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FINANCIAL ENGINEERING MODELS FOR ELECTRICITY MARKETS

Futures pricing, liquidity risks and investment

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SOCRATES : - *And if they were in the habit of conferring honors among themselves on those who were quickest to observe the passing shadows and to remark which of them went before, and which followed after, and which were together; and who were therefore best able to draw conclusions as to the future, do you think that he would care for such honors and glories, or envy the possessors of them?*

- *Would he not say with Homer: "Better to be the poor servant of a poor master, and to endure anything, rather than think as they do and live after their manner"?*

Plato, The Allegory of the Cave

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Notation

Mathematical symbols

bd	Boundary of a set
$\text{CVaR}_{\alpha\%}$	Conditional value at risk at quantile-level $\alpha\%$
$\mathbb{E}_P[.]$	Expected value under the probability measure P
int	Interior of a set
$\mathbb{L}^n$	Lorentz cone (second order cone) $x = (x_1, \dots, x_n) \in \mathbb{L}^n : \ (x_2, \dots, x_n)\ _2 \leq \ x_1\ _2\}$
$\mathbb{I}_{\mathcal{X}}$	Indicator function of the set $\mathcal{X}$
$\mathbb{R}$	Set of real numbers
$\mathbb{R}_+$	Set of nonnegative real numbers
$\mathbb{R}^n$	Set of real column vectors of dimension n
$\mathbb{R}_+^n$	Set of nonnegative real column vectors of dimension n
$\mathbb{R}^{m \times n}$	Set of real matrices of dimension m -by- n
$\sigma(\cdot)$	Standard deviation
$\mathbb{V}\text{ar}[.]$	Variance
$\text{vol}(\cdot)$	Measure of volatility, i.e. $\mathbb{E}[.]/\sigma(\cdot)$
$ \cdot $	Cardinality of a set, $ S $ is the number of element in S
$\ \cdot\ _2$	Vector ℓ_2 -norm, $\ x\ _2 = \sqrt{\sum_{i=1}^n x_i^2}, x \in \mathbb{R}^n$
$\langle \cdot, \cdot \rangle$	Scalar product
$(\cdot)^T$	Transpose operator (for matrix or vector)
$\frac{\partial f}{\partial x}, \nabla_x f$	Partial derivative of f with respect to x
$\partial_x f$	Set of subgradients of f with respect to x

Acronyms

a.e.	almost every
AC	Alternative current
APT	Arbitrage pricing theory
ARA coal	Amsterdam-Rotterdam-Antwerp coal
CAPM	Capital asset pricing model
CCGT	Combined cycle gas turbine
DC	Direct current
EEX	European Energy Exchange
EGT gas	Eon Gastransport
ERCOT	The Electric Reliability Council of Texas market
EU-ETS	European Union emission trading system
FTR	Financial transmission right
GNE	Generalized nash equilibrium
GW	Gigawatt (unit of power): 10^9 W
GWh	Gigawatt hour (unit of energy): $3.6 \times 10^{12} J$
ISO	Independent system operator
KKT	Karush-Kuhn-Tucker
LP	Linear programming
SOCp	Second order cone programming
MW	Megawatt (unit of power): 10^6 W
MWh	Megawatt hour (unit of energy): $3.6 \times 10^9 J$
OCGT	Open cycle gas turbine
PDE	partial differential equation
SDE	Stochastic differential equation
VI	Variational inequalities
VOLL	Value of Lost Load
w.l.o.g.	without lost of generality
w.p. 1	with probability 1

Chapter 1

Introduction

Electricity is a derivative product, structurally related to other commodity assets. Electricity is mainly produced by thermal power stations from the combustion of a fuel (such as nuclear, coal, natural gas, oil or biomass) or resulting from the transformation of other energy sources (as wind, waterfalls or solar). In European countries, thermal generators must compensate for their CO₂ emissions by buying an equivalent amount of carbon allowances (EU-ETS). Each power plant has a cost function that depends on the type of fuel, the technology (combined cycle, generation IV reactors,...), the efficiency and the vintage of the facility. The particular properties of transmission and distribution are another important characteristic of electricity markets. The electric grid is made of interconnected electric transmission lines with thermal capacity limits. The whole electricity system is subject to forced outages (in the generation or the transmission) that, without storage being able to smooth out the shocks, may cause involuntary rationing and rolling blackouts. To mitigate the risks of such events, the market has to meet certain reliability requirements (by operating reserves or congestion management). Also the restructuring of the electricity system has led to many, sometimes quite different, market designs. The organization has profound impacts on the real time operations (the intra-day market), on how energy and transmission are integrated, and on how market is segmented geographically (zonal vs nodal). Finally power markets are susceptible to manipulation and attempts to “corner” the market. The exercise of market power might influence short-term electricity prices. All these factors combine to influence electricity prices in a way that is difficult to fully apprehend. The thesis addresses some important topics arising in restructured electricity markets. A first part of the thesis is dedicated to the pricing of power contingent claims. It particularly focuses on two issues: the relation between power derivatives and related commodity markets, and some implications of the introduction of market frictions (insufficient liquidity) in the theory

of derivatives pricing. The second part elaborates on investment in new generation capacities. It provides new insight on capacity expansion models, on the risks induced by the investment and the criterion to value it.

Power derivatives are contracts traded on power exchanges. They mainly consist of day-ahead, futures and options contracts. The valuation of those derivatives represents a challenging topic due to the very particular nature of electricity. The very limited storage possibilities and transmission idiosyncrasies complicate the pricing of power derivatives. It is widely recognized that the cost-of-carry relationship between spot and forward prices does not hold for electricity implying that, in an arbitrage free modeling framework, the spot price does not play the same important role as for other commodity markets. The pricing of electricity derivatives can be addressed by two approaches. One possibility is to construct a stochastic process describing the random evolution of spot electricity prices and apply the machinery of financial engineering. The alternative is based on fundamentals and the equilibrium determined by the interaction of producers and retailers. In the first chapter of the thesis, we propose a hybrid model, which can be understood as a combination of both the fundamentals of power generation and the stochastic framework. Assuming perfectly competitive market with an inelastic demand structure, the spot electricity prices are determined by the marginal cost of producing the last unit of power. Electricity prices can then be viewed as a derivative of demand, fuel prices and carbon emission price. We extend the Pirrong-Jermakyan [90] model to incorporate the main factors driving the marginal cost and the non-linearities of electricity prices with respect to fuel prices. By analyzing the specificity of the marginal cost in power market, the problem of evaluating power futures can be simplified to make it computationally tractable. The model is then tested on the German European Energy Exchange for “German Month Futures”. The chapter relies on a paper published in the Journal of Energy Markets [34].

Most derivative pricing models rely on the assumption of a perfect market where there are no impediment to trade. The absence of arbitrage is only sustainable in a very liquid market where arbitrageurs can instantaneously exploit all possible mispricings. In the second part of the thesis, we analyze how market frictions may perturb the outcome of the financial market. It is recognized that financial markets deviate, to varying degrees, from the perfect paradigm and that electricity markets remain considerably less liquid than other commodity markets. This is true for energy as well as for transmission contracts. After a preliminary chapter (Chapter 3) where we introduce the notion of risk measure (i.e. a quantification of the risk of a portfolio), we model the incentives for market participants to hedge (minimize the risk associated) their profit by contracting financial products. We first analyze the equilibrium on a liquid market. Our treatment adds to that literature by introducing convex

risk measures that guarantee the absence of arbitrage and hence tie in with the risk-neutral approach. The extreme volatility of electricity makes its financial derivatives crucial instruments for asset managers. We then assess in Chapter 4 the effect of limited liquidity in power exchanges by using an equilibrium model where liquidity contracts prevent agents from hedging up to their desired level. Mathematically, the problem is formulated as a two stage stochastic Generalized Nash Equilibrium with possibly multiple equilibriums. Computing a large panel of solutions, we show how the risk premium and players profits are affected by illiquidity. We also show that the illiquidity of one market can also affect trade in another market.

The last chapter of the thesis discusses investment valuations in generation capacity expansion models. These models optimally select the timing and the levels of investments to meet future demands of electricity. They have a long tradition in the power industry. Designed as optimization problems for the regulated monopoly industry, they can be interpreted as equilibrium models in a competitive environment. While often written as deterministic problems, they can be adapted to accommodate the wide range of uncertainties that currently assail the industry. We focus in this chapter on several topics arising from those models. The first section elaborates on the (accurate) estimation of the distribution of the margin profit in a standard stochastic program. In the next section (published in [35]), we consider a stochastic optimization version of the capacity expansion model where risk is assessed through risk measures. We propose a formulation of the model using the good-deal risk measure from Cochrane and Saá-Requejo [31], which can be seen as an extension of the stochastic discount factor constructed from standard corporate finance theories such as the CAPM and the APT. The resulting model takes the form of a conic optimization program that can be interpreted as a multistage hedging optimization problem in an incomplete market. Finally we briefly extends the stochastic risk averse capacity expansion problem to an equilibrium version, considering risk averse agents that value their profit according to the good-deal theory.

