Fuzzy reliability assessment of bridge piers in presence of scouring

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ABSTRACT: The design of bridge piers placed in riverbeds has to deal with variable hydraulic forces acting on structures with foundations which may be prone to scour effects. The conventional design of drag forces, scour depths and foundation bearing capacity is carried out by means of equations which involve a certain number of uncertainties regarding flow, sediments, structural and geotechnical parameters. A sound reliability assessment of bridges piers against scouring risks must take into account the uncertainties inherent in these parameters.

Due to the phenomenological nature of these uncertainties, a fuzzy approach seems to be the most adequate to deal with to this kind of problems. In the first part of this study, the role of the uncertain parameters is assessed by considering separately their effects on the drag forces and on the general and local scour evaluation. In the second part, the previously obtained responses regarding the drag forces and the scouring depths are used as input data for a fuzzy analysis of the pier foundation reliability.

Different design contexts are studied: embedded foundations and foundations with exposed piles; effects of clear water and live bed scour conditions; different ratios between vertical superimposed and lateral hydraulic loads.

A sensitivity analysis allows to estimate the influence of the various uncertain variables on the partial and on the global responses.

1 INTRODUCTION

In the reliability assessment of bridge piers in riverbed, a primary role is played by the scouring action. A study reported by Hamill (1999) regarding the collapse of 143 bridges between 1847 and 1975 shows, in fact, that 66 bridges collapsed for problems associated to the scouring effect. More recent analyses, concerning the vulnerability of bridges in the Po Valley, show that the most common bridge damage is caused by erosion occurring in riverbed foundations (Ballio et al. 1999).

A safe design of bridges with piers in riverbed cannot be carried out without a careful study of the scouring effects. In this context, the hydraulics studies the flow and its transition characteristics through the bridge waterway and leads to define the drag forces and the scour depths (Hamill 1999), the static firstly work to the design of pile and foundations and then checks their bearing capacity under the most severe loading and riverbed conditions.

Unfortunately, the design computations are not straightforward and they intrinsically involves a great deal of parameters, many of them strongly affected by uncertainties, like irregularity of the channel geometry, variations in water flow, velocity in the contraction zone, shape, dimensions and orientation of obstacles, presence of debris and floating objects.

All these uncertainties must be taken into account and, to their phenomenological nature a fuzzy approach seems to be the most adequate to deal with them (Dordoni 2009).

2 CONVENTIONAL ASSESSMENT OF HYDRAULIC FORCE AND SCOURING

The safety assessment of a bridge must take into account the problems caused by the erosive action of the river stream (Muzzamil et al. 2008, Da Deppo et al. 2008). Generally speaking, the reason of this phenomenon is to be found in the imbalance between the sediment transport capacity of the stream and the actual amount of suspended solids in the fluid, that depends also on the availability of solid particles along the bed to be carried by the fluid motion. In these terms, erosion and deposition of material are opposite manifestations of the same phenomenon.

Piers, abutments and embankments of river bridges can exert a contraction on the flow with respect to its natural conditions, because they occupy a space that would be otherwise available to the stream.

In many situations, when the undisturbed flow is slow, the local contraction of the channel width can increase the velocity of the flow, exposing the material of the riverbed in the proximity of bridge piers (and abutments) to the erosive action.

2.1 *Hydraulic force*

The hydraulic force, exchanged between water streams and solid bodies in relative motion, can be estimated through Equation 1 (Apelt & Isaacs 1968, Blevins 1984):

$$F_D = \frac{1}{2} \cdot \rho \cdot V_2^2 \cdot y_2 \cdot L \cdot C_D \quad (1)$$

where ρ and V_2 are the density and the average flow velocity, y_2 is the average water depth at the contraction and L is the pier dimension in the direction of the flow. The drag coefficient C_D depends on the shape of the piers, the distance between them, the attack angle of the flow and the Reynolds number, which is proportional to the ratio between inertial forces and viscous forces in the fluid motion. The drag coefficient can be estimated experimentally. Unfortunately, in literature, values are available only for profiles with simple geometric shapes; in case of actual bridge piers, the drag coefficient can vary between 0.2 and 2.0 (Blevins 1984). This wide range of variability provides high uncertainty in the estimation of the drag coefficient and force.

2.2 *Scouring assessment*

For the assessment of the scour depth, a common engineering practice suggests that it can be evaluated as the sum of depths due to the three levels in which the phenomenon occurs: contraction scour, local scour and degradation - aggradation of the riverbed (Hamill 1999).

2.2.1 *Contraction scour*

Contraction scour (d_{sc}) is a consequence of the reduction of the available width to the flow due to the presence of the bridge support. With reference to Figures 4 and 5, let be respectively y_1 and b_1 the average water depth and the width of the main channel upstream of the contraction, Q_1 the discharge portion that carries solid particles upstream of the contraction, Q_2 e b_2 the total discharge passing through the waterway and the waterway width, k_1 a coefficient that depends on the amount of sediment present in the channel. On the basis of these quantities, when the undisturbed flow upstream of the contraction contains suspended solid particles, i.e. the case of live-bed conditions, the scour depth can be assessed through the following Equations 2 and 3 (Laursen 1962, Richardson et al. 2001).

$$y_2^{'} = y_1 \cdot \left(\frac{Q_2}{Q_1}\right)^{\frac{6}{7}} \cdot \left(\frac{b_1}{b_2}\right)^{k_1} \quad (2)$$

$$d_{sc} = y_2^{'} - y_1 \quad (3)$$

2.2.2 *Local scour*

Local scour occurs when the flow collides with localized obstacles, such as supports and piers of the bridge. The velocity field of the stream is abruptly distorted, generating vortices and vertical motions that remove material from around the bottom of the obstacle.

Calling F_2 the Froude number relative to the water flow at the beginning of the restriction, K_{1P}, K_{2P}, K_{3P} corrective factors that depend on the shape of the piers and of the river bed and b_p the width of the piers, the depth of local scour (d_{sp}) can be estimated using Equations 4 and 5 (Hamill 1999):

$$F_2 = \frac{V_2}{\sqrt{g \cdot y_2}} \quad (4)$$

$$d_{sp} = 2 \cdot y_2 \cdot K_{1P} \cdot K_{2P} \cdot K_{3P} \cdot \left(\frac{b_p}{y_2}\right)^{0.65} \cdot F_2^{0.43} \quad (5)$$

2.2.3 *Degradation and aggradation*

Degradation and aggradation in the fluvial bed can be due to changes in the river basin, climate trends, or the amount of solid material carried by the flow (Miglio et al. 2009, Goode & Burbank 2009). This process occurs on larger time and space scales respect to the two previous phenomena.

In this paper degradation and aggradation of the fluvial bed is not considered, and the total depth of erosion (d_s) can be approximately assessed as the sum of the first two effects (contraction and localized erosion):

$$d_s = d_{sc} + d_{sp} \quad (6)$$

From what has been previously stated, it is clear that the hydraulic force and scour depth depend from the many uncertainties involved in the problem. Deterministic analysis, based on the Equations 1-6, may be inaccurate, since it does not take into account the uncertainties related to the factors present in the previous equations. A more reliable assessment can be performed using a fuzzy approach.

3 FUZZY THEORY

Fuzzy theory is a branch of Artificial Intelligence that allows to reproduce the approximate reasoning of human mind (Sgambi 2008). The term *fuzzy logic* was introduced by Zadeh (Zadeh 1965, Zadeh 1978). However the idea of a third logical state, in opposition to the ambivalent logic (true or false) theorized by Aristotle, was developed by the Greek philosopher Plato or, in modern times, by the philosopher Lukasiewicz.

The first applications of fuzzy logic were introduced in the engineering fields of control systems, the systems of decisional support, natural language, the recognition of forms and in other various fields. Fuzzy logic has acquired notable importance in civil engineering as well, in problems that require analyses in presence of uncertainty. Some of the major fuzzy applications in civil engineering concern control techniques, structural reliability and the treatment of uncertainties in materials (Biondini et al. 2000, Sgambi & Bontempi 2004, Sgambi 2004).

Fuzzy theory may be applied in the modelling of a system of a know input / output mapping given in terms of a mathematical model y = f(x), where the input variables can not be measured precisely (Piegat, 2001).

Input and output variable in fuzzy theory are represented by fuzzy sets. A fuzzy set A is a set of pair:

$$A = \{(\mu_A(x), x)\} \tag{7}$$

where: μ_A is the membership function which assigns to each element x the grade of its membership to the fuzzy set A. Considering that:

$$\mu_A(x) \in [0,\ 1] \tag{8}$$

The membership function maps the numerical universe X of a given variable in the interval [0, 1].

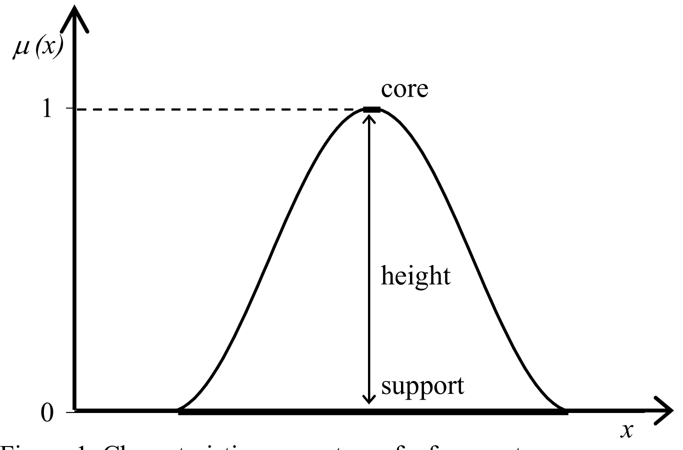

Figure 1. Characteristic parameters of a fuzzy set.

In the practice, various types of membership function are applied (triangular, rectangular, trapezoidal, Gaussian, sigmoid, etc.).

With reference to Figure 1, we recall the characteristic parameters of a membership function: the support, the height and the core. The support of a fuzzy set A is the crisp subset of X whose elements have non-zero membership grades:

$$S(A) = \text{supp}(A) = \{x : \mu_A(x) > 0, \quad x \in X\} \tag{9}$$

The height of a fuzzy set A is the maximal value assumed by the membership function on its support X.

$$h(A) = \text{height}(A) = \sup_{x \in X}(\mu_A(x)) \tag{10}$$

The core of a fuzzy set A is the crisp subset $C(A)$ whose elements all have membership grade equal to one:

$$C(A) = core(A) = \{x : \mu_A(x) = 1, \quad x \in X\} \tag{11}$$

A fuzzy set can be described also by a horizontal representation with the aid of the so-called α-cuts $A^{\geq\alpha}$ of the set. The $A^{\geq\alpha}$ are subsets of A in which:

$$A^{\geq\alpha} = \{x : x \in X, \quad \mu_A(x) \geq \alpha\} \tag{12}$$

as shown in Figure 2. Using a sufficient number of α-cuts, the membership function of a fuzzy set can be reproduced with the required accuracy.

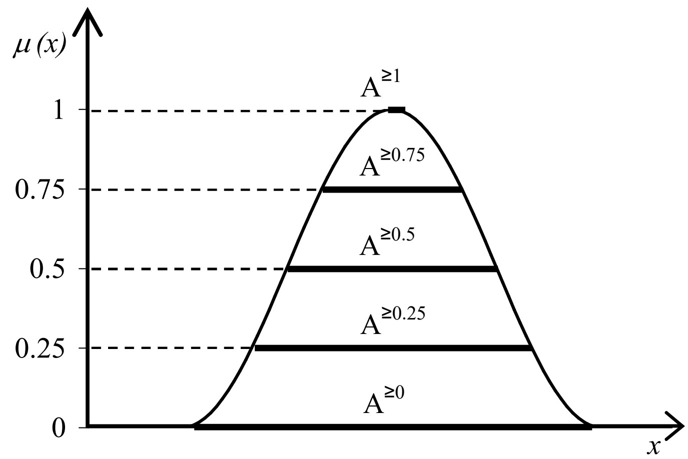

Figure 2. Examples of α-cuts of a fuzzy set A.

Knowing (a) the mathematical model (y = f(x)) that links input (x) and output (y) fuzzy variable and (b) the membership functions of the input, described by means of α-cuts, it is possible to transform the input membership functions into the output one.

Figure 3 shows schematically the mapping process which leads to the fuzzy output. To obtain a deterministic value, associated with the output variable, the membership function $\mu_B(y)$ have to be defuzzified.

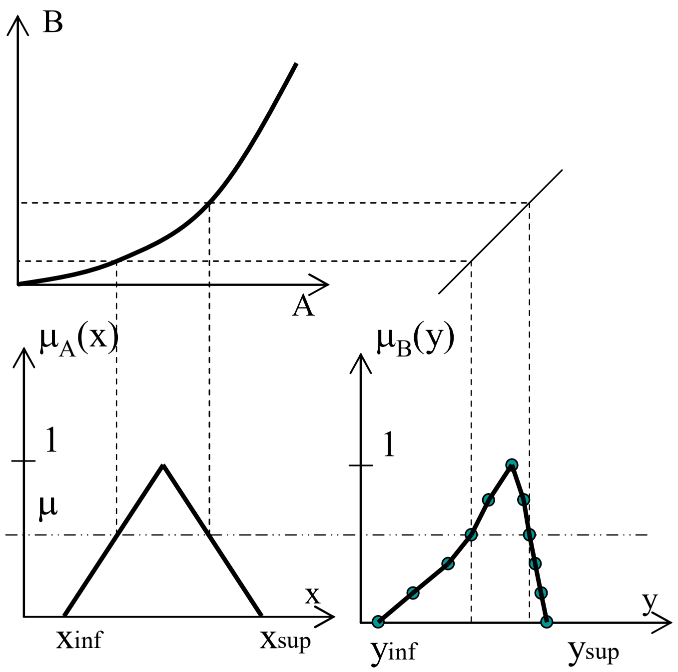

Figure 3. Mapping of a fuzzy input x to a fuzzy output y.

Different defuzzification techniques are proposed in literature: the centre of mass technique (Driankov 1993) is among the most commonly used.

In this method, the procedure links the defuzzified value with the centre of gravity of the membership functions $\mu_B(y)$ and allows to handle this single value as representative of the fuzzy output information. If the membership functions is a continuous function, its centre of gravity is given by:

$$y' = \frac{\int_S y \cdot \mu_B(y)dy}{\int_S \mu_B(y)dy} \tag{13}$$

4 AN APPLICATION

A fuzzy reliability assessment of the hydraulic force and of the scour depth on a typical fluvial bridge is performed.

The bridge and the river geometries are shown in Figure 4 and 5. The riverbed is considered as a wide main channel of rectangular section 64 m wide, 1 m deep.

Two floodplain extend from the sides of the main channel with a slight slope toward the center. The bridge crosses the river in perpendicular direction. The supports and the bridge embankments completely obstruct the floodplains and three 2m-wide piers cause a further restriction of the main channel. The distance among the piers is 16 m and the contracted section is 58 m wide. The dimension of the piers in the other in-plan direction is 12 m.

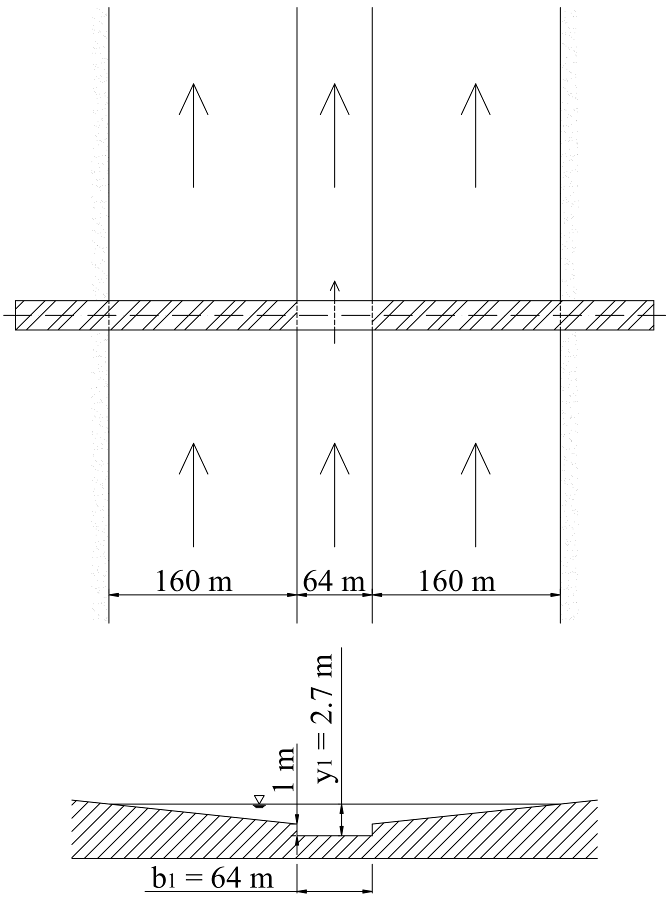

Figure 4. Schematic representation of the fluvial crossing: in-plan and cross-section views.

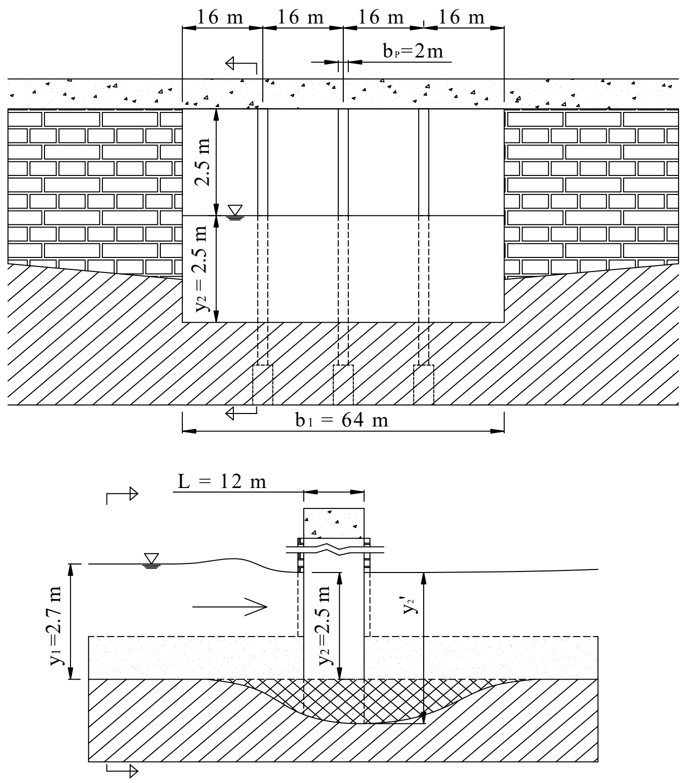

Figure 5. Schematic representation of the restriction and piers positioning: front view and section of the bridge.

Assuming a return period of 200 years, hydraulic evaluations led to asses an average water depth of 2.7 m before restriction and 2,5 m at the restriction.

The average velocity of the stream in the restricted section is assessed to be 2.14 m/s while the discharge in the main channel upstream of the bridge is assessed to be 280 m^3/s.

To understand the influence of the uncertainties in the considered problem, the hydraulic force and the scour depth are assessed, both in the case of pier shaft exposed to the stream (Case A and D) and also in the case of foundation piles exposed to the stream after a possible scour process (Case B end E).

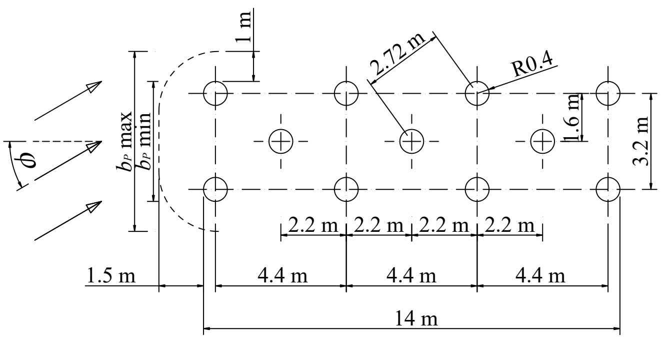

Figure 6. Particular of the foundation piles positioning.

The role of the uncertainties in the assessment of the hydraulic force was finally investigated (Case C) considering, as mentioned before, the high spread of the values concerning the drag coefficient.

The scour profile along the cross section has not been calculated: only the maximum depth at the bottom of the piers was considered (as representative of the riverbed configuration after scour in the most vulnerable locations).

4.1 *Fuzzy evaluation of the hydraulic force*

4.1.1 *Case A) Pier shaft exposed to the stream*

In the Case A) the assessment of the hydraulic force is performed considering only the pier shaft exposed to the stream.

Table 1. Uncertain variables and definition of their membership function.

Variable	Membership function	Deterministic value	Variability	
ρ	Deterministic	1.005 t / m^3	---	---
V_2	Isosceles triangle	2.5 m/s	-10 %	+10 %
Y_2	Isosceles triangle	2.14 m	-10 %	+10 %
L	Isosceles triangle	10.392 m	-5 %	+5 %
C_D	Triangle	1.2	-20 %	+50 %

The assessment of the hydraulic force is provided by Equation 1, considering the uncertainty involved in the definition of some parameters. Table 1 summarizes the membership functions, deterministic values and the variability of the parameters considered. Triangular membership functions were selected. The variability was defined as the relative percentage distance of the triangle vertices with respect to the deterministic reference value. Note that the deterministic value and the variability of the parameter L (length of the pier) are set taking into account the possibility of accumulation of debris carried by the stream on the upstream face of the pier, and considering a skew angle $\varphi = 30°$ between stream and pier alignment.

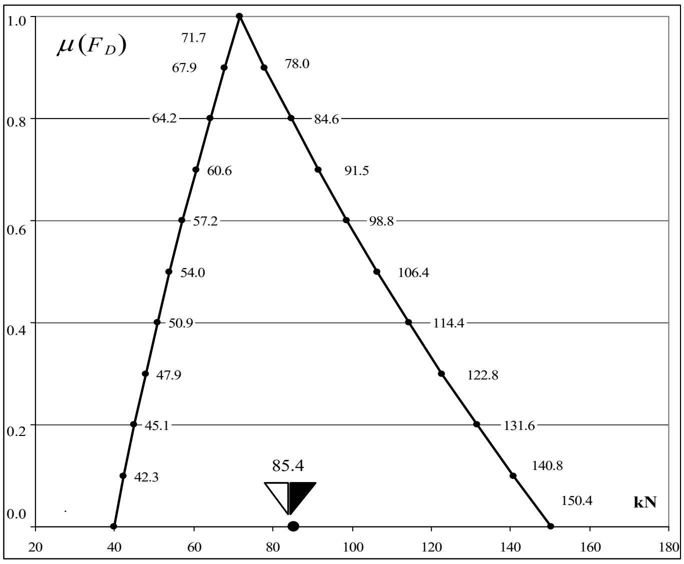

Figure 7. Membership function and defuzzified value for the drag force.

The fuzzy assessment of the drag force brought to the membership function plotted in Figure 7. Such a membership function is non linear, with a deterministic value of 71.7 kN. The support ranges from 40 to 150 kN and the left branch is convex while the right one is concave. Using the centre of mass rule as defuzzification criteria, it is possible to obtain a numerical value for the drag coefficient which takes into account the uncertainties in the input parameters and which results 85.4 kN. This value is 19 % higher than the value estimated with the deterministic approach.

4.1.2 *Case B) Foundation piles exposed to the stream*

If the scour exposes the foundation piles, the hydraulic resistance will depend on the new geometry of the stream obstacle. The geometrical characteristics of the mesh of the foundation piles are shown in Figure 6. To simplify the hydraulic analysis, it was assumed that the flow collides only with the piles, excluding the foundation plinth from the analysis. Table 2 summarizes the membership function, deterministic values and the variability of the parameters considered.

Table 2. Uncertain variables and definition of their membership function.

Variable	Membership function	Deterministic value	Variability	
ρ	Deterministic	1.005 t / m^3	---	---
V_2	Isosceles triangle	2.5 m/s	-20 %	+20 %
Y_2	Isosceles triangle	2.14 m	-20 %	+20 %
C_D	Triangle	1.2	-20 %	+67 %
L	Triangle	12.124 m	-5 %	+5 %

The new configuration of the obstacle locally changes the conditions of motion. For this reason, the uncertainties involved in the water height (y_2) and in the flow speed (V_2) are characterized by a greater spread than in the previous case, since further hydraulic computations could not be performed.

Because of the irregularity of the obstacle, and the increased possibility of debris accumulation in the restricted section, an increased deterministic value and a greater variability, with respect to case A), was assigned to the drag coefficient as well. Figure 8 shows the membership functions of the drag force for the examined cases.

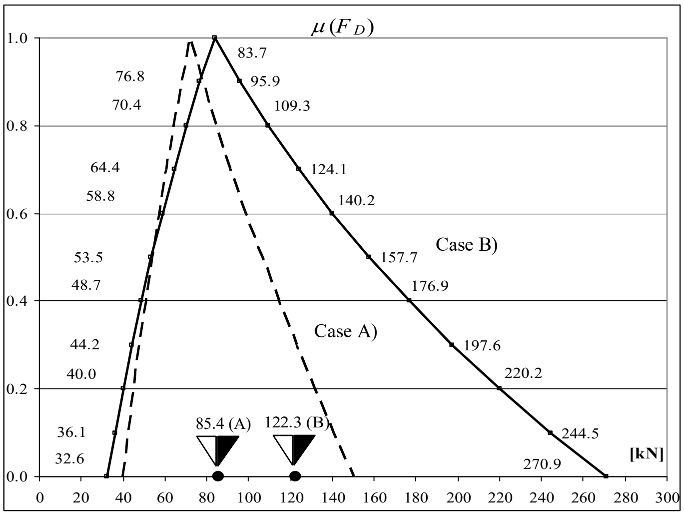

Figure 8. Membership functions and defuzzified values (on the abscissa axis) for the drag force. Comparison between Case A) (piers only exposed) and Case B) (piles exposed).

The two membership functions show a similar shape, however, that of Case B) reflects the greater variability of the data. In fact, in this second case, the support of the membership function ranges from 33 a 270 kN and the defuzzified value is 122.3 kN 46 % greater than the deterministic value. Moreover, it is interesting to remark how the difference between the defuzzified values of the drag force is much higher (44 %) than the difference between the deterministic ones (17 %).

4.1.3 *Case C: Role of drag coefficient C_D*

To highlight the role of uncertainty on the drag coefficient, the analyses reported in the previous sections were reconsidered using the drag coefficient as the only uncertain parameter (IC_D – A and IC_D – B). Two subsequent fuzzy analyses have been performed with a range of variability reduced to ±20 % (RC_D – A and RC_D – B).

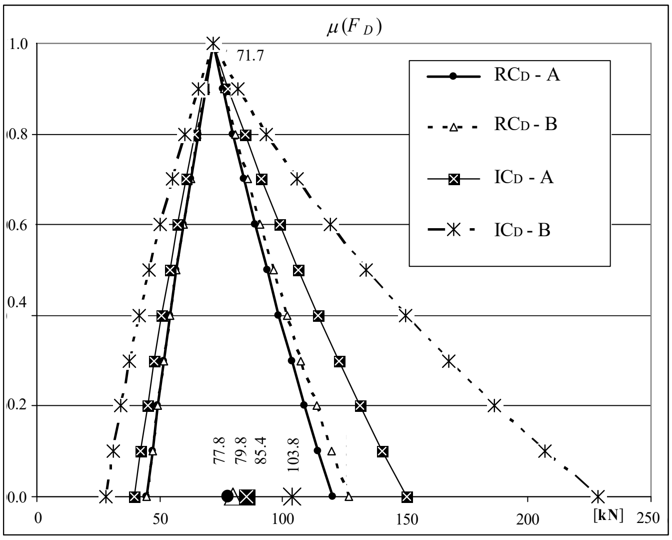

Figure 9. Membership functions and defuzzified values of the drag force (on the abscissa axis), for different uncertainties on the drag coefficient.

The results, in terms of membership function and defuzzified values, are reported in Figure 9, where the considerable influence of the drag coefficient on the curves support and on the centre of gravity are shown.

4.2 *Fuzzy evaluation of the scour depth*

4.2.1 *Case D) Pier shaft exposed to the stream*

The total scour depth is assessed according to Equations 2 – 6 as the sum of contraction and local scour.

Table 3. Uncertain variables and definition of their membership function.

Variable	Membership function	Deterministic value	Variability	
y_1	Isosceles triangle	2.7 m	-10 %	+10 %
Q_1	Isosceles triangle	280 m^3/s	-10 %	+10 %
Q_2	Isosceles triangle	360 m^3/s	-10 %	+10 %
b_1	Deterministic	64 m	---	---
b_2	Deterministic	58 m	---	---
K_1	Deterministic	0.64	---	---
Y_2	Isosceles triangle	2.5 m	-10 %	+10 %
V_2	Isosceles triangle	2.14 m/s	-10 %	+10 %
K_{1P}	Deterministic	1	---	---
K_{2P}	Isosceles triangle	2.2	-15 %	+15 %
K_{3P}	Isosceles triangle	1.1	-5 %	+5 %
g	Deterministic	9.806 m/s^2	---	---
b_P	Triangle	2 m	0 %	+30 %

In Table 3 all the parameters used, their deterministic values and the range of uncertainty assumed are listed. The K_{2P} coefficient is related to the skew angle between the pier and the stream alignment; it was determined according to the method reported by

Hamill (1999) and its variability was based on geometry considerations and a variability of ±10 % of $\varphi = 30°$. K_{3P} was given a small variability, applied to a common-use value for similar cases.

Using the fuzzy theory, it is possible to get the membership function of the total scour depth. As mentioned before this depth is the sum of the contraction scour and local scour effect.

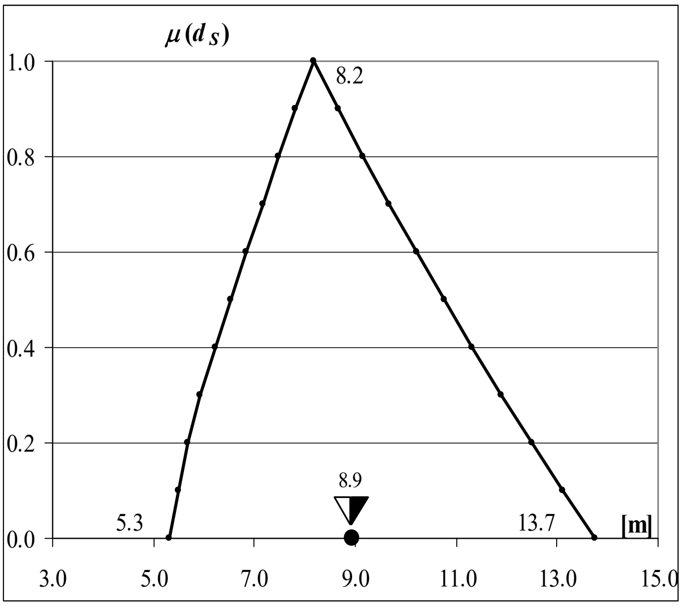

Figure 10. Membership function and defuzzified value for the total scour depth.

The membership function is slightly nonlinear, with a deterministic value equal to 8.2 m. The defuzzified value of the scour depth is equal to 8.9 m, almost 10 % bigger than the deterministic one. It must be referred that, according to the intermediate computations steps, the total scour uncertainty is more influenced by the uncertainty on the local scour than by the contraction scour.

4.2.2 *Case E) Foundation piles exposed to the stream*

When the foundation piles are exposed to the stream, the obstacle changes the geometry. In this case it is not simple to describe how the local conditions of motion vary over the time and with respect to the initial conditions.

Some Authors suggest to consider only the geometry of the piles when the scour is caused primarily by the contraction effect, and only the geometry of the piers when the local scour is stronger. When piles are positioned so as to create a grid, they are particularly effective in capturing debris and floating materials. Such a configuration will create a single obstacle that is larger than the sum of the net dimensions of the piles.

Table 4 summarizes the parameters values assumed in this analysis.

Table 4. Uncertain variables and definition of their membership function.

Variable	Membership function	Deterministic value	Variability	
y_1	Isosceles triangle	2.7 m	-10 %	+10 %
Q_1	Isosceles triangle	280 m^3/s	-10 %	+10 %
Q_2	Isosceles triangle	360 m^3/s	-10 %	+10 %
b_1	Deterministic	64 m	---	---
b_2	Deterministic	58 m	---	---
K_1	Deterministic	0.64	---	---
Y_2	Isosceles triangle	2.5 m	-20 %	+20 %
V_2	Isosceles triangle	2.14 m/s	-20 %	+20 %
K_{1P}	Deterministic	1	---	---
K_{2P}	Isosceles triangle	2.2	-3 %	+3 %
K_{3P}	Isosceles triangle	1.1	-5 %	+5 %
g	Deterministic	9.806 m/s^2	---	---
b_P	Triangle	2 m	0 %	+50 %

Figure 11 shows the membership functions of the total scour depth compared with the previous case. The functions appear similar, although case E) function reports greater scour depth values. In this case, the deterministic value of the membership function is 10.0 m (+22 % than CASE D) while the support spans from 6.8 to 17.7 m. The centre-of-mass value is 11.3 m, +13.7 % than the deterministic value. As for case B), it is possible to note that the greatest influence is exerted by the uncertainties related to local scour.

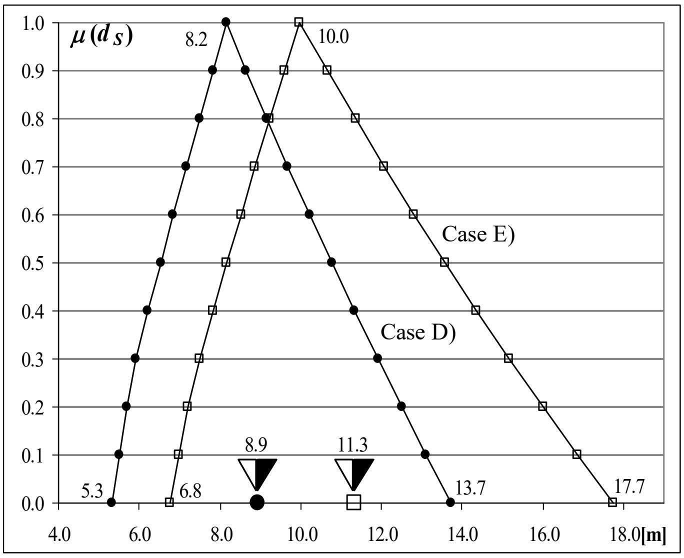

Figure 11. Membership functions and defuzzified values for the scour depth. Comparison between Case D and Case E.

5 CONCLUSION

This paper deals whit the assessment of the hydraulic force and scour dept in bridges with piers in the riverbed. The conventional design uses equations which involve a certain number of uncertainties regarding flow, sediments, structural and geotechnical parameters. By taking into account these uncertainties through a fuzzy approach, the obtained results

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ow a deterministic approach can disregard
s that make the output more severe with re-
structural safety

After introducing the selected models to describe the problem, the fuzzy theory is used to assess hydraulic force and pier scouring in a bridge with piers in the riverbed. The defuzzified output, expressed in terms of centre of mass of output membership functions, assumes values that are noticeably different and more severe than the deterministic results. The results of the cases studied (pier shaft exposed and foundation piles exposed) are summarized in Table 5.

Table 5. Percentage difference between the deterministic analysis and the result of the fuzzy assessment.

	Pier shaft exposed	Foundation piles exposed
Hydraulic force	+19 %	+46 %
Scour depth	+9 %	+13 %

In addition, considering the performed analyses, it is possible to note that the local scour has a prevalent role than contraction scour.

Concerning foundation calculations, the uncertainty about hydraulic load values has a relatively smaller effect than the combination of other parameters, in particular the geotechnical ones. However, the uncertainty concerning scour depth suggests to opt, in a cases like this, for deep foundation systems.

The setting adopted in this work, the different steps and criteria that were followed, could be suitably used for the construction of expert systems in order to evaluate the reliability of bridges and quantify indicators that could help in considering structures during their service life with a proper attention.

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