Spectrum Sensing in Mobile Cognitive Radio: A Bayesian Changepoint Detection Approach

Akpaki S. V. Chede[1,2], François Rottenberg[1], Michel Dossou[2] & Jérôme Louveaux[1]
Université catholique de Louvain[1], Université d'Abomey-Calavi[2]
ICTEAM-ELEN[1], EPAC-LETIA[2]
Belgium[1], Benin[2]

Abstract

Cognitive radio (CR) is the solution to the spectrum scarcity issue faced in wireless communications. Spectrum sensing is a crucial enabler for CR because it gives an insight into the free spectrum. In mobile environments, spectrum sensing is more critical as the spectrum occupancy becomes more dynamic with respect to time and frequency. It would be a good idea to exploit some knowledge about the mobility parameters to make better sensing decisions.
In this work, we consider a mobile CR scenario where some fixed primary users (PU) transmit in the same frequency band, each having its coverage area. A secondary user (SU), the CR node, is moving and continuously looking for an opportunity to use this band. We explore the possibility of using the SU mobility patterns to improve the spectrum sensing. We do that by using the Bayesian changepoint detection approach and assuming that the mobility parameters can be summarized in some a priori knowledge on the average time of spectrum change. Considering that in CR, the power of the signal to be detected is often unknown, we have introduced a low-complexity algorithm that does not rely on this knowledge, in a previous publication. The comparisons with existing algorithms in the literature have shown that the derived algorithm outperforms its non-Bayesian equivalent at low signal to noise ratio (SNR). The current work extends our previous one, presented in IEEE VTC-2021 Spring conference, in several ways. We go further in the comparisons by studying the computational complexity of the proposed algorithm. Moreover, we investigate different ways of computing the optimal detection threshold in practical scenarios of unknown-SNR.

1 Introduction

The ever-growing number of wireless devices, due to the Internet of Things (IoT) explosion, is dramatically leading to spectrum shortage. Dynamic and opportunistic spectrum access should be considered to face this problem. Cognitive radio is the enabling technology for dynamic spectrum access, as it allows a network node to be aware of its electromagnetic environment and adapt its transmission parameters accordingly [1]. This considerably increases the overall spectral efficiency. Spectrum sensing is the process by which the CR node knows the available frequency bands for transmission. As wireless devices may experience mobility, it is important to implement spectrum sensing algorithms that can to quickly detect changes in the spectrum occupancy when a device is moving.
In detection theory, sequential changepoint detection (or quickest detection) aims at detecting, as quickly as possible, a change in the distribution of the observed samples. It is, therefore, suitable for spectrum sensing in a mobile environment. Depending on the time of change (changepoint), there are two changepoint detection frameworks. On the one hand, the *minimax approach* assumes that the changepoint is deterministic and unknown. On the other hand, the *Bayesian approach* assumes that the changepoint is a random variable with a known prior distribution [2]. According to the literature, the optimal detection algorithms in both frameworks are the cumulative sum (CUSUM) and the Shiryaev algorithms. In practical scenarios, where the power of the signal to detect is not known, the generalized likelihood ratio (GLR) test could be applied to the optimal algorithms. It consists in replacing the unknown parameter of the distribution with its maximum likelihood estimate.

In this paper, we study the use of changepoint detection for spectrum sensing in mobile cognitive radio. Assuming that the prior distribution of the changepoint could be, in practice, inferred from mobility parameters and that the SU does not know the PU signal power, we have formerly derived a low-complexity GLR test (termed as *LC GLR-Shiryaev*) for the Bayesian framework. The derived algorithm outperforms its non-Bayesian equivalent at low SNR. This work is an extension of our previous conference paper [3]. As a new contribution, we perform a complexity analysis of the derived algorithm and suggest two methods (adaptive and fixed) for setting the optimal threshold when the SNR is unknown. Finally, we conclude that both methods could give the same performance when a fixed threshold is set correctly. The rest of the paper is organized as follows. The system model is introduced in section 2. Section 3 reminds classical changepoint detection algorithms, including the LC GLR-Shiryaev. Then, in section 4, we carry out the complexity analysis and the optimal threshold setting. Numerical results are given in section 5 in order to compare the complexities of the simulated algorithms and evaluate the performance of the optimal threshold setting. Finally, Section 6 concludes the work and offers a discussion.

2 System Model

2.1 Scenario

We consider a scenario where some fixed PUs are transmitting in the same frequency band, each having its coverage area, as depicted in figure 1. They could be, for example, television (TV) transmitters in a TV broadcasting network or base stations in a mobile network. A mobile SU is looking for opportunities to use the spectrum and setting up a secondary communication. We assume that at the beginning, the SU is not in range of any PU. After a first spectrum sensing, the SU tunes to the frequency band and starts using it.
As it is moving, the spectrum occupancy may change and the SU may see the PU signal appearing unexpectedly. To detect this appearance as quickly as possible, the SU is continuously monitoring the frequency band.

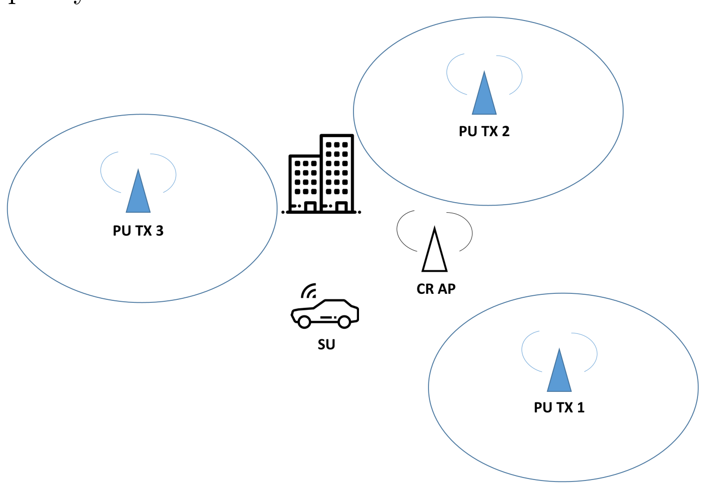


Figure 1: Mobile CR scenario

2.2 Signal model

The signal observed by the SU is denoted by $y[m]$, where m is the time sample of observation. When the PU is not active, $y[m] = w[m]$, where $w[m]$ is the Additive White Gaussian Noise (AWGN) sample. When the PU is active, $y[m] = s[m] + w[m]$, where $s[m]$ is the transmitted signal affected by the channel. We assume that $s[m]$ and $w[m]$ are independent circularly symmetric complex Gaussian variables, $s[m] \sim \mathcal{CN}(0,\, \sigma_s^2)$, $w[m] \sim \mathcal{CN}(0,\, \sigma_w^2)$, and independent of each other.

Initially, the SU is outside of the PUs' coverage areas, and the observed signal (only noise) follows a complex Gaussian distribution with mean 0 and variance σ_w^2. While the SU is moving, at an unknown time sample τ, it gets in range of a PU and the distribution changes to a complex Gaussian distribution with mean 0 and variance $\sigma_s^2 + \sigma_w^2$. As we use the changepoint detection approach, the SU observes each sample of the received signal sequentially. Considering all the received sequence $y_1^n = y[1], ..., y[n]$ up to the current time sample n, it tries to detect the possible change in the distribution by distinguishing between the following two hypotheses:

$$\begin{cases} \mathcal{H}_0: & y[m] = w[m], & m = 1, ..., n \\ \mathcal{H}_1: & \exists\, \tau \in [0, n], \text{ such that} & \\ & y[m] = w[m], & m = 1, ..., \tau \\ & y[m] = s[m] + w[m], & m = \tau + 1, ..., n \end{cases}$$

where τ is the time sample of change (changepoint) and is assumed to be a random variable a priori geometrically distributed*. Moreover, we assume that the parameter p of this geometric prior could be inferred from the SU mobility parameters, with a certain degree of accuracy.

3 Review of existing changepoint detection algorithms

In this section, we introduce some classical changepoint detection algorithms belonging to the deterministic and Bayesian approaches. In general, the changepoint detection algorithm starts taking the observed signal samples. At each time sample n it computes a decision statistic C_n that is compared with a threshold α. When C_n exceeds α, an alarm is raised, and the algorithm stops. The stopping time is denoted by

$$T = \inf\{n \geq 1 : C_n \geq \alpha\}. \tag{1}$$

3.1 Deterministic approach

In the deterministic framework, the optimal algorithm is the well-known CUSUM algorithm, whose decision statistic is given by [4]

$$C_{n,\text{CUS}} = \max_{k \leq n} \sum_{m=k+1}^{n} \ln(L_m), \tag{2}$$

where $L_m = \frac{\sigma_w^2}{\sigma_s^2 + \sigma_w^2} \exp\left(\frac{|y(m)|^2 \sigma_s^2}{\sigma_w^2 (\sigma_s^2 + \sigma_w^2)}\right)$ is the likelihood ratio of the m^{th} observed sample and k is the time sample from which the sum of the likelihood ratio has consistently increased.
In practical scenarios, the PU signal power σ_s^2 is not known by the SU, and the GLR-CUSUM algorithm is used. The decision statistic is defined as [4]

$$C_{n,\text{GLR-CUS}} = \max_{k \leq n} \sup_{\sigma_s^2} \sum_{m=k+1}^{n} \left(\frac{|y(m)|^2 \sigma_s^2}{\sigma_w^2 (\sigma_s^2 + \sigma_w^2)} + \ln \frac{\sigma_w^2}{\sigma_s^2 + \sigma_w^2}\right), \tag{3}$$

where σ_s^2 is replaced by the value that maximizes the CUSUM decision statistic.

3.2 Bayesian approach

In the Bayesian approach, the changepoint τ is assumed to be random with a specific prior distribution [2]. Considering a geometric prior, the Shiryaev algorithm is the optimal algorithm in this approach. Its decision statistic is given by

$$C_{n,\text{SHI}} = \sum_{k=1}^{n} \prod_{m=k}^{n} \left(\frac{1}{1-p}\right) \frac{\sigma_w^2}{\sigma_s^2 + \sigma_w^2} \exp\left(\frac{|y(m)|^2 \sigma_s^2}{\sigma_w^2 (\sigma_s^2 + \sigma_w^2)}\right). \tag{4}$$

* $P(\tau = l) = p(1-p)^l$, $l \geq 0$ where p is the probability of PU appearance at each time sample.

When σ_s^2 is unknown to the SU, the GLR approach can be applied to the Shiryaev algorithm and, as in the GLR-CUSUM case, the statistic $C_{n,\text{SHI}}$ has to be maximized, giving the GLR-Shiryaev statistic

$$C_{n,\text{GLR-SHI}} = \sup_{\sigma_s^2} \sum_{k=1}^{n} \prod_{m=k}^{n} \left(\frac{1}{1-p}\right) \frac{\sigma_w^2}{\sigma_s^2+\sigma_w^2} \exp\left(\frac{|y(m)|^2\,\sigma_s^2}{\sigma_w^2\,(\sigma_s^2+\sigma_w^2)}\right). \tag{5}$$

As this optimization problem is hard to solve analytically, due to non-convexity, we have introduced the so-called low-complexity (LC) GLR-Shiryaev algorithm [3]. This algorithm finds the optimal value $\tilde{\sigma}_{s,k}^2$ that maximizes each term of the sum instead of directly maximizing the sum. Its decision statistic is given by

$$C_{n,\text{LC GLR-SHI}} = \sum_{k=1}^{n} \frac{1}{(1-p)^{n-k+1}} \left(\left(\frac{\sigma_w^2}{\tilde{\sigma}_{s,k}^2+\sigma_w^2}\right)^{n-k+1} \exp\left(\frac{\tilde{\sigma}_{s,k}^2}{\sigma_w^2\,(\tilde{\sigma}_{s,k}^2+\sigma_w^2)} \sum_{m=k}^{n} |y(m)|^2\right)\right), \quad \text{with } \tilde{\sigma}_{s,k}^2 = \left\{\frac{\sum_{m=k}^{n}|y(m)|^2}{n-k+1} - \sigma_w^2\right\}^+. \tag{6}$$

A numerical method for solving the GLR-Shiryaev algorithm is termed M-Shiryaev [5]. It assumes that the SU knows the M possible values of σ_s^2 (termed here as $\sigma_{s,i}^2$ for $i \in [1, M]$), and consists in running M parallel Shiryaev algorithms for each of these values. The PU appearance is declared when any one of the procedures stops:

$$C_{n,\text{SHI},i} = \sum_{k=1}^{n} \frac{1}{(1-p)^{n-k+1}} \left(\left(\frac{\sigma_w^2}{\sigma_{s,i}^2+\sigma_w^2}\right)^{n-k+1} \exp\left(\frac{\sigma_{s,i}^2}{\sigma_w^2\,(\sigma_{s,i}^2+\sigma_w^2)} \sum_{m=k}^{n} |y(m)|^2\right)\right) \quad \text{for } i \in [1, M] \tag{7}$$

A recursive implementation of M-Shiryaev [6] also exists and is given by

$$C_{n,\text{SHI},i} = (1 + C_{n-1,\text{SHI},i}) \left(\frac{1}{1-p}\right) \frac{\sigma_w^2}{\sigma_{s,i}^2+\sigma_w^2} \exp\left(\frac{|y(n)|^2\,\sigma_{s,i}^2}{\sigma_w^2\,(\sigma_{s,i}^2+\sigma_w^2)}\right), \quad \text{for } i \in [1, M]. \tag{8}$$

3.3 Comparison

In [3], we carried out some comparisons between the previously mentioned algorithms in several scenarios. We defined a new metric called the global penalty and given by

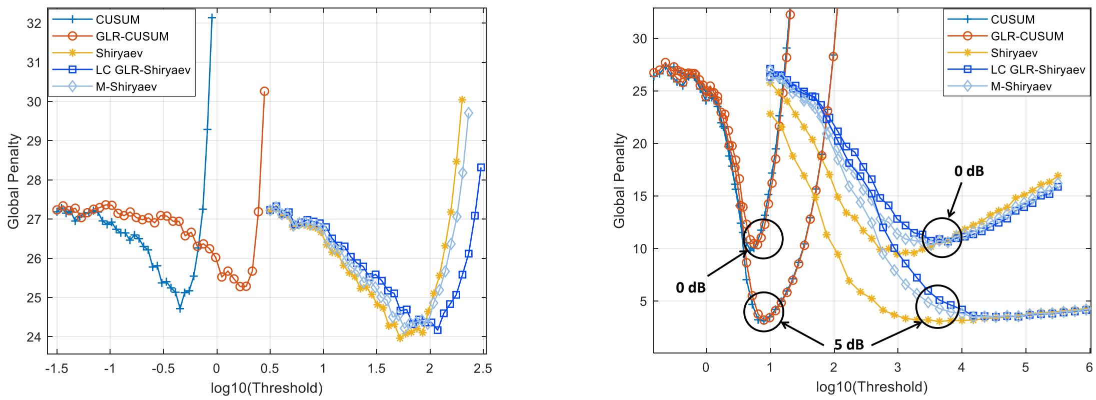
$$G_P = P_{int}\ A_{DD} + P_{wso}\ A_{FAD}, \tag{9}$$

where A_{DD} is the average detection delay, A_{FAD} is the average time between a false alarm and the changepoint, P_{int} and P_{wso} are their respective associated penalties. The global penalty jointly evaluates the costs of interference and spectrum waste induced by the changepoint detection algorithms. When comparing the various algorithms, the derived LC GLR-Shiryaev [3] has better performance (minimum global penalty) than its deterministic counterpart at low SNR, as shown in figure 2. This is due to the prior knowledge of the PU appearance.

It is also seen that LC GLR-Shiryaev and the numerical M-Shiryaev have comparable performances. Studying their computational complexities could give an insight into the best choice of algorithm. Moreover, it is suggested to choose the optimal threshold as the one that minimizes the global penalty. This threshold depends on p and the SNR. Thus, how to set the optimal threshold in an unknown-SNR case is an issue.

4 Complexity analysis and optimal threshold setting

In this section, we evaluate the computational complexity of the LC GLR-Shiryaev and M-Shiryaev algorithms for hardware implementation. Then, we suggest different methods to compute the optimal threshold for changepoint detection algorithms when the SNR is unknown.



(a) p = 0.005, SNR = -10 dB.

(b) p = 0.005, SNR = 0 dB & 5 dB.

Figure 2: G_P vs $log_{10}(\alpha)$ for $p = 0.005$ [3].

4.1 Computational complexity of algorithms

We express the complexity in terms of number of elementary operations needed to run the algorithms at each time sample n. The most used elementary operations are real summation (RS), real multiplication (RM), real division (RD), real exponential function (RE). A real exponentiation a^b can be expressed in terms RM assuming that fast exponentiation algorithms use at most $2\log_2(b)$ RM.

- The complexity of LC GLR-Shiryaev depends on both the computations of the $\tilde{\sigma}^2_{s,k}$ and $C_{n,\text{LC GLR-SHI}}$ (6). To compute each $\tilde{\sigma}^2_{s,k}$, we need $(2n-2k+2)$ RM, $(2n-2k+4)$ RS and 1 RD. Thus, the computation of the n $\tilde{\sigma}^2_{s,k}$ needs $\sum_{k=1}^{n}(2n-2k+2) = (n^2+n)$ RM, $\sum_{k=1}^{n}(2n-2k+4) = (n^2+3n)$ RS and n RD. As the term $\sum_{m=k}^{n}|y(m)|^2$ is already computed in $\tilde{\sigma}^2_{s,k}$, it can be reused in the computation of the decision statistic. Thus, the k^{th} term of $C_{n,\text{LC GLR-SHI}}$ needs $(4+4\log_2(n-k+1))$ RM, 7 RS, 3 RD and 1 RE. To compute and add the n terms, we need $\sum_{k=1}^{n}(4+4\log_2(n-k+1)) = (4n+2n\log_2(n))$ RM, $(7n+n-1)$ RS, $3n$ RD and n RE.
  In total, at each time sample n, LC GLR-Shiryaev needs $(n^2+5n+2n\log_2(n))$ RM, $(n^2+11n-1)$ RS, $4n$ RD and n RE.

- Following the same reasoning, the complexity of both implementations of M-Shiryaev is computed. At each time sample n, M statistics are computed. For the direct implementation (7), the terms $\sum_{m=k}^{n}|y(m)|^2$ and $\frac{1}{(1-p)^{n-k+1}}$ can be computed once and reused in the $M-1$ other statistics. In total, it needs $(n^2+(4M+1)n+(M+1)n\log_2(n))$ RM, $(n^2+(4M+3)n)$ RS, $((2M+1)n)$ RD and (nM) RE. The recursive implementation (8) needs $(5M+2)$ RM, $(3M+2)$ RS, $(2M+1)$ RD and M RE.

The complexity analysis can be summarized in table 1. Numerical comparisons of the complexities are shown in section 5.

4.2 Unknown-SNR optimal threshold setting

As mentioned before, the optimal threshold depends on the SNR value. In this section, we consider how to set the optimal threshold for algorithms which do not have the knowledge of the SNR. We suggest two ways to do that:

Table 1: Complexity analysis

	No. of RM	No. of RS	No. of RD	No. of RE
LC GLR-Shiryaev	n^2+5n $+2n\log_2(n)$	$n^2+11n-1$	$4n$	n
M-Shiryaev (7)	$n^2+(4M+1)n$ $+(M+1)n\log_2(n)$	$n^2+(4M+3)n$	$(2M+1)n$	nM
M-Shiryaev (8)	$5M+2$	$3M+2$	$2M+1$	M

- First, we try to estimate the SNR at each time sample n, based on the observation, and use the optimal threshold (for known SNR) corresponding to the estimated value. This is called *adaptive threshold setting*, as the estimated SNR may change at each time sample and the threshold is updated. We could reuse the SNR estimated in the decision statistic, to compute the threshold. Let us consider the LC GLR-Shiryaev statistic (6), where we compute the $\tilde{\sigma}^2_{s,k}$ that maximize each term because it is difficult to compute the optimal $\tilde{\sigma}^2_s$. Having the $\tilde{\sigma}^2_{s,k}$, how can we get an accurate estimate of σ^2_s? We make two proposals:
  - 1^{st} *proposal:* At every step n, we use the $\tilde{\sigma}^2_{s,k}$ that gives the highest term, as the most reliable estimate of σ^2_s.
  - 2^{nd} *proposal:* We average the current $\tilde{\sigma}^2_{s,k}$ that gives the highest term with the ones of the previous time samples, and use the average as an estimate of σ^2_s.
- Secondly, we suggest to compute numerically the optimal threshold that minimizes G_P in average, assuming a certain SNR distribution. This is called *fixed threshold setting*.

In the next section, we show the performance of each method.

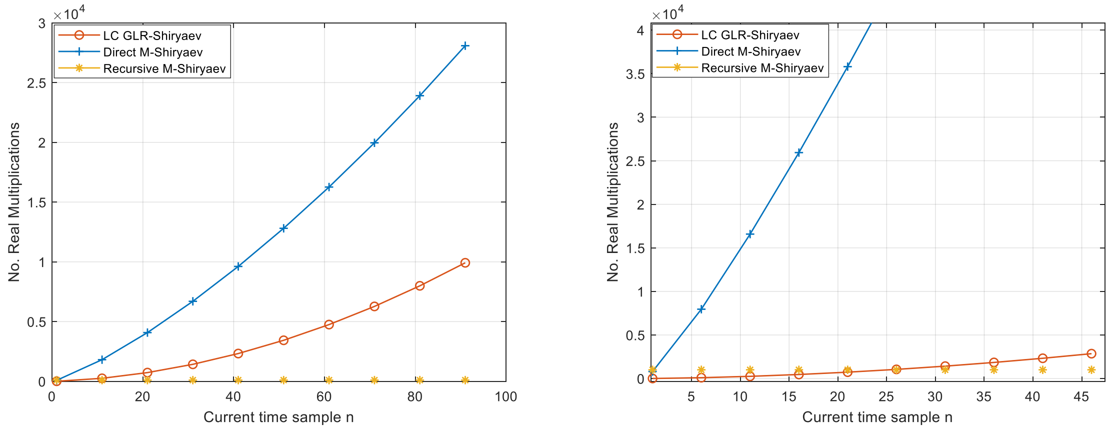
5 Simulation results & discussion

5.1 Complexity simulation

We show some numerical comparisons of the algorithms complexities. For a fixed value of M, we plot the complexity with respect to the number of time samples. We just focus on the number of real multiplications as the other operations have the same trend. As it can be seen in figure 3a, the complexities of LC GLR-Shiryaev and the direct M-Shiryaev grow quadratically ($O(n^2)$) when the number of samples increases, but the recursive M-Shiryaev has a constant complexity ($O(M)$). The direct M-Shiryaev is worse than LC GLR-Shiryaev when M is high but has a closer complexity when M is low. As shown in figure 3b, the only case when LC GLR-Shiryaev and the recursive M-Shiryaev have comparable complexity is when M is high and the number of observed samples is low. To conclude, the recursive M-Shiryaev has the lowest complexity but requires an accurate *a priori* knowledge of the SNR, otherwise the performance will degrade. LC GLR-Shiryaev has the advantage of not requiring any *a priori* knowledge but adds some complexity. A trade-off between complexity and performance should thus be considered when choosing an algorithm among LC GLR-Shiryaev and M-Shiryaev.

5.2 Optimal threshold performance

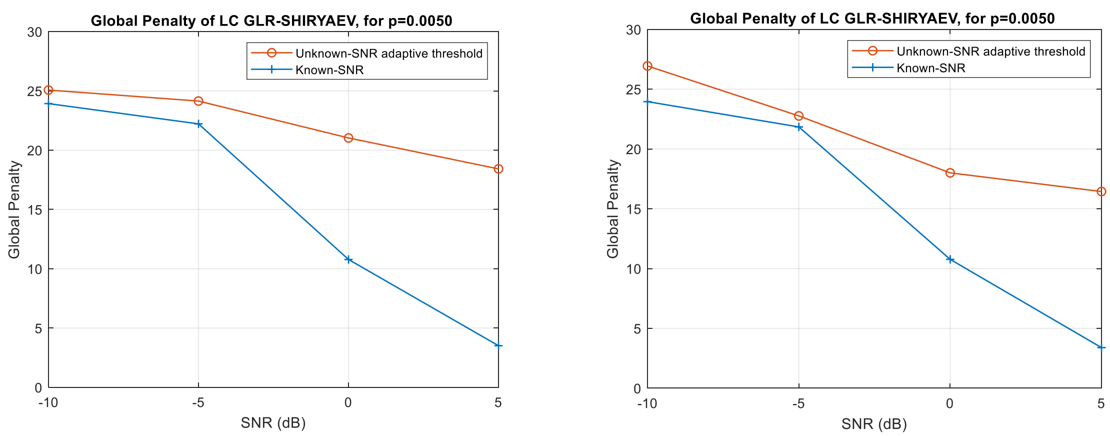
Here, we show the performance of the unknown-SNR threshold methods by using Monte Carlo simulations to compute the average global penalty at various SNR values. We use the following parameters : $p = 0.005$, SNR $= \frac{\sigma_s^2}{\sigma_w^2} \in \{-10, -5, 0, 5\}\, dB$, $P_{int} = 0.5$ and $P_{wso} = 0.14$.



(a) Complexity for $M = 20$ (b) Complexity for $M = 200$

Figure 3: Complexity in terms of No. of Real Multiplications.

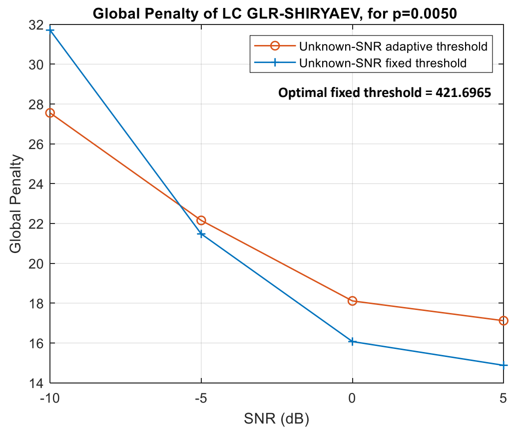
We first compute the global penalty for the adaptive threshold and compare it with the known-SNR case (the optimal one). As shown in figure 4, for the first proposal, the obtained global penalty is closer to the minimal value (known-SNR) at low SNR, but deviates from the minimal value at high SNR. The second proposal improves a bit G_P for high SNR.



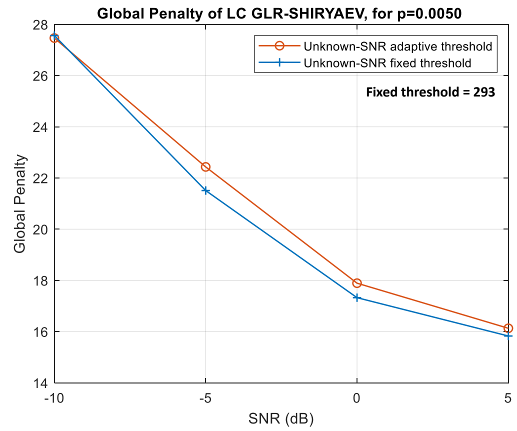
(a) 1^{st} proposal (b) 2^{nd} proposal

Figure 4: Global penalty for unknown-SNR adaptive threshold.

We can see that adaptive threshold setting performs quite well for an unknown SNR method. The poor result at high SNR is due to the fact that a reliable estimate of the SNR cannot be obtained before the changepoint τ. So when the true SNR is low, the successive estimates are closer to it before τ so that G_P is closer to the optimal one. But when the true SNR is high, the estimates before τ are bad and this gives a poor global penalty.
Secondly, we assume a Rayleigh-distributed SNR to compute the optimal fixed threshold. We obtain the result shown in figure 5a, where the optimal fixed threshold method is better than adaptive threshold (2^{nd} proposal) in high SNR, but worse in low SNR. Moreover, it is shown, by varying the distribution parameters, that a fixed threshold can be correctly set to give the slightly same performance as the adaptive threshold. This is shown in shown in figure 5b.



(a)



(b)

Figure 5: Global penalty for unknown-SNR adaptive vs fixed threshold.

6 Conclusion

In this paper, we investigate the use of Bayesian changepoint detection in mobile cognitive radio. We perform a complexity analysis of the previously derived LC GLR-Shiryaev and compare it with the complexity of the M-Shiryaev algorithm. While being less complex than the LC GLR-Shiryaev, the recursive implementation of M-Shiryaev needs an accurate a priori knowledge of the SNR, which is usually not available, to give the same performance. Moreover, we introduce an adaptive and a fixed threshold setting methods for the LC GLR-Shiryaev. Both methods give almost the same performance when the fixed threshold is set correctly. A potential direction for future work is to reduce the computational complexity of the LC GLR-Shiryaev.

References

[1] S. Haykin, “Cognitive radio: brain-empowered wireless communications,” in *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 2, pp. 201-220, feb. 2005.

[2] A. Tartakovsky, I. Nikiforov, and M. Basseville, *Sequential Analysis*. Chapman and Hall/CRC, 1st ed., aug 2014.

[3] A. S. V. Chede, F. Rottenberg, M. Dossou, and J. Louveaux, “Use of Bayesian Changepoint Detection for Spectrum Sensing in Mobile Cognitive Radio,” 2021 IEEE 93rd Vehicular Technology Conference (VTC2021-Spring), Helsinki, Finland (Virtual Event), pp. 1-5, apr 2021, doi: 10.1109/VTC2021-Spring51267.2021.9448935.

[4] L. Lai, Y. Fan, and H. V. Poor, “Quickest detection in cognitive radio: A sequential change detection framework,” in *GLOBECOM - IEEE Global Telecommunications Conference*, pp. 2957–2961, 2008.

[5] J. Geng and L. Lai, “Bayesian quickest detection with unknown post-change parameter,” in *ICASSP, IEEE International Conference on Acoustics, Speech and Signal Processing - Proceedings*, vol. 2016-May, pp. 4169–4173, may 2016.

[6] J. Geng, E. Bayraktar, and L. Lai, “Multi-Chart Detection Procedure for Bayesian Quickest Change-Point Detection with Unknown Post-Change Parameters,” tech. rep., 2017.