a + bx \tag{20}$$

so that his demand function is again given by (9). In this case however from (20) $\frac{\partial x}{\partial s} > 0$ and $\frac{\partial^2 x}{\partial s^2} > 0$ so that the positively sloped demand function in the third quadrant is concave down.

Furthermore we find that since $\frac{\partial x}{\partial P} > 0$ as rise in price will lead to a buyer extending his market position; and since $\frac{\partial x}{\partial S^*} < 0$ an increase in mean expectation will lead to a decrease in demand, results which are the opposite of the risk-averse case.

On the other hand, if $s > 0$ the trader will again be short in futures, his equilibrium market position (with form A expectations) will be given by

$$-\frac{1}{\pi} \text{artan } s + \tfrac{1}{2} = a + bx \tag{21}$$

and the equation to his supply curve will be as in (10). From (21) however, we find that

$$\frac{\partial x}{\partial s} < 0 \qquad \text{and} \qquad \frac{\partial^2 x}{\partial s^2} > 0$$

the negatively sloped supply curve in the first quadrant will be concave up. Furthermore, since $\frac{\partial x}{\partial P} < 0$, a rise in price will mean a contraction of sales by the individual, and with $\frac{\partial x}{\partial S^*} > 0$ an upward revision of expectations will lead to an *upward* shift in the individual's supply function. Hence a market comprising predominantly risk-loving individuals as defined here, with expectations sufficiently diverse to permit a market equilibrium to be achieved, may be unstable in a static sense.

3 : SIMPLE MODEL OF EXPECTED UTILITY MAXIMIZATION

This section is concerned with the derivation of the individual trader's equilibrium position and the nature of his responses to changes in the current futures price and in price expectations, for a simple model of expected utility maximization, with an assumed quadratic utility function.

The trader's prospective return G from holding x units of futures (long or short) is

$$G = x(S-P) \tag{22}$$

where P is the current futures price and S the prospective futures price at the end of the period is a random variable.

The trader's expected return EG from this position is

$$EG = x(S^{*}-P) \tag{23}$$

where S* is the individual's expectation of S.

It is assumed that the individual's utility U is a quadratic function of the value of his current wealth W including prospective returns from his transactions in futures:

$$\begin{aligned} U &= aW + bW^{2} + c \\ &= a(W_{o}+G) + b(W_{o}+G)^{2} + c \end{aligned} \tag{24}$$

where a, b, c are constants and W_o is the initial wealth.

Then expected utility EU is

$$
\begin{aligned}
EU &= \int [aW_o + ax(S-P_o) + bW_o^2 + 2bW_o x(S-P_o) \\
&\quad + bx^2(S-P_o)^2] f(S)\,dS \\
&= aW_o + ax(S^*-P_o) + bW_o^2 + 2bW_o x(S^*-P_o) \\
&\quad + bx^2\sigma^2 + bx^2(S^*-P_o)^2 \qquad (25)
\end{aligned}
$$

where σ^2 is the variance of S.

For EU a maximum

$$U_x = aV + 2bW_oV + 2bx\sigma^2 + 2bxV^2 = 0$$

so that

$$x = \frac{-V(a+2bW_o)}{2b(\sigma^2+V^2)} \qquad (26)$$

where $V = S^* - P_o$, and (26) gives the trader's equilibrium position.

$U_{xx} = 2b(\sigma^2+V^2) < 0$ since $b < 0$ so that (26) represents a maximum.

3.1: Change in Price

If $V > 0$ so that the trader is long in futures his response to a change in the current price is found by differentiating (25) to obtain

$$\frac{\partial x}{\partial P} = \frac{(\sigma^2-V^2)(a+2bW_o)}{2b(\sigma^2+V^2)^2} < 0 \quad \text{if} \quad \sigma < V.$$

Hence a rise in the current futures price *cet.par.* will mean a contraction of the trader's demand.

On the other hand if $V < 0$ so that the trader is short in futures it can be seen that $\frac{\partial x}{\partial P_o}$ from (26) > 0 if $\sigma^2 - V^2 > 0$, i.e. $\sigma > V$ when $V > 0$.

Hence a rise in the current futures price leads to an increase in the quantity of contracts supplied. (Derivation of linear supply and demand curves is trivial.)

3.2: Change in Price Expectations

If the trader is long in futures ($V>0$) the effect on his market position of an upward revision in his mean price expectation is given by $\frac{\partial x}{\partial S^*}$ from (26) and is seen to be positive provided $V^2 > \sigma^2$, that is if $V > \sigma$ when $V > 0$.

On the other hand if $V < 0$ so that the trader is short in futures, it can be seen that $\frac{\partial x}{\partial S^*}$ from (26) is negative if $V^2 < \sigma^2$, that is if $V < \sigma$. Hence an upward revision in the trader's mean price expectation will lead to a reduction in his market position if he is on the short side of the market.

The effect of an increase in the variance of the individual's price expectations is given by $\frac{\partial x}{\partial \sigma^2}$ from (26) and is seen to be negative provided $\frac{a}{2b} < -W_o$, whether $V \gtrless 0$; that is the trader will reduce his market position consequent upon an increase in variance, whether he is a buyer or a seller of futures contracts, provided initial wealth is less than $\frac{-a}{2b}$.

4: GENERAL UTILITY FUNCTION

Suppose that the prospective gain G for an economic agent is

$$G = x(P - S)$$

where that agent has a utility function $U = U(G)$ and $x \gtrless 0$ as $P \gtrless ES$ ($x>0$ is a short position, $x<0$ is a long position).

In order to discover the characteristics of this individual's demand and supply functions for futures contracts we write

$$\frac{dU}{dx} = \frac{dU}{dG}\frac{dG}{dx}$$

$$= U'(P - S)$$

where $U' = U_G$

and we define

$$H(P,x) = E[U'(P - S)] \tag{27}$$

To maximize $U(G)$ we require $H(P,x) = 0$ and $H_x(P,x) < 0$. When $H(P,x) = 0$

$$P = \frac{EU'S}{EU'}$$

$$= ES + \frac{Cov(U',S)}{EU'} \tag{28}$$

$$H_x = E[U''(P-S)^2] < 0$$

when $EU'' < 0$.

In the region of maximum expected utility

$$\frac{dx}{dP} = \frac{-H_P}{H_x} \qquad (29)$$

$$= \frac{-[EU' + EU''x(P-S)]}{EU''(P-S)^2} \qquad (29a)$$

Assuming $EU' > 0$, $EU'' < 0$ then $\frac{dx}{dP} > 0$ if

$$EU' > - EU''x(P-S) \qquad (30)$$

That is the individual's supply function (first quadrant) is positively sloped under the conditions stated, where the current futures price is measured in terms of deviations from ES. The individual's demand function (third quadrant) is positively sloped under the conditions stated, although this four-quadrant result is conventional in that the qûantity demanded is a decreasing function of the current price.

To discover the curvature characteristics of these functions we take a total differential of (29) to obtain

$$\frac{d^2x}{dP^2} = \frac{-1}{H_x} [H_{PP} - 2H_{xP}(\frac{H_P}{H_x}) + H_{xx}(\frac{H_P}{H_x})^2] \qquad (31)$$

where

$$H_{PP} = 2EU''x + EU'''x^2(P-S) \qquad (32)$$

$$H_{xx} = EU'''(P-S)^3 \qquad (33)$$

$$H_{xP} = 2EU''(P-S) + EU'''x(P-S)^2 \qquad (34)$$

Therefore (31) becomes

$$\frac{d^2x}{dP^2} = \frac{-1}{H_x} [2EU'' \{x+2(P-S)\frac{dx}{dP}\}$$

$$+ EU'''(P-S) \{x^2+2x(P-S)^2\frac{dx}{dP} + (P-S)^2(\frac{dx}{dP})^2\}] \qquad (35)$$

Assuming $U''' = 0$

then

$$\frac{d^2x}{dP^2} = \frac{-1}{H_x} [2EU''x + \frac{dx}{dP} 4EU''(P-S)] \qquad (36)$$

When $x > 0$, and assuming that (30) applies we have

$$\frac{d^2x}{dP^2} < 0$$

when $$EU''x < \frac{-dx}{dP} 2EU''(P-S) \tag{37}$$

The inequality in (37) always holds, so that the supply function is concave down, given the assumptions stated above.

Similarly when $x < 0$, then $\frac{d^2x}{dP^2} > 0$ when $EU''x > \frac{-dx}{dP} 2EU''(P-S)$: this inequality always holds, so that the demand function is convex down, on the assumptions employed.

The effect of a change in price expectations on the individual's demand and supply functions is given by

$$H_S = -EU' - EU''x(P-S) \tag{38}$$

which is < 0 if (30) applies.

We see that an increase in price expectations will lead to an increase in demand or to a decrease in supply under the same conditions as $\frac{dx}{dP} > 0$.

Finally, substituting for P from (28) into (27) we obtain

$$H_{\sigma_S} = r\sigma_{U'} \gtreqless 0$$

as $$r \gtreqless 0 \quad (\sigma_{U'} \neq 0)$$

where r is the coefficient of correlation between U' and S. That is an increase in the variance of price expectations will lead to a decrease in demand if $r > 0$, and to a decrease in supply if $r < 0$. Normally we would expect $Cov(U'S) < 0$ for $x > 0$ and $Cov(U'S) > 0$ for $x < 0$.

5: CONCLUSIONS

This paper develops an alternative approach to individual decision-making for a speculator or hedger-speculator, and derives some predictions about the economic behaviour of these agents. It is shown that this approach can be applied to speculative buying and selling activities, and to hedgers who are carrying-charge hedgers or selective hedgers in the sense of Working.

Some comparisons are made between this alternative approach and two models of utility maximization. It was found that demand is a decreasing function of the current price for the risk-averse version of the non-maximizing model, and for the quadratic and general utility models; similarly supply is an increasing function of the current price for all three models, although the concavity characteristics differ, at least for the versions discussed in this paper. On the other hand, for the risk-loving version of the non-maximizing model demand is an increasing function of the current price and supply is a decreasing function of that price.

Although this paper is not concerned with market equilibrium, it is clear that because of the nature of the supply and demand functions derived for the non-maximizing model, a market comprising predominantly risk-averse individuals would have an equilibrium which is statically stable (provided there is diversity of expectations and/or attitude to risk to permit achievement of an equilibrium.) A market comprising predominantly risk-loving individuals however may have an equilibrium which is statically unstable.

It is also found that an upward revision of price expectations will lead to an increase in demand, and to a decrease in supply in the risk-averse case of the non-maximizing theory, and in the quadratic and general utility models. On the other hand, in the risk-loving version of the alternative approach an increase in price expectations will lead to a *decrease* in demand and to an *increase* in supply. Furthermore, an increase in the variance of price expectations will lead to a decrease in demand and a decrease in supply for the risk-averse version of the alternative approach, and in the quadratic and general utility models under stated conditions. In the risk-loving version of the non-maximizing model however, an increase in the variance of expectations leads to an increase in demand and to an increase in supply.

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