The political economy of social security

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Abstract

We consider a two-period overlapping generations model in which individual voters differ not only according to age but also productivity. In such a setting, a (redistributive) Pay-As-You-Go system is politically sustainable, even when the interest rate is larger than the rate of population growth. The medium wages workers (not the lowest) join the retirees to form a majority and vote for a positive level of social security. This level depends on the difference between population growth and interest rate and on the redistributiveness of the benefit rule.

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1 Introduction

The determination of contribution rates and social security benefits is often studied in majority voting models. This approach was initiated by Browning (1975) , who considers a society where people only differ according to age. Within that framework, the decisive voter is the median age individual. The voting equilibrium then leads to an excessive social security budget.

It is more and more recognized that the political forces involved in the debate on social security cannot be represented along the single age axis. There are other dimensions, particularly that of income and, more specifically, income heterogeneity *within* generations. The income dimension is likely to be important when the social security system redistributes not only from younger to older generations, as the pay-as-you-go (PAYG) system often does, but also from high to low wage workers. In other words, introducing differential earning capacities is particularly important when the benefit rule is redistributive rather than actuarially fair.

The voting outcome also depends on the alternatives available for the financing of old age consumption. Consider the hypothetical case where private savings are not available. Then, all individuals vote for a positive tax rate. The old because it determines their benefit level and the young to ensure themselves some consumption during retirement. However, when private savings are available and when their rate of return is higher than that offered by the social security system, individuals will vote against social security. This can occur for at least three reasons. First, if the social security system is of the PAYG type, and the rate of interest is higher than the rate of population growth, private saving is more attractive, at least for a young worker. Second, if the payroll tax implies some deadweight loss, one may get the same bias. Finally, if the social security system is redistributive, those who pay for redistribution, namely workers with earnings above the average level, will prefer private saving. On the contrary, workers with low earning capacities may vote in favor of social security, even when there are some distortions and when the rate of interest is higher than the rate of population growth.

In this paper, we adopt a steady state setting with given rate of interest and population growth. We also assume that the benefit rule is given. Some countries, labelled Bismarckian, such as Germany of France, offer replacement ratios that are stable across income levels, whereas others, labelled Beveridgean, such as Canada or the Netherlands, tend to have replacement ratios that fall as income increases. Table 1 illustrates this distinction for 9 countries.

Table1: Size and redistributiveness of social security

	Half	Average	Twice	Regime*	Spending as % of GDP
Canada	76	44	25	BE	5.4
France	84	84	73	BI	12.5
Germany	76	72	75	BI	12.8
Italy	103 (1/4x)	90	84 (3x)	BI	15.6
Japan	77	56	43	MI	6.6
Netherlands	73	43	25	BE	5.2
New Zealand	75	38	19	BE	5.4
UK	72	50	35	MI	4.4
USA	65	55	32	MI	4.6

*BI: Bismarckian; BE: Beveridgean; MI: mixed.

Source: Johnson (1998)

The benefit rule is taken as given in the same way as one assumes that some people drive on the left or eat with chopsticks and others drive on the right and use forks. In the concluding section, we show that the choice of the benefit rule, either at an earlier constitutional stage or along with the level of social security benefits, can be important. Even from a pure Rawlsian viewpoint, having a benefit rule that is not too redistributive can induce a majority to opt for generous retirement benefits. As Table 1 shows, the most generous systems are also those which redistribute the least.

Following Browning (1975), most of the subsequent literature focuses upon simple majority voting and the median voter outcome. Several variations on the Browning model have been considered; see Myles (1995) for a survey. While these studies have produced different specific results, the fundamental conclusion remains the same: majority voting tends to yield overspending on social security. Among the most representative variations, let us mention Hu (1982) who considers uncertainty of benefits receipts, Boadway and Wildasin (1989) who introduce an explicit capital market, and Veall (1986) who assumes intergenerational altruism.

In all of these studies it is assumed that individuals differ only in age, and that there is some sort of commitment, to preserve past decisions in the future. Tabellini (1990) introduces heterogeneity in a model where there is no such commitment and where individuals are altruistic (children towards parents and parents towards children). In such a setting , a coalition of the young poor and the retired may sustain a positive tax rate.[1]

In our paper, there is no altruism and heterogeneity arises from differ-

[1] Recently, other studies have investigated the issue of intragenerational income heterogeneity. For example, Conde Ruiz and Galasso (1999) explain the simultaneous existence of a linear income tax schedule which redistributes income among the workers and a (redistributive) PAYG system.

ential productivity. As in most work, we assume some commitment. We suppose that individual voters differ not only according to age but also according to productivity. In such a setting, medium wage workers join the retirees to form a majority and vote for a level of social security that is often in excess of that which maximizes lifetime welfare. The majority equilibrium level is shown to depend on the difference between population growth and interest rate and on the redistributiveness of the benefit rule.

2 The model

We consider a small open one sector economy with given interest rate, r_t. At each period of time t, two generations overlap, L_t workers and L_{t-1} retirees with $L_t = L_{t-1}(1+n_t)$, where n_t is the rate of population growth. Individuals differ in two ways: the generation they belong to and their wage w, a continuous variable with support (w_-, w_+), mean $\overline{w}$ and median $w_m < \overline{w}$. Labor supply is given and normalized to 1.

The pension benefit an individual earning w expects to receive is $p_{t+1}(w)$. We assume that $p_{t+1}(w)$ consists of two parts, a (contributory) part that is directly related to individual earning, w, and a (noncontributory) part that is related to average earnings, $\overline{w}$. With a Pay-As-You-Go (PAYG) scheme, the rate of return is the population growth. All these features yield the following expression for $p_{t+1}(w)$:

$$p_{t+1}(w) = (1+n_{t+1})\,\tau_{t+1}\,(\alpha w + (1-\alpha)\,\overline{w}) \qquad (1)$$

where α is the Bismarckian factor, that is the fraction of pension benefits that is related to contributions; we assume $0 \leq \alpha \leq 1$. When $\alpha = 1$, the pension scheme is purely Bismarckian (or contributory); when $\alpha = 0$, pension benefits are uniform and the scheme is Beveridgean. Finally, throughout the paper, we assume dynamic efficiency: $r_t \geqslant n_t > 0, \forall t$.

We first analyze the optimal saving decision of a working individual born in period t with earning w. He is subject to a payroll tax τ_t and expects the future tax rate to be τ_{t+1} when old. He can then allocate his disposable income between consumption c_t and saving s_t. When he retires his consumption d_{t+1} is equal to the gross return of his saving, $(1 + r_{t+1})\, s_t$, and a pension p_{t+1}. Formally, he solves the following program:

$$\max_{s \geq 0} U_t = u\left(c_t\right) + \beta u\left(d_{t+1}\right) \tag{2}$$

subject to:

$$w\left(1 - \tau_t\right) = c_t + s_t \tag{3}$$

and

$$d_{t+1} = \left(1 + r_{t+1}\right) s_t + p_{t+1}(w). \tag{4}$$

In (2), $u\left(.\right)$ is strictly concave and $\beta \leq 1$ is a factor of time preference. Let σ denote the elasticity of substitution between c_t and d_{t+1}. We assume throughout the paper that σ is constant and that $\sigma < 1$, which means that there is not much substitution in consumption, a widely accepted assumption. The coefficient of relative risk aversion, $R_r\left(x\right) = -xu''\left(x\right)/u'\left(x\right)$ is thus also constant and equal to $\varepsilon = 1/\sigma$. Finally, we restrict savings to be non negative.[2]

The first-order condition associated to an interior solution of s_t is the following:

$$-u'(c_t) + \beta u'(d_{t+1})(1 + r_{t+1}) = 0. \tag{5}$$

[2]Allowing negative savings would generate extreme solutions. Young people in favor of the PAYG pension system would vote for a tax rate of 1 and would rely entirely on borrowing to finance their present consumption.

Denoting $s_t^A \geq 0$ the optimal value of s_t, we can define the (indirect) utility function of an individual with income w as:

$$V_t(\tau_t, \tau_{t+1}, w) = u\left(w(1-\tau_t) - s_t^A\right) + \beta u\left((1+r_{t+1}) s_t^A + p_{t+1}(w)\right). \quad (6)$$

We now determine the steady state majority voting equilibrium tax rate. We consider α as given; the problem is therefore unidimensional and, under the condition that preferences are single-peaked, the median voter theorem, which ensures the existence of a Condorcet winner, applies. Individuals vote for τ believing that the value of τ chosen by the majority will hold for ever.[3] Formally, they expect that $\tau_{t+1} = \tau_t, \forall t$. We can thus remove all the time subscripts in the following equations.

In the following, we first derive the preferred tax rate of the retirees and that of the workers. Then, to identify the Condorcet winner, we order these preferred alternatives. The Condorcet winner is the tax rate such that half of the population prefer a higher tax rate and half of the population a lower one. For the time being, there is no tax distortion. Next, we derive the comparative statics of the majority voting solution with respect to some parameters of the model, namely α and n. We then compare the PAYG majority voting equilibrium tax rate with a (collective) fully funded (FF) solution, granted that both systems adopt the same benefit rule. Finally, we solve the problem when taxation generates (quadratic) distortions.

3 Preferred tax rates of the different agents

3.1 The retirees

Each retiree has some non negative private saving, s, with return r. This private saving is the result of past decision and the retirees have no control

[3] Or at least for the next two periods; the tax rate in later periods is of no relevance for the individuals.

over it. The only variable they can manipulate is the tax rate, which determines their pension level. As there is no altruism in this model, they choose the value of τ, τ^R, that maximizes their consumption:

$$d = (1 + r) s + (1 + n) \tau (\alpha w + (1 - \alpha) \overline{w}) . \tag{7}$$

The solution is straightforward: $\tau^R = 1$, the same tax rate for all retirees.[4]

3.2 The workers

A worker with earning w chooses $\tau^A (w)$ which maximizes:

$$v (\tau, w) = u \left(w (1 - \tau) - s^A\right) + \beta u \left((1 + r) s^A + (1 + n) \tau (\alpha w + (1 - \alpha) \overline{w})\right) \tag{8}$$

where $s^A \geq 0$ is the optimal level of private saving and $v (\tau, w) = V (\tau, \tau, w)$.

Note that a worker will always be in favor of a zero tax if:

$$1 + r > (\alpha + (1 - \alpha)\overline{w}/w)(1 + n). \tag{9}$$

This means that the return from private saving is higher than the return from PAYG social security. Without tax distortions, these returns are constant relatively to the value of the tax rate. Therefore, a given individual will either prefer a zero tax rate and positive private saving if (9) holds or a strictly positive tax rate and no private saving in the opposite case.

In other words, for an individual to prefer private saving, his wage must be strictly higher than $\hat{w}$ defined as:

$$\hat{w} = \frac{1 - \alpha}{\frac{1+r}{1+n} - \alpha} \overline{w} \leq \overline{w}. \tag{10}$$

One easily checks that $\hat{w} = \overline{w}$ if $n = r$, $\partial \hat{w} / \partial n > 0$ and $\partial \hat{w} / \partial \alpha < 0$.

This brings our first proposition, which is proved in the appendix.

[4] This result is rather extreme but, introducing tax distortions or some degree of altruism from the old to the young, one would obtain a value of τ^R smaller than 1.

Proposition 1 *The preferred tax rates of the workers have the following properties:*

(i) $\tau^A(w) = 0$ *if* $w > \hat{w}$ *and* $\tau^A(w) > 0$ *if* $w \leqslant \hat{w}$;

(ii) $\dfrac{\partial \tau^A(w)}{\partial w} > 0$ *if* $w < \hat{w}$;

(iii) $Max\ \tau^A(w) = \tau^A(\hat{w}) < 1 = \tau^R$.

The first point of the proposition is related to the discussion above: only workers with a sufficiently low wage level want a positive tax rate. The second point says that preferred tax rates are increasing with income. Intuitively, one sees that with low intertemporal substitution, the lower the level of earnings the lower the most desired tax rate. Consider for example the extreme case where there is no substitution at all: individuals want to equalize their two periods consumptions. Because the rate of return of PAYG social security is decreasing with income, the high wage workers would like to transfer a greater proportion of their income from the first period to the second period than the low wage one.[5] Finally we argue that the maximal preferred tax rate of the workers is lower than 1, the preferred tax rate of the retirees. This simply comes from the fact that individuals want to consume when young. Only for a linear utility function (which corresponds to an infinite intertemporal elasticity of substitution), one would obtain a corner solution with $\tau^A = 1$.

[5] The property that preferred tax rates are increasing with income may seem surprising. Whithin our framework, it results from the (realistic) assumption that the intertemporal elasticity of substitution is low. However, the assumption of a strictly proportional tax is crucial. If instead we had assumed that the poorest individuals are exempted from taxation, they would have a high preferred tax rate, thereby joining the retirees to sustain a generous social security system.

4 The majority voting solution

Because their utility is increasing with the value of the tax rate, preferences of the retirees are obviously single-peaked. We have shown in the proof of proposition 1 that the objective function of the workers is strictly concave, which implies that it is single-peaked. The workers are divided into two classes, those preferring a zero tax rate and positive savings and those preferring a positive tax rate and no saving. The former order the different tax rates in a decreasing way: the higher the tax rate, the worse they are. The latter have an interior preferred value of the tax rate.

Following Proposition 1, a fraction $1/(2+n)$ of citizens, the retirees, is in favor of $\tau^R = 1$. All the workers with earnings above $\hat{w}$ are in favor of a tax of 0. The preferred tax rate for the workers with earnings below $\hat{w}$ increases with w. We can now determine the decisive voter and the majority voting equilibrium tax rate.

Proposition 2 *If $\int_{w_-}^{\hat{w}} f(w)\,dw < n/2(1+n)$, the majority voting equilibrium tax rate, τ^* , is 0. If $\int_{w_-}^{\hat{w}} f(w)\,dw \geq n/2(1+n)$, the majority voting equilibrium tax rate is the rate preferred by the workers with earning $\widetilde{w}$ defined as follows:*

$$\int_{\tilde{w}}^{\hat{w}} f(w)\,dw = \frac{n}{2(1+n)}\ . \tag{11}$$

Proof.

The Condorcet winner is the tax rate such that one half of the total population prefer a higher tax rate and the other half a lower tax rate. The total number of individuals preferring a tax rate higher than 0 is $L + L(1+n)\int_{w_-}^{\hat{w}} f(w)\,dw$, where L is the number of old individuals. The total population is $L + L(1+n) = L(2+n)$. Therefore, if $L+L(1+n)\int_{w_-}^{\hat{w}} f(w)\,dw < L(2+n)/2 \Leftrightarrow \int_{w_-}^{\hat{w}} f(w)\,dw < n/2(1+n)$, less than one half of the total population prefer a strictly positive tax rate and the Condorcet winner is 0. On the contrary, when $\int_{w_-}^{\hat{w}} f(w)\,dw \geq n/2(1+n)$, the individuals preferring a strictly positive tax rate constitutes more than one half of the total population and the Condorcet winner is strictly positive. It is the tax rate preferred by the individual with income $\tilde{w}$ such that the population who prefer a higher tax rate constitutes half of the total population:

$$L + L(1+n)\int_{\tilde{w}}^{\hat{w}} f(w)\,dw = \frac{L(2+n)}{2}. \tag{12}$$

Straightforward manipulations lead to (11). ■

In the first part of the proposition, we argue that the majority voting equilibrium tax rate may be 0, when the number of working individuals in favor of a positive tax rate is too low. This may occur in particular when r is large relatively to n. In this case, the redistributive effect of PAYG social security is dominated by the high return of private saving and even the poorest of the workers may prefer to save privately. When the Condorcet winner is strictly positive, the second part of the proposition says that, in the majority coalition, one finds the retirees but also the workers with medium wages. Such a result is related to Epple and Romano's (1996) "ends against the middle" equilibrium (see also Casamatta and al. (1999)) in which there is a coalition made of the tails of the income distribution. This proposition clearly hinges upon our assumption on the elasticity of substitution. With $\sigma > 1$, preferred tax rates would be decreasing with income and the majority coalition would be composed by the retirees and the poorest working individuals. Figures 1 and 2 illustrate these results for the case $r = n$ and $r > n$ respectively.

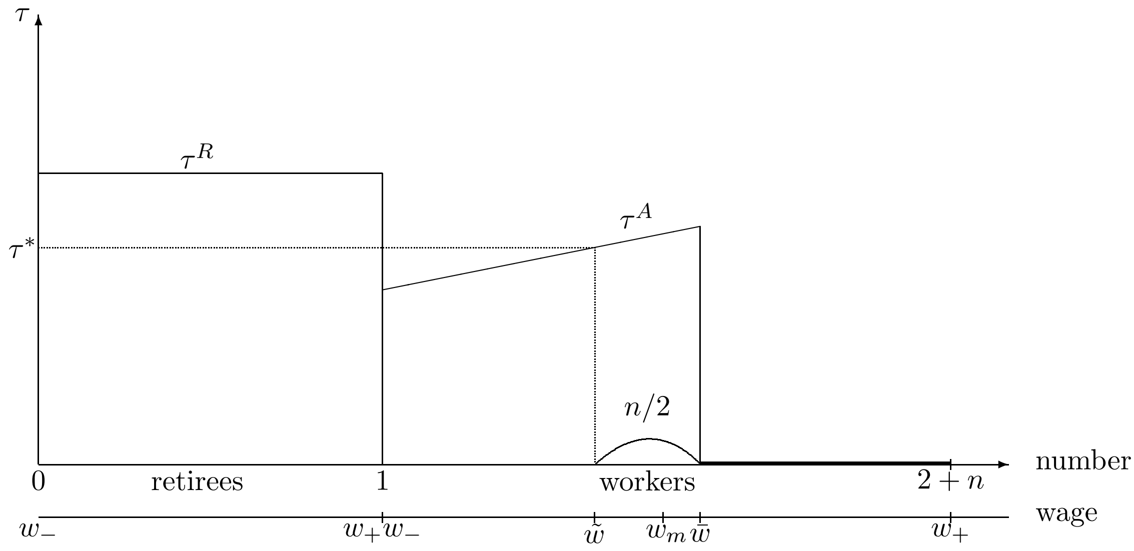

Figure 1: preferred tax rates when $r = n$

5 Comparative statics

In the introduction, we mentioned the conjecture that as the contributive part of social security increases, the equilibrium tax rate tends to increase as well. In fact we have here two effects of α on τ^*: a direct effect, which is positive, and an indirect effect through $\tilde{w}$ which is negative. Consider the case where the Condorcet winner is strictly positive $\left(\int_{w_-}^{\hat{w}} f(w)\, dw \geq n/2\,(1+n)\right)$. Differentiating (11) with respect to α, one gets:

$$\frac{\partial \tilde{w}}{\partial \alpha} = \frac{\partial \hat{w}}{\partial \alpha} \frac{f(\hat{w})}{f(\tilde{w})} \leq 0. \tag{13}$$

Moreover, from the first-order condition on τ^A, we have:

$$\frac{\partial \tau^A}{\partial \alpha}\left(\tilde{w}(\alpha), \alpha\right) = \frac{\beta\,(1+n)\,(\tilde{w} - \overline{w})\, u'(d)\,(1 - R_r(d))}{-D_\tau}. \tag{14}$$

Keeping in mind that $\tilde{w} < \overline{w}$ and that $R_r(.) > 1$, this expression is positive.

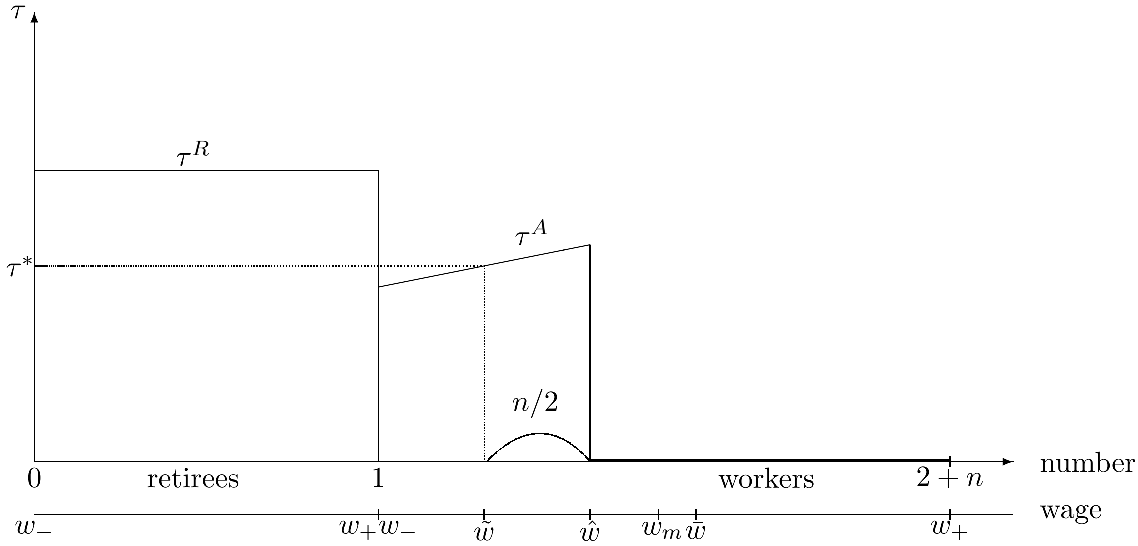

Figure 2: preferred tax rates when $r > n$

And thus, recalling that $\tau^*(\alpha) = \tau^A(\tilde{w}(\alpha), \alpha)$:

$$\underset{?}{\frac{\partial \tau^*}{\partial \alpha}} = \underset{+}{\frac{\partial \tau^A}{\partial \alpha}} (\tilde{w}(\alpha), \alpha) + \underset{+}{\frac{\partial \tau^A}{\partial w}} (\tilde{w}(\alpha), \alpha) \underset{-}{\frac{\partial \tilde{w}}{\partial \alpha}} . \tag{15}$$

Clearly the sign of $\partial \tau^*/\partial \alpha$ is ambiguous except when $r = n$, in which case it is positive (because the decisive voter is indifferent relatively to α and only the first, positive, effect operates). In general, one cannot thus say that the majority voting equilibrium tax rate increases as the Bismarckian factor increases. This indeterminacy comes from two opposite effects. On the first hand, when α rises, the preferred tax rate of the decisive voter rises. But on the other hand, when α changes, the identity of the decisive voter changes as well. When α rises, PAYG public pension becomes less attractive to low wage workers (who support this system). Some of them (the more productive) "switch" to the private sector and the new decisive voter is poorer than before. As preferred tax rates are increasing with income, the tax rate chosen by this new decisive voter is smaller than before.

We should however note that, when $r > n > 0$, the majority voting equilibrium tax rate jumps discontinuously to 0 for α high enough. This comes from the fact that, when α is sufficiently close to 1, the PAYG system operates so little redistribution that even the poorest of the workers does not find it attractive anymore and vote for the abandonment of the system.

Following the same approach, one can also study the comparative statics of $\partial\tau^*/\partial n$. Assuming that $\tau^* > 0$, we have:

$$\frac{\partial \tilde{w}}{\partial n} = \frac{\frac{\partial \hat{w}}{\partial n} f(\hat{w}) - \frac{1}{2(1+n)^2}}{f(\tilde{w})} \gtrless 0. \tag{16}$$

That is the effect of population growth on the pivotal wage cannot be determined.

Moreover, from the first-order condition on τ^*, we have:

$$\frac{\partial \tau^A}{\partial n} \left(\tilde{w}\left(n\right), n\right) = \frac{\beta\left(\alpha \tilde{w} + \left(1-\alpha\right) \overline{w}\right) u'\left(d\right) \left(1 - R_r\left(d\right)\right)}{-D_\tau} \tag{17}$$

which is negative under our assumption on the intertemporal elasticity of substitution.

Consequently, we obtain:

$$\underset{?}{\frac{\partial \tau^*}{\partial n}} = \underset{-}{\frac{\partial \tau^A}{\partial n}} \left(\tilde{w}\left(n\right), n\right) + \underset{+}{\frac{\partial \tau^*}{\partial w}} \left(\tilde{w}\left(n\right), n\right) \underset{?}{\frac{\partial \tilde{w}}{\partial n}}. \tag{18}$$

In words, here again, the comparative statics yields an indeterminate result. A decline in fertility can indeed result in an increase in the majority voting equilibrium tax.

Nevertheless, under the condition that $w_- > \overline{w}/\left(1+r\right)$, for a large decline in fertility, the tax rate falls to zero. The explanation is simple: for n close to 0, $\hat{w}$ is close to $\left(1-\alpha\right)\overline{w}/\left(1+r-\alpha\right)$. Moreover we know that $\hat{w}$ declines with α. Hence the maximal value of $\hat{w}$ when n tends to 0 is

$\overline{w}/(1+r)$. Therefore, if $w_- > \overline{w}/(1+r)$, we are sure that the poorest individual does not support the PAYG pension system so that a majority votes for the abandonment of the system.

6 Pay-as-you-go versus fully funded

Up to now, voting was restricted to PAYG. We would now like to compare the majority voting equilibrium with PAYG with what it could be with an equivalent fully funded scheme. We are not concerned by the transition from one to the other. We just assume that vote takes place in two alternative steady states. To make the comparison fair we assume that $r = n > 0$ and we assume that the benefit rule is the same, which means that the two schemes operate the same degree of redistribution. In other words, we do not compare a PAYG system involving some redistribution to a totally individualized FF system, as it is often done in work on privatization of social security. Actually, an individualized FF scheme here corresponds to private saving.

The big difference between the two systems is that with a FF scheme, the retirees are not concerned by the vote. All the decisions concerning them have been taken: private saving, s, and collective saving through pension funds, if any. Given the open economy assumption, r is also given. Finally, by assumption, the Bismarckian factor, α, is given.

Only the active population takes part to the vote. With our assumption on the elasticity of intertemporal substitution, the decisive voter is that with earning $\breve{w}$ such that

$$\int_{\breve{w}}^{\overline{w}} f(w)\,dw = 1/2. \tag{19}$$

This means that young population is divided in two: those with earnings between $\breve{w}$ and $\overline{w}$ want a tax rate higher than $\tau^A(\breve{w})$ and those with earnings below $\breve{w}$ or above $\overline{w}$ want a lower tax rate. Note that, as soon as $w_m < \overline{w}$, the majority voting equilibrium tax rate is always strictly positive. By comparing this equation with (11) with $\hat{w} = \overline{w}$, it is clear that $\breve{w} < \tilde{w}$. The decisive voter in the FF scheme has a lower wage than the decisive voter in the PAYG system.

This leads to Proposition 3.

Proposition 3 *The majority voting equilibrium for social security is higher with a PAYG than with a fully funded scheme.*

This proposition is not surprising. With PAYG, the retirees want a payroll tax rate as high as possible; as they don't have the majority, they rely on a "minority" of workers that back social security because of its redistributiveness. With a FF system, retirees don't have any leverage on the workers' tax payments.

What about social welfare? In the setting of this model, the comparison is not straightforward as we deal with heterogeneous individuals. In an identical individuals economy, the answer is clear. For $n = r$, the two systems are identical: they give the same rate of return and the voting equilibrium is the same in both system. But with heterogenous individuals, if $\alpha < 1$, the redistributive feature of the collective scheme, PAYG or FF, differentiate them. With a Rawlsian criterion, the FF scheme always dominates the PAYG one as it gives the decisive vote to workers with lower earnings (indeed, $\breve{w} < \tilde{w}$). With a general utilitarian criterion, the comparison is ambiguous, even though in our numerical examples the majority voting equilibrium tax with the FF scheme is always preferable to that with the PAYG scheme; the idea is simple: the former involves only individuals who are at the start of their life.

7 Extension: distortionary payroll taxation

Up to now the tax system was assumed to imply no efficiency loss. Let us see what would be the implication of distortionary taxation on the above results. To keep the presentation simple, we use a quadratic loss function such that the revenue constraint is now:

$$p\left(w\right)=\left(1+n\right)\tau\left(\alpha w+\left(1-\gamma\tau\right)\left(1-\alpha\right)\overline{w}\right), \tag{20}$$

where $\gamma>0$ is the distortion factor. Note that the distortion applies only to the noncontributory part of social security. In other words, we assume that voters see through the budgetary veil that the fraction α of their tax payment is given back to them with a return of n.[6] With this modification, what is the preferred tax rate of the retirees and of the workers?

7.1 The retirees

For the retirees, it is straightforward to show that they will choose a tax rate $\tau^{R}(w)$ defined by:

$$\tau^{R}(w)=\min\left\{\frac{1}{2\gamma}(1+\frac{\alpha}{1-\alpha}\frac{w}{\overline{w}}),1\right\}. \tag{21}$$

The retirees choose the tax rate that maximizes revenue (the top of the Laffer curve) under the constraint that $\tau\leq1$. Note that $\tau^{R}(w)$ is increasing in w. This comes from the fact that tax distortions are related to the Beveridgean

[6] This specification is a reduced form of a more general (and complicated) model where labour supply is endogenous. Indeed the FOC for an interior value of τ would be:

$$\begin{aligned}&-yu'\left(c\right)+\beta\left(1+n\right)\left(\alpha y+\left(1-\alpha\right)\overline{y}\right)u'\left(d\right)+\\ \beta&\left(\tau\left(1+n\right)\left(1-\alpha\right)\int_{w_-}^{w_+}w\partial l^{A}/\partial\tau f\left(w\right)dw\right)u'\left(d\right)=0,\end{aligned}$$

where $\overline{y}=\int_{w_-}^{w_+}yf\left(w\right)dw$, $y=wl^{A}\left(w\right)$ and $l^{A}\left(w\right)$ is the optimal labour supply of an individual with productivity w. It appears clearly that tax distorsions are associated to the third term of this FOC.

part of pensions. Consequently, poor retirees suffer more from a tax increase than the rich.

7.2 The workers

The problem of the workers is dramatically modified when tax distortions are introduced. Indeed, the rate of return of the PAYG system now depends on the value of the tax rate chosen. Consequently, it is now possible that a given individual prefers the PAYG system to private saving for some values of τ and that his preferences change in favor of private saving for some other values. This individual will then choose a positive tax rate, such that the rates of return of public pension and private saving are equalized, and will complement this tax rate with private saving. Therefore, the possibility of a mixed choice, public pension complemented with private saving, cannot be ruled out anymore.

Formally the program of a working individual is the following:

$$\underset{\tau,s}{Max}\, U = u\left(c\right) + \beta u\left(d\right) \tag{22}$$

subject to:

$$w\left(1-\tau\right) = c + s$$

$$d = \left(1+r\right)s + \left(1+n\right)\tau\left(\alpha w + \left(1-\alpha\right)\left(1-\gamma\tau\right)\overline{w}\right) \tag{23}$$

and

$$\tau \geq 0, s \geq 0. \tag{24}$$

In the next proposition, proved in the appendix, we establish that low wage people want a positive tax rate but no private saving, middle wage individuals will make a mixed choice, with both PAYG social security and private saving, and the higher wage individuals will prefer to rely only on private saving.

Proposition 4 *(i)* $\max \tau^A(w) < \min \tau^R(w)$.

Moreover, there exists a value of w*,* $w' < \hat{w}$ *(defined in* (10)*), such that:*

(ii) if $w \leq w'$*,* $\tau^A(w) > 0$*,* $s^A(w) = 0$*,*

(iii) if $w' < w < \hat{w}$*,* $\tau^A(w) > 0$ *and* $s^A(w) > 0$*,*

(iv) if $w \geq \hat{w}$*,* $\tau^A(w) = 0$ *and* $s^A(w) > 0$*,*

(v) if $w < w'$*,* $\partial\tau^A(w)/\partial w > 0$ *and if* $w' < w < \hat{w}$*,* $\partial\tau^A(w)/\partial w < 0$.

In this proposition, we first show that the maximal preferred tax rate of the workers is lower than the minimal preferred tax rate of the retirees. The underlying idea is the same as in the case without distortions, namely that workers do not want too high a tax rate because it decreases their first period consumption. We then show that $\tau^A(w)$ is first increasing and then decreasing with w, being equal to 0 when $w = \hat{w}$. For these workers with $w < \hat{w}$, social security is attractive up to a certain point; its relative return is now equal to $(\alpha + (1-\alpha)(1-\gamma\tau)\overline{w}/w)$, which can be lower than $1+r$ for some τ. For workers with wage close to w_- , there is no saving and the preferred tax rate increases with w. Figure 3 presents this case.

7.3 The majority voting solution

In the appendix, we establish that the objective function of each worker is concave relatively to its arguments, τ and s. Therefore, preferences over τ are single-peaked and the median voter theorem applies. In the next proposition, we identify the Condorcet winner when tax distortions are introduced. In this purpose, we define w^* as the income level satisfying $\tau^A(w_-) = \tau^A(w^*)$.

Proposition 5 *When income taxation gives rise to quadratic distortions, the majority voting equilibrium tax rate satisfies the following conditions:*

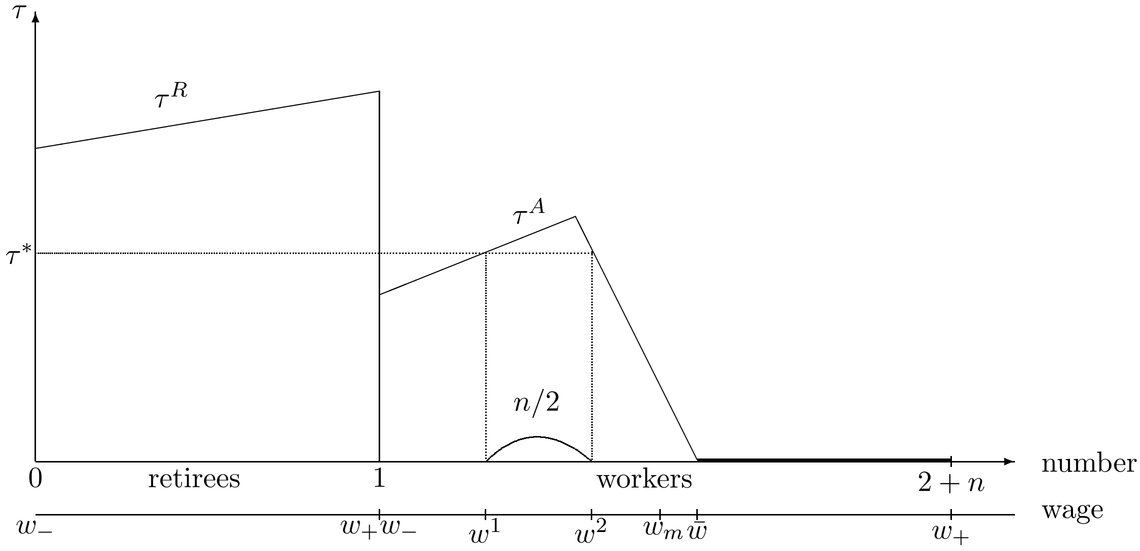

Figure 3: preferred tax rates with quadratic tax distortions when $r = n$

(i) If $\int_{w_-}^{\hat{w}} f(w)\,dw < n/2\,(1+n)$*, the majority voting equilibrium tax rate,* τ^* *, is 0.*

(ii) If $\int_{w_-}^{w^*} f(w)\,dw \leq n/2\,(1+n) \leq \int_{w_-}^{\hat{w}} f(w)\,dw$*, the majority voting equilibrium tax rate is the rate preferred by the workers with earning* w^p *defined as follows:*

$$\int_{w_-}^{w^p} f(w)\,dw = \frac{n}{2\,(1+n)} \ . \tag{25}$$

(iii) If $\int_{w_-}^{w^*} f(w)\,dw > n/2\,(1+n)$*, the majority voting equilibrium tax rate is the rate preferred by the workers with earnings* w^1 *or* w^2 *satisfying:*

$$\int_{w^1}^{w^2} f(w)\,dw = \frac{n}{2\,(1+n)} \ \text{and}\ \tau^A\left(w^1\right) = \tau^A\left(w^2\right). \tag{26}$$

The proof goes along the same lines as the one of proposition (2) so that we do not develop the formal argument here. The idea is that one must find a sufficient number of individuals in the working population to form a majority coalition with the retirees and sustain a positive equilibrium tax rate. Under the condition in (i), the number of young favoring a positive tax rate is not sufficient to form a majority coalition with the old and the PAYG pension system is not sustainable in the steady state. In (ii), the number of young individuals favoring the PAYG scheme is sufficient but the young with income between w^1 and w^2 are not enough to form a majority coalition with the old. Therefore, some young people with income $w > w^2$ belong also to that majority. Finally, in (iii), there are enough people in the interval $\left[w^1, w^2\right]$ and workers with income $w > w^2$ belong to the minority in favor of a lower tax rate than $\tau^A\left(w^1\right) = \tau^A\left(w^2\right)$. It is then clear that the set of workers who join the retirees to form a majority in favor of a positive tax rate is different from what it was without distortion. In particular, when $\int_{w_-}^{\hat{w}} f(w)\,dw$ is not close to $n/2\,(1+n)$, those with earnings equal or just below $\hat{w}$ do not belong to that majority.

8 Concluding comments

In this paper, the more or less Bismarckian feature of the social security system was assumed given. In an earlier paper by Casamatta and al. (1998) that deals with social insurance, the choice of the Bismarckian parameter, α, is made at the constitutional level upon the expectation that the payroll tax is determined later through majority voting. One of the results of that paper is that a positive Bismarckian parameter could be desirable, even though with full control of both α and τ, the constitutional planner would choose α equal to zero. The setting is static and corresponds to what is called here the steady state. The result of a positive α is mainly due to the need of political support. By letting α to be positive, the decisive voter vote for a tax rate that fits better the preferences of the low wage individuals. A

Rawlsian constitutionalist can then favor such a positive α.

Alternatively, we could consider a setting where both α and τ are chosen by majority voting. In this two-dimensional collective choice problem, a Condorcet winner does not generally exist. Therefore, one must give more structure to the political process. Some examples can be found in Casamatta (1999), who studies sequential voting procedures, or in De Donder and Hendricks (1998), who study (two parties) electoral competition.

In future research, we want to study how choosing both τ and α, either sequentially or simultaneously. We also want to study how our model can be used out of steady state, when, for example, a sudden drop in fertility or in productivity occurs. In such a setting, the issue of voting is more complex and depends very much on how expectations are formed. Also, the degree of redistributiveness can be very important when looking at social security reforms. It is often observed that the resistance to change by vested interests is related to entitlements founded on the contributory part of social security. In other words, the choice of α has an effect on political support in the steady state but also on political resistance out of steady state when reforms are contemplated. But this is clearly another story.

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Appendix

A Appendix 1: proof of proposition 1

We first show that the objective function $v(\tau, w)$ is a concave function of τ. This is the case when

$$\begin{aligned}\frac{\partial^2 v}{\partial \tau^2} &= w^2 u''(c) + \beta(1+n)^2(\alpha w + (1-\alpha)\overline{w})^2 u''(d) + \\ &\quad \frac{\partial s^A}{\partial \tau}\left(wu''(c) + \beta(1+n)(1+r)(\alpha w + (1-\alpha)\overline{w})u''(d)\right)\end{aligned} \tag{27}$$

is negative. When the first-order condition on s^A, given by (5), is satisfied, we have:

$$\frac{\partial s^A}{\partial \tau} = -\frac{wu''(c) + \beta(1+n)(1+r)(\alpha w + (1-\alpha)\overline{w})u''(d)}{u''(c) + \beta(1+r)^2 u''(d)} < 0. \tag{28}$$

Simple manipulations lead to:

$$\frac{\partial^2 v}{\partial \tau^2} = \frac{\beta u''(c)u''(d)(w(1+r) - (1+n)(\alpha w + (1-\alpha)\overline{w}))^2}{u''(c) + \beta(1+r)^2 u''(d)} < 0. \tag{29}$$

At points where $s^A = 0$, we have:

$$\frac{\partial^2 v}{\partial \tau^2} = w^2 u''(c) + \beta(1+n)^2(\alpha w + (1-\alpha)\overline{w})^2 u''(d) < 0. \tag{30}$$

Therefore, the objective function $v(\tau, w)$ is strictly concave.

To prove (i), one differentiates $v(\tau, w)$ at $\tau = 0$:

$$\left.\frac{\partial v}{\partial \tau}\right|_{\tau=0} = -wu'(c) + \beta(1+n)(\alpha w + (1-\alpha)\overline{w})u'(d). \tag{31}$$

At $\tau = 0$, everyone saves. Hence, substituting (5) in the previous expression, we obtain:

$$\left.\frac{\partial v}{\partial \tau}\right|_{\tau=0} = u'(c)\left(\frac{(1+n)}{(1+r)}(\alpha w + (1-\alpha)\overline{w}) - w\right). \tag{32}$$

This expression is greater than 0 iff $w \leq \hat{w}$. Therefore, only these individuals will have a strictly positive preferred tax rate.

To prove (ii), let's write the first-order condition on τ for an individual with income $w < \hat{w}$:

$$-wu'(c) + \beta(1+n)(\alpha w + (1-\alpha)\overline{w})u'(d) = 0. \tag{33}$$

Differentiating this expression with respect to w, we obtain:

$$\frac{\partial \tau^A}{\partial w} = \frac{\beta(1+n)(1-\alpha)\frac{\overline{w}}{w}u'(d)(1-\varepsilon)}{D_\tau}, \tag{34}$$

where $D_\tau < 0$ is the second-order derivative of $v(\tau, w)$ with respect to τ. Clearly, under our assumption that $\sigma < 1$, the preferred tax rate of individuals with income below $\hat{w}$ is increasing with w.

To prove (iii), we note that the FOC $\partial v/\partial \tau = 0$ for $w \leq \hat{w}$ implies that workers with less than break-even level of earnings oppose too high a tax rate because it decreases their first period consumption. Indeed, when $\tau \to 1$, $u'(c) \to +\infty$ whereas the limit of $u'(d)$ is finite, which implies that the FOC cannot be satisfied.

B Appendix 2: proof of proposition 4

The first-order conditions associated to the program of a working individual are:

$$-wu'(c) + \beta(1+n)(\alpha w + (1-\alpha)\overline{w}(1-2\gamma\tau))u'(d) + \lambda_\tau = 0 \tag{35}$$

and

$$-u'(c) + \beta(1+r)u'(d) + \lambda_s = 0 \tag{36}$$

where λ_τ and λ_s are the Lagrange multipliers respectively associated to the non-negativity constraints on τ and s.

To solve the program, we have to consider different cases, depending on which constraint binds.

Case 1: $\lambda_\tau = \lambda_s = 0$

In this first case, none of the nonnegativity constraints bind, which means that we have an interior solution for both τ and s. Substituting (36) into (35), we obtain:

$$\tau^A(w) = \frac{(1+n)(\alpha w + (1-\alpha)\overline{w}) - w(1+r)}{2\gamma(1-\alpha)\overline{w}(1+n)}. \tag{37}$$

A necessary and sufficient condition for $\tau^A(w)$ to be positive is that $w < \hat{w} = (1+n)(1-\alpha)\overline{w}/((1+r) - \alpha(1+n))$. All the people with earnings above this threshold will have a preferred tax rate of 0 and will choose to rely exclusively on private savings. For people with wage lower than $\hat{w}$ who save privately, we have:

$$\frac{\partial \tau^A}{\partial w} = \frac{\alpha(1+n) - (1+r)}{2\gamma(1-\alpha)\overline{w}(1+n)} < 0. \tag{38}$$

Consequently, preferred tax rates are decreasing with income.

Using (36), we obtain:

$$\begin{aligned}
\frac{\partial s^A(w)}{\partial w} &= -\frac{\left(-\left(1-\tau^A\right) + w\frac{\partial \tau^A}{\partial w}\right)u''(c)}{u''(c) + (\beta(1+r))^2 u''(d)} \\
&\quad - \frac{\beta(1+r)\left(\frac{\partial \tau^A}{\partial w}(1+n)(\alpha w + (1-\alpha)\overline{w}) + \alpha(1+n)\tau^A\right)u''(d)}{u''(c) + (\beta(1+r))^2 u''(d)} \\
&= \frac{\alpha((1+n)(\alpha w + (1-\alpha)\overline{w}) - w(1+r))}{\gamma(1-\alpha)\overline{w}}.
\end{aligned} \tag{39}$$

This expression is positive for individuals with wage below $\hat{w}$. The function $s^A(w)$ is thus increasing. Moreover, from (36), it is negative for w sufficiently close to 0 and positive for w sufficiently close to $\hat{w}$. Therefore, there exists a value of w, w', such that each individual with income above this value has a strictly positive saving and for people with income below w', the constraint $s \geq 0$ is binding.

To sum up, we have found that all the people with earnings between w' and $\hat{w}$ have an interior solution for both τ and s. These solutions are respectively given by (37) and (36). The other individuals have either a positive preferred tax rate with no savings or positive savings and a zero preferred tax rate. We will analyze these two cases one after the other.

Case 2: $\lambda_\tau = 0, \lambda_s > 0$

As shown before, individuals with earnings less than w' will choose a strictly positive tax rate and no savings. The value of the optimal tax rate is given by the condition (35).

Case 3: $\lambda_\tau > 0, \lambda_s = 0$

The richer individuals (those with earnings above $\hat{w}$) will rely exclusively on savings which value is given by (36).

In order to determine the majority voting solution, we must know how the preferred tax rates vary with income for individuals below w'. We obtain from (35) that:

$$\frac{\partial \tau^A}{\partial w} = \frac{(1-\varepsilon)\left(\beta\alpha\left(1+n\right)u'\left(d\right) - u'\left(c\right)\right) - \alpha\left(1-\alpha\right)\beta\gamma\tau^2\left(1+n\right)^2\overline{w}u''\left(d\right)}{-D_\tau}. \tag{40}$$

For these people who do not save privately, we know from (36) that $u'(c) > \beta(1+r)u'(d)$ which implies that $u'(c) > \beta\alpha(1+n)u'(d)$. Hence, when $\varepsilon > 1$, this expression is positive, so that preferred tax rates are increasing with income.

We next verify that the problem under scrutiny if convex, so that the solutions just described constitute global optima. For this, we just need to show that the objective function $\phi(\tau, s) = u((1-\tau)w-s)+\beta u\left((1+r)s+(1+n)\tau\left(\alpha w+(1-\gamma\tau)(1-\alpha)\overline{w}\right)\right)$ is concave in its argument. The Hessian matrix is:

$$D^2\phi(\tau,s) = \begin{pmatrix} \partial^2\phi(\tau,s)/\partial\tau^2 & \partial^2\phi(\tau,s)/\partial\tau\partial s \\ \partial^2\phi(\tau,s)/\partial\tau\partial s & \partial^2\phi(\tau,s)/\partial s^2 \end{pmatrix}, \tag{41}$$

where

$$\begin{aligned} \partial^2\phi(\tau,s)/\partial\tau^2 &= w^2u''(c) + \beta\left((1+n)\left(\alpha w+(1-2\gamma\tau)(1-\alpha)\overline{w}\right)\right)^2u''(d) \\ &\quad -2\beta\gamma(1+n)(1-\alpha)\overline{w}u'(d) < 0, \end{aligned} \tag{42}$$

$$\partial^2 \phi(\tau, s) / \partial s^2 = u''(c) + \beta (1+r)^2 u''(d) < 0 \tag{43}$$

and

$$\partial^2 \phi(\tau, s) / \partial \tau \partial s = w u''(c) + \beta (1+n)(1+r)(\alpha w + (1-2\gamma\tau)(1-\alpha)\overline{w}) u''(d). \tag{44}$$

One easily verifies that

$$\begin{aligned} \det\left(D^2 \phi(\tau, s)\right) &= \beta u''(c) u''(d) \left(w(1+r) - (1+n)(\alpha w + (1-2\gamma\tau)(1-\alpha)\overline{w})\right)^2 \\ &\quad -2\beta\gamma(1+n)(1-\alpha)\overline{w} u'(d)\left(u''(c) + \beta(1+r)^2 u''(d)\right) \\ &\geq 0. \end{aligned} \tag{45}$$

Therefore the Hessian matrix is negative semidefinite and the objective function is concave.

Finally we show that the maximal preferred tax rate of the workers is lower than the minimal preferred tax rate of the retirees. The maximal preferred tax rate of the workers is:

$$\tau^A\left(w'\right) = \frac{(1+n)\left(\alpha w' + (1-\alpha)\overline{w}\right) - w'(1+r)}{2\gamma(1-\alpha)\overline{w}(1+n)}, \tag{46}$$

and, assuming an interior solution, the minimal preferred tax rate of the retirees is:

$$\tau^R(w_-) = \frac{1}{2\gamma}\left(1 + \frac{\alpha}{1-\alpha}\frac{w_-}{\bar{w}}\right). \tag{47}$$

Therefore, the condition for the maximal preferred tax rate of the workers being higher than the minimal preferred tax rate of the retirees is $\tau^A\left(w'\right) > \tau^R(w_-) \Leftrightarrow w'(\alpha(1+n) - (1+r)) - \alpha w_-(1+n) > 0$. Knowing that $\alpha(1+n) < 1+r$, this is impossible. Hence the result.