COMPETING RISKS PROPORTIONAL-HAZARDS CURE MODEL AND GENERALIZED EXTREME VALUE REGRESSION: AN APPLICATION TO BANK FAILURES AND [...]

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REPRINT | 2021 / 35

ORIGINAL RESEARCH PAPER

Competing risks proportional-hazards cure model and Generalized Extreme Value regression: an application to bank failures and acquisitions in the United States

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ARTICLE HISTORY
Compiled July 1, 2020

ABSTRACT
Several commercial banks in the United States disappeared during the last decades due to failure or acquisition by another entity. From a survival analysis perspective, however, the high censoring rate suggests that some institutions are likely to be immune to failure and/or acquisition. In this study, we use a competing risks proportional-hazards cure model in order to measure the impact of bank-specific and macroeconomic variables on the probabilities of being susceptible to these events (i.e. incidence) and on the survival time of susceptible banks (i.e. latency). Moreover, we propose to model the incidence distribution using Generalized Extreme Value regression and compare the results with the ones obtained by the usual logistic regression model. The proposed methodology is evaluated by means of a simulation study and then applied to a dataset of more than 4000 United States commercial banks spanning the period 1993-2018.

KEYWORDS
Bank failures; Bank acquisitions; Cure model; Propotional-hazards; Competing-risks

1. Introduction

Over the last 40 years, the structure of the commercial bank industry in the United States changed considerably. The number of institutions insured by the Federal Deposit Insurance Corporation (FDIC) has fallen from about 14 000 in 1980 to less than 5 000 in 2018, more than 60 percent. As a consequence of the Savings and Loan crisis, during the period 1982-1995, the federal/state regulatory agencies closed 1 405 commercial banks insured by the FDIC with about \$165 billion in total assets, because they were unable to meet their obligations to depositors and other creditors. The FDIC estimated a total cost linked to the resolution of these failed banks around \$22 billion, which is given by the difference between the amount paid by the deposit insurance fund to insured depositors and the amount estimated to be ultimately recovered from the liquidation of the failed institutions. Between 2007 and 2013, as a consequence of the subprime mortgage crisis, the regulatory agencies closed 393 institutions with about \$226 billion in total assets. The number of failures during this period is much lower

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compared to the previous financial crisis, but the resolution costs estimated by the FDIC almost doubled to $43 billion and the balance of the deposit insurance fund fell to a record low of negative $20.9 billion at year-end 2009 [15]. Despite the large amount of failures, the concentration of the commercial bank industry between 1980 and 2018 is mainly driven by mergers and acquisitions (see in Figure 1). Additionally, after the Savings and Loans crisis, we observe a negative correlation between the number of failures and unassisted mergers. When the number of failures is high during the subprime mortgage crisis, the number of banks that have been acquired is quite small; it increases a little afterwards, but it is quite small compared to the end of the Saving and Loan crisis.

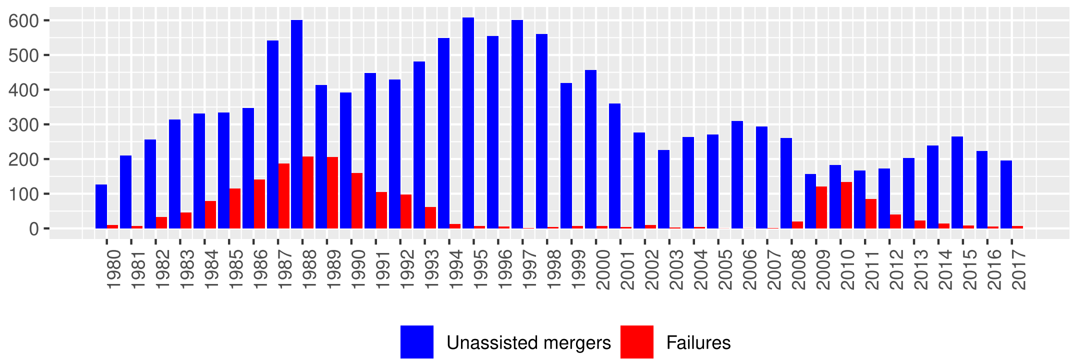

Figure 1. Number of unassisted mergers and failures of FDIC-insured commercial banks during the period 1980-2017. Source: FDIC, Historical Bank Data, https://banks.data.fdic.gov/explore/historical.

The literature about bank failures started growing with the Savings and Loan crisis. Most of the studies used probit/logit regression models to investigate the probability of default or discriminant analysis to separate the healthy banks from the troubled institutions, conditionally on bank-specific information (for a review, see Demirgüç-Kunt [16]). Since then the use of probit/logit regression models is quite a common practice in the literature, as well as the use of bank-specific explanatory variables covering the different dimensions of the CAMEL rating system. The latter is a supervisory tool, formally known as Uniform Financial Institutions Rating System (UFIRS) and developed in 1979 by the Federal Financial Institutions Examination Council (FFIEC), which regulators use during on-site examinations to evaluate the soundness of financial institutions on a uniform basis and to identify the institutions requiring special attention or concern. For this purpose, each financial institution is assigned a composite rating based on the evaluation and rating of five components measured on a numeric scale from 1 to 5, where 1 is the highest rating indicating the strongest performance and the least degree of supervisory concern. These components address the adequacy of capital, the quality of assets, the capability of management, the quality and level of earnings and the adequacy of liquidity. A sixth component, the sensitivity to market risk, was added on December 1996 and the acronym CAMEL was changed to CAMELS. In more recent literature, the logistic regression model and explanatory variables based on the CAMEL(S) rating system have been used by several authors. Cole and White [11] investigate the determinants of U.S. commercial bank failures in 2009 with explanatory variables observed during the years 2004-2008 and find that the CAMELS proxies are important determinants of bank failures, as they were during the 1985-1992 banking crisis. In the same spirit, Balla et al.[5] focus their analysis on U.S. community bank failures and the resulting FDIC losses during the two banking crises. They find that essentially the same variables predict bank failure in both crises and failure probabilities were most influenced by macroeconomic conditions. Altman et al. [2] provide a comprehensive analysis of bank failures in the U.S. and Western Europe during the period 2007-2012. They find that the two models estimated on the samples of U.S. and Western Europe banks predict bank closures in the two regions with a similar level of accuracy, but lower for some Western European countries due to lax bank disclosure laws and their implementation. Betz et al. [7] develop an early-warning model to predict vulnerabilities leading to banks' distress using a sample of European banks observed during the period 2000-2013 and they find that the inclusion of macro-financial imbalance and banking sector vulnerability indicators helps to improve the performance of bank distress predictions during the last financial crisis. Finally, Audrino et al. [4] introduce mixed-data sampling (MIDAS) in the context of logistic regression to improve prediction accuracy in U.S. bank failures during the period 2004-2016, especially for long-term forecasting horizons.

Also the literature about mergers and acquisitions in the banking industry grew considerably since the 1980s (for an exhaustive review of this literature covering over 150 studies, see DeYoung et al. [17]). The two main streams of empirical research studied the determinants of mergers and acquisitions and the financial performance of the merging banks. Several authors employed logit/probit models to investigate bank-specific and sometimes macroeconomic factors affecting the probability of a bank being a target for a merger or acquisition. Focarelli et al. [18] consider a sample of Italian commercial banks observed during the period 1984-1996 and a multinomial logit model to separate the effects on acquisitions from the ones on mergers. Their results are consistent with the hypothesis that expanding revenues from financial services is a strategic objective for mergers, whereas improving the quality of the loan portfolio of the passive bank is central for acquisitions. Akhigbe et al. [1] consider a sample of US banks listed on the stock exchange between 1987 and 2001, and they examine whether the gains of the target banks are conditioned on the probability of being acquired. They find that the likelihood of a bank being acquired is higher for banks that are larger, are more profitable, better capitalized, more liquid, have more non-performing loans, a lower market-to-book multiple and a higher loan concentration. Correa [12] uses a database containing 220 cross-border acquisitions between 1996 and 2003 and shows that cross-country acquisitions are more likely for banks which are large, bad performers, in a small country and where the banking sector is more concentrated. Hernando et al. [20] use a sample of EU-25 observed during the period 1997-2004 and, additionally to the logit model, they use a multinomial logit model to study cross-country effects. Their results suggest that larger, poorly managed and banks operating in less concentrated markets are more likely to be acquired by other banks in the same country. Whereas, the probability of being a target in a cross-border deal is larger for banks that are quoted in the stock market Caiazza et al. [8] use a large dataset of 24,325 banks from all over the world observed between 1992 and 2006, of which 1 484 were involved in domestic and cross-border merger and acquisition deals, and they analyze whether the cross-border target banks are different from those targeted in domestic deals. For this purpose, they employ both a probit and a multinomial probit model that distinguishes targets in domestic operations, targets in cross-border operations and non-targets. They find that the main differences between the domestic and cross-border targets are the characteristics of the countries where the banks operate.

Because the probit/logit models do not include the time-to-event in the modeling efforts and due to their parametric nature, Lane et al. [22] and Whalen [27] proposed to study bank failures using a Cox's proportional-hazards (PH) model [13]. Its main advantage lies in the ability to directly model the survival time. In addition, as a semi-parametric model, it does not require any assumption about the distribution of survival times. In a comparison of actual and predicted times to failure, Lane et al. [22] showed how it tends to identify bankruptcies prior to the actual failure date. Similarly, Molina [23] investigated the determinants of commercial bank failures in Venezuela during the 1994-1995 crisis. In a competing risks framework, Wheelock and Wilson [28] proposed the use of the Cox's PH model to identify the characteristics that make a bank more likely to fail or be acquired, with emphasis on management qualities. Whereas, Hannan and Pilloff [19] investigated the determinants of acquisition by four types of acquirers, depending on their size and the geographic market in which they operate (within or outside the market of the target organization).

Despite its advantages, the Cox's PH model assumes that the entire population will eventually experience the event of interest. However, the small number of defaults and the substantial proportion of right-censored banks suggest the existence of a subpopulation of banks which is likely to be immune to bankruptcy. For this reason, Cole and Gunther [10] introduced the use of the split population survival time model (also known as cure model) into the study of bank failures. Originally developed in biostatistics to study long-term survivors of cancer in clinical trials, cure models assume that the population under study is composed by a cured (i.e. immune) and an uncured (i.e. susceptible) subpopulation. Moreover, they can separate the effects of certain factors on the probability of being susceptible (known as incidence) from the ones influencing the survival time of susceptible subjects (known as latency). For a comprehensive review of cure models the reader may refer to Amico and Van Keilegom [3]. In the framework of U.S. commercial bank failures, Cole and Gunther [10] used a fully parametric cure model to predict survival times from the first quarter of 1986 through the second quarter of 1992. Whereas, Beretta and Heuchenne [6] used a semi-parametric PH cure model with time-varying covariates and a variable selection technique based on its SCAD-penalized likelihood to analyze the defaults occurred during the period 2006-2016.

In cure models, the probability of being susceptible to the event of interest is commonly assumed to follow a logistic regression model, a special case of generalized linear model with a logit link function. As noted by King and Zeng [21], when the event of interest is rare, the use of a logit link function may be problematic due to its symmetry around the value 0.5. The response curve approaches zero at the same rate as it approaches one. According to Czado and Santner [14], the same problem holds for all symmetric link functions and leads to link misspecification, with a potential bias towards the majority class. For the aforementioned reason, Wang and Dey [26] and Calabrese and Osmetti [9] proposed the use of a skewed link function: the inverse of the cumulative density function of the Generalized Extreme Value (GEV) distribution. This function is more flexible and allows the response curve to approach zero and one at different rates. The resulting model is known as binary GEV regression.

The contribution of this paper is twofold. First, since a bank may cease to exist primarily due to bankruptcy or acquisition by another entity, we use a competing risks PH cure model to analyze the effects of bank-specific and macroeconomic covariates on the probabilities of being susceptible to these events (i.e. incidence) and on the survival time of susceptible banks (i.e. latency). For this purpose, we use a dataset of more than 4 000 commercial banks in the United States established before 1993 and observed up to 2018. Second, given the small number of banks facing the two events in our sample, we propose to model the probability to be susceptible using a binary GEV regression.

The rest of the paper is structured as follows. In Section 2 we recall the binary GEV regression model and we conduct a simulation study to compare its finite sample performance with the classical logistic regression model. In Section 3, we present the competing risks PH cure model with the incidence components modeled by the binary GEV regression and we assess its finite sample performance with a simulation study. Finally, in Section 4, we describe the dataset of U.S. commercial banks spanning the period 1993-2018, present the results of our analysis using the proposed methodology, validate its performance using a cross-validation procedure and assess its predictive ability splitting the dataset into training and test sets, covering the periods 1993-2010 and 2011-2018, respectively.

2. The binary GEV regression model

Let $\mathbf{Y}$ be a vector of Bernoulli random variables with parameter π_i, for $i = 1, ..., n$. A classical approach to study the relationship between $\mathbf{Y}$ and a matrix $\mathbf{X}$ of predictor variables, is to estimate π_i using a logistic regression model:

$$\pi(\mathbf{x}_i) = \frac{e^{\mathbf{x}_i \mathbf{b}}}{1 + e^{\mathbf{x}_i \mathbf{b}}},$$

where $\mathbf{x}_i = (1, x_{i,1}, ..., x_{i,p})$ is a vector of explanatory variables and $\mathbf{b}$ is a vector of unknown coefficients, which are estimated using maximum likelihood. However, as noted by King and Zeng [21], when the outcome of interest is a rare event, the use of a logit link function may be inappropriate due to its symmetry around the value 0.5. According to Czado and Santner [14], the same problem holds for all symmetric link functions and leads to link misspecification, with a potential bias towards the majority class. A remedy to this problem, suggested by Wang and Dey [26] and Calabrese and Osmetti [9], is the adoption of a skewed link function: the quantile of the Generalized Extreme Value (GEV) distribution with location, scale and shape parameters equal to 0, 1 and τ, respectively. Depending on the value of the shape parameter $\tau \in \mathbb{R}$, the GEV distribution includes 3 types of distributions as special cases: Gumbel ($\tau = 0$), Fréchet ($\tau > 0$) and Weibull ($\tau < 0$). The relationship between the probability π_i and the explanatory variables is given by the cumulative density function of the GEV distribution:

$$\pi(\mathbf{x}_i) = \begin{cases} \exp\left[-(1 + \tau \mathbf{x}_i \mathbf{b})^{-1/\tau}\right], & \text{if } \tau \neq 0 \text{ and } 1 + \tau \mathbf{x}_i \mathbf{b} > 0 \\ 0, & \text{if } \tau > 0 \text{ and } \mathbf{x}_i \mathbf{b} \leq -1/\tau \\ 1, & \text{if } \tau < 0 \text{ and } \mathbf{x}_i \mathbf{b} \geq -1/\tau \end{cases} .$$

This function is not symmetric around 0, it approaches 0 and 1 at different rates, and is more flexible compared to the response curve of the logistic regression model (see Figure 2). The degree of asymmetry depends on the value of the parameter τ, which governs the tail behavior of the GEV distribution. In order to understand the influence of this parameter, let's consider the case of an unbalanced dataset with many more observations belonging to class $Y = 0$ than to class $Y = 1$. For values of $\tau > 0$, the observations belonging to the majority class will have $\pi(\mathbf{x}_i) = 0$, when $\mathbf{x}_i \mathbf{b} \leq -1/\tau$. As a consequence, some of the observations belonging to the majority class will be excluded from the log-likelihood and the dataset will become more balanced (from an estimation point of view). On the contrary, when $\tau < 0$ the observations belonging to the minority class will have $\pi(\mathbf{x}_i) = 1$, when $\mathbf{x}_i\mathbf{b} \geq -1/\tau$. In this case, some of the observations belonging to the minority class will be excluded from the log-likelihood and the dataset will become more unbalanced (always from an estimation point of view).

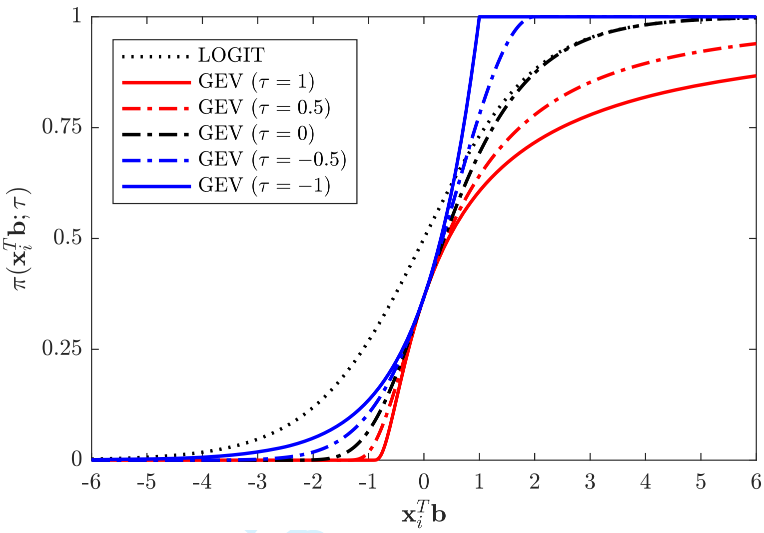

Figure 2. Response curves of the logistic (LOGIT) and GEV regression models.

The parameters $\mathbf{b}$ and τ are estimated using the maximum-likelihood method. The log-likelihood function is

$$\ell(\mathbf{b},\tau) = \sum_{i=1}^{n} y_i \log[\pi(\mathbf{x}_i)] + (1-y_i)\log[1-\pi(\mathbf{x}_i)] \tag{1}$$

and its first derivatives with respect to the parameters of interest are

$$\frac{\partial \ell(\mathbf{b},\tau)}{\partial \mathbf{b}} = \frac{\partial \ell(\mathbf{b},\tau)}{\partial \pi(\mathbf{x}_i)} \frac{\partial \pi(\mathbf{x}_i)}{\partial \mathbf{b}}$$

and

$$\frac{\partial \ell(\mathbf{b},\tau)}{\partial \tau} = \frac{\partial \ell(\mathbf{b},\tau)}{\partial \pi(\mathbf{x}_i)} \frac{\partial \pi(\mathbf{x}_i)}{\partial \tau},$$

where

$$\frac{\partial \ell(\mathbf{b},\tau)}{\partial \pi(\mathbf{x}_i)} = \frac{y_i}{\pi(\mathbf{x}_i)} - \frac{1-y_i}{1-\pi(\mathbf{x}_i)}, \tag{2}$$

$$\frac{\partial \pi(\mathbf{x}_i)}{\partial \mathbf{b}} = \begin{cases} -\frac{\mathbf{x}_i}{1+\tau\mathbf{x}_i\mathbf{b}}\pi(\mathbf{x}_i)\ln[\pi(\mathbf{x}_i)], & \text{if } \tau \neq 0 \text{ and } 1+\tau\mathbf{x}_i\mathbf{b} > 0 \\ 0, & \text{elsewhere} \end{cases} \tag{3}$$

and

$$\frac{\partial \pi(\mathbf{x}_i)}{\partial \tau} = \begin{cases} \left[\frac{\ln(1+\tau\mathbf{x}_i\mathbf{b})}{\tau^2} - \frac{1}{\tau}\frac{\mathbf{x}_i\mathbf{b}}{1+\tau\mathbf{x}_i\mathbf{b}}\right]\pi(\mathbf{x}_i)\ln[\pi(\mathbf{x}_i)], & \text{if } \tau \neq 0 \text{ and } 1+\tau\mathbf{x}_i\mathbf{b} > 0 \\ 0, & \text{elsewhere} \end{cases}. \quad (4)$$

Computational issues. Equations (1)–(4) may not be defined when $\pi(\mathbf{x}_i) = 0$ or $\pi(\mathbf{x}_i) = 1$. In order to solve this issue, in such cases we impose that $\pi(\mathbf{x}_i) = \epsilon$ or $\pi(\mathbf{x}_i) = 1-\epsilon$, where ϵ is the smallest positive floating-point number such that $1+\epsilon \neq 1$.

3. Competing risks GEV-PH cure model

Considering that a bank may disappear due to bankruptcy (event $j = 1$) or acquisition (event $j = 2$), we extend the semi-parametric PH cure model of Sy and Taylor [24] to competing risks. We assume that the banks' population is composed of banks (i) susceptible to both default and acquisition, (ii) susceptible to default only, (iii) susceptible to acquisition only and (iv) immune to both default and merger/acquisition.

Let the observed time be $T = \min(W_1, W_2, C)$, where W_1, W_2 and C are random variables denoting the time to default, acquisition and censoring, respectively. The censoring mechanism is assumed independent from the event times. Furthermore, let the indicator Y_j, for $j \in (1, 2)$, denote whether a bank is susceptible to the j-th event.

The probability to be susceptible to the j-th event, also known as incidence, is modelled by a binary GEV regression:

$$\mathbb{P}(Y_{j,i} = 1|\mathbf{x}_i) = \pi_j(\mathbf{x}_i; \mathbf{b}_j, \tau_j),$$

where $\mathbf{x}_i = (1, x_{i,1}, ..., x_{i,p})$ is a vector of covariates, $\mathbf{b}_j$ a vector of unknown coefficients and τ_j the unknown shape parameter of the GEV distribution. The time to the j-th event for the susceptible banks, also known as latency, is modelled according to a Proportional Hazards (PH) model, with a conditional survival function defined as

$$S_j(t|\mathbf{z}_i) = S(t|Y_j = 1, \mathbf{z}_i) = [S_{0,j}(t)]^{e^{\mathbf{z}_i\boldsymbol{\beta}_j}},$$

where $S_{0,j}(t) = e^{-\int_0^t h_{0,j}(u)du}$ is the baseline survival function, $\mathbf{z}_i$ a vector of covariates and $\boldsymbol{\beta}_j$ a vector of unknown coefficients.

3.1. Estimation

We denote the observed data as $\mathbf{O} = \{(t_i, \delta_{1,i}, \delta_{2,i}, \mathbf{x}_i, \mathbf{z}_i); i = 1, ..., n\}$, where $\delta_{j,i} = 1$, if the i-th bank experienced the j-th event, or $\delta_{j,i} = 0$, otherwise. We further denote the model parameters as $\boldsymbol{\Theta} = [\mathbf{b}_1, \mathbf{b}_2, \tau_1, \tau_2, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, h_{0,1}(t), h_{0,2}(t)]$, where $h_{0,j}(t)$ is the baseline hazard function for the j-th event.

The indicators $\mathbf{y}_j = \{y_{j,i} : i = 1, ..., n\}$ are partially unobserved, because $y_{j,i} = 1$ only when a bank experience one of the events of interest ($\delta_{j,i} = 1$). For this reason, we treat the estimation of the model's parameters as a missing data problem, using an EM algorithm. The complete-data likelihood is defined as the product of an incidence and a latency component: $L_C(\boldsymbol{\Theta}; \mathbf{O}, \mathbf{y}_1, \mathbf{y}_2) = L_C^1(\mathbf{b}_1, \mathbf{b}_2, \tau_1, \tau_2; \mathbf{O}, \mathbf{y}_1, \mathbf{y}_2) \times$

$L_C^2(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, h_{0,1}(t), h_{0,2}(t); \mathbf{O}, \mathbf{y}_1, \mathbf{y}_2)$, where

$$L_C^1(\mathbf{b}_1, \mathbf{b}_2, \tau_1, \tau_2; \mathbf{O}, \mathbf{y}_1, \mathbf{y}_2) = \prod_{i=1}^{n} \pi_1(\mathbf{x}_i)^{y_{1,i}} \pi_2(\mathbf{x}_i)^{y_{2,i}} [1-\pi_1(\mathbf{x}_i)]^{(1-y_{1,i})} [1-\pi_2(\mathbf{x}_i)]^{(1-y_{2,i})}$$

$$L_C^2(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, h_{0,1}(t), h_{0,2}(t); \mathbf{O}, \mathbf{y}_1, \mathbf{y}_2) = \prod_{i=1}^{n} \prod_{j=1}^{2} h_j(t_i|\mathbf{z}_i)^{\delta_{j,i}} S_j(t_i|\mathbf{z}_i)^{y_{j,i}}. \quad (5)$$

In the rest of the paper, we will use ℓ to denote log-likelihoods.

In the E-step, we compute the conditional expectation of the complete-data log-likelihood with respect to $\mathbf{y}_1$ and $\mathbf{y}_2$, given the observed data $\mathbf{O}$ and the current parameter estimates $\hat{\boldsymbol{\Theta}}^{(m)}$. Since it is a linear function of $\mathbf{y}_1$ and $\mathbf{y}_2$, the conditional expectation is equal to $\ell_C\left(\hat{\boldsymbol{\Theta}}^{(m)}; \mathbf{O}, \boldsymbol{\phi}_1^{(m)}, \boldsymbol{\phi}_2^{(m)}\right)$, where

$$\phi_{j,i}^{(m)} = E[y_{j,i}|\mathbf{O}, \boldsymbol{\Theta}^{(m)}] = \begin{cases} 1, & \text{if } \delta_{j,i} = 1 \\ \frac{\pi_j(\mathbf{x}_i) S_j(t_i|\mathbf{z}_i)}{1-\pi_j(\mathbf{x}_i)+\pi_j(\mathbf{x}_i) S_j(t_i|\mathbf{z}_i)}, & \text{if } \delta_{j,i} = 0 \end{cases},$$

which can be seen as the posterior probability to be classified as susceptible to the j-th event.

In the M-step, we compute the parameter estimates for the next iteration of the EM algorithm as $\hat{\boldsymbol{\Theta}}^{(m+1)} = \mathrm{argmax}_{\boldsymbol{\Theta}}\, \ell_C\left(\boldsymbol{\Theta}; \mathbf{O}, \boldsymbol{\phi}_1^{(m)}, \boldsymbol{\phi}_2^{(m)}\right)$. Assuming that the baseline hazard function is piecewise constant $h_{0,j,l} = h_{0,j,l}(t|Y_j = 1)$, for $t \in (t_{(l-1)}, t_{(l)}]$, between the k_j ordered event times $t_{(1_j)} \leq \cdots \leq t_{(k_j)}$, its estimator is given by

$$\hat{h}_{0,j,l}\left(\boldsymbol{\beta}_j; \mathbf{O}, \mathbf{y}_j\right) = \frac{1}{\sum_{i \in R_j(t_{(l)})} y_{j,i} \exp(\mathbf{z}_i' \boldsymbol{\beta}_j)}, \quad (6)$$

where $R_j(t_{(l)})$ is the risk set for the j-th event at time $t_{(l)}^-$ and $l = 1, ..., k_j$. Replacing $h_{0,j}(t)$ in (5) by this estimator, it is possible to derive a partial log-likelihood which does not depend on $h_{0,j}(t)$ any more:

$$\ell_C^3(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2; \mathbf{O}, \mathbf{y}_1, \mathbf{y}_2) = \sum_{j=1}^{2} \sum_{l=1}^{k_j} \left[\mathbf{z}_i' \boldsymbol{\beta}_j - \ln\left(\sum_{i \in R_j(t_{(l)})} y_{j,i} \exp(\mathbf{z}_i' \boldsymbol{\beta}_j)\right)\right].$$

To sum up, starting from some initial values $\hat{\boldsymbol{\Theta}}^{(1)}$, in the E-step we compute $\boldsymbol{\phi}_1^{(m)}$ and $\boldsymbol{\phi}_2^{(m)}$ given $\mathbf{O}$ and $\hat{\boldsymbol{\Theta}}^{(m)}$. Next, in the M-step, we estimate the parameters $\mathbf{b}_j$, $\boldsymbol{\beta}_j$ and τ_j, for $j = \{1, 2\}$,

$$\left(\hat{\mathbf{b}}_1^{(m+1)}, \hat{\mathbf{b}}_2^{(m+1)}, \tau_1^{(m+1)}, \tau_2^{(m+1)}\right) = \underset{\mathbf{b}_1, \mathbf{b}_2, \tau_1, \tau_2}{\mathrm{argmax}}\, \ell_C^1\left(\mathbf{b}_1, \mathbf{b}_2, \tau_1, \tau_2; \mathbf{O}, \boldsymbol{\phi}_1^{(m)}, \boldsymbol{\phi}_2^{(m)}\right),$$

$$\left(\hat{\boldsymbol{\beta}}_1^{(m+1)}, \hat{\boldsymbol{\beta}}_2^{(m+1)}\right) = \underset{\boldsymbol{\beta}_1}{\mathrm{argmax}}\, \ell_C^3\left(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2; \mathbf{O}, \boldsymbol{\phi}_1^{(m)}, \boldsymbol{\phi}_2^{(m)}\right),$$

and the baseline hazard functions $\hat{\mathbf{h}}_{0,j}^{(m+1)} = \left\{ \hat{h}_{0,j,l} \left(\hat{\boldsymbol{\beta}}_j^{(m+1)}; \mathbf{O}, \boldsymbol{\phi}_j^{(m)} \right), \; \forall \; l = 1, ..., k_j \right\}$. The iterations between E-step and M-step continue until the norms of two successive estimates of $\mathbf{b}_1$, $\mathbf{b}_2$, τ_1, τ_2, $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$ are lower than a given tolerance threshold.

In practice, the estimation of the model's parameters reduces to the estimation of two semi-parametric PH cure models, one for each event of interest, treating the acquired (resp. failed) banks as censored when modeling defaults (resp. acquisitions).

3.2. Simulation study

In this section, we present the results of a simulation study conducted to assess the performance of the proposed competing risks GEV-PH cure model using Monte Carlo simulation with 500 replications. The susceptibility indicators (Y_1, Y_2) are generated from a binary GEV regression model with coefficients $\mathbf{b}_1 = (b_{0,1}, 1, -1, 1, -1)'$ and $\mathbf{b}_2 = (b_{0,2}, -1, 1, -1, 1)'$, shape parameters τ_1 and τ_2, and 4 covariates following independent standard normal distributions. The time-to-events (W_1, W_2) are generated from a Cox's proportional hazards model with parameters $\boldsymbol{\beta}_1 = (-1, 1, -1, 1)'$, $\boldsymbol{\beta}_2 = (1, -1, 1, -1)'$, a unit constant baseline hazard function and 4 covariates following independent standard normal distributions. The censoring times are generated from an exponential distribution with parameter λ_C. Finally, the observed data $\{(t_i, \delta_{1,i}, \delta_{2,i}); i = 1, ..., n\}$ are obtained using Algorithm 1.

We consider 5 simulation settings with different levels of censoring and susceptibility to each event, depending on the values of $b_{0,1}$, $b_{0,1}$, τ_1 and τ_2 (see Table 1), and 3 sample sizes ($n = 500, 1000, 2000$). At each replication, we fit two competing risks PH cure models, where the incidence component is modeled by binary GEV or logistic (LOGIT) regression. In Settings 1–4, the susceptibility indicators are generated from a binary GEV regression model, whereas, in Setting 5, from a logistic regression model.

	Cens%	$\bar{\pi}_1$	$\bar{\pi}_2$	$b_{0,1}$	$b_{0,2}$	τ_1	τ_2
Setting 1	0.87	0.11	0.11	-2	-2	1	1
Setting 2	0.83	0.11	0.18	-2	-2	1	-1
Setting 3	0.75	0.11	0.32	-2	-1	1	-1
Setting 4	0.64	0.32	0.32	-1	-1	-1	-1
Setting 5	0.85	0.13	0.13	-3	-3	-	-

Table 1. Simulation settings. Cens% is the percentage of censoring and $\bar{\pi}_1$ (resp. $\bar{\pi}_2$) is the average number of individuals susceptible to event 1 (resp. event 2).

The performance of the two models are measured in terms of Mean Absolute Error (MAE) and Mean Squared Error (MSE) of the estimated probabilities. For the probability to be susceptible (incidence), they are defined as

$$\text{MAE}(\hat{\pi}_j) = \frac{1}{n} \sum_{i=1}^{n} \left| \hat{\pi}_j(\mathbf{x}_i) - \pi_j(\mathbf{x}_i) \right|,$$

and

$$\text{MSE}(\hat{\pi}_j) = \frac{1}{n} \sum_{i=1}^{n} \left[\hat{\pi}_j(\mathbf{x}_i) - \pi_j(\mathbf{x}_i) \right]^2.$$

Whereas, for the survival probability (latency), they are defined as

$$\mathrm{MAE}(\hat{S}_j) = \frac{1}{n}\sum_{i=1}^{n}\left|\hat{S}_j(t_i|\mathbf{z}_i) - S_j(t_i|\mathbf{z}_i)\right|,$$

and

$$\mathrm{MSE}(\hat{S}_j) = \frac{1}{n}\sum_{i=1}^{n}\left[\hat{S}_j(t_i|\mathbf{z}_i) - S_j(t_i|\mathbf{z}_i)\right]^2.$$

In Table 2, we provide the median values of MAE and MSE over the 500 replications for the probability to be susceptible (incidence). In Settings 1–4, the median of the model errors is always lower when the incidence is modeled using GEV regression. Compared to logistic regression (LOGIT), it decreases at a faster pace when the sample size increases; slightly less fast when the average number of 1's is higher, but these results are overall in line with our expectations, since the GEV regression model is more flexible (as we discussed in Section 2). Moreover, when the susceptibility indicators are generated from a logistic regression model (Setting 5), the performance of the GEV and LOGIT model is quite similar.

In Table 3, we provide the median values of MAE and MSE over the 500 replications for the survival probabilities (latency). In all settings, the median of the model errors decreases as the sample size increases. We do not notice an important difference between the two models. Only in situations of high censoring, we observe a slightly better performance when the incidence is modeled by GEV regression.

		$\mathrm{MAE}(\hat{\pi}_1)$			$\mathrm{MAE}(\hat{\pi}_2)$			$\mathrm{MSE}(\hat{\pi}_1)$			$\mathrm{MSE}(\hat{\pi}_2)$		
	N	LOGIT	GEV	rel.%	LOGIT	GEV	rel.%	LOGIT	GEV	rel.%	LOGIT	GEV	rel.%
Setting 1	500	0.0425	0.0367	0.83	0.0424	0.0378	0.82	0.0074	0.0072	0.93	0.0073	0.0078	0.86
	1000	0.0374	0.0237	0.61	0.0372	0.0233	0.60	0.0054	0.0030	0.48	0.0054	0.0029	0.48
	2000	0.0347	0.0129	0.38	0.0348	0.0127	0.36	0.0044	0.0009	0.20	0.0044	0.0009	0.19
Setting 2	500	0.0420	0.0371	0.82	0.0392	0.0292	0.75	0.0072	0.0073	0.91	0.0048	0.0034	0.71
	1000	0.0369	0.0223	0.57	0.0321	0.0201	0.61	0.0053	0.0026	0.44	0.0033	0.0015	0.47
	2000	0.0347	0.0126	0.36	0.0287	0.0143	0.49	0.0044	0.0009	0.19	0.0026	0.0008	0.30
Setting 3	500	0.0427	0.0370	0.86	0.0490	0.0319	0.65	0.0075	0.0075	0.94	0.0056	0.0034	0.58
	1000	0.0374	0.0221	0.58	0.0431	0.0223	0.53	0.0053	0.0027	0.45	0.0042	0.0016	0.40
	2000	0.0350	0.0129	0.37	0.0395	0.0160	0.40	0.0044	0.0009	0.19	0.0035	0.0008	0.23
Setting 4	500	0.0493	0.0335	0.66	0.0487	0.0325	0.68	0.0056	0.0034	0.62	0.0055	0.0034	0.61
	1000	0.0433	0.0232	0.53	0.0429	0.0228	0.53	0.0042	0.0018	0.39	0.0041	0.0017	0.39
	2000	0.0396	0.0162	0.40	0.0397	0.0158	0.39	0.0034	0.0009	0.24	0.0035	0.0009	0.24
Setting 5	500	0.0318	0.0373	1.08	0.0327	0.0365	1.08	0.0029	0.0041	1.20	0.0031	0.0041	1.18
	1000	0.0212	0.0234	1.07	0.0216	0.0241	1.07	0.0013	0.0016	1.14	0.0014	0.0017	1.15
	2000	0.0148	0.0167	1.10	0.0152	0.0171	1.10	0.0006	0.0008	1.23	0.0007	0.0008	1.19

Table 2. Simulation study: median of the model errors for the probabilities to be susceptible. rel.%, is the median of the ratio between the GEV and LOGIT errors.

4. Application to US bank failures and acquisitions

In this section, we analyze a dataset of United States commercial banks insured by the Federal Deposit Insurance Corporation (FDIC), which spans the period from 1993 to 2018. Information about defaults and acquisitions comes from the National In-

	N	MAE($\hat{S}_1$)			MAE($\hat{S}_2$)			MSE($\hat{S}_1$)			MSE($\hat{S}_2$)		
		LOGIT	GEV	rel.%	LOGIT	GEV	rel.%	LOGIT	GEV	rel.%	LOGIT	GEV	rel.%
Setting 1	500	0.0913	0.0768	0.93	0.0867	0.0731	0.91	0.0209	0.0153	0.89	0.0196	0.0139	0.86
	1000	0.0598	0.0515	0.93	0.0584	0.0515	0.90	0.0098	0.0071	0.86	0.0092	0.0071	0.83
	2000	0.0418	0.0357	0.88	0.0426	0.0356	0.87	0.0047	0.0035	0.77	0.0049	0.0035	0.77
Setting 2	500	0.0840	0.0729	0.92	0.0564	0.0562	1.00	0.0188	0.0138	0.86	0.0085	0.0084	0.99
	1000	0.0594	0.0513	0.90	0.0388	0.0385	1.00	0.0099	0.0070	0.83	0.0041	0.0041	1.00
	2000	0.0404	0.0343	0.85	0.0273	0.0271	1.00	0.0045	0.0033	0.74	0.0021	0.0021	1.00
Setting 3	500	0.0796	0.0691	0.95	0.0422	0.0425	1.00	0.0171	0.0129	0.90	0.0046	0.0045	1.00
	1000	0.0545	0.0478	0.94	0.0287	0.0287	0.99	0.0081	0.0064	0.89	0.0022	0.0022	0.99
	2000	0.0392	0.0326	0.86	0.0203	0.0199	0.99	0.0044	0.0031	0.76	0.0011	0.0011	0.99
Setting 4	500	0.0389	0.0391	1.00	0.0383	0.0380	0.99	0.0041	0.0041	1.00	0.0039	0.0039	0.99
	1000	0.0267	0.0268	1.00	0.0273	0.0269	0.99	0.0020	0.0019	1.00	0.0021	0.0020	0.99
	2000	0.0188	0.0186	0.99	0.0189	0.0189	0.99	0.0010	0.0010	0.99	0.0010	0.0010	0.98
Setting 5	500	0.0702	0.0694	1.00	0.0733	0.0729	1.00	0.0131	0.0128	1.00	0.0139	0.0137	1.00
	1000	0.0478	0.0478	1.00	0.0485	0.0486	1.00	0.0062	0.0062	1.00	0.0063	0.0064	1.00
	2000	0.0331	0.0334	1.00	0.0317	0.0316	1.00	0.0031	0.0031	1.00	0.0028	0.0028	1.00

Table 3. Simulation study: median of the model errors for the survival probabilites. rel.%, is the median of the ratio between the GEV and LOGIT errors.

formation Center (NIC)[1] of the Federal Financial Institutions Examination Council (FFIEC), a repository of financial data on institutions for which the Federal Reserve has a supervisory, regulatory, or research interest.

After removing few banks acquired due to regulatory actions, the sample consists of 4 413 banks established before January 1993. During the period under investigation, 294 (6.7%) institutions were closed by the regulator due to failure and 303 (6.9%) institutions have been acquired by another entity. Most of the banks in the dataset are still operative (86%) at the end of 2018. In Figure 3, we provide a bar plot of the number of failures and acquisitions by year. Most failures are concentrated at the end of the Savings and Loan crisis (1993) and during the subprime mortgage crisis (2009-2010). Whereas, acquisitions were numerous after the Savings and Loan crisis and then decreased approaching the subprime crisis. Overall, we can observe a negative relationship between the number of failures and the number of acquisitions over time, failures are lower in periods with many acquisitions.

Similarly to previous studies [2, 4, 5, 10, 11, 16, 25, 28], we use explanatory variables as proxies for the five dimensions of the CAMEL rating system: capital adequacy, assets quality, management efficiency, earnings and liquidity. In particular, as in Beretta and Heuchenne [6], we selected the following variables[2]. **Capital adequacy** is measured by the *Total Equity Capital to Total Assets ratio*, an indicator of financial strength, which serves as a buffer to absorb future losses. **Asset quality** is measured by the *Total Loans to Total Assets ratio*, usually the least liquid and most risky assets, *Non-Performing Assets to Total Assets ratio*, the sum of assets past due 30 days or more (but still accruing interest) and assets in non-accrual status, *Other Real Estate Owned Assets to Total Assets ratio*, the real estate assets acquired by a bank in full or partial satisfaction of a debt previously contracted, and *Allowance for Loan and Lease Losses to Total Assets ratio*, a reserve calculated on the basis of the credit risk to cover future charge-offs. The ability to generate **earnings** is measured by *Return on Assets*, an indicator of a bank's profitability relative to its total assets. **Management efficiency** is measured by the *efficiency ratio*, computed as the ratio of non-interest expenses to net income. Since the objective of a bank is to maximize revenues and minimize costs, a lower value

[1] `https://www.ffiec.gov/npw/FinancialReport/DataDownload`

[2] From yearly balance sheet and income statement data, publicly available on the FDIC's website, under Statistics on Depository Institutions (SDI), `https://www5.fdic.gov/sdi/`.

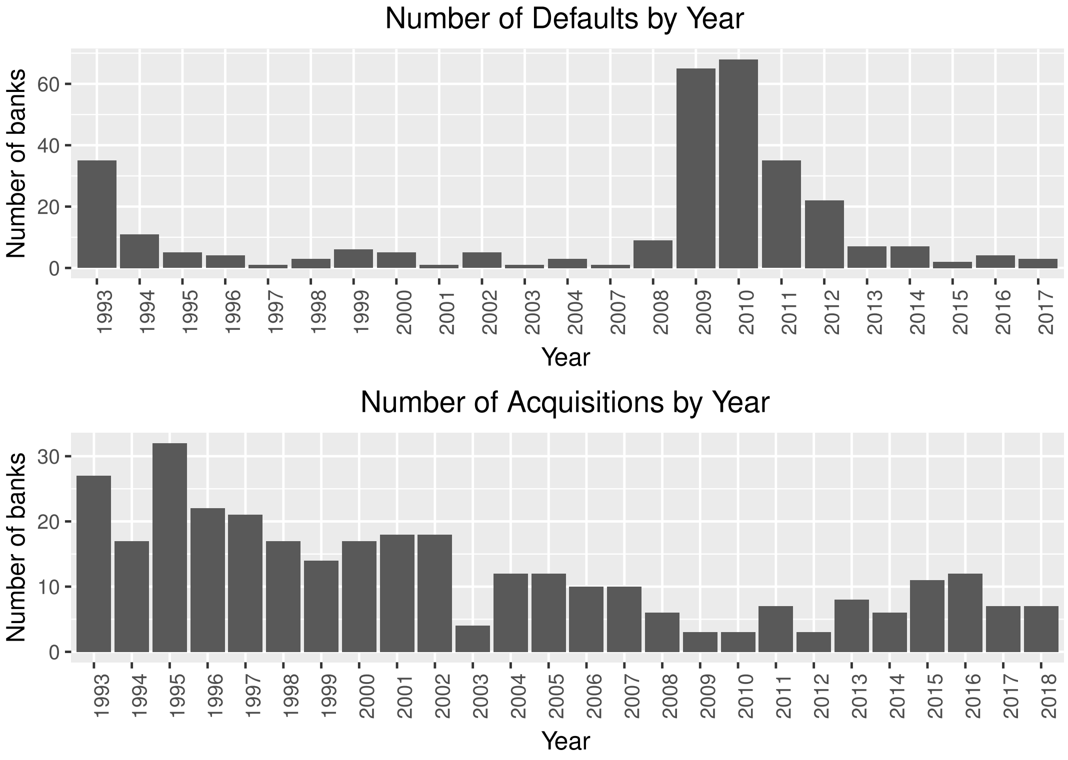

Figure 3. Number of defaults and acquisitions by year.

of this ratio means that a bank is more efficient. **Liquidity** is measured by the *Cash and Balances due from Depository Institutions to Total Assets ratio*, one of the most liquid assets, and *Retail (Core) Deposits to Total Assets ratio*, the most stable source of funding. We also include some **control variables**: the bank's *age*, measured in years since the date of establishment, and the bank's *size*, measured as the logarithm of the bank's total assets. In addition, in order to capture the general economic conditions, we included two **macroeconomic** variables: the yearly *unemployment rate*[3] and the yearly percentage change in *Gross Domestic Product*[4].

All the covariates mentioned above are observed at the end of the year previous to failure, acquisition or censoring (a list and descriptive statistics are provided in Table 4 and 5, respectively).

4.1. Estimation results

In this section, we discuss the estimation results. In Tables 6 and 8, we provide the coefficient estimates and 95% confidence intervals (obtained using the percentile bootstrap method with 1000 replications) obtained with the standard PH cure model, where the incidence is modeled by logistic regression. Moreover, in Tables 7 and 9, we provide the results obtained with the proposed GEV-PH cure model, where the incidence is modeled by GEV regression.

[3] https://www.bls.gov/lau/
[4] https://www.bea.gov/data/gdp/gdp-state

Name	Description
EQ	Total Equity Capital to Total Assets ratio
LOANS	Total Loans to Total Assets ratio
LOANS_CI	Commercial and Industrial Loans to Total Assets ratio
LOANS_RE	Real Estate Loans to Total Assets ratio
OREA	Other Real Estate Owned Assets to Total Assets ratio
NPA	Non-Performing Assets to Total Assets ratio
LLA	Allowance for Loan and Lease Losses to Total Assets ratio
EFF	Efficiency ratio. Non-interest expense less amortization of intangible assets as a percent of net interest income plus non-interest income.
ROA	Return on Assets
CBD	Cash and Balances due from Depository Institutions to Total Assets ratio
COREDEP	Retail (Core) Deposits to Total Assets ratio
SIZE	Bank's size, as logarithm of Total Assets
AGE	Bank's age, in years since the date of establishment
GDP	State Gross Domestic Product growth (yearly % change)
UNEMPLOY	Yearly state unemployment rate

Table 4. United States commercial banks data: explanatory variables.

	FAILURE		ACQUISITION		ALL BANKS	
	Mean	Sd.Dev.	Mean	Sd.Dev.	Mean	Sd.Dev.
EQ	0.0397	0.0320	0.1132	0.0650	0.1135	0.0317
LOANS	0.6889	0.1177	0.6105	0.2106	0.6329	0.1638
LOANS_CI	0.1047	0.0848	0.0948	0.0837	0.0823	0.0620
LOANS_RE	0.5092	0.1781	0.3607	0.2037	0.4404	0.1691
OREA	0.0378	0.0430	0.0031	0.0083	0.0023	0.0071
NPA	0.1042	0.0688	0.0119	0.0155	0.0118	0.0135
LLA	0.0270	0.0152	0.0121	0.0105	0.0085	0.0045
EFF	1.2341	0.6905	0.7321	0.3394	0.6812	0.1938
ROA	-0.0389	0.0306	0.0092	0.0221	0.0097	0.0075
CBD	0.0855	0.0625	0.0750	0.0907	0.0897	0.0875
COREDEP	0.7356	0.1638	0.6764	0.2408	0.7783	0.0998
SIZE	12.0979	1.4953	12.4386	1.9170	12.3475	1.4487
AGE	57.9793	39.0167	57.9904	43.1389	95.7954	32.9149
GDP	0.0202	0.0330	0.0544	0.0256	0.0357	0.0112
UNEMPLOY	0.0773	0.0214	0.0542	0.0163	0.0406	0.0076

Table 5. Explanatory variables: descriptive statistics by event type (failure or acquisition).

Overall, the signs and significance levels of the estimated coefficients are consistent in both models. Considering other factors held constant, a positive (resp. negative) sign in the incidence coefficients indicate that an increase in the relevant variable is associated with an increase (resp. decrease) in the probability to be susceptible to the event of interest. Whereas, a positive (resp. negative) sign in the latency coefficients indicate that an increase in the relevant variable is associated with an increase (resp. decrease) in the hazard and, consequently, with a shorter (resp. longer) survival time for the event of interest. Hereafter, we describe the impacts of the statistically significant variables on the two events (failure and acquisition).

4.1.1. Failure

In Tables 8 and 9, we provide the estimation results for the failure event. In line with the results of Wheelock and Wilson [28], we find that banks with lower capital buffers, higher amounts of non-performing assets, less liquid assets, a less stable source of funding and a smaller size are more susceptible to failure. Not surprisingly we find a positive relationship between the allowances for loan and lease losses and the likelihood of being susceptible to failure, reflecting the higher credit risk of a bank's loan portfolio. Moreover, as a consequence of worst economic conditions, we observe a positive relation between the susceptibility to failure and the state unemployment rate. Finally, we find that among the banks classified as susceptible to failure, the ones with lower levels of allowance for loan and lease losses, less liquid assets, higher amounts of commercial and industrial loans, less earnings and a younger age exhibit a higher failure hazard and, consequently, shorter survival times.

		INCIDENCE			**LATENCY**		
		Estimate	CI (2.5%)	CI (97.5%)	Estimate	CI (2.5%)	CI (97.5%)
FAILURE	(Intercept)	13.2371	6.7206	24.3928			
	EQ	-83.2866	-128.2985	-67.8963	0.4019	-11.2587	7.3670
	LOANS	1.3349	-7.2229	8.1335	0.0355	-1.7756	2.3720
	LOANS_CI	0.6952	-7.3178	11.7915	3.5793	1.0827	5.8822
	LOANS_RE	-2.6752	-9.0166	4.2381	-0.4653	-2.1557	0.8007
	OREA	5.6762	-16.6208	39.3061	-2.8633	-6.2277	2.3117
	NPA	23.6749	9.9602	42.2651	-2.5441	-5.9148	1.0223
	LLA	104.3656	29.0667	266.4567	-17.8761	-34.0970	-4.7322
	EFF	-0.5238	-3.8930	0.2343	0.0041	-0.4345	0.1810
	ROA	-16.7772	-70.5251	26.4951	-9.3797	-17.2589	-3.1909
	CBD	-14.1367	-30.1739	-5.4039	-4.9489	-8.2922	-2.2093
	COREDEP	-9.2155	-16.4423	-6.3852	0.0973	-1.1641	1.1762
	SIZE	-0.6620	-1.3227	-0.3727	-0.0490	-0.1989	0.0770
	AGE	-0.0069	-0.0255	0.0055	-0.0091	-0.0135	-0.0056
	GDP	2.5940	-31.7313	32.4773	1.1406	-4.6109	4.7758
	UNEMPLOY	117.5567	91.4808	243.9680	-2.1231	-15.3646	7.4978

Table 6. Estimation of the standard PH cure model for the failure event. CI, 95% confidence intervals.

		INCIDENCE			**LATENCY**		
		Estimate	CI (2.5%)	CI (97.5%)	Estimate	CI (2.5%)	CI (97.5%)
FAILURE	(tau)	-0.3458	-0.9209	-0.0658			
	(Intercept)	7.1044	4.2645	14.2541			
	EQ	-40.7932	-73.5775	-35.4479	0.1878	-10.9666	7.0573
	LOANS	0.4808	-4.1460	4.1850	0.0932	-1.8306	2.5189
	LOANS_CI	-0.0424	-4.2831	6.2064	3.5777	1.1742	5.8325
	LOANS_RE	-1.2861	-4.8833	2.2772	-0.4867	-2.1406	0.7419
	OREA	1.8138	-9.8359	22.0410	-2.7733	-6.1831	2.1980
	NPA	13.1907	6.9596	24.3898	-2.5555	-5.8172	0.6601
	LLA	51.5121	18.0169	147.2244	-17.9360	-34.2570	-5.3382
	EFF	-0.2745	-2.2711	0.2911	0.0019	-0.4169	0.1690
	ROA	-4.4681	-36.0963	13.5406	-9.4510	-17.3420	-4.0430
	CBD	-6.1780	-17.5201	-2.8784	-5.0270	-8.3469	-2.3049
	COREDEP	-4.7517	-9.8851	-3.2997	0.1162	-1.0723	1.2227
	SIZE	-0.3395	-0.7662	-0.1863	-0.0475	-0.1982	0.0680
	AGE	-0.0040	-0.0134	0.0037	-0.0091	-0.0133	-0.0058
	GDP	1.2121	-14.2198	16.2972	1.0542	-4.2500	5.3280
	UNEMPLOY	60.1916	49.4901	131.5650	-2.1768	-15.7688	7.2510

Table 7. Estimation of the GEV-PH cure model for the failure event. CI, 95% confidence intervals.

4.1.2. Acquisition

In Tables 8 and 9, we provide the estimation results for the acquisition event. The *Equity to Total Assets ratio* has a negative relationship with the probability of being susceptible to acquisition. This finding is consistent with previous studies and several explanations based on past performance, inefficiency and danger of failure have already been proposed [19].

The *Other Real Estate Owned Assets to Total Assets ratio* and the *Non-Performing Assets to Total Assets ratio* have a negative coefficient, as in Wheelock and Wilson [28]. Unsurprisingly, the banks with more assets classified as defaulted or in non-accrual status (when principal/interest payments are late or missing) are less likely to be susceptible to acquisition. Whereas, the *Allowance for Loan and Lease Losses to Total Assets ratio* has a positive relationship with the probability of being susceptible to acquisition. Contrary to the non-performing assets, which are assets effectively in trouble, the allowance is an estimation of the credit risk of a bank's loan portfolio, a general reserve available to absorb expected losses.

We find a positive relationship between bank's inefficiency and the likelihood of being susceptible to acquisition, since the *efficiency ratio* has a positive coefficient. This is consistent with the idea of Hannan and Pilloff [19] that the expected gain from an acquisition is greater when the target bank has an existing management with a poorer performance. But, it is in contrast with Wheelock and Wilson [28], which observe a significant negative relationship with cost inefficiency and a not significant relationship with two measures of technical inefficiency.

The banks with less liquid assets and a less stable source of funding are more susceptible to acquisition, since the *Cash and Balances due from Depository Institutions to Total Assets ratio* and *Retail (core) deposits to Total Assets ratio* have a negative coefficient. In line with previous literature [19, 28], we also find that older and bigger banks are less likely to be acquired.

Regarding the macroeconomic variables, both *unemployment rate* and *annual GDP growth* have a positive coefficient. The economic interpretation is not obvious. Banks in states with weaker economies (in terms of unemployment), but with a higher growth rate, are more likely to be susceptible to acquisition. In contrast, in a previous study in the European Union during the period 1997-2004, Hernando et al. [20] find that acquisitions are more likely during cyclical downturns. A possible explanation may be given by the fact that our sample covers a long period, almost 30 years, containing different economic cycles with different characteristics. Half of the acquisitions are concentrated in the period 1993-2000, which is characterized by a stronger economic growth. Whereas, in the following years, the GDP growth rate in the U.S. exhibit a downward trend until 2018. Looking at Table 5, in fact, we can see that the average GDP growth rate is below the average for the failed banks, which are mostly observed after 2000, and above the average for the acquired ones.

The aforementioned results suggest that troubled banks with lower capitalization, a higher credit risk of the loans portfolio, a higher inefficiency and lower liquidity are more susceptible to acquisition than healthy banks.

Finally, among the banks classified as susceptible to acquisition, we find that the ones with lower capital buffers, higher amounts of loans, less liquid assets, lower amounts of non-performing assets, higher levels of earnings, a younger age and a smaller size are more at risk of acquisition and exhibit shorter survival times. In addition, the banks incorporated in States with a higher unemployment rate and a higher increase in the Gross Domestic Product with respect to the previous year are more at risk of acquisition and exhibit shorter survival times.

4.2. Model validation

In this section, we validate the performance of the proposed competing risks GEV-PH cure model and we compare it with the one of the standard competing risks PH cure

		INCIDENCE			LATENCY		
		Estimate	CI (2.5%)	CI (97.5%)	Estimate	CI (2.5%)	CI (97.5%)
ACQUISITION	(Intercept)	-4.4940	-7.9121	-1.1952			
	EQ	-7.5642	-14.8987	-2.8047	-3.5429	-7.2694	-1.5897
	LOANS	-1.3482	-3.2489	0.9686	1.9988	0.7817	3.0755
	LOANS_CI	2.7143	-0.3594	4.9387	-1.5962	-3.2946	0.4547
	LOANS_RE	-2.0230	-4.0345	-0.3956	-2.1648	-3.2740	-1.1494
	OREA	-26.9500	-66.0321	-6.2319	-10.5601	-31.9443	11.2218
	NPA	-19.4244	-38.3295	-5.8755	-29.4574	-41.5729	-22.6910
	LLA	94.7204	59.7733	143.5417	-7.3435	-23.2919	9.7902
	EFF	1.5724	0.4419	3.0954	-0.1210	-0.7139	0.2866
	ROA	2.6202	-22.8591	33.1639	5.3695	0.4401	19.4150
	CBD	-5.4661	-9.2999	-2.6006	-2.2675	-4.2727	-0.2149
	COREDEP	-4.6984	-6.4673	-3.3204	0.2877	-0.7282	1.0820
	SIZE	-0.1763	-0.3452	-0.0326	-0.1742	-0.2864	-0.1067
	AGE	-0.0173	-0.0232	-0.0121	-0.0070	-0.0108	-0.0034
	GDP	67.1184	52.5170	86.5184	14.4795	9.6845	20.9163
	UNEMPLOY	154.3596	132.1139	187.2850	18.4183	9.5719	30.3097

Table 8. Estimation of the standard PH cure model for the acquisition event. CI, 95% confidence intervals.

		INCIDENCE			LATENCY		
		Estimate	CI (2.5%)	CI (97.5%)	Estimate	CI (2.5%)	CI (97.5%)
ACQUISITION	(tau)	-0.8179	-1.0229	-0.6255			
	(Intercept)	-2.2132	-4.4277	-0.1427			
	EQ	-4.0500	-9.0008	-0.8962	-3.5995	-7.1502	-1.6044
	LOANS	-1.3486	-2.4048	-0.0081	2.0139	0.8085	3.0389
	LOANS_CI	2.2871	0.1138	3.7887	-1.6039	-3.2914	0.2174
	LOANS_RE	-0.7174	-2.0130	0.1515	-2.1783	-3.2983	-1.1781
	OREA	-26.2834	-42.2886	-10.6947	-6.7020	-30.7776	17.7519
	NPA	-11.7681	-21.6249	-4.6422	-29.3905	-41.1973	-22.5866
	LLA	54.3963	37.7617	84.6731	-7.6828	-23.0064	8.7366
	EFF	0.7933	0.2332	1.6238	-0.1086	-0.7498	0.3294
	ROA	-0.3691	-13.8218	12.7540	5.3824	0.2228	18.4384
	CBD	-3.0124	-5.8247	-1.6884	-2.3405	-4.4465	-0.3284
	COREDEP	-3.1569	-4.3945	-2.2643	0.2802	-0.7754	1.1327
	SIZE	-0.1334	-0.2653	-0.0314	-0.1712	-0.2826	-0.1103
	AGE	-0.0096	-0.0155	-0.0060	-0.0071	-0.0107	-0.0034
	GDP	48.2447	34.4664	65.4884	14.0224	9.4546	21.0184
	UNEMPLOY	90.3818	81.0646	117.9610	18.1558	10.3668	29.9891

Table 9. Estimation of the GEV-PH cure model for the acquisition event. CI, 95% confidence intervals.

model, where the probability to be susceptible is modeled by logistic regression. We measure the model's performance in terms of its ability to discriminate between banks susceptible to default and/or acquisition and banks immune from these events. For this purpose, we employ a leave-one-out cross-validation procedure. For $i = 1, ..., n$, we fit the model on all observations except i, which is used to compute the out-of-sample probability to be susceptible to failure $\tilde{\pi}_1(x_i)$ and acquisition $\tilde{\pi}_2(x_i)$. Given the independence assumption (conditional on covariates), we compute the probability for the following classes:

- susceptible to both failure and acquisition $\tilde{\pi}_1(x_i)\tilde{\pi}_2(x_i)$;
- susceptible to failure only $\tilde{\pi}_1(x_i)[1 - \tilde{\pi}_2(x_i)]$;
- susceptible to acquisition only $[1 - \tilde{\pi}_1(x_i)]\tilde{\pi}_2(x_i)$;
- immune to both events $[1 - \tilde{\pi}_1(x_i)][1 - \tilde{\pi}_2(x_i)]$.

On the basis of the out-of-sample classifications and the reference (true) classes, we construct a confusion matrix to assess the model's predictive performance. Notice, however, that we do not know whether the censored banks are susceptible to failure and/or acquisition. We can only assume that the follow-up is long enough to consider them as immune.

In Table 10, we provide the confusion matrices of the out-of-sample classification results for both the standard PH cure (LOGIT) and the proposed GEV-PH cure model. Both models show very few banks classified as susceptible to acquisition (resp. failure), when they actually experienced default (resp. acquisition), and most of the censored banks are classified as immune to both events. However, we observe that some of the banks that have been actually acquired are wrongly classified as immune. But, if we look at the average values of their covariates (see Table 12), we notice that these banks exhibit lower ratios of other real estate owned assets, lower ratios of allowances for loan and lease losses, lower efficiency ratios, grater ratios of core deposits, greater sizes, longer times since establishment, lower state GDP growth rates and lower state unemployment rates compared to the other acquired banks that have been classified as susceptible to failure and/or acquisition. As we discussed, in Section 4.1, these are all characteristics belonging to healthier banks. Thus, we can conclude that the competing risks PH cure model is capable to discriminate between acquisitions of banks in good conditions and troubled banks. Finally, we notice a difference between the standard (LOGIT) and the GEV-PH cure models. Most of the failed banks are classified as susceptible to failure and acquisition. Whereas, in the GEV-PH cure model they are mostly classified as susceptible to failure only. This is due to the fact that, for these banks, the probability of acquisition $\tilde{\pi}_2(x_i)$ is lower than 0.5. We believe that this result is a consequence of the negative τ coefficient, which is significantly lower than zero (see Table 9). As we explained in Section 2, for negative values of the shape parameter τ, some of the observations belonging to the minority class may be excluded from the log-likelihood during the estimation procedure.

Similar results (see Table 11) are obtained using the same cross-validation procedure, but fitting the model using covariates observed two years before failure, acquisition or censoring and then calculating the out-of-sample probabilities using covariates observed with a time lag of one year only. Note that in this case we had to remove 56 banks from the initial sample due to missing data.

		Predicted	immune	failure	acquisition	fail. & acqu.
LOGIT	**Reference**	censored	3782	4	25	5
		failure	12	77	9	196
		acquisition	109	1	148	45
GEV	**Reference**	censored	3787	5	19	5
		failure	15	186	8	85
		acquisition	116	4	143	40

Table 10. Confusion matrices of the cross-validation results (out-of-sample).

5. Conclusion

In this article, we studied the determinants of commercial bank failures and acquisitions in the United States during the period 1993-2018. We used a competing risks proportional-hazards cure model in order to measure the impact of bank-specific and macroeconomic variables on the probabilities of being susceptible (i.e. incidence) to these events and on the survival time of susceptible banks (i.e. latency). Given the

		Predicted	immune	failure	acquisition	fail. & acqu.
LOGIT	**Reference**	censored	3126	11	636	37
		failure	13	52	9	192
		acquisition	72	3	109	97
GEV	**Reference**	censored	3409	18	364	19
		failure	11	156	6	93
		acquisition	93	9	99	80

Table 11. Confusion matrices of the cross-validation results (out-of-sample). When we fit the model we use covariates observed two years before the failure, acquisition or censoring time.

	Acquisition	
	Susceptible	Immune
EQ	0.1120	0.1154
LOANS	0.6141	0.6043
LOANS_CI	0.0964	0.0922
LOANS_RE	0.3361	0.4028
OREA	0.0040	0.0016
NPA	0.0122	0.0114
LLA	0.0135	0.0097
EFF	0.7536	0.6954
ROA	0.0093	0.0091
CBD	0.0746	0.0756
COREDEP	0.6455	0.7289
SIZE	12.3726	12.5512
AGE	47.9461	75.1196
GDP	0.0630	0.0398
UNEMPLOY	0.0593	0.0455

Table 12. Average of the covariates of the acquired banks in the groups susceptible and immune to acquisition.

rarity of failure and acquisition events, instead of using the classical logistic regression model to model the incidence distribution, we proposed the adoption of Generalized Extreme Value (GEV) regression, a more flexible model. We found that banks with a lower capitalization rate, higher level of non-performing assets, higher credit risk of the loans portfolio, lower liquidity, less stable sources of funding and smaller size are more susceptible to failure. Whereas, banks with similar characteristics, but a lower level of troubled assets, higher inefficiency and a younger age are more susceptible to acquisition. Controlling for the macroeconomic conditions, we also found that banks incorporated in states with weaker economies, measured by the unemployment rate, are more susceptible to both failure and acquisition. On the contrary, the probability of being susceptible to acquisition is higher in states with a higher GDP growth rate, presumably due to the fact that most of the acquisitions took place after the Savings and Loan crisis, a period of stronger economic growth.

Using a leave-one-out cross validation procedure, we validated the performance of the proposed methodology, in terms of discriminatory power between banks susceptible to default and/or acquisition and banks immune from these events. The proposed methodology performs reasonably well. We show a good performance in the classification of banks that actually failed, especially when GEV regression is used to model the incidence distribution. Less good in the classification of banks that were actually acquired. But, after a quick analysis of their covariates, we showed how the acquired banks classified as immune are stronger and in better conditions than the other ones. In a way, our model is capable to discriminate between acquisitions under negative and positive circumstances, treating the healthier ones as immune.

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