Utilitarianism and Unequal Longevities: A Remedy?*

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Abstract

In the context of unequal deterministic longevities, classical utilitarianism exhibits, under time-additive individual preferences, a counterintuitive tendency to redistribute resources from short-lived agents towards long-lived agents, against any intuition for compensation. We examine the robustness of that result to the introduction of risky lifetime, and to a broader class of individual preferences. It is shown that classical utilitarianism remains unable to provide, in that broader framework, a general redistribution towards the short-lived. Then, we propose a remedy, which consists in imputing, when solving the social planner's allocation problem, the consumption equivalent of a long life to the consumption of long-lived agents. This compensation-constrained utilitarianism is shown to reduce welfare inequalities across agents with unequal lifetimes.

Keywords: utilitarianism, differential longevity, compensation, redistribution, consumption equivalent.

JEL classification codes: D63, I12, I18, J18.

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1 Introduction

Although widely used by taxation theorists, classical utilitarianism - i.e. the social objective introduced by Bentham (1789), and based on welfarism, consequentialism and sum-ranking - exhibits nonetheless a counterintuitive corollary in the context of unequal deterministic longevity.[1] Under standard assumptions on individual preferences, such as time-additive lifetime welfare, classical utilitarianism recommends a redistribution of resources from *short-lived* agents to *long-lived* agents.

That corollary, which contradicts any intuition of compensation, can be explained as follows. Assume that all individuals have initially the same endowment of material resources, but different longevities, which are known by them. At the laissez-faire, short-lived individuals consume, on average, more resources per period, whereas long-lived agents consume, on average, less per period. Which allocation does classical utilitarianism recommend in that case? Under time-additive lifetime welfare, a utilitarian social planner can hardly distinguish between, on the one hand, one life of x periods, and, on the other hand, x lives of one period.[2] Hence, provided Gossen's First Law (1854) - i.e. the law of declining marginal utility of consumption per period - holds, it is always optimal, for a utilitarian planner, to give the same consumption per period to all agents, whatever their length of life is. As a consequence, long-lived agents benefit, at the utilitarian social optimum, from more resources than the short-lived. Hence, provided life is worth being lived, short-lived individuals are penalized *twice*: once by Nature (shorter life) and once by Bentham (fewer resources).

This double penalization is quite counterintuitive, especially when longevity differentials are exogenous. Clearly, one would like short-lived agents to be *compensated* for their short life, as they cannot be regarded as responsible for this. Classical utilitarianism can hardly do justice to such intuitions. All this does not really come as a surprise: as stressed by Mirrlees (1982), utilitarianism can, at best, serve as an ethical standard in the special case of a society of *identical* individuals, because, in that case, the society as a whole can be regarded as a single individual. However, once some heterogeneity is introduced in the fundamentals (e.g. preferences, handicap, etc.), utilitarianism can only serve as a useful approximation, and may lead to counterintuitive results.[3]

Although expected, that counterintuitive corollary is really problematic, since longevity inequalities are universal.[4] Should we then abandon classical utilitarianism when considering policy issues in which agents have unequal lengths of life, that is, in almost all policy issues? Or is it possible to escape

[1] Note that an earlier formulation of the principle of the largest happiness for the largest number can be found in Beccaria (1764).

[2] For simplicity, we abstract here from pure time preferences. Natural discounting through survival probabilities is assumed throughout this paper.

[3] See Arrow (1971) and Sen (1973) on the utilitarian treatment of handicap.

[4] For instance, according to the United Nations Development Program (2008), the life expectancy of women is, in the U.S., about 5.2 years larger than the one for men in 2007 (80.4 years against 75.2). There exist also large disparities in survival conditions according to the education, the income, the ethnicity, and the employment status (see Rogot *et al* 1992).

from that paradox, for instance by considering a broader domain of individual preferences (e.g. non time-additive lifetime welfare)?

The goal of this paper is twofold. First, we examine, by means of a simple model with risky lifetime, the conditions under which classical utilitarianism redistributes resources from short-lived towards long-lived agents. Then, having shown the generality of those conditions, our second goal is to propose a "remedy" to that counterintuitive corollary of classical utilitarianism.

For those purposes, we focus on a two-period model with risky lifetime, where agents allocate their endowment over the lifecycle, while ignoring the time of their death. We consider two groups of agents, who differ in their life expectancy. In order to isolate the role played by individual preferences, we assume that lifetime welfare can take either a standard time-additive form, or can be a concave transform of the sum of temporal utilities, so as to account for risk-aversion with respect to the length of life.[5] We solve for the laissez-faire equilibrium and for the classical utilitarian optimum, and examine under which conditions classical utilitarianism makes the short-lived worst-off than under the laissez-faire. To do so, we adopt an *ex post* welfarist approach, and compare individual realized lifetime welfare levels, rather than individual expected lifetime welfare levels, contrary to *ex ante* approach.[6]

Then, we explore how one could overcome the general tendency of classical utilitarianism to redistribute resources towards the long-lived. The solution that we propose consists of the addition, in the utilitarian social planner's problem, of *compensation constraints*. The underlying idea is that long-lived agents are advantaged, since their longer life gives them, *ceteris paribus*, a higher capacity to convert resources in terms of welfare, and that this advantage should, on the grounds of fairness, be counted as a part of their consumption bundle. We measure the advantage of long-lived agents by the *consumption-equivalent of a long life*, and count this as part of their consumption.[7] The remedy, which can be called "compensation-constrained utilitarianism", consists of solving a social planning problem where all consumptions - either of short-lived or of long-lived agents -, are homogenized by that procedure.

Anticipating on our results, we show that, under risky lifetime, classical utilitarianism still fails to provide a general compensation to the short-lived individuals. Moreover, the tendency of classical utilitarianism to redistribute resources towards long-lived agents is shown to be a general phenomenon, which is robust to various specifications of individual preferences. Assuming non-additive lifetime welfare does not, in general, suffice to avoid the redistribution towards long-lived agents. We also show that this paradoxical redistribution can be avoided by introducing compensation constraints. Those constraints, by counting the consumption equivalent of the long life as part of the consumption of the long-lived, have a major impact on the direction of redistribution. The undesirable redistribution from short-lived to long-lived agents is, in general,

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[5] This is in line with Bommier (2006) and Bommier *et al* (2011a, 2011b).

[6] On the *ex post* versus *ex ante* approach to welfare economics, see Fleurbaey (2010).

[7] That remedy is close to what Broome (2004) proposes in his attempt to account for the value of longevity in a utilitarian framework, but in a goods metrics (and not utility metrics).

turned into a redistribution from long-lived to short-lived agents.

By those results, the present paper complements the literature on optimal taxation under unequal longevities, such as Bommier (2006) and Bommier *et al* (2011a, 2011b). Our specificity with respect to them is to identify the conditions under which classical utilitarianism yields counterintuitive redistribution, as well as conditions under which some compensation can take place thanks to the homogenization of consumptions. This paper contributes also to the population ethics literature, which already highlighted serious paradoxes faced by classical utilitarianism (see Parfit 1984, Broome 2004, Arrhenius 2011). Our specificity is to focus on a particular paradox, related to the redistributive corollary of classical utilitarianism under unequal longevities, and to propose a solution to it. Finally, we also complement papers on the measurement of welfare variations due to longevity differentials, such as Usher (1973), Nordhaus (2003) and Becker *et al* (2005), which all focused on the monetization of longevity gains. The present paper shows how such a monetization can be used to homogenize consumptions across agents having unequal longevities, in order to avoid counterintuitive redistributive corollaries.

Whereas one may regard the topic of this paper as a purely theoretical contribution, the scope of our results for policy-making in the real world could hardly be overemphasized. Indeed, real world longevity inequalities are substantial, and various aspects of existing social security systems - in particular, pension systems - are instances of distributions from short-lived towards long-lived individuals. Pension systems were designed as ways to smooth consumption across periods and, often, across individuals, in line with classical utilitarianism. Such a smoothing is unambiguously beneficial to those who live long; but it reinforces welfare inequalities between individuals with unequal longevities. The main point of the paper is to highlight the double penalty faced by the short-lived, and to propose a simple solution to overcome it. That solution could easily be introduced in policy-oriented models, as we will show below.

This paper is organized as follows. Section 2 presents the utilitarian redistribution problem under risky and unequal longevities. Section 3 presents the remedy, and contrasts the modified first-best with the classical utilitarian optimum. Numerical illustrations are given in Section 4. Section 5 concludes.

2 The model

We consider a two-period economy with risky lifetime. All agents live the young age (period 1) for sure, but reach the old age (period 2) with a probability π.[8] The length of each period is normalized to 1. There exist two types of agents,

[8] Obviously, a model with $T > 2$ periods would be more appropriate for an empirical study. However, for the particular issue at stake here, such a model would complicate the analysis significantly, without bringing any new result.

$i = 1, 2$, who differ in their survival chances π^i,[9]

$$\pi^1 < \pi^2$$

Without loss of generality, we assume an equal mass of agents of both types, of size normalized to the unit interval.[10]

Each agent has one half of the total endowment W of resources. Agents choose to allocate their endowment across their lifecycle without knowing the time of their death. Agents' preferences are assumed to satisfy the expected utility hypothesis.[11] Moreover, the utility of death is normalized to zero. Hence, the lifetime welfare of an agent of type i, denoted by U^i, takes the form:

$$U^i = \pi^i G\left(u(c^i) + u(d^i)\right) + \left(1 - \pi^i\right) G\left(u(c^i)\right) \quad (1)$$

where c^i and d^i are first- and second-period consumptions for agent $i = 1, 2$, $u(\cdot)$ is the temporal utility of consumption (with $u'(\cdot) > 0$ and $u''(\cdot) < 0$), and $G(\cdot)$ is an increasing and (weakly) concave transform of individual temporal utilities (i.e. $G'(\cdot) > 0$ and $G''(\cdot) \leq 0$).[12]

Regarding the specification of $u(\cdot)$, we assume that the utility function takes the form $u(c) = v(c) + \beta$, where $v(0) = 0$, and $\lim_{c\to\infty} v(c) = \bar{v}$, with $\beta < \bar{v} < \infty$. Moreover, we assume that $G(0) = 0$ and $\lim_{x\to\infty} G(x) = \bar{G} < \infty$. Those assumptions are equivalent to assuming finite welfare differentials between any two lives: the difference between the welfare levels associated with two lives is necessarily *finite*. In other words, a life cannot be, under that assumption, infinitely better (or infinitely worse) than another life.

The standard time-additive lifetime welfare coincides with the case where $G(\cdot)'' = 0$. As shown by Bommier (2006), the linearity of $G(\cdot)$ implies that agents are net risk neutral with respect to the life duration, in the sense that agents with no impatience are strictly indifferent between lotteries with the same expected length of life (under a constant per period consumption). A concave function $G(\cdot)$ allows for net risk aversion with respect to the length of life.

Note that, throughout this paper, we will, when comparing the welfare of individuals, focus exclusively on welfare inequalities in realized terms, that is, we will adopt an *ex post* welfare analysis, in line with Fleurbaey (2010). That approach differs from the usual *ex ante* one, since the perspective of a higher *expected* length of life (or life expectancy) *ex ante* does not necessarily imply the enjoyment of a longer life *ex post*.[13] By adopting here an *ex post* approach, we consider that what really matters for human welfare consists of the *realized* - rather than the expected - outcomes.

[9] For simplicity, we assume here that this probability is exogenous. On optimal tax policy under endogenous survival probabilities through health spending, see Leroux *et al.* (2011).

[10] The mass is sufficiently large, to be able to apply the Law of Large Numbers (see *infra*).

[11] The expected utility hypothesis is an obvious simplification. For an alternative framework based on the moments of utility approach, see Leroux and Ponthiere (2009).

[12] We also assume that $G'(0)$ is finite.

[13] For instance, although women exhibit a higher life expectancy than men, some women have a shorter life than some men, so that the groups with distinct survival prospects do not correspond to the groups with distinct actual longevities.

2.1 The laissez-faire

The problem faced by an agent of type i can be written as:

$$\max_{c^i,d^i} \pi^i G\left(u(c^i)+u(d^i)\right)+\left(1-\pi^i\right) G\left(u(c^i)\right)$$
$$\text{s.t.} \left\{ \begin{array}{c} c^i \leqslant \frac{W}{2}-s^i \\ d^i \leqslant s^i R^i \end{array} \right.$$

where s^i denotes savings and R^i is the return on savings. It is assumed, for simplicity, that the pure interest rate is zero, and that a perfect, class-specific, annuity market exists, which yields an actuarially fair return, so that the return on savings R^i is $1/\pi^i$. The first order conditions yield:

$$\frac{\left[\pi^i G'\left(u(c^i)+u(d^i)\right)+\left(1-\pi^i\right) G'\left(u(c^i)\right)\right]}{G'\left(u(c^i)+u(d^i)\right)} u'(c^i)=u'(d^i)$$

Under a linear $G(\cdot)$, we obtain that $c^i=d^i$, so that substituting into the individual's lifetime budget constraint, laissez-faire levels of consumptions are:

$$c^1=d^1 = \frac{W}{2\left(1+\pi^1\right)} > c^2=d^2 = \frac{W}{2\left(1+\pi^2\right)}$$

Thus, in the laissez-faire, agents with a shorter life expectancy consume more than agents with a high life expectancy, because they face a lower chance to ever be able to consume in the second period. In the special case with deterministic longevity (i.e. $\pi^1=0<\pi^2=1$), the allocation is: $c^1=W/2>c^2=d^2=W/4$, that is, the short-lived agent consumes the entire endowment before dying, whereas the long-lived agent smoothes consumption.

On the contrary, under $G''(\cdot)<0$, the fraction on the LHS of the FOC is larger than one, which leads to $c^i>d^i$, and to the following ranking:

$$d^1 < \frac{W}{2\left(1+\pi^1\right)} < c^1 \quad \text{and} \quad d^2 < \frac{W}{2\left(1+\pi^2\right)} < c^2$$

Note that, under deterministic longevity, the allocation remains the same as under linear $G(\cdot)$: $c^1=W/2>c^2=d^2=W/4$.

Let us now compare welfare levels across agents. Remind that we have here 4 types of agents *ex post*, depending on their life expectancy (i.e. π^1 or π^2) and on the realized longevity (i.e. 1 or 2 periods). For the sake of presentation, let us first focus on the simple case with deterministic longevity.

Proposition 1 *Consider the laissez-faire with deterministic longevity ($\pi^1=0<\pi^2=1$). If $u(0)\geq 0$ (i.e. $\beta\geq 0$), type-2 agents are better off than type-1 agents, whatever the level of W is. If $u(0)<0$ (i.e. $\beta<0$), type-2 agents are better off than type-1 agents if and only if $W>W^S$, where W^S is such that $2v\left(\frac{W^S}{4}\right)+\beta=v\left(\frac{W^S}{2}\right)$.*

Proof. See the Appendix. ■

W^S denotes, in a riskless world, the level of endowment such that short-lived and long-lived individuals have the same lifetime welfare. If $W > W^S$ (resp. $W < W^S$), the long-lived is better off (resp. worse off) than the short-lived.

Thus, under either $u(0) \geq 0$ or $W > W^S$, short-lived agents enjoy, for an *equal* amount of resources, a lower lifetime welfare than long-lived agents. Note that the condition for the disadvantage of short-lived agents is invariant to the transform $G(\cdot)$, that is, to whether lifetime welfare is time-additive or not. Moreover, given that it is only under a very low level of resources that short-lived are advantaged by Nature, we will, in the rest of this paper, pay a larger attention to the case where short-lived are disadvantaged.

Proposition 1 suggests that, even in the absence of uncertainty about the length of life, short-lived agents are disadvantaged with respect to long-lived agents, in the sense that they enjoy, despite the same initial endowment, a lower lifetime welfare level. This is entirely due to the concavity of the temporal utility function $u(\cdot)$. Indeed, under a linear temporal welfare, the realized length of life would have no welfare effect at all.

Turning now to the case with stochastic longevity, we have:

Proposition 2 *Consider the laissez-faire with stochastic longevity ($0 < \pi^1 < \pi^2 < 1$). If $u(0) \geq 0$ (i.e. $\beta \geq 0$), long-lived agents of type i are better off than short-lived agents of type i, whatever W is. If $u(0) < 0$ (i.e. $\beta < 0$), long-lived agents of type i are better off than short-lived agents of type i if and only if $W > W^{Si}$, where W^{Si} is such that $v\left(\frac{W^{Si}}{2(1+\pi^i)}\right) + \beta = 0$.*

Proof. See the Appendix. ■

W^{Si} denotes, in a risky world, the level of endowment such that short-lived and long-lived agents of type i have the same lifetime welfare. If $W > W^{Si}$ (resp. $W < W^{Si}$), the long-lived of type i is better off (resp. worse off) than the short-lived of type i.

As with deterministic longevity, short-lived agents are penalized in comparison to long-lived agents when the intercept of the temporal utility function is non-negative. However, under a negative β, it is only if W exceeds some group-specific threshold W^{Si} that short-lived persons are worse-off than long-lived agents with the same life expectancies. Note that it may possible that $W^{S2} > W > W^{S1}$, so that long-lived agents are better off than short-lived agents for type-1 agents, but that the opposite holds for type-2 agents.[14]

Finally, we compare the threshold W^{Si} under stochastic longevity with its counterpart in absence of risk, i.e. W^S.

Proposition 3 *Comparing the thresholds W^S and W^{Si}, we have: $W^S > W^{Si}$.*

Proof. See the Appendix. ■

[14]This case is related to the existence of perfect group-specific annuity markets. Type-2 agents, having a higher life expectancy, enjoy also lower savings return in case of survival, in comparison to type-1 agents.

The necessary condition for the disadvantage of being short-lived under risky longevity is *weaker* than the corresponding condition under deterministic longevity (Proposition 1), as the threshold of resources W^{Si} is inferior to W^S. Under deterministic longevity, short-lived people can avoid some part of the damage from being short-lived by consuming all their endowment in the first period, because of them perfectly anticipating the time of their death. However, in a risky world, such a behavior is not optimal (as it is then rational to save resources for the case where they survive), so that the mere fact of saving resources for the old days leads to a higher damage from having a short life. Hence, short-lived agents are more penalized in a risky world, since beyond the cost of non-spreading consumption on several periods, there is an additional cost related to the lost resources in case of unanticipated date of death.

2.2 The classical utilitarian optimum

We now examine how a classical utilitarian social planner would allocate resources. The social planner maximizes average lifetime welfare, subject to the budget constraint of the economy:[15]

$$\begin{aligned}
&\max_{c^i,d^i} \sum_{i=1,2} \pi^i G\left(u(c^i)+u(d^i)\right)+\left(1-\pi^i\right) G\left(u(c^i)\right) \\
&\text{s.to} \sum_{i=1,2} \left(c^i+\pi^i d^i\right) \leq W
\end{aligned}$$

The first order conditions are, under $\pi^i > 0$:

$$\begin{aligned}
\left[\pi^i G'\left(u(c^i)+u(d^i)\right)+\left(1-\pi^i\right) G'\left(u(c^i)\right)\right] u'(c^i) &= \lambda \\
u'(d^i) G'\left(u(c^i)+u(d^i)\right) &= \lambda
\end{aligned}$$

where λ is the Lagrange multiplier associated with the resource constraint.

Let us first study of case with deterministic longevity ($\pi_1 = 0 < \pi_2 = 1$). The FOCs are:[16]

$$\begin{aligned}
G'\left(u\left(c^1\right)\right) u'\left(c^1\right) &= G'\left(u\left(c^2\right)+u\left(d^2\right)\right) u'\left(c^2\right) \\
&= G'\left(u\left(c^2\right)+u\left(d^2\right)\right) u'\left(d^2\right)
\end{aligned}$$

Given $G''(.) \leq 0$, per period consumptions are equal for type-2 agents, i.e. $c^2 = d^2$, but the consumptions of type-1 and type-2 agents differ.

Proposition 4 *Under deterministic longevity ($\pi_1 = 0 < \pi_2 = 1$), the classical utilitarian optimum is such that:*
- If $G''(\cdot) = 0$, we have: $c^1 = c^2 = d^2 = W/3$.

[15] For simplicity, we assume that the resources are perfectly transferable across periods. Obviously, in the real world, the possibility, for a government, to transfer resources across periods is quite limited, but here we make that assumption to stay in a first-best world.
[16] For type-2, both FOCs hold, while for type-1, only the first FOC holds.

- *If $G''(\cdot) < 0$, we have:*
  - *under $\beta > -v\left(\frac{W}{3}\right)$, $c^1 > c^2 = d^2$.*
  - *under $\beta = -v\left(\frac{W}{3}\right)$, $c^1 = c^2 = d^2 = W/3$.*
  - *under $\beta < -v\left(\frac{W}{3}\right)$, $c^1 < c^2 = d^2$.*

Proof. See the Appendix. ■

Under standard time-additive lifetime welfare (i.e. $G''(\cdot) = 0$), consumption is smoothed across people and periods. Hence, utilitarianism redistributes resources from short-lived agents to long-lived agents, contrary to any intuition of compensation, which would rather recommend the short-lived's consumption to *exceed* its laissez-faire level, that is, $c^1 > W/2$.[17] Such a compensation is clearly not achieved under classical utilitarianism, which, by leading to $c^1 = W/3 < W/2$, deteriorates the situation of the short-lived even more. However, once $G''(\cdot) < 0$, the utilitarian allocation depends on the size of the intercept β. If $\beta > -v\left(\frac{W}{3}\right)$, the utilitarian planner gives more to the short-lived per period than to the long-lived. Nevertheless, this does not imply that classical utilitarianism necessarily compensates short-lived agents. It is only under strict conditions that such a compensation prevails:[18]

Proposition 5 *Consider the deterministic longevity case. Assume $W > W^S$. Under classical utilitarianism, short-lived agents receive a compensation in comparison with the laissez-faire, i.e. $c^1 > W/2$, if and only if:*

$$\frac{G'\left(v\left(\frac{W}{2}\right)+\beta\right)}{G'\left(2v\left(\frac{W}{4}\right)+2\beta\right)} > \frac{v'\left(\frac{W}{4}\right)}{v'\left(\frac{W}{2}\right)}$$

Proof. See the Appendix. ■

To interpret Proposition 5, let us consider some polar cases. If lifetime welfare is time-additive, the LHS of the condition of Proposition 5 equals 1, whereas the RHS is larger than 1, so that there cannot be any compensation of short-lived agents. It is even worse, as $c^1 = W/3 < W/2$ involves a double penalization of short-lived agents: one by Nature (shorter life) and one by Bentham (lower total consumption). Thus the concavity of the transform $G(\cdot)$ is required for the existence of a compensation of short-lived agents under classical utilitarianism. But although necessary, that condition is not sufficient. The reason why this is not so has to do with the non-linearity of temporal welfare, which supports the opposite of compensation. Indeed, if $v(\cdot)$ was linear, the RHS of the condition in Proposition 5 would be equal to 1, so that, provided W lies strictly above the threshold level W^S, the LHS of the condition would exceed 1, and some compensation of the short-lived would take place. However, temporal utility is likely to be concave, leading to a RHS larger than 1, and this

[17] That redistribution violates, under the conditions of Proposition 1, what Sen (1973) called the Weak Equity Axiom: when an agent has a lower utility than another for all levels of income, the optimal allocation must not give less income to him than to the other.

[18] Under $W \leq W^S$, short-lived agents never receive a compensation: $c^1 < W/2$, because they are better off than long-lived ones.

explains why non-additive lifetime welfare does not suffice, on its own, to bring a compensation of short-lived agents with respect to the laissez-faire.

In general, and despite $G(\cdot)'' < 0$, short-lived individuals suffer, under classical utilitarianism, from a double penalization. The reason is that the transform $G(\cdot)$ is likely to be closer to a linear function than the function $v(\cdot)$, on the grounds that temporal welfare "accumulates" over time periods with a low degree of declining marginal "returns", so that the LHS of the condition must be close to 1, unlike the RHS, where the difference between $v'(\frac{W}{4})$ and $v'(\frac{W}{2})$ is likely to be large, implying that the RHS exceeds 1.[19] Thus classical utilitarianism does not, in general, compensate short-lived agents, and does even penalize them, by transferring resources towards long-lived agents.

Note also that, if $G(\cdot)'' = 0$, the inequality in lifetime welfare between long-lived and short-lived agents is, under $W > W^S$, larger at the utilitarian first-best than under the laissez-faire, as a result of the double penalization of the short-lived. If $G(\cdot)'' < 0$, and if $G(\cdot)$, $u(\cdot)$, W and β are such that the condition of Proposition 5 is not satisfied - which is the most plausible case - then utilitarianism raises lifetime welfare inequalities between long-lived and short-lived agents in comparison with the laissez-faire. In sum, the tendency of classical utilitarianism to redistribute towards the long-lived is robust to the specification of individual preferences. Assuming non-additive lifetime welfare does *not* suffice to make the paradox disappear.[20]

Let us now turn to the case with stochastic longevity.

Proposition 6 *Under stochastic longevity, the utilitarian optimum is such that:*
- If $G''(\cdot) = 0$, we have: $c^1 = d^1 = c^2 = d^2 = \frac{W}{2+\pi^1+\pi^2}$
- If $G''(\cdot) < 0$, we have:
- under $\beta > -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$, $d^2 < d^1 < \frac{W}{2+\pi^1+\pi^2} < c^1 < c^2$ or $d^1 < d^2 <$

[19] Another way to show that the condition of Proposition 5 is likely to be violated is to rewrite it as

$$\frac{G'(y)}{G'((1+k)y)} > \frac{v'(x)}{v'(2x)}$$

with $0 < k < 1$, $x \equiv W/4$ and $y \equiv v(W/2) + \beta$. Note first that $0 < k < 1$ can be justified as follows. The sum of temporal welfares induced by two periods of life with consumption $W/4$ is, under $W > W^S$, necessarily larger than the welfare induced by one period of life with consumption $W/2$, implying $0 < k$, but also necessarily less than twice the welfare induced by one period of life with consumption $W/2$, implying $k < 1$. Back to the condition, one can see that, given $k < 1$, the *relative* change (from the numerator to the denominator) considered in the arguments of $v(\cdot)$ and $G(\cdot)$ is strictly larger on the RHS than on the LHS. Hence the effect of those relative changes on the marginal derivative is larger on the RHS than on the LHS, implying that the RHS exceeds the LHS, contrary to the condition of Proposition 5.

[20] That paradox is close to, but distinct from, another paradox faced by classical utilitarianism: the Repugnant Conclusion (Parfit 1984). That paradox goes as follows: for any population with some extremely low utility per person, it is possible to find a larger population, with an even lower utility per person, such that total welfare is, on agregate, larger than initially. As shown by Arrhenius (2011), that paradox is due to the assumed possibility to substitute the utility of a men by the utility of a newcomer for any utility level. The same kind of strong substituability is at work here, in the sense that our paradox arises from the substituability between the utility of the young and the one of the surviving old, who can be interpreted as a "newcomer".

$\frac{W}{2+\pi^1+\pi^2} < c^2 < c^1$.

- under $\beta = -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$, $c^2 = c^1 = d^1 = d^2 = \frac{W}{2+\pi^1+\pi^2}$.

- under $\beta < -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$, $c^2 < c^1 < \frac{W}{2+\pi^1+\pi^2} < d^1 < d^2$ *or* $c^1 < c^2 < \frac{W}{2+\pi^1+\pi^2} < d^2 < d^1$.

Proof. See the Appendix. ■

As with deterministic longevity, utilitarianism equalizes, under a linear transform $G(\cdot)$, consumptions for all agents and at all periods. Under a concave transform $G(\cdot)$, the optimum depends on the level of the intercept β. Depending on its level, utilitarianism recommends consumption profiles that are increasing or decreasing with the age.[21] Regarding the comparison of type-1 and type-2 agents, utilitarianism does not give an advantage to some type of agents at all periods, but, rather, gives more consumption to some agents at one period and to the other agents at the other period. Hence, it is difficult to see how utilitarianism redistributes resources between different types. Proposition 7 summarizes our results.

Proposition 7 *Under stochastic longevity, utilitarianism affects lifetime welfare inequalities as follows:*

- Under $G''(\cdot) = 0$ *and for any* β*, utilitarianism increases, with respect to the laissez-faire, inequalities between the long-lived and the short lived for type-*2 *agents, and does the opposite for type-*1 *agents.*

- Under $G''(\cdot) < 0$*, the impact of utilitarianism on inequalities between the long-lived and the short-lived within each group is the following:*

- under $\beta > -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$*, utilitarianism has ambiguous effects.*

- under $\beta = -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$*, utilitarianism increases, with respect to the laissez-faire, inequalities between the long-lived and the short-lived for type-*2 *agents, and does the opposite for type-*1 *agents.*

- under $\beta < -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$*, utilitarianism has ambiguous effects.*

Proof. See the Appendix. ■

Whether utilitarianism reinforces or reduces, with respect to the laissez-faire, inequalities between short-lived and long-lived within a group i of agents depends on the precise form of individual preferences, that is, on $G(\cdot)$ and $v(\cdot)$. The redistributive impact of classical utilitarianism between short-lived and long-lived agents depends also on the group i considered, that is, on whether agents have high or low survival chances. If $G(\cdot)$ is linear, utilitarianism raises inequalities between short-lived and long-lived within the type-2 group (the one with high survival chances), but reduces these inequalities in the other group (the one with lower survival chances), even though, in that latter group, some welfare inequalities between short-lived and long-lived agents still remain

[21] If β is sufficiently large, consumption profiles should be declining (to compensate the disutility from a short life), while the opposite holds if β is very low. In that latter case, consumption profiles should be increasing (to compensate the disutility from a long life).

at the optimum. The same result prevails under $G(\cdot)$ non-linear when $\beta = -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$. In the other cases, the utilitarian optimum depends on $u(\cdot)$ and $G(\cdot)$, as well as on W and on π^i, so that no conclusion can be drawn without imposing further assumptions.

Although the above analytical results are incomplete, and, as such, invite some numerical explorations (see below), these suffice to have serious doubts on the capacity of utilitarianism to compensate short-lived agents. In particular, under deterministic longevity, utilitarianism not only fails to compensate the short-lived, but, under mild conditions, leads to a redistribution towards the long-lived. Under stochastic longevity, classical utilitarianism does not provide a compensation to all short-lived individuals. That result is counterintuitive, since short-lived agents, whatever their survival chances were, cannot be taken as responsible for the duration of their life. Thus there exists no obvious reason why they should have, at the social optimum, a lower lifetime welfare than long-lived individuals. Therefore some remedy is to be found, in order to be able to provide a compensation for all short-lived individuals.

3 Compensation-constrained utilitarianism

In order to escape from the counterintuitive redistributive corollary of classical utilitarianism, the solution that we will study here consists of adding *compensation constraints* to the social planning problem. The underlying idea is that the short-lived is, in general, disadvantaged at the laissez-faire with respect to the long-lived, since he derives a lower amount of welfare from an equal amount of resources. Classical utilitarianism tends, in general, to reinforce those welfare inequalities, by redistributing resources towards the long-lived. Thus it does not do justice to the idea of compensation, according to which those who are disadvantaged at the laissez-faire should receive some compensation, aimed at reducing welfare inequalities prevailing at the laissez-faire.

The intuition behind compensation is that individuals should not undergo welfare losses due to damages for which they are not responsible (Fleurbaey and Maniquet 2006). This is the case in our context, where longevity differentials are exogenous, and cause large welfare losses. Hence, there is a strong case for giving more resources to the short-lived, and, even a stronger one for avoiding the double penalty induced, under general conditions, by classical utilitarianism.

Following the idea of compensation, it can be argued that the advantage of the long-lived should be, in the name of fairness, counted as a part of his own consumption basket. This is exactly what compensation constraints amount to do. Our remedy consists thus of carrying out a utilitarian planning problem, where consumptions are homogenized, to take into account the advantage or disadvantage of agents with unequal longevities.[22]

When presenting our remedy, we will proceed in two steps. We will first

[22] Note that our approach does not require uniform preferences, since consumption equivalents can be computed under heterogeneous preferences (see Fleurbaey 2005).

define what we call the consumption equivalent of a long life (subsection 3.1), and, then, redefine the social planner's problem by counting this consumption equivalent as part of the consumption of long-lived agents (subsection 3.2).

3.1 The consumption equivalent of a long life

This subsection examines how one can avoid the counterintuitive corollary of utilitarianism, by introducing compensation constraints. The introduction of compensation constraints amounts to homogenizing the consumptions for unequal longevities, by monetizing the advantage of the long-lived. To define homogeneous consumptions $\tilde{c}^i$ and $\tilde{d}^i$, we will count the consumption-equivalent of a longer life as a part of the long-lived consumption bundle.

For any agent of type i, the consumption-equivalent of a longer life α^i makes him *indifferent* between, on the one hand, a short life with that additional consumption, and, on the other hand, a long life:

$$G\left(u\left(c^i + \alpha^i\right)\right) = G\left(u\left(c^i\right) + u\left(d^i\right)\right) \tag{2}$$

where c^i and d^i are the consumptions chosen *ex ante* by a type-i agent under the laissez-faire (i.e. before knowing whether he survives to period 2 or not). The consumption-equivalent of a long life corresponds to the value, expressed in the (unique) consumption good, of enjoying a long life. Alternatively, α^i can be interpreted as reflecting the consumption equivalent of the continuity of life across periods. In the rest of this paper, we will assume that such a consumption-equivalent exists, in the sense that, for any longevity differential, it is possible to find a compensation in terms of the consumption good.[23]

Since $G(\cdot)$ appears on both sides of (2), we can simplify it to:

$$u\left(c^i + \alpha^i\right) = u\left(c^i\right) + u\left(d^i\right)$$

Hence, the consumption equivalent of a long life α^i is invariant to the transform $G(\cdot)$. This invariance implies that the remedy we propose below is distinct from applying a concave transform to the sum of temporal utilities.

Since α^i is invariant to the transform $G(\cdot)$, its construction can be made on the mere basis of information on the temporal utility function $u(\cdot)$. The construction of the consumption-equivalent of a long life is illustrated on Figure 1 (for type-2 agents). To find the value of α^2, we compute the sum of temporal utilities of a type-2 agent under the laissez-faire, equal to $u(c^2) + u(d^2)$, and look for the level of c^2 that yields the same temporal welfare $u(c^2)$. Hence, the consumption equivalent α^2, is the thick horizontal segment on the left graph of Figure 1.

The sign of the consumption equivalent α^i depends on the conditions guaranteeing that short-lived agents have a lower actual lifetime utility than long-lived

[23] That assumption is far from weak, especially if the longevity differentials considered are large. In that case, a consumption equivalent may not exist.

agents, that is, on the conditions of Proposition 2.[24] Given that temporal utility is concave in consumption, and that resources W are sufficiently large, α^i is likely to be positive: a long life is, *ceteris paribus*, better than a short life, that is, it is better, *for a given amount of resources*, to live long.

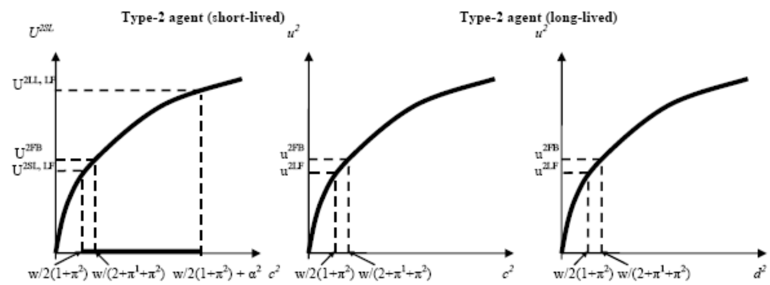

Figure 1: the consumption-equivalent

The level of α^i depends only on the shape of $u(\cdot)$. If, for instance, $u(\cdot)$ was linear and if $u(0) = 0$, we would have $u(c^i + \alpha^i) = W/2(1 + \pi^i) + \alpha^i$, and $u\left(c^i\right) + u\left(d^i\right) = W/(1 + \pi^i)$, so that α^i would be equal to $W/2(1 + \pi^i)$, that is, d^i. To make an agent indifferent between a short and a long life, it suffices to transfer second-period consumption to the first period. Alternatively, if $u(\cdot)$ is affine with an intercept β, we have $u(c^i + \alpha^i) = W/2(1 + \pi^i) + \alpha^i + \beta$, and $u\left(c^i\right) + u\left(d^i\right) = W/(1 + \pi^i) + 2\beta$, which yields $\alpha^i = W/2(1 + \pi^i) + \beta$. However, if $u(\cdot)$ is concave, and if $u(0)$ is not too low or if resources W are sufficiently large, we have $u(W/(1 + \pi^i)) < 2u(W/2(1 + \pi^i))$, so that $\alpha^i > W/2(1 + \pi^i) + \beta$.

The consumption equivalent of a long life α^i consists, by construction, in a measure of the damage suffered by the short-lived agent, or, alternatively, a measure of the advantage of the long-lived agent. As such, the consumption equivalent constitutes an adequate tool for transforming the consumptions of short-lived and long-lived agents into homogeneous, i.e. comparable, variables, which can then serve as a basis for a modified social planner's problem, where compensation for a short life is taken accounted for.

3.2 The modified planning problem

The modified utilitarian problem differs from the standard one (see Section 2.2) in a single aspect: the planner, instead of defining the consumption flows $\left(c^i, d^i\right)$ while not taking into account longevity differentials among agents, will now distinguish between consumptions that are enjoyed by short-lived and by long-lived agents. To do so, the social planner still solves a social welfare maximization

[24] To see this, note that, at the laissez-faire, we have, under $\beta > 0$ or $\beta < 0$ but $W > W^{si}$, $u\left(c^i\right) < u\left(c^i\right) + u\left(d^i\right)$. Hence only a positive α^i can restaure the equality, so that $u\left(c^i + \alpha^i\right) = u\left(c^i\right) + u\left(d^i\right)$.

problem, where the social objective function is now based on *homogenized* consumptions $\left(\tilde{c}^i, \tilde{d}^i\right)$, which take longevity differentials into account.

The homogenization of consumption works as follows: the consumption equivalent of a long life α^i is counted as part of the long-lived agents' consumption, i.e. in $\tilde{d}^i$. Indeed, if α^i measures the advantage of the long-lived, it makes sense to count this as consumption enjoyed by the long-lived and not by the short-lived. Given that it is not possible to know *ex ante* who will live long or not, it makes sense to count α^i as a part of second-period consumption.

Thus, the modified social planner's problem can be written as:

$$\max_{c^i,d^i} \sum_i \left(1-\pi^i\right) G\left(u\left(\tilde{c}^i\right)\right) + \pi^i G\left(u\left(\tilde{c}^i\right) + u(\tilde{d}^i)\right)$$

$$\text{s.to} \begin{cases} \tilde{c}^i = c^i \\ \tilde{d}^i = d^i + \alpha^i \\ \sum_{i=1,2}\left(c^i + \pi^i d^i\right) \leq W \end{cases}$$

where α^i is obtained from expression (2).[25]

Let us briefly discuss the case of deterministic longevity ($\pi^1 = 0 < \pi^2 = 1$). In that case, the FOCs are:

$$\begin{aligned} G'\left(u\left(c^1\right)\right)u'(c^1) &= \lambda \\ G'\left(u\left(c^2\right) + u(d^2+\alpha^2)\right)u'(c^2) &= \lambda \\ G'\left(u\left(c^2\right) + u(d^2+\alpha^2)\right)u'(d^2+\alpha^2) &= \lambda \end{aligned}$$

where λ is the Lagrange multiplier associated with the resource constraint.

Under $G''(.) = 0$, we have: $c^1 = c^2 = d^2 + \alpha^2$, which leads to:

$$\begin{aligned} c^1 &= c^2 = \frac{W+\alpha^2}{3} \\ d^2 &= \frac{W}{3} - \frac{2}{3}\alpha^2 \end{aligned}$$

Type-1 agent's consumption is increased with respect to utilitarian optimum. On the contrary, type-2's second-period consumption is reduced.

Under $G''(\cdot) < 0$, we have:

$$\text{if } \beta \gtreqless -v\left(\left(W+\alpha^2\right)/3\right), \text{ then } c^1 \gtreqless W/3 + \alpha^2/3 \text{ and } c^2 = d^2 \lesseqgtr W/3 - 2\alpha^2/3$$

The size of compensation received by type-1 agents depends strongly on the level of the intercept β.

In the case of stochastic longevity, FOCs can be rearranged as:

$$\begin{aligned} \left[\left(1-\pi^i\right)G'\left(u\left(c^i\right)\right) + \pi^i G'\left(u\left(c^i\right) + u(d^i+\alpha^i)\right)\right]u'(c^i) &= \lambda \\ G'\left(u\left(c^i\right) + u(d^i+\alpha^i)\right)u'(d^i+\alpha^i) &= \lambda \end{aligned}$$

Proposition 8 summarizes our results on the stochastic longevity case.

[25] The standard utilitarian problem coincides with the case where $\tilde{c}^i = c^i$ and $\tilde{d}^i = d^i$ $\forall i$.

Proposition 8 *Under stochastic longevity, the compensation-constrained utilitarian optimum is such that:*

- If $G''(.) = 0$, we have:

$$\begin{aligned} c^1 &= c^2 = \frac{W + \pi^1\alpha^1 + \pi^2\alpha^2}{2 + \pi^1 + \pi^2} = \bar{c} \\ d^1 &= \frac{W + \pi^2\alpha^2 - \alpha^1\left(2 + \pi^2\right)}{2 + \pi^1 + \pi^2} \\ d^2 &= \frac{W + \pi^1\alpha^1 - \alpha^2\left(2 + \pi^1\right)}{2 + \pi^1 + \pi^2} \end{aligned}$$

- If $G''(\cdot) < 0$, we have, under $\alpha^i > 0$:

- if $\beta > -v\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)$, $c^1 > \bar{c} > d^1$; $c^2 > \bar{c} > d^2$

- if $\beta = -v\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)$, $c^1 = c^2 = \bar{c}$; $d^1 = \frac{W+\pi^2\alpha^2-\alpha^1\left(2+\pi^2\right)}{2+\pi^1+\pi^2}$; $d^2 = \frac{W+\pi^1\alpha^1-\alpha^2\left(2+\pi^1\right)}{2+\pi^1+\pi^2}$.

- if $\beta < -v\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)$, $c^1 < \bar{c} < d^1$; $c^2 < \bar{c} < d^2$.

Proof. See the Appendix. ∎

If $G(\cdot)'' = 0$, first-period consumptions are equalized, and this consumption level is also, under either $\alpha^1 > 0$ and/or $\alpha^2 > 0$, higher than under classical utilitarianism, so that short-lived agents are, in comparison with classical utilitarianism, better off. However, d^1 and d^2, which are different for type-1 and type-2 agents, are, in general, smaller than under classical utilitarianism.[26]

Still taking the simplest case of $G''(.) = 0$, let us now compare *ex post* welfare inequalities between long-lived (LL) and short-lived (SL) agents of both types, by comparing lifetime utilities with the utilitarian case:

		compensation-constrained utilitarianism		utilitarianism
Type 1	LL	$u(\bar{c}) + u\left(\frac{W+\pi^2\alpha^2-\alpha^1\left(2+\pi^2\right)}{2+\pi^1+\pi^2}\right)$	$\lesseqgtr$	$2u\left(\frac{W}{2+\pi^1+\pi^2}\right)$
	SL	$u(\bar{c})$	$>$	$u\left(\frac{W}{2+\pi^1+\pi^2}\right)$
Type 2	LL	$u(\bar{c}) + u\left(\frac{W+\pi^1\alpha^1-\alpha^2\left(2+\pi^1\right)}{2+\pi^1+\pi^2}\right)$	$\lesseqgtr$	$2u\left(\frac{W}{2+\pi^1+\pi^2}\right)$
	SL	$u(\bar{c})$	$>$	$u\left(\frac{W}{2+\pi^1+\pi^2}\right)$

Table 1: Some welfare comparisons

Thus compensation-constrained utilitarianism *increases* the welfare of all short-lived agents with respect to classical utilitarianism, while this may not be

[26] This is the case provided $\pi^2\left(\alpha^2 - \alpha^1\right) - 2\alpha^1 < 0$ and $\pi^1\left(\alpha^1 - \alpha^2\right) - 2\alpha^2 < 0$, which are mild conditions (as the difference between α^1 and α^2 cannot be extremely large, given that the difference between π^1 and π^2 must belong to $[0, 1]$).

the case for long-lived agents in comparison with the situation under utilitarianism, depending on the values of α^1 and α^2.

Turning now to the welfare inequalities between long-lived and short-lived agents inside a given group (either type-1 or type-2), we find that inequalities inside group 1 are reduced as compared to utilitarianism if $\pi^2 \left(\alpha^2 - \alpha^1\right) - 2\alpha^1 < 0$ and inequalities inside group 2 are also reduced if $\pi^1 \left(\alpha^1 - \alpha^2\right) - 2\alpha^2 < 0$. This is due to the fact that compensation-constrained utilitarianism raises first-period consumptions and reduces, under mild conditions, second-period consumptions with respect to classical utilitarianism. This reduction of lifetime welfare inequalities within each group occurs under general conditions, that is, if one excludes cases where the differential between consumption equivalents α^1 and α^2 is extremely large (i.e. cases where survival probabilities differ strongly).

Proposition 9 *In comparison with classical utilitarianism and under* $G''(\cdot) = 0$ *and* $\alpha^i > 0$*, compensation-constrained utilitarianism:*

- increases the welfare of short-lived agents of type i, but may or may not increase the welfare of long-lived agents of type i;

- reduces welfare inequalities within each group i between long-lived and short-lived agents, provided $\pi^2 \left(\alpha^2 - \alpha^1\right) - 2\alpha^1 < 0$ *and* $\pi^1 \left(\alpha^1 - \alpha^2\right) - 2\alpha^2 < 0$*.*

Proof. The proof follows from the above table. ■

In general (i.e. if α^1 and α^2 do not differ too much), compensation-constrained utilitarianism reduces inequalities between long-lived and short-lived agents in each group, in comparison to classical utilitarianism.[27] But that reduction of inequalities is not costless. A tension arises between compensation and efficiency. Indeed, because of the impossibility to identify *ex ante* who will be long-lived or short-lived, compensation-constrained utilitarianism counts the entire consumption-equivalent α^i as second-period consumption. As a consequence, d^i is now lower than c^i. This non-equalization of marginal utilities of consumption over time can be regarded as an efficiency loss: from an efficiency perspective, consumption should be smoothed, for each individual, across all periods. This is not the case here, since compensation requires to give a higher first-period consumption to all agents than to survivors in the second period.

Let us now comment on the case where $G''(\cdot) < 0$. In that case, the compensation-constrained utilitarian optimum depends on several determinants, including the intercept of the temporal utility function β. Compensation-constrained utilitarianism recommends decreasing or increasing consumption profiles with the age, depending on the level of β. Additional assumptions on preferences are necessary to provide a more accurate description of the modified utilitarian optimum. This is the task of the next section.

In sum, the introduction of compensation constraints enables us to avoid an undesirable corollary of utilitarianism. The homogenization of consumptions

[27] One exception is when α^1 is much larger than α^2. In that case, there would be a reduction of lifetime welfare inequalities between short-lived and long-lived type-1 agents, but, because of redistribution across types favouring type-2 agents, a rise of lifetime welfare inequalities between the short-lived and the long-lived type-2 agents.

through counting the consumption-equivalent of a long life as long-lived agents' consumption supports a compensation of the short-lived. In particular, this modified approach allows the compensation of the short-lived even when lifetime welfare takes an additive form, that is, even when utilitarianism would imply a double penalization of the short-lived.

4 A numerical illustration

Let us now illustrate how the laissez-faire, the classical utilitarian optimum and the compensation-constrained utilitarian optimum differ. For that purpose, we impose functional forms for $G(\cdot)$ and $u(\cdot)$, and calibrate π^i and W. The temporal utility function $u(\cdot)$ takes the following form:

$$u(c^i) = \frac{c^{i1-\sigma}}{1-\sigma} + \beta$$

Regarding $G(\cdot)$, we rely on a simple functional form:

$$G(x) = \frac{x^{1-\kappa}}{1-\kappa}$$

where κ captures risk-aversion with respect to the length of life.

In order to calibrate preference parameters β, σ and κ, we rely on the empirical literature on the value of a statistical life (VSL). The VSL is the shadow price of reducing the risk of death per unit of risk. Assuming time-additive lifetime welfare (i.e. $\kappa = 0$), the VSL is given by:[28]

$$VSL\left(c^i\right) \equiv \left.\frac{dw}{d\pi^i}\right|_{\bar{U}} = -\left.\frac{\partial U^i/\partial \pi^i}{\partial U^i/\partial w}\right|_{\bar{U}} = \frac{u\left(c^i\right) - u'\left(c^i\right)c^i}{u'\left(c^i\right)}$$

where $U^i = \left(1+\pi^i\right)u\left(c^i\right)$ is the expected lifetime utility of a type-i agent, w is the initial wealth, and $c^i = w/\left(1+\pi^i\right)$ when consumption is smoothed.

Various pairs of preference parameters (β, σ) are compatible with a given VSL estimate, but may lead to very different values of the consumption equivalent of a longer life α^i.[29] Hence, to evaluate the robustness of the social optimum to the form of individual preferences, we consider several pairs (β, σ) compatible with the observed VSL. Substituting for $u(c^i)$ in $VSL(c^i)$ yields:

$$VSL\left(c^i\right) \equiv \beta\left(c^i\right)^{\sigma} + c^i\left(\frac{1}{1-\sigma} - 1\right)$$

[28] This assumption of risk-neutrality with respect to the length of life remains the default calibration given the scarcity of empirical evidence confirming $G''(\cdot) \neq 0$. However, we show below that assuming $G''(\cdot) < 0$ does not affect our results significantly. Note also that the consumption-equivalent of a long life α^i is invariant to the transform $G(\cdot)$.

[29] For an empirical study on that topic, see Ponthiere (2008).

Assuming that the agent's initial endowment equals 10 and that consumption is smoothed across periods, the VSL is:

$$VSL\,(5) = \beta\,(5)^{\sigma} + 5\left(\frac{1}{1-\sigma} - 1\right)$$

Given that the VSL amounts to about 120 times income per head per year (see Miller, 2001), which amounts to $\frac{120}{40}$ income per period of 40 years, we have, given the two-period structure, $VSL\,(5) = \frac{120}{40}\,(5) = 15$. Hence, substituting for different values of σ yields the following pairs of preference parameters: $(4.472, 0.50)$, $(0, 0.75)$, and $(-3.390, 0.85)$.[30]

To illustrate the model of Section 3, we suppose that type-1 agents are males and type-2 agents are females. In the U.S., life expectancy at birth for males is about 75 years, equal here to $1 + \pi^1 = 1.25$.[31] For women, life expectancy is 80.5 years, equal here to $1 + \pi^2 = 1.40$. Thus, one has $\pi^1 = 0.25$ and $\pi^2 = 0.40$. Finally, we assume a total endowment equal to $W = 20$.

Table 2 compares the laissez-faire (LF), the classical utilitarian optimum (CU) and the compensation-constrained utilitarian optimum (CCU) by providing the consumptions and the welfare levels for all agents, of type 1 or type 2, short-lived (i.e. superscript SL) or long-lived (i.e. superscript LL).

When $(\beta, \sigma) = (4.472, 0.5)$, the laissez-faire consumption is smoothed across periods, but is larger for type-1 agents. Short-lived agents are worse-off than long-lived agents who belong to the same group, illustrating the first part of Proposition 2. At the utilitarian optimum, consumption is now equalized at all periods for all agents, so that it reduces welfare inequalities between short-lived and long-lived agents of the type-1 group, whereas it increases them for type-2 agents, in conformity with Proposition 7. Compensation-constrained utilitarianism leads to a corner allocation: the total endowment W should be divided entirely between the first-period consumptions of types 1 and 2, whereas nothing should be left for second-period consumption.[32] Thus, the condition stated in Proposition 9 (point 2) is satisfied, in the sense that the compensation-constrained utilitarian optimum involves, in comparison with classical utilitarianism, a reduction of inequalities between the long-lived and the short-lived. Here the difference in utility between the long lived and the short-lived only results from the positive intercept, β. Only if $\beta = 0$, compensation-constrained utilitarianism yields here a perfect equalization of lifetime welfare across all agents, either short-lived or long-lived, whatever their survival prospects were.

[30] We exclude here cases where $\sigma \geq 1$. The reason is that, under that specification of preferences, it could be the case that some lives are infinitely worse (or better) than other lives, thus contradicting the finite welfare differentials assumption made in Section 2. To see this, take, for instance, the case where $\sigma = 1$. In that case, a life-period with zero consumption, by yielding $u(0) = -\infty$, would make a life infinitely worse than another life with $c, d > 0$.

[31] If a period lasts 40 years, and starting at the age of 25, we obtain that a life expectancy of 75 years involves 65 years (25 years + the first period) + 10 years, equal to 0.25 period.

[32] Note also that the consumption equivalents of a longer life (estimated at the laissez-faire) are such that $\alpha^1 > \alpha^2$, when $\sigma \leq 1$ while $\alpha^i \to +\infty$ when $\sigma > 1$. This was unclear from our theoretical part.

		c^1	c^2	d^1	d^2	U^{1SL}	U^{1LL}	U^{2SL}	U^{2LL}
$\beta = 4.472$	LF	8.00	7.14	8.00	7.14	10.13	20.26	9.82	19.63
$\sigma = 0.5$	CU	7.55	7.55	7.55	7.55	9.97	19.93	9.97	19.93
	CCU	10.00	10.00	0.00	0.00	10.80	15.27	10.80	15.27
$\beta = 0$	LF	8.00	7.14	8.00	7.14	6.73	13.45	6.54	13.08
$\sigma = 0.75$	CU	7.55	7.55	7.55	7.55	6.63	13.26	6.63	13.26
	CCU	10.00	10.00	0.00	0.00	7.11	7.11	7.11	7.11
$\beta = -3.390$	LF	8.00	7.14	8.00	7.14	5.72	11.43	5.56	11.13
$\sigma = 0.85$	CU	7.55	7.55	7.55	7.55	5.64	11.28	5.64	11.28
	CCU	10.00	10.00	0.00	0.00	6.03	2.64	6.03	2.64

Table 2: Outcomes under time-additive lifetime welfare

To examine how sensitive those results are to the preferences, let us compare them with those under $(0, 0.75)$, and $(-3.390, 0.85)$. At the laissez-faire, consumptions are unchanged; only welfare levels vary. The same remark holds under the utilitarian and the compensation-constrained optima. The difference lies in the size of welfare inequalities between short-lived and long-lived agents. Under $(0, 0.75)$, compensation-constrained utilitarianism, by concentrating all resources on first-period consumption, brings a *full* compensation to short-lived agents in comparison with the laissez-faire, as short-lived agents have the same welfare as long-lived agents. However, under $(-3.390, 0.85)$, the compensation is partial, since it is impossible to make the short-lived as well off as the long-lived on the basis of a higher first-period consumption.

We also checked the robustness of our results to alternative assumptions concerning the shape of $G(\cdot)$, and, in particular, focused on the case where $G''(\cdot) < 0$. For the sake of space, those results are not reported here.[33] Welfare inequalities between long-lived and short-lived remain large, so that, still under non-additive lifetime welfare, utilitarianism does not compensate short-lived agents. However, we still find that the compensation-constrained utilitarianism concentrates resources in the young age and as such contributes to compensate the short-lived, to an extent varying with the level of β.

In sum, this section illustrates that, whereas classical utilitarianism can hardly compensate short-lived agents, compensation-constrained utilitarianism can operate such a compensation, by raising first-period consumption while reducing second-period one. That result is robust to the specification of individual preferences. Nonetheless, the size of welfare inequalities between short-lived and long-lived agents depends on those preferences. Some welfare inequalities may still prevail despite the remedy we propose, because of the risky nature of longevity, which prevents a differentiated treatment of agents in the first period.

[33] Under $\kappa = 0.5$ and $\sigma = 0.5$, laissez-faire first-period consumptions are always higher than second-period ones. We find also that per-period consumptions are higher for type-1 agents than for type-2 agents. Under classical utilitarianism, consumption profiles are now always decreasing with age. We find also that $c^1 > c^2$ and $d^1 < d^2$ at the utilitarian optimum, which was unclear in theory. We also find that welfare inequalities between short-lived and long-lived agents are, under classical utilitarianism, smaller than under $\kappa = 0$.

5 Concluding remarks

This paper starts from a paradoxical result of classical utilitarianism: a tendency, under standard assumptions, to redistribute resources from short-lived to long-lived agents, implying, under mild conditions, a double penalization of short-lived agents: one penalty by Nature, one by Bentham.

We firstly proposed a re-examination of the treatment of unequal longevities under utilitarianism, by considering a two-period model with risky lifetime and unequal life expectancies. We identified formal conditions, on preferences and on endowments, under which short-lived agents are worse off than long-lived agents at the laissez-faire. We also showed that the introduction of risk about the length of life weakens the conditions under which short-lived agents are disadvantaged with respect to long-lived agents, because of lost savings due to unanticipated date of death. We also studied the direction of transfers under classical utilitarianism, and identified conditions under which classical utilitarianism can provide a general compensation to short-lived agents with respect to the laissez-faire. Those conditions may only be satisfied under a high degree of concavity of the transform applied to the sum of temporal utilities, which has been neither observed empirically, nor assumed by economists in their models. Hence classical utilitarianism never brings a compensation to the short-lived, and generally implies the opposite: a double penalization of the short-lived.

Secondly, we proposed a remedy to that paradox: the introduction of compensation constraints in the utilitarian planning problem. The underlying idea is to monetize the advantage of the long-lived, and to count it as a part of his consumption, and, then, to solve a utilitarian social planning problem involving so-homogenized consumptions. This was shown to yield, under general conditions, a compensation to the short-lived agents with respect to classical utilitarianism, and, also, with respect to the laissez-faire. Hence, whereas classical utilitarianism exacerbates welfare inequalities caused by Nature, compensation-constrained utilitarianism compensates agents disadvantaged by Nature.

Regarding the relevancy of our results for public policy, it should be stressed that various aspects of existing social security systems lead to redistribution from short-lived towards long-lived agents, and thus exemplify the paradox discussed in this paper. A well-known example is provided by Pay-As-You-Go pensions systems, which lead to transfers from young agents towards older agents. In terms of our model, such systems reinforce welfare inequalities between the long-lived and the short-lived (since the latter is, in case of premature death, only a contributor, but not a beneficiary of that system). Such real-world policies are thus, from an egalitarian perspective, questionable, since they do not do justice to the idea of compensation for a shorter life. This paper argues that some justice can nonetheless be done to the ideal of compensation when designing reforms for the pension system. The key ingredient, as shown in this paper, is to stop treating the young's and the old's consumptions as similar things, but, rather, to carry out the social planning problem on the basis of consumptions that are *homogenized* for longevity inequalities, as proposed in this paper.

While the present study identifies a "paradox", and proposes some solution to it, various deep philosophical issues remain here unexplored, and are worth being mentioned. First, the redistribution from short-lived towards long-lived individuals is schoking only insofar as one believes in the idea of life continuation.[34] Clearly, if one treats a person at the old age as a person distinct from the same person at the young age, the equalization of consumption across ages does no longer look like an anomaly: it is just an equalization of consumption across distinct beings. Secondly, the present study assumed that resources can be used to reduce, at least to some extent, welfare differentials between short-lived and long-lived individuals. This amounts to assuming some degree of substitutability between lifetime and resources. Without such an assumption, compensation would be infeasible. Thus, the existence of a solution to what we regard as a utilitarian anomaly depends on our views on what human well-being really is.

Finally, three extensions of this paper should be mentioned. First, while the social planner obtains here the consumption equivalent of a long life on the basis of laissez-faire choices, one may argue that the social planner should take the consumption equivalent as a *variable* depending on his *own* allocation of resources.[35] Second, one may want to explore the second-best problem, under *non observability* of life expectancies, and, thus, of consumption equivalents. Third, it would be worth considering whether *other* ethical frameworks suffer from the same problem as utilitarianism, to study the compensation for unequal lifetimes *outside utilitarianism*.[36] Much work remains to be done in the future.

6 Appendix

6.1 Proof of Proposition 1

Take the case where $u(0) \geq 0$ (i.e. $\beta \geq 0$). We want to show that $G\left(u\left(\frac{W}{2}\right)\right) < G\left(u\left(\frac{W}{4}\right) + u\left(\frac{W}{4}\right)\right)$ is always true. Given $G'(.) > 0$, that inequality is equivalent to: $u\left(\frac{W}{2}\right) < u\left(\frac{W}{4}\right) + u\left(\frac{W}{4}\right)$. That inequality is, under $u(0) \geq 0$ (i.e. $\beta \geq 0$) and $u''(\cdot) < 0$, always satisfied. However, under $u(0) < 0$ (i.e. $\beta < 0$), that inequality is satisfied only if $2v\left(\frac{W}{4}\right) - v\left(\frac{W}{2}\right) + \beta > 0$. The LHS of that expression is negative at $W = 0$, but tends to $\bar{v} + \beta > 0$ when W tends to infinity. Hence, by continuity, there exists a resource level W^S at which that expression equals zero. For higher resource levels, the above inequality is strictly satisfied, so that short-lived agents are worse off than long-lived agents, despite the equality of endowment $W/2$. That condition is invariant to the transform $G(\cdot)$.

6.2 Proof of Proposition 2

At the laissez-faire, short-lived agents of type i are disadvantaged with respect to the long-lived of type i if and only if: $G\left(u\left(c^i\right)\right) < G\left(u\left(c^i\right) + u\left(d^i\right)\right)$, that

[34] On the difficulties raised by the concept of individual identity, see Parfit (1984).

[35] That alternative approach is not trivial, as the endogeneity of α requires an additional constraint to be imposed, in order to avoid a multiplicity of optima.

[36] A first step in that direction is provided by Fleurbaey *et al* (2011).

is, if and only if $v\left(d^i\right)+\beta>0$. This is always true under $u(0)\geq 0$ (i.e. $\beta\geq 0$) for any $G''(\cdot)\leq 0$. However, under $u(0)<0$, it is not necessarily the case that long-lived are better off than short lived. Indeed, in that case, the LHS is negative at $W=0$. However, as W tends to infinity, the LHS tends to $\bar{v}+\beta>0$. Hence, by continuity, there exists a critical level W^{Si} such that a strict equality holds: $v\left(\frac{W^{Si}}{2(1+\pi^i)}\right)+\beta=0$.

6.3 Proof of Proposition 3

Note that, in the absence of risk, the threshold was defined as $2v\left(\frac{W^S}{4}\right)-v\left(\frac{W^S}{2}\right)+\beta=0$. Under risky lifetime, we have: $v\left(\frac{W^{Si}}{2(1+\pi^i)}\right)=2v\left(\frac{W^S}{4}\right)-v\left(\frac{W^S}{2}\right)$. Combining the definitions of thresholds, we show by contradiction that $W^S>W^{Si}$. Suppose first that we have $W^{Si}=W^S=W$. We would have: $v\left(\frac{W}{2(1+\pi^i)}\right)+v\left(\frac{W}{2}\right)=v\left(\frac{W}{4}\right)+v\left(\frac{W}{4}\right)$, which is false, as the LHS always exceeds the RHS under $0<\pi^i<1$. Assume now that $W^S<W^{Si}$. Hence we should have: $v\left(\frac{W^{Si}}{2(1+\pi^i)}\right)+v\left(\frac{W^S}{2}\right)=v\left(\frac{W^S}{4}\right)+v\left(\frac{W^S}{4}\right)$, which is also false, because the LHS always exceeds the RHS under $0<\pi^i<1$. Therefore $W^S>W^{Si}$.

6.4 Proof of Proposition 4

If $G'(\cdot)$ equals a constant k, we have, from the FOCs, $ku'\left(c^1\right)=ku'\left(c^2\right)=ku'(d^2)$, from which it follows that $c^1=c^2=d^2$.

Let us thus focus on the case where $G''(\cdot)<0$. From the FOCs it is obvious that we have $c^2=d^2$ in all cases.

Assume first that $\beta>-v\left(\frac{W}{3}\right)$. Let us now prove by *reductio ad absurdum* that $c^1>W/3$. For that purpose, assume instead that $c^1\leq W/3$ and $c^2=d^2\geq W/3$. We have $v\left(c^2\right)+v\left(d^2\right)+2\beta>v\left(c^1\right)+\beta$. Hence $G'\left(v\left(c^1\right)+\beta\right)>G'\left(v\left(c^2\right)+v\left(d^2\right)+2\beta\right)$. Moreover, if $c^1\leq W/3$ and $c^2=d^2\geq W/3$, we also have $v'\left(c^1\right)\geq v'(c^2)$. Hence it follows that

$G'\left(v\left(c^1\right)+\beta\right)v'\left(c^1\right)>G'\left(v\left(c^2\right)+v\left(d^2\right)+2\beta\right)v'\left(c^2\right)$, in contradiction with the FOCs. Therefore $c^1>W/3$ and $c^2=d^2<W/3$, so that $c^1>c^2=d^2$.

Assume now $\beta=-v\left(\frac{W}{3}\right)$. In that case, under $c^1=W/3$ and $c^2=d^2=W/3$, we have $v\left(c^2\right)+v\left(d^2\right)+2\beta=0=v\left(c^1\right)+\beta$, so that $G'\left(v\left(\frac{W}{3}\right)+\beta\right)=G'\left(v\left(\frac{W}{3}\right)+v\left(\frac{W}{3}\right)+2\beta\right)$. Hence the FOC of the social planner's problem is satisfied. Thus $c^1=c^2=d^2=W/3$ is the solution.

Finally, take the case where $\beta<-v\left(\frac{W}{3}\right)$. Here again, it can be shown, by *reductio ad absurdum*, that $c^1<c^2=d^2$. The proof being close to the one under $\beta>-v\left(\frac{W}{3}\right)$, it is not reproduced here.

6.5 Proof of Proposition 5

Assume $W>W^S$, so that long-lived agents are better off than short-lived agents at the laissez-faire. Short-lived agents receive a compensation when the utilitarian optimum involves $c^1 > W/2$. This is the case when, from the planner's perspective, we have: $G'\left(u\left(\frac{W}{2}\right)\right)u'\left(\frac{W}{2}\right) > G'\left(u\left(\frac{W}{4}\right)+u\left(\frac{W}{4}\right)\right)u'\left(\frac{W}{4}\right)$, that is, the marginal utility gain from raising the consumption of the short-lived beyond his laissez-faire consumption $W/2$ (i.e. the LHS) exceeds the marginal utility loss from reducing the consumption of the long-lived.

One can rewrite that expression as: $\frac{G'\left(v\left(\frac{W}{2}\right)+\beta\right)}{G'\left(2v\left(\frac{W}{4}\right)+2\beta\right)} > \frac{v'\left(\frac{W}{4}\right)}{v'\left(\frac{W}{2}\right)}$. W^S is such that $2v\left(\frac{W^S}{4}\right)+2\beta = v\left(\frac{W^S}{2}\right)+\beta$. From this, it follows that, if $W \leq W^S$, we have, by definition of W^S, $2v\left(\frac{W}{4}\right)+2\beta \leq v\left(\frac{W}{2}\right)+\beta$. Hence the LHS of the above condition is lower than 1. Given that the RHS exceeds 1 by the concavity of $v(\cdot)$, the condition is not satisfied, so that $c^1 \leq W/2$. However, if $W > W^S$, there may be compensation of the short-lived or not, depending on $v(\cdot)$, $G(\cdot)$, W and β.

6.6 Proof of Proposition 6

The case of G linear is trivial. In this case, $G'(\cdot)$ is equal to a constant and from the first order condition, $c^1 = d^1 = c^2 = d^2$. Replacing into the resource constraint, we obtain the consumption levels, $W/\left(2+\pi^1+\pi^2\right)$.

Let now turn to the case where $G''(\cdot) < 0$.

Assume first that $\beta > -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$. Let us first show that $c^i > \frac{W}{2+\pi^1+\pi^2} > d^i$. Assume instead that $d^i > \frac{W}{2+\pi^1+\pi^2} > c^i$. From the first FOC we have

$\left[\pi^i G'\left(u(c^i)+u(d^i)\right)+\left(1-\pi^i\right)G'\left(u(c^i)\right)\right]u'(c^i) = \lambda$. From the second FOC we have $u'(d^i)G'\left(u(c^i)+u(d^i)\right) = \lambda$. It is easy to see that there must be a contradiction. Given $\beta > -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$, we have $u(c^i)+u(d^i) > u(c^i)$. Hence $G'\left(u(c^i)+u(d^i)\right) < G'\left(u(c^i)\right)$ and $\pi^i G'\left(u(c^i)+u(d^i)\right)+\left(1-\pi^i\right)G'\left(u(c^i)\right) > G'\left(u(c^i)+u(d^i)\right)$. But if $d^i > c^i$, we have also $u'(c^i) > u'(d^i)$. Hence we have:

$\left[\pi^i G'\left(u(c^i)+u(d^i)\right)+\left(1-\pi^i\right)G'\left(u(c^i)\right)\right]u'(c^i) > u'(d^i)G'\left(u(c^i)+u(d^i)\right)$. Therefore, given that the LHS and the RHS of that expression are equal to λ, a contradiction is reached, confirming that $c^i > \frac{W}{2+\pi^1+\pi^2} > d^i$. Thus we have $c^1 > d^1$ and $c^2 > d^2$. Hence 6 rankings are possible:

$$
\begin{aligned}
d^2 &< c^2 < d^1 < c^1; d^1 < c^1 < d^2 < c^2 \\
d^2 &< d^1 < c^2 < c^1; d^1 < d^2 < c^1 < c^2 \\
d^1 &< d^2 < c^2 < c^1; d^2 < d^1 < c^1 < c^2
\end{aligned}
$$

Let us eliminate some of these by contradiction.

Suppose $d^2 < c^2 < d^1 < c^1$. From the FOCs, we have:

$u'(d^1)G'\left(u(c^1)+u(d^1)\right) = u'(d^2)G'\left(u(c^2)+u(d^2)\right)$. But given that $u(c^2)+u(d^2) < u(c^1)+u(d^1)$, it must be true that $G'\left(u(c^1)+u(d^1)\right) < G'\left(u(c^2)+u(d^2)\right)$. But as $u'(d^1) < u'(d^2)$, we have: $u'(d^1)G'\left(u(c^1)+u(d^1)\right) < u'(d^2)G'\left(u(c^2)+u(d^2)\right)$, which contradicts the equality $u'(d^1)G'\left(u(c^1)+u(d^1)\right) = u'(d^2)G'\left(u(c^2)+u(d^2)\right)$. Hence that ranking is not possible.

Similar arguments can be used to show by contradiction that the rankings $d^1 < c^1 < d^2 < c^2$, $d^2 < d^1 < c^2 < c^1$, and $d^1 < d^2 < c^1 < c^2$ are not possible. We are left with two rankings: $d^1 < d^2 < c^2 < c^1$ or $d^2 < d^1 < c^1 < c^2$.

Assume that $\beta = -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$. Hence, if one imposes $c^2 = c^1 = d^1 = d^2 = \frac{W}{2+\pi^1+\pi^2}$, one obtains, by substituting in the FOCs, that

$$\begin{aligned}\left[\pi^1 G'(0) + \left(1-\pi^1\right) G'(0)\right] u'\left(\frac{W}{2+\pi^1+\pi^2}\right) &= \lambda \\ \left[\pi^2 G'(0) + \left(1-\pi^2\right) G'(0))\right] u'\left(\frac{W}{2+\pi^1+\pi^2}\right) &= \lambda \\ u'\left(\frac{W}{2+\pi^1+\pi^2}\right) G'(0) &= \lambda\end{aligned}$$

from which it appears clearly that $c^2 = c^1 = d^1 = d^2 = \frac{W}{2+\pi^1+\pi^2}$ is the unique solution to the planner's problem.

Assume now that $\beta < -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$. Let us first show that $c^i < \frac{W}{2+\pi^1+\pi^2} < d^i$. Assume instead that $d^i < \frac{W}{2+\pi^1+\pi^2} < c^i$. From the first FOC we have $\left[\pi^i G'\left(u(c^i)+u(d^i)\right) + \left(1-\pi^i\right) G'\left(u(c^i)\right)\right] u'(c^i) = \lambda$. From the second FOC we have $u'(d^i) G'\left(u(c^i)+u(d^i)\right) = \lambda$. However, it is easy to see that there must be a contradiction. Given $\beta < -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$, we have $u(c^i)+u(d^i) < u(c^i)$. Hence $G'\left(u(c^i)+u(d^i)\right) > G'\left(u(c^i)\right)$. Hence $\pi^i G'\left(u(c^i)+u(d^i)\right) + \left(1-\pi^i\right) G'\left(u(c^i)\right) < G'\left(u(c^i)+u(d^i)\right)$. But if $d^i < c^i$, we have also $u'(c^i) < u'(d^i)$. Hence it must be the case that $\left[\pi^i G'\left(u(c^i)+u(d^i)\right) + \left(1-\pi^i\right) G'\left(u(c^i)\right)\right] u'(c^i) < u'(d^i) G'\left(u(c^i)+u(d^i)\right)$. Therefore, given that the LHS and the RHS of that expression are equal to λ, a contradiction is reached, confirming that $c^i < \frac{W}{2+\pi^1+\pi^2} < d^i$. Therefore we have $c^1 < d^1$ and $c^2 < d^2$. Hence 6 rankings are possible:

$$\begin{aligned} c^2 &< d^2 < c^1 < d^1; c^1 < d^1 < c^2 < d^2 \\ c^2 &< c^1 < d^2 < d^1; c^1 < c^2 < d^1 < d^2 \\ c^1 &< c^2 < d^2 < d^1; c^2 < c^1 < d^1 < d^2 \end{aligned}$$

By using the same kind of argument as above, we can rule out the first four rankings, and thus we keep $c^1 < c^2 < d^2 < d^1$ and $c^2 < c^1 < d^1 < d^2$.

6.7 Proof of Proposition 7

Assume first that $G(\cdot)$ is linear. Laissez-faire inequalities between a long-lived and a short-lived with the same survival probability π^i are equal to $v\left(W/2\left(1+\pi^i\right)\right)+\beta$, while, in the first-best utilitarian optimum, these inequalities are independent of the survival probability and equal to $v\left(W/\left(2+\pi^1+\pi^2\right)\right)+\beta$. For type-1 agents, it is easy to show that inequalities are higher in the laissez-faire than in the first-best, while for type-2 agents, it is the opposite.

Take now $G(\cdot)$ non-linear. If $\beta > -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$, the difference in lifetime welfare between long-lived and short-lived type-1 agents is, at the utilitarian optimum, smaller than $v\left(\frac{W}{2+\pi^1+\pi^2}\right) + \beta$, as $d^1 < \frac{W}{2+\pi^1+\pi^2}$ at the first-best. At the laissez-faire, that inequality was smaller than $v\left(\frac{W}{2(1+\pi^1)}\right) + \beta$, as $d^1 < \frac{W}{2(1+\pi^1)}$. Given that $\pi^1 < \pi^2$, we have $\frac{W}{2+\pi^1+\pi^2} < \frac{W}{2(1+\pi^1)}$. But this does not allow us to conclude on whether the inequality is reduced or increased by utilitarianism. For type-2 agents, the difference in lifetime welfare between long-lived and short-lived agents is, at the utilitarian optimum, smaller than $v\left(\frac{W}{2+\pi^1+\pi^2}\right) + \beta$, as $d^2 < \frac{W}{2+\pi^1+\pi^2}$ at the first-best. At the laissez-faire, that inequality was smaller than $v\left(\frac{W}{2(1+\pi^2)}\right) + \beta$, as $d^2 < \frac{W}{2(1+\pi^2)}$. Given that $\pi^1 < \pi^2$, we have $\frac{W}{2+\pi^1+\pi^2} > \frac{W}{2(1+\pi^2)}$. Here again, we cannot conclude on whether the inequality is raised or reduced by utilitarianism.

It is only in the special case where $\beta = -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$ that we can say for sure that utilitarianism reduces inequalities between short-lived and long-lived within type-1 agents, and does the opposite for type-2 agents.

The same reasoning as for $\beta > -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$ holds true when $\beta < -v\left(\frac{W}{2+\pi^1+\pi^2}\right)$ and one cannot draw conclusions on how utilitarianism affects inequalities within type-1 agents. The same is true for type-2 agents.

6.8 Proof of Proposition 8

Take the case where $G''(\cdot) = 0$. From the FOCs, we have:

$$u'(c^1) = u'(c^2) = u'(d^1 + \alpha^1) = u'(d^2 + \alpha^2)$$

from which it is trivial to see that, under $\alpha^i > 0$, $c^1 = d^1 + \alpha^1 = c^2 = d^2 + \alpha^2$. Substituting in the resource constraint yields the first part of Proposition 8.

Take now the case where $G''(\cdot) < 0$, and focus on the case where $\alpha^i > 0\ \forall i$.

Let us first show that, under $\beta > -v\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)$, we have $c^i > \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2} > d^i$. Let us show it by contradiction. According to the FOC, we have:

$\left[\left(1-\pi^i\right) G'\left(u\left(c^i\right)\right) + \pi^i G'\left(u\left(c^i\right) + u(d^i+\alpha^i)\right)\right] u'(c^i) = G'\left(u\left(c^i\right) + u(d^i+\alpha^i)\right) u'(d^i + \alpha^i)$. Suppose now $c^i < \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2} < d^i$. Then, under $\alpha^i > 0$, $u'(c^i) > u'(d^i + \alpha^i)$. Moreover, we have $G'\left(u\left(c^i\right)\right) > G'\left(u\left(c^i\right) + u(d^i+\alpha^i)\right)$. This is in contradiction with the FOCs. Hence it must be the case that $c^i > \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2} > d^i$. Therefore we have $c^1 > d^1$ and $c^2 > d^2$. Hence 6 rankings are possible:

$$\begin{aligned} d^2 &< c^2 < d^1 < c^1; d^1 < c^1 < d^2 < c^2 \\ d^2 &< d^1 < c^2 < c^1; d^1 < d^2 < c^1 < c^2 \\ d^1 &< d^2 < c^2 < c^1; d^2 < d^1 < c^1 < c^2 \end{aligned}$$

Note, however, that we have $d^i < c^j$.[37] Hence we can eliminate the rankings

[37] Indeed $c^i > \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2} > d^i$ is true for all i, so that it is also true that $c^i > d^j$.

$d^2 < c^2 < d^1 < c^1$ and $d^1 < c^1 < d^2 < c^2$. However, it is not possible to eliminate the others, since the two types of agents have distinct consumption-equivalents α^1 and α^2. We are left with four rankings: $d^2 < d^1 < c^2 < c^1$, $d^1 < d^2 < c^1 < c^2$, $d^1 < d^2 < c^2 < c^1$ or $d^2 < d^1 < c^1 < c^2$.

Under $\beta = -v\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)$, substituting for $c^1 = c^2 = \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}$; $d^1 = \frac{W+\pi^2\alpha^2-\alpha^1\left(2+\pi^2\right)}{2+\pi^1+\pi^2}$; $d^2 = \frac{W+\pi^1\alpha^1-\alpha^2\left(2+\pi^1\right)}{2+\pi^1+\pi^2}$ in the FOCs yields:[38]

$$\begin{aligned}\left[\left(1-\pi^1\right)G'(0)+\pi^1 G'(0)\right]u'(c^1) &= G'(0)\,u'(d^1+\alpha^1)\\ \left[\left(1-\pi^2\right)G'(u(0))+\pi^2 G'(0)\right]u'(c^2) &= G'(0))\,u'(d^2+\alpha^2)\end{aligned}$$

From it is obvious that $c^1 = c^2 = \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}$; $d^1 = \frac{W+\pi^2\alpha^2-\alpha^1\left(2+\pi^2\right)}{2+\pi^1+\pi^2}$; $d^2 = \frac{W+\pi^1\alpha^1-\alpha^2\left(2+\pi^1\right)}{2+\pi^1+\pi^2}$ is the solution of the problem, as this satisfies the FOCs.

Assume now that $\beta < -v\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)$. Let us first show that $c^i < \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2} < d^i$. Assume instead that $d^i < \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2} < c^i$. From the first FOC we have

$\left[\pi^i G'\left(u(c^i)+u(d^i+\alpha^i)\right)+\left(1-\pi^i\right)G'\left(u(c^i)\right)\right]\, u'(c^i) = \lambda$. From the second FOC we have $u'(d^i+\alpha^i)G'\left(u(c^i)+u(d^i+\alpha^i)\right) = \lambda$. However, it is easy to see that there is a contradiction here. Given $\beta < -v\left(\frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2}\right)$, we have $u(c^i)+u(d^i+\alpha^i) < u(c^i)$. Hence $G'\left(u(c^i)+u(d^i+\alpha^i)\right) > G'\left(u(c^i)\right)$ and

$\pi^i G'\left(u(c^i)+u(d^i+\alpha^i)\right)+\left(1-\pi^i\right)G'\left(u(c^i)\right) < G'\left(u(c^i)+u(d^i+\alpha^i)\right)$. But if $d^i < \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2} < c^i$, we have also $u'(c^i) < u'(d^i+\alpha^i)$. Hence it must be the case that $\left[\pi^i G'\left(u(c^i)+u(d^i+\alpha^i)\right)+\left(1-\pi^i\right)G'\left(u(c^i)\right)\right]u'(c^i) < u'(d^i+\alpha^i)G'\left(u(c^i)+u(d^i+\alpha^i)\right)$. Therefore, a contradiction with the FOCs is reached, confirming that $c^i < \frac{W+\pi^1\alpha^1+\pi^2\alpha^2}{2+\pi^1+\pi^2} < d^i$. Therefore we have $c^1 < d^1$ and $c^2 < d^2$. Hence 6 rankings are possible:

$$\begin{aligned}c^2 &< d^2 < c^1 < d^1; c^1 < d^1 < c^2 < d^2\\ c^2 &< c^1 < d^2 < d^1; c^1 < c^2 < d^1 < d^2\\ c^1 &< c^2 < d^2 < d^1; c^2 < c^1 < d^1 < d^2\end{aligned}$$

By using the same kind of proof as above, we can rule out the first two rankings by contradiction, and thus we keep $c^2 < c^1 < d^2 < d^1$, $c^1 < c^2 < d^1 < d^2$, $c^1 < c^2 < d^2 < d^1$ and $c^2 < c^1 < d^1 < d^2$.

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[38] We rely here on the assumption $G'(0)$ finite.

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