CHAPTER 6

Summary and perspectives I

In this chapter, we summarize the main results of the experiments presented in the previous chapters and draw implications for future research.

1. Linear vs. logarithmic

In our first two studies, two different hypotheses were examined to account for the observation of over- and underestimation in numerosity estimation tasks. According to the first hypothesis (i.e., the bi-directional mapping hypothesis), the occurrence of over- or underestimation depends on the direction of the mapping process that takes place between two differentially scaled representations of numbers (overestimation with a symbolic/linear to non-symbolic/logarithmic mapping and underestimation with a non-symbolic/logarithmic to symbolic/linear mapping). According to the second hypothesis (i.e., the operational momentum hypothesis), the occurrence of the over- and underestimation depends on the direction of the internal spatial movement that has to be made on the representations of numbers (overestimation with a rightward displacement and underestimation with a leftward displacement). The results of our two first studies clearly supported the bi-directional mapping hypothesis. In our first study (Chapter 3), we clearly showed that a symbolic to non-symbolic mapping systematically led to overestimation while a non-symbolic to symbolic mapping systematically led to underestimation. In this study, it was stated that, according to the bi-directional mapping hypothesis, participants should underestimate in the perception task and overestimate in the production task. In

Castronovo and Seron's study (2007a), on another hand, the under- and overestimation were first observed and then resulted in the bi-directional mapping hypothesis. So, one might argue that a circular reasoning was used to explain the outcomes that supported the bi-directional mapping hypothesis. However, as we investigated participants' performance in a new experiment never studied in the literature (i.e., the reproduction task), our first study allowed us to extend and to further support the *"a posteriori"* model proposed by Castronovo and Seron (2007a).

In our second study (Chapter 4), we demonstrated that the overestimation found in numerosity estimation tasks was not due to an operational momentum. Indeed, overestimation of the target numerosity was found with a symbolic-to-non-symbolic numerical mapping, irrespective of the direction of the spatial displacement that has to be made on the mental number line (i.e., from left-to-right or from right-to-left). In this last task, we did, however, not control the distance between the initial display and the target magnitudes. Indeed, this distance was always larger in the leftward condition (i.e., minimum = 254 – 98 = 156) than in the rightward condition (i.e, maximum = 98 – 7 = 91). So, there could be an alternative (and more trivial) interpretation to the anchoring effect proposed to account for the larger overestimation observed in the leftward compared to the rightward condition: the strong tendency to overestimate could be more pronounced when the task is difficult or, in other words, when the distance between the target and the display is larger. This alternative explanation did nevertheless not question the fact that our results supported the bi-directional mapping hypothesis. However, in order to get a clearer picture of the processes involved in numerosity production tasks, it might be worth conducting a new experiment that controls the distance between the initial display and the target magnitude. By using an initial display of 121 dots, the minimum and

maximum distance between the display and the target would have been the same in the two conditions. However, as the ratio separating 98 (i.e., the largest target magnitude) and 121 is very close from 1 (i.e., .81), using 121 as the magnitude of the initial display would not have systematically induce participants to make a leftward displacement on their mental number line.

As mentioned in our introduction, the operational momentum hypothesis was first proposed to account for the over- and underestimation found in approximate arithmetic tasks (e.g., McCrink et al. (2007). Given that the approximate arithmetic tasks used to study this effect did not involve any mapping process between the symbolic and non-symbolic representations of numbers, the over- and underestimation observed in this kind of task cannot be accounted for by the bi-directional mapping hypothesis. However, as noted by McCrink et al. (2007) themselves, the arithmetic estimation errors could merely reflect the compressive scaling of the mental number line. As numerosities are represented farther apart at the low end of the number line than at its high end (see Dehaene, 2001), it is possible that the addition or subtraction of two sections of the mental number line can by itself explain the over- and underestimation observed in non-symbolic arithmetic tasks (Gallistel & Gelman, 1992; McCrink et al., 2007; Restle, 1970) (see Figure 6-1 for an illustration).

Moreover, as the operational momentum has also been observed with Arabic number symbols (Knops et al., 2009; Pinhas & Fischer, 2008), it is possible that attentional displacements along the symbolic numerical representation also affects numerosity perception judgments. However, although every production task involves an additive component which can be seen as the origin of the overestimation error, the perception tasks generally used in the literature do not involve any subtractive

component. So, the underestimation found in such tasks cannot be seen as involving the same logic as the one behind the momentum hypothesis. The bi-directional mapping and the operational momentum hypotheses are therefore not mutually exclusive. They have been postulated in order to account for similar phenomena (over- and underestimation) but in two different tasks that do not imply the same process (mapping from one representation to another vs. computation on a single representation).

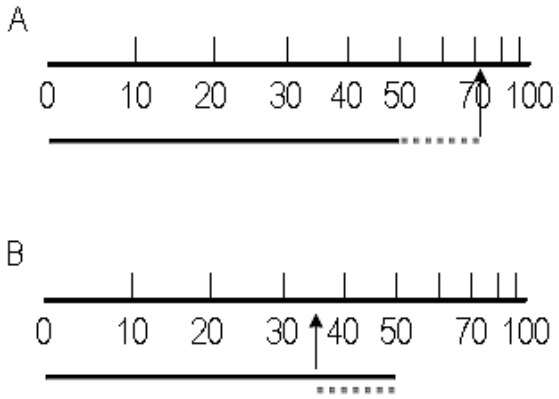


Figure 6-1. The momentum effect observed in approximate arithmetic tasks could be the consequence of the use of the non-symbolic numerical representation: **(A)** the addition of two sections (the numerosity 50 in solid black line and the numerosity 10 in dotted grey line) of the analogue numerical representation might lead, as shown by the black arrow, to the overestimation of the true outcome; **(B)** the subtraction of the two sections might lead to the underestimation of the true outcome.

The observation that the approximate addition and subtraction of two Arabic symbols leads to smaller over- and underestimation errors (Knops et al., 2009; Pinhas & Fischer, 2008) than the addition and subtraction of two non-symbolic numerosities is nevertheless very much in line with some basic concepts of the bi-directional mapping hypothesis; among these, the fact that adults possess different numerical representations (i.e., symbolic vs. non-symbolic) and the fact that these different representations vary in their precision (i.e., linear vs. logarithmic). As the symbolic numerical representation is more precise and linear than the non-symbolic one

(Piazza et al., 2007; Verguts & Fias, 2004), it is indeed normal that participants are less biased when they perform symbolic arithmetic than when they perform non-symbolic arithmetic. In this sense, the over- and underestimation observed in addition and subtraction may actually reflect the use of two differently scaled numerical representations (a non-symbolic representation which is logarithmically compressed and a symbolic representation which is more linear and precise) rather than an operational momentum. Future research should examine this possibility by testing whether individual differences in the mental number line acuity (*w*) (Halberda et al., 2008) could predict the size of the over- and underestimation errors observed in approximate arithmetic tasks.

2. Place-coding vs. summation coding

Besides their differential scaling, it has been suggested that symbolic and non-symbolic representations of numbers also present differential coding, symbolic representation being place-coded and non-symbolic representation summation-coded. In our third study (Chapter 5), we examined whether these different coding steps may shape the calibration of the numerosity estimation process. When the feedback and the responses of the participants were presented in the symbolic notation (Experiment 1), the calibration pattern was of the place-coding type: feedback trials improved numerosity perception judgments. This improvement peaked at the feedback value but also extended to neighbouring numerosities (smaller and larger than the feedback value), with decreasing strength. Inversely, when the feedback and the responses were presented in the non-symbolic format (Experiment 2), the calibration pattern was of the summation-coding type: the feedback trial improved the numerosity production

judgments. This improvement was larger for numerosities smaller than the feedback value than for numerosities larger than the feedback value.

Our results can be used to extend Izard and Dehaene's model (2008) of numerosity estimation. This model was initially proposed to account for the numerosity perception performances and was based on five hypotheses: **(1)** each perceived numerosity is encoded on the mental number line; **(2)** the mental number line is compressive and logarithmically scaled; **(3)** numerical values are represented on the mental number line by Gaussian distributions of activation of constant width (*w*); **(4)** activation on the number line is translated into numerical symbols, by means of a response grid: the number line is divided in segments, each corresponding to a different verbal label; **(5)** the response grid corresponds to an affine transformation of the mental number line.

In geometry, an affine transformation always involves a linear transformation (rotation, scaling or shear) and a translation such as $y=ax+b$ where a corresponds to the linear transformation and b to the translation. Considering the literature on symbolic and non-symbolic numerical representations, we have assumed, in the present thesis, that the linear transformation applied to the mental number line actually corresponds to a change of scaling; the symbolic representation being linearly scaled and the non-symbolic representation being logarithmically scaled. In this sense, the response grid may be considered as the analogue of what we called the symbolic representation of number; and the affine transformation can be seen as the analogue of what we called the numerical mapping process (see Fig. 6-2).

In order to account for the calibration effect, Izard and Dehaene (2008) assumed that participants again applied an affine transformation to their stimulus-response association. In other words, the authors assumed that the feedback trial did

not change the intrinsic variability of the mental number line (*w*), but instead, shifted the numerical mapping process in order to fit it with the feedback value. The third part of research for the present thesis was in agreement with this theory but also extended it by showing that the calibration effect actually depended on the notation used to present the feedback and the participant's response. When both were symbolic (Experiment 1), the calibration effect mainly concerned numerical magnitudes close to the feedback value. When both were non-symbolic (Experiment 2), the calibration effect mainly concerned the numerosities smaller than the feedback value. Because our tasks involved the numerosity estimation process, the symbolic and non-symbolic representations of numerical magnitudes were both activated. However, the model we presented to account for the calibration process does not currently take into account the format of the input stimulus. Only the format of the feedback value and of the participants' responses (i.e., the output variable) was assumed to influence the calibration process. In the study by Roggeman et al. (2007), however, the major features of symbolic and non-symbolic representations of numbers were found not only in a within-modality but also in a between-modality design. In other words, the step-like priming function was observed not only when a dot prime was presented before a target set of dots, but also when a dot prime was presented before a target digit. Similarly, a symmetric priming curve was observed not only when an Arabic digit primed a symbolic number but also when an Arabic digit primed a set of dots. In line with these findings, we could hypothesize that the major factor determining the shape of the calibration pattern is actually the format of the feedback trial. This assumption would greatly benefit from the introduction of two new experimental designs: a perception task involving non-symbolic feedback and a production task involving symbolic feedback. With the introduction of these two

additional experiments, we could further explore the different numerical processing routes.

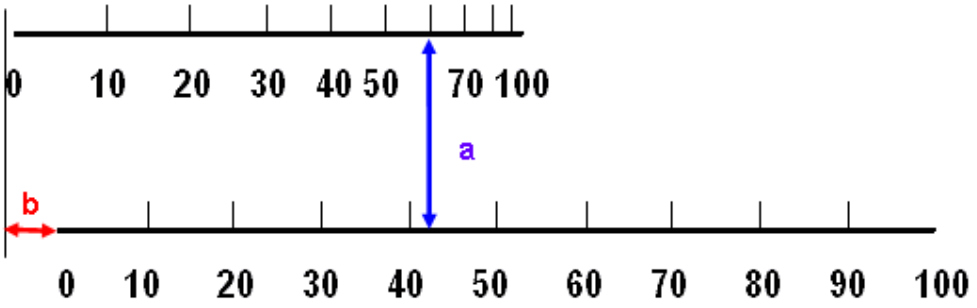


Fig. 6-2. Izard and Dehaene's model of the numerosity estimation process adapted to the bi-directional mapping hypothesis. Parameter *a* corresponds to the linear transformation and parameter *b* to the translation of the affine transformation that characterized the mapping process which associates the symbolic and non-symbolic representations of numbers. When feedback is provided, participants change the value of both parameters in order to fit them to the value of the inducer given. The amount and extend of change is dependent on the format used to present the feedback. When the feedback is symbolic, the calibration mainly occurs in numerosities close to the feedback value. When the feedback is non-symbolic, the calibration effect mainly occurs in numerosities smaller than the feedback value.

3. Conclusions

The data of our first three studies support the idea that the symbolic representation of numbers involves processes that are different in subtle ways from those that are engaged by non-symbolic numerical stimuli (i.e., linear vs. logarithmic scaling; place- vs. summation coding). As we mentioned in our introduction, the neurobiological evidence for a numerosity summation coding and a number-selective system has been demonstrated by single-cell recording within and near the intraparietal cortex of the macaque monkey (Nieder et al., 2002; Nieder & Miller, 2004; Roitman et al., 2007). Moreover, recent research in humans has also described two different cerebral pathways of activation for symbolic and non-symbolic numerosities. While non-symbolic numerosities follow a two-stage code analysis (a

number-summation code system followed by a number-selective system), the symbolic numbers principally and more directly activate the number-selective system (Santens et al., 2010).

Until now, however, we do not know whether the acquisition of symbolic representations either changes pre-existing representations or leads to the construction of novel, format-specific representations of numerical magnitude. So far, both computational models (Verguts & Fias, 2004; Verguts et al., 2005) and training studies in animals (Diester & Nieder, 2007) have assumed that the development of the ability to represent and process numerical symbols involves a process by which symbolic representations of numerical magnitude are mapped onto pre-existing non-symbolic representations. A plausible alternative possibility, however, is that, as postulated in the bi-directional mapping hypothesis, non-symbolic and symbolic representations of numerical magnitude draw on different neurocognitive processes. This possibility has already been supported in the literature by several experimental data showing dissociation between symbolic and non-symbolic numerical processing. Polk, Reed, Keenan, Hogarth, and Anderson (2001), for example, reported the case of a patient with damage to the left supramarginal gyrus (SMG) who was severely impaired in symbolic but not in non-symbolic magnitude processing. Similarly, Lecorre and Carey (2007) found that some children were able to count out exactly eight and nine objects while being unable to use these number words to rapidly estimate arrays of eight to twelve dots. Rousselle and Noël (2007), finally, showed that children with developmental dyscalculia were impaired in symbolic but not in non-symbolic numerical magnitude comparison. The proposal that there is a specific representation for symbolic numbers and exact arithmetic is also supported by brain-activation studies. There is, for example, converging evidence that the left AG and

the left superior temporal gyri are highly involved in symbolic arithmetic. The AG, whose lesion result in the Gerstmann syndrome (Gerstmann, 1930) is indeed activated in calculation (Burbaud, Degreze, Lafon, Franconi, Bouligand, Bioulac, et al.,1995; Rueckert, Lange, Partiot, Appolonio, Litvan, Le Bihan, et al., 1996), is more involved in exact than in approximate calculations[1] (Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Venkatraman, Siong, & Chee, 2006), and is also involved in arithmetical fact retrieval (Delazer, Domahs, Bartha, Brenneis, Lochy, Trieb, et al., 2003; Grabner, Ischebeck, Reishofer, Koschutnig, Delazer, Ebner, et al., 2009). While the AG is assumed to support the retrieval of arithmetical facts, it does not mediate quantity-based operations such as number-comparison estimation and approximation. Taken together, these data therefore suggest that symbolic representations of numbers might be different from non-symbolic representations of numerical magnitude.

To conclude, while current theories on numerical processing suggest that the acquisition of symbolic representations is grounded in the representations of non-symbolic numerical magnitude, the bi-directional mapping hypothesis set out in the present thesis is more in accordance with Carey's idea (2001), according to which the ability to rapidly enumerate small numbers of objects might be a crucial scaffold for the acquisition of an exact representation of symbolic numbers. The bi-directional mapping hypothesis therefore relies on the assumption that adults actually possess two qualitatively different numerical representations. However, the idea that these two representations are mentally represented facing each other is just, according to us, a mere metaphor. Numerosity judgments overall may be seen as deriving from a larger system that capitalizes on information derived from distinct sources, including

[1] In contrast, the IPS is more activated in approximate than in exact calculations (Stanescu-Cosson . Pinel, van De Moortele, Le Bihan, Cohen, & Dehaene 2000) and more strongly activated during subtraction than multiplication (Chochon, Cohen, van de Moortele, & Dehaene 1999; Lee, 2000)

such things as attentional markers (Trick and Pylyshyn, 1993), area (Allik & Tuulmets, 1991) or density information (Durgin, 1995). The continuous power function of numerosity could therefore result from the combination of several different kinds of relevant perceptive information that, though, weighted differently at different levels of numerosity, interact or blend together to form a continuous scale (i.e., the power function of numerosity). Moreover, it is obvious that human's perceptive system presents some limits. Cheatham and White (1954), for example, hypothesized that a central process determines the perceived numerosity and thus the degree of underestimation of extremely fast event sequences. They obtained a similar estimate of subjective rate for auditory and visual stimuli and therefore assumed that this limit was related to the alpha rhythm of brain waves. It is therefore possible that the logarithmic compression of the mental number line originates from the limit of our system. In the numerosity perception tasks, simultaneous displays of multiple objects are presented briefly to prevent sequential counting. This brief stimulus presentation could therefore induce a kind of extinction phenomenon (i.e., a certain percentage of the stimulus is either not attended or not retained in memory until the response is executed), explaining by itself why participants underestimate the numerosity really presented. Participants thus underestimate because their subjective sensation of numerosity is smaller than the obective magnitude really presented (i.e., an addition of quantity on the mental number line does not necessarily imply an addition of the same magnitude on the symbolic representation of numbers). In numerosity production tasks, inversely, participants have to produce more stimuli than necessary to have the subjective impression that they have reached the target number. If individuals have to produce 8 dots, then they are likely to not forget any of them before finishing the response. However, if participants have to produce 98 dots,

then they are very likely to compress their subjective sensation and then to "forget" a number of items already produced. If the participants, during their response, decide whether to continue producing items on the basis of the estimated number of stimulus already produced, then one would expect them to end up producing too many dots. Importantly and interestingly, this explanation of the under- and overestimation is only possible if two differently scaled numerical representations are mapped to each other. According to us, the subjective numerosity sensation corresponds to what we called the non-symbolic representation of numbers while the objective magnitude corresponds to what we named the symbolic representation of numbers. If these two representations were identically scaled, then no under- and overestimation should be observed.

With their study, Izard and Dehaene (2008) demonstrated that it is possible to calibrate the numerosity estimation process. While calibration is present in daily life for small quantity, it is not frequent for larger numbers; this explaining why participants are spontaneously more accurate in estimating small quantities than large quantities. Interestingly, the calibration effect was shown to not affect the intrinsic acuity of the mental number line but was rather shown to shift the numerical mapping process (Izard & Dehaene, 2008). As this numerical mapping ability is disrupted after frontal lobe damage (Revkin et al., 2008), we could hypothesize that executive and strategic processes are involved in external calibration and allow participants to overcome the limits of their system. In their study, Gandini, Lemaire and Dufau (2008) demonstrated that participants can use 4 different strategies to approximately quantify a set of elements: **(1)** *benchmark*: participants visually scanned the stimulus, retrieved a numerical representation in long-term memory, compared the difference between the encoded representation and the retrieved representation, and then

adjusted their answer on the basis of this difference; **(2)** *anchoring*: participants enumerated several dots (via counting), visually estimated the remaining dots based on the first enumeration, and then added the enumerated result and the estimated result; **(3)** *decomposition/recomposition*: participants spotted one group of few dots, up to about four or five items, estimated the number of analogous groups, and then multiplied the number of items primarily subitized by the estimated number of groups; **(4)** *approximate counting*: participants perceived several groups of different sizes and approximately added these groups to produce estimate. The use of these strategies can probably be witnessed by the spontaneous responses used by the participants in numerosity estimation tasks. In Izard and Dehaene's study as well as in our Chapter 5, the most frequent responses produced by the participants corresponded to round numbers (i.e., the decade numbers above 10). Strategically speaking, it is probably easier to anchor, decompose and approximately count round numbers rather than natural numbers such as 11, 12, 13, etc.

However, while two recent studies suggested that prefrontal regions might have a role in mapping abstract numerical symbols onto their quantitative referents (Diester & Nieder, 2007; Grabner et al., 2007), the processes that enable such mapping are still not well understood. Future studies should therefore evaluate the extent to which qualitatively different representations of symbolic and non-symbolic numerical magnitude might exist and should determine how such representations are constructed and mapped to each other.

PART II

OVERVIEW

In addition to being ratio-dependent, the representation of numbers also interacts with two visuo-motor functions. On the one hand, following the recurrent observation that small numbers are preferentially associated with the left side of space while large numbers are preferentially associated with the right side of space (i.e., SNARC effect; Dehaene, Bossini, & Giraux, 1993), numbers were assumed to interact with space. On the other hand, following the observation that children often use their fingers to learn the counting sequence and basic arithmetic operations, numbers were assumed to interact with finger movements (Butterworth, 1999b). As we will see in Chapter 7, these two interactions are assumed to take place in the parietal cortex, a brain area which is often associated with the dorsal visual pathway. The aim of the second part of the present thesis is therefore to examine whether blindness could affect the "shape" of these interactions. Below, we explain further how this aim will be studied throughout the following chapters.

In Chapter 8, we present a study that aimed to investigate the influence of developmental vision on the visuo-spatial and verbal coding accounts in which the SNARC effect takes place.

In Chapter 9, we explore the role of developmental vision in the development of the finger-numeral representation.

Chapter 8 corresponds to a scientific paper which is in preparation while Chapter 9 is an empirical study which has been accepted for publication in the Journal of Experimental Child Psychology. As in the first part of the present thesis, the results of our empirical work are discussed and integrated in a general discussion (Chapter 10).

CHAPTER 7

Interactions between numbers and visuo-motor functions

In the first part of this thesis, the intraparietal cortex was identified as the neural correlate of the mental representation of numbers. However, parietal activations has also been reported when participants are required to spatially orient their attention (Coull & Nobre, 1998), or when they are asked to realize complex sequences of finger movements (Harrington, Rao, Haaland, Bobholz, Mayer, Binderx, & Cox, 2000; Pesenti et al., 2000; Piazza, Mechelli, Butterworth, & Price, 2002; Pinel, Piazza, Le Bihan, & Dehaene, 2004; Zago, Pesenti, Mellet, Crivello, Mazoyer, & Tzourio-Mazoyer, 2001). Likewise, Walsh (2003) proposed that number, space and action shared a common magnitude mechanism supported by overlapping neural circuits within the posterior parietal cortex. The present chapter will show that these visuo-motor functions are not only anatomically close to the area supporting number processing but also interact with it. The first section of this chapter will focus on the interactions that occur between numbers and space whereas the second section will focus on the interactions that exist between numbers and finger movements.

1. Interactions between numbers and space: the SNARC effect

1.1. Behavioural and electrophysiological evidence

The major evidence used to support the fact that numbers and space interact with each other comes from the observation of some compatibility effects between number and space processing in behavioural forced-response paradigms. The so-called SNARC effect (Spatial Numerical Association of Response Codes) indicates

that small numbers are preferentially represented to the left whereas larger numbers are preferentially represented to the right. Hence, in a parity judgment task, responses to smaller numbers are faster when the response is made on the left-side of space, whereas responses to larger numbers are faster when the response is made on the right side of space (Dehaene et al., 1993) (see Fig. 7-1a).

Interactions between numbers and space have been observed not only in parity judgments (Dehaene et al., 1993) but also in number comparison (Dehaene et al., 1990), in tasks requiring judgments of phonemic content of number words (Fias, Brysbaert, Geypens, & d'Ydewalle, 1996) and in non-numerical tasks involving spatial judgments (discrimination of the orientation of a triangle or a line superimposed on a digit: Fias, Lauwereyns & Lammertyn, 2001; Lammertyn, Fias, & Lauwereyns, 2002). Much of the evidence supporting the existence of an interaction between numbers and space also comes from studies using the line bisection task (Calabria & Rossetti, 2005; Fischer, 2001) in which participants tend to perceive a line midpoint to the left of its actual midpoint when this line is composed of small numbers (in Arabic as well as in verbal notation). Inversely, they tend to perceive the line midpoint to the right of its actual midpoint when the line is composed of large numbers (see Fig. 7-1b). Similarly, when each side of a line is flanked by numbers, systematic spatial biases toward the larger number are observed (de Hevia, Girelli, & Vallar, 2006; Fischer, 2001). Number processing distortions have also been found in patients presenting hemi-spatial neglect, who typically tend to ignore the contralesional space (usually left) of their lesion (usually situated in the right parietal cortex). So, in a classic line bisection task, in which the midpoint of a physical line has to be indicated, they neglect the left portion of the line and then tend to displace the midpoint of the line towards the right. This hemi-spatial neglect has been shown

to extend to the representation of numbers: patients systematically displace the actual midpoint of a numerical interval towards the right (i.e., the large numbers) when asked to bisect it (Hoeckner, Moeller, Zauner, Wood, Haider, Gabner, & Nuerk, 2008; Zorzi, Priftis, Meneghello, Marenzi, & Umilta, 2006; Zorzi, Priftis, & Umilta, 2002). However, when small physical lines or small numerical intervals are presented, a "cross-over effect" is observed, which means that, in this case, the bisection performances are deviated towards the left/the smaller numerical values. These spatial and numerical distortions can be improved by wearing prismatic goggles, thereby indicating that numbers and space are intrinsically connected (Rossetti, Jacquin-Courtois, Rode, Ota, Michel, & Boisson, 2004; Rossetti, Rode, Pisella, Farne, Li, Boisson, & Perenin, 1998).

The automatic association between number and space processing is induced not only by numbers but also by other culturally learned sequences such as months of the year or days of the week (Gevers, Reynvoet, & Fias, 2003, 2004). Further, it appears that the interactions between numbers and space are influenced by cultural and contextual factors. Indeed, whereas European people, who write from left-to-right, show a classic SNARC effect, Arabic (Dehaene et al., 1993; Zebian, 2005) or Hebrew (Kazandjian, Cavézian, Zivotofsky, & Chokron, 2010) speaking adults, who write from right-to-left, show a reversed SNARC effect. In the same way, while the spatial-numerical association is oriented from left-to-right in participants asked to imagine the numbers positioned on a ruler, it is oriented from right-to-left in participants asked to classify numbers as hours of the day coming before or after six o'clock (Bachtold, Baumuller, & Brugger, 1998).

The intrinsic connection between numbers and space has even been found in tasks involving goal-directed movements (Fischer, 2003; Fischer, Castel, Dodd, &

Pratt, 2003; Fischer, Warlop, Hill, & Fias, 2004; Schwarz & Keus, 2004). This last wave of research demonstrated that even a non-informative, completely task-irrelevant digit can influence shift of attention (Fischer et al., 2004; Schwarz & Keus, 2004) or pointing movements (Fischer, 2003; Fischer et al., 2003) towards lateralized targets. In the well-known experiment of Fischer and collaborators (Fischer et al., 2003), for example, participants had to react as quickly as possible to a target presented either in the left visual field (LVF) or in the right visual field (RVF) after a single digit number (1, 2, 8 or 9) had been presented as a fixation point. The detection of the LVF target was facilitated after the numbers 1 and 2 had been presented whereas the detection of the RVF target was facilitated after the digits 8 and 9 had been presented (see Fig. 7-1c). These results therefore indicate that the allocation of attention is automatically influenced by the number presented as the fixation point and that similar structures underlie attention shifts across internal spatial representations and external space.

The electrophysiological correlates induced by this effect were described in a recent study by Salillas, El-Yagoubi and Semenza (2008). The data demonstrated that the congruency between number size and location of the target modulates sensory processes (the amplitude of the perceptual P100), but also later cognitive processes (the amplitude of the cognitive P300). According to the authors, P100 modulation indicates that the perception of a digit activates the spatial position of the number on the mental number line and thus generates an orienting of attention towards the left or the right. This cues external spatial representation and is able to amplify the sensory input of the target when it coincides with the cued location. Then, the spatial representation is held in working memory and modifies the relative relevance of a certain location in external space, as signaled by the modulation of the P300. These

data thereofre clearly support the idea that the perception of numbers affects the location of attention to external space (Fischer et al., 2003).

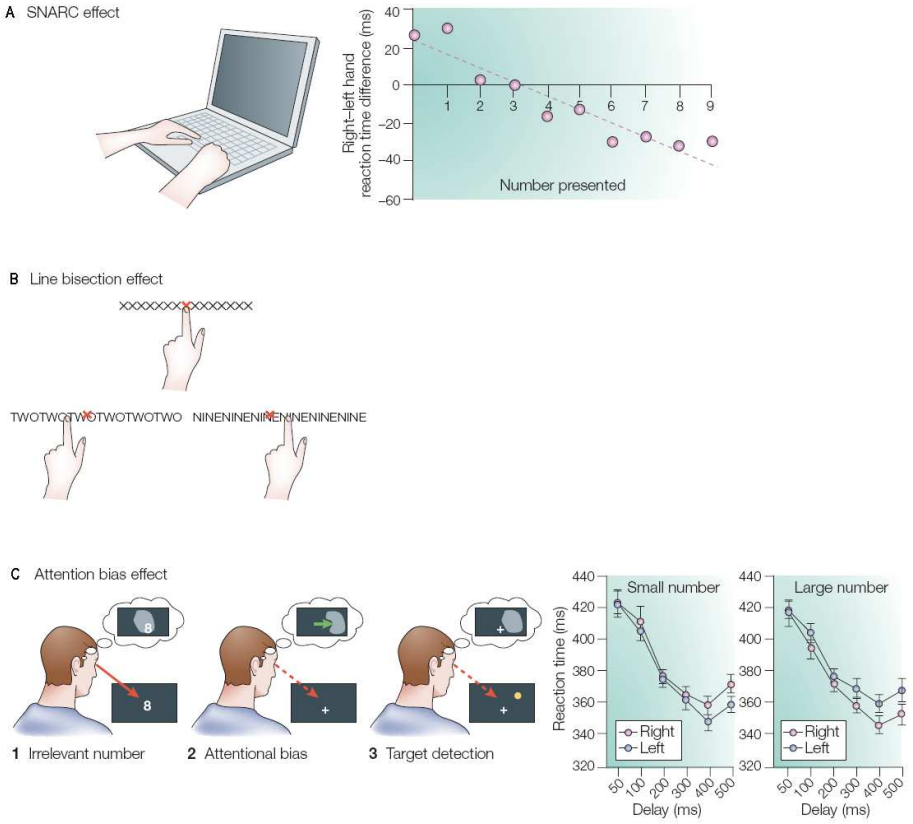


Figure 7-1. SNARC effect. (A) Observation of the SNARC effect in a *parity judgment task*. The graph represents the response time difference between the right-side and left-side responses. Values greater than 0 therefore indicate a left-side advantage whereas values smaller than 0 indicate a right-side advantage. **(B)** Observation of the SNARC effect in a *line bisection task*. When asked to point to the midpoint of a line, participants are accurate when the line is composed of 'X's. Conversely, they tend to displace the midpoint of the line towards the left when the line is composed of small number words (e.g., TWO), and towards the right when the line is composed of large number words (e.g., NINE). **(C)** *Attention bias effect* observed in a target detection task. Participants are faster to detect an LVF target after a small numbers had been presented at fixation whereas they are faster to detect an RVF target after a large number had been presented. *Adapted from Hubbard et al. (2005).*

1.2 Interpretations of the SNARC effect

The SNARC effect was first interpreted as an index of the spatial organization of the mental number line, according to which small numbers are responded to faster

with left-sided responses and large numbers are responded to faster with right sided-responses because this stimulus-response association is congruent with the left-to-right orientation of the mental number line.

However, this idea of a direct association between numbers and a spatial response has been challenged by two different accounts. The first is a computational model proposed by Gevers, Verguts, Reynvoet, Caessens and Fias (2006) which considered that a "categorization layer" exists between the numerical representation and the production of the responses (see Fig. 7-2). The function of this middle layer is to categorize numbers as small or large, odd or even or any given category that is required by the task. Moreover, the model assumes that two different routes map the numerical representation and the response stage. On the one hand, the automatic route is triggered in any numerical context and accounts for the fact that a digit (i.e., small or large) automatically primes a particular response (left or right). On the other hand, the controlled route is activated intentionally by the participants in accordance with the task's instructions (e.g., small numbers → left hands). If the two routes converge on the same response, responses will be faster than if they do not converge. In this case, the response primed by the automatic route must be deprogrammed before the correct response can be initiated, thus explaining by itself the occurrence of the SNARC effect (see also Notebaert, Gevers, Verguts, & Fias, 2006; Santens & Gevers, 2008).

The second account to challenge the idea that a spatially oriented mental number line is necessary to explain the SNARC effect is Proctor and Cho's model (2006), in which stimulus and response alternatives are coded as positive and negative polarity and the authors demonstrated that polarity correspondence is

sufficient to produce the stimulus-response mappings characteristic of the SNARC effect.

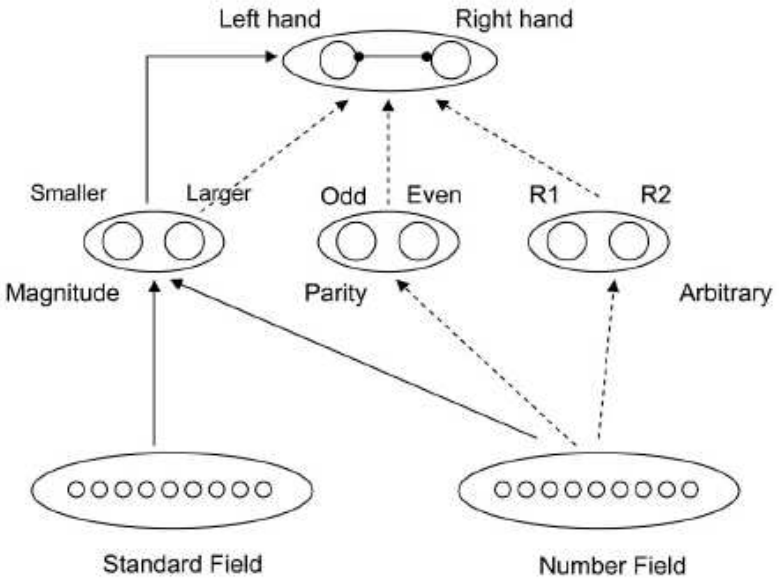


Figure 7-2. Dual route model for the SNARC effect. The bottom layer consists of two fields: the standard field and the number field. The number field codes the target number whereas the standard field is task dependent: in comparison tasks, it codes the standard to which a number has to be compared. In tasks involving a single relevant number (e.g., parity judgments), it codes the mean of the presented numbers. The middle layer receives input from each of these two fields. According to the model, a number is always coded as small or large, irrespective of the task performed. Additional fields can however be activated, depending on the task demands (e.g., the parity field for a parity judgment task). The top layer comprised 2 spatially defined responses (i.e., left and right) connected to each other by lateral inhibition. Full black lines correspond to the automatic route whereas dotted lines correspond to the controlled route. Responses will be faster if the two routes converge on the same response. *Reproduced from Gevers et al. (2006).*

In sum, both accounts suggest that an intermediate step categorizes numerical information before activating a response side. In this step, numbers are coded as small vs. large or as negative vs. positive, this numerical categorization being then directly or indirectly associated with spatially defined responses. Both accounts therefore suggest that the observation of the SNARC effect is not sufficient to conclude that adults represent numbers on a left-to-right oriented mental number line.

1.3 SNARC and SIMON effects : similarities and differences

The Simon effect refers to the fact that the time to respond to a stimulus is faster and more accurate when the position of the stimulus is compatible with the side of the response (e.g., Simon, 1969; Simon & Rudell, 1967; Simon & Wolf, 1963). In a typical Simon task, the participants see lateralized coloured stimuli and are instructed to respond with a left-side response key to one colour and with a right-side response key to the other colour. Although the stimulus location is completely irrelevant to the task, participants' reaction times are faster on spatially corresponding trials (left stimulus position – left side response and right stimulus position – right side response) than on spatially non-corresponding trials (left stimulus position – right side response and right stimulus position – left side response). It is therefore obvious that the Simon and SNARC effects share some important characteristics (Mapelli, Rusconi, & Umiltà, 2003). Firstly, both refer to a spatial compatibility effect between the response location and an attribute of the stimulus (position or magnitude). Secondly, these effects appear not only when the stimulus attribute is task-relevant (for the Simon effect: Umiltà & Nicoletti, 1990; for the SNARC effect: Dehaene et al., 1990) but also when it is task-irrelevant (for the Simon effect: Lu & Proctor, 1994; for the SNARC effect: Fischer et al., 2004). Thirdly, both effects are assumed to originate from the automatic processing of a positional code (Zorzi et al., 2002; Zorzi & Umiltà, 1995) that primes lateralized responses and interferes with the task-relevant response code (i.e., the dual route model). Fourthly, the SNARC and the Simon effects occur across a variety of responses and interact with the response selection stage of action (For the Simon effect: De Jong, Liang, & Lauber, 1994; Valle-Inclàn, 1996; van der Lubbe, Jaskowski, Wauschkuhn, & Verleger, 2001; for the SNARC effect: Daar & Pratt, 2008; Keus, Jenks, & Schwarz, 2005; Keus &

Schwarz, 2005; Wilson & Pratt, 2007). Finally, both effects appear even if participants cross their hands, indicating that the SNARC and the Simon effects originate from the mapping of an input stimulus onto an external frame of reference (for the Simon effect: see Röder, Kusmierek, Spence & Schicke, 2007; for the SNARC effect, see Dehaene et al., 1993; but see Wood, Nuerk, & Willmes, 2006).

Although the SNARC and the Simon effects share various characteristics, several studies also showed that they present a different mechanism: while the SNARC effect is not affected by the speed of response or by the length of the stimulus onset asynchronies (SOA_s) (Daar & Pratt, 2008; Mapelli et al., 2003), the Simon effect only occurs when the speed of response is fast (Mapelli et al., 2003). It could therefore be argued that these two phenomena are spatial in nature but rest on distinct parietal mechanisms (Mapelli et al., 2003; Stoianov, Umiltà, & Zorzi, 2005).

1.4 SNARC and SIMON effects : the role of vision

The development of the numerosity representation seems to depend on the visuo-spatial individuation of objects which rests in the occipito-parietal pathway. Therefore, it could be suggested that the mapping of numbers onto space depends on extensive experience with numbers in the visual modality (Cooper, 1984; Simon, 1997). In line with this, a new wave of research is currently investigating whether number processing in blind people demonstrates the same properties as in sighted adults. At a behavioural level, Castronovo and Seron (2007b) found that blind and sighted participants presented the same distance and SNARC effects in two numerical comparison tasks (to 5 and to 55) and in a parity judgment task (see also Szücs & Csépe, 2005), indicating that the lack of vision does not preclude the development of a left-to-right oriented mental number line. Similarly, blind and sighted people

showed the same bisection effects when asked to haptically explore a rod in order to indicate its midpoint, while listening to a particular number (Cattaneo, Fantino, Tinti, Silvanto, & Vecchi, 2010). Specifically, the presentation of a small number shifted the perceived midline toward the left while the presentation of a large number shifted the perceived midline toward the right. This pattern of results is consistent with the data obtained in the visual modality (Dehaene et al., 1993; Fischer et al., 2003) and therefore supports the idea that the interactions between numbers and space occur regardless of a lack of visual experience.

Szücs and Csépe (2005) showed, like Castronovo and Seron (2007b), that blind and sighted adults both demonstrated robust distance and SNARC effects in a numerical comparison task. Further, the authors found that the EEGs of both groups were dominated by a clear stimulus locked effect between 100 and 300 ms. Due to shorter and less scattered reaction times in the blind, the P300 accompanying the response was more expressed in the blind than in the control group. However, apart from this general amplitude difference, the authors did not find any apparent morphological or distribution discrepancies in ERPs between the two groups of participants.

Different results were nevertheless found by Salilas, Granà, El-Yagoubi and Semenza (2009). By adapting the lateralized target detection paradigm of Fischer et al. (2003) to the auditory modality, the authors showed that the attentional shifts generated by fixation numbers have different electrophysiological correlates in blind and sighted people. While the amplitude of the sensory N100 component was modulated by congruency (as previously demonstrated by Salilas et al., 2008) only in the sighted group, the amplitude of the cognitive P300 was modulated by congruency only in the blind group (see Fig. 7-3).

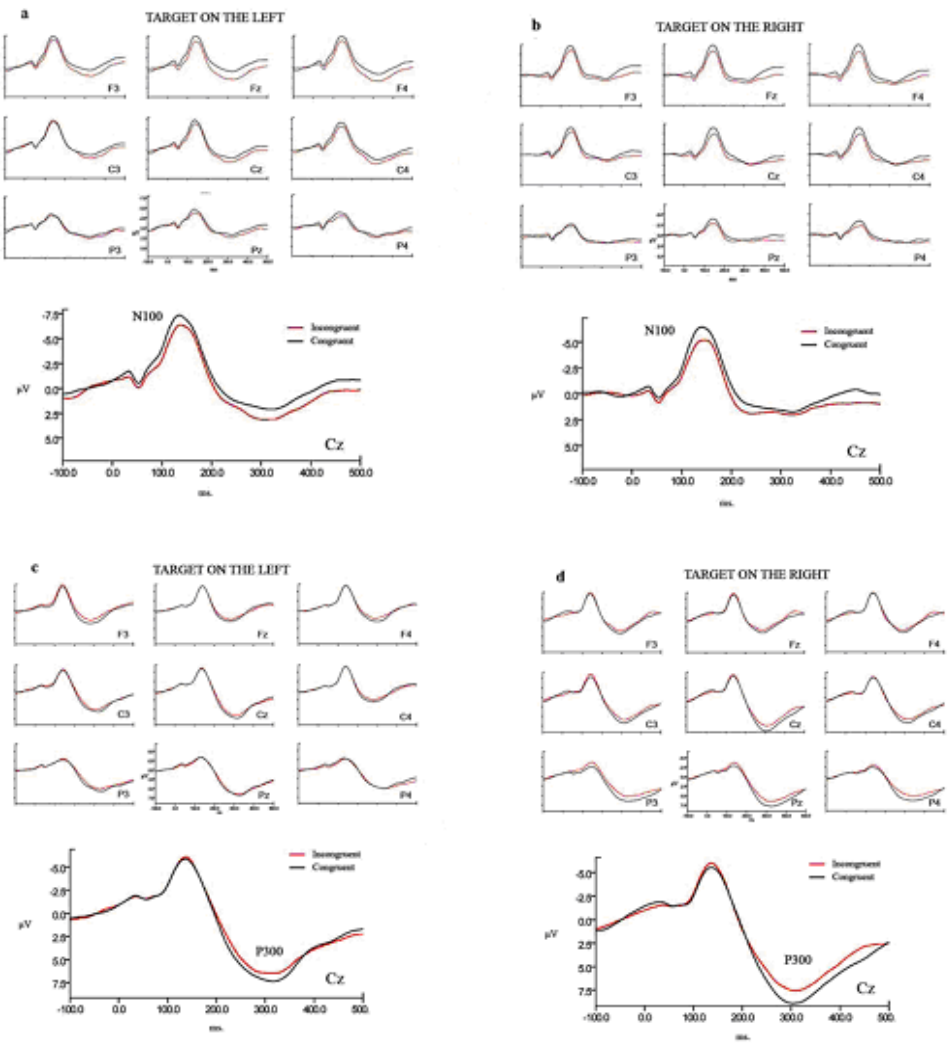


Figure 7-3. ERPs elicited by lateralized auditory targets. (A) target presented on the left – sighted participants; **(B)** target presented on the right – sighted participants; **(C)** target presented on the left – blind participants; **(D)** target presented on the right – blind participants. *Adapted from Salilas et al. (2009).*

The modulation of the N100 component was interpreted as the consequence of a top-down mechanism: the fixation number activated a position on the mental number line which in turn exerted spatial shift of attention over auditory space. The P300, in contrast, was described as reflecting higher cognitive processes such as retrieval and maintenance of a representation in working memory. Accordingly, the absence of

visual input could lead blind people to process numbers in a more cognitive way, relying much more on verbal working memory than on sensory processes.

To sum up, whereas the first experiments on numbers and blindness supported the idea that blind and sighted people present a similar overall spatial organization of the mental number line, the last experiment presented here suggested that blind and sighted people could actually manipulate numbers by using different underlying processes.

At first sight, the absence of vision did neither seem to affect the occurrence of the Simon effect. Indeed, when required to press a left or right response key depending on the bandwidth of a sound presented from a left or a right loudspeaker, late and congenitally blind people reacted in the same way as sighted controls: they performed better when the sound was presented from the same side as the response key. However, when participants performed the task with crossed hands, the Simon effect was reversed but only in the congenitally blind group (Röder et al., 2007) (see Fig. 7-4).

Thus, a very simple experimental manipulation permitted to establish that blind and sighted spatial behaviours were actually grounded on different underlying processes. According to the authors (Röder et al., 2007), the reversed Simon effect observed in blind people occurred because these participants did not remap the sensory information from different modalities onto an external spatial frame of reference. Rather than using stimulus-location / response-location mapping as sighted and late blind did, the congenitally blind used stimulus-location / response-hand mapping. As a consequence, in a classic Simon task, the presentation of a sensory stimulus to blind participants primes the anatomical hand (e.g., left hand) which is congruent with the stimulus location (e.g., left ear presentation), regardless of where

in space that hand is placed (i.e., on the left or right response key). Inversely, in late blind and sighted participants, the sensory stimulus presented from a particular location primes the response key which is the closest from that location, regardless of which anatomical hand is used to press it.

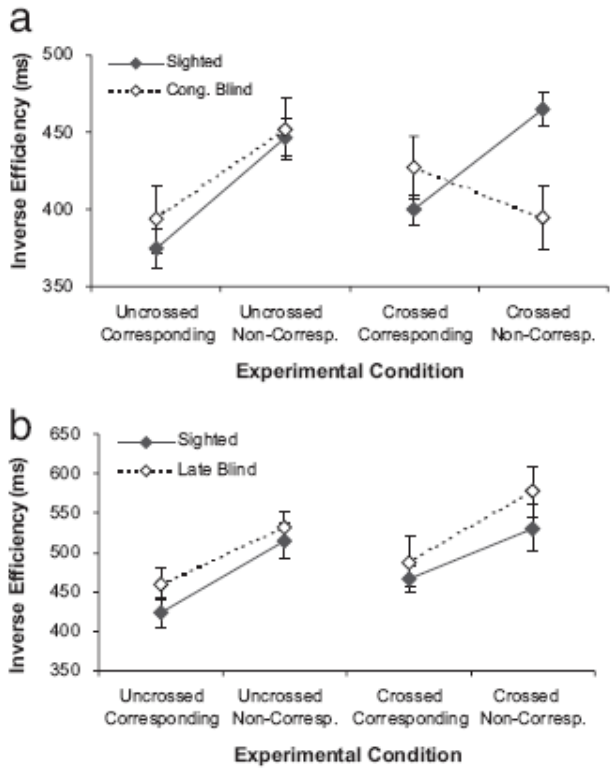


Figure 7-4. Simon effect and reversed Simon effect. (A) Performance of the congenitally blind and their sighted controls. **(B)** Performance of the late blind and their sighted controls. Data on the Y-axis corresponds to the Inverse Efficiency Scores (IES) which are calculated by dividing RTs by correct response rates. IES thus eliminate any potential speed-accuracy trade-off effects in the data. The lower the score, the more efficient the performance. *Adapted from Röder et al. (2007).*

According to recent studies, the use of an external spatial frame of reference develops during early infancy (Bremmer, Holmes, & Spence, 2008) and depends on early visual experience (Collignon, Voss, Lassonde, & Lepore, 2009; Röder, Föcker, Hotting, & Spence, 2008; Röder et al., 2007; Röder, Rösler, & Spence, 2004). Röder et al. (2004), for example, demonstrated that judging the temporal order of two tactile

stimuli, one applied to either hand, was more difficult for sighted and late blind in a crossed when compared to an uncrossed hand posture. By contrast, the performance of early blind participants remained unchanged across posture changes. These results were replicated with auditory and audiotactile stimuli (Collignon, Charbonneau, Lassonde, & Lepore, 2009) and were accounted for by the theory of the automatic external remapping. In sighted and late blind, sensory stimuli are automatically remapped into an external framework. The use of such an external spatial frame induces a conflict between external and body-centred coordinates which indeed decreases performance. However, in the early blind, the absence of the externalization process of touch and audition protects these participants from the detrimental effect of the crossed hand posture. This explanation was supported by an electroencephalographic study showing that the detection of deviant tactile stimuli at the hand induced event-related potentials that varied in crossed when compared to uncrossed condition in sighted participants, whereas changing the posture of the hand had no influence on the early blind participants' brain activity (Röder et al., 2008).

1.5 Conclusions

In summary, the behavioural observations made with healthy participants and neglect patients clearly demonstrated the existence of interactions between numbers and space. More particularly, a preferential association between small numbers and left-sided responses and between large numbers and right-sided responses was reported. The recurrent observation of these associations led to the assumption that numbers were represented on a left-to-right oriented mental number line. According to this model, the SNARC effect is thus triggered by spatial information, or, more particularly, by spatial congruity effects between the stimulus magnitude and the

external spatial coordinate of the response. Interestingly, the construction of this external spatial frame of reference was shown to be dependent on developmental vision. Indeed, in a typical Simon task, when blind participants have to react to a lateralized target with uncrossed hands, their responses are faster and more accurate when the position of the stimulus is compatible with the side of the response. However, when they perform the same task with their hands crossed, the Simon effect reversed, indicating that blind participants actually map the stimulus location with the location of the hand used to respond (Röder et al., 2007). Given that the SNARC effect was observed in blind participants during a numerical comparison task in a classic uncrossed hands condition (Castronovo & Seron, 2007b) and given that the SNARC effect shares some major characteristics with the Simon effect (Mapelli et al., 2003), it could be interesting to examine whether a reversed SNARC effect is observed while blind participants perform a comparison task (i.e., a task that primarily depends on visuo-spatial associations) with their hands crossed.

Moreover, starting from the finding that associations between numbers and space may take different forms according to task demands, Gevers et al. (2006; see also Gevers, Santens, Dhooge, Chen, Van den Bossche, Fias, & Verguts, 2010) postulated that the SNARC effect did not need the metaphor of the mental number line to be explained. Rather, a verbal-spatial model, in which numbers are verbally coded as small or large before a response is produced, was assumed to be sufficient to account for the SNARC effect. As blindness affect the development of the external coordinate system but not the development of semantic labels (i.e., left vs. right), it could also be interesting to examine whether a reversed SNARC effect is observed while blind participants perform a parity judgment task (i.e., a task that primarily

depends on verbal-spatial associations) with their hands crossed. This question will be addressed in Chapter 8.

2. Interactions between numbers and fingers: the role of finger-counting

Starting from the observation that children of every culture use their fingers at an early age while learning enumeration and basic arithmetic abilities (Butterworth, 1999a, 1999b), a large amount of research started to study the role of fingers and finger movements in the acquisition of the number concept. In this section, we will review this research before explaining how the connections between numbers and fingers may lead to the development of a digital representation of numbers.

2.1 Finger-counting in children

The question of how children develop a conceptual understanding of numbers and counting remains an open issue. On the one hand, counting behaviour seems to be guided by the implicit knowledge of five principles (Gallistel & Gelman, 1992; Gelman & Gallistel, 1978): *one-to-one correspondence* (each item has to be enumerated once and only once); *stable order* (numerical labels have to be enumerated in the same order across counting sequences); *abstraction* (any set of items can be counted); *order irrelevance* (the result of counting is the same whatever the order in which the items are counted) and *cardinality* (the last word uttered in a counting sequence represents the total number of items in the counted set). On the other hand, learning the counting sequence can also be divided into five levels (Fuson, Richards & Briars, 1982): the *string level* (counting words are undifferentiated). At this stage, counting is a routine activity without meaning. Children do not distinguish between different components of the counting routine; all

components are equally essential; the *unbreakable chain level* (the counting sequence is understood as the recitation of separate words); the *breakable chain level* (children become able to start counting from arbitrary entry points rather than always starting at one); the *numerable chain level* (the counting words are abstracted still further and acquire a mathematical meaning); and the *bi-directional chain level* (children are able to count up and down, quickly and flexibly, starting at any point). Counting is therefore at first a meaningless activity from which children abstract out certain properties earlier, others later. One property that children learn is that the last number word used in a count is the answer to "how many" items there are. Children first map the word "one" onto its numerosity, achieving this next for the word "two" and the the word "three". At around three and a half years of age, most children induce that there is such mapping for every word in the counting list and thus learn the significance of the counting activity (Wynn, 1990, 1992b). At the developmental level, fingers were assumed to support the emergence of these abilities, and hence, were assumed to play a functional role in the development of a mature counting system (Butterworth, 1999a; 1999b; 2005; Fuson, 1988; Fuson et al., 1982; Gelman & Gallistel, 1978).

The functional and significant role of fingers in children's numerical development was supported by several behavioural studies. More particularly, fingers have been related to the comprehension of several abstract counting principles. First, finger-counting was assumed to help children to *keep track* of the number words uttered while reciting the counting sequence (Fuson et al., 1982; Geary, 1994) or performing basic arithmetic operations (Baroody, 1987; Fuson & Kwon, 1992; Geary, 1994; Ifrah, 2000; Svenson & Sjöberg, 1983). Second, fingers may help children to coordinate the processes of tagging (i.e., attribution of a counting word to each item)

and partitioning (i.e., isolating the items already counted from those which remained to be counted) (Gelman & Gallistel, 1978). In other words, counting on one's fingers was assumed to support the acquisition of the *one-to-one correspondence principle*. This principle appears in gesture before appearing in speech (Graham, 1999) and is less violated when the children perform the gestures themselves than when a puppet gestures as children count aloud (Alibali & DiRusso, 1999). Third, finger-counting was assumed to sustain the assimilation of the *stable-order principle* (i.e., numerical labels have to be enumerated in the same order across counting sequences). According to Wiese (2003a, 2003b), the fact that fingers occupy a fixed position on each hand supports a routine to link fingers to objects in a sequential stable order. Even if counting habits differ substantially between cultures (Pika, Nicoladis, & Marentette, 2009), a child from a particular country always uses the same pattern of fingers to count (i.e., finger-counting) or to represent (i.e., finger-montring[2]; Di Luca & Pesenti, 2008) a specific number. So, each numerosity will progressively be associated with the same hand shape and the same digital configuration so that children will be faster to name numerical finger configurations when these configurations conform to their own finger-counting habit than when they do not (Noël, 2005). Moreover, by always reaching the same finger when counting to a specific number, the child will associate the counting name of that finger with the number of counted objects. The development of these particular associations may help children to understand the *cardinality principle*. The use of finger-counting was also assumed to prompt the understanding of the *10-base numerical system*. Indeed, the fact that we have ten fingers may help children to understand that every symbolic

[2] This concept was introduced by Di Luca and Pesenti (2008) to differentiate the strategy used to count on one's fingers and the strategy used to show numerosities to other people. Interestingly, the finger configurations used for these two different behaviours may sometimes differ from each other (e.g., the thumb and the index finger are used to count to two whereas the index and the middle fingers are used to represent the quantity two).

number beyond the base 10 is in fact a sum and/or a multiple of 10. In addition to the main base-10, the finger-based numerical representation also exhibits a sub-base-5 structure which means that the number of fingers used to represent numbers larger than 5 always include a full hand representation (Marton & Neuman, 1990). In line with this logic, Domahs, Krinzinger and Willmes (2008) provided evidence that finger-based numerical representation still influences children when they perform complex mental calculation involving numbers larger than 10. Indeed, in this study, traces of the 5-base finger-counting system were witnessed by a disproportionate number of split-five errors (i.e., errors deviating from the correct result by exactly ± 5; e.g., 12 – 5 = 2). These split-five errors thus occurred with numbers that cannot be represented by fingers in a one-to-one correspondence and are probably due to a failure to keep track of the number of "full hands" employed to count or calculate. This therefore suggests that embodied counting experiences precede but still influence symbolic numerical processing.

The close connection between fingers and numbers was also put forward in studies demonstrating that finger gnosia was a good predictor of children's mathematical abilities (Fayol, Barouillet & Marinthe, 1998; Marinthe, Fayol, & Barouillet, 2001; Noël, 2005) and in a study providing evidence that finger gnosia training could improve untrained numerical performance (Gracia-Bafalluy & Noël, 2008; but see Fischer, 2010). Children with weaker abilities to identify and discriminate their fingers therefore tend to be less efficient in mathematical tasks than children with a high finger gnosia. Accordingly, understanding of the cardinality principle occurs considerably later in children with William syndrome, who present relative strength in language but strong weakness in visuo-spatial abilities, than in typically developing children (Ansari, Donlan, Thomas, Ewing, Peen & Karmiloff-

Smith, 2003). Moreover, the mathematical achievement of these children is considerably lower than that of their control peers, indicating that an impairment of visuo-spatial cognition may prevent the development of exact number representation.

To sum up, the knowledge of natural numbers seems to be grounded in finger-based representation. Because finger-counting preserves the basic principles of the symbolic numerical system (i.e., cardinality, one-to-one correspondence, stable-order, etc.) as well as higher-order properties such as the base-10 system, fingers have been assumed to be the "*missing link*" between the non-symbolic and symbolic representations of numbers (Fayol & Seron, 2005). The question arises as to whether finger-counting strategies still influence numerical cognition in adults. While a study by Kaufman and collaborators (Kaufman, Vogel, Wood, Kremser, Schoke, Zimmerhackl et al., 2008) indicated that the activation of finger-based representations in numerical tasks was more pronounced in children than in adults, the following section will demonstrate that various aspects of the interaction between fingers and numbers are still present in adults.

2.1 Finger-counting in adults

The idea that fingers and numbers still interact in adulthood is not new. As soon as 1930, Gerstmann described a syndrome characterized by four major symptoms: left-right disorientation, agraphia, acalculia and finger agnosia. According to Gerstmann himself, this association of symptoms followed a focal lesion of the left angular gyrus and originated from a common disorder: the loss of "the finger sense". Whereas the consequence of this disorder is evident in finger agnosia, the presence of the other symptoms was explained by the following observations: **(1)** hands are often used to indicate a left or right orientation; **(2)** writing is accomplished through good

control of finger praxis; and **(3)** finger discrimination is essential for children learning to count. Although this syndrome has been controversially debated since first reported, the existence of an association between numbers and fingers has received additional support from more recent experimental studies. An rTMS study, for example, showed that a transient lesion of the left parietal cortex could disrupt both the finger schema and the processing of numbers (Rusconi, Walsh, & Butterworth, 2005; see also Roux, Boetto, Sacko, Chollet, & Trémoulet, 2003). Similarly, brain imaging studies have revealed that common parietal networks are activated in tasks involving finger movement control and number processing (Pesenti et al., 2000; Piazza, et al., 2002; Pinel et al., 2004; Zago et al., 2001). Finally, Thompson, Abbott, Wheaton, Syngeniotis and Puce (2004) found greater activation of the left IPS when participants had to extract number information from finger movements as compared to lip movements. Together, these experiments confirmed Gerstmann's initial hypothesis (1930) that finger gnosis and numbers may rely on a same neuroanatomical substrate.

The use of fingers to count and to represent numbers is so strongly rooted in children's mathematical development that it was assumed that finger-counting could influence the structure of the mental representation of numbers in adulthood. Fischer, for example, argued that finger-counting habits could affect the association between numbers and space, the "left-starting counters" presenting the classical SNARC effect and the "right-starting counters" presenting no reliable SNARC effect (Fischer, 2008).

Moreover, several traces of the finger-counting behaviour have been observed in adults. As already mentioned, the repetitive and stable-ordered counting experiences progressively lead children to link a specific number name to a specific

finger. The presence of these associations in adults was witnessed in several studies showing that personal finger-counting habits actually affected numerical processing. In a first study, Di Luca and collaborators (Di Luca, Granà, Semenza, Seron & Pesenti, 2006) found that Italian participants identified Arabic digits from 1 to 10 faster and more accurately when the finger used to press the response key corresponded to the typical Italian finger-digit mapping (i.e., from the right thumb to the right little finger for numbers 1 to 5 and from the left thumb to the left little finger for numbers 6 to 10). In a second study, Di Luca and Pesenti (2008) demonstrated that adults, like children (Noël, 2005), named finger configurations more quickly and more accurately when these configurations conformed to their own finger-counting habits than when they did not. Critically, this facilitation effect was not due to a mere perceptual phenomenon but rather stemmed from semantic access. Indeed, comparison of Arabic numbers was improved when a finger configuration was used as a prime. More particularly, targets preceded by a canonical or a non-canonical prime were responded to faster than targets preceded by a neutral prime (i.e., a scrambled hand photograph). This priming effect was stronger in congruent trials (i.e., trials in which prime and target led to the same response), than in incongruent trials (i.e., trials in which prime and target led to a different response) and only generalized to new numerosities when canonical finger configurations were used as primes. As priming effects are usually interpreted as reflecting semantic access, the authors (Di Luca & Pesenti, 2008) assumed that canonical configurations automatically activated the semantic meaning of numbers whereas non-canonical finger configurations did not. These results were extended by three additional studies. The first showed, with a pop-out paradigm, that the speed of processing of canonical configurations did not stem from visuo-perceptual features (Di Luca & Pesenti,

2010). The second demonstrated, with a priming paradigm, that canonical configurations were processed as symbolic stimuli while non-canonical configurations were processed as non-symbolic stimuli (Di Luca et al., 2010). The final one suggested, with a response-compatibility effect paradigm, that canonical finger configurations not only prime numerical comparison of Arabic digits but also underlie simple arithmetic operations in educated adults (Badets, Pesenti, & Olivier, 2010). In this last experiment, participants were asked to verbally solve simple additions. The onset of the response triggered the presentation of either the correct or an incorrect arithmetic result, displayed either as a set of rods or as a canonical configuration of fingers. The results indicated that responses were faster with finger configurations than with rods, but only when the finger configurations represented the correct result of the operation. The authors interpreted these results in line with the *ideomotor theory*: during development, children use their fingers to solve arithmetic problems. The resulting finger configurations then overlap with the arithmetic outcomes of the operations and are therefore reactivated each time the result of the operation has to be retrieved. In this sense, these data are reminiscent of the finger-counting strategy used by children when learning basic arithmetic operations.

Another feature of the finger-counting behaviour which can be observed in adults is the sub-base-five property, where fingers used to represent numbers larger than 5 will always include a full hand representation (Marton & Neuman, 1990). For example, eight will always be represented as {5 + 3} (i.e., 5 fingers of one hand and 3 fingers on the other hand) and never as {4 + 4}. This property of the sub-base-five finger-counting system involves the parity of numbers larger than 5 being not longer straightforwardly assigned: the numbers 6, 8 and 10, although even with respect to the base-10 system, are odd with respect to the base-5 system, as 6 is represented by

{5 + 1}, 8 by {5 + 3}, and 10 by {5 + 5}. Inversely, the numbers 7 and 9 are odd with respect to the base-10 system but are even with respect to the base-5 system, 7 being represented by {5 + 2} and 9 being represented by {5 + 4}. In line with this logic, deaf German speakers, who use German Sign Language (DGS)[3] as their native language, showed a classic MARC-effect (i.e., responses to even numbers are faster with the right hand whereas responses to odd numbers are faster with the left) for the numbers 1-5 but a reversed MARC-effect for the numbers 6-9 (Iversen, Nuerk, Jäger, & Willmes, 2006). Most importantly, this pattern of results was found not only with DGS number signs but also with written number words, thus indicating that finger-based numerical representation influences not only the sub-base-five digital representation of numbers but also the 10-base symbolic numerical system.

A recent study by Domahs and colleagues (Domahs, Moeller, Huber, Willmes, & Nuerk, 2010) showed that sub-base-five effects may also be observed in an Arabic digit comparison task and with cultural groups whose finger-counting system exhibited the sub-base-five property. Indeed, when deaf and hearing German adults (see Fig. 7-5) were required to identify the smaller or the larger number of a pair, responses were slower when at least one number of the pair was larger than 5. According to the authors, this sub-base-five effect is well accounted for by the *generation hypothesis*: finger-based representations for numbers larger than 5 involve the activation of both hands and are therefore more complex to generate than those for numbers smaller or equal to 5. Interestingly, Chinese participants, who did not represent large numbers by activating both hands (see Fig. 7-5), did not exhibit such five break effects, suggesting that the influence of the finger-based numerical

[3] The sub-base-five property is highly salient in DGS (see Fig. 7.5)

representation varies as a function of the finger-counting system in use in a specific culture.

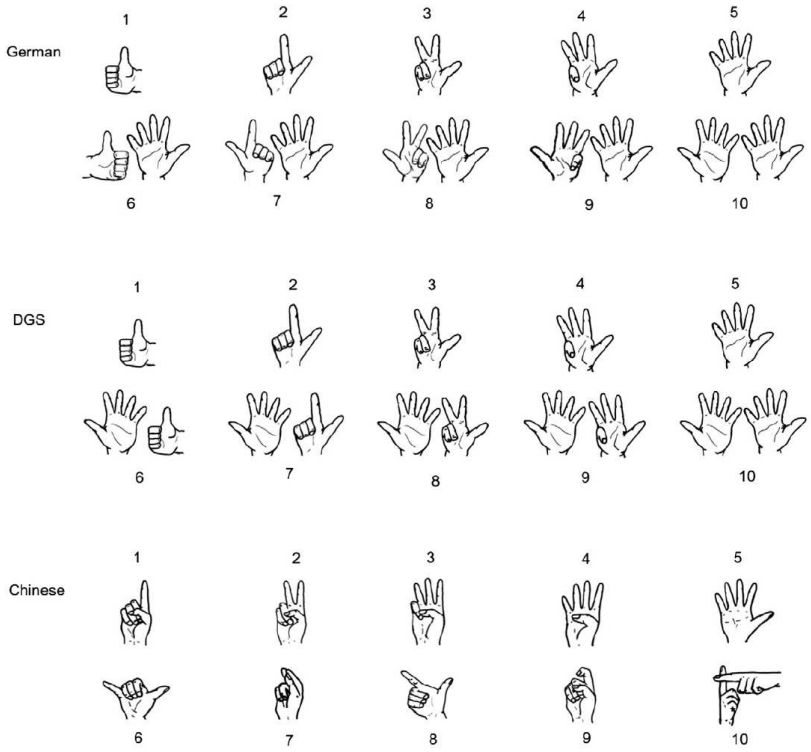


Fig 7-5. Finger-counting systems used in hearing German, deaf German (DGS) and Chinese adults. The saliency of the sub-base-five system varies as a function of the structure of the finger-counting system in use in a specific cultural group. In hearing and deaf German adults, the sub-base-five property is very salient: every number is represented in one-to-one correspondence as finger quantities. In Chinese, on the opposite, only the numbers 1-5 are represented in one-to-one correspondence with fingers; the numbers larger than 5 being coded with only one hand and as numerical symbols. *Reproduced from Domahs et al. (2010).*

A trace of the finger-pointing strategy in adult numerical processing was also evidenced in a study by Andres, Seron and Olivier (2007), who found an increased corticospinal excitability of hand muscles when participants were required to put items in correspondence with elements of an ordered series (i.e., number words or letters). This increase in corticospinal activity was specific to hand muscles because no comparable changes were observed for arm and foot muscles. According to the authors, this specific involvement of hand motor circuits is reminiscent of the use of

fingers to coordinate the act of tagging an object with saying the number words (Alibali & DiRusso, 1999).

Finally, finger-number interactions in adults have been observed beyond this automatic activation of a number and its corresponding canonical finger configurations. Several studies have indeed revealed that processing the magnitude of a number also interfered with grasping movements. Among these, a study by Andres, Davare, Pesenti, Olivier and Seron (2004) demonstrated, with electromyographic recordings, that grip closure was initiated faster in response to small numbers whereas grip opening was initiated faster in response to large numbers (see Lindemann, Abolafia, Girardi, & Bekkering, 2007 for similar results with natural grasping movements and visually presented digits). This interaction between number magnitude and grip aperture occurs at the programming stage of grasping movements (Andres, Ostry, Nicol, & Paus, 2008) and affects the judgment of action towards the object even when no actual action is required, but when participants only have to judge whether they would be able to grasp a rod between their thumb and index finger (Badets, Andres, Di Luca, & Pesenti, 2007). All these studies actually described interactions from semantics (i.e., number magnitude) to sensory-motor processes (i.e., grasping movements) but the inverse interaction (i.e., from sensory-motor processes to semantics) was also reported in a study by Badets and Pesenti (2010). Here, response latency to a parity judgment task was faster when a grip closing preceded a response to a small number or when a grip opening preceded a response to a large number. As this interaction was only found with biological hand movements, the authors (Badets & Pesenti, 2010) postulated that it did not result from a general movement amplitude processing but rather stemmed from the automatic

activation of a code system sharing a dimension of magnitude across numbers and finger movements.

2.3 Conclusions

In childhood, finger-counting helps children to keep track of their counting and understand cardinality, one-to-one correspondence and stable-order principles. Finger-counting strategies also lead to the creation of iconic representations of small numbers (i.e., associations between a specific number and a particular finger) and allow children to infer the base-10 mathematical system. Because traces of the finger-counting behaviour were even found to influence adults' numerical cognition, some authors (e.g., Andres, Di Luca & Pesenti, 2008) assumed that the symbolic numerical system was grounded in these finger-based representations.

To be efficient, finger-counting probably requires a precise knowledge of the position of the fingers on the hand and the recognition of particular hand shapes. Interestingly, these abilities were assumed to take place in the parietal cortex, a brain area often associated with the dorsal visual pathway. So, the question arises as to whether vision is critical or not for the development of the interactions between numbers and fingers. In other words, do blind children use their fingers so often than their sighted peers to establish the symbolic numerical system? To the best of our knowledge, this question has never been investigated and will therefore be the subject of Chapter 9.

Experimental questions

The second part of our thesis led us to investigate: **(1)** the impact of blindness on interactions between numbers and space; **(2)** the impact of blindness on interactions between numbers and fingers.

1. Blindness and interactions between numbers and space

The SNARC effect refers to the fact that the time given to respond to a numerical stimulus is faster when the magnitude of the stimulus (small vs. large) is compatible with the side of the response (left vs. right). Two interpretations of the SNARC effect were proposed in the literature. According to the first view (i.e., the visuo-spatial coding account), the SNARC effect reflects the left-to-right orientation of the mental number line and emerges from an external coordinate system (Dehaene et al., 1993). According to a less widespread view (i.e., the verbal-spatial account), the SNARC effect is not necessarily located at a visuo-spatial level but may also emerge from a categorical level (Gevers et al., 2006, 2010). As blindness affect the development of the external coordinate system (i.e., the visuo-spatial account; Röder et al., 2007) but not the development of language (i.e., the verbal-spatial account), the study of blindness represents a unique opportunity to test the co-existence of these two different models. This is the aim of our Chapter 8.

2. Blindness and interactions between numbers and fingers

Developmental data have shown that fingers play a crucial role in the acquisition of arithmetical abilities in children. However, we do not know whether vision plays a mandatory role in the development of these interactions between fingers and number

processing. To address this issue, we asked blind and sighted children to perform enumeration tasks, an experiment which is presented in Chapter 9. We paid special attention to the spontaneous strategies used by the children to solve these tasks. On the assumption that finger-counting provides a universal ground for the acquisition of mathematical abilities because they draw on visual, proprioceptive and motor representations (Andres et al., 2008), both blind and sighted children should preferentially use their fingers to perform enumeration tasks. In contrast, if finger-number interactions principally arise from the visual recognition of particular hand shapes, then, blind children should not use their fingers as often as their sighted peers.