Math learning disability: what is the core deficit ?

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Ghent, 2011
3th expert meeting -
Mathematics research in the Benelux

Definition (DSM)

- *Developmental dyscalculia is a specific learning difficulty manifested by failure to achieve adequate proficiency in arithmetic despite*
  - *normal intelligence,*
  - *scholastic opportunities,*
  - *emotional stability, and*
  - *sufficient motivation*
- *These difficulties do not result from a sensorial deficit*
- *These difficulties interfere significantly with daily life and/or school success*

prevalence

- In Flanders, MLD, 3978 children (Desoete et al. MLD, 2004)
  - 2.27% in second grade
  - 7.70% in third grade
  - 6.59% in fourth grade
- In Israel, 3029 children 10 y.o. (Gross-Tsur, et al. 1996)
  - 6,5 % (score < two grade below mean & IQ > 80)
- 6 years follow-up (Shalev & al. 2005).
  - Among the 104 DC children
    - 40 % : score < pc 5 standardised test
    - 95% : score < pc 25

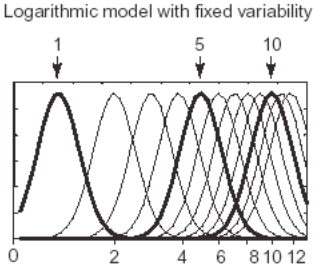
Core deficit

General cognitive factor: WM

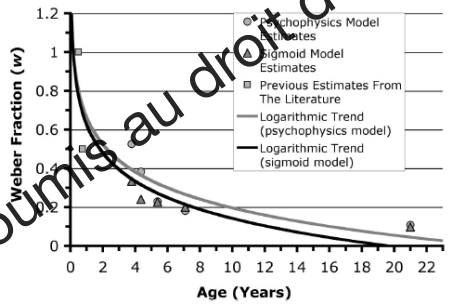
Basic numerical factor: the number sense

A simple story

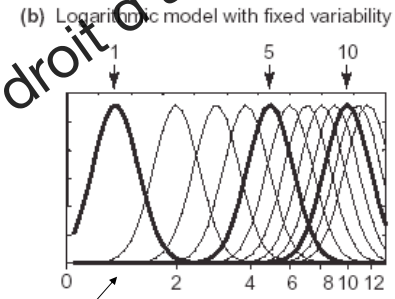
- Babies
  - Parallel individuation system: models of small sets (<4) in WM (or more if chunking: Feigenson & Halberda, 2008)
  - Language-independent system for representing numerosity
    - Analog magnitude system: ANS
    - Weber ratio limit



- Acuity of the ANS increases through development



- Around 3 y.o
  - Learn the counting list
  - Map this counting list to the ANS to build meaning of these words



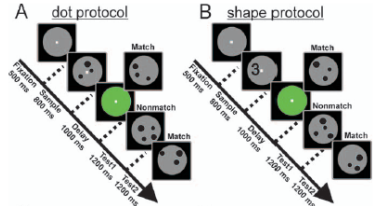
One, two, three, four, five, six, seven ....

Gallistel & Gelman, 1992

Learning symbols in monkeys

Trained 2 monkeys to discriminate small sets, and then pair sets and Arabic digits (1-5)

- set i, 1 s. delay, set j: press a lever if they are the same
- pairing Arabic digits and sets (1-5) for one months

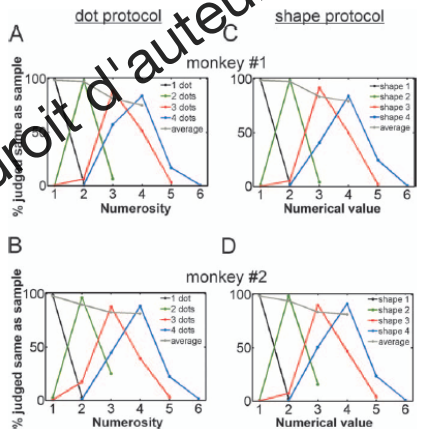


Diester & Nieder, 2007, Plos Biol

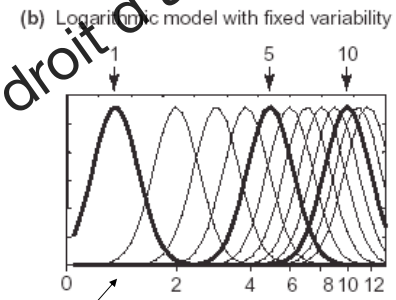
- Comparable performance for both formats in the match-to-sample task

(87 - 88% cr)

Noise in matching digits to sets, even for 2, 3 and 4



- Math learning builts on this ANS representation



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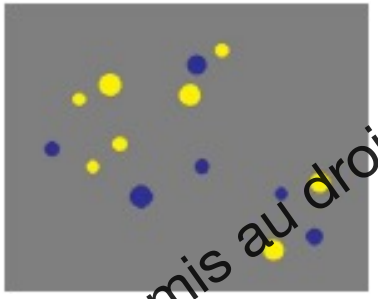
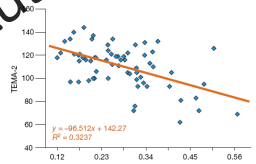
In unselected population

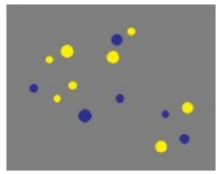


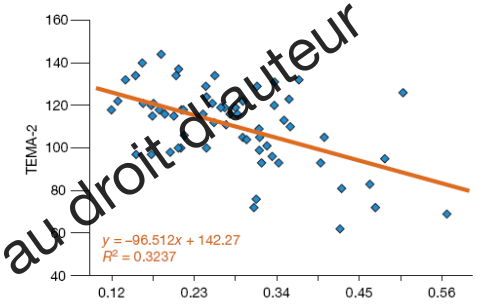


- 64 teenagers of 14 y.o.
- Measure of the acuity of ANS correlated with past school performance in math

Halberda, Mazzocco & Feigenson (2008)

In unselected population





- 64 teenagers of 14 y.o.
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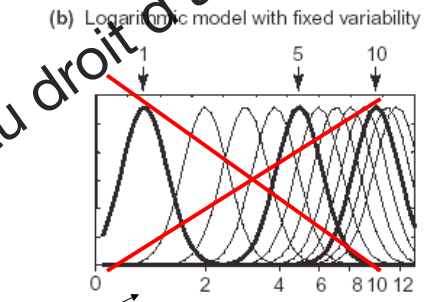
Halberda, Mazzocco & Feigenson (2008)

Developmental dyscalculia

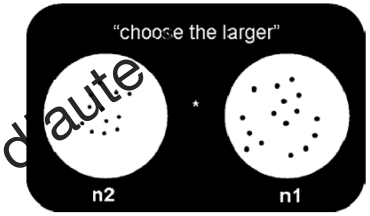
- Deficit of the ANS
  - deficit in learning and processing numerical symbols
  - Deficit in math

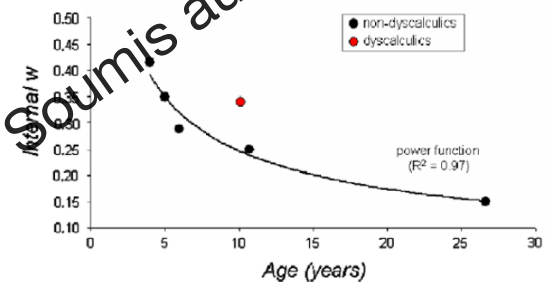


One, two, three, four, five, six, seven ....

Reduced acuity of the ANS in developmental dyscalculia

- Population
  - children from age 3 to 6
  - MLD children (10 y.o.)
  - control children (10 y.o.)
  - 20 adults

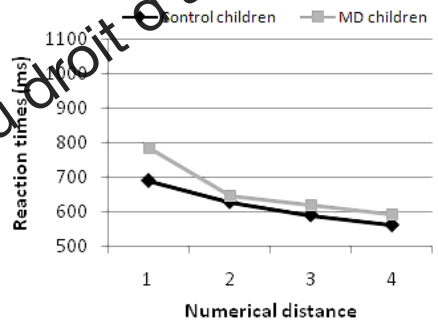




Piazza, M., Facoetti, A., & al, *Cognition*, 2010

- 15 MLD (10-11 y.o.), < pc 15 in X° fluency
- 15 controls (10-11 y.o.)
- Normal reading and IQ

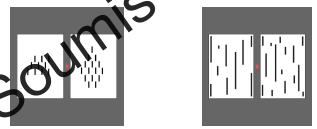
Non canonical dot patterns



Mussolin, Mejias & Noël, 2010, *Cognition*

Yet ...

- Population
  - 45 children with MLD (7 y.o.)
  - 45 control children (7 y.o.)
- Tasks
  - Non-symbolic



Easy condition

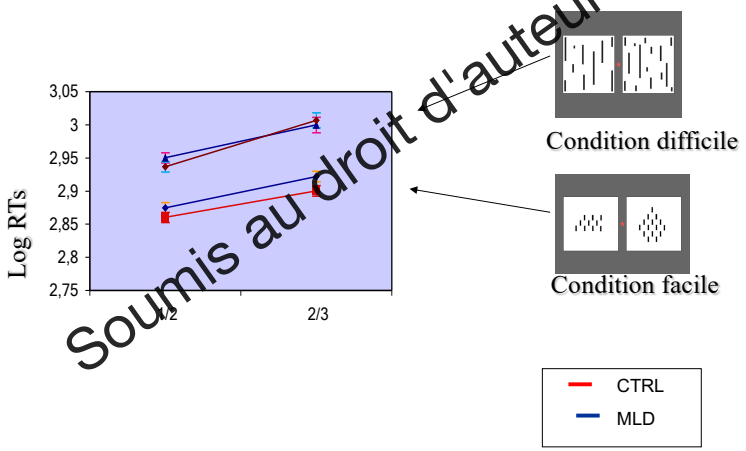
Difficult condition

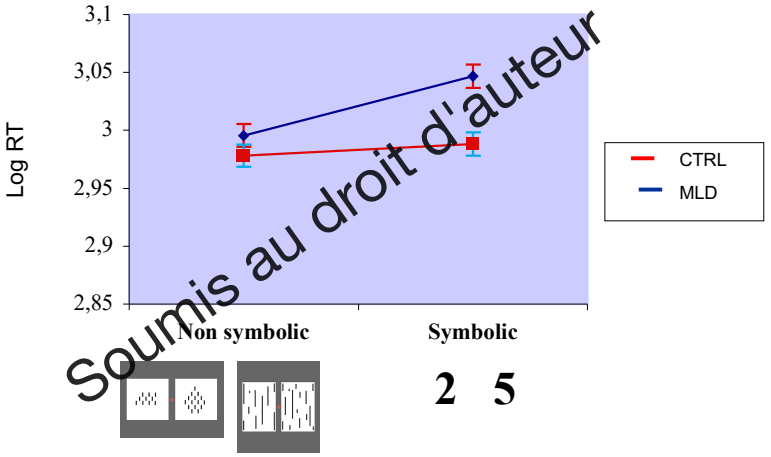
symbolic

2 5

Rousselle & Noël, 2007, *Cognition*

collections comparaison





In unselected populations

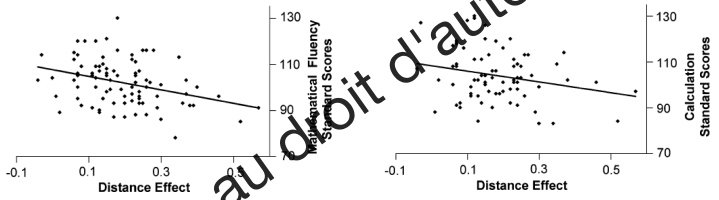
- Population
  - 6 y.o. (n=26), 7 y.o. (n=31), 8 y.o. (n=27)
  - Magnitude comparison
    - Comparison of two symbols (1 - 9)
    - Comparison of two sets (1 - 9 items)
  - Math performance
    - Calculation score (Woodcock Johnson Math)
    - Math fluency (+,-,x in 3')

3 - 7

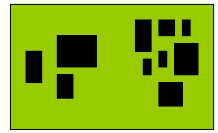
Holloway & Ansari, 2009, *JECP*

Symbolic task correlates with
- Calculation score: $r_{(85)}$=-.222*
- Math fluency:$r_{(86)}$=-.339**

3 - 7



Non-symbolic task does not correlate with
- Calculation score: $r_{(85)}$=-.07
- Math fluency: $r_{(86)}$=.037



=> Not related to the approximate number sense

How can we account for these results ?

A developmental account of MLD ?

	age	symbolic	Non-symbolic
De Smedt & Gilmore (2011)	6 y.o.		
Rousselle & Noël (2007)	7 y.o.		
Landerl & al. (2004)	8-9 y.o.		
Iuculano et al. (2008)	8-9 y.o.		
Piazza	10 y.o.		
Mussolin, Meijns & Noel (2010)	10-11 y.o.		
Price & Ansari (2007)	12 y.o.		

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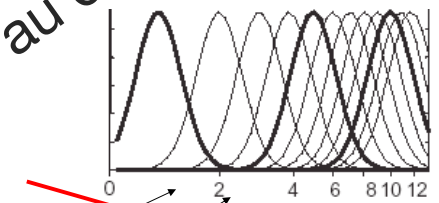
Piazza	10 y.o.	-	MLD<C
Mussolin, Meijns & Noel (2010)	10-11 y.o.	MLD<C	MLD<C
Price & Ansari (2007)	12 y.o.	-	MLD<C

A developmental account

- First deficits are not seen in non-symbolic numerical tasks
  - not due to a deficit of the ANS
- First deficits are seen in symbolic numerical tasks

=> How do children learn the meaning of number symbols (words) ?

- Around 3 y.o
  - Learn the counting list
  - Children must develop a representation of exact natural numbers
    - Seven does not mean "about seven"
  - Meaning of number words are not learned by simple mapping with the ANS

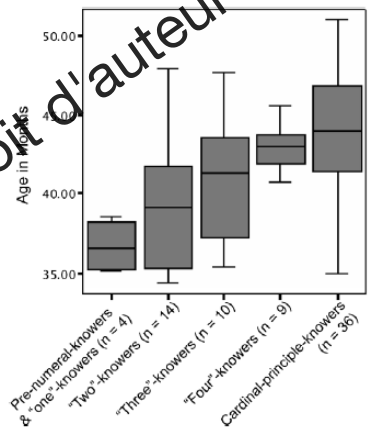


One, two, three, four, five, six, seven ....

- Around 3 y.o
  - Learning the meaning of number words is very slow and sequential : one (± 36 months), two (±6 months later), three (±3 months later) and four in the "give me" task
    - **subset knowers**
  - understand the counting principle and can give any number of items if the word is in their counting list
    - **CP knowers**

Sarnecka & Carey, Cognition, 2008

- **subset knowers**
- **CP knowers**

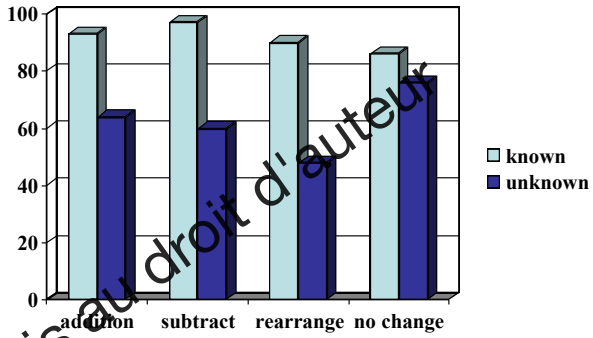


Sarnecka & Carey, *Cognition*, 2008

Transformations that lead to a change of numerical label

- Two sets, one is labeled "*this tray has five sheep*" and then transformed
- Question: "*point to the tray with five / ten sheep*"
- 4 groups
  - no change of the set (start with 2 sets in 2:1 ratio)
  - rearrangement of the set (start with 2 sets in 2:1 ratio)
  - doubling of the set (start with 2 sets of the same size)
  - halving of the set (start with 2 sets of the same size)
- Two types of number words
  - Known
  - Unknown but it the counting list

Condry & Spelke, 2008 *JEP:G*



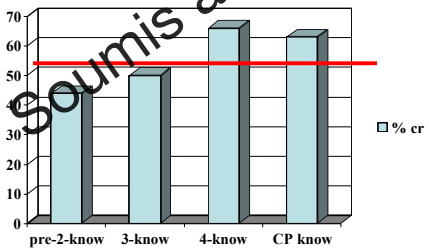
- Correct answers for the "no change" condition
- Correct answers for known words
- Random answers for unknown words,

Subset knowers do not know which transformation affects the use of unknown number words

Condry & Spelke, 2008 JEP:G

The successor function - direction

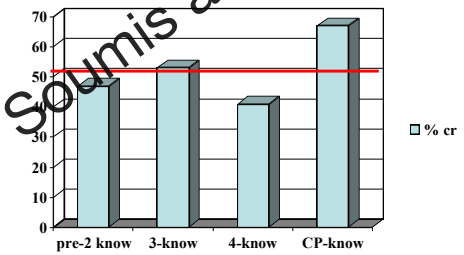
- N items placed on each plate,
  - E.g., *look 5 items here and 5 items there*
- E moves one item from one plate to the other.
- *Now, there is a plate with 4 and a plate with 6, where is 6?*



Sarnecka & Carey, Cognition, 2008

The successor function - unit of change

- the word for N+1 is the one just after N
  - E place N items in a box: "I am putting N (four) items in. How many are there ?
  - Now I am adding one/two more items. Now is there N+1 or N+2 items ? (five or six?)
  - => **only CP knowers > random**



Sarnecka & Carey, *Cognition*, 2008

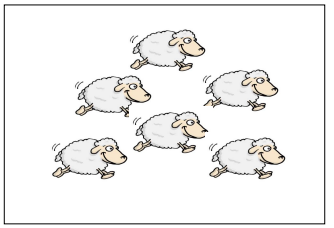
- Only 4-knowers and CP-knowers know that adding one item to a set gives a number word further away rather than before
- only CP-knowers know that adding one item to a set gives the number word just after

  => they master the successor function and the counting principle

Sarnecka & Carey, Cognition, 2008

Mapping number words to the ANS in subset knowers

- Two arrays, ratio of 2:1 (e.g., 3-6, 10-5)
- "*can you point to the card with six sheep?*"



Condry & Spelke, 2008 JEP:G

Mapping number words to the ANS

- Known numbers: 86% cr
- Unknown numbers (in the counting list): 49% cr (chance)

=> **Subset-knowers map known numbers to the ANS but not unknown words**

Condry & Spelke, 2008 *JEP:G*

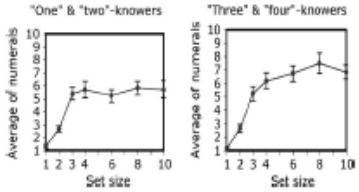
Mapping number words to the ANS: in subset and CP knowers

- 116 children from 3 to 5 y.o.
- All accurate in counting a list of 10 items
- give me task
  - 42 **Subset knowers**
    - 6 one-knowers
    - 14 two-knowers
    - 18 three-knowers
    - 4 four-konwers
  - 71 **CP knowers**

Le Corre & Carey, 2007, Cognition

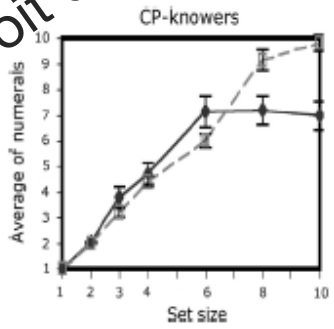
Mapping symbols to ANS

- Estimate without counting (1 s.) the number of dots (1 to 15)
- Average numeral produced according to the size of the target
  - Beyond the # they know, the curve is flat for subset knowers



Le Corre & Carey, 2007, *Cognition*

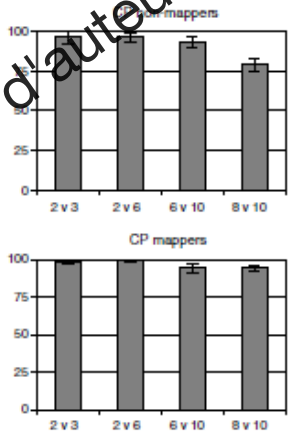
- For the CP knowers, two groups emerge:
  - flat curve : **CP knowers non-mappers**
  - curve increasing with the size of the target: **CP knowers mappers**



Le Corre & Carey, 2007, *Cognition*

ANS

Same ANS for both groups



Le Corre & Carey, 2007, Cognition

COV (SD/mean)

ANS: stable cov (even from 1, Cordes & Gelman, in Campbell (2005)
Stable cov for sets of 6-10 in CP-mappers
But unstable cov for sets of 1 to 4
=> Not based on the same system

Set size	1	2	3	4	6	8	10
CP-non-mappers	0	.02	.14	.25			
CP-mappers	0	0	.01	.12	.22	.24	.18

Le Corre & Carey, 2007, *Cognition*

Summary of the developmental steps

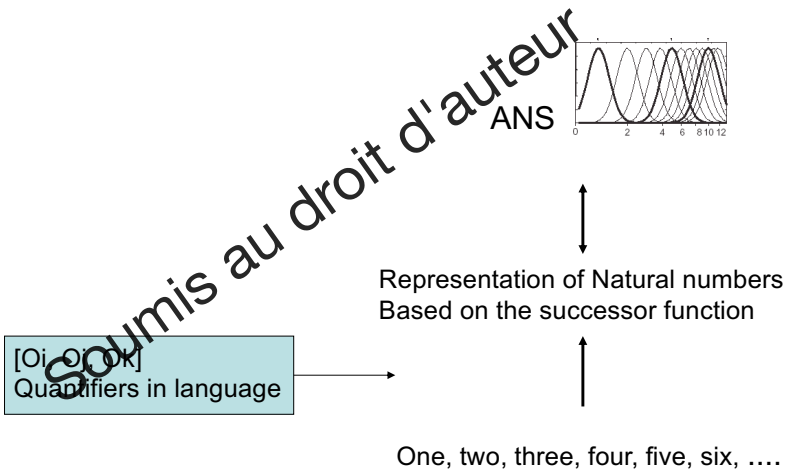
- Innate ANS
- Acquisition of the counting list
- Slow and progressive acquisition of the cardinal meaning of the number words one to four: **subset knowers**
- Understanding of the counting principle and of the cardinal meaning of all the words in the count list, thanks to the understanding of the successor function: **counting principle knowers**
- ± 6 months later: mapping number words to the ANS: **CP mappers**

Le Corre & Carey, 2007, *Cognition*

- Thus, the development of the understanding of the cardinal of number words and the principle of the counting precede the mapping between the ANS and the symbols, so they are not learn by this mapping !
- Mapping sets of 1 to 4 items and number words do not rely on the ANS since it does not present a stable COV
- Hypotheses: based on
  - parallel individuation systems that can create in WM, models of small sets [Oi, Oj, Ok]
  - Semantics of the quantifiers in language (cfr cross-linguistic differences)
- Understanding the meaning of the the first number words leads to discover the succession function, and then the principle of the counting => creation of a representation of natural numbers
- Then mapped onto the ANS

Le Corre & Carey, 2007, Cognition

- A developmental model



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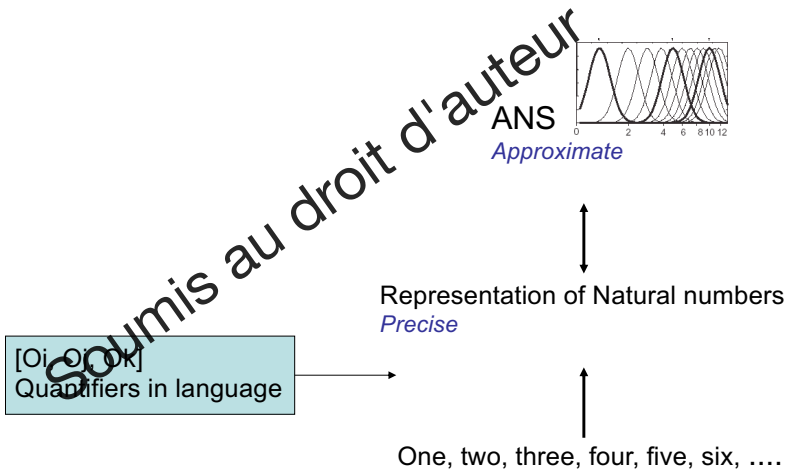
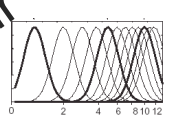


- A developmental model

Secondary deficit in DD

ANS

Approximate



↗ acuity

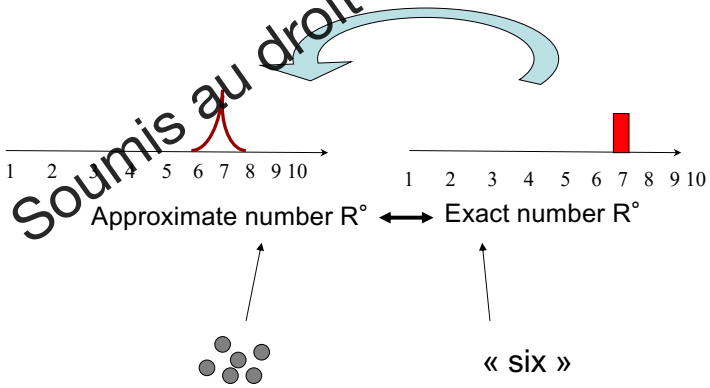
Representation of Natural numbers

Precise

First deficit in DD

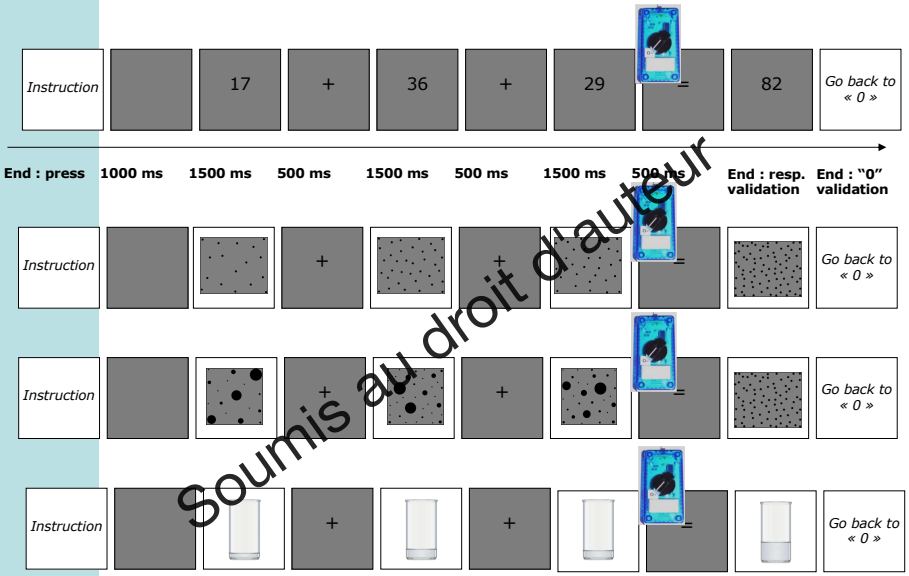
One, two, three, four, five, six, ….

- First deficit is on the symbolic tasks
- Later: deficit on non-symbolic tasks as well
  - Hypoth: connecting the exact number R° with the approximate magnitude system would increase the precision of the later
  - This refinement process would be weaker in MLD children, thus leading to ≠ between MLD and control children



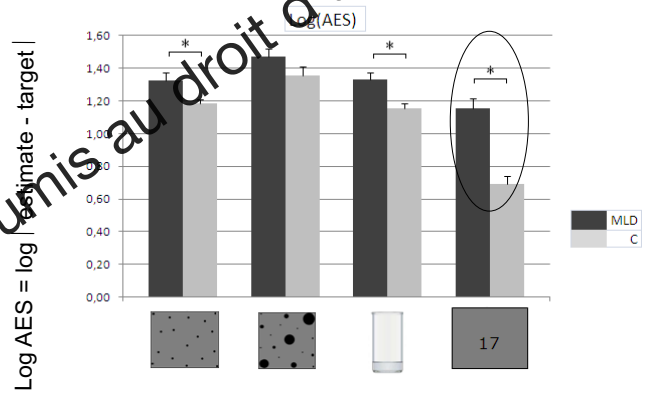
What is the fate of MLD children ?

- MLD: math learning disability
  - Many difficulties in exact calculation
  - Quid in approximate calculation ?



Mejias & Noel, submitted

- 19 MLD (10-11 y.o.)
  - < pc 15 in arith fluency (+,-,x)
  - <pc15 Simonart
  - Teacher: some or severe math diff
- 19 controls (10-11 y.o.)
- Normal reading and IQ

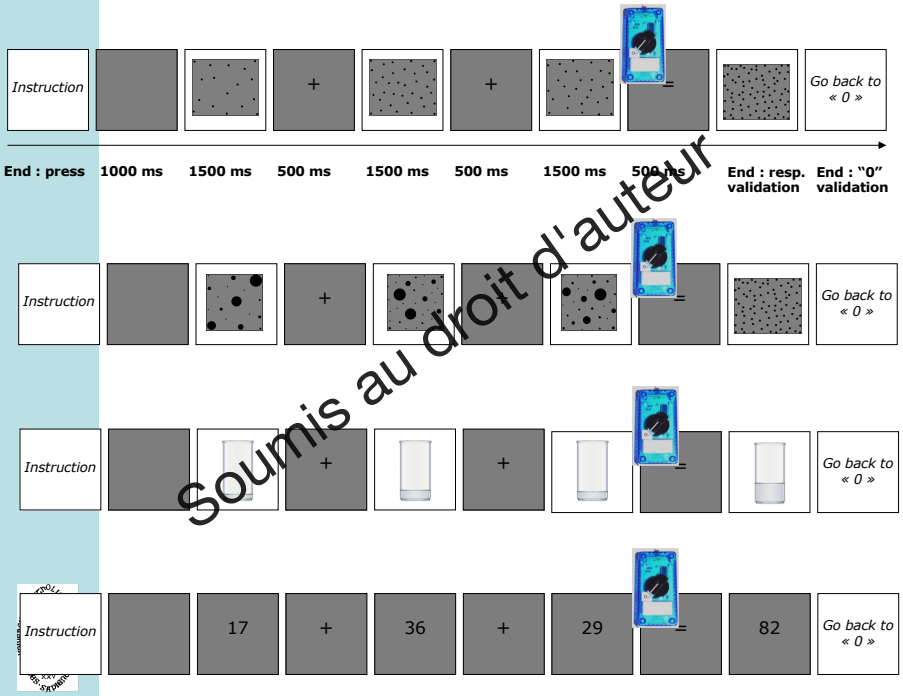


Mejias & Noel, submitted

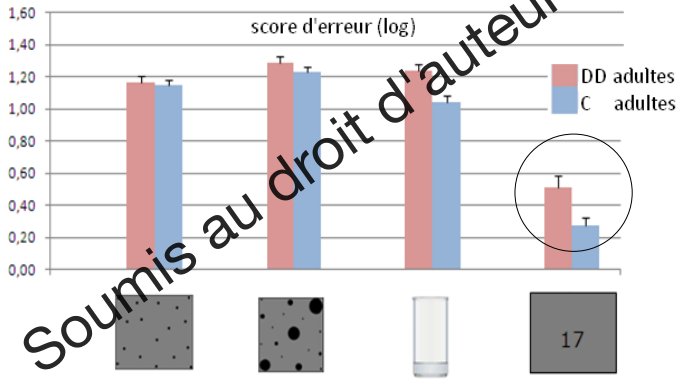
The fate of dyscalculics

- 17 adults who were MLD (25±10 y.o.)
- 17 adults control, same age, same studies or profession
  - Global math battery
    - Error rate: MLD (20±11) > CTR (6±5): t(32)=4.95**
    - RT: MLD (1235±370 s) > CTR (755±284s): t(32)=4.24**

Mejias, Gregoire & Noël, *submitted*



Log AES = log | estimate - target |



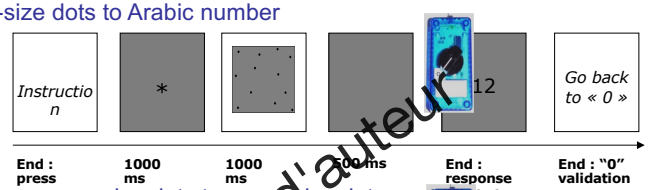
In conclusion

- The ANS deficit seen in dyscalculics
  - is secondary
  - Might even be temporally
- However, heterogeneity is expected

We like simple stories but reality might be more complex

Thanks very much !

Same-size dots to Arabic number



Heterogeneous-size dots to same-size dots

Heterogeneous-size dots to Arabic numbers

Arabic numbers to same-size dots

Munduruku

QuickTime™ et un
décompresseur
sont requis pour visionner cette image.

- Number naming:
- Approximate mapping between words and sets

Pica et al., 2004, *Science*

- Non-symbolic arithmetic tasks: exact subtraction
- N1-N2 (with N1<=6)
- Pointing to the solution: 0,1, or 2

QuickTime™ et un
décompresseur
sont requis pour visionner cette image.

QuickTime™ et un
décompresseur
sont requis pour visionner cette image.

- Large effect of the size of N1
- Better performance when N1<4