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Determination of the top quark mass through the measurement of the $\dfrac{\Gamma(Z\to b\bar{b})}{\Gamma_{\text{hadr}}-\Gamma(Z\to b\bar{b})}$ ratio at high luminosity LEP

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Abstract. We introduce the ratio of $b\bar{b}$ events to the hadronic non $b\bar{b}$ ones as an excellent quantity to determine indirectly the top quark mass at high luminosity LEP. Assuming a realistic impact parameter resolution we conclude that with a few million Z's the top quark mass can be fixed with a precision of about 15 GeV.

1 Introduction and motivation

The influence of the top quark on physics processes through radiative corrections is due to the violation of the decoupling theorem [1] in spontaneously broken theories and manifests itself in terms in the Lagrangian with a quadratic dependence on the top quark mass. In LEP processes those corrections are of two types: corrections to the Z boson propagator and corrections to the $Zb\bar{b}$ vertex.

The *corrections to the Z propagator* or Z self energy are common to all LEP processes and can be included in a common factor and in a redefinition of the Weinberg angle. Up to now, only this first effect has been exploited to constrain the possible values of the top quark mass when analyzed together with the results from other non-LEP experiments, such as the measurement of the W boson mass in $p\bar{p}$ colliders and the Weinberg angle obtained from deep inelastic ν–N scattering. The value of the top quark mass thus obtained is [2]: $m_t = 155 \pm 30$ GeV, where the error due to our ignorance of the Higgs mass is substantial (around 20 GeV when m_H is allowed to vary between 50 and 1000 GeV). On the other hand, in the *corrections to the $Zb\bar{b}$ vertex*, where the leading contribution on the top quark mass is produced by the exchange of longitudinal W's [3], there is no dependence on the Higgs mass. Moreover, the possible new physics contributions to the $Zb\bar{b}$ vertex are much more restricted and different from the contributions to the Z self energy.

Both kinds of corrections enter in the calculation of the $\Gamma(Z\to b\bar{b})$ width, but unfortunately they work in opposite directions for the sensitivity on the top quark mass. Some quantities which are sensitive to the $Zb\bar{b}$ vertex alone are the ratios of the $b\bar{b}$ width to the $s\bar{s}$, $c\bar{c}$ and $x\bar{x}$ widths, namely:

$$R_s \equiv \frac{\Gamma(Z\to b\bar{b})}{\Gamma(Z\to s\bar{s})},\quad R_c \equiv \frac{\Gamma(Z\to b\bar{b})}{\Gamma(Z\to c\bar{c})},\quad R_x \equiv \frac{\Gamma(Z\to b\bar{b})}{\Gamma(Z\to x\bar{x})},$$

where x is any *non b* quark (u, d, s and c). These ratios present cancellations of theoretical uncertainties at different levels. Also, many experimental sources of error, such as the luminosity, vanish. To get a precise measurement of m_t a compromise is necessary between the theoretical sensitivity of one of these observables with m_t and the accuracy that can be reached experimentally. From the theoretical point of view the most sensitive ratios are R_s and R_c [4]. However, to measure R_s is highly difficult due to the necessity of tagging strange events which are very similar to $u\bar{u}$, $d\bar{d}$ events. In the R_c case it is difficult to properly select charmed events because they have an intermediate behaviour between bottom and light quark events. We introduce in this letter the new quantity R_x, which has the advantage of being easier to measure experimentally.

2 Branching ratio R_x in the $\overline{MS}$ scheme

The width $\Gamma(Z\to f\bar{f})$ has been recently calculated in the $\overline{MS}$ renormalization scheme [5] and has been expressed in a compact Born-like form, including the effects of the b quark [4]:

$$\Gamma(Z\to f\bar{f}) = \frac{N_c^f}{48}\frac{\hat{\alpha}}{\hat{s}_W^2\hat{c}_W^2} m_z(\hat{a}_f^2+\hat{v}_f^2)(1+\delta_f^{(0)})(1+\delta_{\text{QED}}^f) \cdot (1+\delta_{\text{QCD}})(1+\delta_\mu^f)(1+\delta_{t\text{QCD}}^f)(1+\delta_b),$$

where $\hat{\alpha}$ is the electromagnetic coupling constant defined at the Z scale; $\hat{s}_W^2$ is the definition of the Weinberg angle in the $\overline{MS}$ scheme and $\hat{a}_f$ and $\hat{v}_f$ are the effective axial and vector coupling constants of the Z boson to the fermions.

The factors are global corrections most of which cancel in the ratios of widths:

• $\delta_f^{(0)}$ contains the small electroweak corrections not absorbed in $\hat{s}_W^2$ and some vertex corrections for massless quarks running in the loop. δ_{QED}^f represents the final state QED corrections. They are less than 0.1%. δ_{QCD} gives the QCD corrections common to all quarks. The error on the hadronic widths is of about 0.3% due to the current error in the determination of the strong coupling constant [2]. δ_μ^f contains the kinematical effects of the external fermion masses and is only important for the b quark (0.5%).

• Triangle loops of bottom and top quarks give an additional $\mathcal{O}(\alpha_s^2)$ correction to the axial-vector current induced rate:

$$\delta_{t\mathrm{QCD}}^f = -\frac{\hat{a}_t \hat{a}_f}{\hat{a}_t^2 + \hat{a}_f^2}\left(\frac{\alpha_s}{\pi}\right)^2 f(m_t),$$

where the function $f(m_t)$ [6] varies from -3 to -5, and the prefactor is of the order of $\sim 10^{-3}$ and has opposite signs for up and down quarks.

• δ_b contains all the interesting effects of the top quark mass in the $Zb\bar{b}$ vertex [4, 3]. It has been parametrized in the form [4]:

$$\delta_b \simeq 10^{-2}\left(-\frac{1}{2}\frac{m_t^2}{m_Z^2} + \frac{1}{5}\right).$$

Taking into account all these factors, R_x has a complicated dependence on m_t in the region under study (90–200 GeV). Its plot is shown in Fig. 1. Nevertheless, we have performed simple linear fit in m_t^2, giving:

$$R_x = 0.28168 - 0.18045 \times 10^{-6}\, m_t^2 \ (\mathrm{GeV}).$$

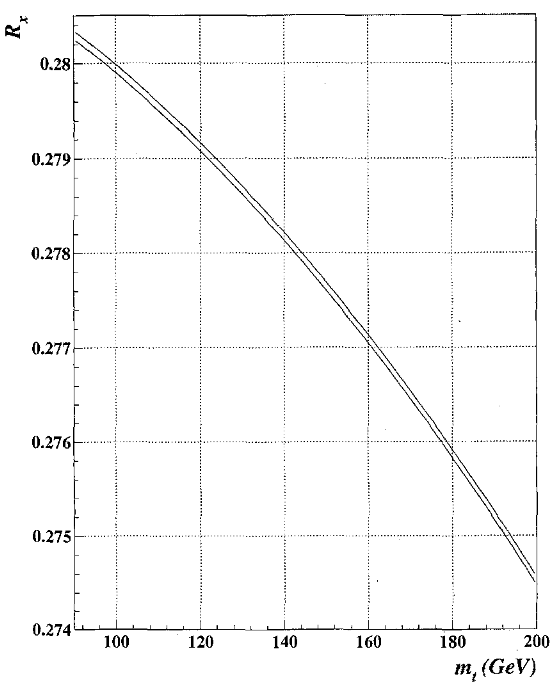

Fig. 1. R_x ratio as a function of m_t. The two lines are for higgs masses of 50 and 1000 GeV

The error introduced with this parametrization in the region under study is practically negligible (less than 0.01%).

3 The experimental method

From the experimental point of view, since we assume that at high luminosity LEP data will be taken at the Z peak, to measure R_x one needs only to count the number of $b\bar{b}$ and non-$b\bar{b}$ events in a given hadronic sample, up to a very good approximation [7]. In order to determine the top quark mass with good precision the error on the ratio must be less than 1%.

To perform directly the counting on the whole sample of hadrons, an efficient method for event tagging is needed. In our simulation study we can distinguish the following steps:

a) Event simulation and selection. For R_x evaluation we simulate 2 classes: b – class for bottom events x – class for *non b* events. These samples are generated by means of JETSET [8] and passed through a fast simulation program of the DELPHI detector [9] that we have tuned with the current data. We analyse the subsample obtained by applying a list of cuts on the track and on the event acceptance parameters already tested with real events as can be seen in [10]. We select only central events ($\theta > 37°$) in order to use the information of the microvertex detector. This region contains over 75% of detected Z's.

b) Choice of discriminant variables. We can divide the variables we use in two classes, general and microvertex variables. The general variables are computed only with particle momenta and multiplicities. They take into account the global topology of the event and select candidate secondary particles only with kinematic cuts. Some of these variables are sphericity, leading momentum, the sum of p_t^2 of secondary particles, etc. The distributions of two of them are shown in Fig. 2a and b. In order to measure the R_x ratio with high precision the presence of a microvertex detector is needed. Several LEP experiments [11] have been equipped with a silicon microstrip device as a microvertex detector. This permits one to use variables sensitive to the decay in flight of the b and c hadrons. These variables are obtained from accurate reconstruction of the particle trajectories near the interaction point. As an example of these variables, we show the projection of the distance between primary and secondary vertex over the most energetic jet axis in the r–ϕ plane (λ_1) and the sum of the impact parameter projection in the r–ϕ plane (ξ), in Figs. 2c and d. The variables we use are independent of the special characteristics of individual channels and are chosen to be highly uncorrelated between each other. A complete list of the variables can be found in [10].

c) Multidimensional Analysis, which maximizes the use of the information by analysing the events in the space of their variables [12, 13]. In our study we have used the 'class likelihood' classification algorithm. From the distribution of the variables for the b and *non b* classes, the joint probabilities that each hadronic event is b and *non b* are computed. Finally, we obtain the classification efficiency

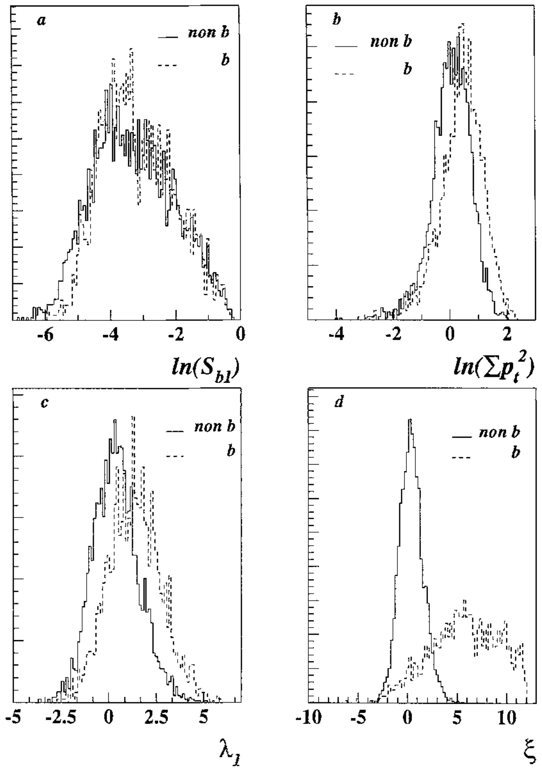

Fig. 2a–d. Distributions of two general variables and two microvertex variables: **a** Logarithm of the sphericity, **b** logarithm of the sum of p_t^2 of the B products, **c** jet 1 decay path (λ_1), and **d** sum of the impact parameter projections (ξ). *Non b* class is plotted in solid line and b class in dashed line

matrix ε which gives the discrepancies between the event flavor before and after the classification procedure in a simulated sample.

4 Estimates of R_x and its error

The proportions of *non b* and b events in a given sample of hadronic events, can be expressed as:

$$f_b = \frac{\Gamma(Z^0 \to b\bar{b})}{\Gamma_{\text{hadr}}}, \quad f_x = \frac{\Gamma(Z^0 \to x\bar{x})}{\Gamma_{\text{hadr}}},$$

respectively, then it follows that

$$R_x = \frac{f_b}{f_x}.$$

On the other hand, f_b and f_x have the following matrix relationship:

$$\begin{pmatrix} f_X \\ f_B \end{pmatrix} = \begin{pmatrix} \varepsilon_{Xx} & \varepsilon_{Xb} \\ \varepsilon_{Bx} & \varepsilon_{Bb} \end{pmatrix} \begin{pmatrix} f_x \\ f_b \end{pmatrix},$$

where f_X and f_B are the proportion of events tagged b or *non b* quarks in the sample and f_x and f_b are the true proportions. So, the error in the ratio is given by:

$$\Delta R_x^2 = \left(\frac{\partial R_x}{\partial f_B}\right)^2 \sigma_{f_B}^2 + \left(\frac{\partial R_x}{\partial \varepsilon_{Xx}}\right)^2 \sigma_{\varepsilon_{Xx}}^2 + \left(\frac{\partial R_x}{\partial \varepsilon_{Bb}}\right)^2 \sigma_{\varepsilon_{Bb}}^2,$$

where we approximate the errors σ_{f_B}, $\sigma_{\varepsilon_{Xx}}$ and $\sigma_{\varepsilon_{Bb}}$ as the variances corresponding to a binomial distribution. In principle, assuming a perfect detector simulation, a large sample of simulated events would permit the determination of the ε matrix with high precision so that the first term in the last expression should dominate. An important point of our study is the control of the systematic errors. We stress that the systematic errors, coming from an imperfect simulation of the variables or border effects coming from the angular acceptance, are small because they mostly cancel in the ratio. The main sources of the systematic errors are the uncertainties on the fragmentation parameters and the misidentification introduced by the classification method. To calculate these effects we have varied the fragmentation parameters in a range compatible with LEP data [14] obtaining a contribution less than 10 GeV to the uncertainty in the top quark mass. Another source of systematic error is the lack of knowledge of theoretical parameters like the Higgs mass and the strong coupling constant α_s. If we vary m_H between 50 GeV and 1000 GeV and α_s between 0.11 and 0.13, the contribution to the systematic error is less than 1 GeV.

5 Results and conclusions

As we have mentioned, R_x is the best ratio to determine the top quark mass because experimentally it is the easiest to measure: one only needs to separate events in two classes b and *non b*. Obviously, any classification algorithm works better the smaller the number of classes. To compare, R_s needs 4 classes and R_c 3 classes. If we assume an impact parameter resolution of the tracks of

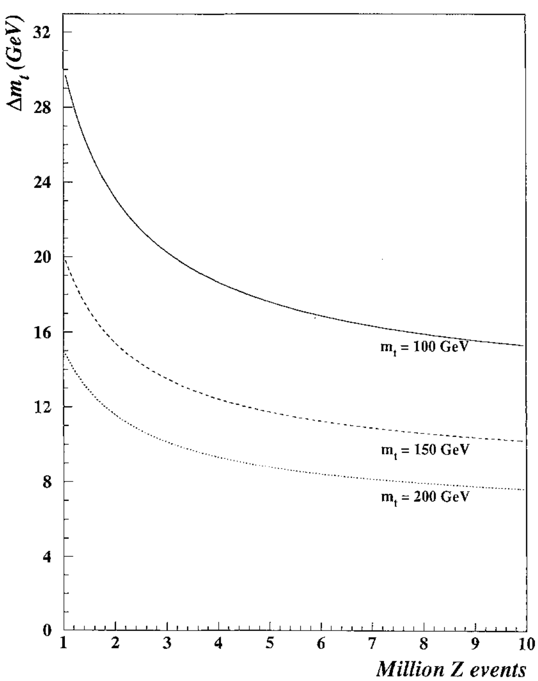

Fig. 3. Top quark mass error as a function of the number of Z events using the R_x ratio

$\sigma_h = \sqrt{(\frac{69}{p_t})^2 + 24^2}$ μm, which is the measured impact parameter resolution for the DELPHI microvertex detector in 1991 [15], the classification matrix obtained applying the likelihood method is:

$$\begin{pmatrix} \varepsilon_{Xx} & \varepsilon_{Bx} \\ \varepsilon_{Xb} & \varepsilon_{Bb} \end{pmatrix} = \begin{pmatrix} 0.901 & 0.175 \\ 0.099 & 0.825 \end{pmatrix}.$$

The statistical uncertainty on the top quark mass computed with the values of this matrix is shown in Fig. 3, as a function of the number of Z events, for three different values of the top quark mass. From the figure, with 2.5 million Z events the error on the top quark mass is then (for $m_t = 150$ GeV):

$$\delta m_t = \pm 15(\text{stat.}) \pm 10(\text{syst.})\ \text{GeV},$$

and an error of about 10 GeV in m_t would be obtained if 4 million Z's are collected.

In summary, if priority is given to an increase in luminosity at LEP, we estimate that the quantity R_x, in which both theoretical and experimental cancellations occur, will test the radiative corrections to the $Zb\bar{b}$ vertex, fixing the top quark mass even better than has been done with current Z scans, testing the Z propagator. Even in the case that the top quark mass is measured in the Tevatron collider at Fermilab, this measurement will be of significant importance because it will be both a strong test of the standard model, determining the top quark mass with LEP measurements alone, and of the Higgs sector when compared with other measurements.

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