Chapter 4
Assessing inequality of educational opportunities in Brazil

Abstract

In this chapter, we discuss advantages and limits of three alternative methodologies which can be used for measuring and comparing inequality of educational opportunities (not gross inequalities anymore), all of which are based on, or try to go beyond, Roemer (1998). The two first alternatives reflect the usual opposition between a dominance approach and an approach based on specific indices. The third alternative is an attempt to make an explicit link with the proposal we outlined in chapter 2, leading us to give a contribution in the form of a 'schooling system fairness index', which aggregates educational fairness information concerning three different grades. We provide an illustration of the three different methodologies using Brazilian data.

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1 Introduction

We continue here the study of test scores inequalities in Brazil initiated in the previous chapter. We already know the intensity of such inequalities and how they are distributed throughout the country. We now apply the distributional analysis toolbox, not to the whole population anymore, but to subsets of the population (groups or types), in such a way that the symmetry or anonymity property is attenuated.[1] More precisely, we discuss different ways of measuring and comparing *inequality of educational opportunities*, instead of gross inequalities as in chapter 3. We discuss different methodologies for measuring and comparing inequality of educational opportunities which are based on, or try to go beyond, Roemer (1998).

Just after this introduction, we provide a methodological discussion (section 2) of two different procedures for measuring inequality of educational opportunity, reviving the usual divide between dominance analysis *versus* specific inequality indices approach. Moreover, we try to provide a more comprehensive, or systemic, evaluation of a schooling system fairness, by means of an index ('SF'), which aggregates educational fairness information concerning three different grades. By introducing this index, we try to bridge an explicit link with the proposal outlined in chapter 2.

Illustrations of the three alternative methodologies are reported in section 3, where we employ the same Brazilian dataset used in chapter 3. The *dominance analysis* reveals a situation of evident inequality of opportunity when types are defined in terms of parental education, while when types are defined in terms of skin color we obtain both inequality of opportunity and (at least weak) equality of opportunity, depending on the types we compare. The *inequality indices* approach shows that, according to the parameters we employ, inequality of opportunity represents 16.07% of overall inequality in Brazil. We observe large variation across regions, just as in the Italian study we use as a benchmark. Brazil's most opportunity-unequal region (South-East) is 176.67% more unequal than Italy's most opportunity-unequal region (Center-South) and 433.71% more so than Italy's least opportunity-unequal region (North). Finally, we calculate a ranking of regions based on a simplified version of the *SF index*.

We conclude in section 4, summarizing the results of Part II.

[1] Symmetry or anonymity is a usual property in distributional analysis. Relaxing it can be interpreted as a challenge to one of the 'further assumptions' listed in chapter 1: that of the 'representative consumer' or 'homogeneity'.

2 Methodological issues

2.1 Recipient unit, attribute, and aggregation method

In this chapter, we keep on working with pupils as the recipient units, and with absolute scores as attributes. As for the aggregation method, we would like to use a procedure which, while respecting usual properties, would also be as compatible as possible, either with the conception of justice stated by Roemer (1998), or with the adaptation of such conception, which we have introduced in chapter 2 (fairness of the schooling system as a whole).

We first discuss two alternative paths for comparing distributions in terms of their inequality of opportunity, both of which do respect standard properties, and are inspired by Roemer's normative criterion. In order to attenuate the inconveniences of the anonymity property, the population is subdivided in groups ("types" in Roemer's terminology), and usual inequality measurement tools are applied to those subgroups.

One of theses paths has been developed in a series of recent paper: Peragine (1999, 2002, 2004b,a) and especially Checchi and Peragine (2005). The alternative path has been developed by a group of French economists: Pistolesi, Lefranc, and Trannoy (2005); Lefranc, Pistolesi, and Trannoy (2006).

While both strategies are clearly inspired by the work of Roemer, there is a net difference between them, which revives the usual distinction between dominance analysis and inequality indices approach. The procedure developed by the Italian economists is an interesting translation of Roemer's intuition: it is based on the use of inequality indices whose aim is to capture the degree of *unfair* inequalities, providing complete rankings of distributions. The procedure developed by the French economists generalizes, in a certain sense, Roemer's concept of EOp. It provides dominance analysis and, as expected, does not necessarily permit to set up complete rankings, but provides more robust conclusions, at least in some pair-wise comparisons.

We briefly present each of them in the next section, followed by a discussion of a systemic approach based on our proposal in chapter 2. Then we assess some advantages and disadvantages of each methodology.[2]

[2]It should be acknowledged the existence of another recent branch of this literature, which tries to split total variance of achievement into circumstance- *versus* choice-related components, but based on *regression techniques* (Devooght, 2004; Cognaux and Gignoux, 2005; Bourguignon, Ferreira, and Menéndez, 2006) and relaxing the assumption of sep-

2.2 Inequality indices

Extending a previous theoretical contribution (Peragine, 2004b) which focused on opportunity inequality dominance criteria, Checchi and Peragine (2005) design a procedure devoted to producing *complete rankings* of *opportunity inequality*. They do so by distinguishing the fraction of inequality which is due to circumstances (unacceptable) and the fraction of inequality which is due to effort (acceptable), and then assessing the absolute magnitude and the relative importance of each of these two kinds of inequality. Sections 1 to 4 in their paper describe in detail their approach and now we offer a brief explanation, emphasizing the points which matter more for us.

The first step is to partition the distribution of an outcome (say, of test scores) according to two criteria: types and "tranches". Belong to a given type all the individuals who have similar circumstances (say, the same family background). Belong to a given tranche all the individuals (from each type) who sit in the same quantile in the distribution of the outcome. As we know from chapter 2, to Roemer the absolute level of effort is irrelevant, and it is the effort relative to the individual's type that matters. So all the individuals who belong to a given tranche are assumed to have expended a similar degree of effort and, hence, deserve to attain the same educational level.

In the second step, each observation of scores is replaced by the arithmetic mean score within a given type and tranche. In other words, they replace the original profile of scores by an artificial profile of scores in which all the intra-type-and-tranche inequality fades away. The rationale for doing so is that such within-type-and-tranche inequality is neither associated with differential opportunities, nor with differential effort. Being irrelevant in Roemer's model, it should be canceled out in an empirical application. In the "artificial" distribution generated thereby, in accordance with Roemer's assumptions, the scores are exclusively a function of the type (previously set by the researcher) and of the effort (inferred by the researcher from the outcome distribution).

The third step is then to evaluate the overall inequality of the artificially generated distribution, as well as the within-tranches inequality and the within-types inequality. Within-tranches inequality is interpreted as inequality due to differential opportunity, since the outcomes of individuals of different types, but who belong to the same tranche (i.e., assumed to have expended similar relative effort), are be-

arability between effort and circumstances , which is adopted in Roemer's framework. This insightful set of papers, which we happened to get in touch with very late in the preparation of this dissertation, will be explored in our future research.

ing compared. In a world of perfect equality of opportunities, there would be no difference from one type to another, within each tranche, in the outcome level obtained. Within-types inequality, in turn, is interpreted as inequality due to differential effort, since the outcomes of individuals of similar types (i.e., those who have similar circumstances), are being compared.

Concretely, the decomposition of overall inequality of the artificially generated distribution into opportunity-inequality and effort-inequality is analogous to the decomposition of overall inequality into between-groups inequality and within-groups inequality (cf. chapter 3). Groups could be defined both as types or as tranches, and Checchi and Peragine discuss both possibilities.[3] We prefer the tranches approach, which in our view is closer to Roemer's conception of EOp, and we will only refer to such approach hereafter. According to such tranches approach, between-groups inequality should be understood as between-tranches inequality (acceptable); and within-groups inequality should be understood as within-tranches inequality (unacceptable).

In order to decompose inequality, Checchi and Peragine (2005) have employed a subgroup decomposable index, namely an index belonging to the generalized entropy class. They have chosen the mean logarithmic deviation (MLD), also known as 2nd Theil's coefficient, and which is precisely the generalized entropy index, $GE(0)$, already employed extensively in our chapter 3. They justify the choice of this index because of a property it respects, namely, path-independence, which is useful in the context of their paper.[4]

[3]The "tranches approach" is stated in their "Definition 2: (...) There is EOp if and only if all those who exerted the same degree of effort have the same chances of achieving the objective, regardless of the type." In our view, this approach is closer to Roemer's conception of EOp than the "types approach" stated in their Definition 1.

[4]A decomposable inequality index can be decomposed in two ways: (i) either subtracting the between-groups inequality from the overall inequality (usually called"smoothing"); (ii) or subtracting the within-groups inequality from the overall inequality (usually called"standardizing"). In the former case, the residual of the subtraction is the within-groups inequality, while in the latter the residual is between-groups inequality. A path-independent index is one which can be decomposed in either ways and exactly the same results are obtained. Among the indices comprised in the GE-class, path-independence holds only for $\alpha = 0$ (Shorrocks, 1984). In Checchi and Peragine (2005), path-independence is important, precisely because they work with two approaches - types and tranches - which correspond in fact to the two ways of decomposing inequality mentioned in this footnote ("standardizing" and "smoothing").

2.3 Dominance analysis

A recent contribution by Pistolesi, Lefranc, and Trannoy (2005) provides an alternative framework for comparing distributions in terms of the degree of equality of opportunity they reflect. Such framework places itself in the tradition of dominance analysis. Again, we limit ourselves to a brief explanation here.

Pistolesi, Lefranc, and Trannoy (2005) summarize the opportunity offered to an individual by the distribution of an outcome, s, conditional on her set of circumstances, t, denoted $F(s|t)$. Considering two groups in a society, $F(s|t)$ and $F(s|t\prime)$, if $F(s|t)$ second-order stochastically dominates $F(s|t\prime)$, then the situation of the former group is unambiguously preferred to that of the latter one.[5] In such case, since it is clear that the distribution of one group is preferred to the distribution of another group, such society presents *inequality of opportunity*.

Logically then, equality of opportunity is satisfied only when the conditional distribution of the outcome of one group does not dominate the conditional distribution of the outcome of another group. They say that "defining equality of opportunity as non-dominance with a second order stochastic dominance criterion is equivalent to saying that an individual choosing among these circumstances is unable to rank them."(Pistolesi, Lefranc, and Trannoy, 2005, p.5). However, two kinds of equality of opportunity are possible:

1. *Weak equality of opportunity*: verified whenever it is not the case that one distribution dominates the other one, and provided that the cumulative distributive functions of the two groups (or equivalently, their GL curves) *cross at least once*. The reasoning is the following: given that they cross, it is not clear from an *ex ante* position (i.e., from a 'veil of ignorance', cf. chapter 2) if it is better to belong to type t or to type $t\prime$.

2. *Strong equality of opportunity*: verified whenever it is not the case that one distribution dominates the other one, and provided that the cumulative distributive functions of the two groups (or GL curves) *are identical*. In this case, any *ex ante* observer would have no clue on choosing whether he would prefer to belong to group t or to group $t\prime$.

They claim strong equality of opportunity is a particular case of the weak equality of opportunity. And they also claim such particu-

[5]We modify the notation employed by Pistolesi, Lefranc, and Trannoy (2005), in favor of one which is closer to the one employed in the remaining of this dissertation. While they use x and s for outcomes and circumstances, respectively, we employ s (for scores) and t (for types), respectively.

lar case corresponds exactly with Roemer's conception of equality of opportunity.

Developing a literature initiated many years ago on the application of *statistical inference to dominance analysis*, a more recent literature has appeared. Contrary to the old one (Bishop, Formby, and Smith, 1991), the concern of the recent literature (Davidson and Duclos, 2000) is not placed anymore in comparing estimates of ordinates of Lorenz curves, but rather in comparing the entire distributions. To do so, the statistical foundations for applying statistical inference to stochastic dominance have been developed. This toolbox is applied by Pistolesi, Lefranc, and Trannoy (2005) in order to, among sets of distributions, finely distinguishing those in which strong equality of opportunity obtains from those in which only the weak version obtains.

2.4 A systemic assessment

In chapter 2, building on Roemer's concept of EOp, but adopting an 'educationist' perspective and taking into account some particular features of the schooling process, we state a concrete proposal of a procedure which provides a basis for the assessment of the degree of fairness of an educational system. In that chapter, we defend the view that, to assess the extent to which educational EOp is verified, we should grant a gradually increasing role to educational responsibility of individuals as they grow up. The assessment scheme we introduce in that chapter is composed of four stages, with a declining importance attributed to circumstances-related inequality as we move from initial grades/ages to more advanced grades/ages.

Such a comprehensive assessment scheme suggests us the idea of evaluating the fairness of a schooling system as a whole, by using test score data regarding *different grades*, and by aggregating somehow the distributional features of those different grades in one single index. Such comprehensive assessment should allow researchers to compare the overall fairness of different schooling systems. A 'schooling system fairness index', could be expressed by the following expression:

$$SF = \frac{1}{\#G} \sum_{1}^{G} SF_g \tag{1}$$

such that:

$$SF_g = \alpha \; I_{g,T} + \beta D_g^{-1} + \gamma P_g \tag{2}$$

where:

- SF is a shortcut for 'Schooling-system Fairness index';
- $\#G$ is the number of grades ($g = 1, \ldots, G$) for which data is available;
- SF_g is the partial SF for each grade g;
- $I_{g,T}$ is a relevant inequality index calculated across T types of pupils enrolled in the g-th grade;
- D^{-1} is a delay-correction term for students enrolled in grade g;
- P is some indicator of participation for pupils of the theoretical cohort corresponding with grade g (intended to be a dropout-correction term);
- α, β, γ are parameters regulating the contribution of each term to each grade's fairness index.

The two corrections terms aim at taking into account performance indicators (of flow), which while not directly expressed in test scores data, are relevant for educational opportunity. A policy-maker responding to the SF index as it is stated above has an incentive not only to improve the fairness provided to 'included students' (captured by the inequality terms, $I_{g,T}$), but also not to exclude students, neither temporarily by retaining them (captured by the delay-correction term), nor permanently (captured by the drop-out-correction term).

Suppose we have data on test scores for pupils of three grades (4^{th}, 8^{th} and 11^{th} grades), on five categories of pupils' parental education, on average delays per grade, and on net enrollment rates per grade. In such case, SF could be calculated with the following parameters:

- $I_{4;N}$ would be an index of equality of opportunity to be calculated according to the 'Italian procedure', but with the number of types tending to the population size. From Roemer (1998) and from our chapter 2, we know that when the number of types tend to the population size ($T \to N$), Roemer's EOp algorithm collapses to straight maximin SEF. In such a case, the calculation of conditional inequality would become redundant, so that, instead of the 'Italian procedure', we could simply compute an inequality index across individuals, as closely related to straight maximin as possible (e.g., $GE(-1)$)[6];

[6]It should be noticed that, strictly speaking, GE-class indices do not have a straightforward normative meaning. However, there is a cardinal correspondence between these indices and SWF-based indices (the Atkinson-class). The reason why we do not use the Atkinson-class directly is related to some desirable properties of the GE-class indices, es-

- $I_{8;5}$ would be an index of equality of opportunity calculated according to the 'Italian procedure', with a finite number of types, say $T = 5$ categories of parental education;
- $I_{11;2}$ would be an index of equality of opportunity calculated according to the 'Italian procedure', with a finite number of types, say $T = 2$ categories of parental education;
- D_g could be set as the average delay of students in grade g. (E.g., In Brazil, in 2005, the average gap ranged from 0.4 years in the state of São Paulo to 1.7 years in the state of Maranhão.);
- P_g could be set as the net enrollment rate in grade g. (E.g., In Brazil, in 2005, the net enrollment rate at primary school ranged from 97.5% in the metropolitan area of São Paulo to 92.9% in the metropolitan area of Salvador.)

According to our proposal of chapter 2, for more advanced grades, the number of types in Roemer's algorithm would converge to 1, such that educational inequality would be acceptable at that stage. This is why we would not include in our index any measure of educational inequality after the 11^{th} grade.

2.5 Methodological issues: an assessment

The opposition between the two first methodologies revives the debate on specific indices *versus* dominance procedure. In fact, the advantages and limits of an approach based on specific indices hold for the 'Italian strategy', while those of the dominance approach hold for the 'French strategy'.

An application of the 'Italian strategy' provides us with numbers (synthetic indices of inequality; fractions of acceptable and of unacceptable inequality), which can be compared within and across regions or countries. That may be a very useful tool to better understand particular patterns of inequality in different places. For example, in later sections of this chapter, we are able to compare Brazilian figures we calculate here with Italian figures calculated by Checchi and Peragine (2005). The drawback is that in the absence of dominance, we can never be sure whether these comparisons would be robust to another index. And this is precisely what the greatest advantage of the French approach.

An interesting feature of the 'Italian approach' is that it is very close in spirit to Roemer's conception of EOp, which is an important

pecially subgroup consistency. Subgroup consistency can be defined as: "the property that if inequality increases in a population subgroup then, other things being equal, inequality increases overall"(Cowell, 2005, p.5).

reference in the normative literature, for reasons exposed in Part I of this dissertation. While an index such as the one the 'Italian approach' offers is indeed a *specific* index, it can not be considered as a fully *arbitrary* one, given that Roemer's theory/algorithm it expresses is normatively meaningful.

What we consider in the 'Italian approach' to be a departure from Roemer's original EOp conception - the use of the $GE(0)$ index instead of one which would be closer in spirit to a maximin across types, such as $GE(-1)$ - seems to us as being in fact a recommendable change, not only due to the technical reason raised by Checchi and Peragine (2005), but also due to two other reasons. The first one is also somewhat technical. When advising on how to choose an inequality index, Cowell (1995, p.65) suggests to take into account what he calls the "discriminatory power of an inequality measure", an idea which is clearly explained as follows: "if very high inequality aversion is specified, nearly all income distributions that are encountered will register high measured inequality, so that it becomes difficult to say whether one is more unequal than another." An index such as $GE(0)$, without being insensitive with respect to inequality, has potentially more discriminatory power than a more more 'extreme' index such as $GE(-1)$.

There is also a more substantial reason to favor the use of $GE(0)$, following Moreno-Ternero (2005). In his defense of "a more equitable proposal for equality of opportunity", while adopting Roemer's algorithm, he refuses to give absolute priority to the worst-off group at each tranche. In other words, he contests the use of a maximin across types in favor of another normative position, which would reduce the weight attributed to the worst-off group, in order to increase the weight of other groups. The idea underlying his proposal is that, while such groups are not the worst-off, they might also face difficulties in terms of opportunities, and thus deserve some "help" too.[7] The inequality index corresponding to maximin in the GE-class would be $GE(-1)$. An index such as $GE(0)$, in turn, would arguably play the role expected by Moreno-Ternero, since it places less weight on inequality at the bottom of the distribution than $GE(-1)$, but still is sensitive to inequality at the bottom.[8]

One limit of the 'Italian procedure' regards its 'second step', where each observation of scores is replaced by the arithmetic mean score

[7]Such a view recalls some inconveniences of maximin / leximin positions mentioned in chapter 2.

[8]As an example of the high sensitivity of $GE(-1)$ to inequality at the bottom of the distribution, we could mention some results of Table 10 in chapter 3. The same ranking is obtained for all indices, except for $GE(-1)$, according to which the North surpasses in terms of equality the South.

within a given type and tranche, aiming at canceling out irrelevant inequality. Clearly, that is an arbitrary discrete approximation of the original continuous distribution. In their paper, Checchi and Peragine (2005) test the impact of such approximation, by comparing the inequality of the original and of the generated distributions, and they find out that the impact is very small. (We make the same test in the empirical section of this paper, reaching a similar result.) However, it is true that the final figures of the 'Italian procedure' (i.e., those expressing opportunity-inequality) obtained both in their study (and in ours) can be criticized on those grounds. Two paths for future research seem to be possible. Either studying the systematic impacts, if any, of different approximations (through sensitivity analysis, etc.) and then selecting the most appropriate among them; or turning to other approaches, such as the regression-based ones, already mentioned before.

According to the 'French approach', Roemer's definition of EOp is only a particular case of a more general conception of EOp in which an individual choosing *ex ante* among different circumstances is unable to rank them (EOp as non-dominance with a second order stochastic dominance criterion). As we said above, according to them, Roemer's position corresponds with "strong EOp", a situation in which the CDFs of every pair of types are identical. For two CDFs to be identical, the frequency distributions from which the CDFs derive must also be identical. We do not interpret Roemer's EOp as requiring identical frequency distributions across every pair of types, at least whenever we take into account Roemer's second-best compromise (or averaging formula)[9]. Indeed, generally, for Roemer's EOp to hold, outcomes must be identical *only in each tranche* and not (necessarily) along the whole distribution. Then, in the averaging formula used to define the actual EOp allocation rule, the same weight is given to each tranche (or quantile), such that after the EOp algorithm has been implemented (i.e., a reallocation of resources has moved society into a new situation) the frequency distributions (and thus the CDFs) of the outcome for each type needs not (necessarily) be identical. So, while it is true that Roemer's EOp without the averaging formula would correspond with 'strong EOp', Roemer's EOp plus the averaging formula does not. In any case, whatever the label given to Roemer's EOp, the 'French approach' is a very welcome dominance procedure for fairness analysis.

With regards to the index, which, based on our proposal of chapter 2, we labeled the 'schooling system fairness index', SF, a few limits and unresolved methodological issues should be mentioned.

The most important limit is that the term reflecting a given grade's

[9]Cf. Equation 2 in chapter 2, or Roemer (1998).

unfairness is only fully meaningful if unfair inequalities has been adequately compensated for in the previous grade. As it is stated right now, it assumes inequality of opportunities have been corrected in previous grades, which is a very particular (probably unrealistic) situation.[10] Having said that, we still believe the index can be useful. Firstly, because it is possible to regulate the parameter T, in order for it to reflect a more realistic situation. If we have reasons to believe not too much inequality has been corrected beforehand, then we could calculate the index attributing a large value to the parameter T. (The limiting case would consist of applying, at each grade, an index corresponding to straight maximin.) But if we believe part of the unfair inequalities has been corrected, then T could comfortably set a smaller T.

Secondly, suppose that over a period of some years, a schooling system becomes fairer at initial grades - an information which is captured in the first term of the index. Then it will gradually make more and more sense to calculate the SF index for such schooling system.

Thirdly, in across-space or over-time comparisons, it may be informative to observe the contribution of each grade's inequality to SF. We could compare, for example, the overall unfairness (as expressed by the SF) of two regions with similar unfair inequality at the 4^{th} grade. Such a comparison would allow us to identify which of the two treats more fairly its students in subsequent grades.

One methodological concern is the weights to be given to the parameters α, β, γ. We have no clue right now on how to set those parameters in a non-arbitrary way. A study on the actual policy trade-offs implied by different values of such parameters is welcome in our future research.

The three approaches mentioned in this chapter are attractive for different reasons and useful for different objectives. In our view, they can be employed in complementary ways, given that each will provide contrasted perspectives on the same issue - of measuring unfair inequalities of test scores. In the remaining of this chapter, we employ each of them in turn.

3 Inequality of educational opportunities in Brazil

We mentioned in chapter 3 that Brazil is a very unequal country in terms of educational quality (according to an international compari-

[10] Such problem would not be solved simply by shifting from an approach based on *grades* to one based on *cohorts*.

son employing PISA data). We studied in some length the pattern of Brazilian test scores inequality using SAEB data, identifying a considerable intra-country variation of the intensity of gross inequality. Now it is time to understand to what extent Brazilian inequalities are fair or unfair, that is, which fraction of inequality is due to choices and which one is due to circumstances, and whether regional variation is sizable.

Studying unfair inequalities is an interesting topic for its own sake, for reasons we mention throughout this dissertation - in fact, in this chapter we bridge an explicit link with Part I, and also with Part III (especially, chapter 6). Besides, studying unfair inequalities is also a complement to the study gross inequalities. By the end of this chapter we will be able to compare *gross inequality*, which was the object of chapter 3, with *opportunity-inequality* and *effort-inequality*, which are ethically connotated conceptions of inequality, and which to our knowledge up to now have been calculated only in a few studies.

3.1 Data, definitions, and descriptive statistics

We use here the SAEB dataset, described in the previous chapter. We report national and regional descriptive statistics regarding only 2001, math, and 8^{th} grade, except when we calculate SF, because in such case we need to use data regarding the three grades.

We need to condition test scores upon individuals' circumstances (*types* in Roemer's terminology). One of the types on which we condition scores is pupils' parental education (the highest education level among parents). Let us mention a few reasons why we use such information to define types. First of all, because pupils' parental education seems to appropriately play the role of a "relevant circumstance". Starting with the so-called "Coleman report" (Coleman et al., 1966), a vast literature in the economics of education has established that the influence of variables related to socio-economic status (SES) on achievement is considerable. Pupil's parental education typically is positively correlated to other SES variables (wealth, occupational status, type of neighborhood where family lives in, and so on), so it can be viewed as a variable summarizing SES. The role of SES as a determinant of achievement is particularly strong in Brazil, as have shown studies based, both on SAEB data (Albernaz, Ferreira, and Franco, 2002), and on PISA data (Waltenberg, 2005). Finally, while not talking about performance in test scores, but about years of schooling, Cognaux and Gignoux (2005) usefully remind us that Brazil has one of the lowest intergenerational educational mobility, reinforcing the

evidence that parental education plays an important in shaping educational opportunities.

Another requirement for a variable to define a type is that it is not under control of the individuals, nor subject to manipulations by them. While the true information on pupil's parental education is not under control of the individuals (in the sense that it is exogenous to a particular individual), the value which is reported can be manipulated by pupils, since often it is a self-reported variable. However, we do not see why pupils would adopt any kind of strategic behavior in this respect when answering a questionnaire such as the one accompanying the SAEB exams. Yet, it should be acknowledged that such variable is subject to misreporting, a problem self-reported variables are typically subject to.

Parental education is usually available in test scores datasets, as well as in other datasets, which ensures comparability. For example, in the specific case of the empirical application we undertake here, we are able to compare our results to those coming from a similar study made using Italian data. In the future, we could also reproduce the exercise using PISA, and other national and international datasets, producing thereby comparable results, which would be helpful to put in perspective the ones we provide in this chapter.

Pupils' parental education is thus a good candidate for a "relevant circumstance", since it has a great influence on the outcome of pupils, it is often available, and for practical matters it is not under control of pupils.

As we need more information than in the previous chapter, there is an increased risk of having to deal with missing observations. In Table 1 are reported some descriptive statistics on scores at both the national and the regional level. We observe in Panel A the descriptive statistics concerning the pooled sample. Panel B shows the statistics for the sub-sample of observations for which information on parental education is missing. There is missing information concerning 3,748 observations out of a total of 50,300, which represents 7.45% of the total (or 7.77% of the weighted observations). Another important feature we can read in this table is that the average score obtained by the pooled sample (243.38) is higher than the average score obtained by the sub-sample (227.13, which corresponds to 93.32% of the pooled sample's average score).[11] It is not surprising that pupils who are not able to report their parents' education obtain lower scores than

[11]The same pattern is observed for regions' statistics. The fraction of missing observations ranges from 6.13% in the South to 8.29% in the North-East; while the ratio between sub-sample average scores and pooled-sample average scores ranges from 92.26% in the South-East to 95.06% in the North-East.

Table 1: Checking impact of missing information.

Panel A: Pooled sample

Region	Mean Score	Std. Dev.	Obs.	Freq.
North	231.86	42.44	7,972	188,469
North-East	228.79	46.13	20,166	800,674
South-East	249.72	51.83	8,672	1,379,759
South	255.34	45.96	6,251	397,108
Mid-West	244.83	45.76	7,239	236,262
Brazil	243.38	49.62	50,300	3,002,272

Panel B: Subsample with missing information on parents' education

Region	Mean Score	Std. Dev.	Obs.	Freq.
North	219.88	36.72	559	14,623
North-East	217.48	41.27	1,671	71,163
South-East	230.4	46.00	636	102,581
South	241.54	39.52	383	25,945
Mid-West	231.59	39.08	499	19,022
Brazil	227.13	43.49	3,748	233,334

those who are able to do so. From now on, we only use the 46,552 observations for which we have information on parents' education, but the slight bias mentioned here should be acknowledged.

In Table 2 we can observe descriptive statistics of scores, by region and by type of pupil. The national average score for the valid observations is 244.75, with standard deviation 49.86. The $46,552$ valid observations expand to $2,768,938$ pupils because of sample weights.[12] The most populated region is by far the South-East ($1,277,178$ weighted observations), more than seven times larger than the least populated region (North, with $173,846$ weighted observations).

As parents' education increase, average scores increase too, both in the national level (ranging from 213.11 to 286.52) and inside each region. The ranking of regions in terms of average scores is the one already mentioned in chapter 3: S $\succ$ SE $\succ$ MW $\succ$ N $\succ$ NE. This overall ranking varies slightly according to the type we turn to. For example, among kids with highly-educated parents (college), the ranking is: SE $\succ$ S $\succ$ MW $\succ$ NE $\succ$ N, while the ranking among kids with poorly educated parents (lacking formal education) is: MW $\succ$ SE $\succ$ S $\succ$ N $\succ$ NE.

3.2 Specific inequality indices

In this section are reported results obtained regarding Brazilian educational data, following the procedure exposed in section 2.2 and previously applied by Checchi and Peragine (2005) to Italian data.

3.2.1 Preliminary checks

The first step is to check how scores vary according to quantiles (deciles here) and to pupils' types (their parents' education here), which is shown at the country level in Table 3. For a given type, we observe similar frequencies across deciles.[13] We also observe that, at any given decile, scores increase with the type (as we move from the top to the bottom of the table). We confirm thereby a pattern which was expected.[14]

[12] In fact, the $46,552$ valid observations expand to $2,768,938.40$, but we omit the decimals in this table and in the remaining ones.

[13] *Frequencies* are *similar*, but not equal, from the construction of the per-type deciles (in terms of weighted frequencies, and not in terms of observations) and the discrete nature of scores (with continuous data, we would have identical frequencies).

[14] We have also made the same calculations for each region. The same pattern observed at the national level is verified, with a few violations of the monotonicity of scores with types: (i) region=SE, decile=1, types= 1, 2, 3; (ii) region=S, decile=1, types= 2, 3; (iii) region=MW, decile=6, types= 1,2.

Table 2: Descriptive statistics related to pupil's mother's education.

Means, Standard Deviations, Frequencies and Number of Observations of Score

Parents' Education	**Region**					**Brazil**
	N	NE	SE	S	MW	
No Formal	216.68	207.40	217.91	216.94	224.89	213.11
	36.41	35.12	34.09	41.12	39.26	36.03
	7,516	61,937	39,905	8,106	8,845	126,309
	253	1,170	188	76	182	1,869
Lower Prim.	223.51	216.95	234.15	243.03	229.88	229.29
	37.49	37.79	41.71	40.68	38.20	40.98
	42,967	234,858	358,861	99,650	56,536	792,873
	1,537	4,600	1,429	1,014	1,169	9,749
Upper Prim.	227.07	224.60	238.58	248.11	237.43	235.85
	39.45	39.79	44.03	41.20	39.55	42.72
	45,317	182,362	362,977	110,756	62,813	764,224
	1,668	3,895	1,710	1,290	1,531	10,094
Secondary	238.48	241.83	259.68	262.75	253.49	253.28
	41.84	46.60	50.91	44.64	44.05	48.58
	54,104	173,069	295,437	93,295	54,752	670,657
	2,337	5,081	2,127	1,561	1,728	12,834
College	253.00	273.05	294.93	289.17	281.71	286.52
	51.43	57.11	54.84	47.83	51.69	55.15
	23,942	77,286	219,998	59,356	34,294	414,876
	1,618	3,749	2,582	1,927	2,130	12,006
Total	232.87	229.90	251.28	256.31	245.99	244.75
	42.73	46.43	51.97	46.23	46.12	49.86
	173,846	729,512	1,277,178	371,163	217,240	2,768,938
	7,413	18,495	8,036	5,868	6,740	46,552

Table 3: Scores according to types and deciles. (Table continues in the next page...)

Means, Standard Deviations, Frequencies and Number of Observations of Score

Parents' Education	**Decile**					**Total**
	1	2	3	4	5	
	158.34	176.95	186.97	195.71	203.95	213.11
No Formal	8.13	4.02	2.96	2.28	2.35	36.03
	12,648	12,652	12,766	12,461	12,637	126,309
	187	170	164	148	175	1,869
	165.19	185.39	198.16	210.58	221.74	229.29
Lower	9.50	4.01	3.43	3.70	2.95	40.98
Primary	79,350	79,718	78,916	79,224	79,246	792,873
	1,025	948	1,022	1,13	972	9,749
	166.59	189.35	203.78	216.81	228.20	235.85
Upper	10.44	4.41	4.07	3.48	3.16	42.72
Primary	76,595	76,534	76,140	76,543	77,293	764,224
	944	967	1,089	996	948	10,094
	174.95	200.45	217.38	231.40	244.45	253.28
Secondary	11.73	5.84	4.20	3.75	3.92	48.58
	67,072	67,235	67,149	66,934	67,017	670,657
	1,084	1,196	1,110	1,121	1,269	12,834
	188.42	223.98	246.35	265.79	282.52	286.52
College	16.07	7.25	5.95	5.34	4.23	55.15
	41,499	42,653	40,336	41,497	41,552	414,876
	861	925	1,055	1,232	1,214	12,006
	171.10	195.63	210.95	224.95	237.30	244.75
Total	14.23	14.57	17.30	19.78	21.75	49.86
	277,164	278,792	275,307	276,659	277,745	2,768,938
	4,101	4,206	4,44	4,627	4,578	46,552

Table 3: (...Continued from previous page)

Means, Standard Deviations, Frequencies and Number of Observations of Score

Parents' Education	**Decile**					**Total**
	6	7	8	9	10	
	213.51	223.85	236.06	254.11	282.71	213.11
No Formal	3.08	3.33	4.12	6.13	17.33	36.03
	12,644	12,627	13,182	12,075	12,615	126,309
	186	206	196	216	221	1,869
	232.88	244.45	256.73	271.62	306.51	229.29
Lower	3.40	3.23	3.74	5.37	21.11	40.98
Primary	79,330	79,342	79,174	79,787	78,786	792,873
	1,012	907	881	932	920	9,749
	240.05	252.90	267.00	282.06	312.07	235.85
Upper	3.51	4.04	4.16	4.73	19.44	42.72
Primary	75,432	76,463	76,480	76,414	76,330	764,224
	1,079	1,076	992	870	1,133	10,094
	258.22	271.68	286.91	305.65	341.97	253.28
Secondary	3.93	3.66	5.09	6.42	20.28	48.58
	66,988	67,152	67,094	67,015	67,001	670,657
	1,328	1,266	1,376	1,369	1,715	12,834
	297.19	312.07	326.88	345.81	376.92	286.52
College	3.73	4.28	4.44	6.94	13.29	55.15
	41,442	41,542	41,407	41,466	41,482	414,876
	1,134	1,408	1,468	1,441	1,268	12,006
	249.77	262.58	276.35	293.10	326.14	244.75
Total	23.16	24.28	25.26	26.85	32.71	49.86
	275,837	277,126	277,337	276,758	276,215	2,768,938
	4,739	4,863	4,913	4,828	5,257	46,552

Another check we have undertaken, following Checchi and Peragine (2005), is a comparison of the original distribution and the artificially generated distribution (the one which cancels out the within-tranche-and-type inequality). The results of this check (not reported here) show the transformation does not affect inequality very much, since the ratio between the inequality - according to $GE(0)$ - of the two distributions is always around 1, both at the national level, and for each region.

3.2.2 Main results

The main results of this section are reported in Table 4, in which we observe the decomposition of overall inequality into opportunity-inequality and effort-inequality.

Based on the parameters we have used, which express a very parsimonious definition of circumstances, we immediately observe two important rankings - columns A and A/C - which equally rank Brazil's regions: N (smaller inequality and smaller fraction) $\succ$ S $\succ$ MW $\succ$ NE $\succ$ SE (large inequality and larger fraction).

In terms of *magnitude of inequality of opportunity*, the result for Brazil's fairest region (North, 0.0009449) is comparable with Italy's results. Inequality of opportunity in that region is 33.27% larger than that of Italy's fairest region (Northern Italy, 0.000709) and 30.91% smaller than that of Italy's unfairest region (Center-Southern Italy, 0.0013677). Thus, those results place Northern Brazil in an intermediary situation between the two Italian regions.

However, when we turn to the second fairest region in Brazil (South, 0.0020801), inequality of opportunity is already much more substantial: 52.08% more than Italy's *un*fairest region. Brazil's unfairest region shows a level of inequality of opportunity (South-East, 0.0037840) which is 176.67% larger than that of Italy's unfairest region (Center-Southern Italy, 0.0013677) and 433.71% larger than that of Italy's fairest region (Northern Italy, 0.000709). So, with the exception of the North, unfairness is much greater in Brazilian regions than in the Italian regions. Considering that only a small fraction of Brazilian pupils study in the Northern region (6.28% of the final weighted sample we used) the overall picture is one of considerable inequality of opportunity in Brazil.

The *fraction of inequality which is due to differential opportunity* amounts to 16.07% of overall inequality in Brazil, and we observe substantial variation across regions. The North is the least unequal one with 5.66%, while the South-East is the most unequal one with 18.06% of overall inequality due to inequality of opportunity.

Table 4: Decomposing inequality into opportunity and effort inequality

	Inequality			Percentage
	...of opportunity (A)	...of effort (B)	Overall (C)	Opportunity/Overall (A/C)
Brazil	0.0032685	0.0170664	0.0203350	16.07%
North	0.0009449	0.0157431	0.0166879	5.66%
North-East	0.0029423	0.0164729	0.0194152	15.15%
South-East	0.0037840	0.0171662	0.0209502	18.06%
South	0.0020801	0.0145051	0.0165852	12.54%
Mid-West	0.0024914	0.0147000	0.0171914	14.49%

Inequality of effort is comparable across regions (column C), ranging from 0.0145051 (South) to 0.0171662 (South-East), with a national level of 0.0170664. These levels of effort inequality are not considerably different from the range found by Checchi and Peragine (2005), namely, 0.0126289 in Northern Italy and 0.0174451 in Center-Southern Italy. So the difference in the magnitude of scores inequality between Italy and Brazil is essentially due to opportunity-inequality, and not to effort-inequality.

3.3 Dominance analysis

In this section, using Brazilian educational data, we apply the procedure exposed in section 2.3, and which has been previously applied by Pistolesi, Lefranc, and Trannoy (2005) to international data on incomes.

According to Pistolesi, Lefranc, and Trannoy (2005) the opportunities offered to an individual of a given type are summarized by the CDF of her type. By plotting CDFs of different types we can verify, for every pair of CDFs, whether there is inequality of opportunity (curves do not cross), weak equality of opportunity (curves cross) or strong equality of opportunity (curves are identical).

We recall that, adapting to our purposes the notation defined by Pistolesi, Lefranc, and Trannoy (2005), we set the conditional scores function as $F(s|t)$, where s stands for individual pupils' score, as usual, and T stands for type. Type is defined as five categories of pupils' parental education. Since we are using exactly the same data as in

the previous section, refer to Tables 2 and 3 for descriptive statistics.[15]

3.3.1 Main results

Figure 1 depicts the CDFs for the 5 types of pupils in Brazil. We can observe that the higher the level of parents' education (*isced*5) the better is the performance of pupils. There is no crossing, which means that we are in a situation of *inequality of opportunity*, for any pair of types we look at. We can observe, though, that the gap across types is not uniform. For example, the difference from the curve associated with parents having higher education (*isced*5) and that of parents having high-school certificate (*isced*4) is larger than the difference between the curves associated with *isced*4 and *isced*3. The largest differences are to be found between the two highest performing types of pupils (*isced*5 and *isced*4) and between the two lowest performing types of pupils (*isced*2 and *isced*1). The same pattern is reproduced almost exactly at each of the 5 macro-regions (regional graphs not reported here).

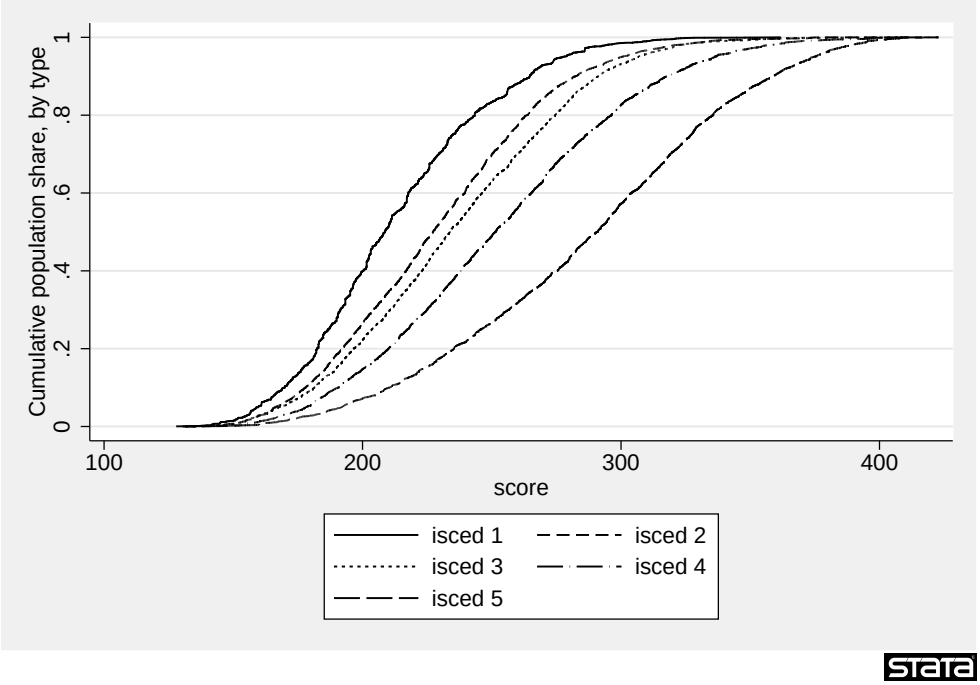

Figure 1: CDFs, with parents' education (isced) defining types

[15]In the CDFs graphics, we use the label *isced* for parental education level, and we have: *isced*1: no formal education; *isced*2: lower primary education; *isced*3: upper primary education; *isced*4: secondary education; *isced*5: college education.

We still have to check whether these results resist statistical inference tests (in our plans for future research), especially whether the types *isced2* and *isced3* are not in a situation of strong equality of opportunity.

3.3.2 An alternative definition of types

We have repeated the exercise using an alternative definition of types: pupil's skin color. Which definition of types - in terms of skin color, of parental education, or of a combination of the two - represents a more legitimate definition of types is an open question, which has recently given rise to an intense debate in Brazil concerning the legitimacy of affirmative action policies for higher education. An affirmative action policy for higher education would certainly benefit relatively well-off non-whites individuals (those who would have surpassed uncountable obstacles to reach the more advanced levels of the Brazilian educational system). While it may be legitimate to grant access to college to non-whites in order to ensure the emergence of diversity among the elite - with possible positive effects, for example, in terms of enhanced motivation for non-white children, and so on -, if the objective is to actually improve opportunities for a larger fraction of non-whites, affirmative action policies would have to be implemented in much earlier stages of the schooling system, possibly at primary school. The objective of the exercise here is simply to contribute to such debate, diagnosing whether inequality of opportunity in terms of skin color can be identified in test scores data, that is, when individuals are still at school.

The variable we employ here originates from a choice made by pupils for defining their skin color themselves, among 5 possible categories: white, mixed, black, Asian, native-Brazilian. Descriptive statistics regarding this variable in its relation with scores are reported in Table 5. There is less missing information on pupils' self-reported skin color (702) than on parents' education (3748), which turns to be an advantage of defining types on the basis of skin color over defining them on the basis of parental education.[16] One characteristic of this definition is that the groups (types) created this way are too different in terms of size: while whites and mixed together amount to 82.74% of the total population, native-Brazilians are only 2.71% of the total, and blacks and Asians amount to less than 8% each. In some regions, the presence of some types is negligible. For example, the sum of blacks (4.08%), Asians (3.76%) and native-Brazilians (1.31%)

[16] Although we have no information on the relative reliability of those two self-reported variables

Table 5: Descriptive statistics related to pupils' skin color

Means, Standard Deviations, Frequencies and Number of Observations of Score

Race			**Region**			**Brazil**
	N	NE	SE	S	MW	
White	234.74	231.22	260.57	259.67	250.36	252.93
	45.36	49.47	53.35	46.88	48.64	52.07
	57,976	260,924	667,992	278,067	99,225	1,364,183
	2,742	6,887	4,482	4,668	3,494	22,273
Mixed	230.81	227.91	239.56	244.53	242.54	235.55
	39.99	44.14	46.73	41.48	43.41	45.13
	92,400	367,518	476,853	78,521	89,581	1,104,874
	3,733	8,947	2,642	964	2,476	18,762
Black	225.94	221.09	225.11	240.60	230.13	225.24
	44.29	40.83	41.33	44.25	38.33	41.56
	12,542	83,869	112,775	16,933	19,292	245,411
	456	1,875	668	253	457	3,709
Asian	242.13	241.46	259.29	255.77	247.69	251.68
	43.26	49.31	58.50	43.93	43.50	53.11
	13,847	50,244	86,226	15,363	18,263	183,943
	637	1,500	592	233	550	3,512
Native	226.34	229.88	246.39	251.64	239.82	238.23
	38.30	43.46	42.60	32.74	43.99	42.88
	7,371	22,368	25,769	4,019	6,324	65,852
	271	611	206	81	173	1,342
Total	232.39	229.20	249.99	255.59	245.17	243.75
	42.39	46.17	51.83	45.99	45.72	49.62
	184,137	784,923	1,369,614	392,903	232,686	2,964,262
	7,839	19,820	8,590	6,199	7,150	49,598

amounts to less than 10% in the South, while in that same region 3/4 of pupils report themselves as being white.

An interesting result with this new definition of types is that clearcut situations of both kinds are found: equality and inequality of opportunity. In fact, we find: (i) equality of opportunity (at least in the weak sense) between whites and Asians, and between mixed and native-Brazilians, and (ii) inequality of opportunity between blacks and any other group, and between the pair mixed/native and the pair white/Asians.

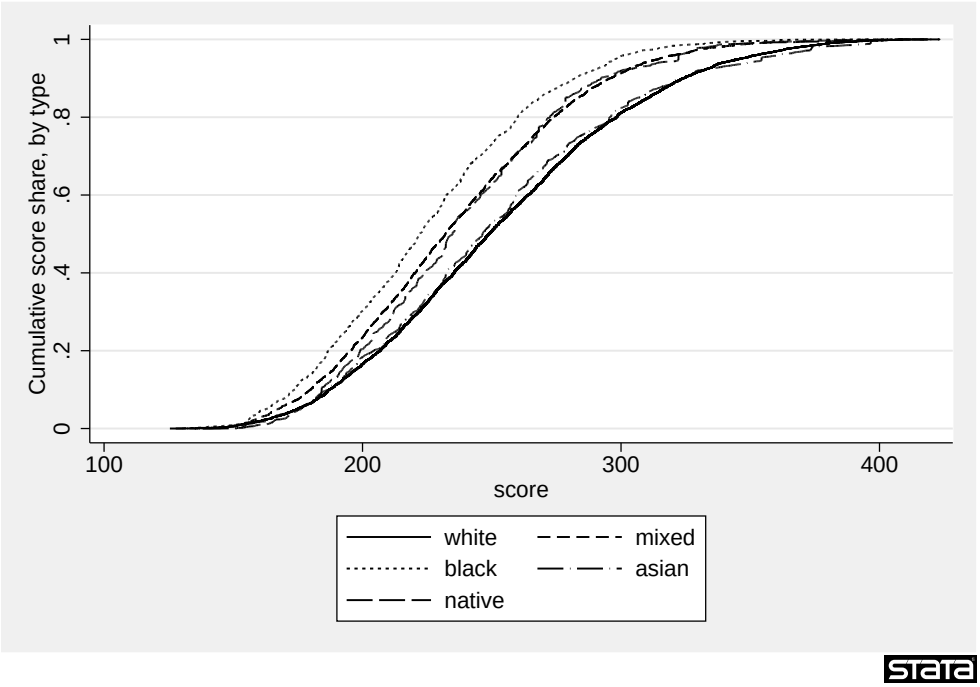

Figure 2: CDFs, with self-reported skin color defining types

Although such results should be verified by statistical inference, they lead us to provisionally conclude that there is inequality of opportunity for achievement in terms of skin color in Brazil.

3.4 Calculating a 'schooling system fairness index'

We have applied the SF index defined above following the example presented in section 2.4, that is, using SAEB test scores for three grades (4^{th}, 8^{th} and 11^{th}) and using data on pupils' parental education to define the types. The parameters α, β, γ have been set to 1, 0, and

0, respectively, which implies that the correction terms, D_g and P_g, have not been taken into account in this version (due to insufficient time to gather appropriate data). As for the inequality index applied to each grade, we have:

- $I_{4;N}$: for the 4^{th} grade, we use $GE(0)$ across individuals;
- $I_{8;5}$: for the 8^{th} grade, we follow the 'Italian procedure', employing $GE(0)$ and 5 types of individuals ($isced1$ to $isced5$);
- $I_{11;2}$: for the 11^{th} grade, we also follow the 'Italian procedure', employing $GE(0)$ and 2 types of individuals;

  (i) 0 stands for 'None of the parents has at least secondary education'; and

  (ii) 1 stands for 'At least one of the parents has at least secondary education';

Table 6: schooling system fairness index

Component	**Region**					**Brazil**
	N	NE	SE	S	MW	
SF_4	0.02483	0.03053	0.03390	0.02645	0.03017	0.03420
Share of SF	93.28%	84.42%	84.07%	87.83%	86.85%	85.91%
Ranking*	1	4	5	2	3	–
SF_8	0.0009449	0.0029423	0.0037840	0.0020801	0.0024914	0.0032685
Share of SF	3.55%	8.14%	9.38%	6.91%	7.17%	8.21%
Ranking*	1	4	5	2	3	–
SF_{11}	0.0008431	0.0026927	0.0026400	0.0015836	0.0020779	0.0023408
Share of SF	3.17%	7.45%	6.55%	5.26%	5.98%	5.88%
Ranking*	1	5	4	2	3	–
SF index	0.0088727	0.0120550	0.0134413	0.0100379	0.0115798	0.0132698
Ranking*	1	4	5	2	3	-
SF x 1,000	8.87	12.06	13.44	10.04	11.58	13.27

* 1= less unequal region; 5 = more unequal region

The main results are summarized in Table 6. In each grade's panel, the line labeled 'Share of SF' indicates the contribution of that grade's inequality index to the overall SF index shown in the bottom of the

table. We also report opportunity-inequality rankings, both for each grade, and for the system as a whole (SF index). Finally, we report in the bottom line the value of SF multiplied by 1,000, simply in order to make it easier to compare the magnitudes of regional indices.

The ranking is stable across the two initial grades, repeating a pattern we already found many times in this chapter and in the previous one: N (smaller inequality) $\succ$ S $\succ$ MW $\succ$ NE $\succ$ SE (larger inequality). When we turn to SF_{11}, there is a rank reversal involving South-East and North-East. Actually, the level of inequality of opportunity is quite similar in those regions for that grade (only the 5^{th} decimals reveal the rank reversal). So, it is not a surprise that when all grades' SFs are aggregated (with equal weight), North-East surpasses the South-East again.

Also deserves some comments the extremely high contribution of SF_4 to SF (at least 84.07%). One reason for that is that the SF index, as it has been calculated here, is assigning relatively much importance to inequality of opportunity in the 4^{th} grade at the expense of inequality of opportunity in the other grades. Indeed, it is hardly surprising that the magnitude of SF_4 is much larger than that of SF_8 and SF_{11}, due to the fact that $I_{4;N}$ has been calculated across individuals, while $I_{8;5}$ and $I_{11;2}$ have been calculated across types of individuals, with a relatively small number of types. However, it should be noticed that in chapter 2 we have precisely defended the view that inequality of opportunity in initial grades is the most serious one from an ethical perspective, and for this reason, Roemer's algorithm - and, analogously, here, inequality measures - should strongly penalize inequality at this level.

Another reason explaining the large contribution of SF_4 to SF is related to the larger scores inequality we previously found for the 4^{th} grade with respect to the two other grades (in chapter 3). As we said there, such pattern could be explained by two different reasons: (i) pupils being naturally more heterogeneous in the initial grades when selection and drop out have not yet taken place, (ii) in the initial grades, schools might not yet have been able to reduce pupils's performance gaps, an objective which is more likely to be achieved in more advanced stages.

In the methodological section, we said that the most important limit of SF is the fact that it assumes inequality of opportunities has been adequately corrected in previous grades, which is a very particular and probably unrealistic assumption. Indeed, it seems unrealistic to assume that the large inequalities of the 4^{th} grade (cf. SF_4) could be compensated and then (possibly) reemerge four years later in the 8^{th} grade - it seems more realistic to believe that they have not been com-

pensated (and possibly even accumulated over time), which would call for a much higher value for T than 5 in $I_{8;5}$, and than 2 in $I_{11;2}$. As we said before, the limiting case would consist of applying for each grade the same index used for the 4^{th} grade. If we do that, the SF becomes simply a weighted sum of each grade's SF. We have computed such index for Brazil: $SF = 0.0248501$ (or 24.85, when it is multiplied by $1,000$), which is much higher than the 13.27 reported in Table 6. The share of SF_4 goes down to 45.88% (still much higher than 1/3), and the shares of SF_8 and SF_{11} raise to 27.63% and 26.50%, respectively.

4 Conclusions

In the methodological sections of this chapter, we discuss advantages and limits of three alternative methodologies which can be used for measuring and comparing inequality of educational opportunities, all of which are based on, or try to go beyond, Roemer (1998). Section 2.5 contains an appraisal of advantages and limits of each approach, as well as open questions regarding the SF index, and section 3 for particular comments related to the applications. The three approaches have relevance for different purposes and, in our view, they can be employed in complementary ways, which is precisely what we did in the empirical sections of the chapter.

The *dominance analysis* reveals a situation of evident inequality of opportunity when types are defined in terms of parental education, while when types are defined in terms of skin color we obtain both inequality of opportunity and (at least weak) equality of opportunity, depending on the types we compare. The *specific indices* approach show that inequality of opportunity represents 16.07% of overall inequality in Brazil, and we observe large variation across regions. Brazil's most opportunity-unequal region (South-East) is 176.67% more unequal than Italy's most opportunity-unequal region (Center-South) and 433.71% more so than Italy's least opportunity-unequal region (North). The ranking based on a simplified version of the SF index, which does not take into account delay- and drop-out-correction, is the following: N (smaller inequality) $\succ$ S $\succ$ MW $\succ$ NE $\succ$ SE (larger inequality).

Various, alternative, specifications can be tested and a series of improvements can be implemented to all three applications (we have mentioned a few in the text). It is also possible to deepen our analysis focusing on particular sub-national units, in order for our results to be more useful for policy use. We plan to accomplish those tasks in the close future.

Table 7: Summary

Region	A	B	C	D	E	F	G	H	I	J
SE	42.7	57.8	46.13	2	2	1	4=	5	5	5
NE	28.2	12.9	26.35	5	5	4	4=	4	4	4
S	14.8	17.6	13.40	1	3	2	1=	2	2	2
MW	7.6	4.6	7.85	3	1	3	1=	3	3	3
N	6.9	7.0	6.28	4	4	5	1=	1	1	1

A: Population share.
B: GDP share.
C: Share in SAEB sample (subsample used in this chapter).
D: Ranking: Average score.
E: Ranking: Average score for mothers with no formal study (*isced1*).
F: Ranking: Average score for mothers with college education (*isced5*).
G: Ranking: Lorenz dominance.
H: Ranking: Gross scores inequality, according to $GE(0)$.
I: Ranking: Inequality of scores-opportunity (Italian procedure, with $I_{8,5}$).
J: Ranking: Simplified version of SF index.

Table 7 provides a summary of the results we have obtained in chapters 3 and 4, and we emphasize the following:

- The two most populated regions (as shown in columns A and C) are the most unequal ones (columns G-J), but a large population alone does not imply high inequality, since the South is the third most-populated in the country, but it presents a relatively low inequality record;
- The Southern region shows encouraging results: it is a relatively rich region, highly populated, presenting high average scores, and finally relatively small inequalities;
- The result found for the North is not so enviable, because, while presenting low inequality, its average scores are low both for the pooled sample (column D) and for the restricted samples of poorly- and highly-educated pupils' parents (columns E-F);
- The Mid-West occupies an intermediary position in almost all the rankings, with the exception of the average scores for poorly educated parents (column E), where it is first;
- The most unequal regions are also those which present more inequality of opportunities (when circumstances are defined in terms of parental education), given that the rankings expressing gross inequalities (columns G and H) coincide with those expressing inequality of opportunity (columns I and J);
- The contrasting positions occupied by the South-East and the North in the ranking of average scores (column D) and in those of inequalities (columns G-J) could misleadingly suggest a kind of "equity-efficiency trade-off"; the opposite results obtained by the South and the North-East play the role of counter-examples.

In the light of a reasonable definition of justice in education (from Part I), and knowing the intensity of inequality and unfairness of Brazil's and its regions' schooling systems (from Part II), we can now proceed to a stage in which we investigate if and how the allocation, or the reallocation, of certain educational resources can contribute to moving educational systems towards a fairer situation (Part III). Education production functions are specified and estimated, in order to express as accurately as possible the relationship between educational resources, educational outcomes, and possible policy alternatives.

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