MARKET INCOMPLETENESS IN REGIONAL ELECTRICITY TRANSMISSION PART II: THE FORWARD AND REAL TIME MARKETS

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Abstract

The paper extends our previous analysis of market incompleteness to the case where there exist both a forward and real time market. The discussion is conducted with reference to a two-stage framework commonly used in stochastic programming and finance, but also familiar to the power industry. The paper considers two market designs. The first organization assumes that physical transmission services are auctioned in the forward market. The second organization supposes a financial market of transmission services. We recover the known perfect hedging property of nodal pricing and show that the flowgate model does not share this property. A standard proposition in finance is that market completeness requires that the number of tradable instruments is equal to or exceeds the number of risk factors. We show that none of the nodal or flowgate models satisfy this property. Other instruments, of the option type, should thus be introduced in these systems in order to better trade risk and complete the market.

1 Introduction

Part I (Smeers (2003)) analyzed the incompleteness of the energy and transmission markets for different organizations of transmission services. Specifically we considered the case of the nodal and flowgate models and introduced the notion of transmission capacity sometimes advocated as a central element of the organization of cross border trade. We analyzed the conditions that make transmission capacity a suitable notion for completing the energy and transmission markets. These conditions are far from innocuous: they indeed require that the ISOs actively assess and update these transmission capacities using nodal prices. In that sense, our notion of transmission capacity is more a repackaging of nodal prices than a substitute for them. The analysis presented in Smeers (2003) is constructed by reference to the forward market. The need for a real time market is more and more often acknowledged. The existence of both a forward and real time market is indeed both part of many restructured systems and an essential ingredient of the nodal and flowgate proposals. This Part II of the paper takes up this enlarged view and extends the analysis of Smeers (2003) to the case where there are both a forward and real time market. We therefore assume that energy and transmission transactions are concluded in the forward market and (at least partially) settled in the real time market. The introduction of a real time market explicitly calls for a representation of uncertainty and, by way of consequence, requires a more involved treatment of market completeness (financial completeness). We introduce uncertainty in Section 2 through a two-stage framework. The modeling of the forward and spot market in the two-stage context is given in Section 3. Section 4 discusses two market designs. In the first one, transmission capacities are initially auctioned in the forward market. We show that physical market completeness, as defined in Smeers (2003), can be achieved both in the nodal and flowgate models, provided the latter allows for aggregate flowgates. We next consider a financial market and show that the nodal system still achieves physical completeness but that the flowgate model requires stronger assumptions to arrive at the same result. We further elaborate on these issues in Section 5 by indicating that none of these approaches achieves market completeness in the sense commonly adopted in finance. Further developments of financial instruments are thus required if financial completeness is to be pursued. The main findings of this second part of the paper are summarized in the conclusion where we also briefly discuss the computational aspects of the models presented in the text. In order to clarify the presentation, problem, equation and figure numberings follow those of Smeers (2003).

2 A two stage framework

The discussion presented in Smeers (2003) is entirely conducted with respect to forward energy and transmission markets. Specifically we only considered completeness of the forward market assuming that all transactions concluded in that market would be settled as such, without deviation between forward and real time. The ever changing nature of electricity makes this assumption quite unrealistic. Deviations in real time from transactions concluded in the forward market are unavoidable. Demand will not be as expected and generators and lines may default. This implies that ISOs will have to reconfigure their system and/or call upon optional resources that they have reserved beforehand. Because of these uncertainties, both the nodal and flowgate models foresee that transmission services be eventually settled in the real time or spot market. We disregard here any distinction between real time, spot and ex post computed prices and assimilate these three notions. Taking stock of these two markets, we accordingly suppose that energy and transmission transactions are concluded in the forward market and settled in the spot market in both the nodal and flowgate models. Our task in this paper is to extend the dual ISO/marketers view of the network presented in Smeers (2003) to the existence of these two markets. We do this by considering the two-stage framework depicted in Figure 2.

In this framework, energy and transmission contracts are concluded in the first stage (stage 0) which represents the forward market. They are then settled in each state of the world $c \in C$ of the second stage (stage 1) after information on contingencies has been revealed. This represents the spot market. This two-stage framework can be traced to both the fields of stochastic programming in mathematical programming (e.g. Birge and Louveaux (1997)) and to introductions to finance theory (e.g. chapter 1 of Duffie (1996)). This section combines the two lines of reasoning. Note that the two-stage framework also has a long tradition in power engineering (see for instance, Bright et al. (1986) for a two-stage model for reactive security and Shirmohammadi et al. (1998) for an overall discussion of security problems in restructured systems).

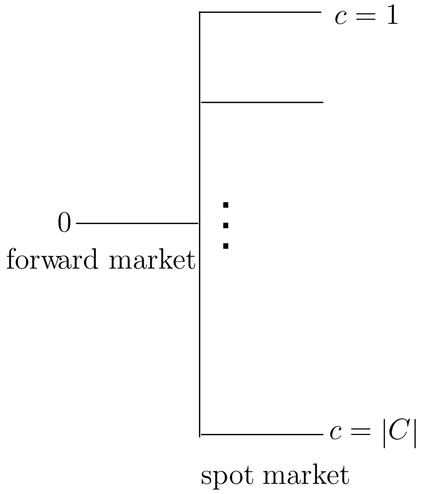

Figure 1: The two stage framework

The standard interpretation of the two stage model in finance is that investors assemble a portfolio of tradable assets in stage 0 while the payoffs from this portfolio accrue in stage 1. In the particular context of transmission across control areas, marketers acquire physical or financial transmission rights in the forward market. Speculators only acquire financial rights. In real time, ISOs take over all transmission resources and operate the system so as to meet the constraints of the network. In order to do so, they may exert physical options procured before. As a by-product of their action, they also compute ex post electricity spot prices that are used to settle transmission services and deviations from energy transactions. Before describing this organization we first recall some notions from Smeers (2003).

The discussion is conducted on a six node network (Chao and Peck (1998)) that can be seen as a two control area system (Figure 3). Zones I and II respectively consist of nodes 1,2,3 and nodes 4,5,6. Lines (1–6) and (2–5) are the sole bottlenecks of the grid as the other lines have infinite capacities. There is one marketer in each zone, purchasing and selling in both zones.

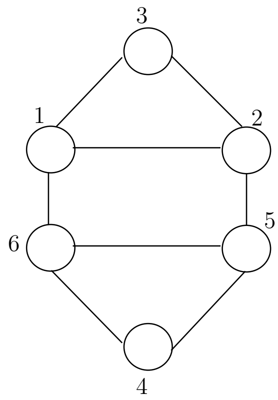

Figure 2:

The vectors of bilateral transactions of these marketers are noted e_I and e_{II}. We abuse notation by writing e for both (e_I, e_{II}) and $e_I + e_{II}$, depending on the context, to refer to the vector of all transactions. We suppose in this paper that all bilateral transactions are concluded in the forward market. The nodal injection/withdrawal of ISO's are noted i_I, i_{II}. We again abuse notation and write i for either (i_I, i_{II}) or $i_I + i_{II}$, depending on the context when there is no risk of ambiguity. ISO transactions take place in real time and are thus contingent on the state of the world. They are upper indexed by c and noted i^c, $c \in C$. Consumers and generators are located at the nodes of the network. They are entirely characterized by their supply and demand curves at the node. Note that we do not associate probabilities to the states of the world.

Discussions relative to the organization of transmission operations are often conducted using power distribution factors (PDF). These are constructed from the DC load flow approximation of the active and reactive load flow equations.

Let Γ be the matrix of power distribution factors. Its elements obey the following properties:

(i) A unit injection at some node k together with the corresponding withdrawal at some arbitrarily selected swing node results in a flow Γ_{mk} on the line m.

(ii) Flows are additive: the combination of injections i_k and i_ℓ from nodes k and ℓ respectively with corresponding withdrawals at the swing node results in a flow $\Gamma_{mk} i_k + \Gamma_{m\ell} i_\ell$ on line m.

Taking the notation on board the extension of the model of Smeers (2003) to the case of uncertainty can now be described as follows. Assume that the behavior of a marketer τ is driven by a utility function $U_\tau(x_\tau^0, (x_\tau^c, c \in C))$ where x_τ^c is the payoff collected by the marketer τ in state of the world c and x_τ^0 is the payoff collected at stage 0. U is not necessarily an expected utility function (recall that there is no probability in the statement of the problem); it suffices that it includes the argument x_τ^c for each state of the world c. We make the standard assumption that U is monotone and strictly concave with respect to $(x_\tau^0, (x_\tau^c, c \in C))$. Let α_τ^c be the marginal utility of marketer τ in state of the world c (by assumption $\alpha_\tau^c > 0$). We assume without loss of generality and in order to simplify the discussion that the marginal utility α_τ^0 of marketer τ in stage 0 is equal to one. Because energy and transmission prices are entirely determined by the e and i^c in this model, α_τ^c can always be expressed as a function of these variables only. We assumed in Smeers (2003) that there is no asymmetry of information or market power (Assumption 1). We also assumed that the mappings L_I, L_{II}, O_I, O_{II} that represent the vectors of marginal opportunity costs suffered by marketers I and II and Systems Operators I and II are monotone (Assumption 2).

Extending the use of the mappings L_I, L_{II}, O_I and O_{II}, we now let $L_\tau^c(e, i^c)$ be the marginal opportunity cost for marketer τ of settling energy transaction e in state of the world c in stage 1. $-L_\tau^c(e, i^c)$ is thus a vector of marginal profits of the same dimension as e. Note that $L_\tau^c(e, i^c)$ may be independent of c in case the energy transaction is perfectly hedged. We will similarly use $O_\tau^c(e, i^c)$

for the marginal opportunity cost for ISO τ of settling transactions i^c in state of the world c of stage 1, given the energy trade e. $-O^c_\tau(e, i^c)$ is thus a vector of marginal profits of the same dimension as i^c. Were it not for transmission costs the behavior of marketer τ would be driven by her marginal disutility with respect to each energy transaction. This marginal disutility is obtained from a combination of the marginal utilities α^c_τ in each state of the world c and of the marginal costs L^c_τ resulting from the transactions. Combining these effects over all states of the world c one obtains the mapping of marginal disutilities of marketer τ

$$\sum_{c \in C} \alpha^c_\tau(e, i^c) L^{cT}_\tau(e, i^c) \tag{1}$$

Note that one cannot infer any property of α^c_τ from those of U. Even continuity cannot be guaranteed as prices and quantities may be discontinuous functions of the exogenous contingency parameters. Transmission is priced on the spot market: let λ^{c*} be the vector of spot values of lines used in state c. The marginal disutility of trader τ after paying for both energy and transmission is equal to

$$\sum_{c \in C} \alpha^c_\tau(e, i^c)(L^{cT}_\tau(e, i^c) + \lambda^{cT} \Gamma^c S) \tag{2}$$

Given this background we now successively model the spot and forward markets.

3 The models of the two stages

3.1 The spot market (the second stage)

Consider the following set of problems (one problem for each $c \in C$, the upper index "1" refers to the stage 1).

Problem VIII^{1,c}

Given e^*, seek i^{c*} s.t. $\Gamma^c(Se^* + i^{c*}) \leq \overline{f}^c$ satisfying

$$O^c_I(e^*, i^{c*})(i^c_I - i^{c*}_I) + O^c_{II}(e^*, i^{c*})(i^c_{II} - i^{c*}_{II}) \geq 0 \tag{3}$$

for all i^c s.t. $\Gamma^c(Se^* + i^c) \leq \overline{f}^c$.

As already argued several times before, Assumption 2 guarantees that each Problem $VIII^{1,c}$ has a solution i^{c*}. Moreover, for each such solution i^{c*}, there exists at least one λ^{c*} that satisfies

$$\lambda^{c*} \geq 0, \qquad \lambda^{c*T}[\overline{f}^c - \Gamma^c(Se^* + i^{c*})] = 0. \tag{4}$$

Problem $VIII^{1,c}$ assumes that the injections/withdrawals i^{c*} of the ISOs clear the market in real time. This assumption is far from trivial but it is directly inspired by current policy debates. This assumption indeed underlies the PJM (Pensylvania Maryland New Jersey) system (see http://www.pjm.com) which was recently mandated by FERC to constitute the basis of the North Eastern RTO (see FERC (http://www.ferc.fed.us) for a discussion of the standard market design issue). Market clearing in the second stage is only possible if there exists a balancing market across the different control areas. Recall that a balancing mechanism is physically required in real time in electricity as generation always needs to equate consumption plus losses. There is a balancing market when the mechanism simulates a market. The balancing market clears if generation and consumption are respectively equal to supply and demand in real time. In short we assume that such a balancing market exists and an equilibrium is reached on it in real time. Procedures for attaining this equilibrium across different systems in interaction are discussed in Cadwalader et al. (1999). An alternative is to assume that an optimization algorithm is used to globally seek the equilibrium on the balancing market. Proposition 8 links the former equilibrium interpretation to the second, centralized view of the balancing market and possibly to the more ad hoc computation of ex post prices that is effectively implemented as a substitute for the real time market. Recall that we do not differentiate between spot and balancing markets.

In order to state Proposition 8, we first introduce the notion of re-dispatching costs. Define $O^{cT}(e^*, i^c) = (O_I^{cT}(e^*, i_I^c, i_{II}^c), O_{II}^{cT}(e^*, i_I^c, i_{II}^c))$. By construction each component of $O^c(e^*, i^c)$ is either a nodal supply or demand curve. Under common assumption, $O^c(e^*, i^c)$ is thus (path) integrable in the sense of Harker and Pang (1990) and its integral is the sum of the generation costs at generations nodes minus the willingness to pay at consumption node. Let $DC_I^c(e^*, i_I^c, i_{II}^c)$ and $DC_{II}^c(e^*, i_I^c, i_{II}^c)$ (Dispatching Cost) be these integrals of $OC_I^c(e^*, i_I^c, i_{II}^c)$ and $OC^c(e^*, i_I^c, i_{II}^c)$ respectively. $DC_I^c(e^*, i_I^c, \tilde{\imath}_{II}^c)$ is the redispatching incurred by ISO I for given bilateral transactions e^* when ISO II undertakes actions $\tilde{\imath}_{II}^c$. We define the joint optimization of the dispatch by all ISOs as the problem

Seek $(i_I^{c*T}, i_{II}^{c*T})$ that solves

$$\begin{aligned} \min DC_I^c(e^*, i_I^c, i_{II}^c) + DC_{II}^c(e^*, i_I^c, i_{II}^c) \\ \text{s.t. } \Gamma^c(Se^* + i_I^c + i_{II}^c) \leq \overline{f}^c. \end{aligned} \tag{5}$$

We can then state the following result

Proposition 1 *Let i^{c*} be the solution to Problem $VIII^{1,c}$ in state of the world c. i^{c*} is also the solution of the joint optimization of the dispatch (Problem (31)) by all ISOs.*

The proof follows trivially from the integrability of $O_\tau^{cT}(e^*, i_I^c, i_{II}^c)$. ■

This well-known result, here applied to the sole spot market, says that this joint optimization simulates a balancing market in equilibrium. It also justifies a joint optimization of the system by all ISOs in order to both efficiently manage differences between the forward and real time market and produce spot prices that can be used to settle those differences. One may wonder about the outcome of the spot market when neither the joint equilibrium nor the centralized optimization can be implemented. This is the case where each ISO tries to resolve the congestion problem through his own means. We explore this problem in the six nodes/two zone example. We shall show in Proposition 9 that refraining from global optimization by the ISOs amounts in this example to resorting to a pair of single ISO actions that does not lead to an equilibrium on the balancing market and does not remove congestion at minimal cost. This result can be extended to more general situations than the two control areas allowed by the six node example and can be adapted to a multi

(> 2) control area context where single, bilateral or multilateral actions by ISOs take place. We briefly comment on this extension after the presentation of Proposition 9. Consider the following set of quasi variational inequality problems (Harker (1991) and Smeers (2003)).

Problem IX^{1,c}
Seek i^{c*} s.t. $\Gamma^c(Se^* + i^{c*}) \leq \overline{f}^c$ satisfying

$$\begin{array}{ll} & O_I^{cT}(e^*, i^{c*})(i_I^c - i_I^{c*}) + O_{II}^{cT}(e^*, i^{c*})(i_{II}^c - i_{II}^{c*}) \geq 0 \\ \text{for all } i_I^c, i_{II}^c \text{ s.t.} & \\ & \Gamma^c(Se^* + i_I^{c*} + i_{II}^c) \leq \overline{f}^c \\ & \Gamma^c(Se^* + i_I^c + i_{II}^{c*}) \leq \overline{f}^c. \end{array} \tag{6}$$

Reproducing the reasoning of Section 3.1 of Smeers (2003) one can show that there exists a solution to this problem. Moreover to each such solution one can associate multipliers λ_I^{c*}, $\lambda_{II}^{c*} \geq 0$ such that

$$\begin{array}{l} \lambda_I^{c*T}[\overline{f}^c - \Gamma^c(Se^* + i_I^{c*} + i_{II}^{c*})] = 0 \\ \lambda_{II}^{c*T}[\overline{f}^c - \Gamma^c(Se^* + i^{c*} + i_{II}^{c*})] = 0. \end{array} \tag{7}$$

Our objective is to show that this set of quasi variational problems describes a situation where each ISO individually tries to manage the deviations between the forward and real time market. In order to see this consider the pair of optimization problems where each ISO tries to relieve congestion at his own minimal dispatching cost, given the actions of the other ISO. Using the same reasoning as in the proof of Proposition 8, one can show that each of these problems is equivalent to a single dispatch optimization problem of the form

Problem CT_I ***(Counter trading*** I***)***

Seek i_I^{c*} that solves

$$\begin{array}{ll} & \min DC_I^c(e^*, i_I^c, \tilde{\imath}_{II}^c) \\ \text{s.t.} & \Gamma^c(Se^* + i_I^c + \tilde{\imath}_{II}^c) \leq \overline{f}^c \\ \text{for given} & \tilde{\imath}_{II}^c \text{ of } ISO\ II \end{array} \tag{8}$$

for ISO I; a similar problem being stated for ISO II.

The interpretation of the quasi variational problems $IX^{1,c}$ is then given in the following proposition.

Proposition 2 *Let i^{c*}, $c \in C$ be a solution to Problem $IX^{1,c}$. It is also a solution to a pair of optimization problems CT_I and CT_{II} where each ISO tries to relieve congestion at his own minimal dispatching cost, given the actions of the other ISO. This solution does not correspond to a joint optimization of the dispatch or to an equilibrium of the spot market. The solution is not unique.*

Proof: Problem $IX^{1,c}$ is equivalent to the pair of problems.

Seek i_τ^{c*} s.t. $\Gamma^c(Se^* + i_\tau^{c*} + i_{\tau' \neq \tau}^{c*}) \leq \overline{f}^c$ satisfying

$$O_\tau^{cT}(e^*, i_\tau^{c*})(i_\tau^c - i_\tau^{c*}) \geq 0$$

for all i_τ^c s.t. $\Gamma^c(Se^* + i_\tau^c + i_{\tau' \neq \tau}^{c*}) \leq \overline{f}^c$, $\tau = I, II$.

We stated, on the basis of the same reasoning as in the proof of Proposition 8, that each of these problems is equivalent to a single ISO dispatch optimization problems CT_τ, $\tau = I, II$ as stated in (34). Moreover one needs $\tilde{e}_\tau = e_\tau^*, \tau = I, II$. The combination of the two problems (34) for $\tau = I$ and II is obviously not equivalent to problem (31) where both ISOs jointly minimize costs to relieve congestion. The multiplicity of solutions in this combination of problems derives from the same application of the results of Harker (1991) as the proof of Proposition 1 of Smeers (2003).

Consider now the extension of this proposition to a multi (> 2) control area where single, bilateral or multilateral actions by ISOs take place. A particular relevant example is the one of a bilateral action by two ISOs controlling the two sides of a congestion in a multi (> 2) control area system. What this extended proposition would say is that anything that is not a joint optimization will not be an equilibrium of the spot market. Moreover, a solution to a set of optimization problems (34) of the type is not in general an equilibrium of the spot market.

The multiplicity of the solution to Problem $IX^{1,c}$ and its associated non-optimality may be claimed to be irrelevant in practice. It is indeed often argued that the optimization of the dispatch involved in nodal prices systems only results in small costs and hence is not important. This view is misleading: the cost of an optimally operated system may be small, while the cost of not operating it optimally may be quite high. In this respect, the objective of using nodal prices is not to recover small congestion costs but to send the right incentives to operate the system in such a way that it only incurs small congestion costs. This remark is especially important because the non-optimal operation of the balancing mechanism is sometimes claimed to be associated with perverse incentives to game the system. Besides this non-optimality, a key issue for the design of the market is that the separate optimization approach does not lead to a single set of spot prices and hence makes it impossible to settle transmission services through spot prices. Specifically, for each particular solution to Problem $IX^{1,c}$, each ISO (or group of ISOs in a more general context) will in general produce different spot prices. This indicates that the market is physically incomplete whenever ISOs only engage in non-global coordination actions. The only possible outcome in reaction to this absence of spot prices is to socialize costs or to allocate them to agents that are arbitrarily defined as responsible for them. These approaches are bound to result in economically unjustified allocation of re-dispatching costs and possibly in perverse incentives to game the system.

Needless to say one cannot achieve an equilibrium in the spot market when this spot market does not exist. It is remarkable that the existence of a balancing and hence of a spot market is not mentioned in ETSO proposals (the European Electric Industry pointed to this very serious flaw). In the absence of this spot or balancing market, ETSO can only recommend cooperation between ISOs to relieve congestion. Full cooperation is a possibility but ETSO also accepts separate joined optimization by subgroups of ISOs. This combines all defects: there is no spot price to settle transmission transactions, and no guarantee of redispatch optimality. This will result in high costs and perverse incentives.

3.2 The forward market (the first stage)

Using the marginal disutilities defined in relation (1) of Section 2, we assume that marketers select their portfolio of transmission rights in the forward market so as to satisfy the following variational inequality:

Given expectations on the λ^{c*}, $c \in C$, marketer τ seeks $e^*_\tau \geq 0$ s.t.

$$\left\{ \sum_c \alpha^c_\tau [L^{cT}_\tau(e^*, i^{c*}) + \lambda^{c*T}\Gamma^c S] \right\} (e_\tau - e^*_\tau) \geq 0 \text{ for all } e_\tau \geq 0. \tag{9}$$

These conditions can easily be rewritten as

$$\left\{ \sum_c [\alpha^c_\tau L^{cT}_\tau(e^*, i^{c*}) + \lambda'^{c*T}_\tau \Gamma^c S] \right\} (e_\tau - e^*_\tau) \geq 0, \text{ for all } e_\tau \geq 0,$$

where $\lambda'^{c*}_\tau = \alpha^c_\tau \lambda^{c*}$ and hence

$$\lambda'^{c*}_\tau \geq 0, \qquad \overline{f}^c - \Gamma^c(Se^* + i^{c*}) \geq 0, \qquad \lambda'^{c*T}_\tau[\overline{f}^c - \Gamma^c(Se^* + i^{c*})] = 0. \tag{10}$$

Except for the case where $\alpha^c_I = \alpha^c_{II}$, $\sum_c \lambda'^{c*T}_I \Gamma^c$ is not equal to $\sum_c \lambda'^{c*T}_{II} \Gamma^c$. This means that the willingness to trade transmission rights is not equal among traders. The market is therefore physically incomplete. Something additional is needed to complete it. We consider the two solutions most commonly proposed in the literature namely to auction physical transmission capacities in the forward market (physical rights) or to introduce a financial market for transmission rights.

4 Completing the market

4.1 Auction physical transmission capacities

The auctioning of transmission capacities, whether defined in nodal or flowgate terms, is either proposed or already in existence in most restructured systems. In the US, PJM (http://www.pjm.com) and NY ISO (New York ISO; http://www.nyiso.conf) auction financial transmission rights. Various auctions of transmission capacities also exist in Europe (e.g. Dutch/German and Belgian borders, Italian borders). As before, we assume that ISOs assemble the transmission capacities or extended flowgates and auction them. As argued before, this assumption is compatible with the flowgate proposal (Chao and Peck (1996), Chao et al (2000)) provided one accepts that flowgates do not necessarily correspond to fixed physical capacities and are in fact extended flowgates. We model the situation through the following variational principle that we interpret later.

4.1.1 Mathematical formulation

Problem X

Seek $(e^*, (i^{c*}, c \in C))$, $e^* \geq 0$ s.t. $\Gamma^c(Se^* + i^{c*}) \leq \overline{f}^c$ satisfying

$$\left[\sum_c \alpha_I^c L_I^{cT}(e^*, i^{c*})\right](e_I - e_I^*) + \left[\sum_c \alpha_{II}^c L_{II}^{cT}(e^*, i^{c*})\right](e_{II} - e_{II}^*) \\ + \sum_c \left[O_I^{cT}(e^*, i^{c*})(i_I^c - i_I^{c*}) + O_{II}^{cT}(e^*, i^{c*})(i_{II}^c - i_{II}^{c*})\right] \geq 0 \tag{11}$$

for all $(e, (i^c, c \in C)), e \geq 0$ s.t. $\Gamma^c(Se + i^c) \leq \overline{f}^c,\ c \in C$.

Assumption 2 of Smeers (2003) and standard conditions on the utility functions do not suffice to prove the continuity of the mapping $\sum_c \alpha_\tau^c L_\tau^{cT}(e, i)$. As a result we cannot a priori guarantee the existence of a solution (e^*, $(i^{c*}, c \in C)$) to Problem X. We therefore simply assume that such a solution exists. To each such solution, one can associate at least one vector of $\lambda^{c*} \geq 0, c \in C$ such that

$$\left[\sum_c \alpha_\tau^c L_\tau^{cT}(e^*, i^{*c}) + \sum_c \lambda^{c*T}\Gamma^c S\right](e_\tau - e_\tau^*) \geq 0, \text{ for all } e_\tau \geq 0,\ \tau = I, II \\ \left[O_\tau^{cT}(e^*, i_\tau^{c*}) + \lambda^{c*T}\Gamma^c\right](i_\tau^c - i_\tau^{c*}) \geq 0 \text{ for all } i_\tau^c,\ c \in C,\ \tau = I, II \\ \overline{f}^c - \Gamma^c(Se^* + i^{c*}) \geq 0, \quad \lambda^{c*T}\left[\overline{f}^c - \Gamma^c(Se^* + i^{c*})\right] = 0, \tag{12}$$

The first relation of (12) can be interpreted as follows. Marketers pay $\sum_c \lambda^{c*T}\Gamma^* Se^*$ at stage 0 for transmission services. This payment is accounted for with a unit marginal utility (recall that we assumed $\alpha_\tau^0 = 1$). Marketers then incur an additional payment $L_\tau^{cT}(e^*, i^{c*})e^*$ when energy transactions are settled. $\sum_c \lambda^{c*T}\Gamma^c S$ can thus be interpreted as the vector of present transmission prices (at stage 0) on the forward market. Because both marketers see the same price of transmission capacities in this forward market, the market is physically complete.

The role of the ISOs in the forward market can be described on the basis of the second and third relations of (12). The i^{c*} component of the solution $(e^*, (i^{c*}, c \in C))$ to Problem X satisfies

$$\sum_c \sum_\tau \left[O_\tau^{cT}(e^*, i_\tau^*) + \lambda^{c*T}\Gamma^c \right] (i_\tau^c - i_\tau^{c*}) \geq 0, \text{ for all } i_\tau^c. \tag{13}$$

Noting from the third relation of (12) that

$$\lambda^{c*T}\Gamma^c i_\tau^{c*} = \lambda^{c*T}\overline{f}^c - \lambda^{c*T}\Gamma^c S e^*, c \in C$$

and from the feasibility condition of (e, i^c) (e.g. last condition of (12)) that

$$\lambda^{c*T}\Gamma^c i_\tau^c \leq \lambda^{c*T}\overline{f}^c - \lambda^{c*T}\Gamma^c S e, c \in C$$

we obtain the following variational inequality

$$\begin{aligned}\textstyle\sum_c \sum_\tau O_\tau^{cT}(e^*, i^{c*})(i_\tau^c - i_\tau^{c*}) - (\sum_c \lambda^{c*T}\Gamma^c S)(e - e^*) \geq 0, \text{ for all } e \geq 0,\\ \text{for all } i_\tau^c, c \in C.\end{aligned} \tag{14}$$

This says that the ISOs maximize the profit accruing from selling transmission rights e^* and buying the energy services i^{c*} in each state of the world c in order to assemble these rights.

4.1.2 Discussion

It remains to cast the auctioned transmission services both in nodal and flowgate terms. The solution to Problem X is directly interpretable in nodal point to point services. ISOs sell bilateral transmission services e^* that they charge at nodal forward price differences ($\sum_c \lambda^{c*T}\Gamma^c S$). For this they buy nodal injection/withdrawal services i^{c*} that they pay at nodal spot prices. Consider now the flowgate version of the model. One can think of at least two interpretations. One is to auction lines capacities contingent on the state of the world. This would result in contingent prices λ^{c*}. As argued by both the proponents and opponents of the flowgate model, this is too complex. The second interpretation is to auction a non-physical flowgate constructed through an aggregation process. This is expressed by relation (14) in which ISOs "assemble" a non-contingent extended flowgate in amount $(\sum_c \lambda^{c*T}\Gamma^c S)e^*$ by buying contingent line and generation services (these latter are not represented in this model). They then auction the extended flowgate capacity to marketers. This interpretation complies with the dual ISO/marketer view of the network: (i) marketers see aggregate flowgates (e.g. one flowgate of capacity $\sum_c \lambda^{c*T}$ $\Gamma^c Se^*$ of unit price that they request according to aggregate PDFs $\sum_c \lambda^{c*T}\Gamma^c$) (ii) system operators see line capacities in different contingencies and take care of these contingencies to produce a firm (physical) aggregate flowgate. As before (see Smeers (2003)), we adopt the extreme view that one only needs a single aggregate flowgate. This is notationally convenient but purely conceptual. The practical issue is whether a small number of firm aggregate flowgates or transmission capacities can be reasonably defined in advance and remain sufficiently stable.

The nodal and flowgate interpretations both assume that one auctions transmission capacities that remain feasible for all contingencies envisaged by the ISOs. This is akin to the arguments in Hogan (2000b) and Ruff (2000) that nodal prices are determined in a security-constrained dispatch (Section 5 in Smeers (2003)) and that flowgates should also be defined to account for these security constraints. This is also related to the issue of revenue adequacy that is extensively discussed in Hogan (2000b). Note that the constraints of (11) guarantee that any transmission service procured in the forward market remains feasible in the spot market. This makes transmission rights firm. It also guarantees revenue adequacy: the congestion charges collected in the spot market are higher than the payment at spot prices of any other bundle of feasible transmission contracts inherited from the forward market. In contrast, a revenue deficiency may arise when the allocation of node to node services inherited from the forward auction is not feasible in real time. This deficiency does not arise in this model because all real time constraints are anticipated by the ISOs in the forward market. The solution to Problem X therefore allows one to generalize the revenue adequacy property of Hogan (1992). But this generalization only holds because one assumes that the ISOs envisage the whole set of possible contingencies in the forward market. This assumption is standard in finance models where one supposes that all agents foresee the same set of states of the world, possibly with different estimates of their likelihood. The assumption may be heroic here. If it is not satisfied, the ISOs may find themselves managing real time contingencies different from those envisaged in the forward market. Revenue adequacy may also fail to be satisfied (see Hogan (2000b)).

As a last remark recall that the above interpretation allows for physical contracts, whether for the nodal or flowgate model. The bilateral contracts e^* negotiated by the marketers, for which they have obtained transmission rights in the auction can indeed be completed in real time. We now turn to another version of the model where financial products are introduced as an alternative tool for managing transmission services.

4.2 Introducing a financial market

The nodal and flowgate models expose marketers to the same transmission risk, namely contingent differences of nodal prices. An extensively discussed issue is whether both systems can offer the same hedges against this risk. Hedging instruments take the form of financial derivatives (forward, options of various types). We hereby introduce forward nodal and flowgate contracts but leave out options contracts for the sake of simplicity.

4.2.1 Mathematical formulation

In order to define nodal and flowgate contracts, consider an arbitrary state of the network represented by the matrix of power distribution factors Γ^0. For any non-negative vector λ^0 of line values, $\lambda^{0T}\Gamma^0$ is a vector of nodal prices. To each node we associate a contract that gives as payoff in state of the world c

the component of $\lambda^{c*T}\Gamma^c$ associated with that node. This contract requires an upfront payment equal to the corresponding component of $\lambda^{0T}\Gamma^0$ in stage 0. A portfolio of these contracts is noted by θ. As is common in finance models we also assume a risk free asset. In order to simplify the presentation, and because of the short horizon covered by this two stage model, we suppose a zero interest rate. The risk free asset thus has a unit value both in stage 0 and in all states of the world c at stage 1. Because there is a financial market, we permit speculators who are not involved in energy or transmission services to also trade these nodal contracts.

Financial contracts introduce a new set of goods in the economy which now trades energy, transmission services and financial transmission contracts. This requires an adaptation of our representation of marketers. These were modeled by a utility function $U_\tau(x_\tau^0, (x_\tau^c, c \in C))$ where x_τ^c is the payoff from the settlement of energy and transmission transactions in state c. They will now be represented by the utility function $U_\tau((x_\tau^0 + y_\tau^0), ((x_\tau^c + y_\tau^c), c \in C)))$ where y_τ^0 and y_τ^c are the revenue accruing from financial contracts. The marginal utilities are still noted α_τ^c but now depend on e, i and y that is $\alpha_\tau^c(e, i, y_\tau^0, (y_\tau^c, c \in C))$. Speculators are also represented by a utility function $V(z^0, (z^c, c \in C))$ where z^0 and z^c respectively denote the revenue accruing to speculators from their trading of financial transmission contracts. We note δ the portfolio of the contracts held by a speculator and $\beta^c(e, i, z^0, (z^c, c \in C))$ his marginal utility in state of world c ($\beta^0(e, i, z^0, (z^c, c \in C)) = 1$). For the sake of simplicity in the presentation, we only assume a single speculator. In this extended set up, a marketer's position in energy and transmission contracts is a solution to the following variational inequality problem:

Problem XI

Seek $e_\tau^* \geq 0$, θ^* satisfying

$$\begin{aligned}&\textstyle\sum_c \alpha_\tau^c \left[L_\tau^{cT}(e^*, i^{c*}) + \lambda^{c*T}\Gamma^c S\right](e_\tau - e_\tau^*)\\ &\quad + \left(\lambda^{0*T}\Gamma^0 - \textstyle\sum_c \alpha_\tau^c \lambda^{c*T}\Gamma^c\right)(\theta_\tau - \theta_\tau^*) \geq 0\\ &\quad \text{for all } e_\tau \geq 0, \theta_\tau, \tau = I, II.\end{aligned} \tag{15}$$

The speculator's position is a solution to

Problem XI'

Seek δ^* satisfying

$$(\lambda^{0*T}\Gamma^0 - \sum_c \beta^c \lambda^{c*T}\Gamma^c)(\delta - \delta^*) \geq 0 \text{ for all } \delta.$$

The term $(\lambda^{0*T}\Gamma^0 - \sum_c \alpha_\tau^c \lambda^{c*T}\Gamma^c)$ appearing in (15) is the contribution to the disutility of marketer τ accruing from her trading financial transmission contracts. It can be explained as follows: $\lambda^{0*T}\Gamma^0\theta_\tau^*$ is the upfront payment for the portfolio of contracts θ_τ^*, and is accounted for with unit marginal utility. $\lambda^{c*T}\Gamma^c\theta_\tau^*$ is the payment accruing from this portfolio of contracts in state of the world c. It is accounted for with marginal utility α_τ^c. Together, these additional terms express that marketer τ takes a position θ_τ^* in financial contracts such that any deviation θ_τ from θ_τ^* can only deteriorate her position. A similar interpretation holds for the speculator.

4.2.2 Physical completeness

Finance models commonly rely on a no arbitrage assumption. In this two stage model, the no arbitrage assumption states that it is impossible at equilibrium to find a portfolio of nodal contracts θ that brings a positive profit in certain states c of stage 1 without investing a positive amount in stage 0. In other words, there does not exist any portfolio θ such that $\lambda^{0*T}\Gamma^0\theta \leq 0$ and $\lambda^{c*T}\Gamma^c\theta \geq 0$ for all c with $\lambda^{c*T}\Gamma^c\theta > 0$ for at least one c. The no arbitrage assumption is at the origin of valuation methods based on so called "risk neutral probability". These notions are used in the following lemma.

Lemma 1 *Suppose a solution to Problem XI exists. Then it satisfies*

$$\lambda^{0*T}\Gamma^0 = \sum_c \alpha_\tau^c \lambda^{c*T}\Gamma^c; \lambda^{0*T}\Gamma^0 = \sum_c \beta^c \lambda^{c*T}\Gamma^c. \tag{16}$$

Moreover, the no arbitrage assumption holds and there exists at least one "risk neutral probability" π such that

$$\lambda^{0*T}\Gamma^0 = \sum_{c\in C} \pi^c(\lambda^{c*T}\Gamma^c); \pi^c > 0, \ c \in C; \ \sum_{c\in C} \pi^c = 1. \tag{17}$$

Proof: The first statement of the lemma is a direct consequence of condition (15). It says that marketer τ takes a position θ_τ^* in financial transmission contracts that results in initial and contingent payments $y_\tau^{0*} = \lambda^{0*T}\Gamma^0\theta_\tau^*$ and $y_\tau^{c*T} = \lambda^{c*T}\Gamma^c\theta_\tau^*$. These payments equalize the first and second stage marginal utilities of the marketer as there would be no solution to Problem XI if this equality were not satisfied. Similarly, there would be no solution to Problem XI if the no arbitrage assumption were violated. As a consequence the second statement of the lemma depends on the existence of a risk free asset and on the no arbitrage assumption. It is an application of the first fundamental theorem of finance theory (e.g. Chapter 1, Duffie (1996)). ■

Suppose that the θ_τ^* and δ^* exist and satisfy a balance condition $\theta_I^* + \theta_{II}^* + \delta^* = 0$ (there is equilibrium on the financial market). Using Lemma 1, one can restate the behavior of marketer τ as obeying the following variational principle.

Seek $e_\tau^* \geq 0$ satisfying

$$\sum_c \left[\alpha_\tau^c L_\tau^{cT}(e_\tau^*, i^{c*}) + \lambda^{0*T}\Gamma^0 S\right](e_\tau - e_\tau^*) \geq 0 \text{ for all } e_\tau \geq 0. \qquad (18)$$

The interpretation of relation (18) is as follows. Marketer τ pays an up-front cost $\lambda^{0*T}\Gamma^0 S e_\tau^*$ for transmission services. This is accounted for with a unit marginal utility. She also incurs the settlement of energy transactions $L_\tau^{cT}(e_\tau^*, i^{c*})e_\tau^*$ in state c of stage 1 . These are accounted with marginal utility α_τ^c. Marketers all pay the same price, $\lambda^{0*T}\Gamma^0 S$, for transmission services. These prices are established in the financial market in stage 0. The market is thus physically complete.

This formulation allows one to recover the perfect hedging property put forward in Hogan (2000b) and Ruff (2000). It says that marketers may, if they so wish, perfectly hedge transmission price risk by covering their energy position by nodal forward contracts. As argued in Hogan (2000b) and Ruff (2000) this property can accommodate changes of PDFs because the nodal contracts are entirely defined in terms of the nodal prices $\lambda^{0T}\Gamma^0$ and $\lambda^{cT}\Gamma^c$. Marketers do not have to bother about the network topology and the associated PDFs Γ^0 and Γ^c that prevail in the forward and real time markets.

Relation (18) only provides an intuitive interpretation of the impact of the financial market. A more rigorous formalization of the situation is given by the generalized complementarity Problem XII (see Harker and Pang (1990) for a definition of generalized complementarity problems) constructed after adding the additional constraints (16) and (17) to the complementary reformulation of Problem X. In order to correctly interpret Problem XII, recall that α^c_τ now depends on an additional vector θ_τ and that finding this vector is part of the statement of the problem.

Problem XII

Seek $(e^* \geq 0, (i^{c*}, c \in C))$; λ^{0*}; $(\lambda^{c*} \geq 0, c \in C); y^{c*}_\tau = \lambda^{c*T}\Gamma^c\theta^*_\tau, c \in C, \tau = I, II$; $\xi^{c*} \geq 1$, $c \in C$ s.t. $\Gamma^c(Se^* + i^{c*}) \leq \overline{f}^c$, $c \in C$, satisfying

$$\left[\sum_c \alpha^c_\tau \left(L^{cT}_\tau(e^*, i^{*c}) + \lambda^{c*T}\Gamma^c S\right)\right] \geq 0 \qquad \tau = I, II \tag{19}$$

$$\left[\sum_c \alpha^c_\tau \left(L^{cT}_\tau(e^*, i^{*c}) + \lambda^{c*T}\Gamma^c S\right)\right] e^*_\tau = 0 \qquad \tau = I, II \tag{20}$$

$$\left[O^{cT}_\tau(e^*, i^{*c}) + \lambda^{c*T}\Gamma^c\right] \geq 0, c \in C, \tau = I, II \tag{21}$$

$$\left[O^{cT}_\tau(e^*, i^{*c}) + \lambda^{c*T}\Gamma^c\right] i^{c*} = 0, c \in C, \tau = I, II \tag{22}$$

$$\lambda^{c*T}\left[\overline{f}^c - \Gamma^c(Se^* + i^*)\right] = 0, \quad c \in C \tag{23}$$

$$\begin{array}{l} \lambda^{0*T}\Gamma^0 = \sum_c \alpha^c_\tau \lambda^{c*T}\Gamma^c, \tau = I, II \\ \left(\sum_{c\in C} \xi^{c*}\right)\lambda^{0*T}\Gamma^0 = \sum_{c\in C} \xi^{c*}\lambda^{c*T}\Gamma^c, \end{array} \tag{24}$$

The constraints (24) added to Problem X force Problem XII to satisfy the conditions (16) and (17) implied by the existence of an equilibrium on the financial transmission market. But they do not help proving that there exists one. Recall that we simply assumed but did not prove in the above that there exists an equilibrium on the financial market. This assumption is standard in finance theory, but probably less satisfactory here where we are interested in a joint equilibrium in both the physical and financial markets. Note, for the rest of this paper that we limit ourselves to that assumption and do not try to prove that Problem XII has a solution. Under this assumption we have the following proposition.

Proposition 3 *Let $(e^*, (i^{c*}, c \in C))$ be part of a solution to Problem XII (assume there exists one). It is also a solution to Problem X after an initial endowment y_τ^{c*}, $c \in C$, generated by the trading of financial contracts, has been given to the marketer τ, $\tau = I, II$.*

Proof: Note that (19), (20), (21), (22), (23) constitue the complementarity reformulation of Problem X after adding the multipliers λ^{c*} (Harker and Pang (1990)). (24) is the reformulation of (16) and (17) where $\xi^c \geq 1$ is introduced to ensure that $\pi^c = \dfrac{\xi^c}{\sum_{c'} \xi^{c'}} > 0$. ∎

One should note that the solutions to Problems X and XII differ by the initial contingent endowments that accrue from the positions taken on the financial market. At the solution to Problem XII, marketer τ indeed pays $\lambda^{0*T}\Gamma^0\theta_\tau^*$ in stage zero and receives $\lambda^{c*T}\Gamma^c\theta_\tau^*$ in state c of stage 1. These endowments which are absent from Problem X modify the marginal utilities and hence the final equilibrium on the market. This modification reflects the outcome of risk trading through the financial market.

4.2.3 Financial completeness

Financial markets are characterized as complete or incomplete (Duffie (1996)). A financially complete market is one where the number of traded financial assets allows one to construct a portfolio θ that can reproduce any vector of contingent payoff p^c, $c \in C$. In our context:

Definition 1 *The market of financial transmission contracts is complete iff there always exists a portfolio θ (of initial up front cost $\lambda^{0*T}\Gamma^0\theta$) such that*

$$\lambda^{c*T}\Gamma^c\theta = p^c, \qquad c \in C \tag{25}$$

for any vector $(p^c, c \in C)$.

Complete markets imply the uniqueness of risk neutral probabilities. This is the second fundamental theorem of finance (Duffie (1996)). We further elaborate on the completeness of the financial transmission contracts in Section 5. At this stage we simply state the following lemma.

Lemma 2 *Suppose the market of financial transmission contracts is complete. Then the risk neutral probability is unique and*

$$\eta\pi^c = \alpha^c_\tau, \qquad c \in C, \tau = I, II. \tag{26}$$

where η is a normalization factor.

Proof: The uniqueness of the risk neutral probability is the second fundamental result of finance (e.g. Duffie, 1996). The equality of α^c_τ to π^c (up to some constant factor) derives from the equilibrium condition achieved by marketers trading risks in a complete financial market. ∎

According to Lemma 2, the completeness of financial transmission contracts implies the equality of the marginal utilities of the marketers in each state of the world (relation (26)). This is interpreted by saying that marketers can, together with the speculators, trade the risk that they incur in the different states of the world. In the usual interpretation of finance theory, π^c is the normalized price of risk in state c and marketers trade risk through the financial market until their marginal utility for money equalize in all states of the world. This allows the market to achieve Pareto optimality. It thus makes sense to define the electricity market as financially complete if the financial transmission contracts are sufficiently diversified to ensure that (25) is satisfied. It is indeed only under this condition that the global, physicial and financial market is Pareto efficient.

Problem $XIII$ and Proposition 11 characterize the solution of the regional transmission problem when the electricity market is financially complete.

Problem XIII

Let $\pi^c > 0$, $c \in C$; $\sum_{c\in C} \pi^c = 1$ be a vector of probabilities of the states of the world. Seek $(e^*, (i^{c*}, c \in C)), e^* \geq 0$ s.t. $\Gamma^c(Se^* + i^{c*}) \leq \overline{f}^c$, $c \in C$, satisfying

$$\left[\sum_c \pi^c L_I^{cT}(e^*, i^{c*})\right](e_I - e_I^*) + \left[\sum_c \pi^c L_{II}^{cT}(e^*, i^{c*})\right](e_{II} - e_{II}^*) \\ + \sum_c \left[O_I^{cT}(e^*, i^{c*})(i_I^c - i_I^{c*}) + O_{II}^{cT}(e^*, i^{c*})(i_{II}^c - i_{II}^{c*})\right] \geq 0 \tag{27}$$

for all $(e, (i^c, c \in C)), e \geq 0$ s.t. $\Gamma^c(Se + i^c) \leq \overline{f}^c, c \in C$.

Proposition 4 *Suppose that the market of financial transmission contracts is complete. Then there exists a unique vector of risk neutral probabilities* $\pi^c > 0, \sum_c \pi^c = 1$ *such that the outcome* $e^*, \lambda^{0*T}\Gamma_0$ *of the market is the solution to Problem* $XIII$.

Proof: Completeness of the financial transmission contracts implies that (24) in Problem XII is automatically insured by the financial market. Problem XII is then rewritten as

Seek $(e^* \geq 0, (i^{c*}, c \in C))$, $(\lambda^{0*}$, $(\lambda^{c*} \geq 0, c \in C))$, s.t. $\Gamma^c(Se^* + i^{c*}) \leq \overline{f}^c$, $c \in C$ satisfying

$$\left[\sum_c \pi^c \left(L_\tau^{cT}(e^*, i^{c*}) + \lambda^{c*T}\Gamma^c S\right)\right] \geq 0, \tau = I, II$$
$$\left[\sum_c \pi^c \left(L_\tau^{cT}(e^*, i^{*c}) + \lambda^{c*T}\Gamma^c S\right)\right] e_\tau^* = 0 \qquad \tau = I, II$$
$$\left[O_\tau^{cT}(e^*, i^{*c}) + \lambda^{c*T}\Gamma^c\right] \geq 0, c \in C, \tau = I, II$$
$$\left[O_\tau^{cT}(e^*, i^{*c}) + \lambda^{c*T}\Gamma^c\right] i^{c*} = 0, c \in C, \tau = I, II$$
$$\lambda^{c*T}\left[\overline{f}^c - \Gamma^c(Se^* + i^{*c})\right] = 0, \quad c \in C.$$

These expressions are the complementarity form of Problem $XIII$ after adding the multipliers λ^{*c} (Harker and Pang (1990)) and multiplying the constraints by π^c. ∎

Note that the unique risk netural probability π appearing in Problem $XIII$ is exogenously given. Given this risk neutral probability, the usual assumption of monotonicity of the mappings guarantee a solution to this problem.

4.2.4 Discussion

The above discussion is entirely conducted in terms of nodal prices ($\lambda^{0T}\Gamma^0$ and $\lambda^{cT}\Gamma^c$). In order to move to the flowgate interpretation, note that the PDF matrix Γ^0 was only used so far as an intermediate tool for constructing a vector $\lambda^{0T}\Gamma^0$ of nodal prices. Any other choice of λ^0 and Γ^0 that results in the same $\lambda^{0T}\Gamma^0$ would be as good. The situation is different in the flowgate model where contracts are based on line capacities associated to a well defined topology. λ^0 and Γ^0 are two constituents that both intervene separately to specify a flowgate transmission contract. In other words the network topology at stage 0 and hence Γ^0 matters. We now examine the financial market associated to the flowgate model. Assume that the relevant contracts are defined in stage 0 on the basis of a given network topology and the associated PDF matrix Γ^0. A financial contract in the flowgate model gives a payoff λ^c for a unit line capacity in state of the world c when the line is up. The payoff when the line is down is not defined in Chao et al (2000). We therefore complete the definition of these authors by specifying that the contract on a line pays 0 when this line is down.

Because flowgate and nodal transmission contracts are different, the marketer's problems need to be revisited. This is done in the following problem.

Problem XIV

Seek $e^*_\tau \geq 0$, θ^* satisfying

$$\sum_c \alpha^c_\tau \left[L^{cT}_\tau(e^*, i^{c*}) + \lambda^{c*T}\Gamma^c S\right](e_\tau - e^*_\tau) + \left(\lambda^{0*T} - \sum_c \alpha^c_\tau \lambda^{c*T}\right)(\theta_\tau - \theta^*_\tau) \geq 0 \tag{28}$$

for all $e_\tau \geq 0, \theta$.

A Problem XIV' analoguous to Problem XI' is defined for the speculator.

Problem XIV'
Seek δ^* satisfying

$$\left(\lambda^{0*T} - \sum_c \alpha^c \lambda^{c*T}\right)(\delta - \delta^*) \geq 0 \text{ for all } \delta.$$

Lemma 3 is the analogue of Lemma 1 for the flowgate system. It characterizes the forward prices of the flowgates in term of their prices in the balancing market.

Lemma 3 *Suppose a solution to Problem XIV exists. Then it satisfies*

$$\lambda^{0*} = \sum_c \alpha_\tau^c \lambda^{c*}. \tag{29}$$

Moreover, the no arbitrage assumption holds and there exists at least one "risk neutral probability" π such that

$$\lambda^{0*} = \sum_{c \in C} \pi^c \lambda^{c*};\ \ \pi^c > 0, c \in C; \sum_{c \in C} \pi^c = 1. \tag{30}$$

Again, the first statement is a direct consequence of condition (28) while the second statement of the lemma depends on the existence of a risk free asset and the no arbitrage assumption which must hold if one assumes an equilibrium on the financial market. In contrast with nodal contracts it is not clear how relations (29) and (30) allow one to derive a unique price of transmission services in stage 0. Specifically, relations (29) and (30) do not imply the equality of $\sum_c \alpha_\tau^c \lambda^{c*T} \Gamma^c$ for $\tau = I$ and II. This can easily be understood by further analyzing the hedging capabilities of the flowgate transmission contracts. Proposition 12 shows that a market that only trades line capacity contracts does not allow marketers to fully hedge their transmission risk when the lines are subject to contingencies.

Proposition 5 *Consider a portfolio $\Gamma^0 Se$ of flowgate contracts associated to energy forward transactions e. The uncovered transmission risk in state c is equal to $\lambda^{cT}(\Gamma^0 - \Gamma^c)Se$. This risk vanishes when there are no line contingencies.*

Proof: Let $\theta = \Gamma^0 Se$ be the portfolio of financial flowgate contracts subscribed to cover transaction e. In contingency c, this contract pays $\lambda^{cT}\Gamma^0 Se$ while the transmission cost is $\lambda^{cT}\Gamma^c Se$. Hence the result. ■

It appears that transmission risks cannot be fully hedged by the sole financial flowgate contracts, except when there are no contingencies. This implies that, except for this case, we are unable to equalize the $\sum_c \alpha_\tau^c \lambda^{c*T}\Gamma^c$ of the different marketers. In consequence we are unable to find a unique transmission price in stage 0. This makes the market physically incomplete.

A relevant question is whether this market incompleteness is sufficiently important to make the flowgate model unpractical, and, if so, whether other financial contracts will not be proposed to complete the market. Alternatively because the residual risk is really a "volume risk" due to reconfiguration of the network, one could envisage that this risk is covered by the ISOs (Chao and Peck (1998), Wilson (1997)). In any case it would seem from this analysis that the flowgate system suffers from hedging defects that the nodal system does not have. As we discuss in the following section the situation is not as clear cut.

5 More on market completeness

The above discussion may clarify some of the statements of the proponents of the nodal model. It is indeed ascertained that one of the major advantages of the nodal system is that it allows for perfect hedging of transmission prices. This certainly requires some condition: perfect hedging is only possible when a sufficiently rich set of financial contracts exist. Specifically a marketer who wants to hedge a transaction between two nodes cannot realistically do so if the market of financial contracts between these two nodes does not exist or is not sufficiently liquid. Absence of one of these conditions results in missing or thin markets and hence leads to a physically incomplete transmission market.

As seen before perfect hedging requires an additional condition in the flowgate model, namely the absence of network contingencies. This is too harsh to be realistic. Perfect hedging will thus never be possible. This phenomenon has been pointed out by Hogan (2000b) and Ruff (2000). It is certainly a disadvantage with respect to the nodal system but possibly not a too serious one. As noted before, physical completeness in the nodal system is indeed only guaranteed if there are enough traded financial transmission contracts. This may also be a difficult condition to fulfill in practice. In any case incomplete markets are commonly accommodated in the real world provided they are not too damaging. It may be that the hedges provided by flowgate contracts are sufficient. Similarly it may be that a subset of the theoretically strictly necessary nodal contracts suffices. In fact we do not really know. The question of the possible coexitence of nodal and flowgate services can again be taken up at this point. Flowgates are again priced on the basis of spot prices and extended flowgates are constructed from spot prices. As in the forward market, it would seem that they are redundant services. In this sense flowgates and extended flowgates could certainly coexist with nodal systems. But the question seems to be deeper if one consider the natural interpretation of flowgates in terms of options (see the Oren/Ruff debate (http://www.ksg.harvard.edu/hepg)). This discussion about the optional nature of flowgate versus nodal contracts, which has been systematically avoided in this paper is of immediate relevance in this context. This suggests some additional elaborations as we now discuss.

Even though the nodal model may exhibit advantages with respect to flowgates when it comes to physically completing the market, it may be a surprise to note that none of the above models and organizations will meet the financial completeness criterion. Indeed the auctioning of transmission services guarantees the equality of transmission prices but not of the marginal utilities of traders in different states of the world. Similarly it is unlikely that the nodal and flowgate transmission financial contracts will be numerous enough to achieve this equality. We may thus end up with the conclusion that none of the proposed systems is powerful enough to financially complete the market. This implies that risk will always be handled in a non-Pareto optimal way, whether perfect hedging of transmission transactions is possible or not. This raises the question of whether there is any hope to achieve financial completeness and hence Pareto optimality. A further analysis of the risks involved may shed some light on the issue.

Part of the attractiveness of the nodal model is that it aggregates all the electricity risks of a bilateral transaction into one. In other words, market risks such as demand and supply uncertainties as well as network risks such as line defaults are aggretated into nodal price risk. But these risk factors are distinct in the real world. Moreover, they are not of the same type. Typically a line risk is like a jump while daily demand uncertainties look more like a diffusion. The flowgate model unbundles some of these risks but fails to provide enough instruments to cover all of them. Interestingly the debate has centered on which system would lead to the smallest number of tradable instruments. The question is unanswered at this stage. But there is another question. Given the different packaging of risk hedges offered by the nodal and flowgate models and the unequal coverage that they allow, will the market prefer a system that offers full coverage of bundles of risks to one that provides individual coverage of only some risk factors ? Moreover given the high number of risk factors, it may be that some of them should simply be assumed by the system operator through an insurance system, as recommended in Wilson (1997).

This suggests an alternative perspective for looking at the nodal and flowgate models. The objective of financial engineering is not necessarily to provide the smallest possible set of instruments that cover global risks. Its aim is to untangle risks factors and to re-bundle them in packages that are attractive to agents operating in the market. In this sense derivative products that offer perfect hedging may not necessarily be what the market wants most. Electricity is a very volatile commodity with a lot of underlying risk factors. Maybe a first question is to see the extent to which one can define elementary derivative products that span each or at least sufficiently small subsets of these risk factors. The next question would then be to re-bundle them in products that the market may desire. This is where the optional character of flowgate may be useful. It is a well-known property that options can be used to complete markets that are initially (financially) incomplete. This is done by introducing options with different strike prices. The flowgate model has so far only introduced options with zero strike price. It therefore did not take full advantage of the optional nature of the contracts that it offers. An analysis of the capability of the flowgate contracts to better span the domain of risk factors of the electricity market is beyond the scope of this paper, but the following last property may shed some line on what is at stake. We did indicate before that the flowgate model in its current form offered inferior hedging properties compared to the nodal system. Suppose that one is able to enlarge the set of instruments traded in the market (for instance by offering flowgate contracts with non-zero strike prices or equivalently with different priorities). Then the following result could be of relevance.

Proposition 6 *Suppose the market of financial flowgate contracts is complete. Then the marginal utilities of the traders are equal in each state of the world and the flowgate contracts allow full hedging of transmission risks. Moreover the price for transmission paid by the marketers is equal in stage 0 and the market is physically complete.*

Proof: Using the same reasoning as for nodal contracts completeness of the financial transmission contracts implies a single risk neutral probability such that $\eta\pi^c = \alpha^c_\tau$, $\tau = I, II, c \in C$ for some η. This implies that $\sum_c \alpha^c_\tau \lambda^{c*T}\Gamma^c$ is equal for both marketers and hence that they can fully hedge the transmission risk in stage 0. ∎

The relevant question thus appears to identify which of the nodal or flowgate, or of the combination of both models is the most capable of producing the bundle of derivative products that the market wants most to trade the numerous risk factors inherent to electricity. This also suggests to look at the problem of the coexistence of the flowgate and nodal systems in an extended way. The formalism presented in these papers suggests that there should be no problem of coexistence. This conjecture could probably be proved by explicitly inserting nodal and flowgate services in the model. (Note that we always derived the nodal and flowgate model from a single mathematical formulation). Another conjecture would be that it is possible to financially complete the market by adding new flowgate services of non zero strike price. This would require a full rewriting of the models. We leave this question for further research.

6 Conclusions

This paper extends the discussion of Smeers (2003) to the situation where there exist both a forward and spot market. Energy and transmission transactions are concluded in the forward market and settled in the spot market. We analyze this organization in two ways. In the first approach, transmission capacities are initially auctioned in the forward market. We show that physical market completeness is achieved both in the nodal and flowgate models, provided the latter allows for aggregate flowgates. The second approach introduces financial transmission contracts. We show that these physically complete the market in the nodal system but that the same results can only be obtained under additional assumptions in the flowgate system. Even though trading transmission capacities (whether through physical or financial contracts) makes the market physically complete (possibly under certain conditions), it does not allow the marketers to fully trade the risks that they are subject to. Specifically the marginal utility of money of marketers can still differ in the states of the world. The market is thus financially incomplete.

It may come to a surprise that financial completeness also seems difficult to achieve with financial transmission contracts. The problem is that electricity involves too many risk factors. Spanning the relevant uncertainty implies a large set of derivative products that may not be immediately at hand. The challenge therefore seems to be in identifying the right set of derivatives products that the market wants in order to trade risk. The flowgate model, because of the natural optional character of the contracts that it defines seems better fit in this respect but this certainly deserves more exploration. The completion of the market of financial transmission contracts will make the nodal and flowgate markets equivalent in terms of hedging possibilities. This appears to be a particularly attractive context for investigating the coexistence of the nodal and flowgate nodals. But whatever the market design selected, the solution always depends on the existence of a real time spot market.

The models discussed in this paper, even though looking relatively straightforward, are computationally unusually complex. First note that these are two stage variational inequality problems, or in other words, variational inequality problems with recourse. Transactions concluded in the forward market are the first stage variables; the actions of the ISOs are the second stage variables. The literature on two-stage variational inequality is not abundant but it is significant. The basic principle is to adapt Benders decomposition and to solve the augmented first stage problem by special bundle or interior point methods (e.g. Lemaréchal et al. (1995) and Denault and Goffin (1999) for examples of both approaches). This extension can easily accommodate the case where the second stage involves different states of the work such as in this paper. This literature applies to monotone variational inequality problems, a quite natural assumption that prolongs the convexity assumptions required by Benders decomposition in stochastic optimisation. Among the different models elaborated in this paper only Problem $XIII$ satisfies this monotonicity property. This problem is constructed using a risk neutral probability and hence is a straightforward extension of standard two-stage stochastic optimization problems to the variational inequality world. We shall not elaborate here on the risk neutral probability. It suffices to say that theory proves its existence on the basis of a "no arbitrage assumption" and that practice estimates it from market data of traded financial derivative products. All models but $XIII$ are significantly more difficult and of a nature unexplored at this stage. First note that in contrast with Problem $XIII$, none of these models involves probabilities. These are indeed replaced by marginal utility functions $\alpha(e, i)$. The utility functions $U(x)$ are defined in a standard way as functions of the payoffs accruing to the agents in the different states of the world. But the variational inequality models of this paper are stated using the marginal utility functions where the payoffs have been replaced by the quantities of transactions on the forward and real time markets namely e and i^c, $c \in C$. Moving from x to (e, i) in the notation hides a considerable complexity. Payoffs in the utility function U are the products of quantities and prices. These are respectively the primal (i) and dual (λ) solutions of the second stage variational inequality problems of our models. Because the dual variables (nodal or flowgate prices) can be seen as (possibly multivalued) functions of the primal variables (nodal injection and withdrawals), our notation $\alpha(e, i)$ is justified. It is used or deriving results under the assumption that an equilibrium exists . But this notation is useless for proving the existence of the equilibrium, let alone for computing it. The significant computational difference between our models and standard extension of stochastic optimisation to the variational inequality world can then be characterized as follows. Payoffs in a stochastic optimisation depend on the primal variables of the second stage problem only. They depend on both primal and dual variables of the second stage in most of our equilibrium problems. This added complexity destroys all convexity properties. Computationally, it makes all models of this paper except Problem $XIII$ two-level equilibrium problems, a domain that is barely explored in computational terms. The added touch of complexity is that, these two-level equilibrium problems are also two-stage stochastic problems.

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