Extending Graphic Statics for User-Controlled Structural Morphogenesis

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Summary

The first geometrical definitions of any structure are of primary importance when considering pertinence and efficiency in structural design processes. Engineering history has taught us how graphic statics can be a very powerful tool since it allows the designer to take shapes and forces into account simultaneously. However, current and past graphic statics methods are more suitable for analysis than structural morphogenesis.

This contribution introduces new graphical methods that can supplement the use of graphic statics during the primary establishment of both the structural shape and its mechanical behaviour. First an explanation is given of the underlying motivations, with a short reminder of traditional graphic statics, and then the proposed design process is outlined with two main concepts: *geometric domains* and *equilibrated operations*. Finally, some interesting applications are set out.

keywords: *structural morphogenesis; structural design methods; graphic statics; stress and shape modelling; graphic handling; strut-and-tie model.*

1. Introduction

1.1 The impact of primary structural definitions

Recent advances in CAD and computational tools have reduced the mathematical complexity inherent in structural design. However, caution is still required when using these tools to define shapes and structural behaviours.

For example, common analysis tools tend to freeze the design process by not facilitating the global reinterrogation or reorientation of initial choices. As far as optimisation tools and automatic form-finding tools are concerned, geometrical changes are the optimised consequences of a very small number of targeted criteria. Further changes by the user are therefore unwelcome. Besides, the results produced might constitute a surprise that designers have to interpret after the event.

Except for physical models, other tools like modellers are seldom concerned with the physical reality of structures. The mechanical answers advocated in order to withstand these shapes generally compel an inappropriate structural behaviour and usually require more material than is really necessary.

These opening remarks emphasise the role of primary definitions of shapes and stresses with a view to ensuring the pertinence and efficiency of building materials. Furthermore, it appears crucial for designers to master the overall structural complexity, and this control is even more essential during the initial structural definitions of shapes and stresses, i.e. before any analysis or optimisation processes.

1.2 A structural morphogenesis approach

There are particular requirements when initially defining the structure. It should always be easy to revisit and revise earlier choices. Therefore the suggested approach encourages (1) the simultaneous graphical handling of geometries and their mechanical incidences, (2) the use of simplifying theories that allow the global control of the structural issue in such a way that the designer's creativity is not hampered by time-consuming calculations, and (3) the designer's permanent control over the structure being shaped.

Graphic statics principles can objectivise these three prerequisites in a very convenient way.

1.3 Classic graphic statics and its present-day uses

Graphic statics is a 150-year-old theory based on Maxwell's reciprocal diagrams [1] which has been greatly elaborated on by many authors [2][3][4]. The great benefit of using graphic statics is that it is an entirely visual method that allows the simultaneous study of geometries and internal forces within a given structural system. Each bar in this system is always represented by two parallel segments, one in the *space diagram* and the other in the *force diagram* in which force amplitudes can be measured graphically.

Since its first theoretical formulations, graphic statics has been enhanced by a large variety of analysis methods: the study of forces in trusses, the stability of masonry arches and domes [5], the study of bending moments in concrete etc. The early definition of structures also gave rise to specific methods [6][7].

Such methods build up both diagrams chronologically. Most operations are executed on partial – thus non-equilibrated – diagrams and equilibrium is only reached once the method has been concluded. For that reason, the use of graphic statics, when the quantity and positions of bars are initially unknown, means that the designer has to have time, good intuition or known typologies in mind – which might not be the case when exploring innovative structural designs. This is the point at which the following methods might be of interest.

2. Overview of the process

The general purpose is to help the structural designer generate and handle strut-and-tie systems by means of the simultaneous control of shapes and stresses. Starting from loads, geometrical constraints and physical constraints, the user applies successive operations to a structure that always remains in equilibrium.

Each diagram is composed of bars, forces and geometrical constraints that are all exclusively defined by the positions of the nodes in both diagrams. To combine graphic statics with computer capabilities offers some advantages, predominantly CAD's precision and speed. For example, as soon as the user moves a node in a diagram, all the other elements that are functions of this node can be instantaneously repositioned.

Furthermore, elements are actually only described here using parallelism or intersection. Just like classic graphic statics, the following methods do not need any algebra or trigonometry. The computations are therefore entirely graphical so that (1) calculations are easier to write and quicker to perform and (2) the user remains visually aware of what is being done.

The first method below explains how displacements of every node can be graphically limited to specific domains in order to meet the geometrical constraints acting on the space diagram or the force diagram. The second method illustrates the basic operations on diagrams which the designer has at his disposal in order to develop the structural system while keeping it in continuous equilibrium.

3. Graphically constructed geometric domains

An introduction to the way nodes are constrained within geometric domains is illustrated in figures 1 to 5. Fundamentally, all planar geometrical constraints can be described using logical combinations (intersection, union or negation) of two different nodal constraint types: a sector acting on the orientation of bars and a radius acting on lengths. That being the case, any structural constraint can literally be defined as a geometrical description. The domains can then be constructed in full using the properties of similar triangles between both diagrams.

The main advantage of this method is that it allows the designer to work out any catenary or reticular structure using predictions rather than empirical trial and error.

4. Equilibrated operations

An introduction to the way structures can be created by successive equilibrated steps is illustrated in figures 6 to 11. The operations presented are quite fundamental, but once combined they can form a new equilibrated operation which on its own can perform a more specific task. Using both diagrams, each operation can be motivated by formal motives or mechanical motives.

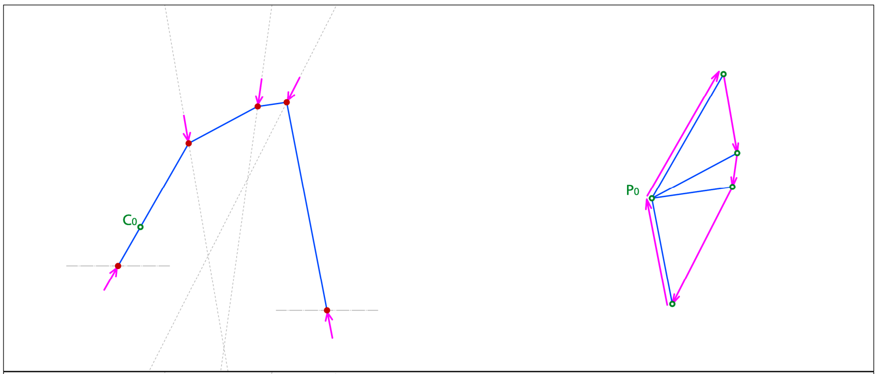

Fig. 1 A catenary withstanding three loads and passing through one given point. The pole P0 is free to move anywhere in the force diagram (right).
Nodes in green are moveable while nodes in red are fully constrained.

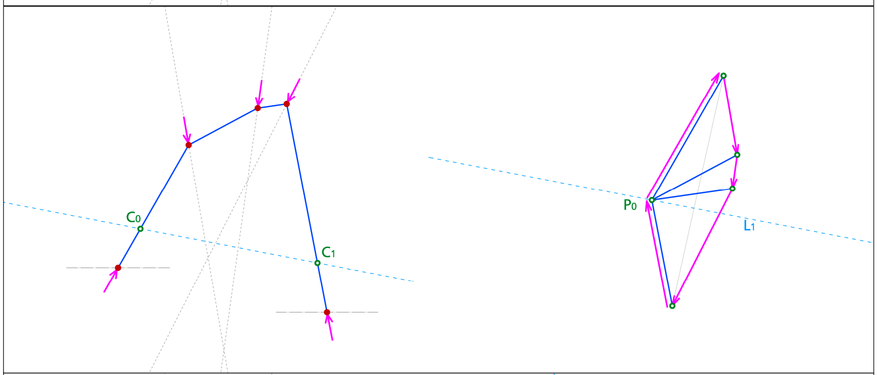

Fig. 2 A catenary passing through two given points. The pole P0 is now restrained on the line L1.

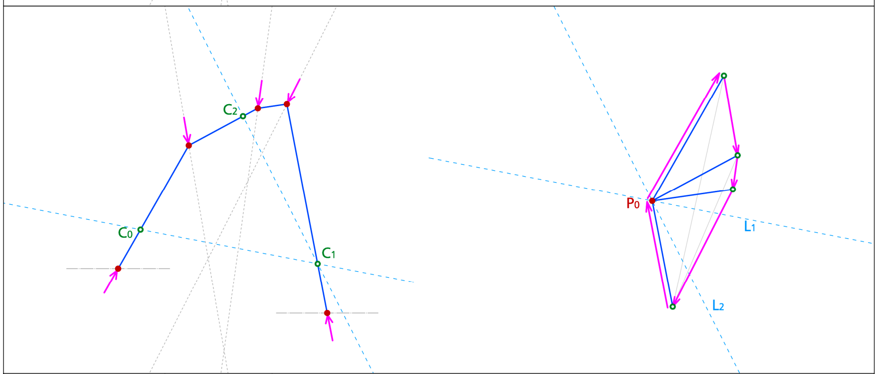

Fig. 3 A catenary passing through three given points. The pole P0 must now be restrained on the intersection of lines L1 and L2. It can no longer be moved by the user.

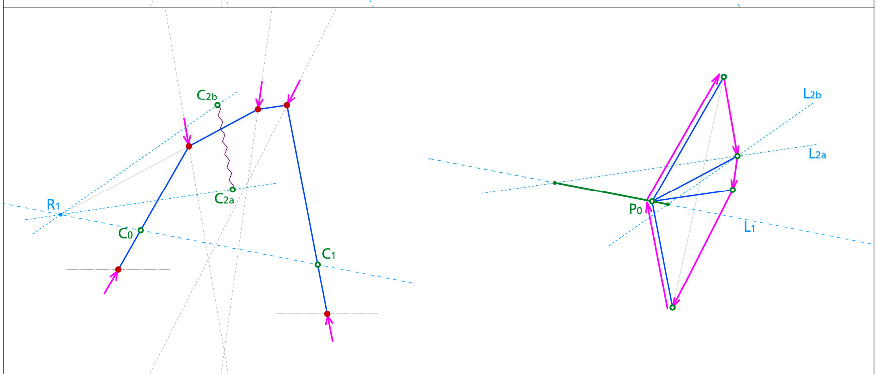

Fig. 4 A catenary passing through two given points and between two other points (C2a and C2b). The pole P0 is restrained on the geometrically constructed green segment. The user can now change the spatial shape by moving P0 inside this segment.

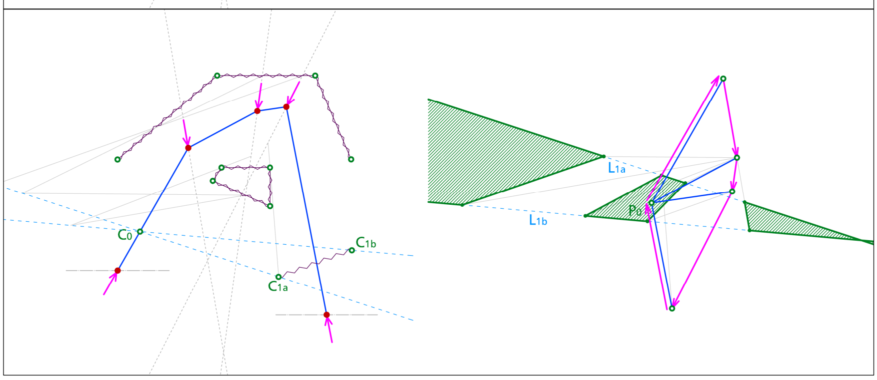

Fig. 5 A catenary subjected to various geometrical constraints. The pole P0 is restrained on the geometrically constructed green areas in order to fulfil those initial constraints. The green area constitutes the domain of the node P0. Every other node has a specific domain as well.

The main characteristic is that the system always remains in equilibrium thanks to the introduction of temporary forces that are waiting for “materialisation”. This feature means that there is no need for the non-equilibrated chronological construction of diagrams. In other words, the designer is now able to develop and combine any equilibrated sub-structures separately. Complex structures, whether they are isostatic or hyperstatic, whether they are prestressed or not, can then easily be produced through an intuitive step-by-step process.

5. Applications and further developments

This standard strut-and-tie model can be applied to various fields, including the concomitant design of sections featuring the arrangement of members while handling bending moments and eccentricities produced by thrust lines and centroids, the design of reinforced or prestressed concrete members by strut-and-tie methods [8], the step-by-step design of tensegrities, the design of three-dimensional equilibrium etc.

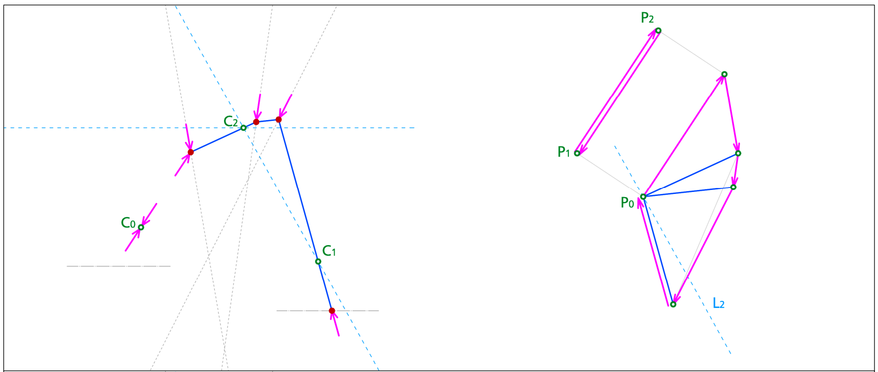

Fig. 6 Starting from figure 3, the goal of this second example is to make the second bar of the catenary (from left) horizontal. First, in order to unlock P0 along one line only, the user can transform the first bar into two equal but opposite forces. The diagrams are then updated.

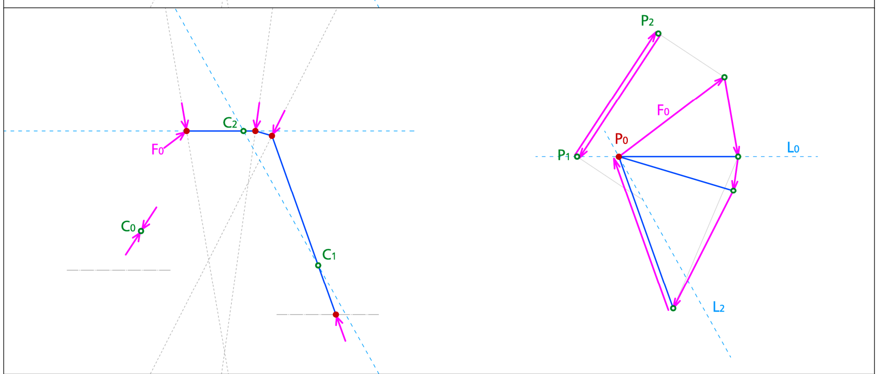

Fig. 7 The second operation moves P0 along L2 until the second bar becomes horizontal. In the meantime both diagrams are automatically re-adjusted to guarantee equilibrium. The main goal is now to transmit F0 to the ground through a new bar member.

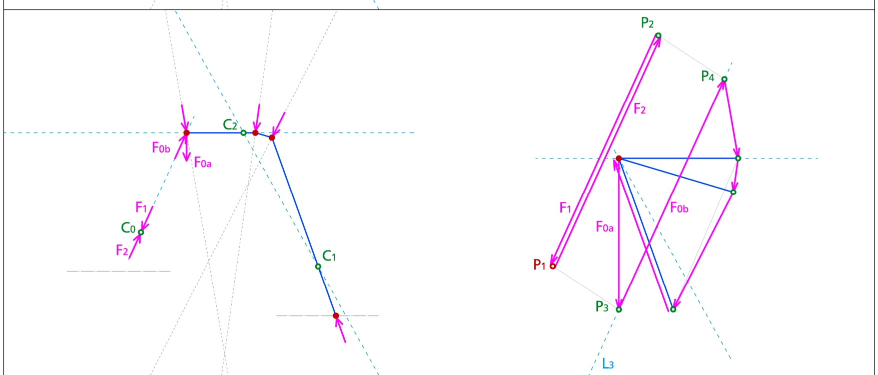

Fig. 8 The third operation simply resolves F0 into two forces that have the desired orientations: one vertical and another aiming at C0. The next operation moves and locks P1 (in the force diagram) in order to match the amplitude and orientation of F1 with F0b.

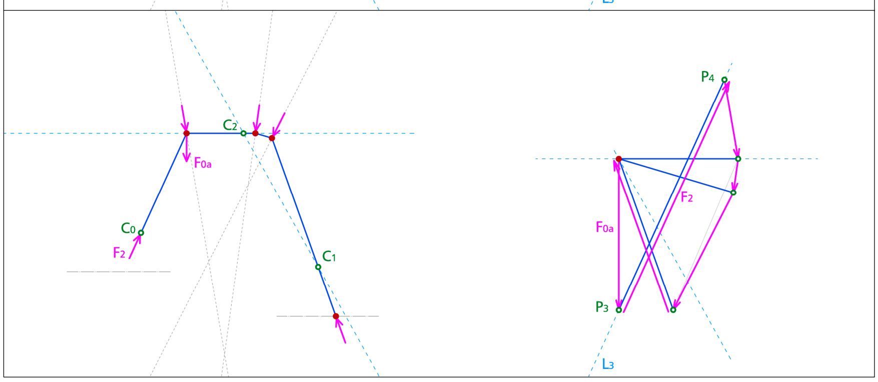

Fig. 9 The node P2 is then merged with P4. After that, the two forces F1 and F0b that were aligned, with the same amplitude and opposites, are transformed into a new bar. The remaining force F0a still has to be transmitted to the ground.

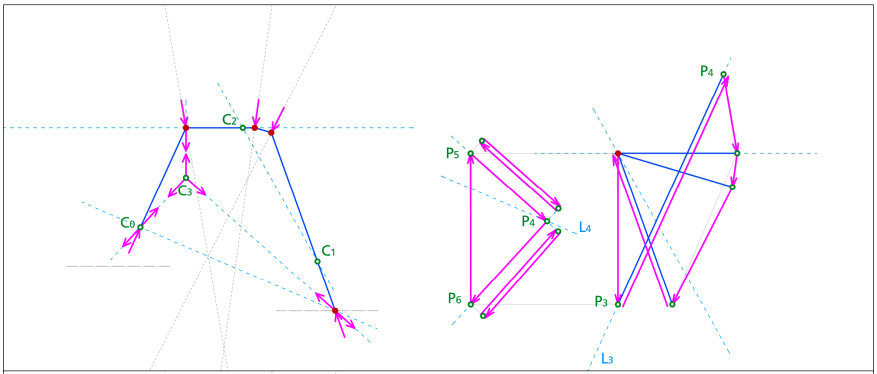

Fig. 10 Three new sub-systems are created: one new node C3 equilibrated with three forces of desired orientations; one pair of equilibrated forces at C0; and a final pair of equilibrated forces at the right supporting node.

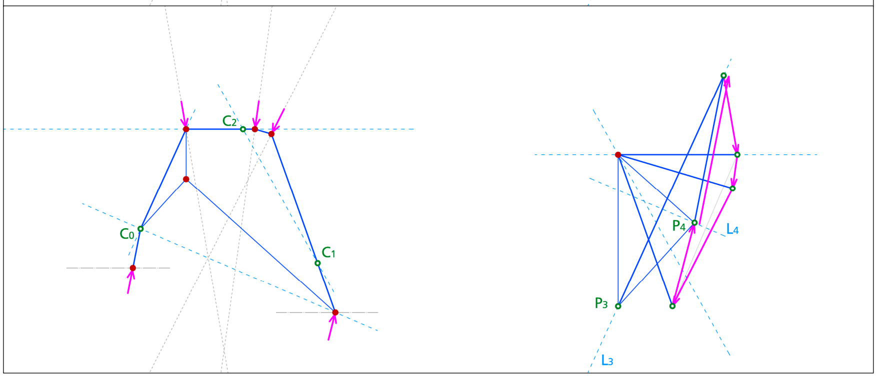

Fig. 11 The user can finally merge adequate pairs of forces into bars and add up resultants on each support. A new equilibrated system with the desired horizontal is now created and ready for further operations. The catenary has changed naturally into a reticular strut-and-tie model.

Furthermore, the computerisation of such non-chronological graphic treatments provides a straightforward environment for more complex parametrical routines, such as long iterations or optimisations of lines of reference.

6. Conclusion

This contribution has presented entirely graphical methods that attempt to extend the use of graphic statics. The first section emphasised the role of the early structural design process, while the next two sections introduced original methods involving geometric domains and equilibrated operations.

The use of such interactive methods means that the preliminary definition of structural geometries can be worked out intuitively at the same time as expected inner stresses. Further developments in this area might constitute an alternative form-finding process in which the designer remains the real driver of each formal and mechanical decision.

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