



UNIVERSITÉ CATHOLIQUE DE LOUVAIN

Faculté des Sciences Appliquées

CERMIN

Unités PCPM & DICE

Parallel field-induced universal conductance fluctuations in open quantum dots

Dissertation présentée en vue de l'obtention du grade de
Docteur en Sciences Appliquées
par

Cédric Gustin

Promoteur:
Prof. Vincent Bayot

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Membres du jury:

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Abstract

Open quantum dots (OQDs) are now commonly used as an experimental tool for the investigation of a particular regime of quantum transport where the electron dynamics is both ballistic and coherent. In particular, the Universal conductance Fluctuations (UCFs), observed in ballistic quantum dots, arise from the complex quantum interferences occurring between electron trajectories that bounce multiple times against the dot walls, before escaping through its leads. Central to quantum interference phenomena is the presence of a magnetic field B that breaks the time-reversal symmetry and changes the phase experienced by electrons in the dot.

OQDs are typically patterned on top of two-dimensional electron gases (2DEGs). Interestingly, when confined to wide GaAs quantum wells (QWs), 2DEGs are known to exhibit a rich physics arising from the interplay of a strong *in-plane* magnetic field, multiple subband occupation, and the finite thickness of the electronic wavefunction.

In this thesis, we use 2DEGs, confined to wide (WQW) and narrow (NQW) quantum wells with one and two occupied subbands at $B = 0$, respectively, to study the parallel field-induced transport in open quantum dots as a function of the well width and the tilt angle of B with respect to the electron gas. Both the WQW and NQW dots feature a rich spectrum of UCFs at intermediate tilt angles and, quite unexpectedly, under a strictly parallel B . Combined with the observation, in the case of the WQW dot, of a reduction in UCFs amplitude at large parallel B , our data indicates that the finite thickness of the electron layer and the orbital effect are responsible for the in-plane field-induced UCFs.

In the second part of this work, we observe a saturation of the UCFs spectral distribution, expressed in terms of an effective tilt angle, as B approaches a strictly parallel configuration, along with the persistence of a limited number of frequency components in the case of the NQW dot. It is found that the saturation angle strongly depends on the width of the 2DEG confining well. Using the results of self-consistent Poisson-Schrödinger simulations, the magnetoconductance is rescaled as

a function of the Fermi level E_F in the 2DEG. A power spectrum analysis of the parallel B UCFs in energy space and its good agreement with theoretical predictions suggest that such a B to E_F mapping is indeed relevant for the interpretation of parallel B -induced UCFs.

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¹Si un théoricien est une personne travaillant sur l'aspect théorique de la recherche, et si l'expérimentateur est principalement intéressé par l'expérience, le simulateur doit donc être le terme approprié pour décrire le chercheur impliqué dans des activités de simulation.

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List of Symbols

Here are the main symbols and acronyms used in this thesis. Consistency with traditional usage has been maintained as far as possible. Vectors are printed in bold-faced fonts.

β	Symmetry (Dyson) index
χ	Electron affinity
ΔE_{os}	Conduction band offset at the GaAs/Al _x Ga _{1-x} As interface
δg^{WL}	Weak localization correction to the conductance
Δ	Mean level spacing
ϵ	Static dielectric constant
ϵ_0	Static dielectric constant in vacuum
$\hat{T}$	Transmission function
$\hbar$	Reduced Planck constant
λ_F	Fermi wavelength
$\mathcal{A}$	Area
$\mathcal{C}$	Cooperon
$\mathcal{D}$, DOS	Density of States
$\mathcal{F}_i$	Fermi-Dirac integral of order i
$\mathcal{H}$, $\hat{H}$	Hamiltonian operator
$\mathcal{R}$	Radius
$\mathcal{V}$	Volume
μ_0	Magnetic constant
μ_B	Bohr magneton
μ_i	Quasi-Fermi level in contact i
μ_s	Maslov index
Ω	Matrix of eigenvectors
ω_c	Cyclotron frequency
Φ	Magnetic flux
Φ_0	Magnetic flux quantum
Φ_B	Barrier height (for Schottky barriers)
Φ_m	Metal work function
Φ_s	Semiconductor work function

Ψ, ψ	Wavefunction
Ψ_i, ψ_i	Wavefunction of the i^{th} excited state
ρ	Charge density
σ	Conductivity
$\sigma(x)$	Standard deviation of variable x
Σ^R	Self-energy
τ	Time constant
τ_{cross}	dot crossing time
τ_{φ}	Phase relaxation time
τ_d	Dwell time
τ_m	Momentum relaxation time
$\text{var}(x)$	Variance of x
Θ	Heavyside step function
θ	Sample tilt angle
θ_{B_c}	Effective tilt angle
∇	Gradient operator
Ω	Spin-orbit frequency
σ	Pauli matrix
$\mathbf{A}$	Magnetic vector potential
$\mathbf{B}$	Magnetic flux density
$\mathbf{e}_z$	Unit vector along the z -direction
$\mathbf{k}$	Wave vector
$\mathbf{k}_F$	Wave vector at the Fermi energy
$\mathbf{L}$	Angular momentum operator
$\mathbf{l}$	Angular momentum
$\mathbf{P}$	Momentum operator
$\mathbf{p}$	Momentum
$\mathbf{r}$	Vector in real space
$\mathbf{S}$	Spin operator
A	Magnetic potential
a	Lattice constant
A_p	Amplitude of focusing peaks in TMF experiments
B	Magnetic field
B^{cal}	Calibration field
B^{limit}	Superconducting magnet field limit
$B_{\parallel}$	Parallel component of the magnetic field
$B_{\perp}$	Perpendicular component of the magnetic field
B_c	Characteristic magnetic field
c	Speed of light in vacuum
$C(\Delta X)$	Autocorrelation function for parameter X
D	Diffusion constant
D_s	Density of trajectories

e	Elementary charge
$E, \mathcal{E}$	Energy
E_0	Ground state energy
E_{corr}	Correlation energy
E_c	Conduction band energy
E_{ex}	Exchange energy
E_F	Fermi energy
E_i	i^{th} eigenenergy
E_S	Characteristic energy
E_{xc}	Exchange-correlation energy
E_Z	Zeeman energy
f	Frequency
f_B	Frequency in magnetic field space
f_E	Frequency in energy space
g	Conductance
g^*	Electron effective Landé factor
G^A	Advanced Green's function
G^R	Retarded Green's function
G^{SC}	Green's function in the semi-classical approximation
g_e	Electron Landé factor in vacuum
h	Planck constant
H_a	Hartree energy
H_{so}	Spin-orbit scattering rate
H_{tr}	Transport scattering rate
I	Electric current
k_B	Boltzmann constant
L	Sample length
l_φ	Phase relaxation length
l_B	Magnetic length
l_e	Electric distance between contacts in a TMF experiment
l_m	Mean free path
l_s	Scattering length
M	Number of transmission modes
M	Number of transverse modes in a conductor
m^*	Electron effective mass
m_e	Electron mass in vacuum
n	Electron density
N_{bounces}	Number of bounces on the dot walls
N_φ	Number of phase-breaking modes
n_s	Electron sheet density
R	Electric resistance
r_c	Cyclotron radius

r_s	Seitz radius
R_{xx}, R_L	Longitudinal resistance
R_{xy}, R_H	Transverse (Hall) resistance
S	Scattering matrix
S	Surface
s	Spin quantum number
S_B	Power spectrum in magnetic field space
S_E	Power spectrum in energy space
S_s	Classical action of an electron trajectory
T	Temperature
t	Time variable
V	Electrostatic potential
v_F	Fermi velocity
V_{s0}	Equilibrium contact potential
W	Sample width
w	Quantum well width
2DEG	Two-dimensional Electron Gas
AFM	Atomic Force Microscope
AL	Antilocalization
DFT	Density Functional Theory
DI	DeIonized
EBL	Electron Beam Lithography
MBE	Molecular Beam Epitaxy
MCF	Magnetoconductance Fluctuation
MIBK	Methylisobutylketone
MOSFET	Metal-oxide-semiconductor field-effect transistor
NQW	Narrow Quantum Well
OQD	Open Quantum Dot
PMMA	Polymethylmethacrylate
QHE	Quantum Hall Effect
QPC	Quantum Point Contact
QW	Quantum Well
RHEED	Reflection High-Energy Electron Diffraction
RMT	Random Matrix Theory
SdH	Shubnikov-de Haas
SEM	Scanning Electron Microscope
SO	Spin-Orbit
SRS	Spin-Rotation Symmetry
TMF	Transverse Magnetic Focusing
TRS	Time-Reversal Symmetry
UCF	Universal Conductance Fluctuation
WL	Weak Localization

WQW	Wide Quantum Well
XWQW	Extra Wide Quantum Well

Preface

The noise is the signal.

Rolf Landauer
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The introductory course in Quantum Mechanics, taught in universities all over the world, is usually a cornerstone in the education of most Physics and Engineering undergraduate students. Either they find this field totally appealing, and adhere immediately to its somewhat puzzling but beautiful mathematical description, or they tend to reject it with the typical argument of inadequacy with the physical and macroscopic phenomena of “real life”. The question of whether the final grade obtained by a student is correlated with his perception of Quantum Mechanics, is of course beyond the scope of this thesis work. Nevertheless, in the case of the undergrad “touched” by the light of Quantum Mechanics, it seems the initial impulse to his interest² usually comes from his first encounter with Louis de Broglie wave-particle duality principle. To know that the electron, this little particle that powers up your small electric appliance, your cellular phone or the computer where these lines are written, can also behave as a wave and produce interference patterns (the famous Stern and Gerlach experiment[124]) comes as a surprise to many. To realize that human bodies are essentially wavefunctions projected on a particular set of eigenstates is also disturbing, to say the least.

It is not too risky to suggest that most, if not all, solid-state physicists one day experienced this particular encounter with Quantum Physics. Beside the wave-particle duality, another major and surprising result of this modern form of mechanics is quantum confinement, *i.e.* the constraint of particles in small regions of space where they can only occupy discretized energy levels. Through their implications on the fabrication and characterization of low-dimensional systems, the particle-

²The answer to the question “Does the interest impulse operator commute with the Hamiltonian ?” is left to the interest reader.

wave duality principle and quantum confinement, both consequences of the differential nature of Schrödinger equation, have definitively shaped the world of research in State Physics in the last decades. And it appears this trend is far from slowing down.

Among the quantum confined systems, two-dimensional electron gases (2DEGs) have probably attracted the most interest from the Solid State Physics community. In the configuration where electrons or holes are confined at the interface(s) between the GaAs and $\text{Al}_x\text{Ga}_{1-x}\text{As}$ III-V compounds³, the availability of high quality, almost disorder-free 2DEGs, characterized using low-temperature, and sub-Kelvin cryogenic techniques, have opened up the possibility of studying new transport regimes and physical phenomena. While it is sometimes forgotten that the Quantum Hall Effect (QHE) was discovered[104] by K. von Klitzing and coworkers in silicon MOS structures, its most famous siblings, like the Fractional QHE[163, 111], composite fermions[81], or most recently in our lab, QHE skyrmions[15, 126] and Wigner crystals[164, 64], strongly rely on III-V 2DEGs.

Partly motivated by the interesting technological opportunities of low-dimensional systems where quantum confinement is obtained along the 3 spatial directions, researchers have also devised methods to further confine the 2DEG in GaAs heterostructures. In the so-called “quantum dots”, only a small number of electrons occupy a discretized set of energy states and behave mostly like the electrons of an atom or small molecule. Their transport properties are characterized by the Coulomb Blockade regime that might prove useful in the next few years for the fabrication of very small, nanoscopic transistors and memory cells. Going one step further, one of the research initiatives in the emerging field of spintronics aims at controlling the spin degree of freedom of electrons confined to quantum dots, hence replacing the actual 1-0 bit status associated with a conducting or non-conducting MOSFET channel with a spin up-spin down logic device.

Central to both spintronic devices and to modern transistors, where only a handful of electrons travel through the 100 nm-wide channel, are the phenomena that play a part in the dynamics of the electron energy states and trajectories, in particular the electron momentum and phase conservation over long distances (at the nanoscopic scale). To study this particular regime of quantum transport where the electron dynamics is both ballistic and coherent, a handy experimental tool is actually proposed by larger quantum dots, populated by a few thousands

³The 2D confinement of charged particles can also be obtained in the metal-oxide-semiconductor field-effect transistor (MOSFET), usually based on silicon technology.

of electrons, and loosely connected to electrons reservoirs by two small openings. As opposed to their Coulomb blockaded counterparts, electrons injected into open quantum dots (OQDs) can bounce many times on the cavity walls before they escape into the reservoirs. Interestingly, when placed in a magnetic field, the dot features Magnetoconductance Fluctuations (MCFs) arising from the complex quantum interferences occurring between electron trajectories.

Transport through OQDs has been the subject of intense research in the last few years. A constant among all the experiments is the orientation of the magnetic field B , always applied perpendicular to the 2DEG in order to maximize the magnetic flux enclosed by electron trajectories. In this thesis, we ask the question of what would be the effect of a parallel field on the electron dynamics in an open quantum dot. An ideal, zero-thickness 2DEG encloses no magnetic flux when placed in a parallel B , and thus, should not modify the quantum interference pattern in OQDs or produce conductance fluctuations. However, it has been known for quite some time that, due to the finite thickness of the 2DEG in realistic structures, the electronic 2D states can be significantly modified due to the competing effects of the Lorentz force and the 2DEG confining potential.

In this thesis, we use two-dimensional electron gases (2DEGs) confined to a wide and a narrow quantum well (WQW and NQW, resp.) as base elements for the study of the relative strength between the perpendicular and parallel field-induced transport regimes in open quantum dots, as a function of the well width and the tilt angle of the field with respect to the 2DEG.

Chapter 1 is the customary part where the essential elements of Mesoscopic Physics are given as a primer to be used in the following chapters. Starting with a general description of ballistic and coherent transport in low-dimensional systems, open quantum dots are introduced as meaningful examples of structures subject to quantum interferences between collision-free electron trajectories. The signatures of these interferences in terms of magnetoconductance fluctuations and weak-localization correction, along with their controversial interpretation in the framework of “Quantum Chaos”, are discussed in the light of recent experiments.

Chapter 2 deals with the various fabrication steps that transform a piece of GaAs substrate into a fully operational open quantum dot. Here, we put particular emphasis on the important processing details that must be attached to standard procedures in order to obtain devices with features below 100 nm. The last portion of Chap. 2 is used for a description of the characterization techniques, both on the cryogenic

and data acquisition side. It also seems that a constant among graduate students involved in cleanroom operation is the inverse proportionality between the numbers of pages devoted to processing and characterization in their thesis, and the hours spent on the lithography instruments. We immediately put the reader's mind at ease: we did not go against this tradition.

After the primer on mesoscopic physics in Chap. 1, the goal of Chapter 3 is to review the current theoretical models that are available for an analysis of the magnetotransport properties of open quantum dots. Here, we deliberately used a pedagogical approach, as we believe a good understanding of the arcanae of the semi-classical approach, the random matrix or diagrammatic perturbation theories are required for a useful interpretation of the experimental results presented in the later chapters. In particular, we tried to do our best to insist upon the links and common physical (while sometimes well hidden) origins of these various models, while summarizing their main results in terms of conductance fluctuations in OQDs. In particular, sections are devoted to the effects of dephasing and thermal smearing on the conductance amplitude. Finally, we briefly introduce the concept of magnetic subbands, *i.e.* the way the electron conduction band in the 2DEG is modified by the presence of a perpendicular or parallel magnetic field.

Next, we come up to our experimental results. In Chapter 4, we present magnetoconductance measurements performed under a strictly parallel B in ballistic quantum dots etched from wide and narrow quantum wells. After a brief description of related experiments recently performed by another group at Harvard University, we start with a complete description of the quantum well samples used in our experimental endeavors. The procedure leading to a near perfect alignment of the magnetic field with respect to the 2DEG is also fully explained. Based on the unexpected observation of conductance fluctuations, we then try to determine the origin of this particular signature of quantum interferences. To achieve this goal, the amplitude of conductance fluctuations are analyzed as a function of temperature. The extracted temperature dependence is purposely compared to measurements performed in a strictly perpendicular configuration of the field. Finally, we evaluate the variance of the conductance fluctuations and compare our experimental data to theoretical models that have recently addressed the issue of an OQD built from a realistic 2DEG (with a finite thickness), and subject to a magnetic field of arbitrary orientation.

Chapter 5 is a temporary journey into the fascinating world of computer simulations of physical phenomena. Using a self-consistent Poisson-Schrödinger model, we determine the effect of a parallel magnetic field

on the conduction band properties of the 2DEG confined to a quantum well. This study is crucial to our understanding of the parallel field conductance measured in Chap. 4. Indeed, a possible interpretation of the observed conductance fluctuations in terms of an orbital effect strongly relies on the precise knowledge of the position of the Fermi level in the well, as a function of both the parallel field and the width of the 2DEG electrostatic confinement. From dispersion relations and Fermi contours, we indeed show that our narrow and wide quantum well samples are differently affected by the orbital motion of the electrons.

Finally, the purpose of Chapter 6 is twofold. First, magnetoconductance measurements are presented for intermediate tilt angles, thus extending the data sets obtained in Chap. 4 for a strictly parallel field. Through a study of the frequency distribution of the fluctuations as a function of the tilt angle, we try to quantify the interplay between perpendicular and parallel field-induced fluctuations. We base our analysis on results of semi-classical theory that target the effect of dephasing on the fluctuation spectrum. Next, the experimental and theoretical results of Chaps. 4 and 5 are combined in the second part of the chapter. The magnetoconductance measurements under a strictly parallel B are rescaled as a function of the field dependent position of the Fermi level in the wells. Their frequency distribution is matched to theoretical models, which further confirms the orbital origin of the parallel field-induced magnetoconductance fluctuations.

Chapter 1

A Primer on Mesoscopic Transport

When John Bardeen went home from Bell Labs that evening of 1947, all he said to his wife was, “I think we discovered something today.”
She was busy getting dinner and didn’t pay much attention to it.

Walter H. Brattain

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1.1 Introduction

It is quite customary in the scientific community to begin a book or a large report with an appropriate citation from a well-known author. As

clearly evidenced by the chapter epigraph, it is not our primary objective to go against this nice tradition. The name of the two famous scientists mentioned in the short citation was here clearly chosen on purpose. Both John Bardeen and Walter Brattain, with their 1947 invention of the contact transistor, not only are among the founding fathers of what is now called “the electronic age”, but also initiated an incredible amount of new and fundamental research fields, encompassing Physical Electronics, Semiconductor Devices Fabrication or Quantum Transport Theory. Most of the scientists involved in these early days were confirmed physicists with a deep knowledge of “Modern Physics”, in particular Quantum Mechanics. Quite amazingly and until recently, theoretical models based on classical laws (like drift and diffusion) were largely sufficient in dealing with the electronic transport properties of most modern semiconductor devices, especially at the industrial, production or end-user levels.

It is only in the last few years, with the increased miniaturization of the electronic components (the so-called “Top-down” approach), in particular with the impressive reduction of the transistors characteristic size in modern microprocessors, that engineers and scientists started to realize the limitations of the classical models of semiconductor transport that they lived with for merely 4 decades. As components started to leave the macroscopic world and entered the sub-micron range in the late 80’s, such simple concepts of Quantum Mechanics as the discreteness of charge, the quantization of motion or the wave-particle duality of the electrons (and holes) became more and more relevant for device physics. For example, it is getting more and more difficult to decently consider as a simple “fluid”, the few tens of electrons that travel across the channel of a modern 100 nm-long transistor channel.

Beside the top-down approach, the last decade has seen the emergence of new research fields such as those dealing with Carbon Nanotubes[54], Molecular Electronics[148] or self-assembling techniques that directly rely on concepts of Quantum Mechanics for the development and production of new devices. In the “Bottom-up” approach, the basic and nanoscopic elements (atoms and molecules) of nature are used as building blocks and combined into more complex structures with the desired functionality.

The term mesoscopic is a catch-all for things that exist on length scales between the nanoscopic and macroscopic realms. While these systems follow the rules of quantum mechanics, they can be experimentally addressed by macroscopic (but sophisticated) techniques. This field combines exciting physics laws with state-of-the-art fabrication techniques or material science, and opens perspectives for nontrivial appli-

cations. As it became the primary focus of governmental and private research agencies, Mesoscopic Physics has evolved into what is currently referred to as Nanoscience, and finally merged with Nanotechnology.

Independently of the approach taken when devising new mesoscopic systems, either the “Top-down” or “Bottom-up” approach, it is crucial to have at one’s disposal some artificial systems that can serve as testing bed for the study of phenomena that look promising with respect to the development of new devices. As an example, Quantum dots patterned on top of two-dimensional electron gases (2DEGs) are now commonly used as an experimental tool for the investigation of a particular regime of quantum transport where the electron dynamics is both ballistic (with no or only a few collisions between particles) and coherent (with quantum interferences between electron trajectories).

The purpose of this chapter is essentially to give a primer on the various concepts that were introduced in this very last sentence. Section 1.2 starts with a discussion of the length scales that are relevant to Mesoscopic Physics. Section 1.3 is devoted to an introduction to Two-dimensional Electron Gases (2DEGs) with particular emphasis on the GaAs-Al_xGa_{1-x}As system used throughout this thesis work. In Sec. 1.4, the main characteristics of ballistic transport are illustrated by two phenomena observed in confined 2DEGs, and crucial to our understanding of the electron dynamics in open quantum dot: conductance quantization in quantum point contacts[168], and transverse magnetic focusing[167, 166, 155]. The second part of this chapter, starting with Sec. 1.5, is entirely dedicated to the basic properties of an open quantum dot and goes from the simple interference pattern observed in the magnetoconductance of an Aharonov-Bohm ring[2] (Sec. 1.5.1) to Quantum Chaos[121, 107] and Universal Conductance Fluctuations[122, 20] (Sec. 1.5.2).

1.2 Characteristic Lengths

As already evidenced in the introductory paragraphs, a clear definition of Mesoscopic Physics is not straightforward. This is mainly because of the interdisciplinary nature of this field, that involves physics, chemistry or biochemistry¹. A simple definition would be the research area concerned with objects for which the nanometer length scale plays a central role. But then immediately arises the question of the choice of the relevant length scales that characterize the systems. In biology, it is reasonable to consider the size of molecule as a pertinent parameter. In Solid State

¹Or is it biophysics ?

Physics, three characteristic lengths must be compared to the physical dimensions of an electron system :

1. Mean free path : In a defect-free crystal, the periodicity of the atomic layout is taken into account by a band structure, where available levels are occupied by electrons up to the Fermi energy. Subject to the Coulomb interaction induced by the ions, the electrons travel across the structure with a constant momentum and a given effective mass. But, as no material is completely free from defect, electrons typically collide with impurities or lattice vibrations (phonons). These scattering events can either be elastic, when no energy transfer occurs, or inelastic when a change in momentum is combined with a change in energy.

The momentum relaxation time τ_m is the characteristic time after which scattering events have sufficiently modified the electron momentum to make it statistically independent of its initial value. Quantitatively, this means that, for a statistical ensemble of electrons with an initial average momentum k , the evolution of k with time follows an exponentially decreasing law, where τ_m is the characteristic time associated to this decay. Alternatively, the mean free path l_m is the distance traveled by an electron before its initial momentum is randomized. It is defined by

$$l_m = v_F \tau_m \quad \text{where} \quad v_F = \frac{\hbar}{m^*} \sqrt{2\pi n_s} \quad (1.1)$$

is the Fermi velocity, with n_s the electron sheet density for a 2DEG, and m^* is the electron effective mass.

The mean free path is directly related to the electron mobility μ through the relation $\mu = e\tau_m/m^*$, where e is the electron charge. If the sample dimensions are smaller than l_m , transport is said to be in the ballistic regime as momentum changes solely occur when the electrons bounce against the sample walls.

2. Fermi wavelength : this length scale is inversely proportional to the electron density and is an image of the extension of the Fermi surface in reciprocal space. In the case of a 2D electron system (more details will follow in Sec. 1.3), the Fermi wavelength is given by $\lambda_F = \sqrt{2\pi/n_s}$, where n_s is the electron sheet density. When λ_F becomes comparable to or greater than one of the sample dimensions, the effect of quantum confinement, along with the wave nature of the electron must be taken into account by solving the Schrödinger equation. More precisely, λ_F is relevant when the level separation

of discretized eigenenergies, solutions to the Schrödinger equation, are fully resolved, *i.e.* comparable to $\hbar/\tau_m$ and to the thermal energy $k_B T$, where k_B is Boltzmann's constant, and T is the temperature.

3. Phase-relaxation length : because of the particle-wave duality, electrons can not only “collide” with each other, with the sample boundaries or with phonons, but can also be subject to quantum interferences similar to those materialized by interference patterns in the famous Stern and Gerlach experiment[124]. Central to quantum interference effect is the phase accumulated by electrons along their trajectories.

Similar to the momentum randomization, phase coherence can be broken by inelastic scattering events such as electron-phonon or electron-electron interactions. The phase relaxation time τ_φ is the characteristic time associated to phase-randomization. In the case of high-mobility semiconductors where $\tau_\varphi \approx \tau_m$, the phase-relaxation length is then defined as $l_\varphi = v_F \tau_\varphi$. In disordered systems however, l_φ is given by $l_\varphi = \sqrt{D \tau_\varphi}$, where $D = v_F^2 \tau_m / 2$ is the diffusion constant.

Signatures of quantum interference start to appear in the transport properties of an electron gas when the sample size is comparable to or smaller than the phase-relaxation length. Under these conditions, the transport regime is said to be “coherent”.

1.3 Two-dimensional Electron Gases : The GaAs-Al_xGa_{1-x}As System

In the last three decades, two main systems have driven research into low-dimensional physics. The first system is the metal-oxide-semiconductor field-effect transistor (MOSFET), invented more than 70 years ago by J. E. Lilienfeld, but only developed on an industrial scale in the 60's. While quite simple with respect to its operation mode, the silicon-based MOSFET has essentially revolutionized solid-state electronics. The second system is the GaAs-Al_xGa_{1-x}As multilayer samples of the III-V family, with selective doped layers, first grown and studied in the late 70's by Dingle, Stormer and Gossard[49]. In both the MOSFET and GaAs-Al_xGa_{1-x}As structures, the electrons are trapped at an interface, but are free to move with metallic-like properties along that surface.

Historically, the MOSFET dominated research in the 70's up to the discovery of the Quantum Hall Effect[104] in 1980, where interest shifted toward III-V's. While silicon based system successfully returned in the 90's with research activities centered on the new SOI (Silicon on Insulator) technology[44], two-dimensional electron (and hole) gases (2DEGs) confined to GaAs-Al_xGa_{1-x}As structures are still the major topic of investigation for thousands of solid-state physicists around the world, with new transport phenomena coming to attention every year: the Fractional Quantum Hall Effect[163, 111], Composite Fermions[81], the signature of Skyrmions in the electronic and thermodynamic properties of 2DEGs[15, 126], metal-insulator transition[108] or, very recently, the formation of a Wigner crystal in interacting bilayer hole systems[164, 64].

The mobility of bulk GaAs is quite high at 77 K, but is unfortunately reduced at high temperature because of optical phonon scattering, and at low temperatures by carrier freeze-out, electron-electron interactions and, principally, by the scattering of the mobile carriers with charged impurities. Fortunately, the use of modulation-doped heterojunctions and quantum wells can significantly reduce this Coulomb scattering.

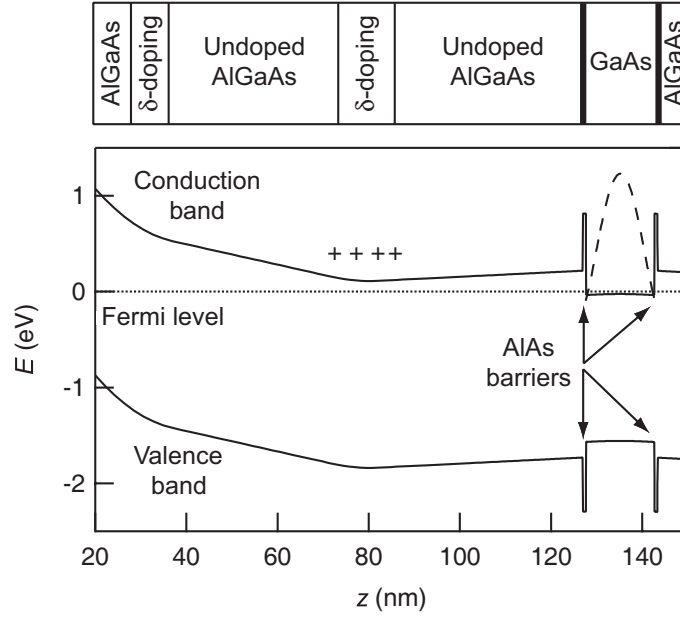


Figure 1.1: Conduction and valence band diagrams of a GaAs-AlAs quantum well structure where a 2DEG is confined some 140 nm below the sample surface. The dashed line in the well represents the spatial distribution of the electron density.

The principle of operation is quite simple. As shown in the upper panel of Fig. 1.1 that outlines the growth structure of a typical quantum well sample, electrons are trapped in an ultra-pure and undoped GaAs layer, surrounded by thin AlAs barriers. Going toward the top of the structure, two highly doped and ultra-thin layers are sandwiched by large layers of undoped $\text{Al}_x\text{Ga}_{1-x}\text{As}$. Figure 1.1 shows the self-consistently calculated conduction and valence profiles of the structure. As electrons are liberated from the donor sites by thermal excitation or light, they move freely to the GaAs layer, where they remain physically separated from the ionized donors located in the δ -doped layers. As a result, scattering is greatly reduced and mobilities greater than $1 \times 10^7 \text{ cm}^2/\text{Vs}$ can be obtained.

Because the width of the quantum well, or that of the triangular electrostatic confinement at the GaAs- $\text{Al}_x\text{Ga}_{1-x}\text{As}$ interface (15 to 100 nm), is on the order of the Fermi wavelength of the electrons ($\approx 50 \text{ nm}$), quantization plays a major role in these systems (Sec. 1.2). The energy levels are discretized and, at the low densities observed in semiconductor structures, electrons occupy a small number of transverse subbands. In the case of a simple, 15 nm-thick GaAs- $\text{Al}_x\text{Ga}_{1-x}\text{As}$ quantum well, only the ground state is populated for an electron sheet density $n_s = 3.2 \times 10^{11} \text{ cm}^{-2}$.

1.4 Ballistic Transport

Associated with the high mobility obtained in 2D GaAs- $\text{Al}_x\text{Ga}_{1-x}\text{As}$ systems are electron mean free paths of a few microns. As a result, electrons can move with very few collisions over very long distances. It also means that a ballistic transport regime is expected if the electrons can be further confined to regions of lower dimensions, along one or the two directions of the 2D plane.

In Chapter 2, we will more deeply discuss the fabrication methods that are available to further confine the 2DEG. In the present introduction to mesoscopic transport, it is sufficient to know that two methods are commonly used. The first one, called etching, is based on the selective removal of sections of the sample where electronic transport should not be permitted. The second method uses metallic gates, located at the surface of the sample, to electrostatically depopulate the 2DEG.

With these two techniques, one and zero-dimensional systems can be created, with dimensions smaller than the mean-free path, hence featuring signatures of a ballistic transport regime.

1.4.1 Quantum Point Contacts

The first example of a ballistic system presented here is also a fundamental building block for the quantum structures studied in this thesis work. The quantum point contact (QPC) is a short 1D quantum system, adiabatically coupled to two electron reservoirs[168]. A scanning electron micrograph of a quantum point contact formed by depletion gates is shown in the Inset to Fig. 1.2. In the direction perpendicular to current propagation, one or more transmission modes are supported, depending on the width of the constriction.

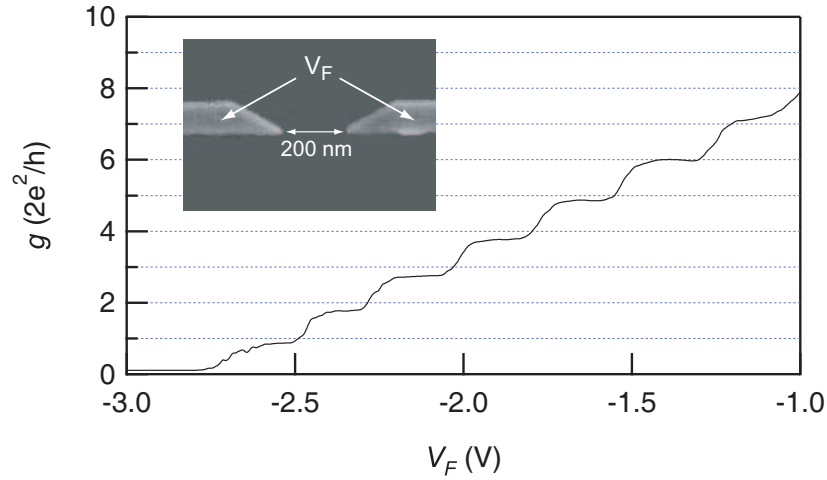


Figure 1.2: Conductance through a quantum point contact. Inset: SEM micrograph of the electrostatic gates on top of the GaAs-Al_xGa_{1-x}As 2DEG. (Courtesy of F. Faniel)

As the gate voltage V_F is made more negative on the metallic gates, the narrow channel gets slowly depleted and the discrete number of transverse modes is reduced. Quite surprisingly, the conductance g , measured as a function of V_F and shown in Fig. 1.2, does not follow the ohmic scaling law $g = \sigma W/L$ where σ is the conductivity, W the width of the sample and L its length. Instead, g is quantized as a function of V_F in units of $2e^2/h$, where e is the electron charge and h is Planck's constant.

This quantization, first reported by van Wees *et al.* in their famous 1988 paper[168], is an experimental manifestation of the Landauer formula that reads

$$g = \frac{2e^2}{h} MT, \quad (1.2)$$

where M is the number of transverse modes and T represents the probability for an electron injected at one end of the sample to be ballistically transmitted to the other end.

As a side note, we mention that conductance steps of e^2/h can be observed in QPCs when spin degeneracy is lifted, using the Zeeman effect induced by a magnetic field, for example. More recently, there has been much research activity linked to the observation of conductance steps at non integer values of e^2/h . In particular, the 0.7 structure[46] evidenced in QPCs has been attributed to the presence of a Kondo-like correlated state in the vicinity of the point contact.

1.4.2 Transverse Electron Focusing

Classically, a magnetic field B applied perpendicular to the 2DEG is expected to bend electron trajectories through the Lorentz force. Moreover, if the electrons are confined to a structure whose dimensions are smaller than the mean free path, it is possible with B to control the reflections of the particles on the structure walls, in such a way that they travel ballistically between an injecting and collecting contacts. An example of such a transverse magnetic focusing (TMF) experiment[167, 166] is presented in Fig. 1.3(a).

Forcing a current between opening a and an external contact injects electrons toward the elliptic barrier. After specular reflection, the carriers are focused onto either contact c or b , inducing a voltage in the corresponding lead. The magnetic field B , applied perpendicular to the plane of the elliptical structure, causes a curvature of the electron (or hole) trajectories which move along a circle with the cyclotron radius given for a 2DEG by

$$r_c = \hbar \frac{\sqrt{2\pi n_s}}{eB}. \quad (1.3)$$

Figure 1.3(b) shows the voltage drop versus B due to focusing effect. It features distinctive peaks, called “commensurability oscillations”, corresponding to particular trajectories, visible in Fig. 1.3(c), where the electrons bounce on the elliptical wall before reaching the collector.

Similar results have been obtained in many different geometries like triangular cavities[117], in antidot arrays[85] or with other ternary compounds like InGaAs associated to InP[84] or InAs[77]. The temperature dependence of focusing peaks has also been addressed both theoretically and experimentally[84]. It has been reported[169, 86, 85] that the peak amplitude varies exponentially with the distance l_e between contacts

$$A_p = A_0 e^{-\frac{l_e}{l_s}}, \quad (1.4)$$

where l_s is a characteristic scattering length. Using the mean free path l_m for l_s does fit the experimental data at high temperature. However, unlike l_m , the characteristic length l_s remains T -dependent at low temperatures, meaning that l_s and l_m are not identical. The difference between the two T -dependences was attributed to a higher sensitivity of focusing phenomena to small-angle scattering.

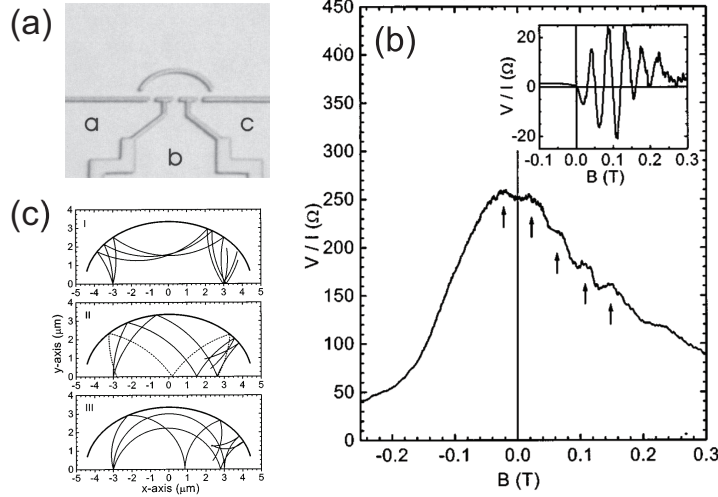


Figure 1.3: (a) Optical photograph of the elliptic focusing geometry. Three apertures, a , b , and c allow injection or detection of carriers. (b) Voltage measured at c with current injection at a vs. magnetic field B . Extrema are indicated by arrows. Inset: voltage vs. B measured in a TMF geometry without elliptic barrier. (c) Calculated electron trajectories corresponding to the extrema in Panel (b). (From Heremans *et al.*, Ref. [87])

Combining the effects of the thermal smearing on the Fermi surface, and of the finite relaxation time τ_m , it is possible to derive a relation[84, 86, 85] giving the amplitude of the focusing peaks as a function of temperature:

$$A_p(T) = A_0 [(aT/B)/\sinh(aT/B)] e^{-\pi/\omega_c\tau_m}, \quad (1.5)$$

where the factor with the hyperbolic sine describes the effect of the thermal smearing, while the last factor is related to momentum relaxation. The parameter a is given by

$$a = 2\pi k_F m^*/(e\hbar), \quad (1.6)$$

where k_F is the amplitude of the Fermi wavevector, and m^* is the electron effective mass.

Because phonon scattering is almost irrelevant in the temperature ranges where our measurements are performed ($T < 4$ K), the major factor in our case for the T -dependence of the focusing peak amplitude is the thermal smearing of the Fermi surface.

1.5 Open Quantum Dots

Open quantum dots are somewhat unique among the low-dimensional systems that are commonly encountered in Solid State Physics, as they stand between fully open mesoscopic systems such as carbon nanotubes[54], and fully confined structures like nearly-closed quantum dots in the Coulomb blockade regime[100, 38]. Because of this intermediate position, open quantum dots, usually patterned on top of 2DEGs, are now commonly used as an handy experimental tool for the investigation of a particular regime of transport where the electron dynamics is both ballistic and coherent.

In the previous section, we gave two examples of structures (QPCs and TMF systems) where the ballistic nature of electron transport adds clear signatures to the device conductance. In both cases, the mean-free path is commensurate with the average distance traveled by electrons between the input and output leads. While quite similar to the TMF structure of Sec. 1.4.2 where electron keep bouncing on the elliptic walls (Fig. 1.3(c)), open quantum dots simultaneously add one level of complexity and one major simplification to the focusing geometry:

1. An open quantum dot has only two contact leads that connect the cavity to the 2D electron reservoirs. At least one transmission mode is available in each of the dot openings. It differentiates the open dot from its closed counterpart that operates in the quantum limit with transport occurring through tunneling. Also, an open dot is sufficiently large to accommodate thousands of electrons, as opposed to the zero-dimensional quantum box that operates in the Coulomb blockade regime.
2. Beside the mean-free path, the size of the open dot is carefully chosen so that the electron trajectories are smaller than the phase-relaxation length l_φ , introduced in Sec. 1.2. The goal is of course to enhance quantum interferences and study their influence on the dot transport properties as a function of its geometry.

Name	Symbol	Formula	Value
Area	A		3.0 μm^2
Effective mass (GaAs)	m^*		0.067 m_0
Electron sheet density	n_s		$2.0 \times 10^{11} \text{ cm}^{-2}$
Fermi wavelength	λ_F	$\sqrt{2\pi/n_s}$	60 nm
Fermi energy	E_F	$2\pi\hbar^2 n_s/m^*$	7 meV
Fermi velocity	v_F	$\hbar\sqrt{2\pi n_s}/m^*$	$2 \times 10^5 \text{ m s}^{-1}$
Mean level spacing	Δ	$2\pi\hbar^2/m^* A$	2.3 μeV
Dwell time	τ_d	$2\pi\hbar/\Delta N$	0.9 ns ($N = 2$)
Crossing time	τ_{cross}	$\sqrt{A}/v_F$	6 ps
Wall bounces	N_{bounce}	$\tau_d/\tau_{\text{cross}}$	150 ($N = 2$)
Cyclotron radius	r_c	$\hbar\sqrt{2\pi n_s}/eB$	75 nm ($B = 1 \text{ T}$)
Magnetic length	l_B	$\sqrt{\hbar/eB}$	25.6 nm ($B = 1 \text{ T}$)

Table 1.1: Some characteristic scales for Open Quantum Dots. The dot area and electron sheet density are typical values for the samples used in the thesis.

Table 1.1 summarizes the important scales, characteristic lengths and energies relevant to open quantum dots.

Among other macroscopic signatures of quantum interferometry, the Universal Conductance Fluctuations (UCFs), observed in the magnetoconductance of ballistic quantum dots, have been the subject of intensive research[122, 91, 162, 65] in the last few years. The fluctuations arise from the complex quantum interferences occurring between electron trajectories that bounce multiple times against the dot walls before escaping through its leads. Before we immerse ourself into the world of interference patterns in open dots, it seems relevant to illustrate the wave-particle duality of the electron with the most basic interferometer.

1.5.1 Basic Quantum Interferometry

Figure 1.4(a) shows a SEM micrograph of a ring structure, called the Aharonov-Bohm (A-B) interferometer[2]. In the original experiment, the small 820 nm ring had been etched in a high quality gold film. To emphasize that research teams are still investigating the A-B effect, some 30 years after its inception, the picture of Figure 1.4(a) was borrowed from a recent publication of Yau *et al.* who studied hole transport in

GaAs system with a strong spin-orbit interaction[175].

Experimentally, the conductance of the ring oscillates as a function of the magnetic field, applied perpendicular to the ring. Once a slowly varying background magnetoconductance has been removed from the raw data curve (Fig. 1.4(b)), periodic oscillations are clearly visible in Fig. 1.4(c). These oscillations arise from interferences between the waves traversing the two arms in the ring. The difference between the two phases accumulated along the trajectories in the upper and lower arm is given by[47]

$$\Delta\phi = \frac{e}{\hbar} \oint \mathbf{A} \cdot d\mathbf{l} = \frac{eBS}{\hbar}, \quad (1.7)$$

where $\mathbf{A}$ is the vector potential and S is the area enclosed by the ring. Here, h/e is the flux quantum.

As B is increased, so does the magnetic flux BS enclosed by the ring, and the conductance oscillates following the rule[2]

$$g = g_0 + g^* \cos\left(\frac{eBS}{\hbar} + \phi_0\right), \quad (1.8)$$

with a periodicity in field given by $B_p = h/(eS)$.

Before we close this section, we point out that a mean-free path longer than the ring arms is not a requirement for the observation of A-B oscillations, as there are only two paths available to electrons. Unlike the TMF experiment where multiple specular reflections occur on the walls as a function of the device geometry, the only relevant parameter in an A-B ring is not its shape but the area it encloses.

1.5.2 Universal Conductance Fluctuations

Back to the open quantum dots, Figure 1.5(a) shows an example of a device, with the shape of a Bunimovich cavity[30, 31, 8, 9], obtained by wet etching of a GaAs- $\text{Al}_x\text{Ga}_{1-x}\text{As}$ quantum well sample (more details on the fabrication techniques will be given in Chapter 2). A typical electron trajectory connecting the two leads, with multiple bounces on the dot walls, is also schematically represented. It is tempting to draw a parallel with the A-B ring: semi-classically, in an open quantum dot subject to a perpendicular magnetic field, a large set of electrons crosses the dot following paths made accessible by the geometry of the cavity. Along these trajectories, the electrons accumulate phase. If two paths are associated, a specific Aharonov-Bohm phase can be calculated, giving rise to oscillations in the magnetoconductance g with a specific periodicity. As many trajectories contribute to the total current across the dot, g is

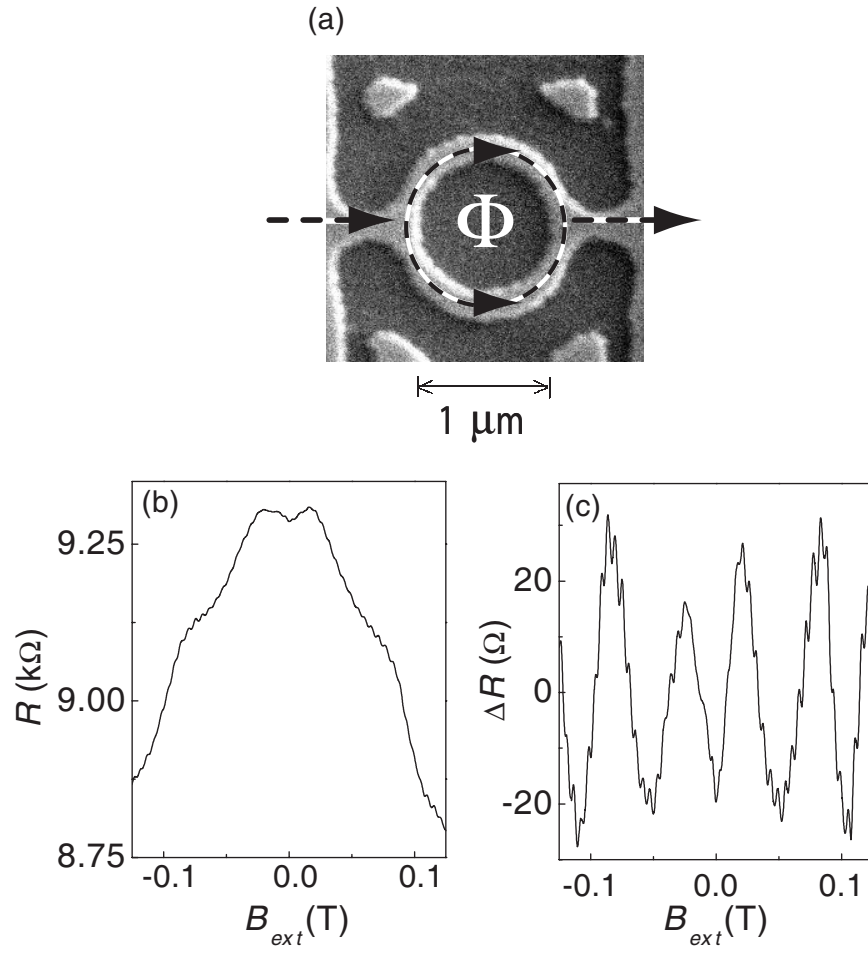


Figure 1.4: (a) Scanning electron microscope picture of a ring structure. (b) A typical trace of the Aharonov-Bohm oscillations measured in GaAs 2D holes. (c) Data in (b) after subtraction of a smooth background signal. (From J.-B. Yau *et al.*, Ref. [175])

expected to reflect the complete spectrum of A-B phases and bear the signature of all the individual oscillations.

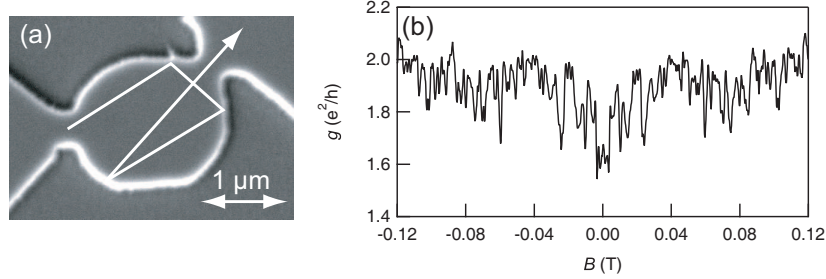


Figure 1.5: (a) SEM micrograph of a Bunimovich dot patterned on a GaAs- $\text{Al}_x\text{Ga}_{1-x}\text{As}$ quantum well and example of an electron trajectory giving rise to Universal Conductance Fluctuations. (b) Magnetoconductance of the dot at $T = 0.3\text{K}$, showing reproducible fluctuations.

A typical magnetoconductance measurement performed in the open quantum dot of Fig. 1.5(a) is shown in Fig. 1.5(b). Superimposed upon a slowly varying background are aperiodic but reproducible conductance fluctuations. Unlike the A-B ring, the stability over time of the conductance fluctuations stems from the coherent nature of transport in the dot. Moreover, as transport is also ballistic, electron paths are solely determined by the dot geometry, the electron density, and the location of impurities in the sample, which remain unchanged from one magnetoconductance measurement to another, as long as the sample is not subject to some temperature cycling that modifies the charge and dopant distribution.

Immediately, the question arises as to which electron trajectories are probed by the magnetic field. At low temperatures, the current passing through the dot is contributed by electrons with energies close to the Fermi level. Moreover, electron behavior in the dot itself is governed by the Schrödinger equation. If one temporarily sets aside the Coulomb repulsion between electrons, it is possible to evaluate the 2D occupation probability $|\Psi(x)|^2$ for conduction electrons at the Fermi level E_F , as shown in Fig. 1.6 for the Bunimovich dot. Here, we neglected the coupling between the cavity and the 2D reservoirs, and supposed that the dot was fully isolated from the external world. On Fig. 1.6, the interference pattern indicates the dot regions that are accessible to electron trajectories, at least for energies close to E_F . As the magnetic field is changed or if any parameter that affects the A-B phase is modified, $|\Psi(x)|^2$ can be significantly altered and open new electron paths.

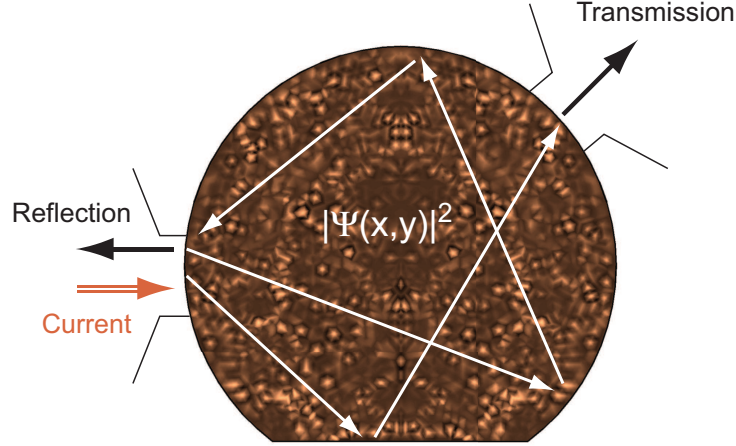
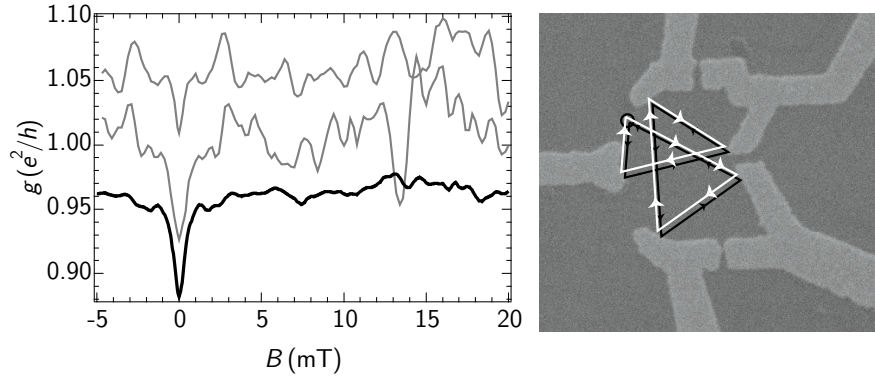


Figure 1.6: Electron wavefunction of a chaotic open dot at the Fermi level.

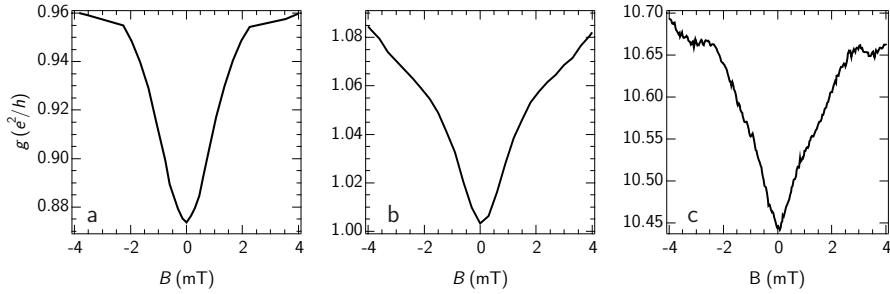
1.5.3 Weak Localization

The cavity of Fig. 1.6, the Bunimovich billiard, is known to have a chaotic dynamics in the macroscopic world. This means that, independently of the initial conditions, the phase space is covered evenly by electron trajectories. Therefore, quantum transport in chaotic open dots is also said to be ergodic, and is a manifestation of the so-called “Quantum Chaos”[121, 107]. On the opposite, transport is “regular” in other dots (circles, squares, ...) with stable trajectories contributing to the major part of the total current. This difference between regular and “quantum chaotic” systems will be more thoroughly discussed in Chap. 3. Nevertheless, clear signatures of the chaotic or non-chaotic nature of transport in open dots have been found in the weak localization correction to the dot magnetoconductance[37].

In Sec. 1.5.2, it was shown that UCFs were essentially aperiodic in the magnetic field. They also disappear when multiple measurements are averaged over a statistical ensemble of independent dot configurations, *i.e.* by changing the Fermi energy or the dot shape using electrostatic gates, as represented in Fig. 1.7(a). On the two upper magnetoconductance traces, a negative peak, the weak localization (WL) correction, is visible around $B = 0$ T and remains even after an average is calculated (bold trace of Fig. 1.7(a)). Weak localization of electron waves is the result of an enhancement of backscattering from an impurity center in a disordered environment or by the cavity boundaries in QDs. This en-



(a) Magnetoconductance of the open dot as represented in SEM micrograph. The two upper traces, offset for clarity, are individual measurements showing UCFs. The lower curve is the average of many such measurements obtained by slightly changing the dot shape using the split gate contacts visible on the SEM micrograph in the right panel.



(b) Magnetoconductance line shape around $B = 0$ T for (from left to right) chaotic, mixed and integrable device shapes.

Figure 1.7: Universal Conductance Fluctuations and Weak Localization Peak in split-gate open quantum dots. (From Chang *et al.*, Ref. [37]).

hancement of backscattering suppresses the electrical conductivity of a quantum conductor, as compared to the value expected from the classical Drude theory of transport. This correction to the conductivity is generated by the constructive interference between pairs of electron waves that scatter from the same impurities or bounce on the same walls in the dot, *i.e.* two electron waves arriving together at a particular impurity site, scattering from it, then following a closed-loop random-walk path, visiting the same set of impurities, either in a clockwise or anti-clockwise configuration, and finally scattering into the exactly backward direction. An example of electron paths involved in such a non-local scattering process is schematically shown in Fig. 1.8. Of course, the observation of such an interference effect requires phase-coherent propagation along the closed-loop part of the geometrical trajectory. Also, similar to the Aharonov-Bohm ring, a small magnetic field modifies the interference pattern between pairs of electron waves, breaking time-reversal symmetry, canceling the constructive effects on average, and resulting in a disappearance of the conductance peak at small B , as seen on Fig. 1.7(a).

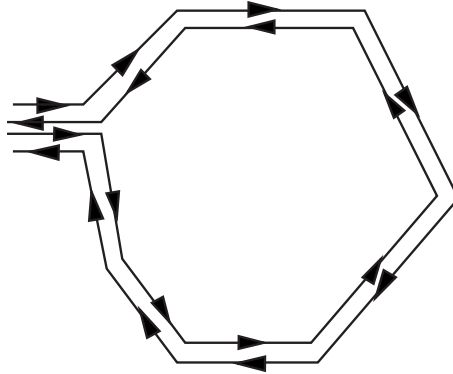


Figure 1.8: Enhanced backscattering due to the coherent interference of electrons traveling along the same trajectory in opposite directions.

Interestingly, it has been predicted and verified experimentally[37] that the *shape* of this weak-localization peak depends on the chaotic nature of transport in the dot. Figure 1.7(b) shows three localization peaks measured in (a) chaotic, (b) mixed and (c) integrable dots. The line shape of the weak localization correction changes from Lorentzian for the chaotic system, to triangular in integrable cavities. More interesting maybe is the mixed case (b) where no clear line shape is observed. As such a shape of the weak localization peak appears rather frequently in measurements, it also means that the most common configuration

encountered when characterizing open dots is that of a mixed phase space[103, 144, 131], neither fully chaotic nor fully regular. More details will follow in Chapter 3 where the theoretical models of quantum transport are presented.

Chapter 2

Fabrication of Open Quantum Dots

La perfection est atteinte non quand il ne reste rien à ajouter, mais quand il ne reste rien à enlever.

Antoine de Saint-Exupéry

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2.1 Introduction

The success or failure of a measurement session performed on a GaAs- $\text{Al}_x\text{Ga}_{1-x}\text{As}$ open quantum dot strongly depends on the optimization of the successive processing steps that transform a piece of GaAs wafer into a full-featured, *working* nanostructure. While many labs share well-known fabrication techniques[172] which will be described succinctly in the next sections, some recipes (some people would call them black magic) finely-tuned for GaAs processing on the nanoscale are always required on a per-structure and per-sample basis.

There are two constants, two pieces of equipment that can't be ignored in modern GaAs processing methods. The first one, Molecular Beam Epitaxy (MBE) is the most versatile technique for growing epitaxial layers with atomic precision. It is the building block for the confinement of electrons to a two-dimensional electron gas (2DEG). The second equipment is the Scanning Electron Microscope (SEM), whose lithography capabilities are used to further restrict the lateral extent of the 2DEG to both a Hall bar mesa and to the dot itself, and to define electrical and electrostatic contacts on the sample. Thanks to the team of Prof. Shayegan from Princeton University who provides us with extra high quality 2DEGs, our processing work at UCL is "limited" to the steps that involve SEM lithography.

In Section 2.2, we first give an overview of the modern tools and methods currently used in the preparation of GaAs open dots. After a general introduction to the material properties of the GaAs- $\text{Al}_x\text{Ga}_{1-x}\text{As}$ compounds in Sec. 2.2.1, we shortly discuss the molecular beam epitaxy and Electron Beam Lithography (EBL) techniques in Sec. 2.2.2 and 2.2.3, respectively. The processing steps are explained in Sec. 2.2.4 in terms of the associated lithography layouts, the sample-specific process parameters, and sample areas they are focused on. When necessary, the appropriate theoretical elements are provided and finer technical details are emphasized if they differ from common practice. Section 2.2.4.5 presents some images of completed samples, with SEM micrographs of open quantum dots.

Next, Section 2.3 is devoted to an inventory of the low-temperature equipment and data acquisition tools used for the characterization of the open quantum dots. In particular, Sec. 2.3.1 revolves around the add-on to the cryostat sample holders that allows for the precise control of the sample tilting angle, central to the magnetoconductance measurements presented in this work. Finally, in Sec. 2.3.2, we give details on the magnetoconductance measurement techniques and the data acquisition tools that were developed during the course of this thesis work.

We summarize our results in Sec. 2.4.

2.2 Fabrication Techniques

As already mentioned in Sec. 2.1, the complete sequence of operations that leads to the fabrication of an open quantum dot is divided into two independent process steps centered around important pieces of equipment

1. The preparation of GaAs-Al_xGa_{1-x}As heterostructures by molecular beam epitaxy, where the electrons are confined to a 2D plane.
2. The lateral confinement of the electron layer into a billiard-shaped nanostructure by means of high resolution EBL, performed on parts or the entirety of a small piece of heterostructure and combined with sample etching or the deposition of electrostatic gates.

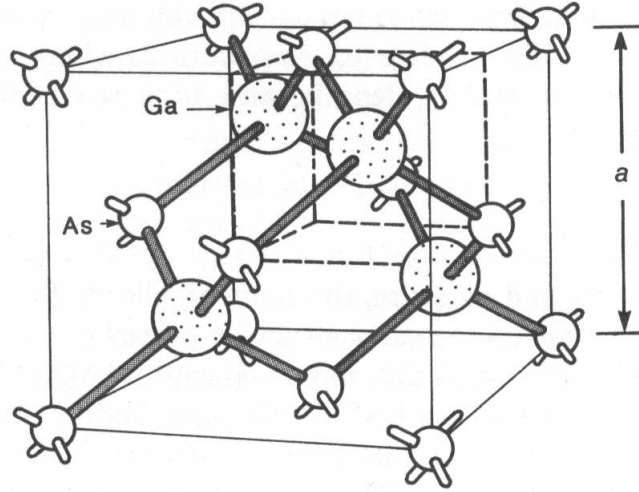
2.2.1 Materials

Beside Silicon (Si) which is used for doping purposes, the three main atomic species composing our samples are Gallium (Ga), Arsenide (As) and Aluminum (Al). Together, they form the ternary compound Al_xGa_{1-x}As ($0 \leq x \leq 1$) with the binary compounds GaAs and AlAs as particular cases. These are called III-V semiconductors, because of the group III origin of the Ga and As species, and the group V origin of As. The crystals of GaAs and AlAs are of the Zincblende type, with tetrahedrally coordinated atoms (Fig. 2.1(a)). The lattice constants at room temperature are 0.56533 nm for GaAs and 0.56611 nm for AlAs, which makes them almost perfectly latticed-matched.

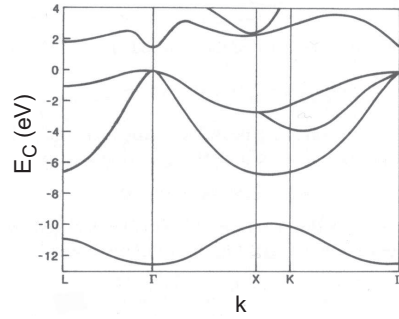
The band structures of the GaAs and AlAs binary compounds are shown in Figs. 2.1(b) and 2.1(c), respectively. The following features should be noted :

- For both GaAs and AlAs, there is an energy gap E_g across the complete Brillouin zone between the filled valence band states and the empty conduction bands. The band gap energy, which is characteristic of semiconductors, amounts to 1.42 eV for GaAs and 2.23 eV for AlAs. In the case of the ternary compound Al_xGa_{1-x}As, a useful expression giving the value of the band gap energy (in eV) as a function of the molar fraction is taken from Adachi[1] and reads

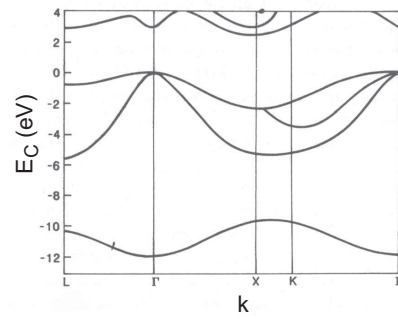
$$E_g(x) = 1.424 + 1.266x + 0.266x^2, \quad (2.1)$$



(a) Zincblende structure for GaAs.



(b) Energy band structure for GaAs.



(c) Energy band structure for AlAs.

Figure 2.1: Material properties for the GaAs and AlAs binary compounds. (From Kelly, Ref. [102])

valid at $T = 300$ K for $x < 0.45$. Similarly, the temperature coefficient of the band gap energy is given by

$$\frac{\partial E_g(x, T)}{\partial T} = -3.95 - 1.15x. \quad (2.2)$$

- The lowest conduction band is always non-degenerate, while the highest energy states for the valence bands at the Γ -point are three-fold degenerate¹.
- As evidenced by the band structures of Figs. 2.1(b) and 2.1(c), GaAs is direct-gap material, with the minimum in the energy gap occurring at the Γ point. On the opposite, AlAs is an indirect semiconductor, with the minimum in the conduction band located at the X point. In the ternary alloy $\text{Al}_x\text{Ga}_{1-x}\text{As}$, the conduction band minimum moves up relative to the valence band as x varies from 0 to 1, with the transition from direct to indirect gap taking place for $x = 0.38$.
- Near the band minima, the conduction band dispersion relations are approximately parabolic. By expanding the band energies about these minima, one obtains an effective mass tensor. In the case of GaAs, the Fermi surface is spherical in reciprocal space around the Γ point, giving an effective mass $m^* = 0.067m_0$, where m_0 is the free-electron mass. For AlAs, the Fermi surfaces are ellipsoidal and can be characterized by a longitudinal effective mass $m_l^* = 1.1m_0$ and two transverse masses $m_t^* = 0.19m_0$.

2.2.2 Molecular Beam Epitaxy

The growth of quantized structures where an electron gas is confined at the interface(s) between GaAs and $\text{Al}_x\text{Ga}_{1-x}\text{As}$ is usually performed using a technique called Molecular Beam Epitaxy (MBE). Developed in the early 70's at Bell Labs by Cho and Arthur[41], MBE is a very versatile method of growing atomic layers, where collimated beams of heated Al, Ga and As components, along with various dopants, escape into a ultra-high vacuum (UHV: typically $< 10^{-10}$ mbar) and are directed toward the surface of a GaAs wafer, maintained at a temperature of approximately 600°C (Fig. 2.2).

Changes in the doping concentration or crystal composition are obtained with an atomic layer precision by controlling the shutters located

¹This is only true in the absence of higher-order effects like spin-orbit interactions (See Sec. 3.3.3 for details).

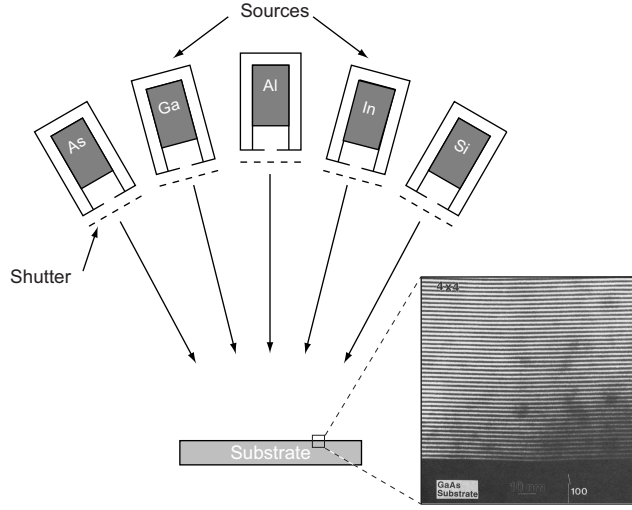


Figure 2.2: Molecular beam epitaxy: materials and dopants are evaporated onto a GaAs substrate under ultra high-vacuum conditions. Inset: High precision scanning micrograph of MBE-grown layers of GaAs and $\text{Al}_x\text{Ga}_{1-x}\text{As}$ (AT&T Bell Laboratories, Ref. [158]).

in front of the individual cells. In principle, it is quite difficult to control the concentration of atomic species arriving at the substrate. Practically, one uses electrons and ions as probes to monitor the surface and deposition quality during the growth process[176]. The ion-based technique is usually mass spectroscopy while among electron-based methods, the most commonly used is Reflection High-Energy Electron Diffraction (RHEED). In a typical RHEED system, an electron gun produces a high-energy beam (15 keV) directed toward the substrate. The reflected electron diffraction pattern can be used to determine the surface geometry by monitoring RHEED oscillations during the growth of GaAs/AlAs heterostructure.

The highly detailed control of crystal growth in MBE chambers is illustrated by the inset to Fig. 2.2. On this scanning electron micrograph of the cross section of a III-V structure, an extremely regular superlattice of GaAs and $\text{Al}_x\text{Ga}_{1-x}\text{As}$ is clearly visible. Because of the almost-perfect lattice match between the GaAs (5.65 Å) and AlAs (5.66 Å) compounds, elastic strain is negligible between the layers and allows for the growth of complex, crystal-like heterostructures over several hundreds of nanometer.

The samples used in this thesis were grown by MBE on (100)-oriented GaAs by Drs. E. De Poortere and M. B. Santos from Princeton Univer-

sity. Details of the growth sequence and doping concentrations of the quantum well samples will be given and discussed further down the text (Sec. 4.4). As a reference, the MBE growth structures of the 2DEGs used in this work, including the doping profiles, are available in Appendix B. Here, we emphasize that the 2DEGs are confined to quantum wells, as opposed to heterojunctions, with widths of 15, 45 and 68 nm. The purpose of this gradient in the 2DEG layer thickness will become apparent when we present in Chapter 4 our magnetoconductance measurements performed in open quantum dots subject to a parallel component of the magnetic field.

2.2.3 Electron Beam Lithography

Given a GaAs substrate containing epitaxially grown multilayers with one or multiple 2DEGs, the next processing steps involve the transfer of a pattern to the sample surface, in order to either remove selected parts of the surface or to make/deposit conducting or electrostatic contacts. There are a few methods commonly used to perform lithography on semiconductor surfaces and to create fine patterns. Beside the optical and X-ray techniques, electron beam lithography (EBL) is now the preferred tool in most solid-state research laboratories because of its nanoscale precision and high flexibility.

Figure 2.3 shows a schematic of a Scanning Electron Microscope (SEM) with lithography capability. A complete description of operation of a SEM is beyond the scope of this chapter and is thoroughly covered in reference materials[140, 172]. Here we summarize the role of its main components. Electrons are emitted by the electron gun from a conducting material. The emission can be obtained either by thermionic effect (with a filament usually made of tungsten or LaB_6), or by applying an electric field sufficiently strong to permit electron tunneling across the conductor barriers (field emission microscope). Next, the electron beam is focused by a complex system of condenser and objective lenses and deflected to a particular region of the sample using electrostatic plates. Between the condenser lenses and the x - y plates, a beam blanker is inserted in the column and allows for a fast on/off switching of the electron beam.

The equipment available at the Microelectronics Lab of the Université Catholique de Louvain is a Philips-FEI XL-30 SFEG microscope combined to a Raith Elphy Plus hardware module that drives the beam blanker and the x - y deflection plates through two high frequency (1MHz) 16bit digital-analog converters (DACs). As an add-on to the FEG microscope, a laser interferometry stage allows for a precise positioning

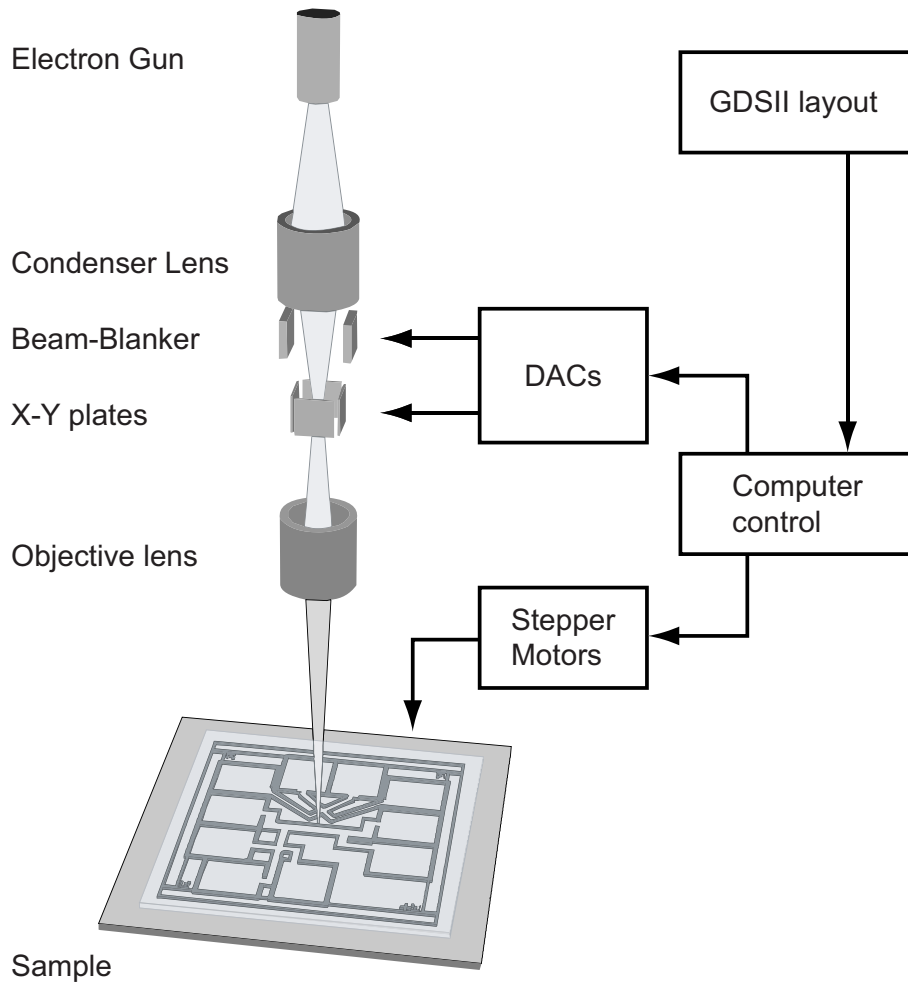


Figure 2.3: Schematic of the SE microscope and lithography setup

of the sample with respect to the focused electron beam. As opposed to the industry-oriented (and much more expensive) e-beam writers, the lithography process is semi-automatic, with a computer controlling both the beam scanning system and the transfer of the lithography pattern from a layered structure onto the sample surface.

The resist used in our SEM lithography processes is the organic polymethylmethacrylate (PMMA), available in the mono and poly-disperse forms, and typically diluted in chlorobenzene at concentrations of 3 and 6%. In liquid form, the resist is deposited at the sample surface by high-speed spin-coating (5000rpm). After a short baking (a few minutes at 160°), the solvent is completely evaporated and the resist becomes hard and polymerized to some extent. The film thickness strongly depends on the concentration of the polymer in its solvent, and on the rate at which the sample is spin-coated. Using ellipsometry, we measured thicknesses of ~150 and ~350 nm for the 950K polydisperse PMMA² with dilutions of 3 and 6% in chlorobenzene, respectively.

When exposed to an electron beam at reasonable dose, the PMMA film acts as a positive resist: the exposed polymer chains are broken into smaller organic units that can be easily dissolved by solvents, the most common one being methylisobutylketone (MIBK) combined to isopropanol. The unexposed surfaces remain unattacked. Above a critical exposure dose (3.5 mC/cm²), PMMA acts as a negative resist and polymerizes further, so that solvents can only remove those parts that have not been exposed. In our SEM lithographies, PMMA was constantly used in its positive resist configuration.

The de Broglie formula states that the wavelength of a particle varies inversely with its momentum:

$$\lambda = \frac{h}{p}. \quad (2.3)$$

Therefore, unlike in imaging mode, the highest possible electron energy permitted by the microscope is typically used for lithography in order to achieve the highest precision. As an example, 10 keV electrons have a wavelength of about 0.1 Å, which sets the limit on the precision that can be intrinsically achieved by EBL. Nevertheless, we emphasize that the linewidth is usually much larger than this intrinsic limit, as scattered electrons can induce changes in the PMMA polymerization at boundaries of exposed areas, a phenomenon known as “proximity effect”.

After development, the requested pattern is printed in the resist at the surface, with the PMMA fully removed of the exposed areas. It now

²The PMMA is commercially available from MicroChem Corp., Newton, MA 02464, USA. The resist data sheets are available on <http://www.microchem.com>.

remains to exploit this layout, either by etching the exposed surface or by depositing metallic materials. A commented description of the successive EBL steps involved in our process follows in the next section. A standard process sheet for a GaAs-Al_xGa_{1-x}As device, with the EBL parameters used at each step, is also available in Appendix A as reference material.

2.2.4 Fabrication Steps

Beside its higher precision in linewidth³, the main advantage of EBL over the classical technique of optical lithography is its higher flexibility. As the electron beam directly writes onto the sample surface, no mask is required. It is thus straightforward to frequently adapt the lithography patterns to the Hall bar and/or nanostructure geometries by simply changing the lithography layers drawn on the computer. The drawback of EBL is the obvious difference in speed and throughput with respect to optical lithography. However, as only 4 to 5 lithography steps are required for the preparation of an open quantum dot sample, the versatility of EBL, used in all our processing steps, by far outweighs the time that could be saved by using optical lithography.

The starting point of processes aiming at the fabrication of an open quantum dot is a $3 \times 3 \text{ cm}^2$ piece of GaAs-Al_xGa_{1-x}As sample, cut from the main wafer supply with the help of a diamond.

A typical process[172], as outlined in the Table of Appendix A, is composed of 4 independent steps, divided into two groups:

1. Etches: where selected regions of the sample are subject to one or more etchants. The result of the etching process is the selective removal of a given thickness of the material in contact with the etchant.
2. Lift-offs: where one or multiple layers of metal are evaporated both at the surface of the PMMA and of the exposed semiconductor, and selectively removed from areas defined by the lithography.

2.2.4.1 Ohmic Contacts

As explained in Chapter 1, the main goal and main advantage of fabricating 2DEGs with III-V semiconductor materials is to obtain the highest

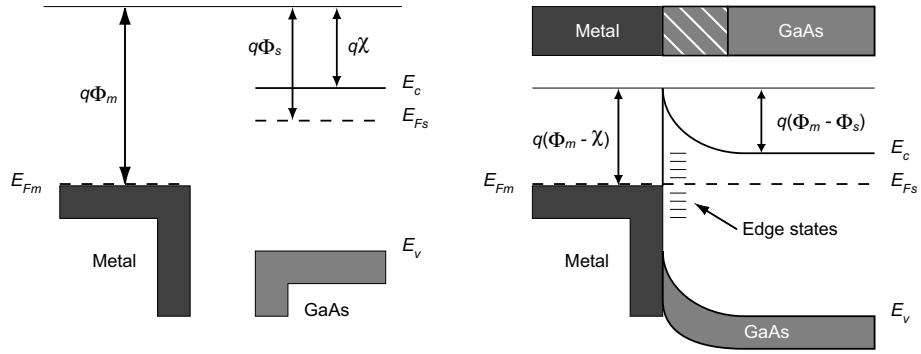
³The difference in lithographic linewidth between EBL and optical techniques is rapidly fading away, as a precision of 80 nm is now common practice with modern (and very expensive) optical steppers operating in the deep UV spectrum.

possible mobilities. To achieve this objective, the electrons are confined at GaAs-AlAs interfaces which are typically located a few hundreds nanometers below the sample surface. As a result, there is no physical link between the 2DEG and the contact pads patterned on top of the surface and used as current injectors into the structure. To make it worst, the simple fact of putting in close contact such metallic pads with the low resistivity semiconductor (GaAs) layer located at the surface creates a barrier to current injection into the 2DEG. The physical origin of this so-called Schottky barrier, is illustrated by the band structures of Fig. 2.4(a). When a metal is brought in contact with the GaAs surface, charge transfer occurs rapidly between the two materials up to the point where the two Fermi levels are at equilibrium. For example, for a n-type semiconductor with a work function Φ_s and a metal with a work function $\Phi_m > \Phi_s$, the alignment of the Fermi levels gives rise to an equilibrium contact potential $V_{s0} = \Phi_m - \Phi_s$ and a total barrier height $\Phi_B = \Phi_m - \chi$, where χ is the electron affinity in the semiconductor. Both the height and width of the barrier strongly depend on the ability of the semiconductor to provide electrons for the charge transfer, and is thus a function of its doping concentration. The Schottky barrier needs to be overcome for the current to flow into the 2DEG.

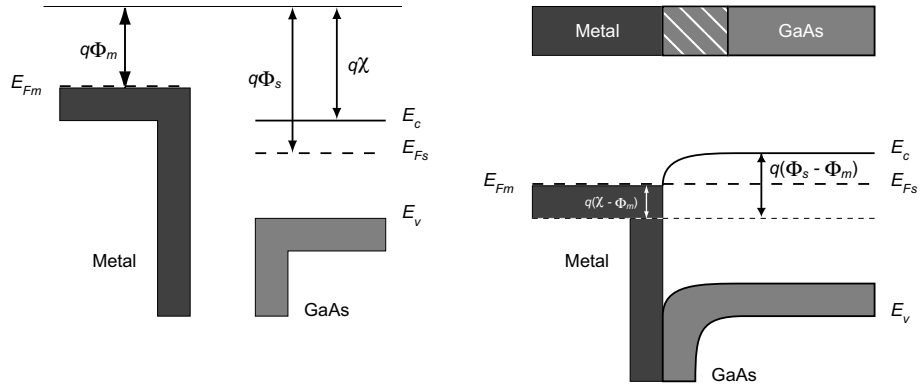
In the case of a realistic semiconductor like GaAs, the metal-semiconductor surface is not perfect on the atomic scale but includes incomplete covalent bonds that generate *surface states* around the Fermi level E_F , as represented on Fig. 2.4(a). These surface states strongly influence the position of E_F with respect to the conduction band E_c at the interface. For example, in n-type GaAs, a collection of surface states are located ~ 0.8 eV below E_c and *pin* E_F (hence the expression “Fermi level pinning”) above the conduction band, independently of the work function of the metal.

In order to solve both the problem of the Schottky barrier and the low conductivity GaAs path between the 2DEG and metallic pads at the surface, a process step that involves the deposition and rapid thermal annealing of a suitable metallic alloy is commonly used. The process steps are outlined on Fig. 2.4(c). After deposition of a thick PMMA layer, contact pads are exposed by EBL using the layout sketched on Fig. 2.4(d). The relatively large magnification ($\times 400$) requires the use of the stitching technique to assemble the individual exposure fields (size: $216 \mu\text{m}$) and cover the entire surface of the sample.

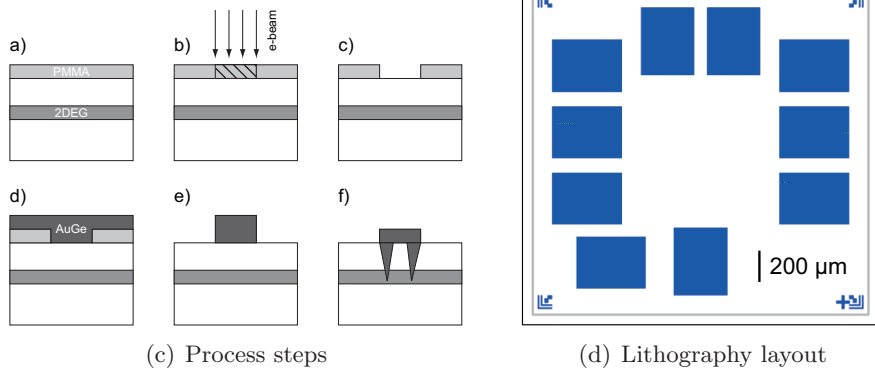
Development exposes the contact area that are subsequently covered by evaporation with a NiAuGe alloy. After metalization, lift-off is performed by soaking the sample in acetone, which leaves only the metal deposited onto the exposed GaAs surface. Ultrasonics are also used for



(a) Schottky barriers



(b) Ohmic contacts



(c) Process steps

(d) Lithography layout

Figure 2.4: Fabrication of ohmic contacts.

a few seconds to get rid of any trace of PMMA or metal in unexposed regions. Following lift-off, rinsing under ultra-pure, deionized (DI) water and drying, the sample is thermally annealed at 460°C for 6 minutes with a rise time of 30 seconds in a 95% N₂ - 5% H₂ atmosphere.

The choice of the materials composing the alloy, the precise sequence of their deposition outlined in Table A.1, and the high temperature annealing serve multiple purposes:

1. The germanium (Ge) contained in the alloy diffuses at high temperature into the GaAs, occupying Ga sites left empty by Ga atoms that accumulate in the Au layer. Through this diffusion process, the Ge atoms locally dope the semiconductor at concentrations of up to $1 \times 10^{21} \text{ cm}^{-3}$. High conduction paths are formed between the surface and the 2DEG (Fig. 2.4(c)).
2. While the Au serves as buffer for the Ga atoms migrating from the semiconductor surface, it also protects the metallic contacts against oxidation and minimizes the overall contact resistance when bonding is performed. These two points are further optimized by the deposition of an extra cap layer of pure Au at the end of the metalization.
3. Two layers of nickel are evaporated during the process. The first one, in direct contact with the sample surface, is used as a wetting layer for the subsequent Au and Ge layers. It also serves as a barrier that somewhat controls the diffusion speed of Ga and Ge atoms into the semiconductor and Au layers, respectively. The second load of Ni acts as a barrier against diffusion of potentially oxidant atoms into the Au located at the surface of the metallic pads.

Following the annealing process, highly conductive spikes of heavily Ge-doped GaAs connect the surface and the 2DEG (Fig. 2.4(c)). While the Schottky barrier that originates from the GaAs surface states is taken care of, another barrier remains between the ohmic spikes and the 2DEG. But because the height and width of this barrier are strongly reduced by the high doping concentration, tunneling processes are expected to occur and to contribute to current flow between the contact pads and the 2DEG.

2.2.4.2 Hall Bar Geometry: Mesa Etching

There are two forms of etching, either wet etching using liquid chemicals, or the dry form using plasmas. Both techniques have their advantages

and disadvantages. In order to avoid some possible contaminations with the Silicon on Insulator (SOI) fabrication line in the Microelectronics lab, it was decided to completely isolate the GaAs process from the SOI one that intensively uses plasma etching. Wet etching has also the advantage of being easily and quickly implemented. As a result, only wet etching was used for the preparation of the open quantum dot samples.

The most popular wet etchant for GaAs is composed of a blend of sulfuric acid, hydrogen peroxide, and DI water. The relative etch rate is a function of the components proportions and has been thoroughly covered in the literature (See Chapter 5 of R. Williams[172] and references therein). A typical composition is 1:8:240 $\text{H}_2\text{SO}_4:\text{H}_2\text{O}_2:\text{H}_2\text{O}$ for an etching rate of ~ 100 nm/min. Before starting any etch on a target sample, a calibration is usually performed on a dummy piece of GaAs wafer. The etching depth is then measured on the dummy sample using a Dektak profilometer and/or an Atomic Force Microscope (AFM).

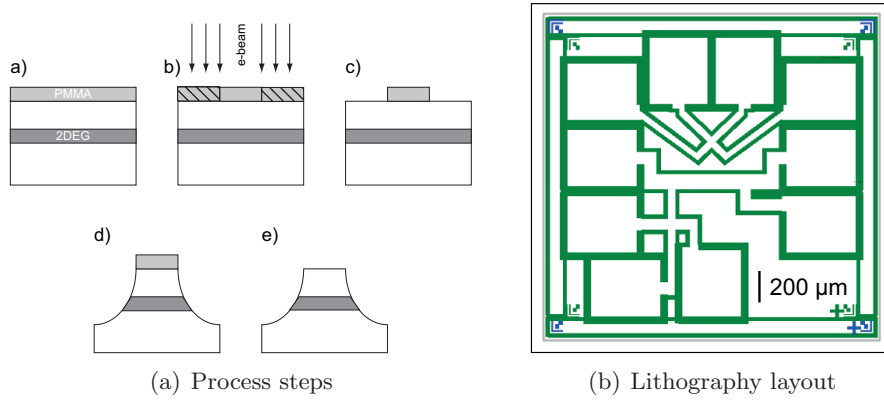


Figure 2.5: Etching process and Hall bar geometry (e-beam layout).

Once ohmic contacts have been deposited on top of the surface and a conduction channel established with the 2DEG, the second processing step involves the restriction of the 2DEG lateral extension. The goal is to form a Hall bar “Mesa” by etching the structure as shown in Fig. 2.5(a). While it would be feasible to fully expose and etch away the large parts of the sample surface where electron transport should not occur, it is more efficient with respect to lithography exposure time, to only expose narrow regions at the Hall bar boundaries.

A typical lithography layout for the Mesa etch is presented in Fig. 2.5(b). The dark regions are the exposed surfaces where wet etching is performed after development. Alignment with the ohmic contact layout is enforced by means of marks located at the corner of the de-

vice and clearly visible in Fig. 2.4(d). The etching depth is chosen large enough in such a way that the etched region extends beyond the 2DEG, typically located 100 nm below the surface (Fig. 2.5(a) d-e). After the desired etching time, the sample is soaked into DI water at boiling temperature to quickly wash away any trace of acid. Finally, the PMMA is removed in a MIBK:isopropanol bath.

2.2.4.3 Open Quantum Dot: Shallow Etch

Intrinsically, the shallow etching step that leads to the confinement of the 2DEG to the open quantum dot is rather similar to the Mesa etching step of Sec. 2.2.4.2. The lithography layouts are shown in Fig. 2.6 with close-ups of the dot area in Fig. 2.6(b). Similar to the previous lithography, the layout are aligned to the Hall bar geometry using alignment marks visible on Fig. 2.6(a). Because of the precision of the laser stage on the position of the sample with respect to electron beam, it is sufficient to calibrate and align the exposure field using marks located at the edges of the sample, a few hundred microns away from the dot center. However, for an increased accuracy of the field alignment, it is also possible to scan marks located at the corner of the $43.7\ \mu\text{m}$ exposure field shown in Fig. 2.6(b).

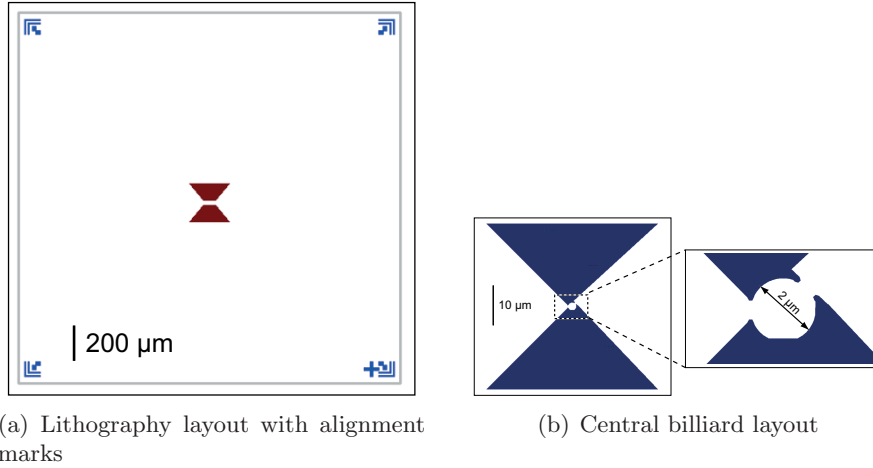


Figure 2.6: Shallow etch - Dot lithography (e-beam layout).

Beside the size of the exposure field (magnification $\times 2000$, size: $43.7\ \mu\text{m}$), there are nevertheless major differences between the deep Mesa etch of Sec. 2.2.4.2 and the shallow etch performed for the dot creation:

1. As opposed to the deep etch that extends beyond the 2DEG, shallow etching is usually interrupted at the level of the $\text{Al}_x\text{Ga}_{1-x}\text{As}$ layer located between the two delta dopings above the 2DEG (See Appendix B for details on the growth structures of the 2DEGs used in this thesis). The purpose of this smaller etching depth is to avoid the formation of surface states at the boundaries between the etched and unpatterned regions. Unlike the Hall bar Mesa, the small size of the dot makes it particularly sensitive to surface states. Stopping the wet etching between the two doping layers guarantees that the etched regions are forbidden to electrons while preserving the electrical integrity of the dot.
2. For the shallow etching of the open dot, extra care must be taken to remove any trace of water on the sample before the deposition of the PMMA resist. Water can cause a poor adhesion of the resist on the sample surface, which can result in an unexpected etching of unexposed sample areas. This can prove devastating for the dot geometry, as the etchant can penetrate a few nanometers below the PMMA. “Underetched” structures can be avoided by drying the sample overnight in oven at 140°C .
3. Finally, deep sub-micron structures can be strongly affected by profiles resulting from an anisotropic etching of crystalline planes. Due to the polar nature of GaAs, grooves can appear with slopes up to 45° , depending on the crystallographic orientation of the GaAs substrate and of the etched trench itself. For an etch depth of 100 nm, a similar reduction of the dot lateral dimensions can sometimes be observed.

2.2.4.4 Surface Electrostatic Gates

The process step (Fig. 2.7(a)) where electrostatic gates are deposited on top of the sample is quite similar to the lithography devoted to the ohmic contacts (Sec. 2.2.4.1). Once again, the corresponding EBL parameters are outlined in Table A.1 of Appendix A. As opposed to the ohmic contacts, only two metallic species are evaporated, while thermal annealing is skipped. A small chromium layer acts as a wetting agent between the sample surface and the 35 nm-thick gold layer. The electrostatic gate covers the entirety of the main Hall bar that contains the open dot, as can be seen from the lithography layout presented in Fig. 2.7(b).

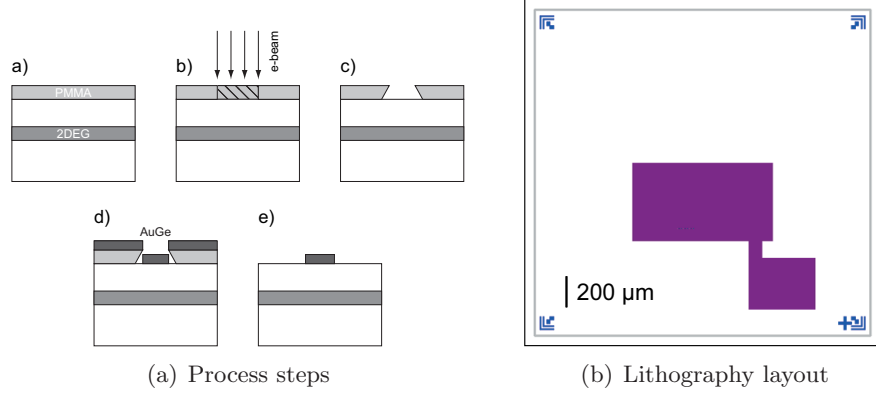


Figure 2.7: Electrostatic gates (process steps and e-beam layout).

2.2.4.5 Bonding and Packaging

Once the lithography steps have been successfully performed, the sample is glued to a non-magnetic epoxy sample holder. Gold wires are soldered between the pins of the sample holder and the ohmic/electrostatic contact pads on the sample. Silver epoxy is conveniently used as a soldering agent. Available in semi-liquid form, silver epoxy solidifies and becomes conductive as the solvent fully evaporates and cross-linking of polymer chains occurs when the sample is put in an oven for about an hour.

Figure 2.8 shows a view of a completed sample with the silver epoxy clearly visible on the ohmic and electrostatic contact pads. The lower part of the sample is occupied by the main Hall bar with the open quantum dot at its center. The Hall bar is covered by an electrostatic gate that extends up to the four adjacent ohmic contacts. The upper part of the sample is devoted to a second Hall bar with a different geometry. The purpose of this second Hall bar is to precisely determine the tilt angle between the 2DEG and an external magnetic field, using transverse magnetoresistance measurements. More details concerning the evaluation of tilt angle will be given in Sec. 4.5.

Examples of open quantum dots that were produced during the course of this thesis are presented in Fig. 2.9. The first SEM micrograph (Fig. 2.9(a)) shows a 1 μm -radius Bunimovich cavity[30] that was obtained using the process steps outlined in Sec. 2.2.4. The dark regions are areas of the sample that were etched away after a successful electron beam lithography of the dot region. Clearly visible is the anisotropic deformation of the cavity geometry due to a combination of proximity effect and anisotropic etching.

The SEM micrograph of Fig. 2.9(b) shows a completely different

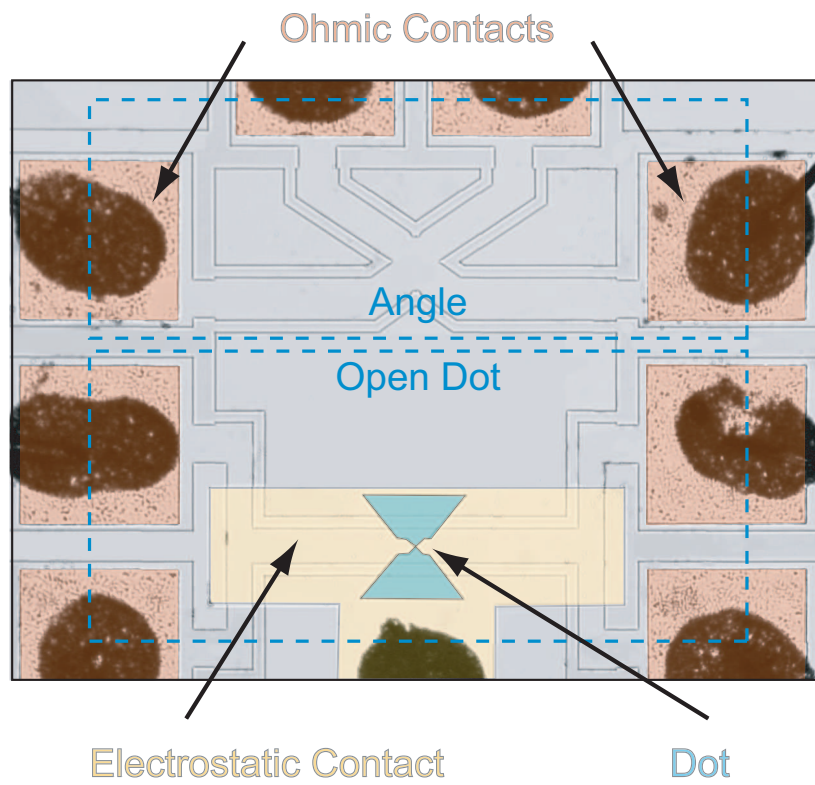


Figure 2.8: View of a completed sample with conductive epoxy bond pads

type of open quantum dot where the lateral electrostatic confinement is obtained by polarizing electrostatic split gates. This technique has the advantage of an independent control of the dot shape, of its area, and of the strength of its coupling to the electron reservoirs (through the width of the dot openings). Unfortunately, the use of individual electrostatic gates, as opposed to a single contact deposited on top of an etched dot, requires extremely “quiet” 2DEGs. Quietness is here a synonym for stability with respect to switching noise. Empirically, switching noise is associated with an electrostatic environment, surrounding the 2DEG, that fluctuates in discrete switching events with a typical time period ranging from a few milliseconds to several hours. The 2DEGs samples used in this work were rather sensitive to switching noise, with timescales of only a few seconds. Of critical importance are switching events that involve particles located in the vicinity of the dot openings. For split gate dots similar to Fig. 2.9(b), this extra sensitivity translates into a noise signal with an amplitude on the order of the magnetoconductance itself when the width of the dot openings is electrostatically controlled with only one transmission mode between the dot and the electron reservoirs. After some unconvincing attempts with electrostatically driven dots, we reverted to etched open dots that turned out to be much more robust with respect to switching events.

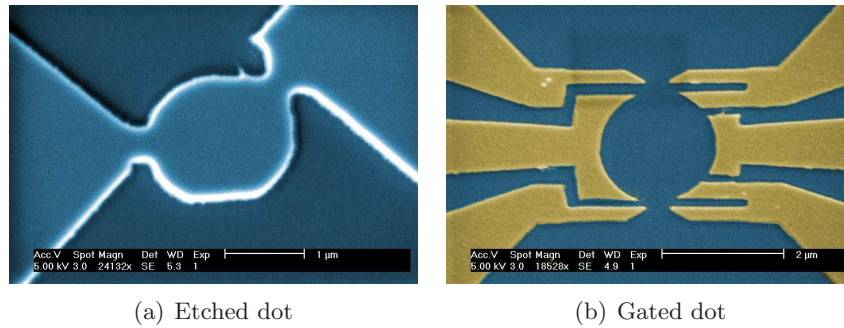


Figure 2.9: SEM micrographs of open quantum dots (a) produced by wet etching, and (b) where the electron confinement is obtained using electrostatic split gates.

2.3 Characterization Techniques

Most of the magnetotransport measurements performed on our open quantum dots took place in a Cryogenics ^3He cryostat with a base temperature of 300 mK. Some additional measurements were collected in

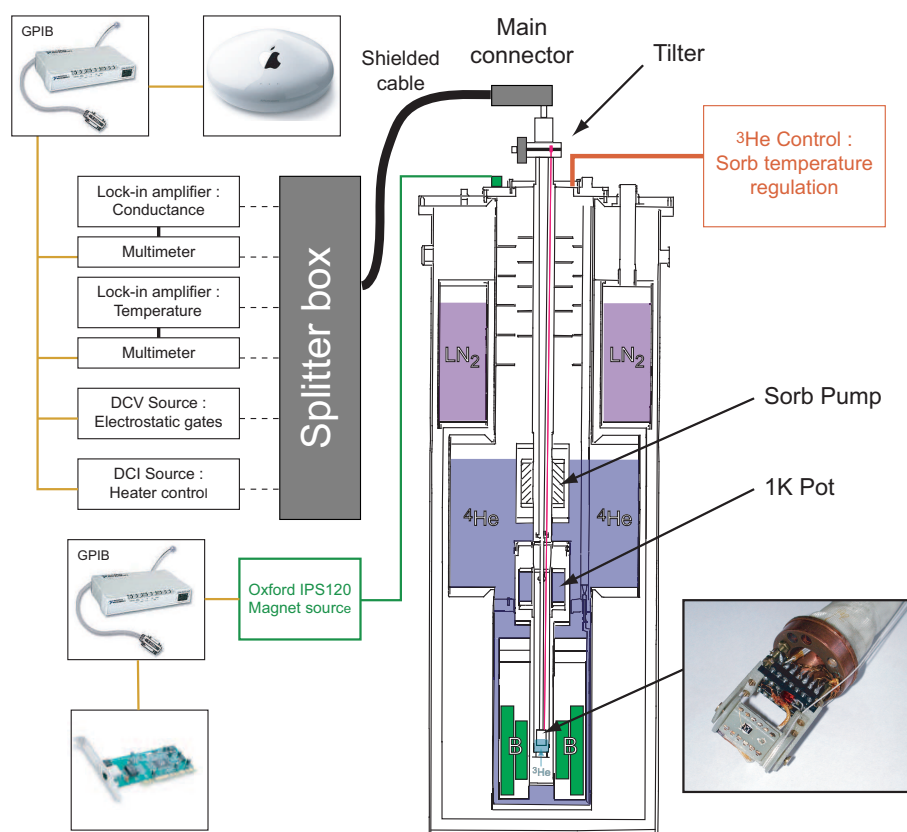
an Oxford Instruments Kelvinox 300 Dilution Refrigerator (commonly called the “Bayotron”) with a base temperature of 30 mK. A schematic view of the complete ^3He measurement systems, including the data acquisition tools, is shown in Fig. 2.10. A superconducting solenoid is inserted in the cryostats and, in its actual configuration, provides a magnetic field with a maximum value of 9 T at 4.2 K. Other solenoids are also available with maximum fields up to 17 T at 2.2 K.

Both cryostats are equipped with a tilting stage (See the closeup of the sample holder in the Inset to Fig. 2.10) which can be rotated *in-situ*. The tilt angle θ is defined as the angle between the applied magnetic field and the normal to the sample (the plane of the 2DEG). The rotating sample holder, made of Stycast epoxy, can accommodate standard non-magnetic, multiple pin (DIP) sockets. While a carefully done wiring of the tilter provides an optimized cooling of the sample, we point out that the sample is not in contact with the cold finger of the mixing chamber in the dilution refrigerator. In the ^3He refrigerator, the device is directly immersed into the ^3He liquid bath.

In the next paragraph, we briefly describe the main working principles of the ^3He cryostat and comment on its thermometry setup. A more complete introduction to the operation of a ^3He or dilution refrigerator can be found in the numerous textbooks devoted to cryogenics[143, 171], or in the course notes of J. P. Issi and V. Bayot.

2.3.1 Cryogenics

The operation of a ^3He cryostat is based on an efficient use of the properties of the ^3He phase diagram, in particular the equilibrium between its vapor and liquid phases. The boiling point of ^3He at ambient pressure is $T = 3.19$ K. As the vapor pressure is lowered below 10^{-3} mbar above the liquid phase, the ^3He bath reaches the temperature of 0.3 K. Unlike the ^4He isotope where a superfluid transition occurs at 2.17 K, the ^3He liquid phase remains normal down to 2.6 mK. Starting from its gaseous form, ^3He is first condensed as it is put in contact with the external surface of a 1K pot. In practice, the 1K pot is a reservoir containing pumped ^4He , giving a temperature of $\sim 1.5 - 2$ K. Once the ^3He has fully condensed, its vapor pressure is lowered. Unlike the 1K pot where the pressure is controlled with a rotary pump, the high market price of ^3He requires a pumping technique where no molecule of the precious gas is lost into the air. This is typically achieved using a “sorb”, which acts as a cryopump by adsorbing the ^3He vapor at its surface, when cooled down from 30 K down to 4.2 K. When no liquid ^3He remains in the sample chamber, the sorb pump temperature can be increased to 30 K

Figure 2.10: Schematic of the ^3He cryostat with the tilter.

to release the adsorbed ^3He particles, starting a new condensation cycle. Depending on the amount of ^3He available in the system, the efficiency of both the sorb pump and condensation step, and the measurement characteristics (magnetic field sweep speed, regulation at $T > 0.3$ K, loading of a new sample), a typical operating cycle can last about 24 hours.

For a precise control of the temperature, a heater and a thermometer are placed in the ^3He chamber. While the sorb and 1K pot temperatures are monitored with a Carbon Glass sensor, a RuO_2 sensor gives the sample temperature. We point out that with the inset currently used in the ^3He cryostat, only the sample is dipped into the ^3He liquid phase while the RuO_2 sensor is placed higher in the sample chamber, in contact with the vapor phase and the copper heat sink, itself in contact with the ^3He bath. As a result, a small difference (of a few percents) between the sample and thermometer temperatures is observed. However, we emphasize that this ΔT is relatively small, on the order of 20 mK at the base temperature of the cryostat. A solution to circumvent this small error in the determination of the sample temperature is to measure the vapor pressure, and to convert its value into the effective temperature of the ^3He bath, using reference tables[171].

2.3.2 Transport Measurements and Data Acquisition

For magnetotransport measurements, we use a standard AC lock-in technique. A low frequency (< 20 Hz) current is injected into one of the ohmic contacts in the Hall bar containing the open quantum dot (Fig. 2.8, lower part) and collected in the opposite contact. The two other ohmic contacts in the Hall bar serve as voltage probes where an AC signal is measured at the same frequency as the input current. Beside the bandpass filtering capabilities of the lock-in amplifiers, further noise reduction is obtained by means of 5000 pF π -filters attached to the wiring systems at the top of the cryostat.

The magnetic field is produced by applying a current through the 4.2 K superconducting solenoid. An Oxford IPS120 magnet source controls both the target field and sweeping rate. In order to fully resolve the rich spectrum of conductance fluctuations measured in open quantum dots, very small sweep speed down to 0.1 Gauss/s are used for the most demanding transport measurements. Because sweeping the magnetic field induces a small hysteresis cycle, the exact value of the applied magnetic field is extracted from the transverse magnetoresistance measured on the second Hall bar (Fig. 2.8, upper part). More details concerning the conversion from resistance to magnetic field will

be given in Chapter 4, along with a thorough discussion in Chapter 6 of the evaluation of the tilt angle, using the same transverse measurements on the adjacent Hall bar.

The magnet source, the various multimeters, current and voltage sources required for an complete analysis of the transport properties of an open dot sample, cooled down to 0.3 K in the ^3He cryostat, are all driven by two GPIB controllers connected to a workstation. The first controller, attached to the magnet source, communicates wirelessly with the computer, while a more classical ethernet cable connects the second controller to the workstation. Based on a previous program created by V. Bayot, a complete and highly flexible data acquisition framework, called NanoFreeze, was developed for the Wavemetrics Igor Pro software. A screenshot of a typical acquisition session is shown in Fig. 2.11. The user and programming manuals for NanoFreeze are available elsewhere.

2.4 Summary

The successful fabrication of mesoscopic devices, in particular open quantum dots, strongly depends on the aptitude of the researcher to find a good balance between proven process recipes, new techniques, a good knowledge of the materials and instruments involved, ... combined with a reasonable amount of luck. In this chapter, we discussed the various processing steps (deep and shallow etches, ohmic contacts, electrostatic gates,...) that were used for our open quantum dots, putting much emphasis on the electron beam lithography technique. After a description of the cryogenic systems, we also introduced the measurement and data acquisition methods relevant to an analysis of the transport properties of ballistic dots. Sample specific details related to particular measurement techniques will be given later in the text as we study the behavior of open dots subject to a parallel (Chap. 4) or tilted (Chap. 6) magnetic field.

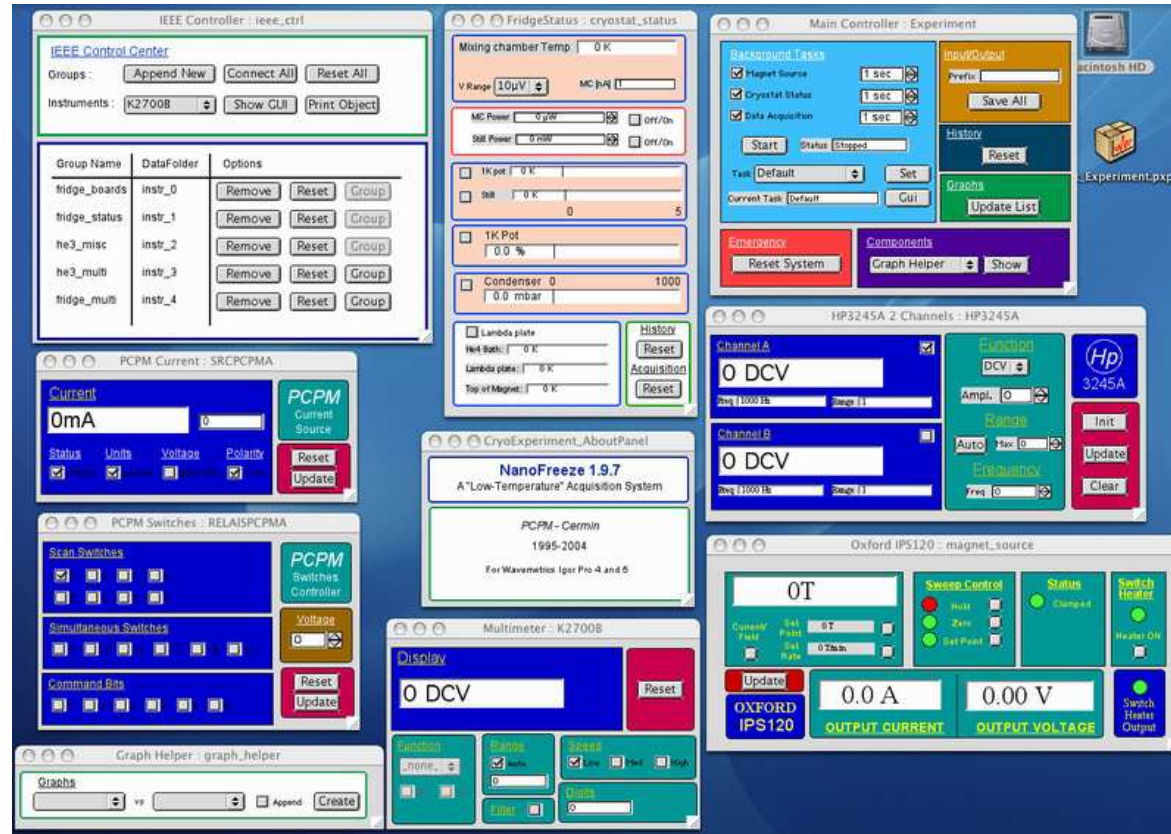


Figure 2.11: Screenshot of the NanoFreeze Data Acquisition program, running under Wavemetrics Igor 5 (MacOS X Edition).

Chapter 3

Elements of Quantum Transport in Open Quantum Dots

Hang on for some metaphysics. The Aneristic Principle is that of ORDER, the Eristic Principle is that of DISORDER. On the surface, the Universe seems (to the ignorant) to be ordered; this is the ANERISTIC ILLUSION. Actually, what order is “there” is imposed on primal chaos in the same sense that a person’s name is draped over its actual self. It is the job of the scientist to implement this principle in a practical manner, and some are quite brilliant at it. But on closer examination, order dissolves into disorder, which is the ERISTIC ILLUSION.

Malaclypse the Younger
PRINCIPIA DISCORDIA (1980)

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3.1 Introduction

In the common language, the word “Chaos” is usually associated to the notions of “disorder” and “complexity”, what the Bible calls the “original chaos”. Chaos can describe at the same time a spatial layout, like a pile of rocks or the coastline of a country, or an immaterial topic in the social, economical or political domains, the most famous example being the stock markets. In Physics, the “Chaos” terminology was usually applied to a phenomenon that does not follow any known rule or equation, and therefore, is unpredictable. For quite some time, the “chaotic” and “random” adjectives were equivalent in most people’s mind.

Since the seminal works of H. Poincaré, B. Mandelbrot or M. Feigenbaum¹, the exact meaning of “Chaos” has dramatically changed in the scientific or scientifically knowledgeable community. It now describes the behavior of certain nonlinear dynamical systems that (under certain conditions) exhibit some phenomena characterized by sensitivity to initial conditions. Examples of such systems include turbulences in fluid flows (the atmosphere, aerodynamics of planes, water flow through pipes), the orbital motion of planets, instabilities of chemical reactions, in neural networks or in population growths.

In what is now commonly called “Mathematical Chaos”, chaotic systems are deterministic and thus orderly in some sense. Instabilities arise from non-linearities either inherent to the equation of motion, or from boundary conditions in otherwise linear systems. A famous example is the Lorentz attractor arising from the fully deterministic system of

¹For a review, see Refs. [120] and [72].

differential equations

$$\frac{dx}{dt} = \alpha(y - x) \quad (3.1)$$

$$\frac{dy}{dt} = x(\beta - z) - y \quad (3.2)$$

$$\frac{dz}{dt} = xy - \gamma z \quad (3.3)$$

where $\alpha = 10$, $\beta = 27$ and $\gamma = 8/3$. Here, x , y and z are the cartesian coordinates in 3D space, and t is the time. A projection of the “Lorentz strange attractor”, solution to Eq. 3.1, is presented in Fig. 3.1 for $0 < t < 50$ s. In this particular case of deterministic systems, the transition to chaos occurs through sporadic events, *i.e.* by means of a dynamic regime that remains periodic most of the time, but suddenly moves out of stability for a short period of time before returning to a stable behavior. Another scenario for a transition to chaos is that of quasi-periodicity: through a continuous, external parameter, a periodic regime can smoothly become quasi-periodic, then can lose its stability and become chaotic.

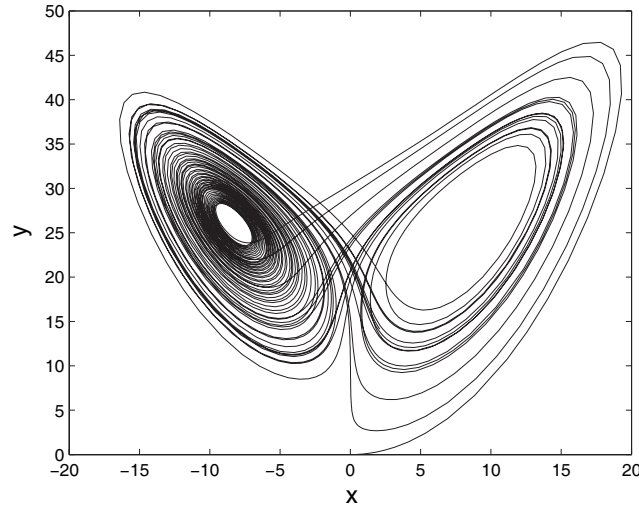


Figure 3.1: Projection of the Lorentz strange attractor onto the x - y plane. The investigated time range is $0 < t < 50$ s.

Quantum chaotic systems definitively belong to this second category. For quite some years now, pioneering works have raised the issue of whether chaotic dynamics, as observed in the classical world,

has its counterpart in quantum systems. More precisely, if Quantum Chaos refers to the quantum signature of systems which classically exhibit chaotic dynamics, can such a regime have experimentally observable consequences ? And if the answer is yes, what tools are available to describe this regime ?

It has been found that the phenomenon of quantum chaos can manifest itself in many different physical systems, among others the level repulsion in nuclei, wave properties in microwave cavities or, as we saw in Chap. 1, in GaAs-Al_xGa_{1-x}As microstructures such as open quantum dots. Interestingly, the available theoretical tools are not peculiar to a given system, but can be successfully applied (with minor changes) to a wide range of quantum chaotic systems or, to be more precise, to systems featuring signatures of quantum chaos under precise circumstances.

The primary purpose of this chapter is to give a gentle introduction to these theoretical tools, using open quantum dots (OQDs) as their main target. To understand the origin and provide a quantitative description of the conductance fluctuations observed in OQDs, various approaches have been used, all of them starting from the Schrödinger equation, and involving a statistical analysis of electron trajectories and wave scattering in the system. Each model yields its own set of useful results, with frequent overlaps with conclusions from other models. In the course of this thesis, it will be sometimes necessary to invoke two models to explain some experimental observations. It would be a bit of an overkill to fully describe these models. In Sec. 3.2, we try to give the necessary elements for the reader to take advantage of the most recent theoretical advances available in the literature, whose main references will be used in Chapters 4 and 6.

One of most common method for a description of quantum chaos in OQDs relies on the semi-classical approach[95, 13], where the conductance is calculated from the contribution of all classical trajectories connecting the leads to the dot. After a general description of the theory of the scattering matrix in Sec. 3.2.1, we describe the main results of the semi-classical theory in Sec. 3.2.2. Random Matrix Theory[16, 7] is covered in Sec. 3.2.3, including a discussion of dephasing in OQDs and of the effect of thermal smearing on the conductance fluctuations. Supersymmetry[58] is briefly introduced in Sec. 3.2.5, while Sec. 3.2.7 is devoted to Diagrammatic Perturbation Theory[129, 47].

In the second part of this chapter, we theoretically discuss some effects of a magnetic field on the static properties of a two-dimensional electron gas (2DEG), in particular the concept of transverse magnetic modes in two-dimensional electron gases (2DEGs), that will appear frequently in this thesis. Starting in Sec. 3.3.1 with Landau levels for un-

confined electrons, the orbital effect arising from the interplay between magnetic and electrostatic confinement is introduced in Sec. 3.3.2. We close this chapter with a discussion of spin-orbit coupling in Sec. 3.3.3.

3.2 Models of Transport through Quantum Dots

Central to the various models of quantum transport in mesoscopic systems is the scattering theory where the conductance is calculated from the transmission probabilities of electrons across a ballistic device. Figure 3.2 shows the geometry of a ballistic conductor of width W and length L , connected to electron reservoirs through leads with 2 and 1 transmission modes in the left handside and right handside contacts, respectively. If the dimensions of the conductor were large, the conductance would be given by Ohm's law $g = \sigma W/L$, where the conductivity σ does not depend on the sample dimensions but is material specific. However, as we go to smaller dimensions, the conductance becomes independent of both W and L , and is only a function of the number of transverse modes M in the conductor, *i.e.* the Landauer formula[109] reads

$$g = \frac{I}{(\mu_1 - \mu_2)/e} = \frac{2e^2}{h} MT, \quad (3.4)$$

where I is the current flowing through the sample. Here, μ_1 and μ_2 are the quasi-Fermi levels in contact 1 and 2, respectively, and T is the average probability for an electron injected in contact 1 to be transmitted to contact 2.

Equation 3.4 is strictly valid in the ballistic regime, *i.e.* when the size of the conductor is much smaller than the momentum relaxation length (see Sec. 1.2). Additionally, if transport is coherent (the sample size is smaller than the phase coherence length), it is possible to calculate the transmission function T from the Schrödinger equation using the concept of the scattering matrix, or S-matrix.

3.2.1 Scattering Matrices and Green's Functions

The concept of scattering matrices (see Fisher *et al.*[66] and references therein) is commonly used in the theory of electron transport in mesoscopic systems. Here, we basically follow the outline of Chapter 3 in Datta[47] to succinctly introduce the theory of scattering matrices.

For a ballistic and coherent conductor, a scattering matrix S connects the amplitudes of the incoming and outgoing waves passing through the leads. From Fig. 3.2, if a total of three modes contribute to the current,

S is a 3×3 matrix with

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}. \quad (3.5)$$

Generalizing the problem from 3 to N modes in each lead, S is a $2N \times 2N$ matrix which has the block structure

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}, \quad (3.6)$$

where r and r' are $N \times N$ reflection matrices (for the left and right contacts, resp.), and t and t' are transmission matrices (from left to right and right to left, resp.).

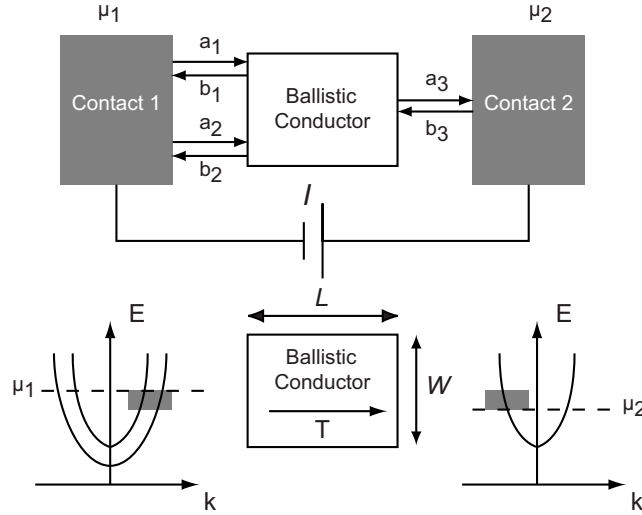


Figure 3.2: Schematic of a ballistic conductor connected to two electron reservoirs. The left handside reservoir is in equilibrium at the Quasi-Fermi level μ_1 , while the second reservoir is at energy μ_2 .

The transmission probability from mode m to mode n is given by the squared amplitude of the corresponding element in the S matrix:

$$T_{m \rightarrow n} = |s_{mn}|^2. \quad (3.7)$$

Macroscopically, the transmission between lead p and q is obtained by summing the transmission probability $T_{m \rightarrow n}$ over all modes m in lead p

and all modes n in lead q :

$$\hat{T}_{p \rightarrow q} = \sum_{m \in p} \sum_{n \in q} T_{m \rightarrow n}. \quad (3.8)$$

Because of current conservation through the conductor, the S matrix is unitary

$$S^\dagger S = I = S S^\dagger, \quad (3.9)$$

where the $\dagger$ superscript denotes the conjugate transpose of matrix S , and I is the identity matrix.

For a conductor with a given geometry, the S -matrix can be readily obtained from the Schrödinger equation

$$\left[\frac{(i\hbar\nabla + e\mathbf{A})^2}{2m^*} + U(\mathbf{r}) \right] \Psi_i(\mathbf{r}) = \mathcal{E}_i \Psi_i(\mathbf{r}), \quad (3.10)$$

where $\mathbf{A}$ is the vector potential and U the potential energy. Here, $\mathcal{E}_i$ and $\Psi_i(\mathbf{r})$ are the eigenenergies and eigenvectors of the Hamiltonian, respectively. While it is possible to evaluate S -matrices using Eq. 3.10 on homogenous subsections of the conductor, and subsequently combine them with full or partial coherence, the preferred method relies on the powerful concept of Green's functions (for a review of the Green's function formalism, see for example the standard texts by Inkson[93] and Economou[57]).

Commonly used in many physical fields like signal theory and electromagnetics, a Green's function in the context of the Schrödinger equation is basically the wavefunction at position $\mathbf{r}$ resulting from a point-like excitation applied at position $\mathbf{r}'$. More formally, the Green's function $G(\mathbf{r}, \mathbf{r}')$ is defined by rewriting Eq. 3.10 as

$$\left[E - \frac{(i\hbar\nabla + e\mathbf{A})^2}{2m^*} - U(\mathbf{r}) \right] G(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}'), \quad (3.11)$$

where $\delta(\mathbf{r})$ is the delta function.

In the simple case of a constant potential energy U_0 , the two solutions G^R and G^A to Eq. 3.11 are given by plane waves with

$$\begin{aligned} G^R(\mathbf{r}, \mathbf{r}') &= -\frac{i}{\hbar\nu} \exp(ik|\mathbf{r} - \mathbf{r}'|) \\ G^A(\mathbf{r}, \mathbf{r}') &= +\frac{i}{\hbar\nu} \exp(-ik|\mathbf{r} - \mathbf{r}'|) \end{aligned} \quad (3.12)$$

where

$$k = \frac{\sqrt{2m^*(E - U_0)}}{\hbar} \quad \text{and} \quad \nu = \frac{\hbar k}{m^*}. \quad (3.13)$$

Here, G^R is the retarded Green's function and corresponds to an outgoing wave coming from the excitation point $\mathbf{r}'$. On the opposite, the advanced Green's function G^A is an incoming wave disappearing at $\mathbf{r}'$.

The main advantage of using Green's functions instead of S -matrices lies in the way boundary conditions can be taken into account. A common method is to use an imaginary energy term η (with $\eta > 0$) in Eq. 3.11 that now reads

$$\left[E - \frac{(i\hbar\nabla + e\mathbf{A})^2}{2m^*} - U(\mathbf{r}) + i\eta \right] G(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}'). \quad (3.14)$$

The imaginary energy η makes the retarded Green's function G^R the only acceptable solution to Eq. 3.14, as G should decrease in amplitude when one moves away from the excitation point $\mathbf{r}'$.

There is of course a direct connection between the Green's function and S -matrix formalisms. The Fisher-Lee relation connects the scattering element s_{pq} between lead p and lead q to the retarded Green's function G_{pq}^R and reads

$$s_{pq} = \delta_{pq} + i\hbar\sqrt{\nu_p\nu_q} G_{pq}^R, \quad (3.15)$$

where ν_p, ν_q are the electron velocities in lead p and q given by Eq. 3.13, and G_{pq}^R is evaluated at points located in the two leads.

There is however a major problem with Eq. 3.14 and the way boundary conditions at the conductor leads are replaced by the imaginary term η . Because the leads are electron reservoirs that extend semi-infinitively in space, the retarded Green's function is a matrix with infinite dimension. The way to solve this issue is to partition the full Green's function into sub-matrices

$$G^R = \begin{bmatrix} G_p^R & G_{pC}^R \\ G_{Cp}^R & G_C^R \end{bmatrix} \quad (3.16)$$

where G_p^R is the Green's function of the isolated lead, G_C^R of the isolated conductor, while G_{pC}^R and G_{Cp}^R are the contributions of the coupling between lead p and the conductor. It can be shown[47] that the contributions of lead p to the total Green's function can be merged into an energy term Σ^R , called the self-energy, in such a way that the retarded

Green's function G_C^R of the conductor alone becomes a solution of the finite-dimension system

$$G_C^R = [EI - H_C - \Sigma^R]^{-1}, \quad (3.17)$$

where I is the identity matrix and H_C is the Hamiltonian describing the isolated conductor.

Going one step further, it is possible to rewrite Eq. 3.17 in a slightly different form. If G_0 is the Green's function that corresponds to the unperturbed, isolated system characterized by the Hamiltonian H_C , one has

$$\begin{aligned} G^R &= (G_0^{-1} - \alpha)^{-1} = (I - G_0\alpha)^{-1} G_0 \\ &= G_0 + G_0\alpha G_0 + G_0\alpha G_0\alpha G_0 + \cdots, \end{aligned} \quad (3.18)$$

where one made use of the convergence point of an infinite geometric series. The C subscript to the Green's function G_C was also dropped, as it is clear that calculations are performed in the conductor only.

Recursively, Eq. 3.18 can be rewritten as

$$G_{i+1}^R = G_0 (1 + \alpha G_i^R), \quad (3.19)$$

where G_{i+1}^R and G_i^R are the retarded Green's functions at steps $i+1$ and i of a recursive process starting with the unperturbed Green's function G_0 . Eq. 3.19 is commonly called the Dyson Equation. In such a framework, α is a matrix composed of hopping elements connecting nearest neighbors

$$\alpha_{ij} \propto \begin{cases} \exp [i\mathbf{A}(\mathbf{r}_i - \mathbf{r}_j)/\hbar] & \text{if } i \text{ and } j \text{ are adjacent points,} \\ 0 & \text{otherwise,} \end{cases} \quad (3.20)$$

where $\mathbf{A}$ is the strength of the coupling.

Practically, in a fully quantum mechanical calculation of the conductance through a structure, the Hamiltonian H_C is solved using a standard tight-binding method, with G_R computed recursively from Eq. 3.19 on adjacent subdomains of the conductor. Once G^R has been found, the transmission functions can be readily obtained using Eqs. 3.15 and 3.8. The conductance is then deduced from the Landauer Formula (Eqs. 3.4, 3.7 and 3.8).

3.2.2 Semi-classical Approach

Beside a fully quantum mechanical calculation, another method for evaluating transport properties is based on an approximation of the conductance by a sum of all classical trajectories connecting the sample

leads, each of them carrying the quantum mechanical phase. In the semi-classical approach (see Jalabert *et al.*[95], Baranger *et al.*[13] and references therein) applied to an open quantum dot subject to a perpendicular magnetic field B , the Green's function of Eq. 3.17 is replaced by its semi-classical limit

$$G^{SC}(\mathbf{r}, \mathbf{r}', k_F) = \frac{2\pi}{(2\pi i \hbar)^{3/2}} \sum_s \sqrt{|D_s(\mathbf{r}, \mathbf{r}', k_F)|} \exp\left(\frac{i}{\hbar} S_s(\mathbf{r}, \mathbf{r}', k_F) - i\mu_s \frac{\pi}{2}\right), \quad (3.21)$$

where

$$S_s(\mathbf{r}, \mathbf{r}', k_F) = \hbar \left(k_F l_s + \frac{2\pi A_s B}{\phi_0} \right), \quad (3.22)$$

is the classical action of an electron trajectory of length l_s , connecting the $\mathbf{r}$ and $\mathbf{r}'$ points and enclosing the area A_s . Here, k_F is the Fermi wave vector, $\phi_0 = h/e$ is the flux quantum, μ_s is the Maslov index², and $D_s(\mathbf{r}, \mathbf{r}', k_F)$ is the density of trajectories belonging to an ensemble of traces connecting the dot entrance and exit leads with the same number of bounces[76].

A thorough description of conductance calculation in the semi-classical framework is beyond the scope of this thesis. Instead, we concentrate on the main results obtained by the semi-classical theory for the statistics of conductance fluctuations in open quantum dots.

In the case of a chaotic ballistic dot with strong coupling to the electron reservoirs through entrance and exit openings, most electrons entering the cavity will escape within a single characteristic time, function of the time it takes to cross the dot ballistically. Classically, transport is ergodic as all electron trajectories are scrambled whatever the initial conditions, especially the distribution of the momentum at the dot openings. This gives rise to an exponentially decreasing probability distribution for an electron to remain in the dot for at least a time t

$$P(t) \propto e^{-\tau_d^{-1}t}, \quad (3.23)$$

for sufficiently long times. Here, τ_d is here the dwell time. The corresponding distribution of signed area enclosed by the electron trajectories has the form

$$P(A_s) \propto e^{-2\pi\alpha|A_s|}, \quad (3.24)$$

²The Maslov index is the number of sign changes in the Jacobian along a given trajectory.

where $\alpha = \sqrt{2\kappa N}$ is a characteristic inverse area, and κ is a geometry-specific factor.

Central to the analysis of experimental manifestations of chaos in OQDs (conductance fluctuations, weak localization correction) is the conductance autocorrelation which is a measure of the sensitivity of coherent interferences in the dot with respect to an external parameter:

$$C(\Delta X) = \langle \delta g(X + \Delta X) \delta g(X) \rangle_X, \quad (3.25)$$

where $\delta g(X) = g(X) - \langle g \rangle(B)$. Here, $\langle f(X) \rangle_X$ is the average of function f performed with respect to the variable X . On the opposite, $\langle f \rangle(X)$ is the average of a set of independent $f(X)$ curves. For example, in the case of the magnetoconductance $g(B)$, $\langle g \rangle(B)$ can be obtained by averaging $g(B)$ over ensembles of dot shapes, created by changing the voltage applied to electrostatic gates.

If B is the trajectory probing parameter, semi-classical theory yields the following correlation function at zero temperature ($T = 0$) for a chaotic quantum dot connected to electron reservoirs through N transmission modes

$$C(B) = \frac{1}{4} \frac{1}{\left[\left(\frac{B}{B_c} \right)^2 + 1 \right]^2}, \quad (3.26)$$

where B_c is a characteristic field. By Fourier transforming this Lorentzian-squared autocorrelation function, one obtains the power spectral density of conductance fluctuations:

$$S_B(f_B) = S_B(0) (1 + 2\pi B_c |f_B|) e^{-2\pi B_c |f_B|}. \quad (3.27)$$

Even though our work will be mainly focused on magnetoconductance fluctuations in the regime where time-reversal symmetry is already broken, it is worth mentioning another important result of semi-classical theory. Similar to the correlation function of Eq. 3.26, an exponentially decreasing distribution of areas, as in Eq. 3.24 for chaotic classical dynamics, gives rise to a weak localization correction to the magnetoconductance[14] with a Lorentzian line shape

$$\langle g \rangle(B) = \langle g \rangle(B \neq 0) - \frac{\langle g \rangle(B \neq 0) - \langle g \rangle(B = 0)}{1 + (2B/B_c)^2}. \quad (3.28)$$

The validity of Eq. 3.28 has been verified[37] in magnetoconductance measurements performed in OQDs with various shapes. Striking differences were found in the shape of the weak localization peak for transport through chaotic cavities (stadium with a Lorentzian line shape) and

non-chaotic dots (circle with an unusual triangular line shape). However, while the experiment of Ref. [37] tends to support the relevancy of the semi-classical approach in the treatment of transport through OQD, some authors have shown[23] that the use of the semi-classical approach can lead to contradictory results when compared to the dot magnetoconductance obtained through quantum mechanical calculations (Eq. 3.17). More precisely, the semi-classical approximation of Eq. 3.21 does not guarantee the unitarity of the scattering matrix required by current conservation (Eq. 3.9). As a result, both the weak localization (WL) correction and autocorrelation function can be strongly sensitive to geometry specific, non-universal features of the system. They can also bear signatures of a mixed regime of transport with longer trajectories in the device that are chaotic, intermixed with short regular trajectories that can sometimes dominate the WL response or conductance fluctuations[103, 144, 3, 21, 131].

In spite of these objections, it has been found in numerous experiments[122, 77, 42] that the predictions of semi-classical theory, in particular the power spectrum of conductance fluctuations (Eq. 3.27), are generally in good agreement with experimental data sets, and can yield some important information on the strength of dephasing in the dot, as it will be discussed later in this chapter.

3.2.3 Random Matrix Theory

Another approach to transport in mesoscopic systems is Random Matrix Theory (RMT) which was originally developed for the modeling of complex nuclear scattering[125]. There exist many excellent reviews of RMT, the most relevant one for quantum transport in OQD being the monographs by Beenakker[16] and Alhassid[7] combined with some seminal articles[27, 28] by Brouwer and co-workers. However, as many of the theoretical results needed for this thesis originate from RMT, in particular recent developments on WL and conductance fluctuations in parallel magnetic fields[45], we will give here a short introduction to the calculation process along with its main conclusions.

The central idea in RMT lies in the universality of the statistical description of some quantum transport properties in mesoscopic systems. For open quantum dots with ergodic properties similar to those required in the semi-classical approach (Sec. 3.2.2), the conductance can be described statically without any specific details on the dot geometry. In the scattering matrix approach of Sec. 3.2.1, the dot is thus considered as a black box, and the transmission function $\hat{T}$ to be used in Landauer formula (Eq. 3.4) can be readily obtained from Eq. 3.8 using a random

ensemble of S matrices obeying some required symmetries.

In Sec. 3.2.1, the S -matrix was described in Eq. 3.6 using a block structure composed of reflection and transmission matrices from and to the dot opening leads. An altogether different representation of the $2N \times 2N$ S -matrix is its eigenvalue-eigenvector decomposition

$$S = \Omega \text{diag} \left(e^{i\phi_1}, e^{i\phi_2}, \dots, e^{i\phi_{2N}} \right) \Omega^\dagger, \quad (3.29)$$

where the real numbers ϕ_n are the scattering phase shifts, and Ω is the eigenvector matrix of dimension $2N \times 2N$. It was found[24] that the correlations of the phase shifts ϕ_n in the case of chaotic scattering can be described by the distribution function

$$P([\phi_n]) = \prod_{n < m} |\exp(i\phi_n - i\phi_m)|^\beta, \quad (3.30)$$

where β is the symmetry index that counts the number of degree of freedom in the matrix elements of the the eigenvector matrix Ω . Its value can be $\beta = 1, 2$ or 4 . Physically, $\beta = 1$ applies to a system where both time-reversal symmetry (TRS) and the electron spin are conserved. $\beta = 2$ corresponds to a system where time-reversal symmetry is broken, and $\beta = 4$ if spin-rotation symmetry (SRS) is also broken (see Sec. 3.3.3 for a discussion of spin-orbit coupling). The eigenvector matrix Ω is orthogonal, unitary or symplectic for $\beta = 1, 2$ or 4 , respectively. The corresponding S -matrix is symmetric, unitary and self-dual, respectively.

To $P([\phi_n])$ in Eq. 3.30 corresponds an ensemble of scattering matrices that are *uniformly* distributed according to a “measure” $d\mu(S)$. Because the eigenvalues ϕ_n are distributed according to Eq. 3.30 with the eigenvalue density on the unit circle $e^{i\phi_n}$, the ensemble of S -matrices is called “circular” with the addition of the adjective orthogonal, unitary and symplectic for $\beta = 1, 2$ or 4 . For example, the circular orthogonal ensemble (COE), associated to $\beta = 1$, is the set of uniformly distributed, unitary and symmetric S -matrices.

Depending on the symmetries in the system, it is thus sufficient to select an ensemble of N scattering matrices (with N large enough: $N \rightarrow \infty$), with some properties set by the symmetry index β , in order to calculate the statistics of some macroscopic transport values, like moments of the conductance. The exact calculation of the first two moments, the average conductance $\langle g \rangle_B$ and the conductance variance $\text{var}(g)$, in the circular orthogonal (COE, $\beta = 1$) and unitary (CUE, $\beta = 2$) ensembles can be found in the literature[16]. The variance of conductance fluctuations, the square of their standard deviation $\sigma(g)$, is defined as

$$\text{var}(g) = \sigma^2(g) = \langle \delta g^2 \rangle_B - \langle \delta g \rangle_B^2. \quad (3.31)$$

	TRS	SRS	Type	$\langle g \rangle_B [e^2/h]$	$\text{var}(g) [e^4/h^2]$
$\beta = 1$	yes	yes	COE	$\frac{2N}{2N+1}$	$\frac{4N(N+1)^2}{(2N+1)^2(2N+3)}$
$\beta = 2$	no	yes	CUE	N	$\frac{N^2}{4N^2-1}$

Table 3.1: Values of the first two conductance moments $\langle g \rangle$ and $\text{var}(g)$ obtained by RMT, along with the symmetry properties of the scattering and eigenvector matrices. Here, TRS stands for time-reversal symmetry, SRS for spin-rotation symmetry, and N is the number of transmission modes in the two dot openings.

Table 3.1 summarizes the values of $\langle g \rangle_B$ and $\text{var}(g)$ obtained by RMT, along with the symmetry properties of the scattering matrices. N is the number of transmission modes in the two dot openings.

3.2.4 Dephasing

Dephasing is the loss of quantum coherence. On average, after a period of time called the phase-relaxation time τ_φ , introduced in Sec. 1.2, electrons lose the “memory” of the phase they carry along their trajectories in the dot, and do not contribute to interferences anymore. To take into account dephasing in models of transport through mesoscopic devices, a most widespread method is to append a fictitious voltage probe[32], called “ ϕ -lead”, to the structure. The time scale that characterizes dephasing is the phase relaxation time τ_φ and is related to the number of phase-breaking modes in the ϕ -lead by the relation

$$N_\varphi = \frac{2\pi\hbar}{\tau_\varphi\Delta}, \quad (3.32)$$

where Δ is the mean level spacing given by

$$\Delta = \frac{2\pi\hbar^2}{m^*A}. \quad (3.33)$$

The addition of the fictitious voltage probe to the scattering matrices[27, 28] modifies the expressions of the weak localization correction $\delta g^{\text{WL}} = \langle g \rangle_{B=0} - \langle g \rangle_{B \neq 0}$ and conductance variance $\text{var}(g)$ given by RMT

in Table 3.1. The weak localization correction becomes

$$\delta g^{\text{WL}} = \frac{e^2}{h} \frac{N}{2N + N_\varphi} \quad \text{for } N \geq 2. \quad (3.34)$$

The accuracy of Eq. 3.34 improves as N becomes larger than 2. Moreover, if N_φ is on the order of, or much greater than N , δg will significantly decrease with an increasing dot area ($N_\varphi \propto A$).

Taking into account dephasing, the variance of conductance fluctuations in the regime of broken time-reversal symmetry ($\beta = 2$) is given by the expression[27, 28]

$$\text{var}(g) = \frac{e^4}{h^2} \frac{N^2}{\left(\sqrt{4N^2 - 1} + N_\varphi\right)^2} \quad \text{for } N \geq 2. \quad (3.35)$$

There are many dephasing mechanisms that are relevant to transport in open quantum dots. The most important ones are now rapidly discussed.

3.2.4.1 Electron-electron interaction

In semiconductors, there are many types of interactions between quasi-particles. At high temperature, transport is dominated by interactions between the electrons and excited modes of the lattice. However, as the temperature decreases, electron-phonon scattering rates become small compared to electron-electron interaction, where an electron is scattered by the fluctuating potential that it feels due to the other electrons.

In the absence of magnetic impurities, two types of inelastic electron-electron interaction play a role in dephasing. The first one has its origins in strongly inelastic collisions where the energy transfer is typically on the order of $k_B T$, where k_B is Boltzmann's constant, and T is the temperature. Such a high energetic collision can induce a 2π phase shift for an electron, causing a dephasing rate proportional to T^2 . More quantitatively, in a 2DEG, the dephasing rate is given by the expression[177]

$$\tau_{ee}^{-1} = \frac{\pi}{4} \frac{(k_B T)^2}{\hbar E_F} \ln \frac{E_F}{k_B T}, \quad (3.36)$$

for $k_B T \ll E_F$, where E_F is the Fermi energy.

3.2.4.2 Nyquist noise

The second mechanism of electron-electron interaction involves low-energy, quasi-elastic collisions and is only relevant for two-dimensional

(or lower) structures. Because of their low energy, many collisions are required to shift the electron phase by 2π . In a *disordered* 2DEG, the so-called Nyquist dephasing rate[10] is

$$\tau_{\varphi N}^{-1} = \frac{k_B T}{2\pi\hbar} \lambda_F l_e \ln \frac{\pi l_e}{\lambda_F}, \quad (3.37)$$

where λ_F is the Fermi wavelength and l_e is the elastic mean free path.

The phase relaxation times τ_{ee} and $\tau_{\varphi N}$ coming from inelastic e-e interactions and Nyquist noise, resp., are usually combined into a single τ_φ using Matthiesen's law

$$\tau_\varphi^{-1} \approx \tau_{ee}^{-1} + \tau_{\varphi N}^{-1}. \quad (3.38)$$

3.2.4.3 High frequency radiations

Altshuler *et al.* have addressed[10] the effect of the high frequency electric fields to dephasing in semiconductor structures. It was shown that the phase relaxation time is inversely proportional to f^4 , where f is the signal frequency. However, there are experimental evidences[90] that high frequency radiations are not the dominant player for dephasing in OQD as their impact on τ_φ is masked by other effects like electron heating and thermal broadening (see the discussion in Sec. 3.2.6).

3.2.5 Supersymmetry

The supersymmetry method is a set of techniques that provide a way of calculating ensemble average over products of Green's functions. Supersymmetry was originally introduced for disordered systems by Efetov[58]. It was also used extensively for open quantum dots, and provides an independent confirmation of the major results obtained by Random Matrix Theory or with the semi-classical method. Among other interesting outcomes is an expression[58] of the conductance autocorrelation function, as defined in Eq. 3.25, for both magnetic field B and energy E :

$$C_0(B, E) = \frac{1}{4} \frac{1}{\left[\left(\frac{B}{B_c} \right)^2 + \frac{N_\varphi}{2N} + 1 \right]^2 + \left(\frac{\pi E}{N\Delta} \right)^2}, \quad (3.39)$$

where E and B are obviously correlated ($C_0(B, E) \neq C_0(E)C_0(B)$).

In the absence of dephasing $N_\varphi = 0$, the magnetic field autocorrelation function $C_0(B, E = 0)$ is identical to Eq. 3.26 obtained semi-classically. In the case of an energy dependence and in the absence of

dephasing, the autocorrelation function is still a Lorentzian with

$$C_0(B=0, E) = \frac{1}{4} \frac{1}{1 + \left(\frac{\pi E}{N\Delta}\right)^2}, \quad (3.40)$$

and by Fourier transforming C_0 , the energy power spectrum reads

$$S_E(f_E) = S_E(0)e^{-2\pi E_S|f_E|}, \quad (3.41)$$

where E_S is a characteristic energy.

3.2.6 Thermal Smearing

The expressions of δg and $\text{var}(g)$ obtained in the framework of RMT and given by Eqs. 3.34 and 3.35 are only valid for zero temperature, where only the electrons located at the Fermi energy E_F contribute to transport. In real experiments though, additional carriers spread out in energy by a few $k_B T$'s around E_F can contribute to the total current (physically this translates into the condition $k_B T \gg \Delta$). More precisely, the conductance must be expressed as a weighted average of the conductances at all energies

$$g = - \int f'(E)g(E)dE, \quad (3.42)$$

where f' is the first-order derivative of the Fermi-Dirac distribution

$$f' = \frac{d}{dE} \frac{1}{1 + \exp\left(\frac{E-E_F}{k_B T}\right)}. \quad (3.43)$$

Similarly, the variance of conductance fluctuations is also affected by thermal smearing[59]. In a general way, at temperature T , the correlation function $C(B, E, T)$ of Eq. 3.39, given by supersymmetry[58] for the $T = 0$ magnetoconductance $g(B, E)$, requires a triple integration in energy space

$$C_0(B, E, T) = \int \int f'(E_1)g(B, E' - E_1)dE_1 \int f'(E_2)g(B, E' + E - E_2)dE_1 dE'. \quad (3.44)$$

For the variance of conductance fluctuations, this translates into

$$\begin{aligned} \text{var}(g) &= \langle \delta g^2 \rangle - \langle \delta g \rangle^2 = \sigma^2(g) \\ &= \int_0^\infty \int_0^\infty f'(E)f'(E')C_0(E, E')dEdE', \end{aligned} \quad (3.45)$$

where

$$C_0(E, E') = \langle g \rangle^2 / [(N + N_\varphi)^2 + 4\pi(E - E')^2 / \Delta^2] \quad (3.46)$$

is the conductance correlator whose expression can also be obtained from RMT[69].

Expressions 3.45 and 3.46 will be used extensively in Chapter 4 when we study the magnetotransport properties of OQDs in parallel magnetic fields.

3.2.7 Diagrammatic Perturbation Theory

The diagrammatic perturbation theory[129, 47] is yet another elegant method for calculating ensemble averages of transport properties. The starting point is an expression of the conductance in terms of the retarded Green's function which is an extension to the Fisher-Lee relation[66] given by Eq. 3.15. In 2D, the conductance reads

$$g = \frac{2e^2}{h} \hbar^2 \sum_{\mathbf{k}, \mathbf{k}'} \nu_x(\mathbf{k}') \nu_y(\mathbf{k}) |G^R(\mathbf{k}', \mathbf{k})|^2, \quad (3.47)$$

where ν_x and ν_y are the electron velocities along the x and y axes. The retarded Green's function is given by Eq. 3.17, where the Hamiltonian H_C in reciprocal space is

$$H_C(\mathbf{k}, \mathbf{k}') = \frac{\hbar^2 k^2}{2m^*} \delta_{\mathbf{k}, \mathbf{k}'} + U(\mathbf{k}, \mathbf{k}'), \quad (3.48)$$

and where $U(\mathbf{k}, \mathbf{k}')$ is a scattering potential with randomly distributed impurities, that only depends on $\mathbf{q} = \mathbf{k} - \mathbf{k}'$, so that it is possible to use the notation $U(\mathbf{k}, \mathbf{k}') = U_{\mathbf{q}}$. Typically, it is assumed that impurity scattering is isotropic, so that U has a zero mean and is delta-correlated :

$$\langle U_{\mathbf{q}} \rangle = 0, \quad \langle U_{\mathbf{q}} U_{\mathbf{q}'} \rangle = U_0^2 \delta_{\mathbf{q}, -\mathbf{q}'}, \quad (3.49)$$

with the average performed on impurity distributions.

The Green's function is then given by a Dyson equation similar to Eq. 3.19 in reciprocal space:

$$\begin{aligned} G^R(\mathbf{k}', \mathbf{k}) &= \delta_{\mathbf{k}, \mathbf{k}'} G_0(\mathbf{k}) + G_0(\mathbf{k}') U_{\mathbf{q}} G_0(\mathbf{k}) \\ &\quad + G_0(\mathbf{k}') U_{\mathbf{q}_2} G_0(\mathbf{k} + \mathbf{q}_1) U_{\mathbf{q}_1} G_0(\mathbf{k}) + \dots \end{aligned} \quad (3.50)$$

where G_0 is the Green's function of the unperturbed system, *i.e.* with $U_0 = 0$.

$$\begin{array}{c} \text{k} \\ \xrightarrow{\quad G^R \quad} \text{k}' = \end{array} \begin{array}{c} \text{k} \\ \xrightarrow{\quad G_0 \quad} \text{k}' + \end{array} \begin{array}{c} \text{k} \\ \xrightarrow{\quad \uparrow q \quad} \text{k}' + \end{array} \begin{array}{c} \text{k} \\ \xrightarrow{\quad \uparrow q_1 \quad} \text{k}' + \end{array} \begin{array}{c} \text{k} \\ \xrightarrow{\quad \uparrow q_2 \quad} \text{k}' + \end{array} \dots$$

Figure 3.3: Diagrammatic representation of the Dyson equation for the Green's function $G^R(\mathbf{k}', \mathbf{k})$. Here, G_0 is the Green's function of the unperturbed (free propagator: $U_0 = 0$) system.

The diagram illustrates the expansion of the retarded Green's function G^R in terms of the bare Green's function G_0 and interaction vertices q_i . The expansion is shown in two rows.

Top Row: The retarded Green's function G^R is represented by a thick black arrow pointing from k to k' . This is equal to the bare Green's function G_0 (a thin black arrow from k to k') plus a series of terms involving interaction vertices q_i . The first term is G_0 with an upward arrow labeled q_1 from k to k' . The second term is G_0 with two upward arrows labeled q_1 and q_2 from k to k' . The third term is G_0 with two upward arrows labeled q_3 and q_4 from k to k' . The expansion continues with an ellipsis.

Bottom Row: The expansion is further detailed by showing the interaction vertices q_i as either isolated or as part of a ladder structure. The first term is G_0 with an isolated upward arrow labeled q_1 . The second term is G_0 with an isolated upward arrow labeled q_2 . The third term is G_0 with an isolated upward arrow labeled q_3 . The fourth term is G_0 with an isolated upward arrow labeled q_4 . The fifth term is G_0 with a ladder structure (two vertical dashed lines) labeled "Ladder". The sixth term is G_0 with a ladder structure (two vertical dashed lines) labeled "Ladder". The seventh term is G_0 with a maximally crossed structure (two crossing dashed lines) labeled "Maximally Crossed".

Figure 3.4: Diagrammatic representations of the first and second order contributions to $|G^R(\mathbf{k}', \mathbf{k})|^2$. Ensemble averaging is equivalent to connecting scattering lines and leads by means of three different topologies: isolated, ladder or maximally crossed diagrams.

A diagrammatic representation of Eq. 3.50 is shown in Fig. 3.3. To satisfy momentum conservation, the following relations must be fulfilled

$$\mathbf{k} = \mathbf{k}' \quad \mathbf{k} + \mathbf{q} = \mathbf{k}' \quad \mathbf{k} + \mathbf{q}_1 + \mathbf{q}_2 = \mathbf{k}'. \quad (3.51)$$

To calculate the average conductance across the structure using Eq. 3.47, it is necessary to evaluate the ensemble average of the square magnitude of the Green's function, $\langle |G^R(\mathbf{k}', \mathbf{k})|^2 \rangle$, where

$$|G^R(\mathbf{k}', \mathbf{k})|^2 = G^R(\mathbf{k}', \mathbf{k}) G^R(\mathbf{k}', \mathbf{k})^*. \quad (3.52)$$

Diagrammatically, the ensemble average of the product of a Green's function by its complex conjugate is obtained by reversing the representation of the second function, and by closing all the $\mathbf{q}$ arrows two-by-two, as can be seen in Fig. 3.4. Interestingly, the diagrams stemming from ensemble averaging can be sorted into three different topological families, each contributing in a different manner to the average conductance:

1. The isolated, or closed-loop diagrams. They actually correspond to the $|G^R(\mathbf{k}', \mathbf{k})|^2$ contribution to the $\langle |G^R(\mathbf{k}', \mathbf{k})|^2 \rangle$ term which represents the total average conductance. It can be shown[47] that closed-loop diagrams give rise to the standard Drude conductivity term

$$\sigma_0 = \frac{n_s e^2 \tau_m}{m^*} \quad (3.53)$$

where n_s is the electron sheet density and τ_m is the momentum relaxation time.

2. The second group is that of the ladder diagrams. Once again, it can be shown that these diagrams do not contribute to the average conductance.
3. The last group is composed of maximally crossed diagrams. The diagram shown in Fig. 3.4 is composed of two scattering lines and two propagator lines G_0 . The total contribution to g of this type of diagrams represent the weak localization correction to the Drude conductivity[129]. Quantitatively, the WL correction is given by

$$\langle g \rangle_{B \neq 0} - \langle g \rangle_{B=0} \propto \int d\mathbf{r} d\mathbf{r}' \sum_i \mathcal{C}_i(\mathbf{r}, \mathbf{r}') \quad (3.54)$$

where $\mathcal{C}_i$ is called a Cooperon[129, 47], and is solution to the equation in reciprocal space

$$\mathcal{C}(\beta) = \frac{1/D\tau_m}{(1/D\tau_\varphi) + \beta^2} \quad (3.55)$$

equivalent to the differential equation

$$\left[-D(\nabla - 2i\mathbf{A})^2 + \frac{1}{\tau_\varphi} \right] \mathcal{C}(\mathbf{r}, \mathbf{r}') = \frac{1}{\tau_m} \delta(\mathbf{r} - \mathbf{r}') \quad (3.56)$$

in real space. Here, D is the diffusion constant and $\mathbf{A}$ is the vector potential. Solving Eq. 3.56 leads to a WL correction identical to Eq. 3.34, obtained by RMT[129]. Similarly, the variance of conductance fluctuations can be evaluated from

$$\text{var}(g) \propto \int d\mathbf{r} d\mathbf{r}' \sum_i |\mathcal{C}_i(\mathbf{r}, \mathbf{r}')|^2, \quad (3.57)$$

and is equivalent to Eq. 3.35.

3.3 Properties of 2DEGs in Magnetic Fields

In this section, we introduce the concept of transverse magnetic subbands that will be constantly used throughout this work. The influence of a magnetic field B on the in-plane motion of electrons in an heterostructure or quantum well is a result of the finite width of the 2DEG. This effect has already been discussed in various contexts[152, 83]. Here, we quickly review the static properties of the 2D electrons as a function of the strength of the electrostatic confinement, and of the orientation of B with respect to the 2DEG. We essentially follow the developments outlined in Chapter 1 of Datta[47], with some additional changes to the expression of the analytical confinement potential. Finally, we spend a few words introducing the concept of Spin-Orbit (SO) interaction, and the way it manifests itself in GaAs-Al_xGa_{1-x}As 2DEGs, along with its strength with respect to B .

3.3.1 Magnetic Subbands in a Perpendicular B : Landau Levels

We consider a 2DEG located in the x - y plane in coordinate space. A magnetic field $\mathbf{B} = (0, B_{\parallel}, B_{\perp})$ can be tilted in the y - z plane. In the Coulomb gauge, the vector potential is

$$\mathbf{A} = zB_{\parallel}\mathbf{e}_x + xB_{\perp}\mathbf{e}_y, \quad (3.58)$$

where $\mathbf{e}_x$ and $\mathbf{e}_z$ are the unit vectors along the x and z -axes, respectively.

The 3D Schrödinger equation reads

$$\left[\frac{(i\hbar\nabla + e\mathbf{A})^2}{2m^*} + V(z) \right] \Psi_n(\mathbf{k}, \mathbf{r}) = \mathcal{E}_n(\mathbf{k}) \Psi_n(\mathbf{k}, \mathbf{r}), \quad (3.59)$$

where $\mathcal{E}_n(\mathbf{k})$ and $\Psi_n(\mathbf{k}, \mathbf{r})$ are the eigenvalues and eigenvectors of the Hamiltonian, and $V(z)$ is the electrostatic confinement energy of the 2DEG in the quantum well or heterostructure.

In the case where B is strictly perpendicular to the 2DEG ($\mathbf{B} = (0, 0, B_\perp)$), the wavefunctions can be separated into independent components along x , y and z :

$$\Psi_n(\mathbf{k}, \mathbf{r}) = \phi_{n_x}^x(x, k_y) e^{-ik_y y} \phi_{n_z}^z(z), \quad (3.60)$$

with the eigenenergies having the form

$$\mathcal{E}_n(\mathbf{k}) = E_{n_x}^x(k_y) + E_{n_z}^z. \quad (3.61)$$

The eigenvectors $\phi_{n_x}^x$, $\phi_{n_z}^z$ and eigenvalues $E_{n_x}^x$ and $E_{n_z}^z$ are solutions to the 1D Schrödinger equations

$$\left[-\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial x^2} + \frac{(eB_\perp x + \hbar k_y)^2}{2m^*} \right] \phi_{n_x}^x(x, k_y) = E_{n_x}^x(k_y) \phi_{n_x}^x(x, k_y) \quad (3.62)$$

$$\left[-\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial z^2} + V(z) \right] \phi_{n_z}^z(z) = E_{n_z}^z \phi_{n_z}^z(z). \quad (3.63)$$

For the sake of the calculation, we suppose that the confinement energy $V(z)$ is parabolic, centered at $z = z_0$, and can be written as

$$V(z) = \frac{1}{2} m^* \omega_0^2 (z - z_0)^2. \quad (3.64)$$

Here, we strongly emphasize that, in realistic structures, $V(z)$ is usually far from parabolic. In GaAs-Al_xGa_{1-x}As heterostructures, it has a triangular shape while it has a rectangular profile for quantum wells. However, solving the Schrödinger equation for the Hamiltonian given by Eq. 3.59 with a parabolic confinement has the advantage of yielding an analytical solution whose expression directly reflects the interplay between the magnetic and electrostatic confinements.

Plugging the parabolic expression of $V(z)$ into Eq. 3.63, one gets

$$\left[-\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial x^2} + \frac{1}{2} m^* \omega_c^2 (x + x_{k_y})^2 \right] \phi_{n_x}^x(x, k_y) = E_{n_x}^x(k_y) \phi_{n_x}^x(x, k_y) \quad (3.65)$$

$$\left[-\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial z^2} + \frac{1}{2} m^* \omega_0^2 (z - z_0)^2 \right] \phi_{n_z}^z(z) = E_{n_z}^z \phi_{n_z}^z(z), \quad (3.66)$$

where

$$x_{k_y} = \frac{\hbar k_y}{qB_\perp} \quad \text{and} \quad \omega_c = \frac{eB_\perp}{m^*} \quad (3.67)$$

is the cyclotron frequency.

The solutions to both Eqs. 3.65 and 3.66 are well-known:

$$\phi_{n_x}^x(x, k_y) = \exp\left(-\frac{m^*\omega_c [x + x_{k_y}]^2}{\hbar}\right) H_{n_x}\left(\sqrt{m^*\omega_c/\hbar} [x + x_{k_y}]\right) \quad (3.68)$$

$$\phi_{n_z}^z(z) = \exp\left(-\frac{m^*\omega_0(z - z_0)^2}{\hbar}\right) H_{n_z}\left(\sqrt{m^*\omega_0/\hbar}(z - z_0)\right) \quad (3.69)$$

$$E_{n_x}^x(k_y) = \left(n_x + \frac{1}{2}\right) \hbar\omega_c \quad \text{and} \quad E_{n_z}^z = \left(n_z + \frac{1}{2}\right) \hbar\omega_0, \quad (3.70)$$

where $H_n(x)$ is the n th order Hermite polynomial.

The total solution is thus a combination of two sets of Landau levels, one set classically induced by the perpendicular field $B_\perp$ (the magnetic subbands) while the second one (the electric subbands) comes from the electrostatic confinement of the 2DEG (the ω_0 coefficient). As we said earlier in the text, the confinement energy is usually far from having a parabolic form in real life samples. This will strongly affect the spectrum of eigensolutions along the z direction. In order to find $\phi_{n_z}^z(z)$ and $E_{n_z}^z$ for an arbitrary potential, a perturbation theory analysis or a numerical self-consistent technique can be used.

3.3.2 Magnetic Subbands in a Parallel B : Orbital Effect

In the case of a strictly parallel magnetic field ($\mathbf{B} = (0, B_\parallel, 0)$), the 3D wavefunctions can be written as

$$\Psi_n(\mathbf{k}, \mathbf{r}) = e^{-i(k_x x + k_y y)} \phi_{n_z}^z(z, k_x), \quad (3.71)$$

with the eigenenergies separated into

$$\mathcal{E}_n(\mathbf{k}) = \frac{\hbar^2 k_y^2}{2m^*} + E_{n_z}^z(k_x). \quad (3.72)$$

As opposed to the strictly perpendicular case, only a single 1D Schrödinger equation must be solved. Under the assumption of a parabolic confinement energy $V(z)$ (Eq. 3.64), Equation 3.59 is simplified to

$$\left[-\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial z^2} + \frac{(eB_\parallel z + \hbar k_x)^2}{2m^*} + \frac{1}{2} m^* \omega_0^2 (z - z_0)^2 \right] \phi_{n_z}^z(z, k_x) = E_{n_z}^z(k_x) \phi_{n_z}^z(z, k_x), \quad (3.73)$$

which can be rewritten as

$$\left[-\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial z^2} + \frac{1}{2} m^* \frac{\omega_0^2 \omega_c^2}{\omega_0^2 + \omega_c^2} z_{k_x}^2 + \frac{1}{2} m^* (\omega_0^2 + \omega_c^2) \left(z - z_0 + \frac{\omega_c^2}{\omega_0^2 + \omega_c^2} z_{k_x} \right)^2 \right] \phi_{n_z}^z(z, k_x) = E_{n_z}^z(k_x) \phi_{n_z}^z(z, k_x), \quad (3.74)$$

where

$$z_{k_x} = z_0 + \frac{\hbar k_x}{eB_{\parallel}} \quad \text{and} \quad \omega_c = \frac{eB_{\parallel}}{m^*}. \quad (3.75)$$

Once again, this is basically a 1D Schrödinger equation with parabolic potential. The wavefunctions are Hermite polynomials with

$$\phi_{n_z}^z(z, k_x) = \exp \left(-\frac{1}{2} \left[q + \frac{\omega_c^2}{\omega_0^2 + \omega_c^2} q_{k_x} \right]^2 \right) H_{n_z} \left(q + \frac{\omega_c^2}{\omega_0^2 + \omega_c^2} q_{k_x} \right) \quad (3.76)$$

and

$$\begin{aligned} E_{n_z}^z(k_x) &= \left(n_z + \frac{1}{2} \right) \hbar \sqrt{\omega_0^2 + \omega_c^2} + \frac{1}{2} m^* \frac{\omega_c^2 \omega_0^2}{\omega_0^2 + \omega_c^2} z_{k_x}^2 \\ &= \left(n_z + \frac{1}{2} \right) \hbar \sqrt{\omega_0^2 + \omega_c^2} + \frac{1}{2m^*} \frac{\omega_0^2}{\omega_0^2 + \omega_c^2} (\hbar k_x + m^* \omega_c z_0)^2, \end{aligned} \quad (3.77)$$

where

$$q^2 = m^* \sqrt{\omega_0^2 + \omega_c^2} (z - z_0)^2 / \hbar \quad \text{and} \quad q_{k_x}^2 = m^* \sqrt{\omega_0^2 + \omega_c^2} z_{k_x}^2 / \hbar. \quad (3.78)$$

As opposed to the strictly perpendicular case, there is a strong interplay between the magnetic (ω_c) and electrostatic (ω_0) confinements. As a result, there is a mixing between the magnetic and electric subbands. The dispersion relation $E_{n_z}^z(k_x)$, given by Eq. 3.77, has the k_x parabolic dependence of a free electron gas with a major difference: the effective mass is renormalized by a factor that depends on the relative amplitudes of the two confinements:

$$m^* \rightarrow m^* \left(1 + \frac{\omega_c^2}{\omega_0^2} \right). \quad (3.79)$$

Moreover, $E_{n_z}^z(k_x)$ has also a Landau level-like term, which is linearly proportional to $B_{\parallel}$ through the cyclotron frequency ω_c . Similarly, because of the q_{k_x} factor defined in Eq. 3.78, the value of $B_{\parallel}$ determines

the position of the center of the wavefunction $\phi_{n_z}^z(z, k_x)$ for a given k_x (Eq. 3.76).

Therefore, as $B_{\parallel}$ increases, the position of the subband minimum in the dispersion relations, located at $k_x = -m^*\omega_c z_0/\hbar$, moves linearly in $\mathbf{k}$ -space with $B_{\parallel}$. At the same time, a linear shift in energy of the subband minimum is observed. Self-consistently, because the electron density in the 2DEG remains constant, charges must be transferred from the upper occupied subbands into the lower ones due to the renormalization of the effective mass with increasing magnetic field. As a result, it is possible to alter the individual subband populations to the point where higher lying subbands are completely depleted.

We must once again emphasize that the dispersion relations given by Eq. 3.77 are only valid in the theoretical case of a parabolic confinement energy, chosen here for educational purposes. For an arbitrary confinement $V(z)$ of characteristic width d , perturbation theory[157] yields an effective Hamiltonian that takes the form

$$\mathcal{H} = \frac{\mathbf{p}^2}{2m^*} - p_{\perp}^2 \gamma(B_{\parallel}) + p_{\perp}^3 \beta(B_{\parallel}), \quad (3.80)$$

where

$$\gamma(B_{\parallel}) \simeq \frac{1}{m^*} \left(\frac{d}{l_B} \right)^4 \quad \text{and} \quad \beta(B_{\parallel}) \simeq \frac{d}{m^*} \left(\frac{d}{l_B} \right)^6, \quad (3.81)$$

valid for a weak to intermediate magnetic field $B_{\parallel}$. Here, $\mathbf{p} = -i\hbar\nabla - e\mathbf{A}$ is the momentum operator, $p_{\perp}$ its component perpendicular to $B_{\parallel}$, and $l_B = \sqrt{\hbar/eB_{\parallel}}$ is the magnetic length. Perturbation theory[157] was applied up to the third order in $\mathbf{p}$ using the $B_{\parallel} = 0$ eigensolutions.

The second term in the Hamiltonian of Eq. 3.80, proportional to k_x^2 , gives rise to the increase of the effective mass. When self-consistency is included[82, 157], the same second-order term in $p_{\perp}$ also induces a *quadratic* energy shift of the subband minima, as opposed to a linear dependence calculated earlier for a parabolic confinement (Eq. 3.76), and a linear displacement with increasing $B_{\parallel}$ of their position along k_x . We will shown later in this work (Chap. 4) that the third term in Eq. 3.80, proportional to k_x^3 , is related to time-reversal symmetry breaking by a parallel field[63]. It will be central to the analysis of the magnetotransport properties of open quantum dots subject to parallel fields.

3.3.3 Spin-Orbit Coupling in GaAs-Al_xGa_{1-x}As 2DEGs

In the expression of the multi-particle Hamiltonian of an electron system in a semiconductor crystal, the interaction between electrons and protons is represented by a sum of Coulomb potentials $V(r) = -e^2/r$. While these terms provide an excellent first-order approximation, they do not take into account relativistic effects. Since the electron velocity is typically $\gamma = v/c \simeq \mathcal{O}(\alpha)$, where v is the electron velocity, c the speed of light, and $\alpha = 1/137$ the fine-structure constant, there are two additional corrections of order $(v/c)^2$ to be included in the Hamiltonian for a fully relativistic description of the system. Rigorously, relativistic effects are fully taken equation in the Dirac equation[51, 52]. However, the so-called fine-structure corrections to the Hamiltonian can be conveniently introduced[43] by starting with the Taylor development of the Dirac equation with respect to v/c

$$H = m_e c^2 + V(r) + \frac{\mathbf{p}^2}{2m_e} - \frac{\mathbf{p}^4}{8m_e^3 c^2} + \frac{1}{2m_e^2 c^2} \frac{1}{r} \frac{dV(r)}{dr} \mathbf{S} \cdot \mathbf{L} + \dots \quad (3.82)$$

where m_e is the electron mass in vacuum, $\mathbf{L}$ is the angular momentum operator (a generator of infinitesimal rotations) and $\mathbf{S}$ is the spin operator.

Beside the energy $m_e c^2$ of an electron at rest, the first correction to the classical expression $p^2/2m_e$ of the kinetic energy can be simply deduced from the relativistic expression of the energy of an electron with momentum p

$$E = \sqrt{p^2 + m_e^2 c^2}. \quad (3.83)$$

Using a Taylor development with respect to $p/m_e c$, the energy E becomes

$$E = m_e c^2 + \frac{p^2}{2m_e} - \frac{p^4}{8m_e^3 c^2} + \mathcal{O}(p^6). \quad (3.84)$$

The first correction $-p^4/8m_e^3 c^2$ thus arises from the relativistic variation of the electron mass with its velocity.

The second relativistic correction, whose effects can be much more pronounced in semiconductor systems, is called the spin-orbit interaction. A complete mathematical description of spin-orbit interaction would require complete relativistic kinematics, as provided by the Dirac equation[51, 52]. However, it is possible to give a general understanding of its main concepts using simple arguments. First, let's consider an electron moving at a velocity $\mathbf{v}$ with respect to a fixed proton. Inversely, in the electron rest frame, the proton will be moving at a velocity $-\mathbf{v}$ and

will generate a magnetic field whose expression is given by the theory of special relativity[94], and reads

$$\mathbf{B} = -\frac{1}{c^2} \mathbf{v} \times \mathbf{E} = \frac{-e}{c} \frac{\mathbf{v} \times \mathbf{r}}{r^3}, \quad (3.85)$$

where $\mathbf{E} = \nabla V$ is the electrostatic field. The interaction of the magnetic moment μ of the electron with this magnetic field leads to an additional term in the Hamiltonian, called spin-orbit energy

$$H_{\text{so}} = -\mu \cdot \mathbf{B} = \frac{e}{m_e c r^3} \mu \cdot (\mathbf{p} \times \mathbf{r}) = -\frac{e}{m_e c} \frac{\mu \cdot \mathbf{l}}{r^3}, \quad (3.86)$$

where $\mathbf{l}$ is the angular momentum in coordinate space.

In the framework of quantum theory[43], the expression of H_{so} is thus expected to be

$$\mathcal{H}_{\text{so}} = \left(-\frac{e}{m_e c}\right) \left(-\frac{e}{m_e c}\right) \mathbf{S} \cdot \mathbf{L} = \frac{e^2}{m_e^2 c^2 r^3} \mathbf{S} \cdot \mathbf{L}, \quad (3.87)$$

where $\mathbf{L}$ is the angular momentum operator (a generator of infinitesimal rotations) and $\mathbf{S}$ is the spin operator.

However, this simple argument does not take into account the complex motion of the electron which is not necessarily rectilinear. As a result, the proton in the rest frame of the electron does not have a fixed velocity with respect to the center of the coordinate systems. This gives rise to a 1/2 correction factor to the spin-orbit energy, which actually reads

$$\mathcal{H}_{\text{so}} = \frac{e^2}{2m_e^2 c^2 r^3} \mathbf{S} \cdot \mathbf{L}. \quad (3.88)$$

Expressing $\mathbf{S}$ in terms of the Pauli matrices σ , the total Hamiltonian of conduction band electrons in a 2DEG is given by

$$\mathcal{H} = \frac{\hbar^2 k^2}{2m^*} + \hbar (\sigma \cdot \boldsymbol{\Omega}), \quad (3.89)$$

where the total SO frequency $\boldsymbol{\Omega}$ is an odd function of $\mathbf{k}$. In 2DEGs, $\boldsymbol{\Omega}$ can be written as the sum of three contributions: a linear $\boldsymbol{\Omega}_D^1$ and a cubic $\boldsymbol{\Omega}_D^3$ Dresselhaus term[53], and a Rashba term[33] $\boldsymbol{\Omega}_R$, where

$$\boldsymbol{\Omega}_D^1 = \alpha_1 (-k_x \mathbf{e}_x + k_y \mathbf{e}_y) / \hbar, \quad (3.90)$$

$$\boldsymbol{\Omega}_D^3 = \alpha_2 (k_y \mathbf{e}_x - k_x \mathbf{e}_y) / \hbar, \quad (3.91)$$

$$\boldsymbol{\Omega}_R = \gamma (k_x k_y^2 \mathbf{e}_x - k_x^2 k_y \mathbf{e}_y) / \hbar. \quad (3.92)$$

The Dresselhaus terms come from the lack of inversion symmetry of the crystal, particularly present in the GaAs Zincblende structure

(Sec. 2.2.1). Here, α_1 depends on the width of the potential profile in the quantum well, or at the GaAs-Al_xGa_{1-x}As hetero-interface, while α_2 is a material parameter characterizing the polarity of the bond between the Ga and As atoms. On the contrary, the Rashba contribution to SO coupling originates from the confining electric field of the quantum structure itself. As a result, it is a pure 2D term which is irrelevant in bulk semiconductors. The corresponding magnetic field always lies in the plane of the 2DEG, hence the $\mathbf{e}_x$ and $\mathbf{e}_y$ components to $\mathbf{\Omega}_R$.

The three spin-orbit terms can be tuned by means of a gate voltage that changes the electron density n_s in the 2DEG. Typically, at low densities, the cubic Dresselhaus term $\mathbf{\Omega}_D^3$ is quite weak with respect to the linear term $\mathbf{\Omega}_D^1$, but becomes somewhat stronger at higher densities. Interestingly, experiments[135, 39] have shown that, in quasi-symmetric wells, the Rashba term $\mathbf{\Omega}_R$ can be tuned at fixed densities with an externally gate voltage that breaks the well symmetry. However, for triangular wells, $\mathbf{\Omega}_R$ is independent of the gate voltage, as long as the density remains unchanged. Furthermore, because of their linear k -dependence, the linear Dresselhaus and Rashba terms have similar variations of their amplitude with respect to n_s .

Intimately associated with spin-orbit coupling is the spin-orbit scattering, where elastic collisions induce a randomization of the spin. In the presence of scattering, the electron scatters before it reaches an eigenstate of the system. This results in a change of the direction of the effective magnetic field induced by the spin-orbit coupling. The electron now precesses around a new direction, thus randomizing its spin.

It was shown in Sec. 1.5.3 that the constructive interference of time-reversed electron paths give rise to a quantum reduction of the conduction, the so-called weak localization (WL) effect. However, when strong spin dephasing is present, the spin-dephased paths destructively interfere, and cause an enhanced conductivity. Similar to WL, a perpendicular magnetic field destroys the interference effect, resulting in an antilocalization (AL) signature in the magnetoconductance around $B = 0$ T. A nice example of a such an AL peak is shown in Fig. 3.5 in the case of a bismuth nanocavity[79].

Weak antilocalization was first observed in disordered metallic samples[18], where spin-orbit coupling is associated with the electric field of a remote impurity. The situation is commonly referred to as the Elliot mechanism[60]. On the opposite, in semiconductors, the elastic scattering and its corresponding spin-dephasing are mainly associated with the dipole field due to inversion symmetry (Dresselhaus term) or the interfacial field (Rashba term). This is the so-called D'yakonov-Perel' (DP) mechanism[56].

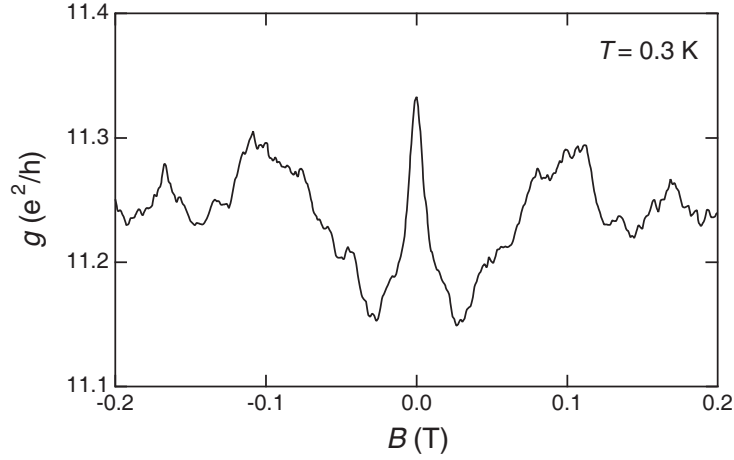


Figure 3.5: Antilocalization correction to the conductance measured in a bismuth nanocavity. (Courtesy of B. Hackens, Ref. [79])

The common method to discriminate the Elliot from the DP mechanism is to study the dependence of the spin-orbit scattering rate H_{so} with respect to both the electron density and transport scattering rate $H_{tr} = e/4\pi\hbar n_s \mu^2$, where μ is the mobility. The value of H_{so} can be readily extracted[88] from the weak antilocalization correction to the conductance. At fixed density, the Elliot mechanism is characterized by a linear dependence between H_{so} and H_{tr} , while H_{so} is independent of H_{tr} in the case of the DP mechanism. Moreover, in the DP mechanism, the Dresselhaus term induces a quadratic dependence of H_{so} on the carrier density, while H_{so} is independent of n_s for the Rashba term. Using these theoretical results, it was possible to interpret numerous antilocalization measurements in 2DEGs. For example, it was shown that spin-orbit scattering in GaAs-AlGaAs MOSFETs[55] could be linked to a DP mechanism caused by a crystal electric field (Dresselhaus term). Similarly, in InAs quantum structures[39], weak AL measurements suggested that a predominant role was played by the interfacial Rashba field, with a strong enhancement of SO coupling by the asymmetry of the quantum confinement.

More recently, it has been shown[132, 29] that SO coupling could be controlled *in situ* by means of a geometry-dependent top-gate voltage. SO effects have also been reported in mesoscopic systems like open quantum dots[68, 178, 5, 6]. Of particular interest are recent measurements of the anisotropy of SO-induced spin-splitting in quantum dots by means of resonant tunneling spectroscopy[106], or theoretical investigations[62]

of the spin relaxation in lateral dots in a strong in-plane magnetic field. The effect of SO coupling on the transport properties of open quantum dots will be more completely discussed in Chapter 4 when we introduce our magnetoconductance measurements performed on open quantum dots in a parallel magnetic field.

3.4 Summary

In this chapter, we did our best to give an introduction to the “state of the art” theoretical models that are currently available for the analysis of quantum transport in ballistic systems, in particular open quantum dots. Each model, from the semi-classical approach, Random Matrix Theory (RMT), Supersymmetry, to the diagrammatic perturbation technique, has its own characteristics in terms of the mathematical tools that are used. However, they are all deeply rooted to the theory of scattering matrices and, as such, yield identical or complementary expressions for transport properties like the weak localization correction to the conductance, or the variance of conductance fluctuations. The most recent development of RMT and diagrammatic perturbation theory, including the effects of dephasing and thermal smearing, will prove to be central to the conductance analysis of our open quantum dots subject to tilted magnetic fields.

Finally, we theoretically discussed the effects of a magnetic field on the static properties of a two-dimensional electron gas (2DEG), with strong emphasis on spin-orbit coupling and on the concept of transverse magnetic modes, that will appear frequently in the next chapters.

Chapter 4

Transport under a Strictly Parallel Field

All we can do is to search for the falsity of our best theory.

Karl Popper

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4.1 Introduction

In Chapter 1, we tried to give a gentle introduction to the fascinating field of quantum transport in semiconductor nanostructures, using open quantum dots (OQDs) as meaningful examples. In particular, we put much emphasis on the particular regime where transport is both ballistic and coherent, giving rise to complex interferences between electron trajectories in the dot.

Central to quantum interference phenomena is the presence of an external parameter that, when modified, affects the accumulated phase along the trajectories, changes the interference pattern and breaks time-reversal symmetry (TRS). Macroscopically, quantum interferences are materialized by Universal Conductance Fluctuations (UCFs) or a weak localization (WL) correction to the magnetoresistance across the dot[16]. Usually, a magnetic field B is the preferred tool for breaking time-reversal symmetry and changing the phase experienced by electrons in the dot, although UCFs can also be observed by changing the position of the Fermi level E_F with respect to the bottom of the conduction band edge in the 2D quantum well by means of an electrostatic gate.

Until recently, in all experiments on open dots, B was applied perpendicular to the two-dimensional electron gas (2DEG) in order to maximize the magnetic flux enclosed by electron trajectories in the cavity. In Chapter 3, we theoretically described two effects that an in-plane component of the magnetic field can have on the conduction band structure of electrons in a 2DEG. Both the spin-orbit coupling and the orbital effect can significantly modify the electron dispersion relation, inducing characteristic signatures in the magnetoconductance of the 2DEG[132, 83, 178].

If we use the simplified view of open quantum dots as pure 2D structures, it is clear that an in-plane field can only couple to the electron field in the form of a Zeeman energy term¹. For example, truly 2D systems would not feel the orbital effect of an in-plane field on WL or UCFs phenomena, as the paths within the plane enclose no magnetic

¹The Zeeman energy actually depends on the magnitude of the B and not on its orientation with respect to the plane of the 2DEG.

flux. However, if we take into account the true physical nature of the 2DEG, in particular the finite thickness of the 2D electron layer or relativistic effects like spin-orbit (SO) coupling, it is not irrational to expect new signatures of an in-plane field in the transport properties of open dots.

This is the reason why, almost concurrently, members of Prof. V. Bayot's group (including the author) in Louvain-la-Neuve and researchers from Prof. C. M. Marcus team in Harvard started to investigate the possible effects of a parallel component of the magnetic field on interference phenomena in open quantum dots. Interestingly, the two groups followed different paths for their experimental setup, in particular in the way the parallel field is produced. This surprisingly lead to complementary results that could be analyzed in terms of new theoretical models, developed in the last few months to study the influence of a parallel field on the correlation properties of UCFs.

Therefore, it seems appropriate to start with a description of the experiment performed by J. Folk *et al.* [68] from the Marcus group, in order to clearly isolate the differences with our experimental setup, both in terms of the magnetic field geometries and of the sample properties, in particular the shape of the 2DEG confinement potential. Once this distinction has been made, it becomes much easier to describe the theoretical concepts that play a central and complementary role in each type of experiment.

As a result, the outline of the chapter is as follows: in Sec. 4.2, we describe Folk *et al.* experiment and succinctly discuss its main conclusions. In Sec. 4.3, we pave the way for the measurements that will be presented in this chapter: based on our experimental setup and on recent theoretical studies[82], we discuss the general requirements on both the measurement infrastructure and sample characteristics in order to observe the signature of an orbital effect in OQDs. The samples used throughout our measurements are then presented in Sec. 4.4 followed in Sec. 4.5, by the distinctive features of our experimental setup with *in-situ* tilting of the magnetic field with respect to the plane of the 2DEG. We put particular emphasis on the techniques used for a near-perfect field alignment. In Sec. 4.6, we validate the fabrication techniques used for the preparation of the open dots by classically measuring their magnetoconductance under a perpendicular field. Next, the magnetic field is tilted in the plane of the 2DEG and the unexpected observation of magnetoconductance fluctuations is thoroughly discussed in Sec. 4.7. To isolate the true coherent nature of the parallel B -induced fluctuations, the temperature dependence of their amplitude is studied in Sec. 4.8. Finally, in Sec. 4.10, we apply recent results of Random Matrix The-

ory (RMT) and Diagrammatic Perturbation Techniques (DPT) to link the observed B -dependence of the fluctuations variance with a strong orbital effect in the 2DEG and a breaking of time-reversal symmetry by the parallel field. We draw conclusions in Sec. 4.11.

4.2 Harvard Experiment: Spin Orbit Effect and UCFs in a Parallel Magnetic Field

The main target of the work[68] performed in 2000 by the Marcus group was initially to study the dependence on parallel magnetic field of conductance fluctuations in QDs and to use the results as a probe of spin degeneracy in the dot. In these experiments, the applied magnetic field originates from the combination of two independent magnets. The open dot is oriented with the 2DEG parallel to the primary field, generated by a standard cryogenic solenoid. A perpendicular field coil is also attached to the Inner Vacuum Chamber of the cryostat, with its main axis perpendicular to the open dot. A schematic describing the field orientations is shown in Figs. 4.1(a) and (b). This second solenoid generates much smaller fields (up to 250 mT) than what can be applied in the plane of the 2DEG through the main magnet (up to 7 T). It is crucial for comparison purpose with our experimental setup to keep in mind that, in Folk *et al.* experiment, the small perpendicular field still acts as the main parameter that probes the cavity phase and generates UCFs. The main function of the parallel solenoid is to lift spin degeneracy via Zeeman splitting.

The quantum dots used in this experiment were fabricated on a 2DEG using single GaAs-Al_xGa_{1-x}As heterostructures, and measurements were made on a few dots with areas ranging from 1 to 8 μm^2 . Using surface depletion gates, statistics of conductance fluctuations were calculated over ensembles of dot shapes by varying B_{perp} but keeping B_{par} constant. Samples of the B_{perp} -induced UCFs variance $\text{var}(g)$ were then gathered over discrete values of B_{par} .

Figure 4.1(c) shows some of these variances as a function of the parallel field in the case of a 1 μm^2 dot, at a few different temperatures and number of transmission modes in the dot openings. It was found that the amplitude of conductance fluctuations decreases to finally saturate with the increasing B_{par} , by a factor of ~ 2 at the lowest temperatures and ~ 3 at the higher temperatures. This factor actually depends on the dot size and can reach values of ~ 4 to 5 for a 8 μm^2 dot. Reduction of the conductance variance due to magnetic field-induced Zeeman splitting, observed previously in disordered systems, can amount to a factor

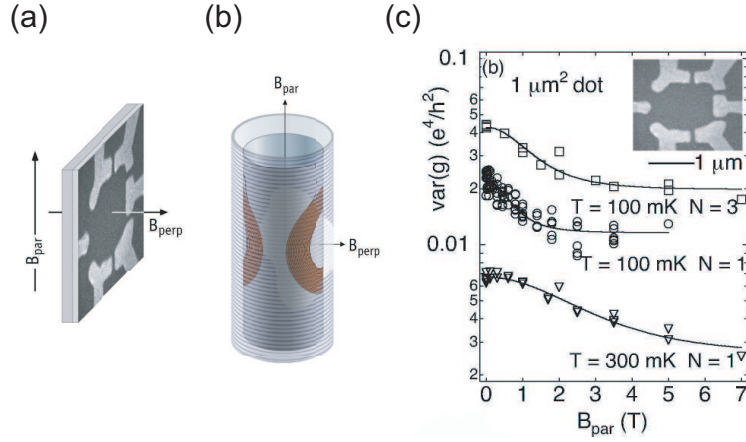


Figure 4.1: (a) Schematic indicating the orientations of the perpendicular and parallel components of B with respect to the open dot. (b) Illustration showing the placement of the superconducting split coil and primary solenoid used to generate B_{perp} and B_{par} , resp. (c) Variance of conductance fluctuations as a function of parallel field B_{par} and indicated temperatures and number of modes in the dot openings. (From Folk *et al.*, Ref. [68])

of 2 as spin degeneracy is broken and fluctuations of the spin levels add randomly instead of constructively. To explain the fact that $\text{var}(g)$ is reduced in some dots by significantly more than a factor of 2, Folk *et al.* proposed an interpretation based on other mechanisms than the simple spin degeneracy lifting by the Zeeman energy. In Ref. [80], Halperin and coworkers argue that spin-orbit effects, even if they are negligible for $B_{\text{par}} = 0$, can become important in the presence of a parallel component of the field. In the presence of a strong spin-orbit coupling, the two spin states are mixed, which theoretically induces a modification of the statistics of Random Matrices. In that case, the UCFs variance can be reduced by another factor of 2, giving a total factor of 4 with respect to the $B_{\text{par}} = 0$ variance.

By identifying an antilocalization (AL) signature in the magnetoconductance of the OQDs using the same experimental setup, the Harvard group subsequently showed[178] that spin-orbit coupling can indeed be enhanced by a parallel B . Interestingly, the strength of the spin orbit coupling, hence the presence or absence of an AL peak, strongly depends on the size of the dot. It was found and proved theoretically[29] that large enough ($\sim 8 \mu\text{m}^2$) dots or a local change in the electron density

(with a central electrostatic gate) were necessary to increase SO coupling up to values macroscopically observable in conductance measurements, either in the WL peak or by a reduction of the UCFs variance.

4.3 What if B is strictly parallel to the 2DEG ?

It is interesting to mention that the initial goal of the experiment[68] by Folk *et al.* was to explore ways of emitting spin currents from OQDs, a key component for the development of spintronic devices. In the preceding section, it was shown that in an in-plane field, the Fermi levels for spin-up and spin-down electrons are split by the Zeeman energy. As the contributions from the two spin states to the conductance fluctuations become statistically independent, very different interference patterns are observed for opposite spin directions. It thus seems natural to try to use an open quantum dot as a tunable spin emitter using gate voltage configurations where one of the two spin species is suppressed by destructive interferences.

Spin filtering using OQDs, demonstrated by the same group, does not require a precise alignment of the main solenoid (the one that generated B_{par}) in the plane of the 2DEG, as conductance fluctuations still originate from the perpendicular component of B . The parallel component B_{par} of the field serves only as an interference tuning parameter, through Zeeman energy splitting and enhancement of the spin-orbit coupling. And indeed, the experimental setup of Fig. 4.1(a) has the advantage of decoupling the parallel and perpendicular component of B but typically presents an alignment error on B_{par} of about 1° with respect to the 2DEG².

Therefore, the question arises of what would happen to the magnetotransport properties of an open quantum dot if it was placed into a strictly parallel magnetic field. Using a truly 2D approximation, the open dot would not feel the orbital effect of an in-plane field at all, and should not display any macroscopic signature of quantum interferences. On the opposite, taking into account the finite thickness of the 2DEG, one could naively think that a magnetic flux could thread through the 2D electron layer and produce conductance fluctuations by means of a strong orbital effect, like in the perpendicular field configuration.

In this simple picture, the exact fingerprint (amplitude, frequency

²The alignment error apparently has been corrected in a later publication of the Marcus group by borrowing our idea of an Hall effect measurement performed on an adjacent Hall bar. Nevertheless, they never claim an alignment precision comparable to the values that will be discussed in Sec. 4.4.

distribution, temperature dependence, strength with respect to dephasing) of these hypothetical fluctuations is unknown. However, it is quite easy to enumerate a few requirements on both the experimental setup and sample characteristics that are mandatory in order to maximize the chances of observing parallel B -induced fluctuations:

1. To make sure that a perpendicular component of the field does not interfere in the dot magnetotransport properties under a parallel field, it is crucial to come up with an experimental framework where the field can be precisely aligned with the 2DEG.
2. Because a “perfect” alignment cannot be attained, the experimental setup must include a method for a numerical evaluation of the alignment error.
3. If the key player for transport through OQD in parallel B is indeed the magnetic flux through the 2DEG, it seems relevant to study the effect of the width of the electron layer, hence to select 2DEG samples with various strengths of quantum confinement.

With these requirements clearly stated, we now describe the samples and measurement infrastructure used in the rest of this thesis work.

4.4 Sample Descriptions

Using the fabrication techniques described in Chapter 2, two identical ballistic quantum dots were prepared using electron beam lithography and wet etching. The dots have the shape of a Bunimovich billiard[30, 31] with a radius of $1\ \mu\text{m}$ and a flat section that extends between -10 and 10° on the unit circle, resulting in a dot area of $3\ \mu\text{m}^2$. Similar to the stadium dot[122], transport is expected to be fully ergodic in Bunimovich cavities. Nevertheless, we understand that full ergodicity is never completely achieved in ballistic quantum dots and that transport is determined by a mixed phase space of both chaotic and stable electron trajectories[103, 144, 48, 141, 130].

A scanning electron microscope (SEM) micrograph of such an OQD was presented in Fig. 2.9(b) as an example of a typical billiard realized through wet etching techniques. A schematic of the dot sample is also provided in the forefront of Fig. 4.2(a) with a close-up on the cavity itself in Panel (b). Two openings allow current to pass through the device by means of the contacts represented as gold rectangles in Fig. 4.2(a). Two additional metallic gates, located on each side of the device, are also available for measurements of the potential drop across the dot in

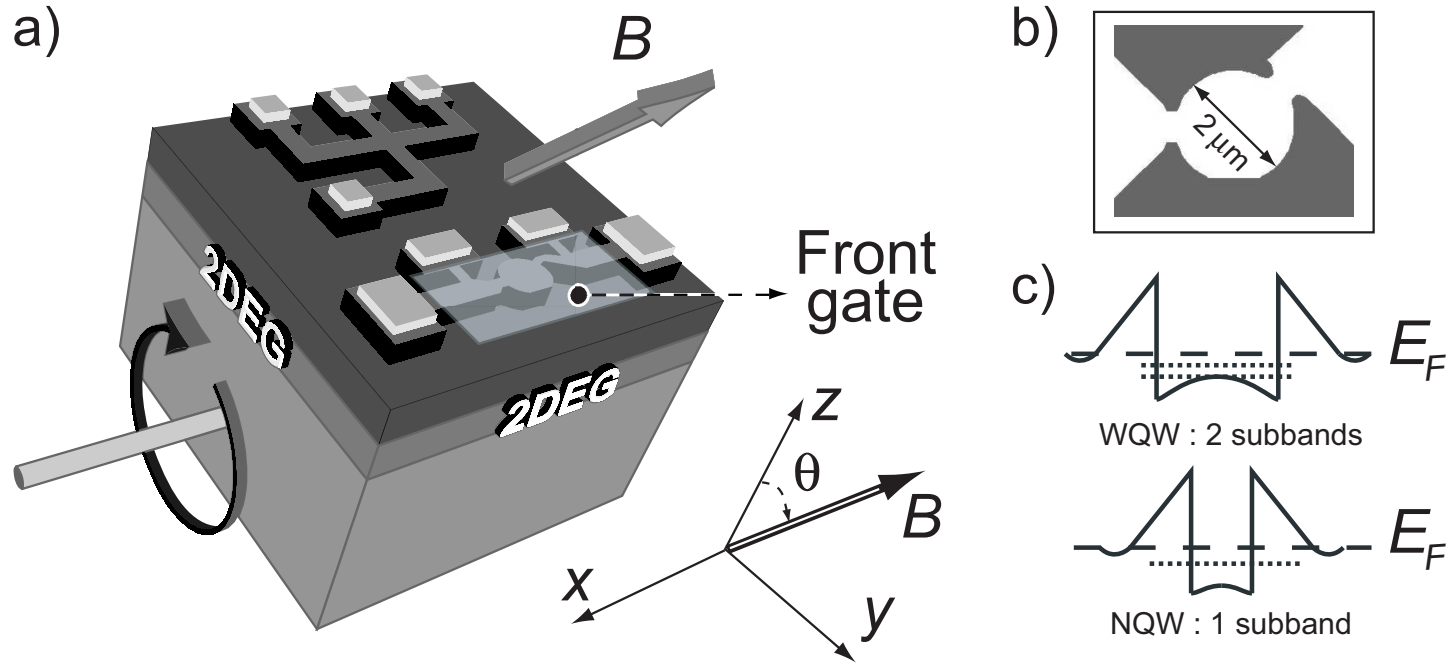


Figure 4.2: (a) Experimental setup with *in-situ* tilting of the sample. (b) Schematic diagram of the dot. Dark regions represent etched areas of the 2DEG. (c) Conduction band profiles for the wide and narrow QWs. The dashed lines represent the position of E_F . The lowest occupied subbands at $B = 0$ are displayed as dotted lines.

a standard 4-contacts configuration. A Cr/Au electrostatic gate, deposited on top of both the dot and its measuring voltage leads, is used as a means to control the width of the dot openings. A second Hall bar, located on the same wafer, 300 μm away from the dot (Fig. 4.2(a)) and not covered by the Cr/Au electrostatic gate, is used for measurement of the transverse magnetoresistance R_{xy} of the *unpatterned* 2D electron gas. Its main purpose is the determination and control of the angle between the magnetic field and the 2DEG, which will be discussed later in this chapter.

Based on our discussion of the requirements for an experiment targeting transport under a strictly parallel field, the dots are fabricated on two high-mobility 2DEG confined to 15 and 45 nm-thick quantum wells, whose characteristics are now fully discussed.

4.4.1 Quantum Wells

The high-quality GaAs-Al_{0.3}Ga_{0.7}As 2DEG samples were provided to us by Prof. Mansour Shayegan from Princeton University, Princeton, N. J., USA. We have fabricated our open dots on various substrates, including simple GaAs heterojunctions. Two of these were used to make the dots studied in this dissertation, as they feature the most reproducible and physically sound results with respect to the OQD electron dynamics in a parallel magnetic field.

The first 2DEG is confined to a 15 nm-thick quantum well (NQW), located 100 nm below the surface. This NQW sample has a mobility $\mu \sim 5 \times 10^5 \text{ cm}^2/\text{Vs}$. The second 2DEG is confined to a 45 nm-thick quantum well. This very high-quality and well characterized [160, 161, 113] WQW is located 150 nm below the surface and its mobility is $\mu \sim 1.6 \times 10^6 \text{ cm}^2/\text{Vs}$. Both the NQW and WQW 2DEGs have electron sheet densities $n_s = 3.2 \times 10^{11} \text{ cm}^{-2}$ when not subject to an electrostatic gate voltage ($V_F = 0 \text{ V}$). The MBE growth structures of the NQW and WQW samples, including the doping profiles, are given in Sections B.1 and B.2 of Appendix B, respectively.

Because of the difference in thickness between the two QWs, one and two transversal subbands are populated by electrons at $B = 0$ in the NQW and WQW, respectively, as schematically represented in Fig. 4.2(b). Throughout the measurements, negative gate voltages V_F with values ranging from -0.2 to -0.3 V were applied to the NQW dot in order to set the number N_1 and N_2 of transmission modes in the openings just above the quantum limit ($N_1, N_2 > 1$) while keeping the average resistance across the dot large enough to maximize the fluctuations amplitude with respect to the intrinsic noise of the measurement

system. In the case of the WQW dot, positive gate voltages V_F with values around 0.6 V were used. The positive potential applied for the WQW dot was required in order to have at least one transmission mode in each of the two quantum point contacts.

The two dots were characterized in the ^3He cryostat (see Sec. 2.3) with a base temperature $T = 0.3$ K after having been cooled down in the dark. The NQW dot magnetoconductance was also measured in the dilution refrigerator down to 30 mK but only in the perpendicular field configuration. For all measurements, a standard, 4-contact low-frequency (typically < 20 Hz) lock-in technique, as described in Chap. 2, was used to measure their conductance with a current of 1 nA. The voltage across the sample was kept well below kT. To make sure that we were indeed in the linear regime, measurements were reproduced with values of the current between 0.1 and 10 nA, without any noticeable difference in the magnetoconductance across the dot.

4.4.2 Subband Densities

To evaluate the effect of the electrostatic gate voltage on the electron sheet densities in the NQW and WQW 2DEGs, longitudinal and transverse magnetoresistance measurements were performed in the main Hall bar³ of both samples with the magnetic field perpendicular to the 2DEG.

Figure 4.3(a) shows Fast Fourier Transforms of Shubnikov-de Haas (SdH) oscillations in the NQW longitudinal magnetoresistance R_{xx} for a few gate voltages V_F . A R_{xx} curve is presented for $V_F = -0.25$ V in Fig. 4.3(b). The electron sheet densities n_s in the occupied subbands of the 2DEG correspond to peak maxima in the Fourier transform of magnetoresistance expressed as a function of $1/B$, as successive SdH oscillations occur at values B_1 and B_2 of the magnetic field related by

$$\frac{n_s h}{2eB_1} - \frac{n_s h}{2eB_2} = 1. \quad (4.1)$$

To evaluate n_s in the NQW, R_{xx} is typically Fourier transformed for $0 < B < 0.5$ T, in order to avoid the spin degeneracy lifting which becomes clearly visible on the R_{xx} trace of Fig. 4.3(b) at fields higher than 0.8 T. As expected from negative gate voltages, the sheet density in the 2DEG decreases from $3.2 \times 10^{11} \text{ cm}^{-2}$ at $V_F = 0$ V to $2.2 \times 10^{11} \text{ cm}^{-2}$ at $V_F = -0.25$ V. Using the same FFT technique at intermediate values of V_F in the range $[0, -0.3]$, we obtain a linear dependence of n_s with respect to V_F ($n_s = (3.2 - 4V_F) \times 10^{11} \text{ cm}^{-2}$).

³Based on the lithography layouts shown in Chap. 2, current injectors and voltage probes were carefully selected among the metallic contacts in such a way that no

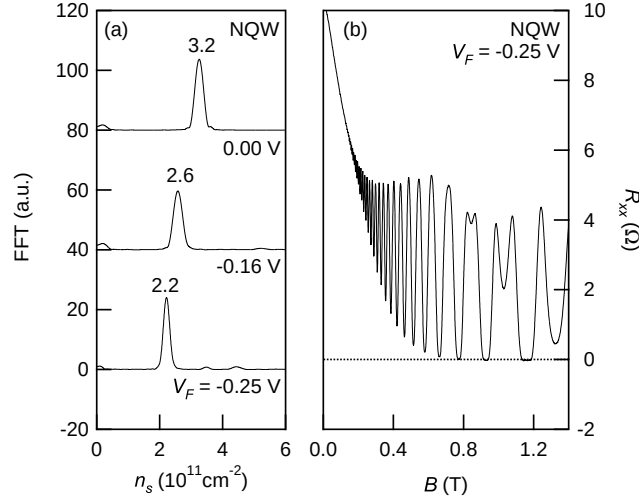


Figure 4.3: (a) Fast Fourier Transform (FFT) of Shubnikov-de Haas oscillations in the NQW at indicated gate voltages V_F . (b) Corresponding longitudinal magnetoresistance in the adjacent Hall bar for $V_F = -0.25$ V.

The action of a positive voltage is far from linear in the case of the WQW 2DEG. As can be shown in Fig. 4.4, the density is essentially independent of gate voltage. Secondly, two peaks, corresponding to the two occupant subbands in the WQW, are clearly visible at $n_s = 1.2 \times 10^{11}$ and $2.0 \times 10^{11} \text{ cm}^{-2}$. Minor peaks, symmetrically located at both sides of the $n_s = 2.0 \times 10^{11} \text{ cm}^{-2}$ peak give the density of the spin-resolved lower subband at high B . On the $V_F = 0.65$ V FFT curve of Fig. 4.4(a), an additional peak around $3.2 \times 10^{11} \text{ cm}^{-2}$ represents the sum of the two subband densities. We point out that, due to a defective ohmic contact in the main Hall bar, the longitudinal magnetoresistance R_{xx} for $V_F = 0.65$ V had to be measured in the WQW sample with one of the voltage probes traversing the OQD. This unfortunately adds a Hall contribution (R_{xy}) to the magnetoresistance, and could explain why R_{xx} has a linear dependence with respect to B in Fig. 4.4(c). A second explanation for the linear component in the longitudinal magnetoresistance R_{xx} could also be the presence of non-negligible channels of parallel conduction in the WQW sample.

We understand that a top gate voltage should induce a linear variation of n_s with V_F , as this is the case in the NQW. However, for the

current flows through the dot.

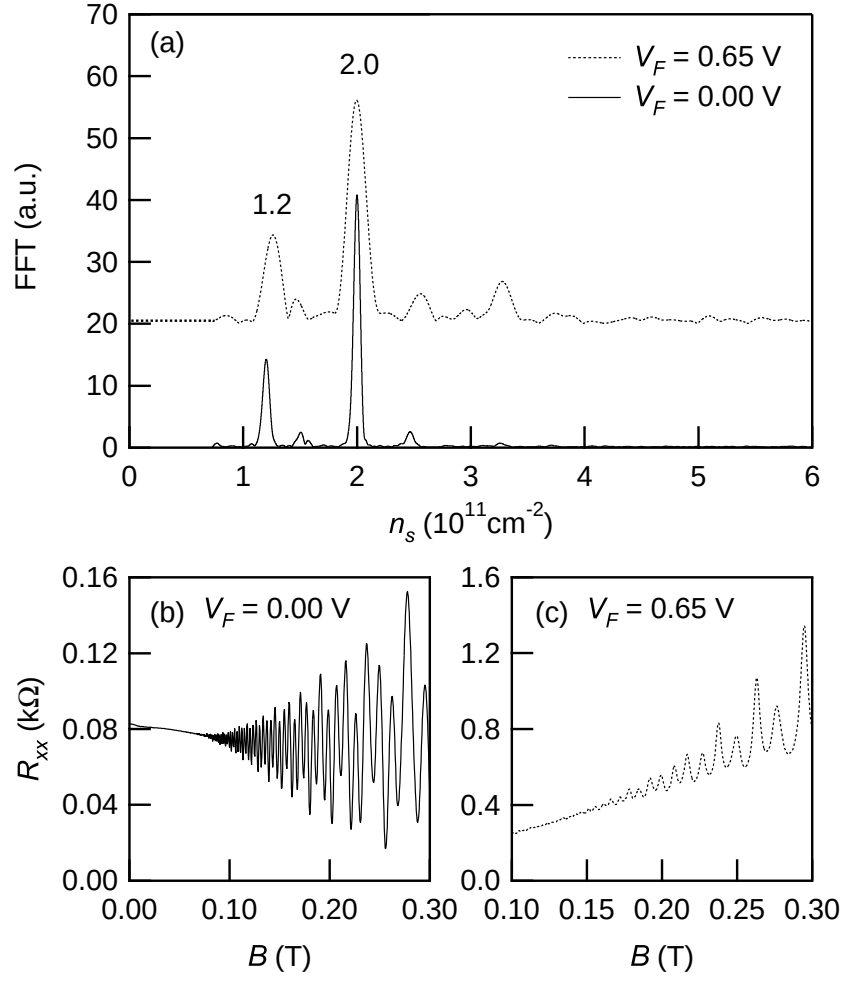


Figure 4.4: (a) Fast Fourier Transform (FFT) of Shubnikov-de Haas oscillations in the WQW at indicated gate voltages V_F . (b)-(c) Corresponding magnetoresistance for $V_F = 0.00 \text{ V}$ and -0.65 V , respectively.

2DEG	w [nm]	n_s [10^{11} cm^{-2}]	μ [$10^5 \text{ cm}^2/\text{Vs}$]	Subbands	l_m [μm]
NQW	15	$3.2 + 4 \times V_F$	5	1	4
WQW	45	$1.2 + 2.0$	16	2	10

Table 4.1: Sample parameters: width w of the quantum well, electron density n_s and its dependence on gate voltage, mobility μ , number of occupied subbands and mean-free path l_m .

WQW, we suspect that the presence of surface states reduces the effect of the electrostatic gate on the subband densities. While a positive V_F of 0.65 V seems not to be strong enough to affect n_s or to change the dot effective area, it is sufficient to open at least one transmission mode across the dot.

Rough estimates of the mean free-path based on the mobilities and average subband densities measured in our NQW and WQW samples give values of 4 μm in the NQW and 10 μm in the WQW - larger than the lateral dimensions of the dots. This comfortably places the two dots in the ballistic regime⁴.

Before discussing the magnetic field alignment with the samples, we summarize in Table 4.4.2 the main parameters of the NQW and WQW 2DEGs.

4.5 Field Calibration and Alignment

The devices are mounted on the sample holder described in Chap. 2. The samples can be tilted *in-situ* with respect to the magnetic field B . The tilt angle θ between the magnetic field and the normal to the 2DEG is precisely controlled by monitoring the Hall resistance $R_{xy}(\theta, B)$ of the parent 2DEG on the Hall bar located on the same piece of wafer, a few hundred microns away from the dot (Fig. 4.2).

Additionally, $R_{xy}(B)$ serves a second purpose: because of an hysteresis phenomenon in the superconducting solenoid, the value of B returned by the magnet source, proportional to the applied current, is not strictly

⁴Concerning scattering in the WQW 2DEG, it is worth noting that in absence of strong intersubband scattering[61, 115], transport in the two occupied subbands will be characterized by two independent mean-free paths. They can be extracted from measurements of commensurability oscillations[118]. However, a precise evaluation of the mean free-path is not critical to UCFs in open dots as long as it is larger than the dot dimensions, which is obviously the case in our samples.

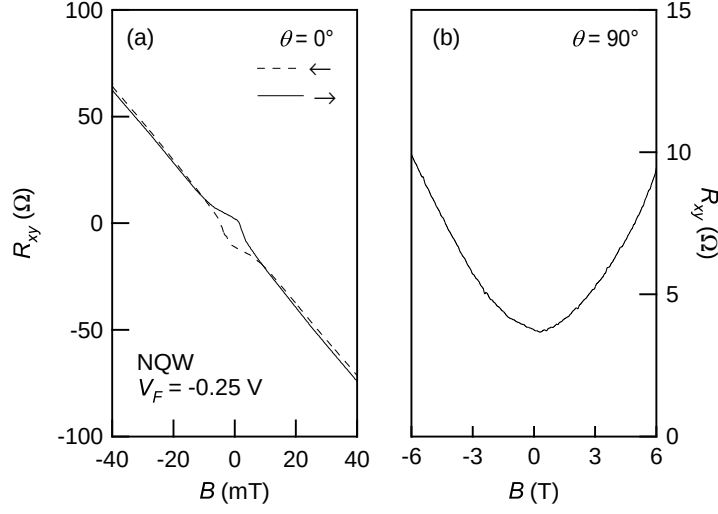


Figure 4.5: Transverse magnetoresistance of the 2DEG measured on the adjacent Hall bar for $V_F = -0.25$ V under a strictly (a) perpendicular and (b) parallel magnetic field.

equal to the real magnetic field applied to the sample. This is clearly visible in Fig. 4.5(a) that shows the transverse magnetoresistance of the NQW under a strictly perpendicular B . For a given sweeping range and sweeping speed of the magnetic field, we thus perform the following calibration procedure:

1. The sample is brought to a strictly perpendicular configuration by maximizing $R_{xy}(\theta, B^{\text{cal}})$ at a given calibration field B^{cal} .
2. The field is swept back and forth at the desired speed between B^{cal} and $-B^{\text{cal}}$.
3. A best linear fit is calculated for the R_{xy} dataset obtained by merging the upstream and downstream magnetoresistance.

$$R_{xy}(B) = \alpha B + R_{\text{res}}, \quad (4.2)$$

Of particular interest is the value R_{res} of the linear function for $B = 0$. It corresponds to a longitudinal component of the resistance induced by a small imperfection in the Hall bar geometry.

4. The magnetic field can now be recalibrated in other measurements using Eq.4.2: for a given value of B given by the source, its associated transverse magnetoresistance R_{xy} is an image of the real field given by $B = (R_{xy} - R_{\text{res}})/\alpha$.

Beside serving as a magnetic field calibration tool, the main purpose of the R_{xy} measurement on the adjacent bar is to precisely control the alignment of B with the plane of the 2DEG. In the case of a perfect parallel alignment, one would expect a zero transverse magnetoresistance R_{xy} at all fields. The reality is slightly different, as can be seen in Fig. 4.5(b) for the NQW under a strictly parallel field. Similar to the residual resistance R_{res} observed under a strictly perpendicular B , the parabolic-like R_{xy} is contributed by a small and unavoidable longitudinal component of the sample's resistance at zero magnetic field. Interestingly, this parabolic dependence has already been observed in low density quasi-two-dimensional layers[146] and is a direct consequence of the orbital effect induced by the parallel field.

When tilting the sample toward a perfectly parallel configuration, we use the following procedure:

1. Starting with a perpendicular field, B is brought to B^{cal} and the sample is crudely rotated until $R_{xy} \simeq 0$. The purpose of this procedure is to avoid any resistance overflow of the lock-in instruments in the next steps.
2. B is then increased to the experiment target field B_0 , without exceeding the superconductor magnet field limit B^{limit} . Once the magnet has reached its target, the sample is slowly rotated and the measured R_{xy} is finely tuned until it reaches 0.
3. The field is swept between $-B_0$ and B_0 . R_{xy} is measured on the adjacent bar.
4. The field is now at a value of B_0 . The sample tilt angle is carefully adjusted in such a way that the magnetoresistance on the multimeter now reads

$$R_{xy}^{\text{new}}(B_0) = \frac{R_{xy}(B_0 - R_{\text{res}}/\alpha) + R_{xy}(-B_0 - R_{\text{res}}/\alpha)}{2}, \quad (4.3)$$

where we took into account the longitudinal correction that was isolated during calibration.

5. The last two steps should be repeated until $R_{xy}(B)$ is perfectly symmetric with respect to $B = -R_{\text{res}}/\alpha$, as shown in Fig. 4.5(b).

From the ratio of α with the slope of a best linear fit of the measured $R_{xy}(B)$, an estimation of the alignment error can be obtained. To avoid any confusion on the true significance of an “alignment error”, it is necessary to enumerate first the possible causes of an error on the sample alignment:

1. a first source of error comes from the settling accuracy that can be obtained with the mechanical tilter. With which precision can we mechanically approach the perfect symmetry condition on $R_{xy}(B)$ and adjust its value to $R_{xy}^{\text{new}}(B_0)$, as determined by Eq. 4.3 ? By finely rotating the tilter back and forth around its target position, an arbitrary low precision can be achieved, that only depends on the time spent in the alignment procedure. With the alignment finely tuned over a period of 2 to 3 hours, a precision better than 0.01° can be obtained. However, for a standard magnetoconductance measurement in the strictly parallel B configuration, performed by a standard graduate student (like this author), an error on the order of 0.05° was considered to be acceptable.
2. a second source of alignment error originates from the error on the measurement of the transverse resistance R_{xy} , contributed by the data acquisition system (multimeters, lock-in, noise filtering in the cryostat). However it is clear, as evidenced by the absence of noise in the trace of Fig. 4.5(b), that this effect is quite negligible with respect to the mechanical settling accuracy.
3. Finally, the stability in time of the tilter position might also deteriorate the global accuracy on the sample alignment, mainly through frictions. But similar to the error on R_{xy} , repeated measurements of the transverse resistance as in Fig. 4.5(b) showed a great stability over time.

The “alignment error” is thus solely determined by the accuracy chosen by the operator for the mechanical settling of the tilter. For the measurements presented in this chapter, the alignment error was kept below 0.05° .

4.6 Perpendicular Field-Induced UCFs

We now turn to our measurements of the NQW and WQW dot magneto-transport properties. In order to validate our fabrication techniques and to verify that the dots are indeed in the ballistic and coherent regime, we first measure their magnetoconductance in a strictly perpendicular field.

Figure 4.6 shows the results of typical magnetoconductance measurements performed at $T = 0.3$ K on the NQW dot under a perpendicular magnetic field ($\theta = 0^\circ$) only. As a reminder, in our coordinate system represented in Fig. 4.2(a), θ is the angle between the magnetic field and the normal to the 2DEG. $\theta = 90^\circ$ thus corresponds to a strictly parallel

B , and $\theta = 0^\circ$ to a strictly perpendicular configuration. We also used a large perpendicular field ($B_\perp$) domain ranging from -0.15 to 0.15 T to confirm that the magnetoconductance through the dot is indeed symmetric in B , as expected for UCFs.

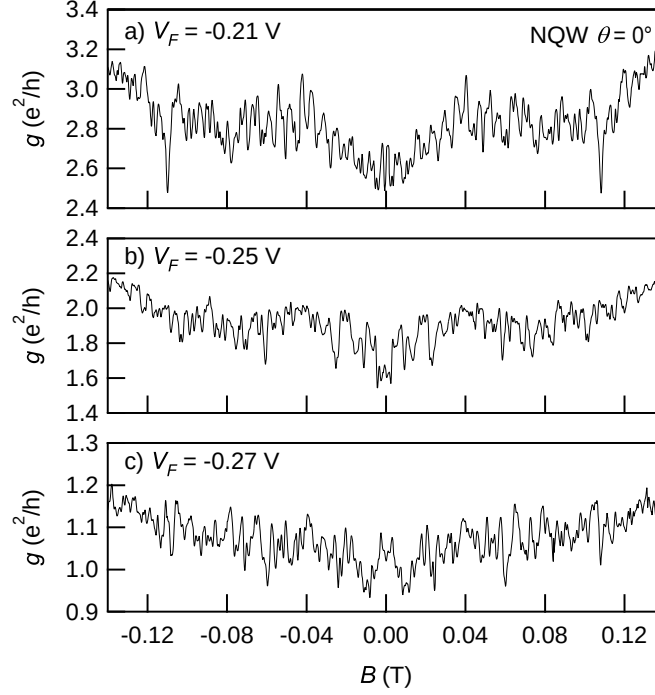


Figure 4.6: (a)-(c) Magnetoconductances of the NQW dot under a strictly perpendicular B at indicated values of the gate voltage V_F .

The conductance traces are presented for three different values of the gate voltage V_F , each corresponding to three dot areas, and to a particular number N_1 and N_2 of transmission modes in the dot openings. More precisely, if we suppose that the number of modes is equal in the two openings ($N = N_1 + N_2 = 2N_1 = 2N_2$), then both N and the average conductance $\langle g \rangle$ across the dot decrease from 3 to 2 between $V_F = -0.21$ and -0.25 V before reaching the quantum limit ($N_1 = N_2 = 1$) for $V_F = -0.27$ V. Here N_1 and N_2 are the number of *spin-degenerate* transmission modes in the dots openings, and $\langle g \rangle$ is the average over B of the conductance $g(B)$.

For all gate voltages, reproducible conductance fluctuations with no clear periodic pattern are superimposed upon a slowly varying background. The investigated B -range is limited to 0.15 T in order to avoid

the coexistence of both interference-induced fluctuations and Shubnikov-de Haas oscillations that appear at higher magnetic fields when the cyclotron radius becomes commensurate with the dot size[141]. As expected from RMT, the fluctuation amplitude increases with $V_F = -0.27$ to -0.21 V as the amplitude is, to first order, proportional to the number of transmission modes, hence to the average conductance $\langle g \rangle$. The background magnetoresistance seen on all magnetoconductance traces is commonly observed in open quantum dots. It corresponds to a magnetic focusing effect between the two dot openings with electron trajectories on the order of, or smaller than, the mean-free path l_m , and will be discussed later.

Moving to the WQW dot, Figures 4.7(a) and (c) show the low- T , raw magnetoconductances of the WQW dot for top-gate biases $V_F = 0.65$ and 0.53 V, respectively. Since we showed, with the NQW, that $g(B)$ is indeed symmetric in B , we now restrict the magnetoconductance traces to the positive B domain only, unless expressly indicated.

Similar to the NQW, $g(B)$ reveals the presence of reproducible fluctuations superimposed on a slowly varying background magnetoconductance. The two gate voltages used for Fig. 4.7 correspond to the most “stable” configurations, both in terms of signal noise and stability over time, that could be obtained. The average conductances for $V_F = 0.65$ and 0.53 V are $\langle g \rangle = 5$ and $1.6 e^2/h$, respectively. Taking into account the occupation of the two lowest subbands in the WQW, a good estimate for the number of transmission modes is $N \simeq 10$ at $V_F = 0.65$. With $V_F = 0.53$ V, the assumption of an equal number of transmission modes in the two dot openings is not compatible with an average conductance of $1.6 e^2/h$. In this particular case, we propose the configuration $N_1 = 1$ and $N_2 = 4$, yielding a total number of modes $N = 5$. We strongly emphasize that, with these occupation numbers, the upper subband is already fully depleted in the vicinity of the narrowest dot opening.

To better visualize the high-frequency fluctuations, we also calculate the conductance variation $\delta g = g - g^*$ in the WQW for $\theta = 0^\circ$ and display it for $V_F = 0.65$ and 0.53 V in Figs. 4.7(b) and (d), respectively. Here g^* is the average, slowly varying, low-frequency background magnetoconductance of Figs. 4.7(a) and (c). As it will be discussed in Sec. 4.8, g^* is also the magnetoconductance that remains at high temperature ($T \geq 2.5$ K). Practically, δg is extracted from $g(B)$ using a high-pass filter with a cutoff frequency f_{cut} . Its purpose is to remove from the magnetoconductance the low-frequency oscillations associated to a focusing effect, and to keep only the high-frequency signal related to quantum interferences. Figure 4.8(a) shows the standard deviation $\sigma(g)$ of the $V_F = 0.65$ V magnetoconductance measured at $\theta = 0^\circ$ in

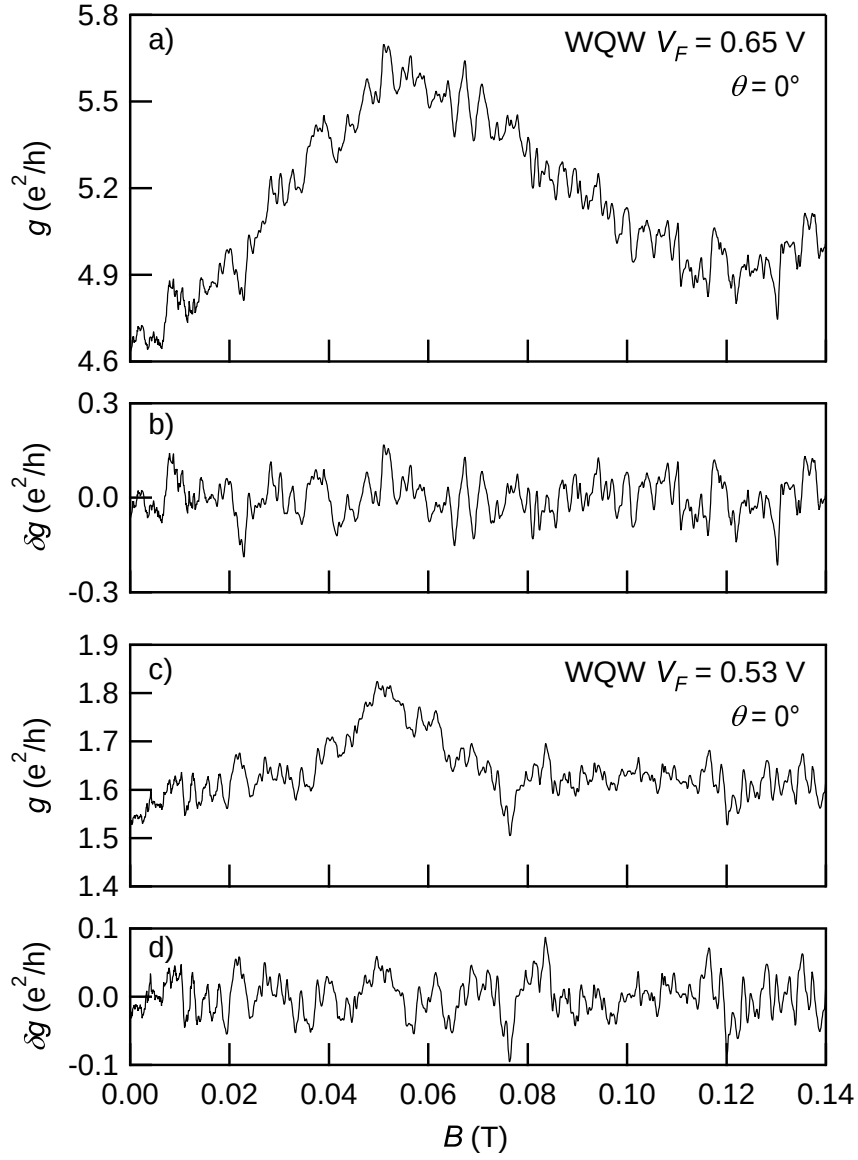


Figure 4.7: Magnetoconductance of the WQW dot under a strictly perpendicular B for $V_F = 0.65$ V (Panel (a)) and 0.53 V (Panel (c)). (b) and (d) The conductance fluctuations in the dot for the same configurations (see text).

the WQW (Fig. 4.7). Here, $\sigma(g)$ first decreases rapidly then saturates above 10 T^{-1} . To evaluate δg , we thus choose a cutoff frequency beyond the saturation point of $\sigma(g)$, namely $f_{\text{cut}} = 15 \text{ T}^{-1}$, valid for both the NQW and WQW dot at $\theta = 0^\circ$.

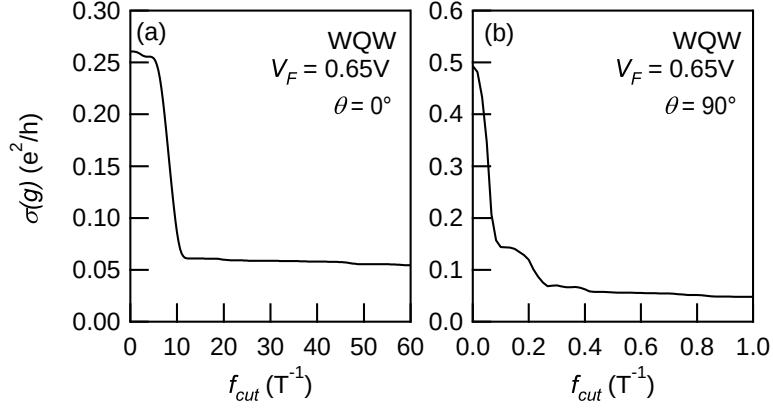


Figure 4.8: Standard deviation of the WQW dot magnetoconductance as a function of the cutoff frequency of a high-pass filter (a) under a strictly perpendicular and (b) parallel B for $V_F = 0.65 \text{ V}$.

Now, looking back at Figs. 4.7(b) and (d), the fluctuation fingerprint is quite similar at $V_F = 0.65$ and 0.53 V , the only difference being their amplitude that decreases by a factor of 3 between 0.53 and 0.65 V , in accordance with the reduction of the average conductance $\langle g \rangle$ from 5 to $1.6 e^2/h$.

4.7 Fluctuations under a Strictly Parallel Field

As a next step, the sample is tilted away from its perpendicular ($\theta = 0^\circ$) configuration and is brought to a strictly parallel ($\theta = 90^\circ$) position with respect to the magnetic field using the procedure described in Sec. 4.5.

4.7.1 Narrow Quantum Well

Using the same presentation as for the $\theta = 0^\circ$ magnetoconductances, $g(B)$ of the NQW dot subject to strictly parallel field at $V_F = -0.25 \text{ V}$ is shown in Fig. 4.9 for $V_F = -0.21, -0.25$ and -0.27 V . Surprisingly, fluctuations are still present for $\theta = 90^\circ$ even though a parallel field should have no effect on a purely 2D open quantum dot. However, while fluctuations are still clearly visible for $\theta = 90^\circ$, their main frequencies are

much smaller than that observed for $\theta = 0^\circ$: only a few, quasi-periodic oscillations are present in the range $0 \lesssim B \lesssim 6$ T. We also note that fluctuations are present for the three values of V_F : the conductances share similar periodicities with maybe more complexity in the fluctuation fingerprint for $V_F = -0.21$ V than for -0.27 V.

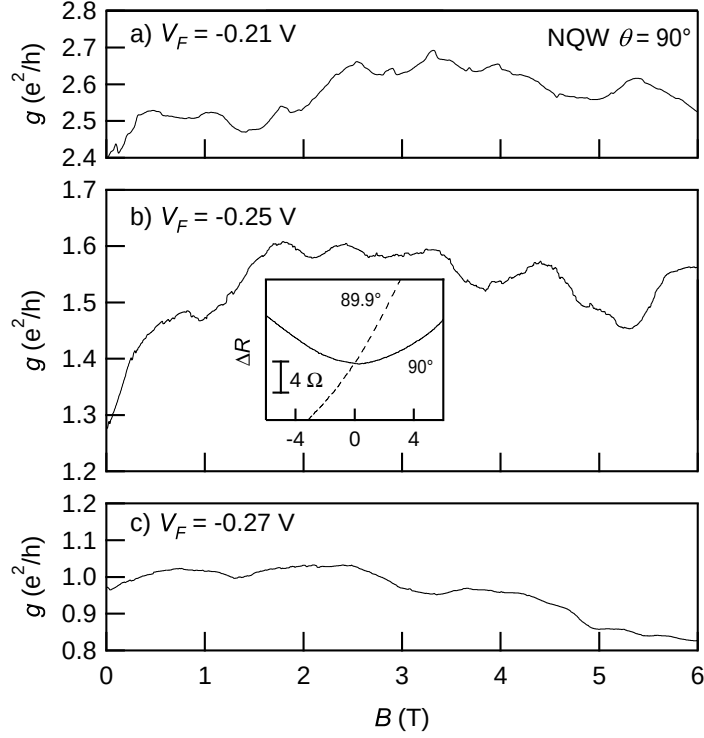


Figure 4.9: (a)-(c) Magnetoconductance of the NQW dot under a strictly parallel B at indicated values of the gate voltage V_F . Inset to panel (a): Transverse resistances of the parent 2DEG measured on the adjacent Hall bar for $V_F = -0.25$ V.

The first natural reaction is to wonder if these $\theta = 90^\circ$ fluctuations could not be $\theta = 0^\circ$ leftovers induced by an imperfect alignment of the field with the plane of the 2DEG. To invalidate this assertion, the inset to Fig. 4.9(b) shows the transverse resistance measured on the adjacent Hall bar at what is supposed to be a perfectly parallel configuration, along with the trace taken at an angle slightly tilted away from 90° ($\theta = 89.9^\circ$). In the case of the strictly parallel configuration, the calculated angle is $\theta = 89.98^\circ$, well within the selected upper boundary on the alignment error.

As thoroughly discussed in Sec. 4.5, a strictly parallel B corresponds to a symmetric, quasi-parabolic B -dependence of the ΔR_{xy} resistance. The difference between the $\theta = 89.9$ and 90° curves along with the very small error on the $\theta = 90^\circ$ target (0.02°) demonstrate the precision achieved with the experimental setup. When the total field B is equal to 6 T, such an error would correspond to a residual and negligible perpendicular component of B on the order of $B \sin(0.02) = 3.5$ mT. Although it is not 100% safe to affirm at this point that the fluctuations at $\theta = 90^\circ$ are solely induced by the parallel B , our confidence on the precision of the field alignment would tend to rule out the effect of a residual perpendicular component of the field. Nevertheless, it is also important to point out that, near $\theta = 90^\circ$, the magnitude of the perpendicular component of B , function of $\cos \theta$, strongly changes with θ . However, the magnetoconductance remains essentially unchanged when the sample is slightly tilted away from $\theta = 90^\circ$, an evidence that directly points toward the parallel field as the source of the fluctuations. We will return to this discussion in Chapter 6 when we study the magnetoconductance of OQDs under tilted magnetic fields.

4.7.2 Wide Quantum Well

Similar to the NQW dot, the WQW sample is now tilted into a strictly parallel configuration. Using the same layout as Fig. 4.7, Fig. 4.10(a) shows the low- T magnetoconductance of the WQW dot at $\theta = 90^\circ$ for a top-gate bias (a) $V_F = 0.65$ V and (c) 0.53 V. It reveals the presence of reproducible UCFs superposed on a slowly varying background magnetoconductance, which we first discuss using the $V_F = 0.65$ V trace.

The zero-magnetic field average conductance $\langle g \rangle$ is $5 e^2/h$. As B is swept up, the magnetoconductance decreases monotonically to a value of $4 e^2/h$ around 4 T and remains relatively constant for higher fields. As opposed to the $\theta = 0^\circ$ background of Fig. 4.7(a), the reduction in $\langle g \rangle$ is not linked to focusing effects in the dot. Instead, it indicates that the threshold of magnetic field-induced subband depopulation is around $B = 4$ T.

The onset of subband depopulation is clearly visible on the transverse resistance measurement performed on the adjacent Hall bar and shown in the inset to Fig. 4.10(a). The drop in the transverse resistance around $B \approx 4$ T is a result of the reduction of inter-subband scattering as the upper subband becomes fully depleted[61, 115]. We note that the magnetic depopulation is expected to occur at essentially the same B in the unpatterned 2DEG and in the interior of the dot where the lateral quantum confinement is negligible. However, in the vicinity of the dot

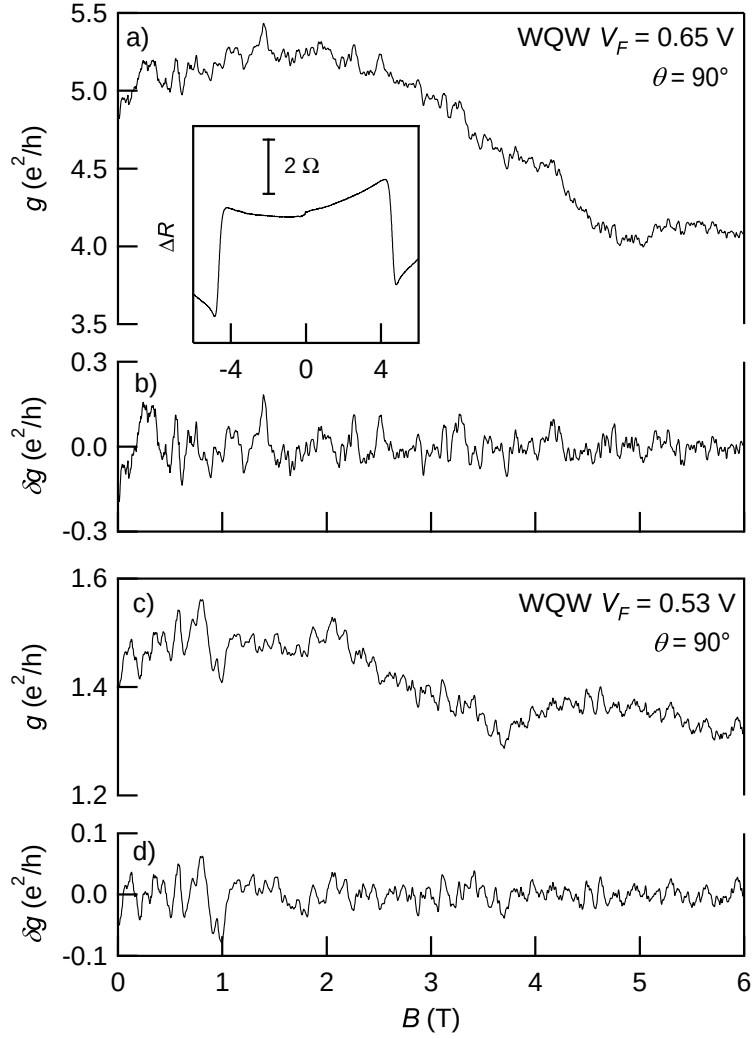


Figure 4.10: Magnetoconductance of the WQW dot under a strictly parallel B for $V_F = 0.65$ V (Panel (a)) and 0.53 V (Panel (c)). (b) and (d) UCFs of the dot for the same configurations (see text). Inset: Transverse resistance of the parent 2DEG measured on the adjacent Hall bar, at indicated tilt angles.

2DEG	V_F	n_s	$B = 0$ T			$B = 6$ T			f_{cut}	
			N1	N2	N	N1	N2	N	0°	90°
NQW	-0.21	2.4	3	3	6	3	3	6	15	0
NQW	-0.25	2.2	2	2	4	2	2	4	15	0
NQW	-0.27	2.1	1	1	2	1	1	2	15	0
WQW	0.65	$2.0 + 1.2$	5	5	10	4	4	8	15	0.5
WQW	0.53	$2.0 + 1.2$	1	3	4	1	2	3	15	0.5

Table 4.2: Mode occupation in the NQW and WQW dots for $\theta = 90^\circ$, as a function of the parallel magnetic field B : gate voltage V_F , electron density n_s [10^{11} cm^{-2}], number of modes in the first (N_1) and second (N_2) QPC, total of modes N across the dot, cutoff frequencies [T^{-1}] used in the extraction of δg at $\theta = 0^\circ$ and 90° .

openings, the additional lateral confinement generates two sets of one-dimensional (1D) states, each from an occupied subband.

Around the dot openings, the effect of B is to slowly depopulate these 1D states[145, 17]. In a simple picture, above the onset of 2D subband depopulation ($B > 4$ T), it is expected that the 1D states of the upper subband are located above E_F and only the lower subband 1D states contribute to the transmission modes. Hence, in Fig. 4.7(a), the 8 transmission modes in each dot opening, associated with the $\langle g \rangle = 4 e^2/h$ average conductance at high B , are contributed only by the 1D states of the lower subband. Similarly, for $V_F = 0.53$ V, the contribution of the lower subband is 3 modes, as the $\theta = 90^\circ$ background magnetoresistance $\langle g \rangle$ in Fig. 4.7(c) is reduced from 1.5 to $1.3 e^2/h$ between $B = 0$ and 6 T. This is in agreement with the assumption we made earlier in the text that the upper subband was depleted at $B = 0$ T in the vicinity of the narrower opening for $V_F = 0.53$ V.

To clarify this point, Table 4.2 summarizes the mode occupation in the NQW and WQW dot, in the presence or absence of a parallel B .

To isolate the fluctuations $\delta g(B)$, the background magnetoconductance g^* is subtracted from the raw data using a high-pass filter with a cutoff frequency of 0.5 T^{-1} (see also Table 4.2). Similar to the $\theta = 0^\circ$ case, the cutoff frequency is selected beyond the onset of saturation observed in the standard deviation of the high-pass filtered conductance fluctuations (Fig. 4.8(b)).

For the $\theta = 90^\circ$ data shown in Fig. 4.10(b) and (d) for $V_F = 0.65$ and 0.53 V, respectively, a significant reduction in the UCFs amplitude

with increasing B is easily distinguishable. For example with $V_F = 0.65$, while δg has a value (standard deviation) of about $0.07 e^2/h$ near $B = 0$, it barely reaches $0.03 e^2/h$ for $B \gtrsim 5$ T. This is reminiscent of the amplitude reduction of the *perpendicular* field-induced UCFs, observed by Folk *et al.*[68] in quantum dots patterned on GaAs-Al_xGa_{1-x}As heterojunctions. As presented in Sec. 4.2, this effect was subsequently attributed[80] to an enhancement of spin-orbit coupling with increasing parallel B . Because of the symmetry of the QW and the absence in our sample of a crossover between weak localization and antilocalization[132] in the $\theta = 0^\circ$ magnetoconductances of the NQW (Fig. 4.6) and WQW (Fig. 4.7), we can rule out the possibility that spin-orbit coupling plays a role in our experiment with respect to the amplitude of parallel B -induced fluctuations.

Before we discuss in detail the variance of the UCFs and their origin, especially in the WQW sample, it is interesting to compare the $\theta = 90^\circ$ magnetoconductance of the WQW dot (Fig. 4.10) with similar data obtained for the NQW dot (Fig. 4.9). The difference is striking: while fluctuations are clearly visible for the NQW dot, they have a much poorer frequency spectrum, *i.e.* higher frequency components are visible in the WQW dot but only a few oscillations are present in the range $0 \lesssim B \lesssim 6$ T in the NQW dot. The observed difference in fluctuation frequencies between the NQW and WQW dots distinctively highlights the role played by the QW thickness in the creation of UCFs.

As we are now investigating the possible origins of the parallel B fluctuations, we first have to confirm that these low-frequency oscillations cannot be simply linked to a ballistic effect[167, 155] across the open dot but instead, are generated by interference effects between electron trajectories.

4.8 Temperature Dependence

To achieve this goal, we investigated the T dependence of the magnetoconductance fluctuations in the NQW and WQW dots. The strength of the fluctuations as a function of temperature is indeed a good indication of their origin. The electron mean free-path, the key player in electron focusing [84, 77], is much less sensitive to temperature than the phase relaxation length that limits the UCFs amplitude [42, 90, 138].

4.8.1 Narrow Quantum Well

Figure 4.11(a) shows the temperature dependence of the fluctuations amplitude $\delta g(B) = g(B) - g^*(B)$ in the NQW dot under a perpendicu-

lar B only for $V_F = -0.25$ V. Upon inspection of Fig. 4.11(a), it is clear that $\delta g(B)$ rapidly decreases with increasing temperature. This behavior points toward interference effects as the source of the fluctuations. More quantitatively, the fluctuation amplitude, expressed in terms of its standard deviation $\sigma(g)$, is reduced from $0.076 e^2/h$ at $T = 0.3$ K to $0.002 e^2/h$ at $T = 2.1$ K (Fig. 4.12[◇]).

The T dependence of fluctuations generated by a strictly parallel magnetic field in the NQW dot is presented in Fig. 4.11(b). Clearly visible at $T = 0.3$ K are the relatively low frequency, almost periodic, fluctuations superimposed upon a slowly varying background. As the temperature is increased, the fluctuations progressively vanish, leaving only the slowly varying background magnetoconductance at $T = 2.8$ K. Because of the small number of fluctuation periods observed at $\theta = 90^\circ$ and their limited frequency distribution compared to their $B_\perp$ -generated counterparts, it is impossible to extract the standard deviation of the fluctuations. Therefore, the discussion must be limited to a simple analysis of a few oscillations on a selected B -range (Fig. 4.11(b), gray area), where the peak to peak amplitude decreases from 0.12 to $0.02 e^2/h$ between $T = 0.3$ and 2.8 K. The four data points are illustrated in Fig. 4.12[■]. The best power law fit gives a $T^{-0.8}$ temperature dependence, close to the $T^{-0.6}$ law observed for the perpendicular B -induced fluctuations. Even though this comparison is not fully conclusive, it nevertheless seems reasonable to give a common electron coherence origin to these similar reductions of the fluctuation amplitude under both a perpendicular and parallel B . Further arguments in favor of the quantum interference origin of the fluctuations will be given later in this thesis.

To compare the T -dependence of the $\theta = 0^\circ$ and $\theta = 90^\circ$ fluctuations, we now calculate τ_φ in the perpendicular magnetic field configuration. Using a result of RMT[69] discussed in Sec.3.2.6 and valid for interference-induced UCFs, τ_φ can be extracted from the variance of the conductance fluctuations using the generalized formula

$$\begin{aligned} \text{var}(g) &= \langle \delta g^2 \rangle - \langle \delta g \rangle^2 = \sigma^2(g) \\ &= \int_0^\infty \int_0^\infty f'(E) f'(E') C(E, E') dE dE', \end{aligned} \quad (4.4)$$

where f' is the first derivative of the Fermi distribution. As a reminder,

$$C(E, E') = \langle g \rangle^2 / [(N + N_\varphi)^2 + 4\pi(E - E')^2 / \Delta^2] \quad (4.5)$$

is the conductance correlator, $\Delta = 2\pi\hbar^2 / Am^*$ the mean level spacing, m^* the electron effective mass, $\langle g \rangle$ is the average conductance, A the dot area, and $N_\varphi = 2\pi\hbar / (\tau_\varphi \Delta)$ and N are the number of dephasing

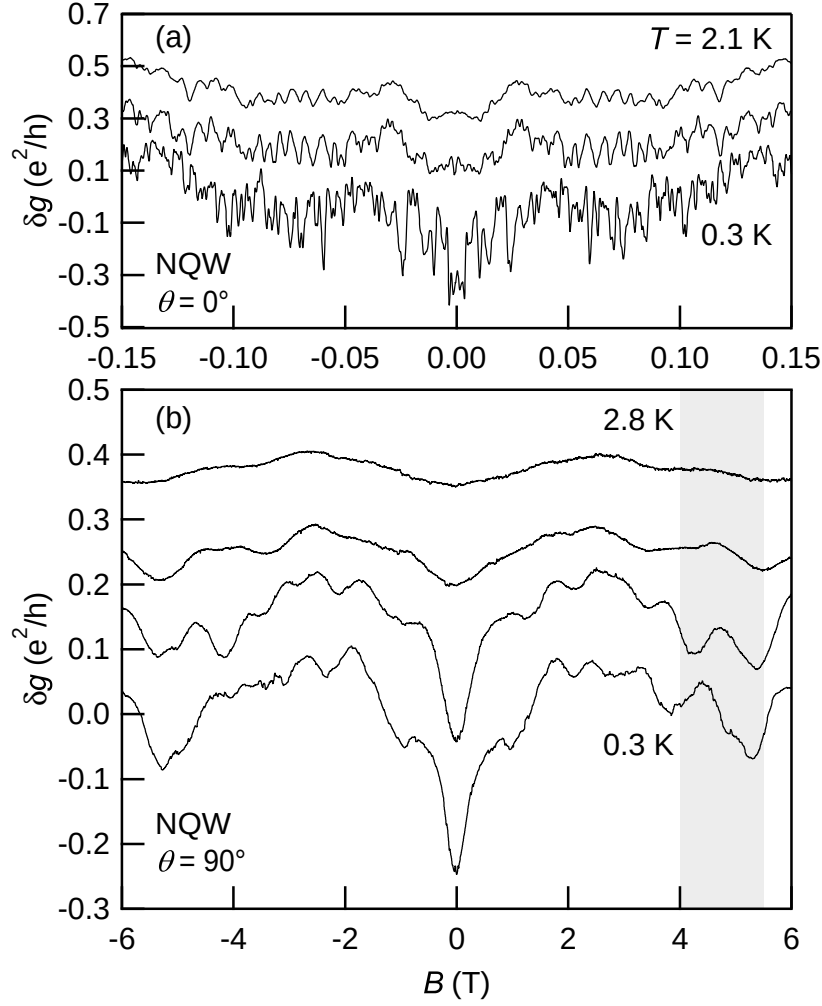


Figure 4.11: (a) T dependence of the NQW dot magnetoconductance under a strictly *perpendicular* B . The temperatures are 2.1, 1.1 and 0.3 K from top to bottom. (b) Temperature dependence of the NQW dot magnetoconductance under a strictly *parallel* B . The temperatures are 2.8, 1.5, 0.8, and 0.3 K from top to bottom, respectively. The gray area indicates the B -range for which the peak-to-peak amplitude of the fluctuations is determined (see text). The traces in panels (a) and (b) are shifted in steps of $0.1 e^2/h$ with respect to the $T = 0.3$ K curves.

channels and of transmission modes in the dot openings, respectively. Thermal smearing was explicitly included as the mean level spacing is not negligible with respect to the thermal energy [$\Delta/(k_B T) \simeq 0.1$ at $T = 0.3$ K].

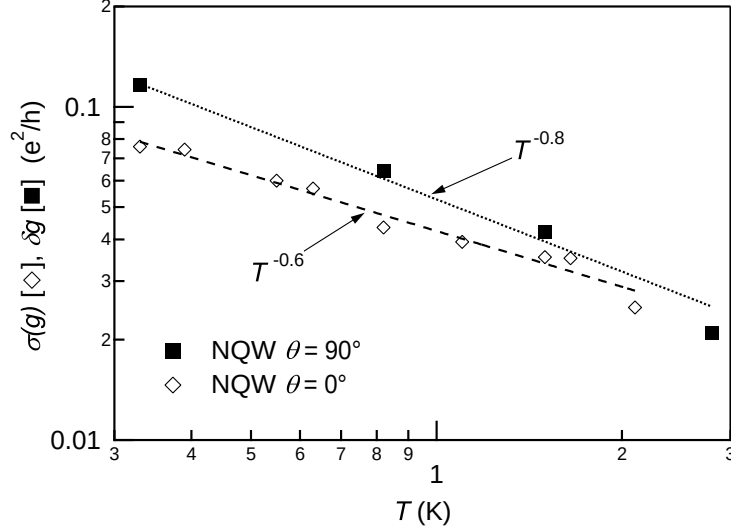


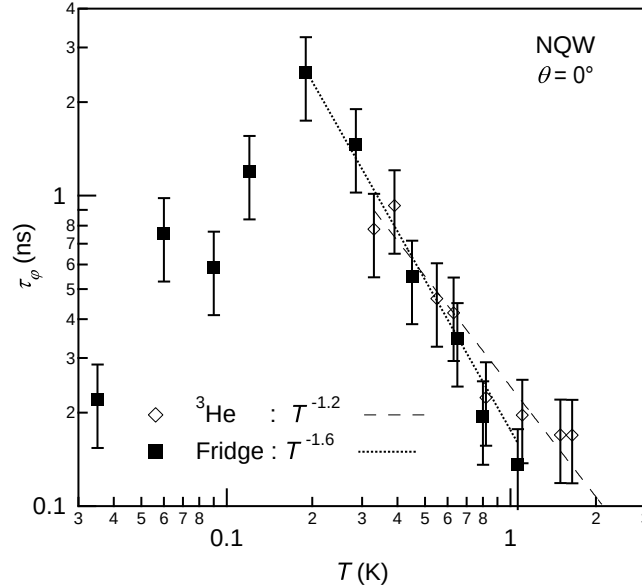
Figure 4.12: Fluctuation amplitude vs. T at $\theta = 0^\circ$ (standard deviation, $\diamond$) and $\theta = 90^\circ$ (peak to peak, $\blacksquare$) in the NQW for $V_F = -0.25$ V. The dashed curves are two-parameter power law fits to data points.

The values of τ_φ calculated from Eq. (4.4) are shown on Fig. 4.13. We point out that the τ_φ dataset is composed of values extracted from measurements independently performed in the ^3He ($\diamond$) and dilution refrigerator ($\blacksquare$) cryostats. Down to 0.2 K, τ_φ follows a $T^{-1.2}$ power law ($T^{-1.6}$ if we consider only the dilution refrigerator dataset), consistent with results from the literature[92, 138]. The corresponding number of dephasing channels N_φ changes from 3 to 20 when the temperature T increases from 0.3 to 2.8 K. At $T = 0.3$ K, N_φ is thus close to the number of transmission modes in the dot openings ($N \simeq 4$ for $\langle g \rangle \simeq 2 e^2/h$). Surprisingly, below 0.2 K, τ_φ does not keep increasing nor does it saturate, as it has been observed in other experiments on OQDs[92, 138]. We have two explanations for this unexpected behavior.

1. We suspect that the electron temperature below 0.2 K starts to significantly deviate from the temperature returned by the sensor: the sensor temperature is an image of the lattice temperature

when a good thermal matching exists between the sample and the sensor⁵ but there is no guarantee that the lattice and electron temperatures are equal. A method to precisely evaluate the electron temperature of the dot reservoirs would be to close the dot beyond the quantum limit and to measure the Coulomb Blockade peak width[154].

2. The method to evaluate τ_φ , based on Eq. (4.4), is only strictly valid when the number of modes, including the virtual dephasing channels N_φ , is significantly greater than 1. But for $T < 0.2$ K, N_φ is precisely on the order or smaller than 1. This raises some doubt on the validity of the RMT model for the calculation of τ_φ below 0.2 K.



the $T > 0.2$ K power law dependences of τ_φ in the ^3He and dilution refrigerator. Moreover, a similar temperature dependence is observed in the fluctuations amplitude of both the $\theta = 0^\circ$ and $\theta = 90^\circ$ magnetoconductances. This definitely supports our claim that a common origin, namely quantum interferences, is responsible for the fluctuations observed in both configurations.

4.8.2 Wide Quantum Well

Similar to the NQW dot, the magnetoconductance curves of the WQW dot at $V_F = 0.65$ V for the temperature range $0.3 \lesssim T \lesssim 2.5$ K, shown on Fig. 4.14, give insight on the strength of the $\theta = 0^\circ$ and $\theta = 90^\circ$ fluctuations with respect to T . The curves are presented here for a symmetric B -range.

Apart from the difference in the shape of the background magnetoconductance, the $\theta = 0^\circ$ fingerprint in the WQW dot (Fig. 4.14(a)) is similar to the fluctuations observed at $\theta = 0^\circ$ in the NQW dot (Fig. 4.11(a)). The variance of the fluctuations $\delta g(B)$ is constant over the investigated B -range ($\text{var}(g) = 3.4 \times 10^{-3} e^4/h^2$) and $N_\varphi \simeq 27$ at $T = 0.3$ K. As stated earlier in the text, N_φ in the fictitious voltage probe model[32] only depends upon the dephasing time τ_φ and the mean-level spacing Δ , which is identical for the NQW and WQW dots because they have the same shape and area. The larger N_φ observed for the WQW dot is thus caused by an increased dephasing that can have several origins[174, 151]: higher electron-electron scattering rate (see Sec. 3.2.4), Nyquist dephasing[10], switching noise or inter-subband scattering[61, 115].

In the T -dependence of the $\theta = 90^\circ$ traces (Fig. 4.14(b)), some features are noteworthy. First, unlike the fluctuation pattern, the background magnetoconductance, associated with subband depopulation, remains relatively unchanged with increasing T . Since the 1D confinement energy around the dot openings is much larger than $k_B T$, even at 2.5 K, the effects of magnetic subband depopulation on the background magnetoconductance are still strong at these relatively high temperatures. Second, we note that above $T = 2.5$ K, the $\theta = 90^\circ$ magnetoconductance is featureless and no longer depends on T .

In a more quantitative way, and following the same procedure used for the $\theta = 0^\circ$ magnetoconductance in the NQW dot, it is possible to obtain an estimate of τ_φ for the $\theta = 0^\circ$ fluctuations in the WQW dot. It is found that τ_φ decreases with the temperature according to a $T^{-0.8}$ power law (Fig. 4.15, Δ). This is comparable to the $T^{-1.2}$ dependence of the $\theta = 0^\circ$ fluctuations observed in the NQW dot (Fig. 4.13), with

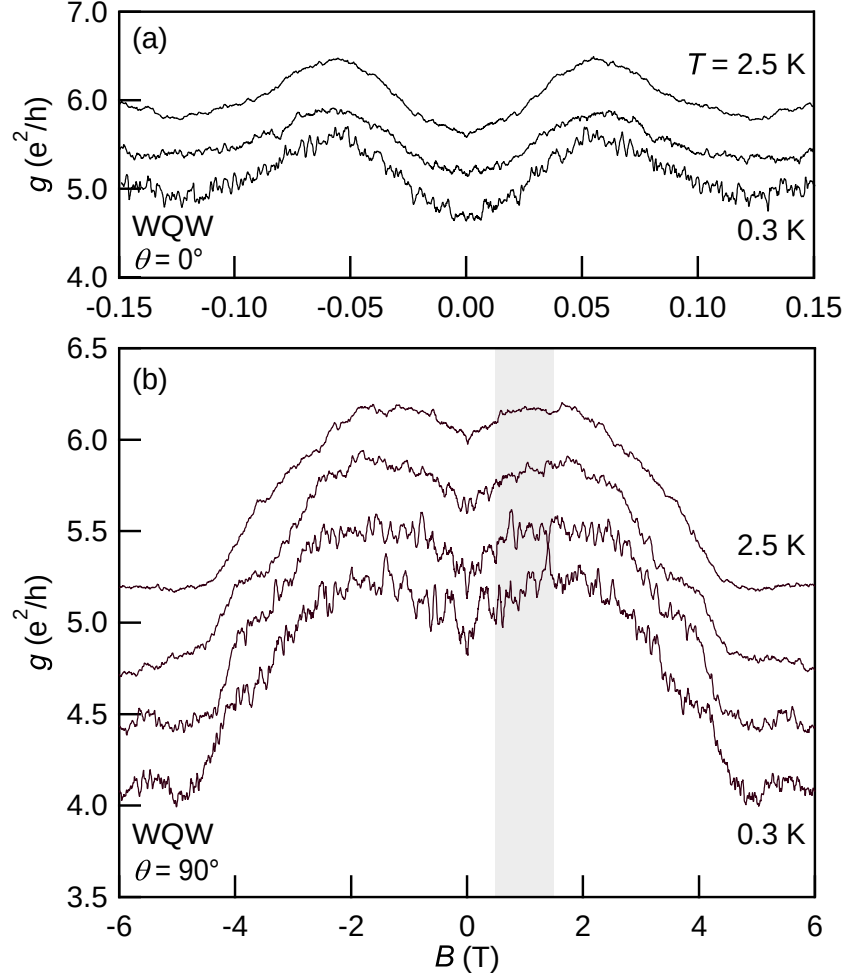


Figure 4.14: (a) Temperature dependence of the WQW dot magnetoconductance under a strictly *perpendicular* B . The temperatures are 2.5, 1.5 and 0.3 K from top to bottom. (b) Magnetoconductance traces for the WQW dot under a strictly *parallel* B at various T 's. From the upper to the lower curve, the temperatures are 0.3, 0.6, 1.5 and 2.5 K, respectively. The gray area indicates the B -range for which the fluctuations amplitude is determined (see text). The traces in panels (a) and (b) are shifted in steps of $0.4 e^2/h$ with respect to the $T = 0.3$ K magnetoconductance curves.

probably a greater dephasing taking place in the WQW dot than in the NQW dot, as discussed earlier in the section.

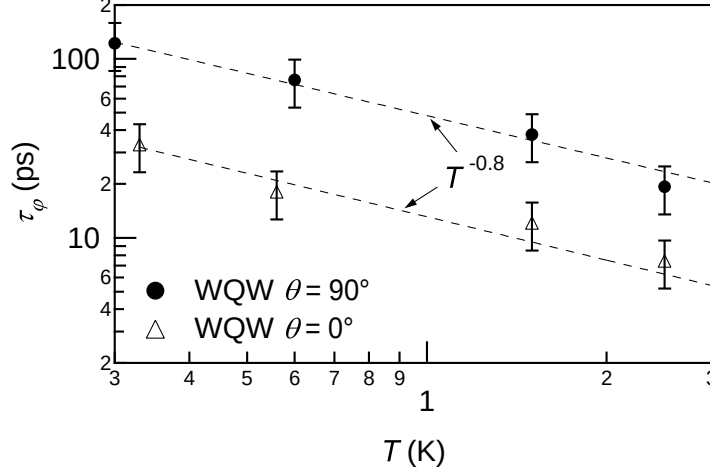


Figure 4.15: T dependence of the dephasing time τ_ϕ extracted from Eq. (4.4) for the $\theta = 0^\circ$ ($\triangle$) and $\theta = 90^\circ$ ($\bullet$) fluctuations in the WQW at $V_F = 0.65$ V. The dashed lines are two-parameter power law fits to the data points.

The reduction of fluctuations amplitude with increasing parallel magnetic field, clearly visible on the $T = 0.3$ K trace of Fig. 4.14(b) will be thoroughly discussed in Sec. 4.10. But precisely because this amplitude is not constant with B , a quantitative analysis using Eq. (4.4) has to be performed in a small B -range, as opposed to the $\theta = 0^\circ$ fluctuations where the full B -range can be used. For the gray area displayed in Fig. 4.14, the standard deviation of fluctuations amplitude decreases from 0.066 to $0.013 e^2/h$ between 0.3 and 2.5 K. The corresponding dephasing time τ_ϕ follows a $T^{-0.8}$ power law (Fig. 4.15, $\bullet$). This behavior is consistent with the amplitude reduction of both the parallel *and* perpendicular B -induced fluctuations observed in the NQW dot [Fig. 4.13]. The similar T dependencies for the fluctuations observed at $\theta = 90^\circ$ in the NQW and WQW dots confirm that they have a common origin, *i.e.* coherent interference effects as opposed to focusing. As a result, we will now use the term “UCFs” to characterize both the perpendicular and parallel B -induced fluctuations observed in the NQW and WQW dot.

Before closing this section, we would like to compare the τ_ϕ calculations of Figs. 4.13 and 4.15 to a recent work[78] by B. Hackens *et*

al. where it is proposed that the dwell time is the limiting factor for electron interference effects in QDs at low temperature[90, 138]. More precisely, the authors use the following form of Matthiesen’s rule to derive the effective phase relaxation time τ_φ in an QD:

$$\frac{1}{\tau_\varphi} = \frac{1}{\tau_\varphi^{\text{int}}} + \frac{1}{\tau_d}, \quad (4.6)$$

where $\tau_d = 2\pi\hbar/\Delta N$ is the dot dwell time, and $\tau_\varphi^{\text{int}}$ is an intrinsic phase relaxation time that typically follows a power law with respect to temperature ($\tau_\varphi^{\text{int}} = \alpha T^{-\beta}$). Because all τ ’s must be positive, the condition for a saturation of τ_φ with decreasing T is $\tau_\varphi = \tau_d$. In our dots, taken into account the number of transmission modes given in Table 4.2, we obtain $\tau_d = 0.6$ ns for the NQW dot at $V_F = -0.25$ V and $\tau_d = 0.3$ ns for the WQW dot at $V_F = 0.65$ V. Comparing these values with the traces of τ_φ of Figs. 4.13 and 4.15, we note that τ_φ at $T = 0.3$ K is still below the saturation value of $\tau_d = 0.3$ ns. In the case of the NQW, if one sets aside the $T < 0.2$ K domain where there is an uncertainty on the electron temperature, a few points in Fig. 4.13 have values of τ_φ greater than τ_d . However, as already mentioned in Sec. 4.8.1, the number of dephasing channels N_φ around $T = 0.3$ K is close to 1 which is the lower validity limit of Eq. (4.4). As a result, the calculated value of τ_φ might be overestimated at very low-temperatures. We thus conclude that our data, while at the limit of the range of validity of the models, are not inconsistent with the paper[78] of Hackens *et al.*.

4.9 Orbital Effect

Now that we have the confirmation that both the perpendicular and parallel B fluctuations bear the signature of UCFs, and cannot be simply linked to a ballistic effect[167, 155], we discuss the origin of the parallel UCFs. Qualitatively, we understand the data presented in Fig. 4.10 as follows. As discussed in Sec. 3.3.2, the influence of a parallel B on electron dynamics in the dot is twofold[82, 127]. First, it lifts the symmetry in momentum space of the electron dispersion relation. Second, it causes an anisotropic effective mass enhancement which increases the 2D density of states and, for a 2D gas with a fixed sheet density n_s , induces a magnetic depopulation of the electronic levels by reducing the Fermi energy calculated from the bottom of the conduction band. Both effects are direct consequences of the finite thickness of realistic 2DEGs, and of the orbital motion of the electrons. From a “classical” point of view, electron trajectories are not only located in the plane of

the 2DEG, bouncing at the dot walls, but also have a component that lies along the transverse direction of the QW[152, 98]. These pseudo three-dimensional orbital trajectories enclose a flux that becomes non-negligible at high B and contribute to the creation of a UCFs pattern. If we consider the WQW dot, the magnetic depopulation of the 2DEG in a parallel B can also be cast in a simple model, sufficient to evaluate the energy shift of the Fermi level E_F with increasing B .

Assuming two occupied subbands at $B = 0$, the position of E_F with respect to the bottom of the lower conduction subband is given to first order by[83]

$$E_F(B) = \frac{\pi \hbar^2 n_s}{2m^*} + \frac{\Delta E_{21}^0}{2} - \frac{m^* \omega_c^2}{4} (\langle z^2 \rangle_1 - \langle z^2 \rangle_0), \quad (4.7)$$

where m^* is the electron effective mass at $B = 0$, ΔE_{21}^0 is the subband separation at $B = 0$ and $\omega_c = eB/m^*$ is the cyclotron frequency. Here $\langle z^2 \rangle_0$ and $\langle z^2 \rangle_1$ represent the square of the mean widths of the lower and upper subband ground-state wavefunctions at $B=0$, respectively. The Fermi energy experiences a quadratic shift in B which is compensated by an increase of the density of states so that n_s remains constant. Because of the large number of electrons in the dot, it is reasonable to consider that this quasi-2D model also applies to the dot itself. Applying Eq. 4.7 to the WQW dot and using a simple, infinite square potential of width $w = 45$ nm, $\Delta E_{21}^0 = 3\hbar^2\pi^2/2m^*w^2$ and E_F at $B = 0$ are 8.3 and 9.8 meV, respectively. For a subband depopulation that is complete at $B \approx 4$ T, as evidenced by the Inset to Fig. 4.10(a), Eq. 4.7 then gives a non negligible quadratic shift in B of $0.1 B^2$ [meV T⁻²]. This magnetic field-induced shift of E_F is very similar in amplitude with energy ranges covered in other UCFs measurements[101] where E_F was changed by means of an electrostatic gate.

For the NQW dot, the E_F shift is expected to be about 9 times smaller than for the WQW dot, as $E_F(B) \propto \langle z^2 \rangle_0 B^2$ when a single subband is occupied. As a result, fewer electron trajectories are probed by the magnetic field and the UCFs frequency is strongly reduced as seen in Fig. 4.9. These findings, based on a simple model of subband depopulation, further confirm that a parallel B -induced orbital effect can play a similar role as a perpendicular B or an electrostatic gate, and generate UCFs like those observed in Figs. 4.9 and 4.10.

Before closing this section, one crucial topic remains to be discussed: during the presentation in Sec. 4.7 of the strictly parallel B measurements, we never tackled the question of the precise orientation of B in the 2D plane. We suspect that the orientation of the in-plane field can have a twofold influence on the UCFs fingerprint in the dot:

1. As the parallel B couples to the 1D states around the dot openings, it is expected that the 1D energy levels[145] will depend on the orientation of B . As a result, electron injection into the dot might be modified, thus changing the UCFs pattern, especially for stable, wave-scarring[19, 21] electron trajectories associated to a mixed (chaotic and regular) phase space. However, if transport is mainly ergodic, the UCFs power spectrum should remain unchanged as long as the orientation of B does not modify the multi subband occupation of the 1D states, *i.e.* the number of transmission modes in the dot openings.
2. In the regime of strong in-plane magnetic field B , it has been recently shown[106, 62] that the spin splitting and spin relaxation rates can vary with the in-plane orientation of B due to the crystalline anisotropy of spin-orbit coupling[62]. As spin relaxation can be enhanced by a parallel B , like in the Harvard experiment described in Sec. 4.2, giving rise to a reduction of the UCFs variance, it is thus expected that the amplitude of the variance reduction will also depend on the orientation of the parallel B .

These questions, related to the orientation of the parallel B , remain essentially unexplored and, unfortunately, were not further investigated in this work.

4.10 Variance Calculations

In Sec. 4.7.2, we observed a significant reduction, with increasing B , of the $\theta = 90^\circ$ UCFs amplitude $\delta g(B)$ in the WQW dot. This is clearly visible in Figs. 4.10(b) and (d). Unfortunately, such an observation is impossible with the NQW dot as the parallel B fluctuations are not rich enough frequency-wise.

In order to analyze the amplitude reduction in the WQW dot in a more quantitative way, we now perform a variance analysis of its $\theta = 90^\circ$ traces.

4.10.1 Variance of Parallel B UCFs

Starting from the $\delta g(B)$ curves of Figs. 4.10(b) and (d), the variance is obtained by calculating the standard deviation of the UCFs on half-overlapping fixed-size boxes. These boxes of sizes ΔB are moved along the B axis, a procedure commonly used[122] for power spectra calculations. In Fig. 4.16, the variance of UCFs in the WQW dot at $V_F = 0.65$

V is represented as a function of the box size ΔB . For a real variance calculation, a compromise is found between small ΔB 's that tend to give too much weight to sharp local variations of $\delta g(B)$, and large boxes that excessively smooth out the variance. Typically, we use box sizes ΔB of 0.5 to 1 T when calculating the variance of parallel- B induced fluctuations.

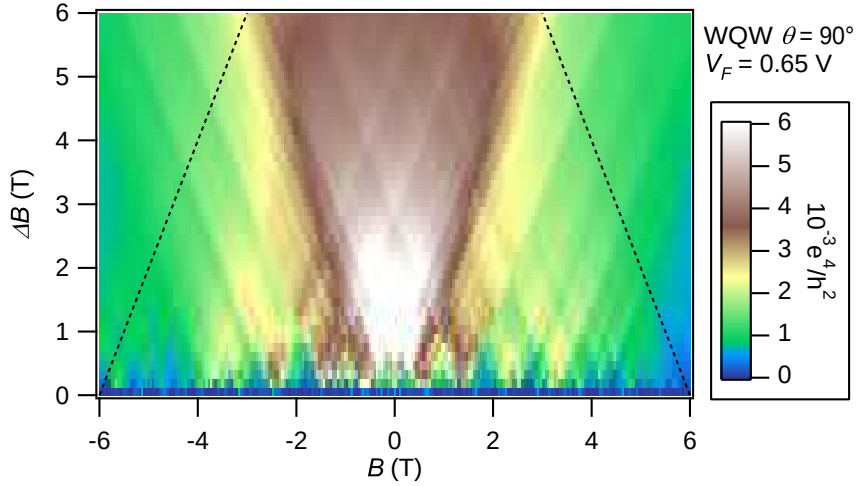


Figure 4.16: Variance of UCFs in the WQW dot at $V_F = 0.65$ V as a function of the size of the half-overlapping boxes (ΔB). The dotted lines are the limits of the confidence zone where the variance calculation at a given ΔB is not influenced by periodic boundary conditions (see text).

Figure 4.17 (solid curves) shows the variance of the UCFs in the WQW dot at $V_F = 0.65$ V (main panel) and 0.53 V (inset), extracted with $\Delta B = 1$ T from magnetoconductance curves similar to Figs. 4.10(b) and (d). Both at $V_F = 0.65$ and 0.53 V, our data analysis confirms the qualitative observation made on the magnetoconductance data, namely a reduction of the UCFs amplitude with increasing B . The variance reduction factor in the range $1 \leq B \leq 6$ T is 6 at $V_F = 0.65$ V, and 5 at $V_F = 0.53$ V.

4.10.2 Conductance Correlator for Parallel B Transport

Earlier in the text, we used Eq. 4.4 to evaluate the temperature dependence of the phase relaxation time τ_φ . The equation given by Random Matrix Theory in Sec. 3.2.4 reads

$$\text{var}(g) = \int_0^\infty \int_0^\infty f'(E)f'(E')C(E, E')dEdE', \quad (4.8)$$

f' is the first derivative of the Fermi distribution. The double integration is performed in energy-space. Here $C(E, E')$ is a conductance correlator whose expression is given in Eq. 4.5 and is only valid for a strictly perpendicular B .

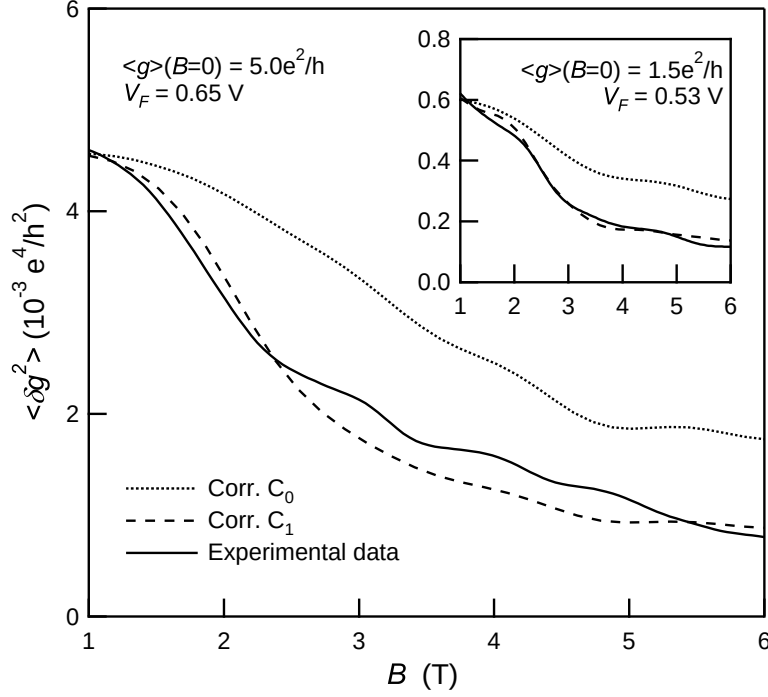


Figure 4.17: Variance of UCFs in the WQW dot at $V_F = 0.65$ V. The dotted and dashed curves are theoretical variances calculated by integration (Eq. 4.8) of the C_0 (Eq. 4.9) and C_1 (Eq. 4.14) correlators, respectively (see text). Inset: Similar set of curves at $V_F = 0.53$ V.

Fortunately, a new expression for $C(E, E')$ was recently derived[45] in the case of a system subject to spin-orbit coupling and to a magnetic field of arbitrary orientation:

$$C_0(E, E', B) = \langle g(B) \rangle^2 (V_D(E, E', B) + V_C(E, E', B)), \quad (4.9)$$

where the full expressions of V_D and V_C can be found in Appendix C. Because of the symmetry of the QW and the absence in our sample of a crossover between weak localization and antilocalization[132], spin-orbit coupling was considered to be negligible in our experiment, which leaves only two B dependent factors in V_D and V_C . More precisely, for a strictly parallel B and no spin-orbit coupling, V_D and V_C are functions of

1. The Zeeman energy

$$E^Z = \mu_B g B, \quad (4.10)$$

2. The total number of modes in the dot, including the N_φ dephasing channels

$$N_T = \frac{\langle g \rangle}{2e^2/h} + N_\varphi \quad (4.11)$$

where μ_B is Bohr's magneton and g is Landé factor (-0.44 in GaAs).

Using the slowly varying background magnetoconductance of Figs. 4.10(a) and (c) for the average conductance $\langle g(B) \rangle$ and the values of N_φ calculated in Sec. 4.8.2 from the $\theta = 0^\circ$ UCFs, the integration of C_0 (Eq. 4.8) yields the dotted curves shown in Fig. 4.17. The combined effect of B on the reduction of the average conductance $\langle g \rangle$ (through subband depopulation) and on the conductance correlator term ($V_D + V_C$) thus accounts for a reduction of the variance with increasing magnetic field. However, the reduction factor at $B = 6$ T given by Eq. 4.8 is two times smaller than the experimental one.

4.10.3 Orbital Effect Correction to the Variance

Beside the effective mass renormalization and the magnetic shift of E_F (Eq. 4.7), it has been recently demonstrated[63, 128] using Diagrammatic Perturbation Theory (DPT) that a parallel B can also break the time-reversal symmetry of the system, much like a perpendicular field would do in a weak localization correction to the conductance. More precisely, the presence of a parallel B adds a new decay rate τ_B^{-1} to the Cooperon[129, 47] equation (see Sec. 3.2.7) describing transport in the dot:

$$[D(\nabla + 2i\mathbf{A})^2 - i\omega + \tau_B^{-1}(B)] \mathcal{C}_i(\omega, \mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}'), \quad (4.12)$$

where $D = v_F^2 \tau_m / 2$ is the diffusion constant with τ_m the phase relaxation time, and $\mathbf{A}$ is the vector potential.

The decay rate reads:

$$\tau_B^{-1}(B) = \frac{\tau p_F^6}{8\hbar^2} \beta^2(B), \quad (4.13)$$

where $\tau = m_0^* L / p_F$ is the dot crossing time and $p_F = \hbar \sqrt{2\pi n_s}$ is the Fermi momentum. At low fields (up to ≈ 2.5 T), $\beta(B)$ can be approximated[179] by a simple cubic term $\beta(B) = \beta_0 B^3$. Physically, $\beta(B)$ represents a non-parabolic correction to the electron dispersion

relation in the 2DEG, induced by the parallel B . As a result, the conductance correlator acquires an additional B -dependent factor

$$C_1(E, E', B) \propto \int d\mathbf{r} d\mathbf{r}' \sum_i |\mathcal{C}_i(0, \mathbf{r}, \mathbf{r}')|^2 \quad (4.14)$$

$$= C_0 \left(1 + \left(1 + \frac{\tau_B^{-1}(B)}{\tau_{esc}^{-1}} \right)^{-2} \right), \quad (4.15)$$

where $\tau_{esc} = N\Delta/h$ is the escape rate of electrons. The correlator $C_0(E, E', B)$ was defined in Eq. 4.9. By integrating $C_1(E, E', B)$ using Eq. 4.8, the dashed curves in Fig. 4.17 are obtained for both the $V_F = 0.65$ V and 0.53 V cases. The value of the fit parameter β_0 is 2.3×10^{-4} and $0.7 \times 10^{-4} [m_0^* p_F]^{-1} T^{-3}$, respectively. These values of β_0 are consistent with theoretical and experimental parameters recently obtained[179] for single-subband GaAs-Al_xGa_{1-x}As heterojunction dots. Good agreement is found between the theoretical (dashed) and experimental (plain) curves. In the $V_F = 0.65$ V case, small deviations are observed above $B = 2.5$ T where $\tau_B^{-1}(B)$ becomes negligible with respect to τ_{esc}^{-1} in Eq. 4.14. We attribute this behavior to the breaking of the cubic B dependence of $\beta(B)$ when the magnetic confinement becomes commensurate with the 2DEG electrostatic potential. This is particularly true for a 45 nm wide quantum well where the orbital effect induced by the parallel B is much stronger than for a 2DEG confined to a simple GaAs-Al_xGa_{1-x}As heterojunction.

Before closing this section, it seems appropriate to summarize the various transport regimes that can be encountered with open quantum dots subject to a parallel magnetic field. Both the Harvard measurements of Sec. 4.2 and our variance traces of Sec. 4.10.1 only probe subsets of the possible system symmetries covered by the theoretical results of RMT[5, 6, 45] that take into account the SO coupling, the Zeeman energy, and the orbital effect. To better understand the role played by each type of excitation, the RMT formulation[5, 6] introduces three symmetry parameters that determine the variance of the UCFs *for a constant average conductance* $\langle g \rangle$

$$\text{var}(g) \propto \frac{s}{\beta \Sigma}, \quad (4.16)$$

where $s = \{1, 2\}$ describes the Kramers degeneracy, $\sigma = \{1, 2\}$ characterizes spin mixing, and $\beta = \{1, 2\}$ is the Dyson parameter for TRS.

In the Harvard experiment, TRS is broken ($\beta = 2$) at $B_{\parallel} = 0$ T by a small perpendicular field $B_{\perp}$. As $B_{\parallel}$ is increased, spin degeneracy is first

	Without SO coupling	With SO coupling
$B_{\perp} \neq 0$		Harvard Experiment
	$\beta = 2$	$\beta = 2$
	$s = 2 \rightarrow 1$	$s = 2 \rightarrow 1$
	$\Sigma = 1$	$\Sigma = 1 \rightarrow 2$
$B_{\perp} = 0$	UCL Experiment	
	$\beta = 1 \rightarrow 2$	$\beta = 1 \rightarrow 2$
	$s = 2 \rightarrow 1$	$s = 2 \rightarrow 1$
	$\Sigma = 1$	$\Sigma = 1 \rightarrow 2$

Table 4.3: Symmetries of the RMT system for an open quantum dot subject to a parallel magnetic field, as a function of the presence of SO coupling and of a perpendicular component $B_{\perp}$ of the field.

lifted by Zeeman coupling ($s = 2 \rightarrow 1$), with spin mixing taking place at higher $B_{\parallel}$ ($\Sigma = 1 \rightarrow 2$) due to SO coupling[80]. On the opposite, in our strictly parallel B experiment, TRS is not broken ($\beta = 1$) at $B_{\parallel} = 0$ T. Increasing $B_{\parallel}$ induces a change of s from 2 to 1, with a Zeeman lifting of spin degeneracy. As SO coupling was considered negligible in our WQW sample, no spin mixing is expected and Σ thus remains at a value of 1. However, it was mentioned earlier that the parallel B can break TRS, thus creating a β crossover from 1 to 2. The result is, for both experiments, a maximum reduction factor of 4 for the variance, consistent with the variance traces of Figs. 4.1(c) and 4.17, even though two different phenomena are involved in this reduction, namely a $B_{\parallel}$ -enhanced spin mixing in the Harvard experiment, and TRS breaking by the parallel B in our measurements.

Of course, the complementary symmetry configurations are also experimentally accessible, and have been recently investigated[180] with OQDs built from high density wafers where SO coupling was strong enough for the magnetoconductance to exhibit antilocalization. The various system symmetries are summarized in Table 4.10.3, as a function of the presence of both SO coupling and of a small perpendicular component of B .

Combining the parameters of Table 4.10.3 with Eq. 4.16 yields estimates of the expected reduction factor for the UCFs variance, while the precise dependence of $\text{var}(g)$ with respect to $B_{\parallel}$ is given for all symmetries by Eqs. 4.8 and 4.14, using the full expressions of V_D and V_C ,

including the effects of the Zeeman and SO couplings, as found in Appendix C.

Here, it is worth mentioning that the efficiency of the spin mixing, characterized by the $\Sigma = 1 \rightarrow 2$ crossover, strongly depends on both the dot area and the strength of the SO coupling, as shown in the recent experiments[180] of Zumbühl and coworkers.

4.11 Summary

In this chapter, we have investigated the electronic transport properties of ballistic quantum dots subject to both a perpendicular and parallel magnetic field B with respect to the 2DEG. The transition from the two field configurations is performed *in-situ*. Two dots were patterned in two-dimensional electron gases (2DEGs), confined to 15 and 45 nm-thick GaAs quantum wells (QW) with one and two occupied subbands at $B = 0$, respectively. For both dots we observe unexpected magnetoconductance fluctuations when the magnetic field is strictly parallel to the 2DEG. The differences between the rich spectrum of parallel B -induced UCFs in the WQW dot and the UCFs measured in the NQW dot emphasizes the role played by the finite thickness and width of the 2DEG. A precise control of the field alignment, by means of a transverse magnetoresistance measurement on an adjacent Hall bar discards the presence of an unaccounted for perpendicular component of B as the source of the fluctuations.

The observation of strong similarities in the temperature dependence of the perpendicular and parallel B -induced magnetoconductance in both the NQW and WQW dots, confirm that fluctuations have a common origin, *i.e.* coherent interference effects as opposed to focusing.

An analysis of both the magnetoconductance and variance of the fluctuations validate our theory that they can indeed be attributed to the orbital effect. Comparison between the experiment and recent theoretical models highlights the complementary role played by the B -induced subband depopulation, the variation of the Fermi energy induced by the orbital effect, and time-reversal symmetry breaking for the reduction of the UCFs variance.

Chapter 5

Self-Consistent 2D Electron Subbands of Quantum Wells in a Parallel Magnetic Field

Debugging is twice as hard as writing the code in the first place. Therefore, if you write the code as cleverly as possible, you are, by definition, not smart enough to debug it.

Brian W. Kernighan

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5.1 Introduction

In chapter 4, the variation of the Fermi level with respect to the parallel component of the magnetic field was isolated as the main source for the complex modifications of the electron conductance in the quantum dots. Moreover, a simple model based on the confinement of the 2DEG in an infinite square well potential, was introduced to account for the depopulation of electron subbands by a parallel B in the WQW sample. As already mentioned in the previous chapter, it is indeed well known that the finite thickness of the quantum wells confining the 2DEGs induces an orbital motion of the electrons[157]. Its most important consequence is a modification of the subband energy dispersion relations combined with a diamagnetic shift in reciprocal space of the position of the Fermi energy E_F with respect to the subband minima. Using arguments based on the well width and the onset of upper subband depopulation, a non-negligible quadratic shift of $0.1B^2$ [meVT⁻²] in B was obtained in Sec. 4.9 for the position of E_F in the WQW dot with respect to the bottom of the conduction band. We subsequently concluded that a parallel B , through the orbital effect it induces, can play the same exact role as a polarizing top gate in the production of UCFs, similar to those observed in Fig. 4.10.

However it is clear that this model of a 2DEG confined to an infinite square well, while giving a qualitative interpretation of the differences in the UCFs frequency distribution between the NQW and WQW dots, presents some severe limitations:

1. It is based on the false assumption that the 2DEG is confined to a square well with infinite walls and, as such, is completely isolated from the rest of the structure.

2. As a corollary, the position of E_F with respect to the bottom of the well is solely determined by the choice of the 2DEG sheet density n_s . The charge distribution is also strictly symmetric and independent of the gate voltage V_F at the top of the structure.
3. The growth structure of the GaAs-Al_xGa_{1-x}As samples is also completely ignored, in particular the position, concentration and ionization status of the doping layers.
4. The raw estimate of the variation of the Fermi level with increasing B strongly depends on the spatial extension $\langle z^2 \rangle$ of the wavefunctions in the occupied subbands. In the case of the WQW, it is possible to exploit the onset of subband depopulation, determined experimentally, as a mean of extracting these values¹. However, in the case of the NQW where only one subband is occupied at $B = 0$ T, no experimental data is available for the evaluation of $\langle z_0^2 \rangle$. The only solution is then to use the expected value of $\langle z_0^2 \rangle = W/2$ for the ground state wavefunction of a particle in a infinite square well of width W . Once again, this estimate does not take into account the finite height of the well barriers and the possible asymmetric deformation of the confining potential by the gate voltage V_F .

In this chapter, these limitations are overcome and the interplay between magnetic and electrostatic energy is investigated quantitatively in terms of the electronic structure of the 2DEGs confined in both the narrow and wide QWs, but also in an additional, extra wide QW that serves as a theoretical testing ground for further experimental studies. Our calculations are based on the self-consistent solution[156, 133] of the coupled Schrödinger and Poisson equations. This standard procedure has already been used successfully to study shell-filling effects in quantum dots[134, 97], the relevancy of lowest order approximations in electronic structure calculations of GaAs-Al_xGa_{1-x}As heterostructures in parallel magnetic field[82], and in explaining the single-to-double layer transitions in bilayer electron systems[153, 112]. Using fully self-consistent calculations[99, 152], we precisely determine the conduction energy dispersion relations and the amplitude of the diamagnetic shift induced by the orbital effect. We discuss these values as a function of the well width and show that the $E_F(B)$ relation, the key player for the

¹More precisely, the onset of subband depopulation gives an estimate of the difference of wavefunction spatial extension between the ground state and first excited state, $\langle z_1^2 \rangle - \langle z_0^2 \rangle$, as defined by Eq. 4.7.

parallel B -induced UCFs, strongly depends on both W and the total electron density in the well.

The ultimate purpose of this chapter is to clear up the path for a remapping of the magnetic field onto a self-consistently calculated effective Fermi energy. This variable will be derived and subsequently used in Chap. 6 to express the parallel B -induced UCFs and study their frequency distribution in this new energy space.

5.2 Self-Consistent Poisson-Schrödinger Model

5.2.1 General Description

The model used in this chapter allows for the evaluation of the static properties of a 2DEG immersed in a magnetic field $\mathbf{B}$ parallel to the plane of the electron gas itself. More precisely, we consider structures with a translation invariance along two directions of the coordinates space, say $\mathbf{e}_x$ and $\mathbf{e}_y$, and a magnetic field strictly located along the $\mathbf{e}_y$ direction, i.e. with components $\mathbf{B} = (0, B, 0)$, where B can be position dependent, but is here kept constant across the structure for simplicity. The 2DEG is formed at the GaAs-Al_xGa_{1-x}As interface perpendicular to $\mathbf{e}_z$. This coordinate system is consistent with the one shown on Fig. 4.2(a).

In the most general formulation, an electron gas of N particles is described by an eigenstate Ψ , solution to the 3D stationary Schrödinger equation in coordinate space

$$\hat{\mathcal{H}}\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = \mathcal{E}\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N), \quad (5.1)$$

where $\mathcal{E}$ is the total energy of the system. The many-body Hamiltonian [43, 93, 119, 96] $\hat{\mathcal{H}}$ is given in the absence of an external magnetic field by

$$\hat{\mathcal{H}} = \left[-\frac{\hbar^2}{2m_e} \sum_{i=1}^N \nabla_i^2 + \sum_{i=1}^N V(\mathbf{r}_i) + \frac{1}{2} \sum_{i,j}^N \frac{e^2}{|\mathbf{r}_i - \mathbf{r}_j|} \right]. \quad (5.2)$$

In Eq. 5.2, the first term is the kinetic energy, the second represents the Coulomb interaction of the electrons with the nuclei², and the third and last term is the repulsion between pairs of electrons.

²In the Born approximation, the nuclei are considered as fixed with respect to the electrons due the difference in mass between the two types of particles. The Coulomb interaction between electrons and nuclei can then be readily replaced by a pseudo-potential V .

Because of this electron-electron interaction factor, the many-body wavefunction Φ , with its $3N$ degrees of freedom, is *a priori* not separable. However, in order to conveniently write $\hat{\mathcal{H}}$ as a sum of N Hamiltonians with 3 degrees of freedom, a commonly used assumption called the Hartree approximation[26, 50] states that Ψ can be cast into a product of N single-electron wavefunctions

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = \Psi_1(\mathbf{r}_1)\Psi_1(\mathbf{r}_2) \dots \Psi_1(\mathbf{r}_N). \quad (5.3)$$

With the additional restriction that the total energy $\mathcal{E}$ of the system must be minimized and that Ψ must be antisymmetric in order to follow the Fermi statistics, a variational rewrite of Eq. 5.2 leads to the Hartree-Fock equations[26, 67]

$$\left[-\frac{\hbar^2}{2m_e} \nabla^2 + V(\mathbf{r}) + \sum_{j=1}^N e^2 \int \frac{\Psi_j^*(\mathbf{r}')\Psi_j(\mathbf{r}')d\mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|} \right] \Psi_i(\mathbf{r}) - \sum_{j=1}^N e^2 \Psi_j(\mathbf{r}) \int \frac{\Psi_j^*(\mathbf{r}')\Psi_i(\mathbf{r}')d\mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|} = \mathcal{E}_i \Psi_i(\mathbf{r}) \quad i = 1 \dots N. \quad (5.4)$$

The last term in the left hand side member of Eq. 5.4 represents the *exchange interaction* and is a direct consequence of the Fermi principle, i.e. electrons with the same spin cannot share the same orbital quantum state. Unfortunately, the Hartree-Fock approximation is well known for its overestimation of the exact exchange energy. The difference between the exact total energy and the Hartree-Fock eigensolution to Eq. 5.4 is called the *correlation energy*. One way to overcome this limitation and to better take into account both the exchange and correlation interaction is to use linear combinations of Hartree-Fock wavefunctions. However, this method is computationally intensive.

Fortunately, an alternative method to the Hartree-Fock approximation consists in evaluating the electron density defined by

$$n(\mathbf{r}) = N \int d\mathbf{r}_2 \dots \int d\mathbf{r}_N \Psi^*(\mathbf{r}, \mathbf{r}_2, \dots, \mathbf{r}_N) \Psi(\mathbf{r}, \mathbf{r}_2, \dots, \mathbf{r}_N), \quad (5.5)$$

which is a more manageable variable than Φ , with only 3 degrees of freedom.

In the formalism of the density functional theory[96], the Hohenberg-Kohn theorem[89] enunciates that the ground state of an interactive electron gas in an external potential $V_{ext}(\mathbf{r})$ can be expressed as a functional of the local density $n(\mathbf{r})$. A reformulation of Eq. 5.1 then leads to

the so-called Kohn-Sham equations[105]

$$\left[-\frac{\hbar^2}{2m_e} \nabla^2 + [E_c(\mathbf{r}) + E_{xc}(n(\mathbf{r}))] \right] \Psi_i(\mathbf{r}) = \mathcal{E}_i \Psi_i(\mathbf{r}), \quad (5.6)$$

where

$$n(\mathbf{r}) = \sum_{i=1}^N |\Phi_i(\mathbf{r})|^2, \quad (5.7)$$

and

$$E_c(\mathbf{r}) = E_{ext}(\mathbf{r}) + E_H(\mathbf{r}) = E_{ext}(\mathbf{r}) + \frac{e^2}{4\pi\epsilon_0} \int \frac{n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r}' \quad (5.8)$$

is the conduction band energy, E_{ext} is the external energy term coming from the interaction of the electrons with the ions, and E_H is the Hartree energy.

In the local density approximation, where the electron gas is treated as homogeneous, the exchange correlation interaction E_{xc} is given by

$$E_{xc}(n(\mathbf{r})) = \frac{\partial}{\partial n(\mathbf{r})} \int n(\mathbf{r}) \epsilon_{xc}(n(\mathbf{r})) d\mathbf{r}. \quad (5.9)$$

$\epsilon_{xc}(n(\mathbf{r}))$ represents the exchange-correlation energy per electron of a uniform electron gas of density n . This energy has been precisely evaluated using quantum Monte-Carlo techniques by Ceperley and Adler[34]. Simple numerical approximations are also available, and will be discussed in Sec. 5.2.5.

5.2.2 Electro-Magnetic Interplay

In the case of electrons in a semiconductor subject to magnetic field $\mathbf{B}$, it is necessary to slightly adapt the Kohn-Sham equation[105] by making the following approximations (Eq. 5.6):

1. The periodicity of the crystal lattice is taken into account by replacing the mass m_e with the electron effective mass $m^*(\mathbf{r})$ of the material, which can be position-dependent. Moreover, because the N particles in the systems are now the electrons in the Brillouin zone, the index i of the wavefunction Ψ is replaced by the 3D wave vector $\mathbf{k}$. Similarly, $\sum_i$ is converted to a sum in $\mathbf{k}$ -space that spans the entire Brillouin zone.
2. The conduction band energy is split into two terms

$$E_c(\mathbf{r}) = -e\varphi(\mathbf{r}) + \Delta E_{os}(\mathbf{r}), \quad (5.10)$$

where $\Delta E_{os}(\mathbf{r})$ is the conduction band offset at the interfaces between GaAs and $\text{Al}_x\text{Ga}_{1-x}\text{As}$. The electrostatic potential $\varphi(\mathbf{r})$ combines the contributions to the total energy of the external voltage bias, of the ionized impurities in the structure and of the repulsive electron-electron interactions. As a result, $\varphi(\mathbf{r})$ is readily obtained by solving Poisson equation.

3. An extra term, proportional to the vector potential $\mathbf{A}$ ($\mathbf{B} = \nabla \times \mathbf{A}$) is added to the kinetic operator in the Hamiltonian.

The Kohn-Sham equation[105] now reads

$$\left[\frac{(i\hbar\nabla + e\mathbf{A})}{2m^*} - e\varphi(\mathbf{r}) + \Delta E_{os}(\mathbf{r}) + E_{xc}(n(\mathbf{r})) \right] \Psi_i(\mathbf{k}, \mathbf{r}) = \mathcal{E}_i(\mathbf{k}) \Psi_i(\mathbf{k}, \mathbf{r}), \quad (5.11)$$

where the index to the wavefunction Ψ_i and energy $\mathcal{E}_i$ now denotes the i^{th} solution to the eigenvalue problem.

In Eq. 5.11, $\varphi(\mathbf{r})$ is the solution to Poisson equation

$$\nabla [\epsilon(\mathbf{r}) \nabla \varphi(\mathbf{r})] = -\rho(\mathbf{r}), \quad (5.12)$$

where $\epsilon(\mathbf{r}) = \epsilon_0\epsilon_r(\mathbf{r})$ is the dielectric constant of the material at position $\mathbf{r}$ (ϵ_r and ϵ_0 are the material effective permittivity and static dielectric constant in vacuum, resp.).

The total charge density $\rho(\mathbf{r})$ is given by the signed sum of both free charges and ionized impurities

$$\rho(\mathbf{r}) = e [p(\mathbf{r}) - n(\mathbf{r}) + N_D^+(\mathbf{r}) - N_A^-(\mathbf{r})] \quad (5.13)$$

where $p(\mathbf{r})$ is the hole concentration, $N_D^+(\mathbf{r})$ and $N_A^-(\mathbf{r})$ the ionized donors and acceptors concentrations, respectively.

In our model, $p(\mathbf{r})$, $N_D^+(\mathbf{r})$ and $N_A^-(\mathbf{r})$ are three-dimensional (bulk) charges that follow the Fermi statistics of bulk (3D) free charges and impurities. Therefore, their value at position $\mathbf{r}$ only depends on the conduction band energy $E_c(\mathbf{r})$ and on material specific parameters, like $\epsilon(\mathbf{r})$, $E_{os}(\mathbf{r})$ or the doping concentrations, through the differential equation 5.12. This will be further developed in the following section.

As far as electrons are concerned, quantization is fully taken into account in a region formed by the GaAs quantum well, the AlAs barriers, and a small part of the $\text{Al}_x\text{Ga}_{1-x}\text{As}$ layers surrounding the barriers. A bulk expression is used in the other parts of the sample. In quantized regions, the free electrons density $n(\mathbf{r})$ is spatially distributed according

to the Euclidean norm of the wavefunctions, solutions to Eq. 5.1. Assuming zero temperature ($T = 0$ K), using a sum in $\mathbf{k}$ -space, and adding a factor of 2 for spin degeneracy, Eq. 5.7 becomes

$$n(\mathbf{r}) = 2 \sum_{\mathbf{k} \text{ occupied}} |\Psi(\mathbf{k}, \mathbf{r})|^2. \quad (5.14)$$

In most cases, and in particular in the effective mass approximation, it is straightforward to evaluate the number of available states per unit volume at energy E in the interval $[E, E + dE]$, commonly called density of states $\mathcal{D}(E)$, and to rewrite the sum in $\mathbf{k}$ -space in Eq. 5.14 as an integral of energy, so that Eq. 5.14 becomes for $T \neq 0$

$$n(\mathbf{r}) = \sum_i \int_{\mathcal{E}_i}^{+\infty} \mathcal{D}(E) f(E) dE |\Psi_i(\mathbf{r})|^2, \quad (5.15)$$

where the sum is performed over the eigenstates, solutions to Eq. 5.1, and $f(E)$ is the Fermi-Dirac distribution

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k_B T}\right)}, \quad (5.16)$$

where k_B is Boltzmann's constant, T the temperature and E_F the Fermi energy.

An analytical expression of the density of states $\mathcal{D}(E)$ can be easily calculated for different levels of quantization in the effective mass approximation. However it is critical for the discussion to emphasize the approximations that were implicitly made to transform Eq. 5.14 into Eq. 5.15: basically, both the eigenenergies $\mathcal{E}_i$ and the Euclidean norm of the eigenfunctions $|\Psi_i(\mathbf{r})|^2$ were assumed to be independent of $\mathbf{k}$. This allowed for the conversion of the discretized sum in $\mathbf{k}$ -space into a continuous integration in energy space. Unfortunately, this approximation is only valid when no magnetic field is present, i.e. when the eigenfunctions in the directions with no spatial confinement can basically be written as plane waves.

For $\mathbf{B} = \nabla \times \mathbf{A} \neq 0$, the $(i\hbar\nabla + q\mathbf{A})^2$ term in Eq. 5.11 induces a coupling between the vector potential $\mathbf{a}$ and the momentum operator $\mathbf{p}$, which in turn means a non-trivial coupling between $\mathbf{k}$ and $\mathbf{r}$ in the Hamiltonian $\hat{\mathcal{H}}$. The result is a Schrödinger equation whose eigensolutions will non-trivially³ depend on one or more components of the wave vector $\mathbf{k}$.

³By trivially, we mean that the dependence with respect to $\mathbf{k}$ of the wavefunctions and eigenenergies departs from simple plane wave solutions, namely $e^{-i\mathbf{k}\cdot\mathbf{r}}$ and $\hbar^2 k^2 / 2m^*$.

5.2.3 Schrödinger Equation with a Parallel Field

In the case of a parallel field $\mathbf{B} = B\mathbf{e}_y$ strictly parallel to the 2DEG, it seems appropriate to use the Coulomb Gauge⁴ where the vector potential has the form $\mathbf{A} = Bz\mathbf{e}_x$.

Using the argument of translation invariance of the growth structure in the $(\mathbf{e}_x, \mathbf{e}_y)$ plane, the eigenstates $\Psi_i(\mathbf{k}, \mathbf{r})$, solution to Eq. 5.11 can be written as

$$\Psi_i(\mathbf{k}, \mathbf{r}) = \psi_i(k_x, z)e^{-ik_x x}e^{-ik_y y}, \quad (5.17)$$

and the corresponding eigenvalues as

$$\mathcal{E}_i(\mathbf{k}) = E_i(k_x) + \frac{\hbar^2 k_y^2}{2m^*}, \quad (5.18)$$

The Schrödinger equation can then be simplified into a 1D equation, and becomes

$$\hat{H}\psi_i(k_x, z) = E_i(k_x)\psi_i(k_x, z), \quad (5.19)$$

with

$$\hat{H} = -\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial z^2} + \frac{1}{2m^*} (eBz - \hbar k_x)^2 + E_c(z) + E_{xc}(z). \quad (5.20)$$

The first term in the Hamiltonian is the kinetic energy operator in the zero field case. The second term comes from the presence of parallel B and expresses well the coupling that exists between the coordinate z in real space and the component k_x along $\mathbf{e}_x$ of the wave vector in reciprocal space. It is equivalent to a parabolic confinement energy whose intensity is proportional to B . The center of this “magnetic well” is also shifted in real space by a factor $z_0 = \hbar k_x / (eB)$.

As a reminder, the conduction band energy

$$E_c(z) = -e\varphi(z) + \Delta E_{os}(z) \quad (5.21)$$

is the electrostatic potential $-e\varphi(z)$ with a correction term $\Delta E_{os}(z)$ that takes into account the GaAs-Al_xGa_{1-x}As band alignment. Both are functions of the real space coordinate z only in Eq. 5.10. Here, $E_c(z)$ is an expression of the position of the conduction band edge with respect to the Fermi energy E_F and, as such, represents the *electrostatic* confinement, as opposed to the *magnetic* confinement of the $(eBz - \hbar k_x)^2$ term in the Hamiltonian (Eq. 5.20).

⁴Schrödinger equation, like Maxwell equations, is gauge invariant. This gives the opportunity to choose a gauge that simplifies in a most convenient way the solution to the differential equation.

5.2.4 Poisson Equation and Charge Calculation

The total charge density constitutes the right hand side member of Poisson equation (Eq. 5.12) and includes the contribution of both free charges and ionized impurities. Because the simplified Schrödinger equation of electrons in a strictly parallel B only depends on the spatial variable z , Poisson equation becomes a 1D equation

$$\frac{\partial}{\partial z} \left[\epsilon(z) \frac{\partial}{\partial z} \varphi(z) \right] = -e \left(p(z) - n(z) + N_D^+(z) - N_A^-(z) \right), \quad (5.22)$$

where charge quantization is only considered for the free-electron density $n(z)$ in selected regions of the structure. In the non-quantized parts of the sample and in the delta-doping layers for $n(z)$, across the full structure for $p(z)$, $N_D^+(z)$ and $N_A^-(z)$, we use a Thomas-Fermi approximation[173] with the usual expressions for the equilibrium charge densities in bulk semiconductors :

- In the bulk, the electron density in the conduction band is given by

$$n = N_c \mathcal{F}_{\frac{1}{2}}(\eta_c), \quad (5.23)$$

where

$$\eta_c = \frac{E_F - E_c}{k_B T} \quad \text{and} \quad N_c = 2 \left(\frac{2\pi m_n^* k_B T}{h^2} \right)^{\frac{3}{2}}. \quad (5.24)$$

N_c is the effective density of states for electrons and $\mathcal{F}_{\frac{1}{2}}$ is the Fermi-Dirac integral of order 1/2 whose general expression reads

$$\mathcal{F}_j(\eta) = \frac{1}{j!} \int_0^{+\infty} \frac{x^j dx}{1 + \exp(x - \eta)}. \quad (5.25)$$

- Similarly, the hole concentration in the Thomas-Fermi approximation is given by the expression

$$p = N_v \mathcal{F}_{\frac{1}{2}}(\eta_v), \quad (5.26)$$

with

$$\eta_v = \frac{E_v - E_F}{k_B T} \quad \text{and} \quad N_v = 2 \left(\frac{2\pi m_p^* k_B T}{h^2} \right)^{\frac{3}{2}}. \quad (5.27)$$

The valence band energy E_v is related to the conduction band edge energy E_c through the relation

$$E_v = E_c - E_g, \quad (5.28)$$

where E_g is band-gap energy.

- For impurities, the ratio of ionized acceptors or donors to their respective total concentrations is given by

$$\frac{N_D^+}{N_D} = \frac{1}{1 + g_D \exp\left(\frac{E_F - E_D}{k_B T}\right)} \quad \text{and} \quad \frac{N_A^-}{N_A} = \frac{1}{1 + g_A \exp\left(\frac{E_A - E_F}{k_B T}\right)}, \quad (5.29)$$

for N_D donors and N_A acceptors per unit volume, respectively. g_D and g_A are the donors and acceptors degeneracy factors whose value is 2 in GaAs-Al_xGa_{1-x}As compounds.

In the quantized regions surrounding the wells, and in the absence of a magnetic field ($B = 0$), the electron concentration is given by Eq. 5.15 specialized to the case of a 2D confinement

$$n(z) = \sum_i \int_{E_i}^{+\infty} \mathcal{D}_{2D}^i(E) f(E) dE |\psi_i(z)|^2. \quad (5.30)$$

The density of states $\mathcal{D}_{2D}^i(E)$ contributed by the i^{th} eigenstate in a 2D confinement is given by

$$\mathcal{D}_{2D}^i(E) = \frac{m^*}{\pi \hbar^2} \Theta(E - E_i), \quad (5.31)$$

where $\Theta(E - E_i)$ is the Heaviside step function.

Combining Eqs. 5.30 and 5.31, an analytical expression of the 2D electron density reads

$$n(z) = \sum_i \frac{k_B T m_n^*}{\pi \hbar^2} \ln \left(\exp \left(\frac{E_F - E_i}{k_B T} \right) + 1 \right). \quad (5.32)$$

In the presence of a parallel B , the eigenenergies $E_i(k_x)$ depend on k_x in a non-trivial (non-parabolic) way. As a result, it is impossible to obtain an analytical expression of the 2D density of states similar to Eq. 5.31. General numerical techniques based on the division of the Brillouin zone into tetrahedrons are available[22, 116] and commonly used in Density Functional Theory (DFT). They make no assumption on the analytical form of the Hamiltonian or the shape of the Fermi energy surface. However, for a 2DEG subject to a parallel B , a simple analytical expression of the Hamiltonian is known (Eq. 5.20) which brings some simplifications to the evaluation of the electron density in quantized regions.

Closely following the arguments of Simserides[150], which we refer to for extended developments, we start with the general expression of

the density of states

$$\mathcal{D}(E) = \sum_{i,k_x,k_y} \delta(E - \mathcal{E}_i(k_x, k_y)), \quad (5.33)$$

where $\mathcal{E}_i(k_x, k_y)$ is the i^{th} eigenenergie of the 3D Hamiltonian. Using Eq. 5.18 and integrating over k_y , equation 5.33 becomes

$$\mathcal{D}(E) = \sum_{i,k_x} \frac{\sqrt{2m_n^*}}{2\pi\hbar} \frac{1}{\sqrt{E - E_i(k_x)}} \Theta(E - E_i(k_x)). \quad (5.34)$$

It is important to point out that, unlike in the zero-field case, the density of states given by Eq. 5.34 is not a step-like function anymore.

Combining Eqs. 5.30 and 5.34, and replacing the discrete sum over k_y by an integral in the first Brillouin zone, the electron density now reads

$$\begin{aligned} n(z) &= \sum_i \sqrt{\frac{2m^*k_B T}{\hbar^2}} \frac{1}{2\pi^2} \int_{-\infty}^{+\infty} |\psi_i(k_x, z)|^2 dk_x \\ &\quad \int_0^{+\infty} \frac{1}{\sqrt{E}} \left[1 + \exp\left(\frac{E + E_i(k_x) - E_F}{k_B T}\right) \right]^{-1} dE \\ &= \sum_i \sqrt{\frac{2\pi m^* k_B T}{\hbar^2}} \frac{1}{2\pi^2} \int_{-\infty}^{+\infty} |\psi_i(k_x, z)|^2 \mathcal{F}_{-1/2}\left(-\frac{E_i(k_x)}{k_B T}\right) dk_x, \end{aligned} \quad (5.35)$$

where we used the definition of the Fermi integral given in Eq. 5.25.

5.2.5 Numerical Issues

By self-consistently solving the Poisson differential equation 5.22 and the Schrödinger eigenvalue problem[156, 133] of Eq. 5.19 with its field-dependent Hamiltonian given in Eq. 5.20, it is possible to obtain the static properties of the electron gas confined to the quantum wells whose growth structures were presented in Chapter 2. Of particular interest are the dispersion relations of the one or multiple conduction subbands occupied by electrons and the onset of subband depopulation observed experimentally around $B = 4$ T in the wide quantum well sample.

5.2.5.1 Self-Consistent Algorithm

The block-diagram of Fig. 5.1 summarizes the self-consistent scheme used for the calculations, whose main steps are now succinctly described:

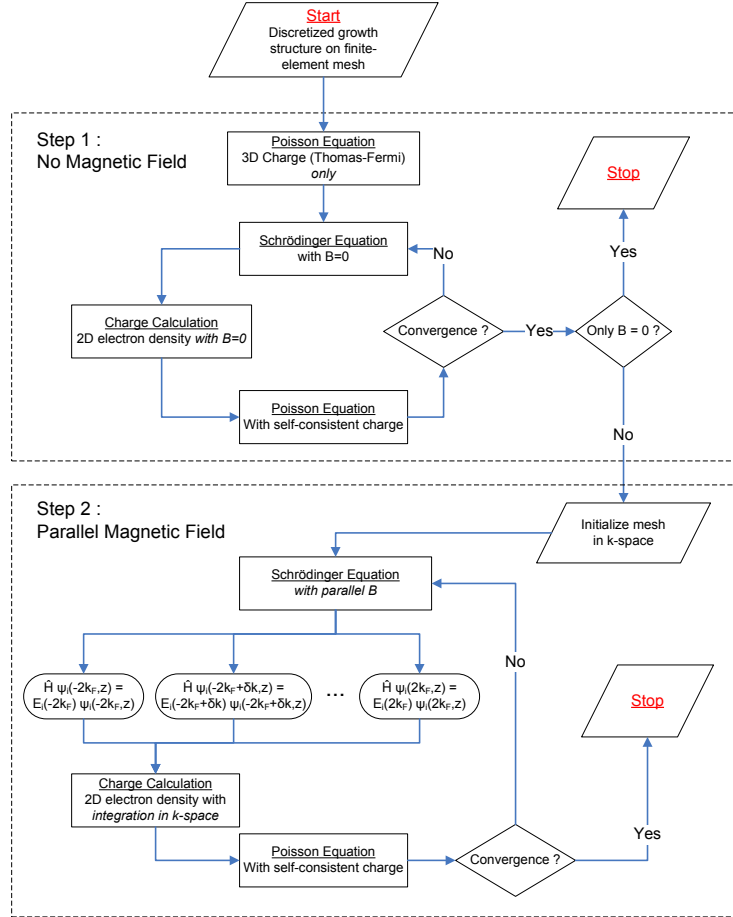


Figure 5.1: Flowchart of the Poisson-Schrödinger self-consistent scheme. After initialization of the finite-element mesh, the problem is first solved in the case of a zero magnetic field until convergence is fully reached. The second step consists in calculating the eigenstates and eigenenergies of the multiple iterations at a given B of the Schrödinger equation on the discretized mesh in $\mathbf{k}$ -space. These solution are subsequently combined in order to evaluate the 2D electron density. Once the Poisson equation has been solved with the new total charge, the loop is restarted until convergence is achieved.

1. The algorithm starts with a discretization on a mesh of the GaAs-Al_xGa_{1-x}As growth structure. Because of translational invariance of the material properties in the x - y plane, a 1D mesh is sufficient.
2. As a first step, the non-linear Poisson (Eq. 5.22) equation is solved using Thomas-Fermi (bulk) approximation for all charge species, including the electrons in the quantized regions. More precisely, the right hand-side member of Eq. 5.22 is fully determined by the charge densities defined in Eqs. 5.23 for electrons, 5.26 for holes and 5.29 for the donor and acceptor concentrations. The electrostatic potential $\varphi(z)$ is the only independent variable in the problem as both the conduction and valence band energies E_c and E_v depend on $\varphi(z)$ through Eqs. 5.21 and 5.28, respectively.
3. $\varphi(z)$ in the Thomas-Fermi approximation is used as a first guess to zero magnetic field 1D Schrödinger equation (Eqs. 5.19 and 5.20 with $B \equiv 0$). Because there is no coupling between the z and k_x coordinates, a single instance of the equation has to be solved in the quantized regions and the eigenenergies have a classical parabolic shape $\mathcal{E}_i(\mathbf{k}) = E_i + \hbar^2(k_x^2 + k_y^2)/(2m^*)$ in the k_x - k_y plane.
4. The eigenstates and eigenenergies are subsequently used to compute the electron charge densities in the quantized regions using Eq. 5.32.
5. With the updated total charge density, Poisson Equation (Eq. 5.22) is solved non-linearly.
6. The self-consistent loop is restarted until convergence is achieved.
7. If the problem deals with a non-zero magnetic field, a homogeneous 1D mesh is created in $\mathbf{k}$ -space to cover a range of k_x between $-2k_F^0$ and $2k_F^0$, where k_F^0 is the Fermi wavevector calculated at $B = 0$ in the previous self-consistent step. A mesh of 50 values of k_x is typically required[11] to maintain a constant electron density for all B values.
8. Multiple iterations of the full Schrödinger equation (Eq. 5.22) for all discretized values of k_x are solved in parallel in the quantized regions using the solutions to the $B = 0$ problem as first guesses.
9. Once the complete spectrum of eigenstates $\psi_i(k_x, z)$ and eigenenergies $E_i(k_x)$ has been determined on the k_x mesh, the electron density can be updated for the quantized regions using Eq. 5.35.

The integration in $\mathbf{k}$ -space is performed numerically between $-2k_F^0$ and $2k_F^0$.

10. Poisson Equation (Eq. 5.22) is then solved non-linearly with the new total charge density.
11. A solution is obtained when the appropriate convergence conditions are fulfilled. Otherwise, the self-consistent loop is restarted.

5.2.5.2 Finite Elements Method

Numerically, we rely on the Finite Elements Method for the solution of both the Schrödinger and Poisson differential equations[123, 136]. A complete description of the discretization of differential equations on a finite elements mesh is beyond the scope of this thesis. However, it is interesting to point out that for one dimensional problems, the finite elements method is equivalent to the simpler finite differences method as long as linear functions are used as a basis for the finite elements space. For more details, we refer the reader to introductory textbooks, like Ref. [149].

The 1D finite elements mesh used throughout the self-consistent scheme is computed by the Femlab package, based on a topographical description of the growth structure. In particular, a higher grid point density is applied to the regions surrounding the quantum wells, where a higher numerical precision is required for the determination of the 2DEG eigenstates and confinement potential. A typical mesh consists of around 300 points. For better accuracy, second-order Lagrange polynomials are used as basis elements.

To numerically solve the Schrödinger eigenvalue problem, we use an algorithm based on the Implicitly Restarted Arnoldi Method[165]. The solution to the non-linear Poisson equation is obtained through a conjugate gradient linear solver combined to Newton iterations[139].

In the self-consistent loop, the electrostatic potential is under-relaxed at each external iteration

$$\varphi_{i+1}(z) = (1 - \alpha)\varphi_i(z) + \alpha\varphi_{\text{exact}}(z), \quad (5.36)$$

where φ_i is the self-consistent potential at step i and φ_{exact} is the exact solution to Poisson equation. The relaxation factor α is on the order of 0.01 to reach self-consistency, but strongly depends on the growth structure (in particular the doping layers) and on the temperature. Usually, a careful selection of α is required to optimize the convergence.

To test the convergence of the outer self-consistent loop, a twofold criterion is used:

1. The variation of eigenenergies between consecutive iterations must be smaller than a selected threshold. More precisely

$$\max \left(\left| E_i^{j+1} - E_i^j \right| \right) \leq E_{th}, \quad (5.37)$$

where E_i^j is the i^{th} eigenenergie calculated at step j . A value of 10^{-7} eV is typically selected for the convergence parameter E_{th} .

2. The residual on Poisson equation with the damped electrostatic potential must be inferior to a given tolerance

$$\sum_i \left| \frac{1}{\epsilon(z)} \frac{\partial}{\partial z} \left[\epsilon(z) \frac{\partial}{\partial z} \varphi(z) \right] + \frac{\rho(z)}{\epsilon(z)} \right|_{z=z_i} \leq \text{tol}, \quad (5.38)$$

where $\varphi(z)$ is the under-relaxed potential given by Eq. 5.36 and the sum is performed on the mesh points in the structure. For the choice of the tolerance, a common value is $\text{tol} = 10^5$.

5.2.5.3 Boundary conditions

Of fundamental interest for the solutions to both Poisson and Schrödinger equations is the choice of appropriate boundary conditions. For Schrödinger equation, we define regions of space around the quantum wells (2D) in such a way that the approximation of eigenfunctions vanishing ($\psi(z) = 0$) at their boundaries is physically valid. Practically, we compute the eigenstates up to the the $\text{Al}_x\text{Ga}_{1-x}\text{As}$ layers surrounding the confined (2D) regions and enforce the $\psi(z) = 0$ boundary condition a few hundred nanometers away from the AlAs barriers.

For Poisson differential equation, the uniqueness of its solution relies on two boundary conditions, one at the top of the structure subject to the gate voltage V_F , and one that deals with the GaAs bulk. In the bulk, we use a von Neumann condition that states that the normal component of the electric field vanishes at a given distance

$$\left. \frac{\partial \varphi(z)}{\partial z} \right|_{z=z_0} = 0 \quad (5.39)$$

Typically, a good approximate of the extension of the transition region through which the field relaxes is given for a bulk p-doped semiconductor with N_A acceptors per unit volume by

$$z_0 = \sqrt{\frac{2\epsilon}{N_A} \frac{E_g}{e}} \quad (5.40)$$

In our calculations, we discretize the growth structure up to a distance z_0 from the surface given by Eq. 5.40.

At the exposed surfaces, a Dirichlet boundary condition is enforced. Its value is determined by the occupancy of surface states and the resulting pinning ψ_s of the Fermi level at the interface. When electrostatic gates are deposited on top of the sample with a potential V_F applied to the metallic contact, the conduction band edge energy at the surface is given by

$$E_c|_{z=0} = \varphi_s - V_F \quad (5.41)$$

The height of the Schottky barrier φ_s is used in our calculations as a degree of freedom. Surface states in GaAs give a typical value of φ_s around 0.8 eV. In our model, φ_s is slightly adjusted around this target value in order to obtain a perfect agreement between the simulated and experimental 2DEG sheet density n_s for a given range of gate voltages V_F . As a side note, it is interesting to point out that the modeling of surface states by a Dirichlet boundary condition is a simple but rather crude approximation. A more exact numerical formulation of the effect of surface gates involves a precise knowledge of the acceptors and/or donors-like interface states. More complex models exist based either on a combination of a pinned potential and frozen charges[110], or on the numerical solution of Schrödinger equation for the impurity states at the exposed surface[40].

5.2.5.4 Exchange-Correlation Energy

For the exchange-correlation energy, we use a local density approximation as parametrized by Perdew and Zunger[137] and based on the Quantum Monte-Carlo calculations of Ceperley and Adler[34]. In this framework, the exchange-correlation energy E_{xc} defined in Eq. 5.9 can be written as the sum of an exchange E_{ex} and correlation term E_{corr} , functions of the local electron concentration $n(z)$

$$E_{xc} = -\frac{0.611}{r_s} + \mu_{corr} - \frac{r_s}{3} \frac{d\mu_{corr}(r_s)}{dr_s}, \quad (5.42)$$

with

$$\mu_{corr}(r_s) = \begin{cases} \frac{A}{1 + B\sqrt{r_s} + Cr_s} & \text{if } r_s \geq 1, \\ D + E \ln r_s + Fr_s + Gr_s \ln r_s & \text{if } r_s < 1, \end{cases} \quad (5.43)$$

and

$$\begin{aligned} A &= -0.1423, \quad B = 1.0529, \quad C = 0.3334 \\ D &= -0.0480, \quad E = 0.0311, \quad F = -0.0116, \quad G = 0.0020. \end{aligned} \quad (5.44)$$

In these expressions, r_s is the Seitz radius

$$r_s = \left[\frac{3}{4\pi n(z)} \right]^{1/3} \frac{1}{a_0^*}, \quad (5.45)$$

expressed in terms of the effective Bohr radius $a_0^* = \epsilon m_0 a_0 / (\epsilon_0 m^*)$, where $\epsilon = 13.2\epsilon_0$ is the permittivity of bulk GaAs and a_0 is the Bohr radius. We point out that E_{xc} in Eq. (5.42) is expressed in scaled atomic units $H_a^* = H_a * (\epsilon_0^2 m^*) / (\epsilon^2 m_0)$, where $H_a = 27.2$ eV is the Hartree energy.

5.2.5.5 Band Offset and Energy Gap Properties

As described by Eq. 5.10, the conduction band energy E_c includes the offset due to the band discontinuity at the interface GaAs-Al_xGa_{1-x}As. In order to calculate the hole concentration as in Eq. 5.26, a knowledge of the material's bandgap energy is also required. For the purpose of the simulations, a useful expression, giving the value of the GaAs-Al_xGa_{1-x}As bandgap[73] as a function of the temperature and the molar fraction, is taken from Adachi[1] and reads

$$E_g(x) = 1.424 + 1.266x + 0.266x^2, \quad (5.46)$$

valid at $T = 300$ K for $x < 0.45$, with

$$E_g(T) = E_g(T = 300 \text{ K}) - \frac{5.405 \times 10^{-4} T^2}{T + 204}. \quad (5.47)$$

From experimental results, it seems that there is a general agreement around a value of 3/2 for the offset share ratio between conduction and valence band. Therefore we associate 65% of the bandgap to the conduction band offset energy and the remaining 35% to the valence band offset.

5.2.5.6 Fermi-Dirac integral

For numerical applications, one generally does not explicitly evaluate the Fermi-Dirac of order -1/2 to obtain the electron density in the quantized

regions subject to parallel field, as in Eq. 5.35. Instead, it is computationally interesting to use an approximation based on rational functions, as proposed by Antia[12]

$$\mathcal{F}_j(\eta) = \frac{1}{j!} \int_0^{+\infty} \frac{x^j dx}{1 + \exp(x - \eta)} \simeq \begin{cases} e^\eta \mathcal{R}_{l_1, m_1}^1(e^\eta) & \text{for } \eta < 2 \\ \eta^{i+1} \mathcal{R}_{l_2, m_2}^2(\eta^{-2}) & \text{for } \eta \geq 2 \end{cases} \quad (5.48)$$

with

$$\begin{aligned} \mathcal{R}_{l_1, m_1}^1(x) &= \frac{a_0 + a_1 x + \dots + a_{l_1} x^{l_1}}{b_0 + b_1 x + \dots + b_{m_1} x^{m_1}} \\ \mathcal{R}_{l_2, m_2}^2(x) &= \frac{c_0 + c_1 x + \dots + c_{l_2} x^{l_2}}{d_0 + d_1 x + \dots + d_{m_2} x^{m_2}}. \end{aligned} \quad (5.49)$$

l_1, m_1, l_2, m_2 are integers whose values are function of the accuracy obtained with the rational approximation. The corresponding coefficient sets, a_i, b_i, c_i and d_i , have been tabulated for various orders j and precisions of the Fermi-Dirac integral and are available in Ref. [12].

5.2.5.7 Delta-doped Layers

All the samples investigated in this thesis include one or more δ -doped layers in their MBE growth process, located either between the surface and 2DEG or in the GaAs bulk. There are many advantages of using δ -doping instead of the more conventional (and larger) modulation doped layers, the main ones being an increase of the electrons density in the 2DEG, a higher mobility and hence better transport performances, or a greater control on the conduction band structure and symmetry of the 2DEG confining well through the Fermi level pinning induced by the high concentration of ionized donors in the δ -doped layers.

The standard way of growing δ -dopings is to highly dope (with a group IV specie, usually silicon) one or more narrow ($\simeq 20 \text{ \AA}$) layers of GaAs and to confine them into wells with AlAs barriers. This unfortunately results in a conduction band edge that tends to be close to the X valley in the vicinity of the δ -doped layers. In these circumstances, DX centers appear. The most important outcome is the production of deep donor levels that give rise to two types of electronic states: a shallow and delocalized state, and a more localized level near the donors.

The study of DX centers in δ -doped layers and their impact on device performances has been the subject of intensive research[114, 35] and is definitively a topic of its own. The way to deal with δ -doping

in self-consistent simulations of heterostructures is to include the doping layers in the quantized regions where the Schrödinger equation is solved[25]. This presents the disadvantage of significantly increasing the computation time. Because we are mainly interested in the dispersion relations of the quantum wells and not by the exact doping profile, we have replaced the δ -doping layers in our simulations with an equivalent layer of $\text{Al}_x\text{Ga}_{1-x}\text{As}$ in such a way that both the total dopant concentration and the distance between the surface and the 2DEG are preserved. However, because DX centers tend to trap electrons in the localized levels, we had to slightly decrease the donors concentration per unit area in $\text{Al}_x\text{Ga}_{1-x}\text{As}$ layers with respect to their δ -doping counterparts. This was usually performed through a simple trial and error procedure where the donors concentrations are modified in the doping layers right above and below the 2DEG until the required symmetry of the well and electron sheet density are obtained.

5.3 Orbital Effect in the Narrow and Wide Quantum Wells

Now that a complete description of the self-consistent Poisson-Schrödinger model has been provided in the previous paragraphs, calculations can be performed on the NQW and WQW samples using the growth structures outlined in Figs. B.1 and B.2 (Appendix B), respectively. Throughout the calculations, the temperature is kept at a constant value $T = 0.3$ K across the structure.

We understand that, with such a simple 1D model, phenomena resulting from the complex geometry of the cavity are not fully taken into account. In particular, the effect of static impurities, located near the cavity boundaries, on the reflection of electron trajectories, or the “smoothness” of the walls for etched and electrostatic dots are neglected in these Poisson-Schrödinger simulations. Similarly, the electrostatic potential inside the dot is solely determined by the conduction band profile of the 2DEG, thus neglecting the additional confinement in the plane of the electron layer. A more complete self-consistent treatment would of course require the solution of the full 3D Poisson-Schrödinger problem[134, 97], with appropriate electrostatic and transport models for the potential profile near both reflecting cavity walls and dot openings[147, 142].

In the case of the NQW, simulations are performed for a wide range of gate voltages V_F between 0 and -0.5 V. The experimental values of the 2DEG sheet density n_s versus V_F obtained by standard Shubnikov-

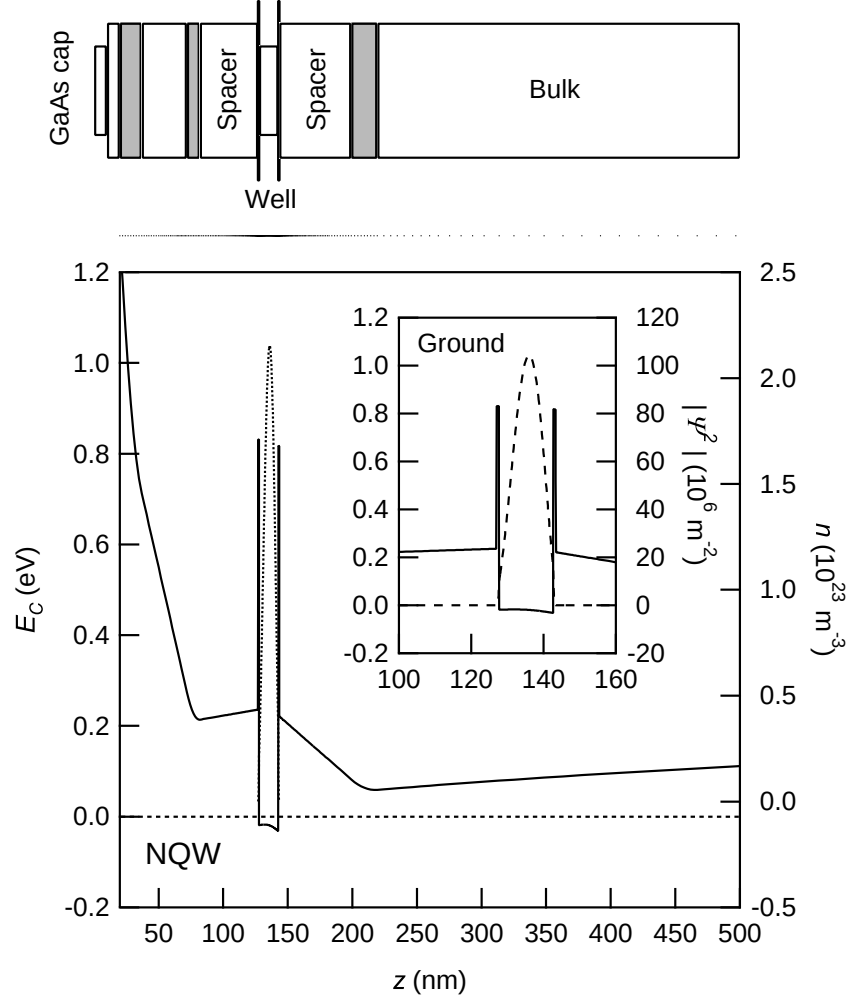


Figure 5.2: Conduction band energy (plain curve) and electron density in the well (dashed curve) for the NQW sample. The conduction band edge E_c is calculated with respect to the Fermi energy $E_F = 0$ eV. A schematic of the growth materials is provided on top of the figure. The height of the layers is proportional to the bandgap of the material (wider for AlAs, narrower for GaAs). Doping layers are shown as gray surfaces. The density of mesh points used for the calculation is presented in the space between the figure and the growth structure. Inset: Closeup of the conduction energy (plain) in the vicinity of the well along with the groundstate wavefunction (dashed curve).

de Haas measurements (Fig. 4.3) are used to finely tune the height of the Schottky barrier φ_s at the sample surface in order to have an excellent agreement with the self-consistently calculated densities in the well. When the metallic gate is grounded ($V_F = 0$ V), we also suppose that the narrow well, hence the electron density, was designed to be symmetric. Therefore, the donor concentrations in the doping layers are slightly adapted, as explained in Sec. 5.2.5 to enforce such a spatial symmetry.

On the contrary, in the case of the WQW, it was mentioned in Chap. 4 that a positive V_F was required to have at least one transmission mode across the dot openings. Moreover, it appeared that this positive gate voltage had not significant influence on the 2DEG sheet density. As result, it seems reasonable to limit the self-consistent analysis of the WQW conduction band properties to the zero gate voltage case ($V_F = 0$ V). Similar to case of the narrow well, we also suppose that the WQW is symmetric when no potential is applied to the metallic gate. The Schottky barrier and doping concentrations are adapted accordingly.

5.3.1 Band Structure at Zero Field

Figure 5.2 shows the conduction band edge energy E_c across the sample in the case of the NQW subject to a gate voltage $V_F = -0.25$ V. No parallel field is applied to the 2DEG. The Fermi level is used as a reference energy ($E_F = 0$ eV). A schematic of the growth structure including the position and width of the doping layers (hatched surfaces) is provided in the upper portion of Fig. 5.2. To easily discriminate among the various types of material, the height of the growth layers is proportional to the material bandgap energy, the narrowest for GaAs and the largest for the AlAs barriers confining the 2DEG. A typical density of mesh points used for the self-consistent simulations is presented in the space between the sample schematic and the E_c figure. While the density of points is very high from the top of the structure up to the doping layer located to the right of the well, the distance between grid points becomes larger as we move farther away into the bulk. Although not visible in Fig. 5.2, at least 50 grid points are devoted to the GaAs well and its AlAs barriers.

Starting from the top of the structure, the conduction energy E_c is pinned at the surface with energy corresponding the sum of the Schottky barrier and the gate potential qV_F . We point out that the Dirichlet boundary condition was enforced at $z = 10$ nm, i.e. at the beginning of the GaAs-Al_{0.3}Ga_{0.7}As layer, as the GaAs cap does not play any role in the conduction band profile, and only acts as a protection against oxydation of the Al specie present in the other growth layers.

Going deeper into the structure, E_c has a positive value at the locations of the two doping layers, meaning that both are fully ionized. A kink in the conduction energy is clearly visible around $z = 80$ nm and corresponds to the high donor concentration in the second doping layer. As expected, a 2DEG is confined to the 15 nm-wide quantum well located 140 nm below the surface. The electron density in the well is represented as a dotted curve in the main panel to Fig. 5.2. Both the AlAs/GaAs/AlAs barrier and the electron density are slightly asymmetric with respect to the center of well, a result of the negative potential $V_F = -0.25$ V applied to the electrostatic gate. An integration along z of n in the well gives a sheet density $n_s = 2.2 \times 10^{11} \text{ cm}^{-2}$, in agreement with the experimental value obtained under the same conditions.

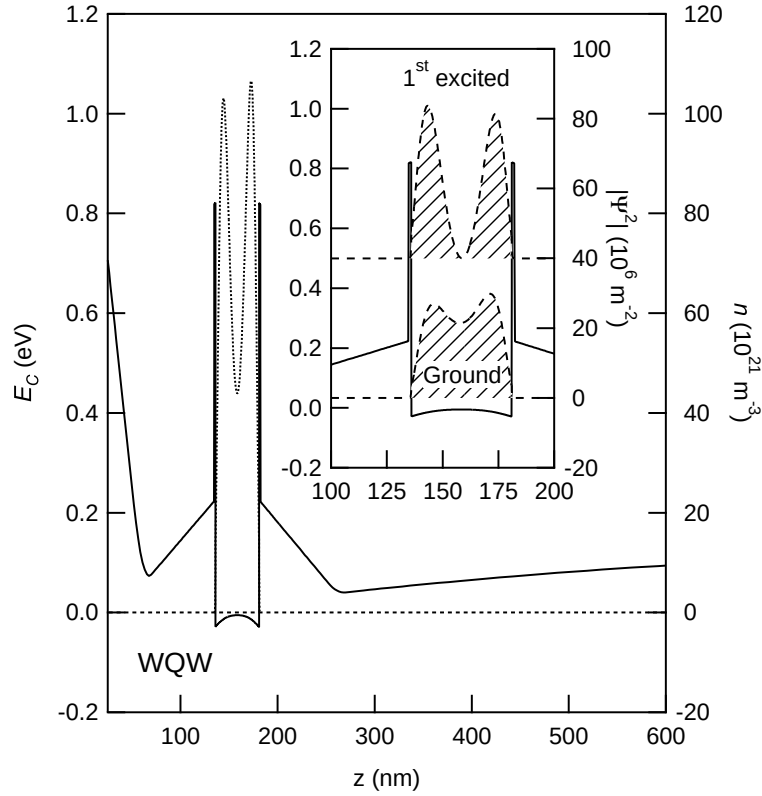


Figure 5.3: Conduction energy (plain curve) and electron density in the well (dashed curve) for the WQW sample. Inset: Closeup of the conduction energy (plain) in the well superposed upon the the groundstate and first excited state wavefunctions (dashed curve).

Located some 100 nm after the quantum well, the third δ -doping layer is also fully ionized and creates yet another kink in the conduction band energy. It is followed by a depletion region in the bulk⁵ where the electrostatic potential slowly relaxes.

In the inset to Fig. 5.2, more details are provided on the shape of the confinement energy in the well. Also represented as a dotted curve is the groundstate wavefunction, more precisely the occupation probability $|\Psi(x)|^2$. With no applied magnetic field, $T = 0.3$ K, and $V_F = -0.25$ V, the two lowest eigenenergies in the well are $E_0 = -7.38$ meV and $E_1 = 3.94$ meV, meaning that only the lowest subband is populated by electrons at low T . This is also in agreement with Shubnikov-de Haas measurements performed on the 2DEG.

A similar set of curves can be obtained from self-consistent simulations of the WQW sample and are shown in Fig. 5.3. The major differences with the NQW conduction band profile E_c of Fig. 5.2 is the symmetry of the confinement potential in the well and the shape of 2DEG electron density. Because of the larger (45 nm) width of the quantum well, two density maxima appear near the AlAs barrier while n has now a minimum at the well center. Calculating the eigenenergies of the system, we find that $E_0 = -7.31$ meV, $E_1 = -4.18$ meV and $E_2 = 7.96$ meV. This confirms that the two lowest subbands are indeed occupied in the WQW. The groundstate and 1st excited state wavefunctions ($|\Psi(x)|^2$) are shown in the inset to Fig. 5.3. The shape of the electron density profile in the well comes from the linear combination of the two wavefunctions, with weighting factor proportional to the corresponding eigenstates, as defined in Eq. 5.30.

The effect of the gate voltages V_F on the subband energies in the NQW was also investigated self-consistently. Figure 5.4(a) shows the effect of V_F on the conduction band energy E_c (plain) and electron density profile n (dashed curves) in the NQW. For $V_F = 0.00$ V, the well is almost perfectly symmetric, as expected from the choice of the Schottky barrier height and doping concentrations in the simulation. So is the electron density whose maximum is located at the center of the well. As more negative voltages are applied to the top of the structure, a deformation of E_c slowly appears and the maximum of the electron density shifts towards one of the AlAs barrier while n globally decreases in amplitude. The total sheet density n_s , obtained by a numerical integration of n in the well, is shown in 5.4(b) as a function of V_F . The linear fit, represented as a dashed line, is in good agreement with the measured

⁵The bulk typically presents a residual p-doping with an acceptor concentration $N_A = 2 \times 10^{14} \text{ cm}^{-3}$.

values of the density (Fig. 4.3(a)). This demonstrates the robustness of the self-consistent model and the relevance of the external parameters (φ_s , doping concentrations) that were carefully adjusted to get the best agreement between experiments and simulations.

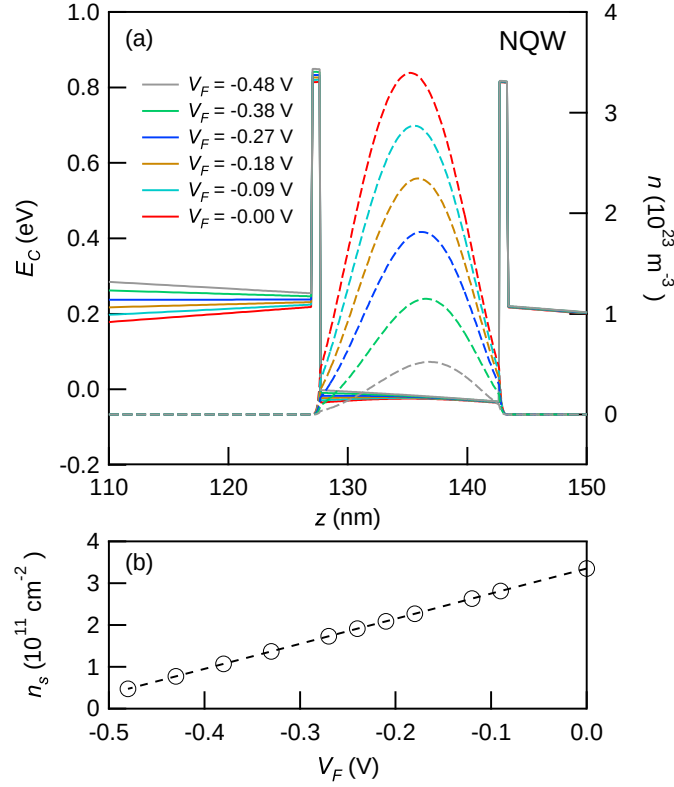


Figure 5.4: (a) Conduction band energy and electron density in the NQW as a function of the gate voltage V_F . (b) 2DEG sheet density in the NQW as a function of the gate voltage for selected V_F 's ($\circ$). The dashed line is a linear fit to the data set.

5.3.2 Dispersion Relations in a Parallel Field

We now turn our attention to the effect of a parallel field on the conduction band energy profiles in both the NQW and the WQW. Figure 5.5 shows the $E_c(k_x)$ dispersion relation of the conduction energy for the NQW as a function of the parallel magnetic field B . At $B = 0$, $E_c(k_x)$ has the expected $\hbar^2 k_x^2 / (2m^*)$ parabolic profile. When B is in-

creased, the bottom of the conduction band is pushed upward in energy and slightly shifted laterally in $\mathbf{k}$ -space. This is a result of the increasing density of states and of the well-known diamagnetic shift. Because the electrostatic confinement in the NQW is quite strong with respect to the magnetic confinement, the dispersion relation is only slightly modified by the parallel B and keeps its parabolic shape even at high fields.

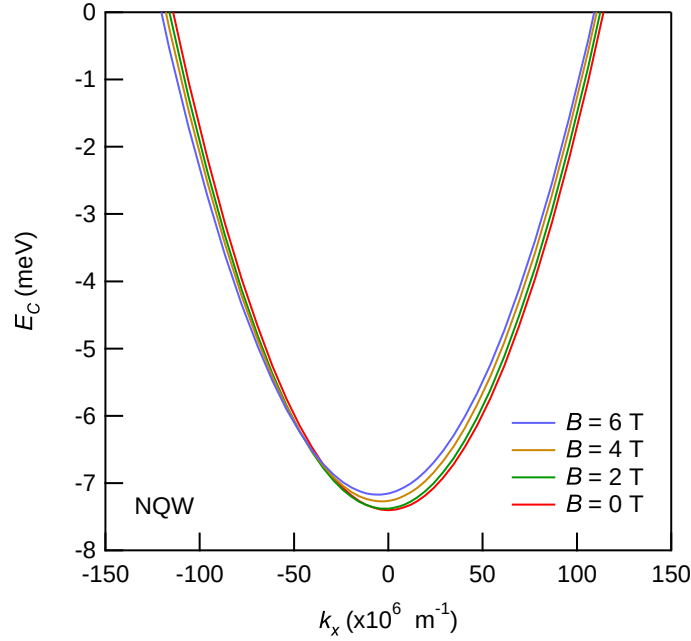


Figure 5.5: Dispersion relations of the unpatterned 2DEG confined in the NQW sample at indicated values of the parallel B . The 2DEG has a sheet density $n_s = 2.2 \times 10^{11} \text{ cm}^{-2}$. The conduction band edge E_c is calculated with respect to the Fermi energy $E_F = 0$.

The difference is striking when one compares the NQW results (Fig. 5.5) to the dispersion relations of the two occupied subbands in the WQW (Fig. 5.6). Similar to the NQW conduction band, the bottom of the upper subband in the WQW is also lifted in energy with increasing B , but the amplitude of the shift is much larger than with the NQW. Quantitatively, between 0 and 6 T, the energy shift is 9 meV for the upper subband in the WQW while only 0.2 meV in the NQW. As expected, the bottom of the upper subband rises above the Fermi level around 4.2 T, marking the onset of subband depopulation and consistent with the experiment of Sec. 4.7.2. This is a direct consequence of

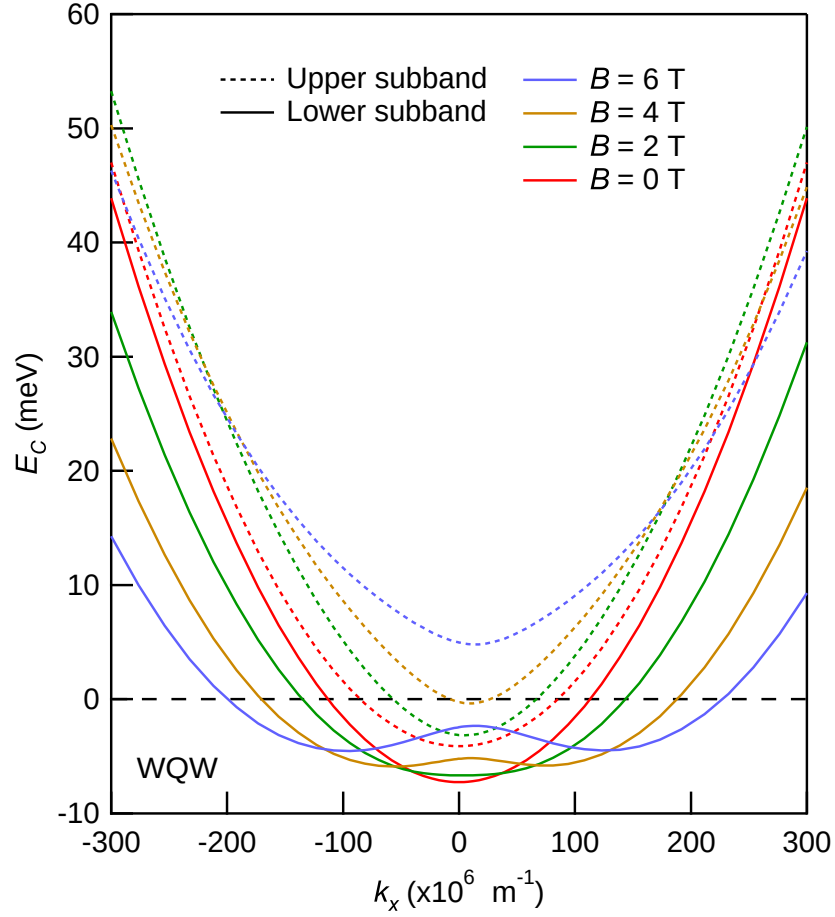


Figure 5.6: Dispersion relations of the unpatterned 2DEG confined in the WQW sample at indicated values of parallel B . The conduction band edge E_c is calculated with respect to the Fermi energy $E_F = 0$. The 2DEG has a sheet density $n_s = 3.2 \times 10^{11} \text{ cm}^{-2}$ with two occupied subbands at $B = 0$. The lower (upper) subband $E_c(k_x)$ relations are represented as plain (dashed) curves.

the increase in the subband density of states resulting from the electron effective mass renormalization, i.e. the change in the curvature of the dispersion relation. This renormalization is more pronounced in the case of the WQW upper subband than for the NQW ($1.36m^*$ and $1.05m^*$ at $B = 6$ T, resp.).

Comparing the two subbands in the WQW, $E_c(k_x)$ for the upper band keeps a parabolic shape even at high fields ($B = 6$ T) as opposed to the curvature of the lower subband which strongly varies with increasing parallel B . More precisely, the local energy minimum observed at $B = 0$ splits into a local maximum and two local minima as the parallel B is increased.

5.3.3 Fermi Surfaces and Subband Depopulation

To give a more visual understanding of the effect of a parallel B on the conduction band properties of the narrow and wide QWs, we now discuss the Fermi contours. Figure 5.7 shows the self-consistently calculated Fermi lines, with increasing B , of the NQW occupied state (a), and of the lower (b) and upper (c) WQW subbands. In both dots, the $B = 0$ perfectly circular contours get slowly deformed as the parallel B is increased. In the case of the NQW, the distortion of the Fermi line is very limited. On the contrary, both the upper and lower subband contours in the WQW are strongly distorted. While the lower subband contour takes a 'peanut'-like shape, the ellipsoidal surface enclosed by the upper subband contour decreases with increasing B and disappears above 4 T.

It is clear from the Fermi contours of Fig. 5.7 that, for our two samples, there are fundamental differences in the variation of the Fermi level with respect to the bottom of the occupied subbands. Because transport in the open dots occurs through electrons with energies close to E_F , this also means that trajectories in the dots, located in reciprocal space around the Fermi contours, will be more strongly affected by the parallel B in the WQW dot.

5.3.4 Fermi Level Shifts

Figure 5.8(a) shows E_F as a function of B for the three indicated subbands. Numerically, $E_F(B) = -E_c(B, k_x = k_{\min})$, where $k_{\min}$ is the location of the subband minimum at a given B in the dispersion relations of Figs. 5.5 and 5.6. In the case of the NQW, the variation of E_F with B is small [0.2 meV, see inset to Fig. 5.8(a)] compared to the values of 3 and 9 meV calculated for the lower and upper subbands in

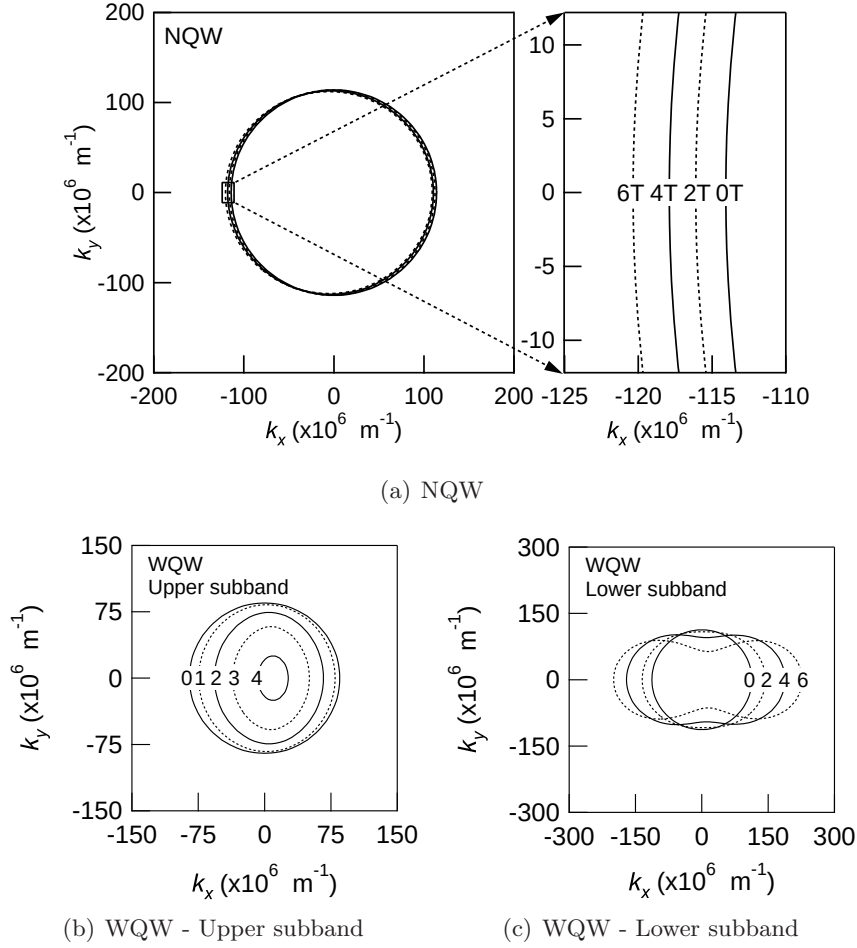


Figure 5.7: Self-consistent Fermi lines at indicated values of the magnetic field B and for (a) the ground state of the NQW, (b) the upper and (c) lower occupied subbands of the WQW sample. The right panel in subfigure (a) is a closeup of the Fermi lines in the small window indicated on the left panel. In subfigure (b), the range of investigated B 's is limited to 4 T, as the subband is fully depleted for $B > 4.2$ T.

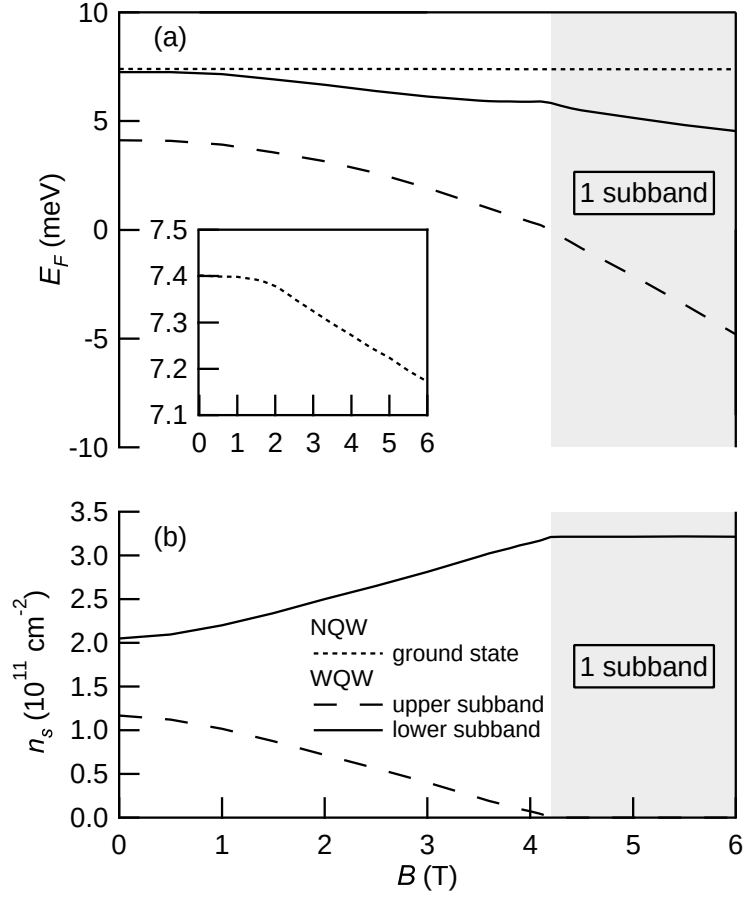


Figure 5.8: (a) Self-consistent calculation of the variation of E_F with B for the ground state of the NQW (inset) and for the two occupied subbands of the WQW. Here E_F is calculated with respect to the corresponding conduction band edges. (b) Calculated sheet densities n_s of the lower and upper occupied subbands in the WQW with increasing magnetic field B . The total density is $3.2 \times 10^{11} \text{ cm}^{-2}$. The gray regions indicate the B range where only one subband is occupied.

the WQW, respectively. Another interesting feature in Fig. 5.8(a) is the subband depopulation that occurs around 4.2 T, consistent with the value of B where the sudden drop in transverse magnetoresistance is observed in the inset to Fig. 4.10(a). The gray region in the figure indicates the B range where only one subband is occupied, *i.e.* where the Fermi level of the upper band (dashed curve) has negative values. The upper band depopulation in the WQW is even more visible in Fig. 5.8(b) that shows the electron sheet densities contributed by the lower (plain curve) and upper (dashed curve) subbands. Because the magnetic field has no effect on the total charge in the well, electrons are slowly transferred from the upper to lower band as B increases, while the total sheet density remains constant.

5.4 An Extreme Case: Extra-Wide Quantum Well

From the main results of the preceding section, in particular the $E_F(B)$ curves in Fig. 5.8, it is now quite clear that the width of the quantum well and the number of occupied electron subbands play a critical role in the variation of E_F with increasing parallel B , and are central to the presence of conductance fluctuations in the dots. Moreover, the upper subband depopulation, observed both numerically and experimentally in the WQW, and the value of its onset field are also key-players for the reduction of the UCFs amplitude with increasing B . It was indeed shown in Chap. 4 that the variance of parallel B -induced UCFs depends, among other parameters, on the average conductance $\langle g \rangle$ across the dot. And this is precisely the subband depopulation, present in the WQW, that induces a reduction of the number of transmission mode in the dot openings, hence of the average conductance $\langle g \rangle$.

It thus seems natural to try to go one step further, at least numerically, and consider even larger quantum wells where the orbital effect is expected to be even stronger than in the 45 nm WQW. In this section, we present the result of self-consistent simulations performed on sample M270, a 68 nm wide quantum (XWQW, standing for “eXtra” wide quantum well) whose growth structure is outlined in Fig. B.3. Apart from the larger well width, the major differences with the WQW are the lower 2DEG electron density ($n_s = 1.1 \times 10^{11} \text{ cm}^{-2}$ compared to $3.2 \times 10^{11} \text{ cm}^{-2}$ in the WQW) and the greater distance between the surface of the sample and the 2DEG. Similar to the WQW, Shubnikov-de Haas measurements show that two transverse subbands are occupied at $B = 0 \text{ T}$ (Fig. 4.4(a)). The lower sheet density is an interesting feature

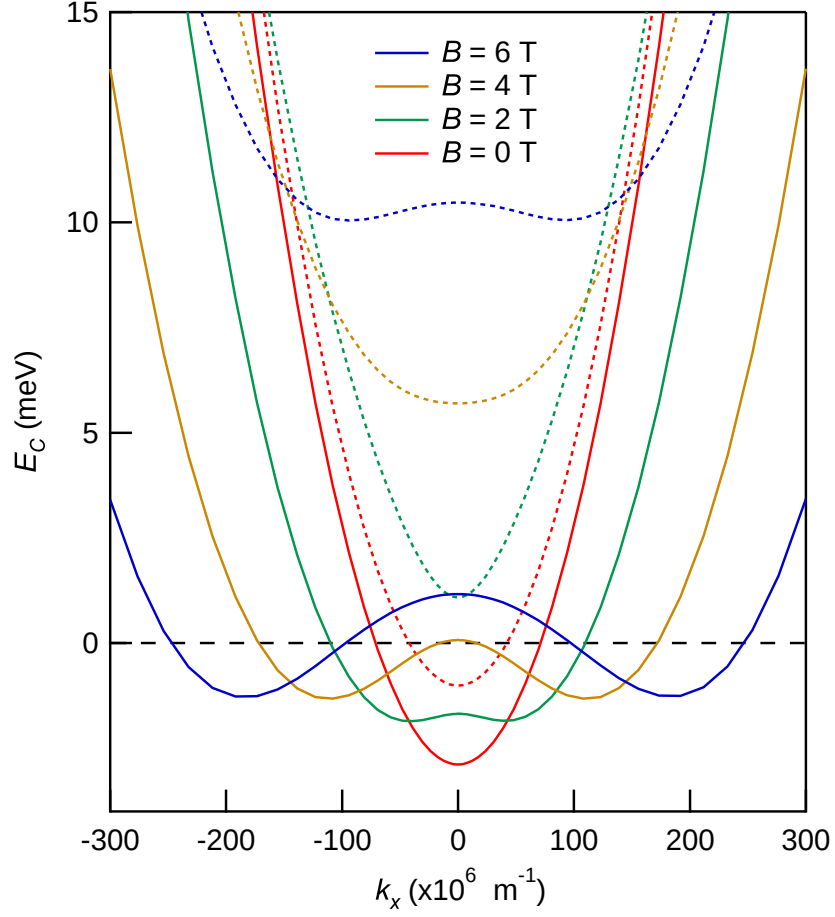


Figure 5.9: Dispersion relations of the unpatterned 2DEG confined to the XWQW sample at indicated values of parallel B . The conduction band edge E_c is calculated with respect to the Fermi energy $E_F = 0$. The 2DEG has a sheet density $n_s = 1.1 \times 10^{11} \text{ cm}^{-2}$ with two occupied subbands at $B = 0$. The lower (upper) subband $E_c(k_x)$ relations are represented as plain (dashed) curves.

as it means that most electrons are located in the lower subband. It is thus reasonable to expect a depopulation field significantly smaller than in the case of the WQW. The greater distance to the surface is more problematic as it induces some numerical instabilities in the self-consistent scheme caused by undesirable electron transfers between the 2DEG and the doping layers. Experimentally, it is also expected that the deeper position of the 2DEG in the growth structure will make it more difficult to control the width of the dot openings, whether they are defined by wet etching or controlled by means of electrostatic gates.

Leaving the experimental details of the fabrication of a XWQW dot to our successors, Figure 5.9 shows the $E_c(k_x)$ dispersion relation of the conduction energy for the XWQW as a function of the parallel magnetic field B . Compared to the corresponding dispersion relation calculated for the WQW (Fig. 5.6), the energy shift of the upper subband with increasing B is quite similar in amplitude (12 meV between 0 and 6 T) to the WQW case. The main difference is the value of B corresponding to the onset of upper subband depopulation (around $B = 1$ T), much lower than in the WQW, as expected from the lower density in the extra wide well. Of particular interest is the shape of dispersion relation for the lower occupied subband. The minimum of the $E_c(k_x)$ relation, located at $k_x = 0$ for $B = 0$, quickly becomes a local maximum whose value rises above the Fermi level when $B \sim 4$ T.

The transition from an energy minimum to a local maximum is even more visually appealing on Figure 5.10 where Fermi lines are represented for the XWQW lower subband with increasing B . Around $B = 3.7$ T, the “peanut”-like shaped Fermi contour spreads into two independent Fermi “pockets” that move further apart from the $k_x = 0$ center as B increases. The saddle point occurs at $B = 3.7$ T and is represented in the inset to Fig. 5.10. It is the onset of a transition from a single-layer to a bilayer electron system[153, 112] that comes from the competition between the well width and the magnetic length

$$l_B = \sqrt{\frac{\hbar}{eB}} \quad (5.50)$$

When the well width becomes larger than the magnetic length, the magnetic confinement starts to dominate the electrostatic one. As a result, spatial quantization is not the dominant factor anymore and the subband dispersion relations in the well tends to approach those of bulk Landau levels. Classically, at large enough fields, the electron orbits are governed by the Lorentz force with the $E(k_x)$ curves becoming flat and separated by an energy gap equal to $\hbar\omega_c$. The saddle point condition

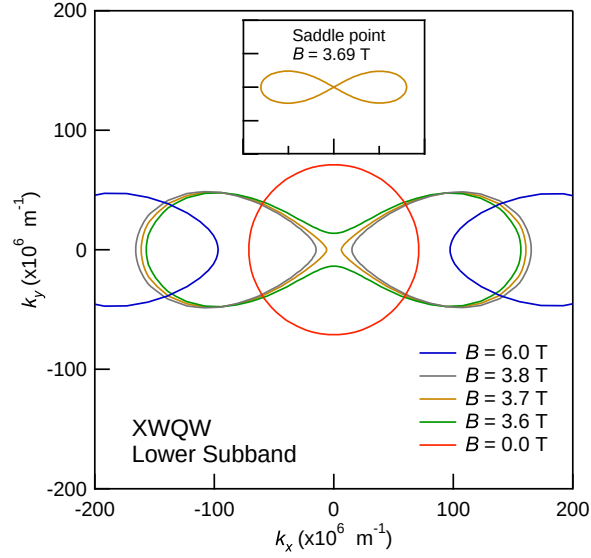


Figure 5.10: Self-consistent Fermi contours at indicated values of the magnetic field B and for the ground state of the XWQW. Inset: Fermi line at the value of B corresponding to the saddle point condition.

also corresponds to a singularity of the density of states, called van Hove singularity, similar to the one observed in 1D systems.

This transition from a single to a bilayer electron gas is clearly visible in Fig. 5.11 that shows the electron density profile in the XWQW as a function of the parallel B . At low fields, the electrons occupy almost homogeneously a great part of the well width. Up to $B = 3$ T, the density profile is barely affected by the parallel field and remains virtually unchanged. As the field approaches the saddle point condition of the Fermi contour in Fig. 5.10, the electrons move toward the AlAs boundaries, depopulating the center of the well. Ultimately, at even higher fields that were not numerically investigated in this work, a full single to bilayer transition is expected to be achieved, with a complete separation of the two electron gases.

As a further illustration of the competition between the well width and the magnetic length, the groundstate wavefunctions with $k_x = k_F$ are presented in Fig. 5.12 as a function of B for both the NQW (panel (a)) and the XWQW (panel (b)). Only a small shift of the wave maximum is visible in the Inset to Fig. 5.12 in the case of the NQW. On the opposite, the Lorentz force strongly affects the XWQW, pushing the

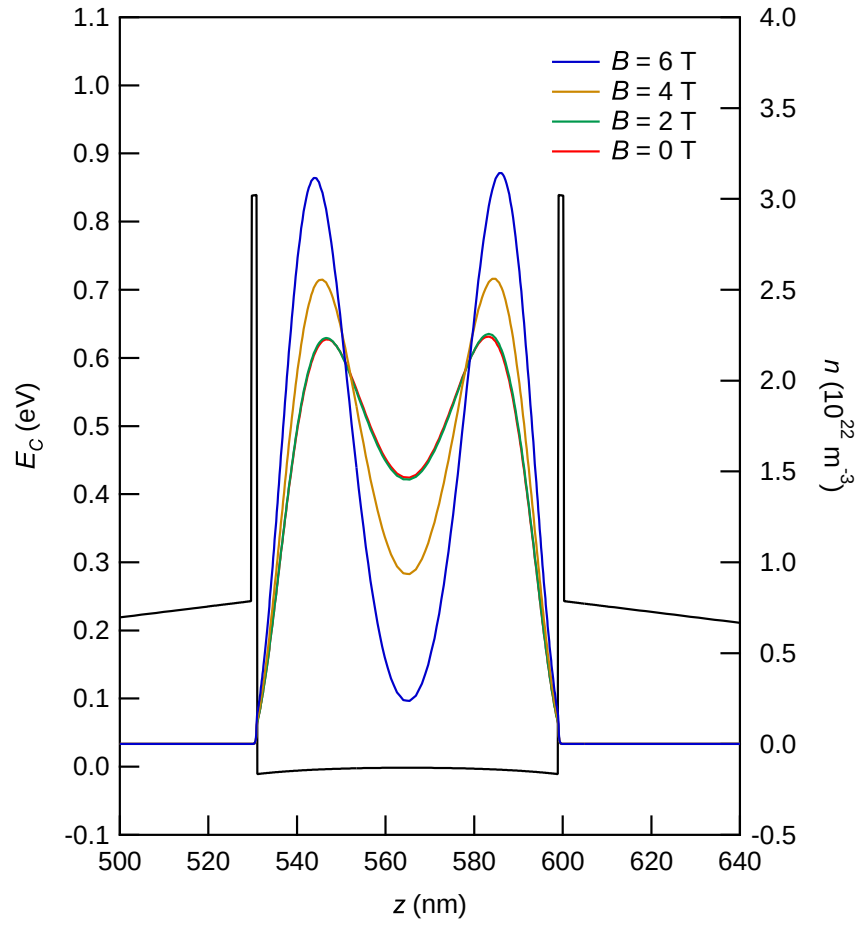


Figure 5.11: Conduction band energy and electron density calculated for the XWQW at indicated values of the parallel B .

wavefunctions toward the AlAs barrier. Because the electron density comes from an integration of the wavefunctions in $\mathbf{k}$ -space between $-k_F$ and k_F , as given by Eq. 5.35, the bilayer transition of Fig. 5.11 will only be relevant in the case of the XWQW and won't affect the NQW, at least in the experimental limits of modern superconducting solenoids.

Closing the discussion on the XWQW, Figure 5.13.(a) shows the position of the Fermi level E_F with respect to the conduction band edge as a function of B for the two indicated subbands. The variation of E_F with B for the upper band (dashed curve) is similar to the WQW case (Fig. 5.8.(a)) and gives a precise value of $B = 1.5$ T for the onset of subband depopulation, much smaller than the value of 4.2 T calculated in the WQW. More interesting is the E_F (plain) curve of the lower subband, in particular the saturation of E_F that occurs at a value of the field corresponding to the saddle point condition⁶. If we once again link the observation of parallel B -induces UCFs to a variation of E_F through the orbital effect, it seems natural to predict that no UCFs should be observed in a XWQW dot beyond the saddle point condition. This affirmation will be further discussed in the next chapter where we confront the results of the self-consistent simulations with the UCFs measurements of Chapter 4.

5.5 Summary

Motivated by the central role played by the position of the Fermi level in the observation of parallel B -induced UCFs, we have developed self-consistent Poisson-Schrödinger simulations of 1D magnetic subbands for electrons in realistic quantum wells. The results of our calculations show that a parallel field has a very different effect on the subbands dispersion relations and Fermi contours of the NQW and WQW 2DEGs. More precisely, the position of the Fermi level E_F with respect to the bottom of the subbands does not vary significantly in the NQW with increasing B . On the contrary, in the WQW, a strong variation of E_F is needed to accommodate the electron depopulation of the upper subband. Similarly, the Fermi contours of the upper and lower subbands in the WQW are strongly deformed by the parallel field, but remain circular in the NQW.

⁶At $B = 3.69$ T, a numerical instability due to the van Hove singularity of the DOS gives rise to a 0.1 meV error on the value of E_F , hence the small “kink” in the $E_F(B)$ curve.

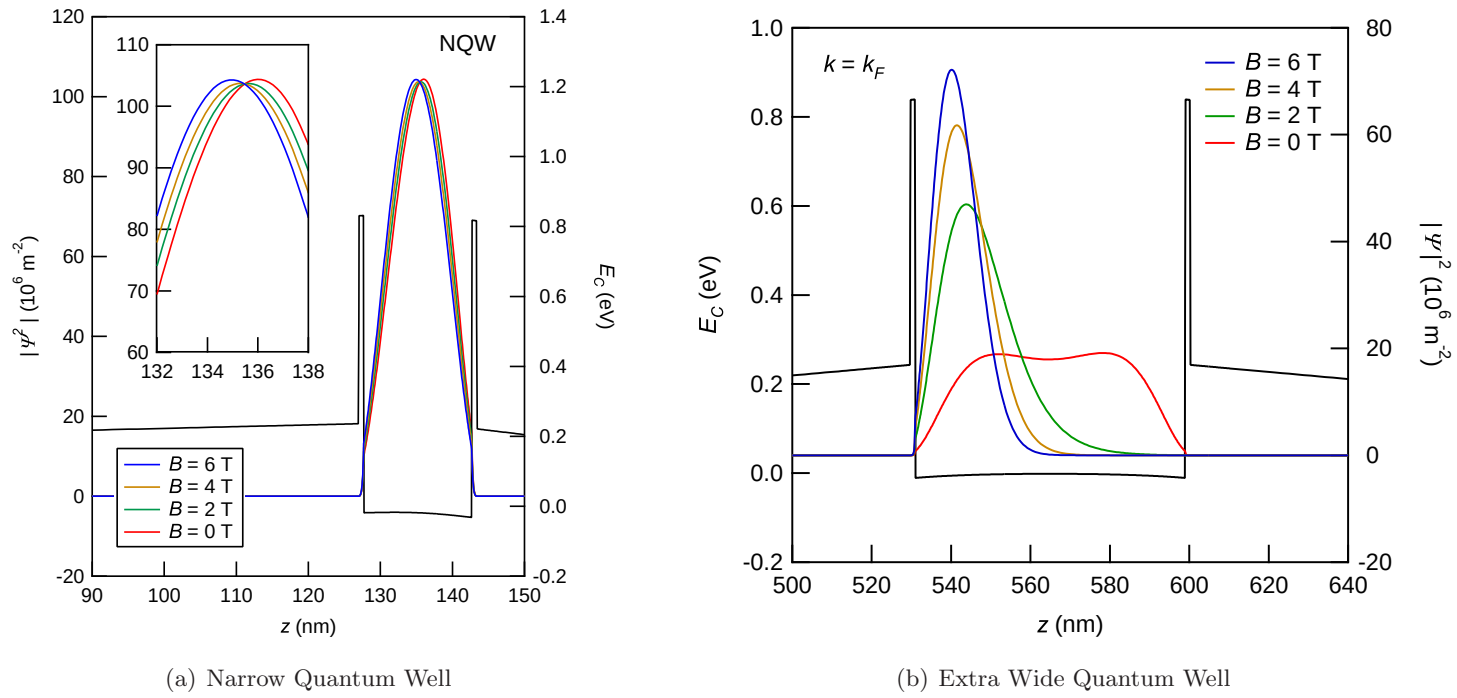
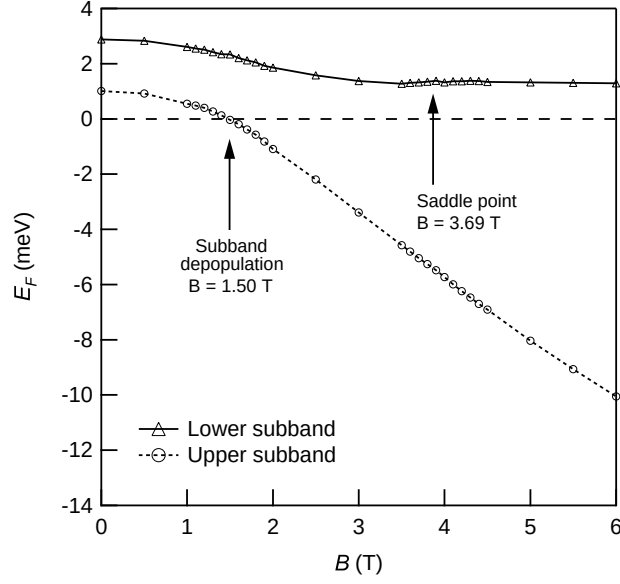
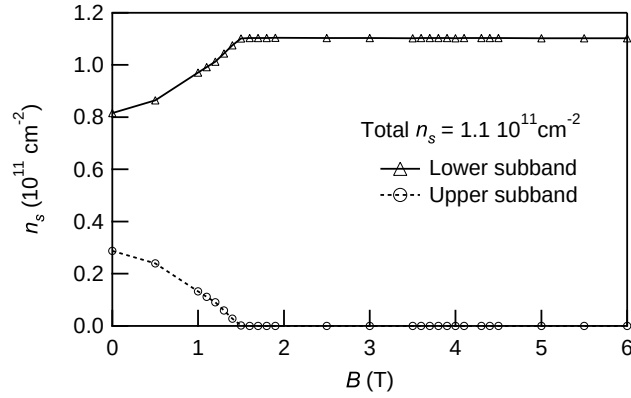


Figure 5.12: Groundstate wavefunction of the 2DEG in (a) the NQW and (b) the XWQW at indicated values of the parallel field B and for $k_x = k_F$. Inset to panel (a) : Closeup on the wavefunctions around their maxima.

Extending the calculations to the case of an “extra” wide quantum well, we pointed out that the observation of a single to bilayer transition of the electron system, associated with a saddle point condition of the Fermi contours, can lead to a saturation of E_F at high B ’s. Such a transition could significantly alter or even suppress the parallel B UCFs in a XWQW dot.



(a) Fermi Level



(b) Subband Densities

Figure 5.13: (a) Self-consistent calculation of $E_F(B)$ for the two occupied subbands in the XWQW. Similar to the cases of the NQW and WQW, E_F is calculated with respect to the corresponding conduction band edges. (b) Calculated sheet densities n_s of the lower and upper occupied subbands in the XWQW with increasing B . The total density is $1.1 \times 10^{11} \text{ cm}^{-2}$. The values of B where the self-consistent calculation was performed are indicated with circles.

Chapter 6

Tilted Fields, Frequency Analysis and UCFs in Energy Space

Pourquoi nous retirer et abandonner la partie, quand il nous reste tant d'êtres à décevoir ?

Emil Cioran

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6.1 Introduction

In Chapter 4, we presented experimental evidences that conductance fluctuations can indeed be observed for ballistic quantum dots under a strictly parallel B . This came as a surprise as, up to our measurements, a varying electrostatic gate voltage or a magnetic field applied perpendicular to the 2DEG were the only known methods of generating UCFs in open quantum dots. A careful analysis of the temperature dependence of magnetoconductance proved that the parallel B -induced fluctuations were sharing fundamental properties in terms of dephasing with their perpendicular B counterparts. The natural conclusion was that the parallel B fluctuations were indeed UCFs induced by coherent interferences between electron trajectories in the dot. To further strengthen our argument, a confrontation of the experimental UCFs variance vs. B against recent results from random matrix theory confirmed that the orbital effect, i.e. the interplay between the magnetic and electrostatic confinement of the 2DEG was playing a central role for this new type of UCFs.

However, by focusing our attention to the strictly parallel field configuration and by basing our analysis on the UCFs variance only, we (temporarily) set aside two potentially interesting topics: First, the question arise of what would be the effect of having both a parallel and perpendicular component of B (*i.e.* a tilted field) on the UCFs. The use of tilted fields is not uncommon in Solid State Physics. It has already proved successful in the study of various quantum Hall states displayed by the 2DEGs at low temperatures and in the presence of a high B [159, 112]. In the case of ballistic quantum dots, tilted-field magnetotransport is potentially interesting as it might shed light on the (dis-)similarities between the phase spaces probed by either a perpendicular or parallel field and on the role played by the finite thickness of the 2DEG. More precisely, it is expected that, at intermediate fields, the fluctuations induced by the parallel component of the field will be superimposed upon some perpendicular B UCFs. Keeping in mind the great differences between the strictly parallel B fluctuations observed in the NQW and WQW dots, it seems legitimate to try to characterize the relative strength of those two sets of fluctuations in terms of the well width. As a result, tilted magnetic fields will be thoroughly investigated in Sec. 6.2.

The second major field of interest whose study was temporarily delayed in Chap. 4 deals with the analysis of the UCFs in terms of their frequency distribution. Methods based on UCFs power spectra calculations are quite customary for the determination of the temperature-

dependent dephasing time[42] or to study shape-induced fluctuations[36] in open quantum dots. In particular, when combined to variance calculations, power spectra methods turned out to be essential tools in recent studies on the origins of the observed τ_φ saturation in open quantum dots[78]. In Sec. 6.3, we analyze the power spectra of the magnetoconductance fluctuations in tilted magnetic fields, presented in Sec. 6.2. We subsequently extract an effective tilt angle, which is a useful parameter to analyze the dependence of the UCFs fingerprint on the well width.

Finally, in the ultimate section of this thesis work, we combine the various experimental results of Chap. 4 and Sec. 6.2 with the major conclusions of our simulations performed in Chap. 5. In Sec. 6.4, the magnetoconductance measurements under a strictly parallel B are rescaled as a function of the field dependent position of the Fermi level in the wells. We show that a frequency analysis of the parallel B -induced UCFs in *energy space* is in good agreement with predictions from semi-classical theory[95]. From these findings, we draw our conclusions in Sec. 6.5.

6.2 UCFs in Tilted Magnetic Fields

To investigate the effects of a tilted magnetic field, we now return to our two ballistic quantum dots patterned on top of 2DEGs confined to a narrow and a wide quantum well, respectively. The characteristics of the two dots were detailed in Chapter 4, and are summarized in Table 4.4.2. Here, we point out that, unlike the strictly parallel B experiments of Chap. 4, the complete set of magnetoconductance measurements in a tilted field were performed in the ^3He cryostat (base temperature $T = 0.3$ K).

We use the same sample holder mechanism where the sample can be tilted *in-situ* with respect to the magnetic field. To control the tilt angle with a high precision, we monitor the transverse magnetoresistance $R_{xy}(\theta, B)$ of the 2DEG on the adjacent Hall bar. In Chap. 4, the sample was rotated to cancel R_{xy} in order to reach a strictly parallel B configuration¹. For magnetoconductance measurements at intermediate angles, the sample tilting procedure is somewhat enhanced as the value of the tilt angle θ is now extracted from the ratio of the slopes of $R_{xy}(\theta, B)$ and $R_{xy}(0, B)$ with respect to B . More precisely, for a target tilt angle θ , the following procedure is used :

1. The sample is brought to a strictly perpendicular configuration

¹Within mechanical and operating precision, see Sec. 4.5 for details on the alignment accuracy.

by maximizing $R_{xy}(\theta, B^{\text{cal}})$ at a given calibration field B^{cal} . The resistance maximum $R_{xy}(0, B^{\text{cal}})$ is carefully written down.

2. Without changing the tilt angle, B is brought to 0 T and the residual longitudinal component R_{xy}^{res} of the transverse magnetoresistance is measured on the multimeter.
3. B is brought back to B^{cal} and the sample is crudely rotated until $R_{xy} = R_{xy}(0, B^{\text{cal}}) \cos \theta$. The purpose of this procedure is to avoid any resistance overflow of the lock-in instruments in the next steps.
4. B is increased to the experiment target field $B_0 = B^{\text{cal}} / \cos \theta$, without exceeding the superconductor magnet field limit B^{limit} . Once the magnet has reached its target, the sample is slowly rotated and the measured R_{xy} is finely tuned until it reaches the “Algebra 101”² value given by

$$R_{xy} = \frac{\min((B^{\text{cal}} / \cos \theta), B^{\text{limit}})}{B^{\text{cal}}} R_{xy}(0, B^{\text{cal}}) \cos \theta + (1 - \cos \theta) R_{xy}^{\text{res}}. \quad (6.1)$$

Throughout our measurements, we kept B^{cal} large enough to maintain a good precision on the measured value $R_{xy}(0, B^{\text{cal}})$ but, at the same time, small enough to remain in the linear regime where $R_{xy}(0, B) \propto B$, thus avoiding the first plateaus of the Quantum Hall effect. A value of 0.1 T for B^{cal} was found to be a good compromise.

To evaluate the actual tilt angle, and compare it to its target value θ , we proceed as follows :

1. $R_{xy}(\theta, B)$ is measured on the adjacent bar between $-B_0$ and B_0 . Examples of transverse magnetoconductance traces at various tilt angles are shown in Fig. 6.1.
2. A curve fitting of $R_{xy}(\theta, B)$ against a second-order polynomial is performed. The second-order term comes from the contribution by a small and unavoidable longitudinal component of the sample’s resistance, still present in a strictly parallel configuration, as thoroughly discussed in Sec. 4.5. The linear term is the Hall resistance, induced by the perpendicular component of the field.

²Algebra 101 is the introductory course usually taught to freshmen (1st year students) in US colleges.

3. The linear coefficient is compared to the Hall coefficient of a calibrated measurement of $R_{xy}(0, B)$, performed at $\theta = 0^\circ$ at the same temperature and gate voltage V_F configuration. The actual tilt angle is given by the inverse cosine of the ratio of the two coefficients.

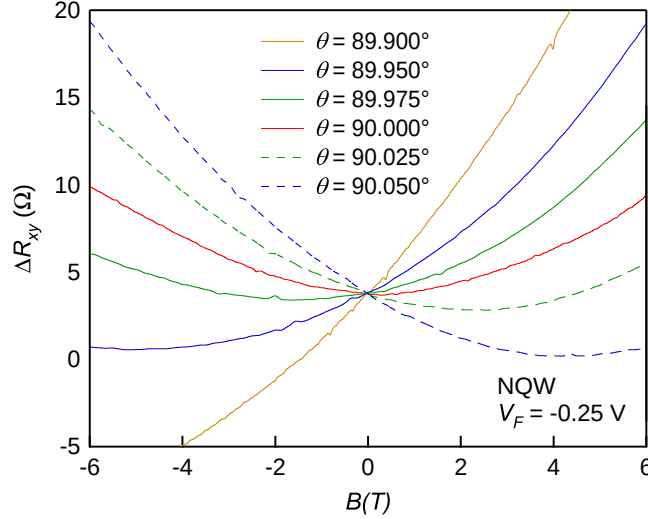


Figure 6.1: Transverse resistance of the parent 2DEG at indicated tilt angles, measured on the adjacent Hall bar.

Because of the $\cos \theta$ term, the precision on the sample alignment at the desired tilt angle θ improves with increasing θ , and is always kept below a value of 0.05° in the configuration where B is almost parallel to the 2DEG. At intermediate angles ($70 \leq \theta \leq 90^\circ$), we evaluate the error to be about 1% of the difference between the target θ and 90° .

Figure 6.1 shows the transverse resistance measured on the adjacent Hall bar in the NQW sample between $\theta = 89.900$ and 90.050° . A strictly parallel B corresponds to a symmetric, quasi-parabolic B -dependence of the resistance. The difference between the $\theta = 89.975$ and 90° curves demonstrates the precision achieved on the angle selection with the experimental setup.

6.2.1 Narrow Quantum Well

We now turn to the magnetoconductance measurements performed on the two open dots under a tilted B starting, as in Chap. 4, with the narrow QW sample.

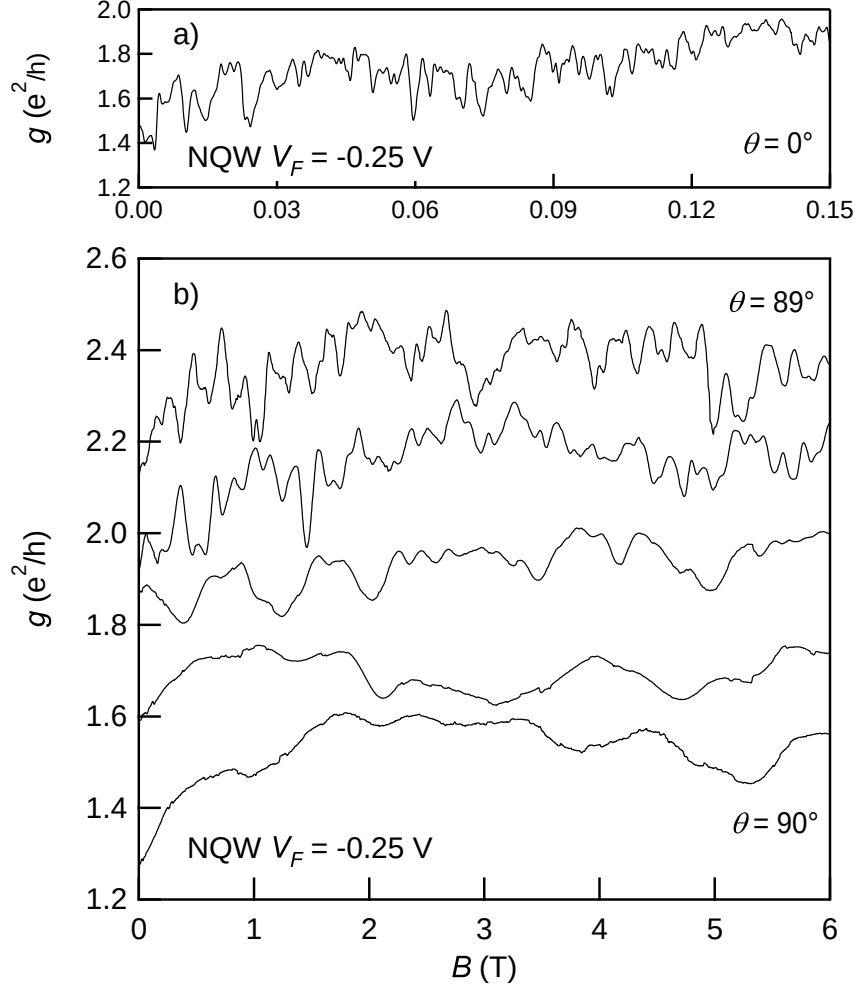


Figure 6.2: (a) Magnetoconductance g of the NQW dot at $T = 0.3$ K under a strictly perpendicular B . (b) Magnetoconductance of the NQW dot for tilt angles close to 90° ($\theta = 89, 89.3, 89.6, 89.9$, and 90° from the upper to lower curve). The traces are shifted for clarity in steps of $0.2 e^2/h$ with respect to the $\theta = 90^\circ$ curve.

For comparison purpose, Figure 6.2(a) shows a typical magnetoconductance measurement performed at $T = 0.3$ K on the NQW dot under a perpendicular magnetic field only, similar to those extensively presented in Chap. 4. As expected, reproducible UCFs with no clear periodic pattern are superimposed upon a slowly varying background.

As a next step, the magnetic field is tilted away from its perpendicular configuration and the magnetoconductance is measured at various θ 's as shown on Fig. 6.2(b). We point out that the curves are presented as a function of the *total* magnetic field. The upper trace corresponds to the $\theta = 89^\circ$ configuration. For smaller tilt angles, the magnetoconductance, with B rescaled by a factor $\cos\theta$, is identical to the $\theta = 0^\circ$ curve of Fig. 6.2(a), meaning that for $\theta < 89^\circ$, the observed UCFs are solely contributed by the *perpendicular* component of field and that the parallel B has only a negligible effect. On the contrary, for $\theta \geq 89^\circ$, the conductance fluctuations first shift to lower frequencies ($\theta = 89.3$ and 89.6°), as expected for perpendicular- B induced UCFs, but saturate at $\theta = 89.9^\circ$. While the $\theta = 89.9$ and 90° traces are not identical, they definitively share common features, like some major frequency components.

6.2.2 Wide Quantum Well

We now focus our attention on the WQW sample. Figure 6.3(a) shows the magnetoconductance of the WQW dot under a strictly perpendicular B at $T = 0.3$ K, starting with the $V_F = 0.65$ V gate voltage configuration. The average conductance $\langle g \rangle$ across the dot is $5.2 e^2/h$. Following the $\theta = 0^\circ$ measurements (Chap. 4), the sample is tilted in the magnetic field. Magnetoconductances at high θ 's, including $\theta = 90^\circ$, are shown on Fig. 6.3(b). It is important to note that the magnetoconductance traces illustrated in this figure are once again expressed in terms of the *total* magnetic field. Comparing these curves to their counterparts obtained for the NQW dot [Fig. 6.2(b)], two main characteristics can be pointed out. First, the UCFs have a much richer frequency spectrum, *i.e.* higher frequency components are present in the WQW dot. Second, this frequency distribution of the UCFs is not significantly modified as the sample is slightly tilted away from the 90° position. If the UCFs had their origin in the perpendicular component of the field, one would expect the apparition of higher frequency fluctuations, contributed by the perpendicular field. Although a shift toward higher frequencies occurs in the magnetoconductance of the NQW dot on Fig. 6.2, this is not the case with the WQW dot. This observation suggests that the finite width of the QW plays a significant role for the parallel B -induced UCFs and it reveals the importance of tilted-magnetic field magnetotransport

studies.

Still superposed upon the UCFs in the WQW dot is the sudden drop in $\langle g \rangle$ that occurs around $B = 4.2$ T for $\theta = 90^\circ$. This drop is slightly shifted toward smaller B 's when θ decreases. In contrast, the magnetoconductance in the NQW dot fluctuates over the entire B -range around its average value determined by the dot openings. In Chapter 4, this reduction of $\langle g \rangle$ in the WQW dot was attributed to the magnetic subband depopulation of the upper electronic subband and the corresponding reduction of the number of transmission modes in the dot openings[145, 17]. As this effect is linked to the presence of a parallel component of B , the onset of subband depopulation is thus expected to be proportional to $B/\sin\theta$, which explains why the $\langle g \rangle$ drop occurs at smaller fields for $\theta = 89^\circ$ (upper curve Fig. 6.3(b)) of than in the strictly perpendicular configuration ($\theta = 90^\circ$, lower curve of Fig. 6.3(b)).

As mentioned in Chapter 4, the WQW dot transport properties were tested for various gate voltage configurations, with particular emphasis on the $V_F = 0.65$ and 0.53 V values. Figure 6.4 presents a set of magnetoconductance traces under tilted fields in the WQW dot for $V_F = 0.53$ V. As opposed to the $V_F = 0.65$ V configuration, the average conductance $\langle g \rangle$ across the dot is here much smaller. A comparison with the tilted B conductances of Figure 6.3.(b) shows that the same saturation of the UCFs frequency distribution with increasing θ is observed. This effect is thus independent of the applied gate voltage, hence of $\langle g \rangle$. As a result, we have an additional confirmation that the key player in the UCFs fingerprint for tilt angles near $\theta = 90^\circ$ is not the geometrical properties of the dot or the number of transmission mode in its openings, but instead is deeply rooted in the strength of the 2DEG electrostatic confinement, *i.e.* the well width.

6.3 UCF Frequency Analysis

6.3.1 Power Spectrum in B -space

We now turn to a more quantitative study of the frequency distribution of both the NQW and WQW UCFs as a function of the tilt angle θ . From the comparison of Fig. 6.2(b) with Figs. 6.3(b) and 6.4(b), it is clear that the two dots react differently to the presence of a tilted B . To analyze the role played by the parallel component of the field on the production of UCFs, we evaluate the power spectrum of the magnetoconductance curves at tilt angles close to $\theta = 90^\circ$. As described in Sec. 3.2.2, the shape of our open quantum dots was carefully selected in order to maximize the chances of having an ergodic transport regime in the cavities. When such

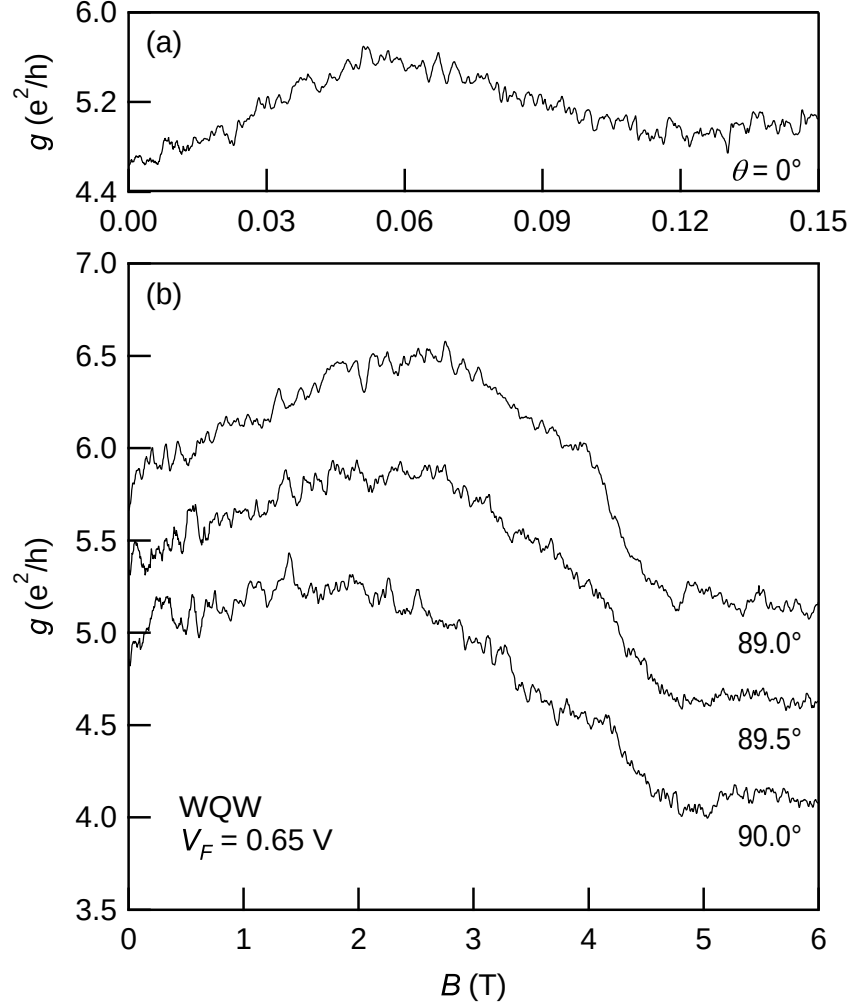


Figure 6.3: (a) Low-temperature magnetoconductance of the WQW dot under a strictly perpendicular B and for $V_F = 0.65$ V. (b) Magnetoconductance of the WQW dot at indicated tilt angles, with $V_F = 0.65$ V. The traces are shifted in steps of $0.5 e^2/h$ with respect to the $\theta = 90^\circ$ magnetoconductance curve.

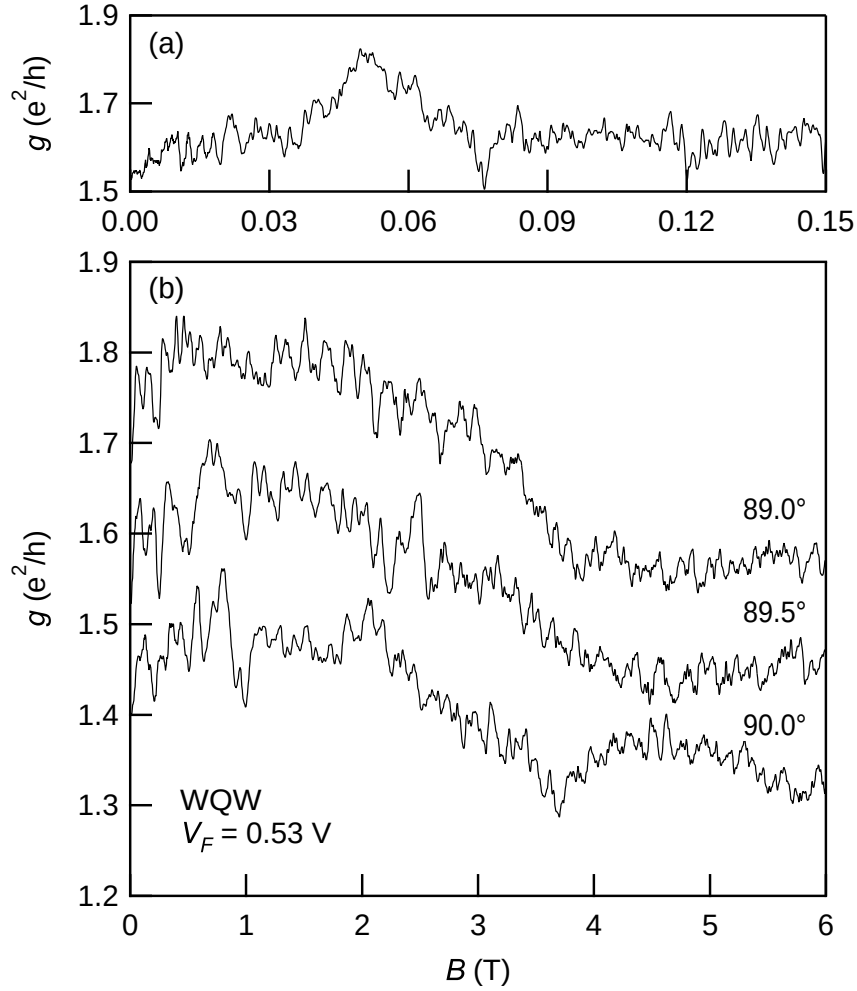


Figure 6.4: (a) Low-temperature magnetoconductance of the WQW dot under a strictly perpendicular B and for $V_F = 0.53$ V. (b) Magnetoconductance of the WQW dot at indicated tilt angles, with $V_F = 0.53$ V. The traces are shifted in steps of $0.15 e^2/h$ with respect to the $\theta = 90^\circ$ magnetoconductance curve.

conditions are fulfilled, the autocorrelation function of the conductance fluctuations with respect to magnetic field is Lorentzian-squared[170] and the corresponding UCFs power spectrum reads

$$S_B(f_B) = S_B(0) [1 + 2\pi B_c f_B] e^{-2\pi B_c f_B}, \quad (6.2)$$

where f_B is the frequency in [1/T] and B_c is a correlation field.

Numerically, a power spectrum can be easily obtained by calculating the Fast Fourier Transform (FFT) of the magnetoconductance $g(B)$. However, when applied to ballistic transport properties, extra care must be taken in the way the FFT is evaluated. Figure 6.5.(a) shows the FFT of the $\theta = 90^\circ$ magnetoconductances in the NQW (plain curve) and WQW (dashed curve) dots at the indicated gate voltages. Similar to the variance analysis of Chap. 4, a slowly varying, low frequency background was removed from the raw magnetoconductances of Fig. 6.2.(b) (NQW) and 6.3(b) (WQW).

The difference between the two power spectra is striking as it is now clearly visible that parallel B -induced UCFs in the NQW dot are limited to a few selected frequencies in the lower part of the spectrum while the UCFs frequency distribution in the WQW dot has a much larger spectral extension. Figure 6.5.(b) gives a closeup of the NQW dot power spectrum for $V_F = -0.25$ V, with two other power spectra at the additional gate voltages $V_F = -0.21$ V and -0.27 V. Clearly, only 3 to 4 majors peaks can be isolated and are marked with vertical lines on fig. 6.5.(b). Their respective contributions to the UCFs are shown on Figs.6.5.(d) and (e). This is a confirmation of the “almost”-periodic nature of the parallel B -induced UCFs observed in the NQW dot (Fig. 6.2.(b)) as opposed to the aperiodicity of the $\theta = 0^\circ$ fluctuations in the WQW (Fig. 6.3.(b)) or the $\theta = 0^\circ$ UCFs in both dots (Figs. 6.2.(a) and Figs. 6.3.(a)).

Similar power spectra, limited to a few frequencies, have already been observed in open quantum dots and were attributed to a selection by the cavity of a restricted number of stable electron trajectories[4, 19, 65, 48] in the dot (the so-called “Wave Scarring” effect) that could be probed by the magnetic field. However, in this particular case, the main peaks were located at much higher frequencies and could be linked to trajectories encompassing a large part of the cavity area. Similar to the periodicity of Aharonov-Bohm oscillations, peaks in the UCFs power spectrum are expected at frequencies given by

$$f_B(B) = \frac{eS}{2\pi\hbar}, \quad (6.3)$$

where S is the area enclosed by stable electron trajectories in the dot. For cavities with sizes on the order of a few μm^2 , Eq. 6.3 yields frequency contributions in the range 100 to 1000 T^{-1} , as observed in Ref. [141].

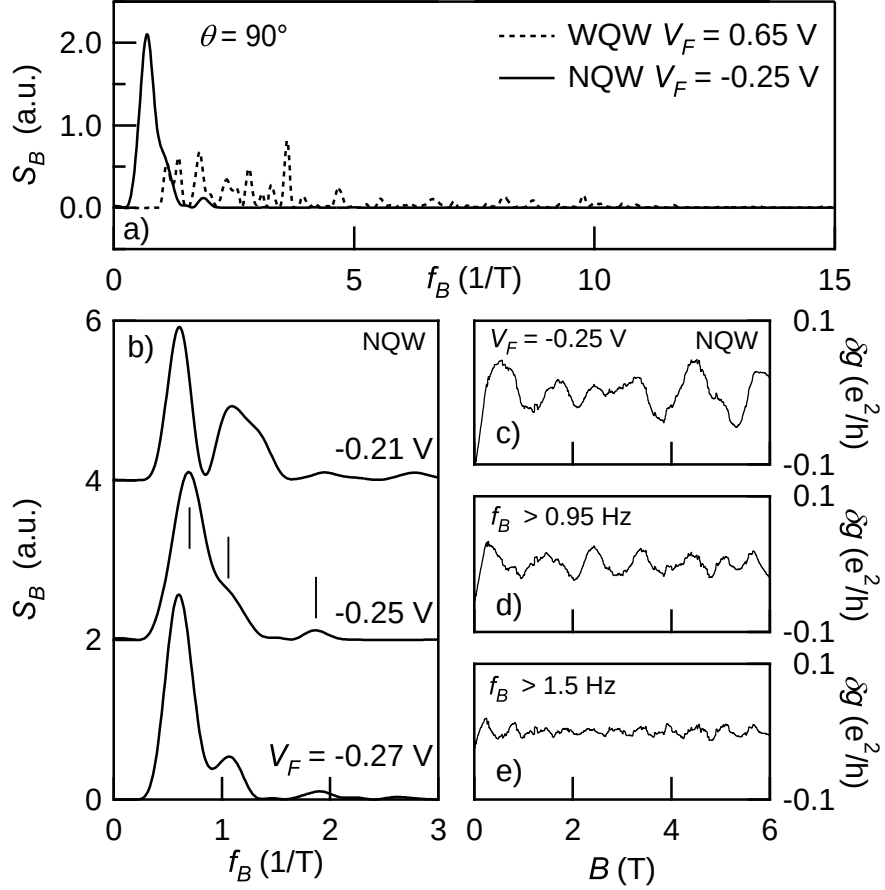


Figure 6.5: (a) Fourier transforms of the UCFs in the WQW (dotted curve) and NQW (plain curve) dots. To obtain the UCFs, a slowly varying background was removed from the raw $\theta = 90^\circ$ magnetoconductances of Figs. 6.3.(b) and 6.2.(b), respectively. (b) Fourier transforms of the UCFs in the NQW dot at indicated V_F 's. The traces are shifted in steps of 2 [a.u.] with respect to the $V_F = -0.27$ V curve. At $V_F = -0.25$ V, the 3 main frequency peaks are indicated by vertical bars. (c) UCFs and (d-e) main frequency components of the NQW dot magnetoconductance for $V_F = -0.25$ V.

On the contrary, for the parallel field power spectra in the NQW dot, the peaks have their maxima for $f_B < 3 \text{ T}^{-1}$. These values would correspond to trajectory-delimited areas ridiculously small compared to the dot size, with characteristic lengths two orders of magnitude smaller than the cavity radius. Quantitatively, an Aharonov-Bohm frequency of 3 T^{-1} is contributed by a trajectory loop defining an area $A = 2\pi\hbar f_B/q$ of $1.4 \times 10^{-3} \mu\text{m}^2$, even smaller than a semi-classical trajectory that would be partly located in the 15 nm-thick well itself. Therefore, the stable orbits argument cannot be safely applied in this case. Instead, the parallel B seems to have a stronger impact on the *speed* with which the cavity trajectories are probed by electrons. The stronger the electrostatic confinement of the 2DEG, the larger the portion of the phase space that can be explored for a given range of the parallel magnetic field. We will return to this important question later in Sec. 6.4 when we perform a remapping of the UCFs in energy space.

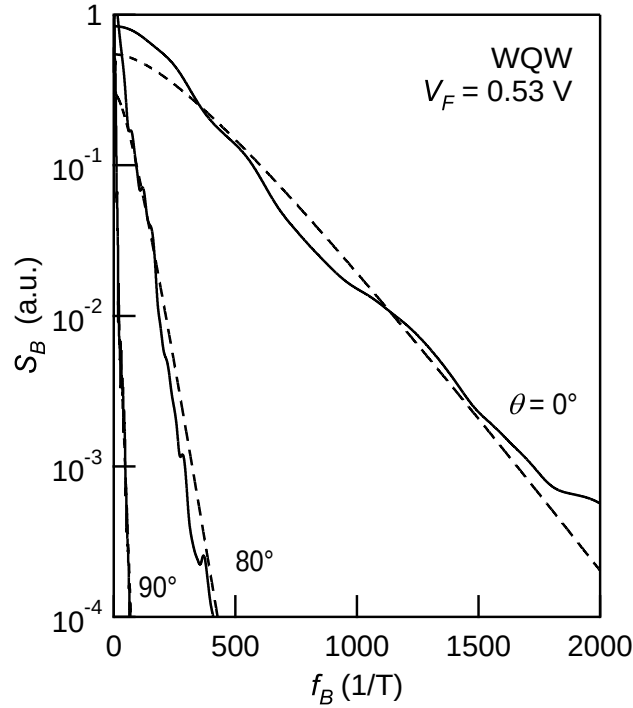


Figure 6.6: Power spectra of the WQW dot UCFs for $\theta = 0, 89$, and 90° . The dashed curves are fits against the predictions of RMT. (see text).

In the meantime, the calculation of power spectra, that could be compared to the theoretical prediction of Eq. 6.2, requires some averaging techniques to smooth out the UCFs frequency peaks particularly visible in the case of the NQW dot. We therefore perform an average power spectrum analysis by evaluating the FFT of the magnetoconductance on half-overlapping fixed-size boxes along the B -axis. This procedure is similar to the technique used for variance analysis in Chap. 4 and is quite common in the literature[122]. Figure 6.6 shows a sample of these average power spectra at indicated tilt angles for the WQW dot. We emphasize that the calculations are based on magnetoconductances expressed in terms of the *total* field. For the three tilt angles displayed on Fig. 6.6, the power spectra are nicely fitted over more than 3 decades against the theoretical model of Eq. 6.2.

6.3.2 Effective Tilt Angle

For each magnetoconductance curve at a tilt angle θ , a similar power spectrum $S_B(f_B)$ can be evaluated. A correlation field $B_c(\theta)$ can then be calculated by fitting the experimental power spectrum with Eq. (6.2). To express this correlation field in terms of a more physical entity, we introduce an effective tilt angle $\theta_{B_c}(\theta)$, extracted from $B_c(\theta)$ using the expression

$$B_c(\theta) = B_c(\theta = 0) / |\cos \theta_{B_c}|. \quad (6.4)$$

In Fig. 6.7, values of θ_{B_c} obtained from Eq. (6.4) for both the NQW and WQW dots are compared to the real tilt angle of the sample θ , extracted from the 2DEG transverse magnetoresistance in the Hall bar located near the dot. In both the NQW and WQW dots, θ_{B_c} has a linear dependence on θ at low tilt angles but saturates as $\theta \rightarrow 90^\circ$. The onset of saturation is observed at $\theta = 89^\circ$ with $V_F = 0.65$ V ($\theta = 88.5^\circ$ with $V_F = 0.53$ V) in the WQW dot, compared to $\theta = 89.9^\circ$ in the NQW dot (upper inset to Fig. 6.7). For UCFs created by a perpendicular B , one would expect a strict linear dependence $\theta_{B_c} \equiv \theta$ at all tilt angles. The observed saturation thus indicates that above some QW-dependent tilt angle, UCFs are mainly generated by the in-plane component of B .

For an ideal, zero-thickness 2DEG, a parallel field should have no effect on electron dynamics in the dots: classically, it induces no Lorentz force that could bend the electron paths, and quantum mechanically, it produces no magnetic flux, hence no interferences among trajectories. Therefore, the UCFs that dominate above the saturation angle must have a different origin from the fluctuations generated by the perpendicular field. Since the major difference between the two samples is the

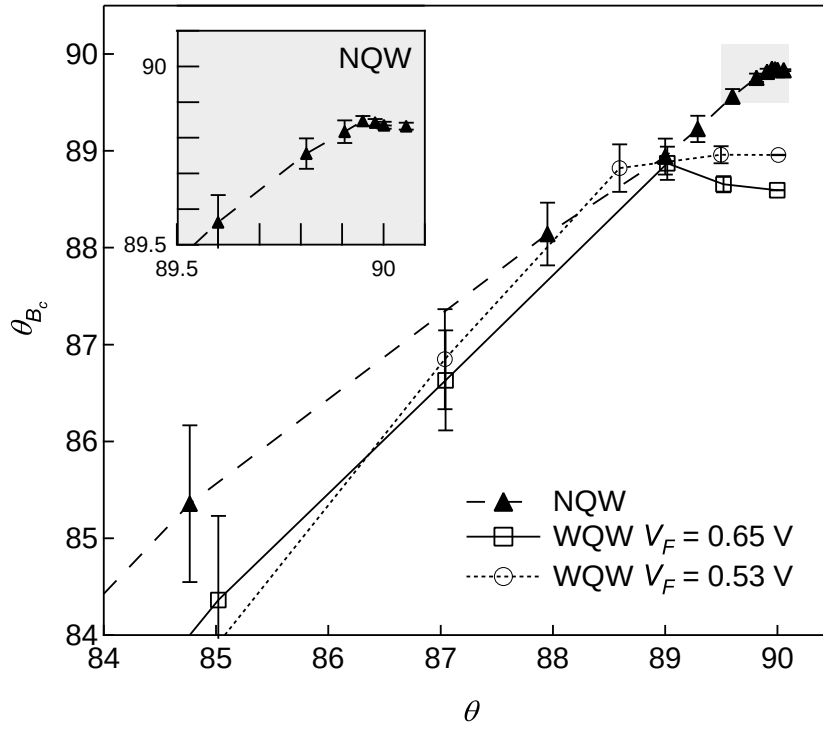


Figure 6.7: The effective tilt angle θ_{B_c} calculated from the correlation field B_c (see text) is shown as a function of the angle θ extracted from Hall measurements. Here θ_{B_c} is calculated for $\theta \geq 84^\circ$ for both the WQW ($\square$) and NQW ($\blacktriangle$) dots. The plain and dotted curves are guide to the eye. Upper inset: Zoom into the $\theta \geq 89.5^\circ$ range for the NQW dot.

width of the QWs, the different values of the saturation angle in the NQW and WQW dots indicates that a parallel field has a stronger effect with respect to the production of UCFs when the 2DEG is weakly confined in the QWs. It is thus natural to invoke the interplay between the electrostatic and magnetic energies as the source of these parallel B -induced UCFs. The value of the saturation angle would then be an indication of the threshold point where the magnetic confinement of the 2DEG becomes commensurate in strength with the electrostatic one, *i.e.* where the parallel field-induced fluctuations start to dominate, then completely replace the perpendicular ones.

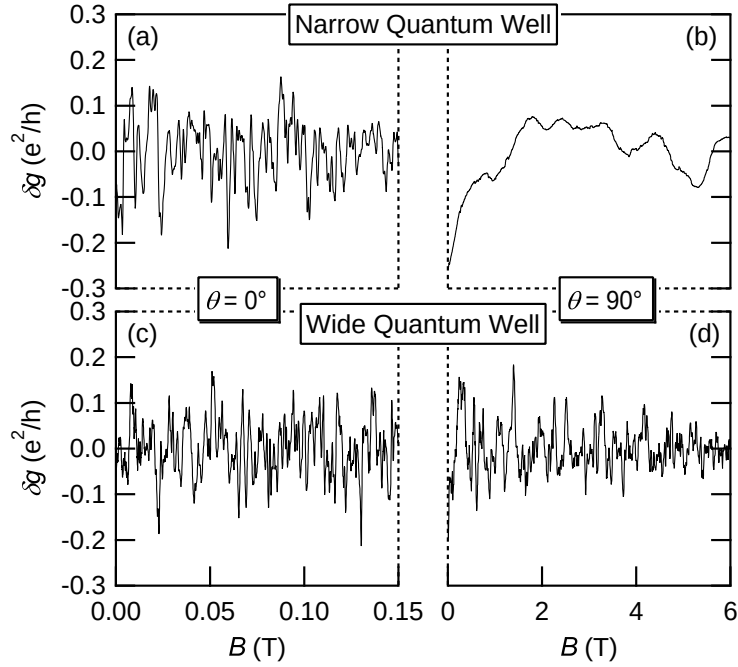


Figure 6.8: UCFs of the NQW (upper row) and WQW (lower row) under a strictly perpendicular (left column) and parallel (right column) B . To isolate the UCFs, a slowly varying background corresponding to the high- T magnetoconductances (Figs. 4.11 and 4.14) was removed from the raw curves of Figs. 6.2 and 6.3, at the indicated tilt angles.

This closes the discussion of the open dot transport properties under tilted fields. For the last section of this chapter, we return to the $\theta = 0^\circ$ and $\theta = 90^\circ$ configurations. As a reminder, Figure 6.8 summarizes the main results of Chap. 4 obtained when the NQW (upper

row) and WQW (lower row) dots are subject to a strictly perpendicular (left column) and parallel (right column) magnetic field. For all the traces, the corresponding, temperature-independent background was subtracted, leaving only the fluctuations induced by electron coherent effects. The three important observations are: the presence of UCFs in a strictly parallel field (panels b and d), the difference in the UCFs frequency distribution between the NQW and WQW dot (all panels), and the reduction of the UCFs amplitude with increasing B for the strictly parallel field configuration in the WQW dot (panel d).

6.4 E_F -induced UCFs

6.4.1 Effective Fermi Energy

We now discuss the experimental results of Chap. 4 and Sec. 6.2 in light of the self-consistent simulations of Chap. 5. In Sec. 4.9, it was shown that the presence of UCFs under a strictly parallel B could be linked to the non-negligible changes in the position of E_F with respect to the conduction band edge as B increases. This variation of $E_F(B)$, particularly large in the case of the WQW, is similar to the variation of the Fermi level obtained by sweeping the voltage on electrostatic gates[101].

It is therefore relevant to present the $\theta = 90^\circ$ magnetoconductances of Fig. 6.8 in terms of energy instead of magnetic field. The idea is to obtain a mapping between B and the variation δE_F of the Fermi level with respect to the conduction subband edges in the QWs. Because only a single band is occupied by electrons in the NQW sample, δE_F can be readily calculated using the $E_F(B)$ curve in Fig. 5.8(a)

$$\delta E_F(B) = |E_F(B) - E_F(B = 0)|. \quad (6.5)$$

Unfortunately, this relation is not directly applicable to the WQW as two subbands contribute to transport in the dot, at least until the onset of subband depopulation is reached. As a result, the two $E_F(B)$ curves of Fig. 5.8(a) for the WQW upper and lower subbands must be combined here in a meaningful way to obtain a single $\delta E_F(B)$ relation. To our knowledge, such a relation has never been derived in the literature. More generally, quantum transport in multiple subband ballistic systems remains a somewhat unexplored field on the theoretical point of view, with the major endeavours[118, 82] focusing on the properties of 2DEG itself and unfortunately not on nanostructures with additional degrees of confinement (quantum wires or open dots).

Therefore, we now investigate a homebrewed and physically sound model leading to a single $\delta E_F(B)$. The model takes into account the relative occupancy of the 2DEG electrons in the occupied subbands. For this purpose, we propose the following empirical arguments: (1) Macroscopically, it is reasonable to consider that the average magnetoconductance $\langle g \rangle$ across the dot at $\theta = 90^\circ$ is a good image of δE_F . This is supported by the fact that $\langle g \rangle$ is strongly influenced by the transfer of electrons from the upper to the lower subband as B increases. In particular, when the upper band is fully depleted, $\langle g \rangle$ remains constant. (2) At lower magnetic fields, a sensible hypothesis would be that the contributions to δE_F of the two occupied subbands are proportional to their respective weights in the total sheet density n_s . (3) Finally, because the UCFS are created by the variation of E_F with B and do not depend on its absolute value at a given field, it seems reasonable to extract δE_F from a density weighted average of first order derivatives of $E_F(B)$ for the two occupied subbands. Based on these arguments, we propose

$$\delta E_F(B) = \left| \int_0^B \left(\frac{\partial E_F^{up}(B')}{\partial B'} \frac{n_s^{up}(B')}{n_s(B')} + \frac{\partial E_F^{low}(B')}{\partial B'} \frac{n_s^{low}(B')}{n_s(B')} \right) dB' \right| \quad (6.6)$$

for the WQW, where $n_s^{up}(B)$, $n_s^{low}(B)$ and $n_s(B) = n_s^{up}(B) + n_s^{low}(B)$ are the upper subband, lower subband and total sheet densities at a given B , respectively. Here E_F^{up} and E_F^{low} are the positions of the Fermi level with respect to the bottom of the upper and lower subbands, as represented on Fig. 5.13.(b).

We now apply this density-balanced model to the various quantum wells whose dispersion relations were calculated self-consistently in Chap. 5. Figure 6.9(a) shows the calculated δE_F obtained from Eqs. 6.5 and 6.6 for the narrow, wide and extra wide quantum well under a strictly parallel B . The difference in energy variation between the NQW and WQW wells is striking, and particularly emphasized by the first-order derivatives of δE_F shown in Fig. 6.9(b), as there is one order of magnitude between δE_F at $B = 6$ T. The onset of subband depopulation in the WQW is also clearly marked by a kink in δE_F .

Of major interest is the particular case of the extra-wide quantum well (XWQW), where δE_F has a similar B -dependence as the WQW at low fields, but saturates around $B = 4$ T. Even though this value is close to the field where the onset of subband depopulation occurs in the WQW, the coincidence is purely accidental. The characteristic field in the XWQW is actually the value of B where the saddle point condition occurs in the XWQW lower subband Fermi contour, as shown

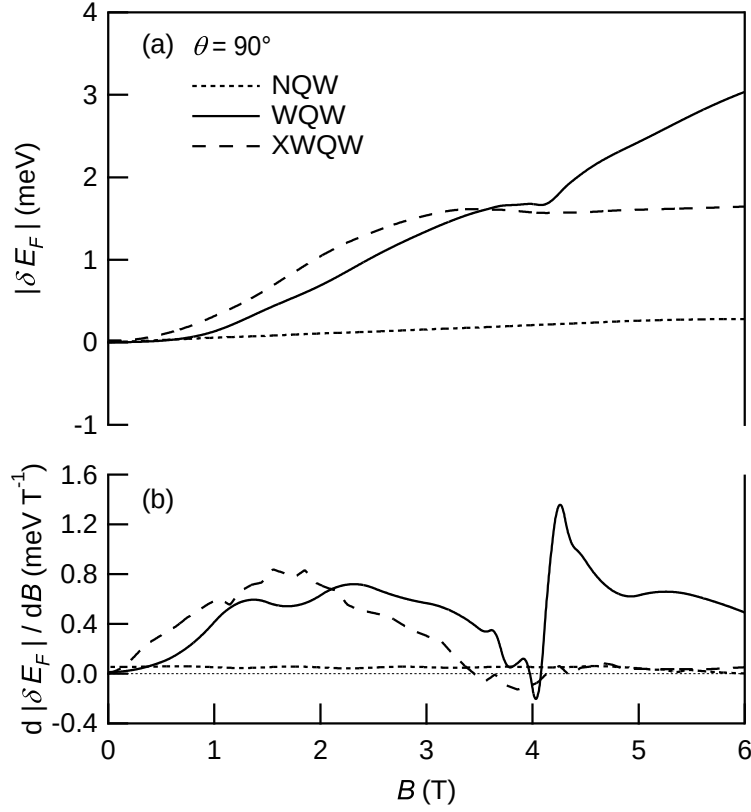


Figure 6.9: (a) Variation with magnetic field B of the self-consistent effective Fermi energy δE_F in the narrow (dotted curve), wide (plain curve) and extra wide (dashed curve) quantum wells. For the WQW and XWQW, the two $E_F(B)$ relations for the upper and lower occupied subbands shown in Figs. 5.8(a) and 5.13.(a), resp., were merged into a single $\delta E_F(B)$ using Eq. 6.6. (b) First-order derivatives with respect to magnetic field of the effective Fermi energies shown in Panel (a). The fine dotted trace is the zero ($d|\delta E_F(B)|/dB \equiv 0$) line.

in Fig. 5.10. As soon as the transition from a single to a bilayer electron gas[153, 112] has started, following the van Hove singularity in the density of states, E_f changes only marginally in the lower subband (Fig. 5.13.(a)) and δE_F saturates in the XWQW (Fig. 5.13.(b)).

6.4.2 UCFs in Energy-space

We next use the mapping relation between B and δE_F illustrated in Fig. 6.9 to rescale the $\theta = 90^\circ$ magnetoconductance curves of Fig. 6.8 [panels (b) and (d)].

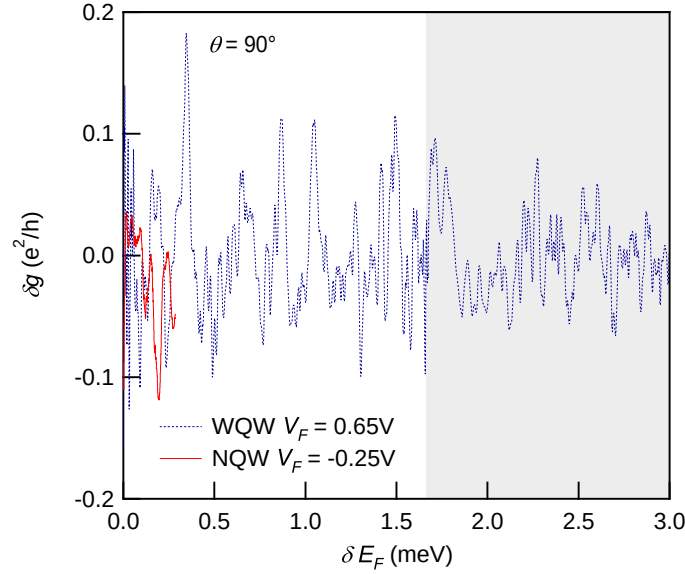


Figure 6.10: UCFs of the narrow and wide quantum wells as a function of δE_F . The gray region indicates the energy range where only one subband is occupied in the WQW sample.

Figure 6.10 shows the UCFs in the full energy range (from 0 to 3.0 meV) covered by δE_F in the WQW dot. By rescaling B into δE_F , it becomes apparent that the parallel magnetic field only probes a small subset of UCFs in the NQW dot compared to what happens with the WQW dot, as we already implied in Sec. 6.3.1 from the discussion of the frequency peaks in the power spectra for the NQW dot (Fig. 6.5). Moreover, the energy-scaled fluctuations have now similar periodicities, meaning that δE_F is indeed the ruling parameter. We point out that, even though the two dots have the same shape and area, it is natural that

no exact or even sensible superposition of the rescaled UCFs is observed in Fig. 6.10 as the dots do not share the same sheet density or number of transmission modes in the dot openings. However, the observation of common periodicities in the UCFs strongly reinforces our interpretation that these UCFs have their origin in the probing of electron trajectories in the dot by the Fermi level, with a strength that depends on the well width and on the parallel B -induced orbital effect.

A few words on the XWQW. We unfortunately did not perform magnetoconductance measurements on a XWQW dot. Hopefully, this loophole in our experimental data set will be addressed in the next few months. As a result, it is impossible to use the δE_F relation, shown in Fig. 6.9, to perform a remapping of the XWQW magnetoconductance into E_F -space. However, it seems safe to make some predictions on the *shape* of the experimental UCFs fingerprint:

1. For $B < 3.5$ T (the onset of single to bilayer electron gas transition), UCFs are expected to be similar to those observed in the WQW, with a reduction in their variance with increasing B , as investigated in Chap. 4.
2. A drop in the average conductance $\langle g \rangle$ across the dot, due to a reduction in the number of transmission modes in the dot openings, is expected at the onset of subband depopulation. In the XWQW, it was shown through self-consistent simulations in Sec. 5.4 that this transition should occur around $B = 1.5$ T, although the exact position of the onset field depends on the electron density in the 2DEG whose value might change for non-zero gate voltages V_F .
3. More importantly, as δE_F does not increase significantly above $B > 3.5$ T, it is expected that UCFs will disappear or, at least, should have their power spectrum shifting to much smaller frequencies.

While predictions 1 and 2 will surely be confirmed by magnetoconductance measurements, we are eager to see if prediction 3 will be verified. If this were unfortunately not the case, it would mean that our simple model of an effective Fermi level, as proposed in Eq. 6.6, does not fully take into account the complex interplay between the electrostatic and magnetic confinements of the 2DEG on one side, and the coherent interferences between electron trajectories in the dot on the other side. In particular, it might be interesting to consider the shift in $\mathbf{k}$ -space of the XWQW Fermi contours beyond the saddle point condition, as represented in Fig. 5.10. For the observation of UCFs, the way

electrons are injected in the dot and the distribution of their $\mathbf{k}$ -vectors in the vicinity of the quantum point contacts might play an additional role. In which case, more complex theoretical tools, like RMT with multi-subband Hamiltonians, will be required to fully describe the UCFS fingerprint in WQW dots beyond the single to bilayer transition.

6.4.3 Power Spectrum in E_F -space

In Sec. 6.3, a frequency analysis of the UCFS in tilted magnetic fields was performed and an effective tilt angle θ_{B_c} was extracted from the power spectrum of $\delta g(B)$. It was found that θ_{B_c} saturates as $\theta \rightarrow 90^\circ$. As discussed earlier in the text, it seems more appropriate in the case of the $\theta = 90^\circ$ UCFS to perform a power spectrum analysis in energy space, using the $B \rightarrow \delta E_F$ mapping of Fig. 6.9. Because the $\theta = 90^\circ$ UCFS spectrum in the NQW dot has a limited frequency distribution, we restrict our analysis to the $\delta g(\delta E_F)$ conductance obtained in the WQW dot [Fig. 6.10, dotted curve]. An average power spectrum in energy space $S_E(f_E)$ is calculated using the same procedure as in Sec. 6.3 for the power spectrum $S_B(f_B)$ in magnetic field space. As seen in Sec. 3.2.5, and similar to the result of Eq. (6.2), the UCFS autocorrelation function in energy space is a Lorentzian[58, 59] that yields an exponentially decreasing power spectrum density

$$S_E(f_E) = S_E(0)e^{-2\pi E_S f_E}, \quad (6.7)$$

where E_S is a characteristic energy.

Figure 6.11 shows power spectra in energy space of the WQW dot UCFS $\delta g(\delta E_F)$ at $\theta = 90^\circ$ and $V_F = 0.65$ V. A good agreement between the experimental $S_E(f_E)$ and the theoretical prediction of Eq. 6.7 is obtained by evaluating and fitting the power spectrum on two distinct domains, delimited by the onset of subband depopulation in the WQW. The two domains are those depicted in Fig. 6.9 and correspond to $0 < \delta E_F < 1.7$ meV and $1.7 < \delta E_F < 3$ meV. On Fig. 6.11, the set of red curves represent the power spectrum and the fit to the dotted curve of Fig. 6.9(b) for $0 < \delta E_F < 1.7$ meV, where two subbands are occupied. The blue curves were obtained in the energy domain $1.7 < \delta E_F < 3$ meV, where the upper subband is fully depleted. The calculated characteristic energies E_S are 0.010 and 0.013 meV for the one and two occupied subbands configurations, respectively. Interestingly, this increase of E_S with the number of occupied subbands is expected, as the semi-classical theory predicts³ that $E_S \propto N\Delta/\pi$. Because Δ

³The same expression for E_S can also be obtained through the semi-classical

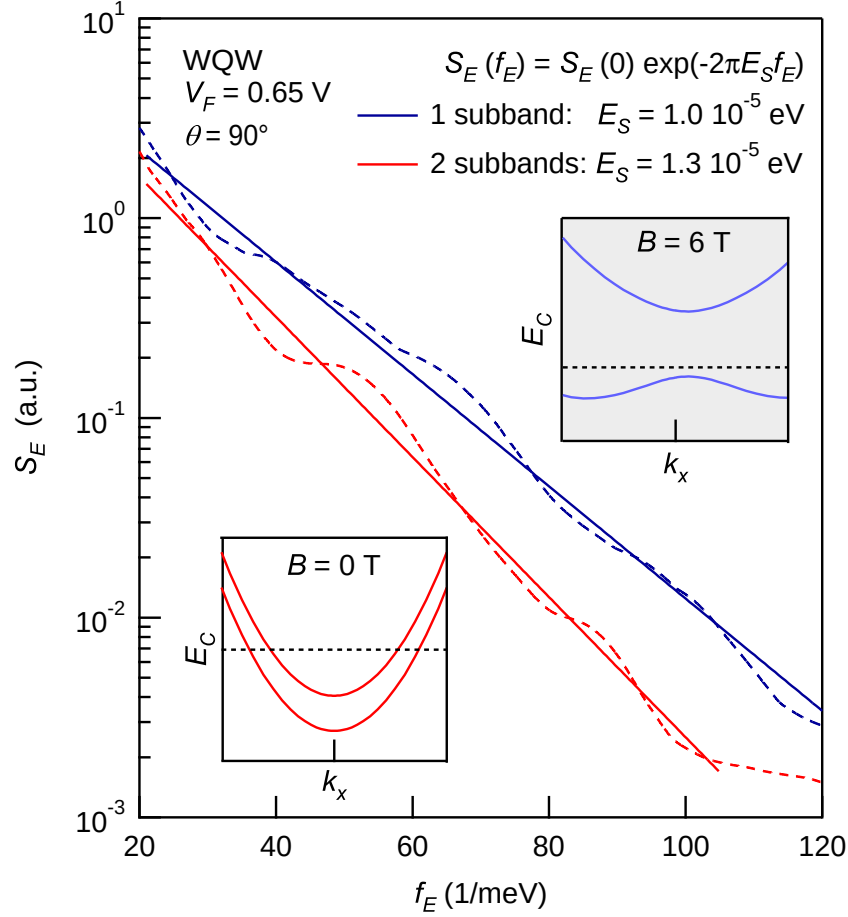


Figure 6.11: Frequency analysis as a function of energy of the WQW dot parallel B -induced UCFs for $V_F = 0.65 \text{ V}$. Here $\delta g(\delta E_F)$ is Fourier transformed on the selected energy ranges (see text) to obtain the average power spectra S_E shown in the figure as dashed red and blue curves. The plain red and blue curves are one-parameter fits (see text) to these $S_E(f_E)$ spectra. Insets: Dispersion relations of the wide quantum well at $B = 0$ ($\delta g(E_F) = 0$, two occupied subbands, lower panel) and $B = 6 \text{ T}$ ($\delta g(E_F) = 3 \text{ meV}$, one occupied subband, upper panel). The position of the Fermi level is indicated as a dotted line.

remains constant, the only parameter that can influence the value of E_S for parallel B -induced UCFs is the number of transmission modes. The drop in the background magnetoconductance around $B = 4.2$ T, observed when the WQW dot is subject to a parallel field (Fig. 6.3(b)), is an image of the reduction of N as the upper subband gets depopulated and does not contribute anymore to the transmission modes into the dot. Using an effective radius of $0.75 \mu\text{m}$ for the dot, that takes into account the depletion region induced by the sample wet etching, the mean-level spacing is 4×10^{-3} meV. Further assuming that N is reduced from 10 to 8 as the magnetoconductance g decreases from 5 to $4 e^2/h$ because of subband depopulation (see Table 4.2 and Fig. 6.3(b)), the theoretical value of E_S is expected to drop from 0.0128 to 0.0102 meV, which is clearly in good agreement with the values of E_s extracted from the experimental power spectra.

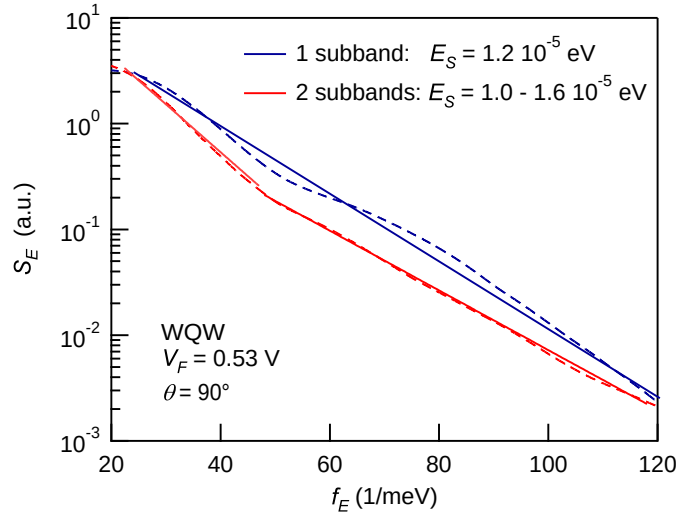


Figure 6.12: Frequency analysis as a function of energy of the WQW dot parallel B -induced UCFs for $V_F = 0.53$ V. The color conventions are similar to those of Fig. 6.11. The plain red and blue curves are one-parameter fits (see text) to the $S_E(f_E)$ spectra.

Next, we analyse the $\theta = 90^\circ$ magnetoconductance measured in the same WQW at a different gate voltage $V_F = 0.53$ V (Fig. 6.4.(b)). As it

approach[58] as both Supersymmetry and semi-classical approximations[95] give the same expressions for the UCFs autocorrelation function in magnetic field and energy spaces.

appeared in Sec. 4.4.2 that the density in the 2DEG was not significantly modified by the gate voltage, we keep using the $B \rightarrow \delta E_F$ mapping of Fig. 6.9 to transform $\delta g(B)$ from the magnetic field to the Effective Fermi level space. The corresponding power spectra and their fits against Eq. 6.7 are both displayed in Fig. 6.12. The color convention is the same as for Fig. 6.11 and corresponds to the domains depicted in Fig. 6.9.

We immediately point out that, unlike the $V_F = 0.65$ V UCFs, fitting against Eq. 6.7 is not straightforward with the $V_F = 0.53$ V power spectra. In particular, two fitting traces are provided for the energy domain corresponding to the two subband case ($\delta E_F < 1.7$ meV), yielding characteristic energy values E_S of 0.016 and 0.010 meV at low and high frequencies, respectively. The extraction of E_S for the 1 subband case ($\delta E_F > 1.7$ meV) is even more complicated. We finally settled on a fit performed on the full frequency range, giving a characteristic energy $E_S = 0.012$ meV.

Surprisingly, these values of the characteristic energy E_S are quite comparable to the ones obtained for $V_F = 0.65$ V. We would have expected E_S to scale as N , *i.e.* the number of transmission modes in dot, hence as the average conductance $\langle g \rangle$ across the dot. Because $\langle g \rangle$ is reduced from 5 to $1.5 e^2/h$ at $B = 0$ T between $V_F = 0.65$ and 0.53 V, E_S should normally be roughly 2-3 times smaller at $V_F = 0.53$ V. However, such an argument does not take into account two side effects of the variation of the gate voltage V_F :

1. While it is unquestionable that N is reduced from $V_F = 0.65$ to 0.53 V, it is not clear that the mean-level spacing remains constant in the dot. If V_F modifies the number of transmission mode in the dot openings, there is no reason to think that the dot area A is not reduced at smaller gate voltages. As a reminder, one of the WQW dot opening is already in the quantum limit ($R > 1 M\Omega$, less than 1 transmission mode) at $V_F = 0.35$ V. For the value of E_S , the reduction in N would then be compensated by a corresponding increase of the mean-level spacing Δ .
2. We discussed in Chap. 4 the possible configurations for the transmission modes in the QPCs in the case of the WQW for $\theta = 0^\circ$ and $V_F = 0.53$ V. In order to explain the difference in the measured values of $\langle g \rangle$ at low and high B s, an asymmetric occupation of the QPCs was proposed, with the upper subband already fully depleted at $B = 0$ in one of the dot openings. If this is truly the case, it means that the coupling between the dot and the electron reservoirs mostly occurs through the lower subband at $V_F = 0.53$ V. As a result, the $B \rightarrow \delta E_F$ mapping might not be as relevant as for

$V_F = 0.65$ V, where $\langle g \rangle = 5 e^2/h$. Even though E_F keeps probing the same trajectories in the dot when B is increased, access to some of these trajectories might not be permitted by the QPC with a single occupied subband. This could also explain why there is no clear difference in the characteristic energies E_S extracted from the power law fit of the two power spectra in Fig. 6.12, and why two characteristic energies can be extracted from the power spectrum in the 2 occupied subband energy domain (plain red curves in Fig. 6.12).

A solution to lift the uncertainties on the dot area and on the number of transmission modes in the dot openings require the use of individual depletion gates for the confinement of the 2DEG in the open dot, as opposed to the wet etching we applied as fabrication technique. The observation and tuning with electrostatic gates of the e^2/h steps in the conductance directly yield the number of transmission modes in the two QPCs. To get an estimate of the dot area, a common technique is to close the two dot openings beyond the quantum limit (e^2/h) and to measure the separation between successive Coulomb Blockade peaks.

The drawback of the depletion gates techniques, beside the added complexity of the measurement infrastructure, is a non-negligible increase of the noise level in the magnetoconductance measurements. In this respect, it seems that the actual tendency is to grow GaAs structures that are finely tuned in such a way that the nanostructure is as close to the surface as possible to have the 2DEG more easily controlled by the depletion gates, and as insensitive to switching noise as possible. While the 2DEG samples were of superb quality with respect to mobility, width and charge distribution in the quantum wells, an acceptable level of noise could only be obtained with etched open dots. Time will tell if the depletion gates technique can be successfully applied to the forthcoming XWQW dot.

6.5 Summary

In this chapter, we have investigated UCFS generated in ballistic quantum dots in tilted magnetic fields. We compared the magnetotransport properties of the NQW and WQW quantum dots as a function of the well width and the tilt angle. The differences between the rich spectrum of parallel B -induced UCFS in the WQW dot and the UCFS measured in the NQW dot were analyzed by means of an effective tilt angle θ_{B_c} , extracted from the power spectra of the magnetoconductance curves. It was found that θ_{B_c} saturates as $\theta \rightarrow 90^\circ$. The strong dependence of this

saturation angle on the well width confirms that the interplay between the electrostatic and magnetic confinements of the 2DEG is the source of the parallel B -induced UCFs.

Using the results of Self-consistent Poisson-Schrödinger simulations, a remapping of B into a self-consistently calculated effective Fermi energy δE_F revealed that the parallel magnetic field only probes a limited set of UCFs in the NQW dot compared to the WQW sample. Predictions were also made on the expected aspect of the magnetoconductance under a parallel field for an open dot built from an extra wide 68 nm-thick quantum well sample. Furthermore, a good agreement was found between the power spectrum analysis in energy space of the WQW magnetoconductance at $\theta = 90^\circ$ and theory. This is a confirmation of the relevance of the δE_F rescaling for the interpretation of parallel B -induced UCFs.

Conclusions and Perspectives

Si c'est peu, si c'est rien
Qu'ils décident eux-mêmes
Je n'espère plus rien
Mais je m'en vais sereine
Mais je m'en vais sereine...

Barbara

A MOURIR POUR MOURIR

Someone having to draw conclusions at the end of a thesis spawning many years, might be in the same position as a drunkard in front of a half-full bottle: neither is the bottle completely full nor totally empty. But while the empty space in the bottle is useless aether, what really matters is what is left behind to quench one's thirst.

Open quantum dots built from GaAs/Al_xGa_{1-x}As two-dimensional electron gases (2DEGs) are funny little experimental toys, not only because they give access to a particular transport regime where electrons move ballistically *and* coherently in the cavity, but also because the universal fluctuations observed in their magnetoconductance bears macroscopic signatures of fundamental microscopic phenomena occurring in the dot, like the loss of phase coherence, wave scarring effects or spin-orbit coupling.

The most significant result of this thesis is the addition of a new entry to this list: the electron orbital effect, a direct consequence of the presence of a parallel magnetic field B combined to the finite width of the 2DEG in realistic structures. This conclusion emerged from the unexpected observation of reproducible, feature-rich magnetoconductance fluctuations (MCFs) in open dots subject to a magnetic field *strictly* parallel to the 2DEG. To isolate the origin of these fluctuations and compare them with their perpendicular field-induced counterparts, our journey in “Quantum Chaos” has experienced four major stages.

First came the fabrication of the open quantum dots. If the key player for transport through OQD in parallel fields was indeed the magnetic flux through the 2DEG, it seemed relevant to carefully choose the width of the electron layer, hence to select 2DEG samples with various strengths of quantum confinement. As a result, two dots were patterned in two-dimensional electron gases (2DEGs), confined to a 15 (NQW) and 45 nm-thick (WQW) GaAs quantum wells with one and two occupied subbands at zero magnetic field, respectively. Even more crucial to the experiment was the use of a sample holder that could be tilted *in situ*, thus ensuring a precise control of the field alignment, by means of a transverse magnetoresistance measurement on an adjacent Hall bar. This is one of the major achievements of this thesis to have successfully discarded, by means of careful alignment procedures, the presence of an unaccounted for, perpendicular component of B as the source of the parallel field-induced fluctuations.

The second major stage involved the comparison of the perpendicular and parallel B -induced magnetoconductances obtained in both the NQW and WQW dots. Through an analysis of the temperature dependence of their oscillation amplitude, based on dephasing models in the framework of Random Matrix Theory (RMT), strong similarities were found between the two conductance sets. This allowed us to confirm that the parallel B fluctuations had indeed a coherent origin, *i.e.* quantum interference effects as opposed to focusing. Even more rewarding was the analysis of the variance of the parallel B and the comparison between the experimental data and recent theoretical models. Here our experimental results brought a clear confirmation to the theoretical proposal of the complementary role played by the Zeeman energy, the B -induced subband depopulation, the variation of the Fermi energy induced by the orbital effect, and time-reversal symmetry breaking for the reduction of the MCFs variance. It also validated our theory that, through the orbital effect, a parallel magnetic field can play a role similar to a perpendicular B , or to an electrostatic gate in probing electron trajectories in the dot, as long as the strength of the electrostatic potential of the 2DEG is commensurate with the confinement induced by the parallel magnetic field.

It was one of the initial goals of this thesis when it started many years ago, to try to combine experimental work with a more theoretical approach based on computer simulations of the transport properties of mesoscopic systems. While these ambitious projects were somewhat amended, self-consistent Poisson-Schrödinger calculations of the 2D magnetic subbands for electrons in realistic quantum wells have shed some light on the microscopic parameter that proved to be the key player

in parallel field MCFs. More precisely, we showed that the width of the quantum well has a critical effect on the way a parallel field modifies the subbands dispersion relations and Fermi contours of the NQW and WQW 2DEGs. In particular, it is now clear that a multiple subband system is necessary to guarantee a large enough variation of the position of the Fermi level with respect to the bottom of the conduction band in the quantum well.

Moreover, the extension of our calculations to the case of an “extra” wide quantum well allowed us to venture a guess on the transport properties of an open dot beyond the 2DEG single to bilayer transition associated to a strong in-plane field. We will come back to this later when outlining the perspectives on future works.

Finally, in Chapter 6, we returned to experimental work by analyzing in great details the frequency distribution of the magnetoconductance fluctuations in the NQW and WQW quantum dots as a function of the tilt angle. Similar to the strictly parallel configuration of Chap. 4, the mechanical precision achieved with the tilting stage, combined to powerful procedures for angle calculation, lead to the calculation of an effective tilt angle θ_{B_c} , extracted from the power spectra of the magnetoconductance curves. Moreover, the observation of the saturation of θ_{B_c} as the sample approaches the parallel configuration, coupled to the strong dependence of this saturation angle on the well width, further confirmed that the interplay between the electrostatic and magnetic confinements of the 2DEG is the source of the parallel B -induced UCFs.

The last part of this thesis work is maybe the most appealing one to the PhD student who started his graduate life doing calculations on solid-state systems before changing his profile to that of an experimentalist. Using the results our self-consistent Poisson-Schrödinger simulations, we proposed a remapping of the magnetic field into a self-consistently calculated effective Fermi energy δE_F , based on the dispersion relations and the subband densities in the wells. Such a transformation from magnetic field to energy space revealed that the parallel magnetic field only probes a limited set of UCFs in the NQW dot compared to the WQW sample. The relevance of the δE_F rescaling for the interpretation of parallel B -induced UCFs was further confirmed by a power spectrum analysis in energy space of the WQW magnetoconductance in a strictly parallel field.

While the results of this work are numerous, and hopefully sufficient to justify the many liters of liquid helium that were consumed during the various measurement sessions, there is fortunately no shortage of tasks that can be tackled to further improve our understanding of the electron dynamics in open quantum dots subject to a parallel field. A large num-

ber of perspectives might come from improvements to some limitations in our fabrication procedures. In particular, a transition from a wet etching-based techniques to a confinement of the electron gas by means of individual electrostatic gates will help a lot in precisely controlling the number of transmission modes in the dot openings. Moreover a precise knowledge of the subband contributions to the 1D states at the dot openings, based on self-consistent simulations extended to a 1D confinement, will improve the accuracy of the variance calculations based on the RMT model of time-reversal symmetry breaking by a parallel field.

Recently, there has been a renewed interest for the study of spin relaxation in quantum dots, following measurements of the anisotropy of spin splitting by means of resonant tunneling spectroscopy[106]. In the regime of strong in-plane magnetic field B , the spin splitting and spin relaxation rates vary with the in-plane orientation of B due to the crystalline anisotropy of spin-orbit coupling[62]. Interestingly, the effect of the orientation of the in-plane field on the transport properties of open quantum dots remains essentially unexplored. Beside its anisotropic effect on spin-orbit scattering, it is expected that the orientation of the parallel field will affect the multi subband occupation of 1D states in the dot openings, hence modifying the way electrons are injected into the cavity. Moreover, as the electron dynamics of realistic OQDs is characterized by a mixed (chaotic and regular) phase, the orientation of B might also play a role in the way classical, wave-scarring trajectories are probed.

When contemplating measurements of the electronic spin relaxation in open quantum dots subject to an in-plane field, it is worth considering other type of structures beside GaAs-AlGaAs 2DEGs. In particular, InGaAs-InAlAs 2DEGs have proved to be useful in the study of the saturation, at low temperatures, of the phase relaxation time in OQDs[77, 78]. Moreover, because of their higher electron density, SO coupling induced by the asymmetric interfacial Rashba field is expected to be stronger in these dots than in GaAs structures. Therefore, an interesting experiment would be to explore the fundamental symmetries[5] of InGaAs dots, in terms of both SO coupling and in-plane fields, and compare the UCF fingerprints with their GaAs counterparts[180].

Beside InGaAs structures, it is also worth mentioning the efforts undertaken by members in our group to fabricate an OQD out of p-type material, *i.e.* where transport occurs through holes, with the perspectives of a strong enhancement of spin-orbit coupling.

More conceptually, we also believe that further experimental work should be performed on similar dots subject to a strong *total* magnetic field, *i.e.* large enough to concentrate all electrons in the lowest Lan-

dau level (filling factor $\nu = 1$) in a perpendicular field configuration. The motivation for this type of measurements has its origin in works by Fromhold *et al.* [70, 71] where a transition from classical electron dynamics to a fully chaotic regime was observed in resonant tunneling diodes, in the presence of a high tilted magnetic field. Major changes in the resonances observed in the device current-voltage characteristics indicated a transition from stable orbits between the well barriers to strongly chaotic motion, as the tilt angle moves away from a perpendicular configuration. It might be interesting to consider similar measurements in OQDs with a large enough confinement potential, and possibly isolate the signature of a chaotic transition in the MCFs pattern.

But in the near future, the most promising perspective is certainly the on-going project of measurements on an open quantum dot built from an “extra” wide (68 nm) quantum well. We showed in self-consistent simulations that the predicted transition from a single to bilayer electron system, associated with a saddle point condition of the Fermi contours, can lead to a saturation of E_F at high fields. We are much eager to see in the next few months if such a transition significantly alters or even suppresses the parallel B UCFs in a extra-wide quantum well dot, which would confirm our orbital interpretation of parallel B -induced UCFs. I personally have full confidence in my colleagues at PCPM to give this task a positive outcome.

Appendix A

Process Sheet

The following table describes the complete succession of processing steps involved in the preparation of an open quantum dot sample based on a GaAs two-dimensional electron gas. A standard process is composed of 4 majors stages:

1. Ohmic contacts
2. Hall bar deep etch
3. Dot shallow etch
4. Electrostatic top gate

These steps were more completely discussed in Chapter [2](#), which we refer the reader to for further details.

Table A.1: Process sheet

Step	Description	Products		Parameters	Notes										
Ohmic contacts															
1	Sample cutting	Trichloroethylene, Acetone, Methanol		Sample dimensions $3 \times 3 \text{ cm}^2$	Boiling temperature										
2	Standard cleaning			10' immersion + 10' ultrasonic bath											
3	Sample sticking	Carbon paint, onto a Si wafer		15' instant backing at 100°C	Rinser-dry										
4	Backing	PMMA 6% 950K in chlorobenzene		60' at 140°C	Better if overnight										
5	PMMA spin-coating			5000 rpm - 5000 rpm/s - 60 s	Thickness : 300-350 nm										
6	SEM Lithography			<table><tr><td colspan="2">Large field lithography</td></tr><tr><td>Magnification</td><td>400</td></tr><tr><td>Aperture</td><td>5</td></tr><tr><td>Spot size</td><td>4</td></tr><tr><td>Pixel</td><td>7</td></tr><tr><td>Sensitivity ($\mu\text{C}/\text{cm}^2$)</td><td>250</td></tr></table>	Large field lithography		Magnification	400	Aperture	5	Spot size	4	Pixel	7	Sensitivity ($\mu\text{C}/\text{cm}^2$)
Large field lithography															
Magnification	400														
Aperture	5														
Spot size	4														
Pixel	7														
Sensitivity ($\mu\text{C}/\text{cm}^2$)	250														
7	Development														
		Standard recipe													
		1:3 MIBK - Isopropanol	1' 30"												
		Isopropanol	30"												
		Water (rinsing)	10'												
continued on next page															

Table A.1 – continued from previous page

Step	Step Description	Products		Parameters	Notes	
8	Metalization			$P < 2 \times 10^{-6}$ mbar	Sample holder as far as possible from crucibles	
		Ni	13 nm			
		Ge	50 nm			
		Au	100 nm			
		Ni	15 nm			
		Au	50 nm			
9	Lift-off	Acetone		5' at boiling temperature + ultra-sonic bath if required	Boiling temperature	
10	Standard cleaning	Trichloroethylene, Acetone, Methanol		10' immersion + 10' ultrasonic bath		
11	Rapid Thermal An-nealing (RTA)			6' at 460°C under 5% H_2 - 95% N_2		Initial rise to base temperature in 30"
12	Standard cleaning	Trichloroethylene, Acetone, Methanol		10' immersion + 10' ultrasonic bath		Boiling temperature
<i>Hall bar deep etch</i>						
13	Standard cleaning	Trichloroethylene, Acetone, Methanol		10' immersion + 10' ultrasonic bath	Boiling temperature	
14	Sample sticking	Carbon paint, onto a Si wafer		15' instant backing at 100°C	Rinser-dry	
15	Backing			60' at 140°C	Better if overnight	
16	PMMA spin-coating	PMMA 6% 950K in chlorobenzene		5000 rpm - 5000 rpm/s - 60 s	Thickness : 300-350 nm	
continued on next page						

Table A.1 – continued from previous page							
Step	Step Description	Products		Parameters	Notes		
17	SEM Lithography			Current ≈ 1 nA, dwelltime $\approx 1.5 \mu\text{s}$			
18	Development					Large field lithography	
						Magnification	400
						Aperture	5
						Spot size	4
						Pixel	7
		Sensitivity ($\mu\text{C}/\text{cm}^2$)	220				
18	Development	Standard recipe					
		1:3 MIBK - Isopropanol	1' 30"				
		Isopropanol	30"				
		Water (rinsing)	10'				
18	Wet etching	1:8:240 $\text{H}_2\text{SO}_4\text{:H}_2\text{O}_2\text{:H}_2\text{O}$		3' + 10' rinsing in water			
19	PMMA cleaning	Acetone		5' at boiling temperature			
20	Standard cleaning	Trichloroethylene, Acetone, Methanol		10' immersion + 10' ultrasonic bath			
Boiling temperature							
Dot shallow etch							
21	Standard cleaning	Trichloroethylene, Acetone, Methanol		10' immersion + 10' ultrasonic bath			
22	Sample sticking	Carbon paint, onto a Si wafer		15' instant backing at 100°C			
23	Backing			60' at 140°C			
Boiling temperature							
Rinser-dry							
Better if overnight							
continued on next page							

Table A.1 – continued from previous page						
Step	Step Description	Products		Parameters		Notes
24	PMMA spin-coating	PMMA 3% 950K in chlorobenzene		6100 rpm - 6100 rpm/s - 60 s		Thickness : 130 nm
25	SEM Lithography					
				</		

Table A.1 – continued from previous page						
Step	Step Description	Products		Parameters		Notes
<i>Electrostatic top gate</i>						
30	Standard cleaning	Trichloroethylene, Acetone, Methanol		10' immersion + 10' ultrasonic bath		Boiling temperature
31	Sample sticking	Carbon paint, onto a Si wafer		15' instant backing at 100°C		Rinser-dry
32	Backing			60' at 140°C		Better if overnight
33	PMMA spin-coating	PMMA 6% 950K in chlorobenzene		5000 rpm - 5000 rpm/s - 60 s		Thickness : 300-350 nm
34	SEM Lithography			Large field lithography Magnification 400 Aperture 5 Spot size 4 Pixel 7 Sensitivity ($\mu\text{C}/\text{cm}^2$) 250		Current ≈ 1 nA, dwelltime ≈ 1.5 μs
35	Development	Standard recipe 1:3 MIBK - Isopropanol 1' 30" Isopropanol 30" Water (rinsing) 10'				
36	Metalization	Au contacts Cr 4 nm Au 35 nm		$P < 2 \times 10^{-6}$ mbar		Sample holder as far as possible from crucibles
continued on next page						

Table A.1 – continued from previous page				
Step	Step Description	Products	Parameters	Notes
37	Lift-off	Acetone	5' at boiling temperature + ultra-sonic bath if required	Boiling temperature
38	Standard cleaning	Trichloroethylene, Acetone, Methanol	10' immersion + 10' ultrasonic bath	

Appendix B

Growth Structures of the QW Samples

In this appendix, schematics of the MBE growth structures are provided for the three GaAs/AlGaAs samples used in this thesis. The structures were grown on semi-insulating, (100)-oriented GaAs substrates at Princeton University. A complete description of the rationale behind the choice, position and width of each layer is beyond the scope of this work. The main purpose of this complex design is the fabrication of 2DEGs with very high mobilities, and a complete control of the shape and width of the confining potentials.

Nevertheless, it seems interesting to mention the main common characteristics of the growth structures :

1. Starting from the GaAs substrate, a superlattice is grown first. Its role is essentially to trap impurities coming from the substrate, that might unintentionally migrate to the quantum well itself and modify the properties of the 2DEG.
2. Follow one or two δ -doped superlattices. By pinning the conduction band at the Fermi level, they guarantee a good spatial symmetry of the quantum well while providing some electrons to the 2DEG.
3. Next comes the well, with a layer of undoped GaAs enclosed by two narrow AlAs barriers.
4. The next superlattice with δ -doping, closer to the 2DEG is a twin to the other superlattice described in point 2. It also pins the conduction band near the Fermi level (at the impurity levels), and determines the electron concentration in the well.

5. The role of the doped superlattice close to the surface is to passivate the surface states at the GaAs-air interface.
6. Finally, a GaAs cap-layer serves as a protection against surface oxidation.

B.1 M276 Narrow Quantum Well

Confidential material. Please contact the author for information.

Figure B.1: M276 - Narrow Quantum Well - Growth Structure

B.2 M231 Wide Quantum Well

Confidential material. Please contact the author for information.

Figure B.2: M231 - Wide Quantum Well - Growth Structure

B.3 M270 Extra Wide Quantum Well

Confidential material. Please contact the author for information.

Figure B.3: M270 - Extra Wide Quantum Well - Growth Structure

Appendix C

Conductance Correlator with Parallel Magnetic Field and Spin-Orbit Coupling

A major result of Random Matrix Theory (RMT) applied to quantum transport in open quantum dots (OQDs) is an expression of the variance of conductance fluctuations that takes into account both dephasing and thermal smearing. In Sec. 3.2.6, it was shown that the variance is obtained from a double integration in energy-space that reads :

$$\text{var}(g) = \int_0^\infty \int_0^\infty f'(E)f'(E')C(E, E')dEdE', \quad (\text{C.1})$$

where f' is the first derivative of the Fermi distribution

$$f' = \frac{d}{dE} \frac{1}{1 + \exp\left(\frac{E-E_F}{k_B T}\right)}. \quad (\text{C.2})$$

with E_F the Fermi energy, k_B Boltzmann constant, and T the temperature. In Eq. C.1, $C(E, E')$ is a conductance correlator whose expression is given by

$$C(E, E') = \langle g \rangle^2 / [(N + N_\varphi)^2 + 4\pi(E - E')^2/\Delta^2]. \quad (\text{C.3})$$

where $\langle g \rangle$ is the average conductance, N is the total number of transmission modes in the dot openings and $N_\varphi = 2\pi\hbar/(\tau_\varphi\Delta)$ the number of dephasing channels. Finally, τ_φ is the phase relaxation time and

$\Delta = 2\pi\hbar^2/Am^*$ the mean level spacing, with A the dot area and m^* the electron effective mass.

Equation C.3 is only valid for UCFs generated by a strictly perpendicular B in a dot with broken time-reversal symmetry, thus for fields beyond the weak localization peak observed in the conductance (Sec. 1.5.2). Therefore, Eq. C.1, combined to the correlator given Eq. C.3, can not be used *as is* to explain the reduction of the fluctuations variance observed in the WQW dot in a strictly parallel field (Fig 4.16).

Fortunately, a new expression for $C(E, E')$ was recently derived by Cremers *et al.* [45] in the case of a system subject to spin-orbit coupling and to a magnetic field of arbitrary orientation. Here, we summarize the main results of this quite imposing publication, and describe the simplifications that can be made to match our experimental configuration, namely a system with a strictly parallel field and no spin-orbit interaction.

As explained in Sec. 3.2.3, Random Matrix theory requires an analysis of the symmetries of the scattering matrix, which are function of the relative magnitudes of characteristic energy scales. For a dot, located in the x-y plane, with spin-orbit coupling and a magnetic field of arbitrary orientation $\mathbf{B} = (B_1, B_2, B_3)$, these energy scales are

$$\begin{aligned}\varepsilon_B &= \kappa E_{\text{Th}} (eB_3 L^2 / 2\hbar)^2, \\ \varepsilon_{\perp}^{\text{so}} &= \kappa E_{\text{Th}} (L^2 / 4\lambda^2)^2, \\ \varepsilon_{\parallel}^{\text{so}} &= \kappa' (L^2 / 4\lambda^2) \varepsilon_{\perp}^{\text{so}}, \\ \varepsilon^Z &= \mu_B g B, \\ \varepsilon_{\perp}^Z &= \frac{\kappa'' (\varepsilon^Z)^2}{E_{\text{Th}}} (L^2 / 4\lambda^2).\end{aligned}\tag{C.4}$$

Here, $E_{\text{Th}} = \hbar v_F l_m / 2L^2$ is the Thouless energy, with v_F the Fermi velocity, l_m the mean free path and L the dot radius, $\lambda^2 = \lambda_1 \lambda_2$, and κ , κ' , and κ'' are coefficients of order unity. The two length scales λ_1 and λ_2 are associated with spin-orbit coupling along the two directions in the plane of the quantum dot, and characterize the length at which the spin of an initially polarized electron would precess with an angle 2π . The coefficients κ' and κ'' depend on the sample geometry and on the ratio λ_1/λ_2 of the spin-orbit lengths. The coefficient κ'' also depends on the direction of the magnetic field. The coefficient κ depends on the sample geometry only. For a circular quantum dot with radius L , mean free path $l \ll L$, Fermi velocity v_F , and $\lambda_1 = \lambda_2 = \lambda$, one has $\kappa = 2$, $\kappa' = 1/3$ and $\kappa'' \approx 0.292$.

In Eq. C.4, ε_B is the characteristic energy associated with the per-

pendicular component of the magnetic field, $\varepsilon_{\perp}^{\text{so}}$ and $\varepsilon_{\parallel}^{\text{so}}$ with spin-orbit coupling, ε^Z is the Zeeman energy, with μ_B Bohr's magneton and g the Landé factor (-0.44 in GaAs). Finally, $\varepsilon_{\perp}^Z$ takes care of the changes in the spin-orbit coupling when a magnetic field is applied[80].

The characteristic energies of Eq. C.4 can be rescaled to

$$\begin{aligned} x^2 &= \pi\varepsilon_B/\Delta, \\ a_{\perp}^2 &= \pi\varepsilon_{\perp}^{\text{so}}/\Delta, \\ a^2 &= \pi\varepsilon_{\parallel}^{\text{so}}/\Delta, \\ b &= \pi\varepsilon^Z/\Delta, \\ b_{\perp}^2 &= \pi\varepsilon_{\perp}^Z/\Delta. \end{aligned} \quad (\text{C.5})$$

with respect to the mean-level spacing Δ .

If we define the corresponding quantities x' , b' , and $b'_{\perp}$ at a different value $\mathbf{B}'$ of the magnetic field, the conductance correlator is given by

$$C[g(\varepsilon, \mathbf{B}), g(\varepsilon', \mathbf{B}')] = \left(\frac{e^2}{h}\right)^2 \left(\frac{N_1 N_2}{N_1 + N_2}\right)^2 \times (V_D + V_C), \quad (\text{C.6})$$

where N_1 and N_2 are the number of transmission modes in the two dot openings, and where

$$V_D = \sum_{\mu, \nu, \mu', \nu' = \pm} D_{\mu\nu; \mu'\nu'} D_{\nu'\mu'; \nu\mu} = \frac{\Xi_D}{\Upsilon_D} \quad (\text{C.7})$$

$$V_C = \sum_{\mu, \nu, \mu', \nu' = \pm} (\mathcal{TCT})_{\mu\nu; \mu'\nu'} (\mathcal{TCT})_{\mu'\nu'; \mu\nu} = \frac{\Xi_C}{\Upsilon_C}, \quad (\text{C.8})$$

Here we abbreviated

$$\begin{aligned} \Xi_D &= 2 \left| K_D(4a^4 - L_D^2) + 4a^2bb' - L_D(b^2 + b'^2) \right|^2 \\ &\quad + 2 \left| K_D(b^2 + b'^2) + K_D^2 L_D - 4a_{\perp}^2 L_D(x - x')^2 \right|^2 \\ &\quad + 8a_{\perp}^2 (x - x')^2 \left((b + b')^2 + |L_D + 2a^2|^2 \right) \\ &\quad \left((b - b')^2 + |L_D - 2a^2|^2 \right) \\ &\quad + 8 \left| K_D bb' + a^2(K_D^2 - 4a_{\perp}^2(x - x')^2) \right|^2 \\ &\quad + 4 \left| b(b^2 - b'^2) - K_D(2a^2b' - bL_D) \right|^2 \\ &\quad + 4 \left| b'(b'^2 - b^2) - K_D(2a^2b - b'L_D) \right|^2 \\ &\quad + 8 \left| a^2(b^2 + b'^2) - bb'L_D \right|^2 \end{aligned} \quad (\text{C.9})$$

$$\begin{aligned}
\Xi_C = & 2 \left| K_C(4a^4 - L_C^2) - 4a^2bb' - L_C(b^2 + b'^2) \right|^2 \\
& + 2 \left| K_C(b^2 + b'^2) + K_C^2 L_C - 4a_\perp^2 L_C(x + x')^2 \right|^2 \\
& + 8a_\perp^2(x + x')^2 \left((b - b')^2 + |L_C + 2a^2|^2 \right) \\
& \quad \left((b + b')^2 + |L_C - 2a^2|^2 \right) \\
& + 8 \left| K_C bb' + a^2(-K_C^2 + 4a_\perp^2(x + x')^2) \right|^2 \\
& + 4 \left| b(b^2 - b'^2) + K_C(2a^2b' + bL_C) \right|^2 \\
& + 4 \left| b'(b'^2 - b^2) + K_C(2a^2b + b'L_C) \right|^2 \\
& + 8 \left| a^2(b^2 + b'^2) + bb'L_C \right|^2
\end{aligned} \tag{C.10}$$

$$\begin{aligned}
\Upsilon_D = & \left| (b^2 - b'^2)^2 + 2K_D(-4a^2bb' + (b^2 + b'^2)L_D) \right. \\
& \left. + (L_D^2 - 4a^4)(K_D^2 - 4a_\perp^2(x - x')^2) \right|^2
\end{aligned} \tag{C.11}$$

$$\begin{aligned}
\Upsilon_C = & \left| (b^2 - b'^2)^2 + 2K_C(4a^2bb' + (b^2 + b'^2)L_C) \right. \\
& \left. + (L_C^2 - 4a^4)(K_C^2 - 4a_\perp^2(x + x')^2) \right|^2
\end{aligned} \tag{C.12}$$

and

$$\begin{aligned}
K_D &= N + \frac{2\pi i(\varepsilon - \varepsilon')}{\Delta} + \frac{1}{2}(x - x')^2 + 2(a^2 + a_\perp^2) + \frac{1}{2}(b_\perp + b'_\perp)^2, \\
L_D &= N + \frac{2\pi i(\varepsilon - \varepsilon')}{\Delta} + \frac{1}{2}(x - x')^2 + 2a^2 + \frac{1}{2}(b_\perp - b'_\perp)^2, \\
K_C &= N + \frac{2\pi i(\varepsilon - \varepsilon')}{\Delta} + \frac{1}{2}(x + x')^2 + 2(a^2 + a_\perp^2) + \frac{1}{2}(b_\perp - b'_\perp)^2, \\
L_C &= N + \frac{2\pi i(\varepsilon - \varepsilon')}{\Delta} + \frac{1}{2}(x + x')^2 + 2a^2 + \frac{1}{2}(b_\perp + b'_\perp)^2.
\end{aligned}$$

In the case of an open dot with no spin-orbit coupling and a strictly parallel field, one has $x = a_\perp = a = b_\perp = 0$. Therefore, we get

$$K_D = L_D = K_C = L_C = N + \frac{2\pi i(\varepsilon - \varepsilon')}{\Delta} \tag{C.13}$$

When plugged into Equations C.9, C.10, C.11 and C.11, Eq. C.13 yields an expression of the correlator $C[g(\varepsilon, \mathbf{B}), g(\varepsilon', \mathbf{B}')]$ that has no dependency upon a fitting parameter.

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