

Estimate of the societal Energy Return on Investment (EROI)

Elise Dupont^{*1}, Marc Germain², and Hervé Jeanmart¹

¹iMMC (Institute of Mechanics, Materials and Civil Engineering), Université catholique de Louvain

²LEM-CNRS (UMR 9221), Université de Lille and IRES, Université catholique de Louvain

Abstract

Access to abundant and affordable primary energy resources has been recognised as an essential factor for the prosperity of human societies. To measure their quality, understood as the ease with which the energy system can extract and transform them into a form useful to society, the concept of Energy Return On Investment (EROI) is widely used. Yet, so far, very few estimates of this indicator exist at a society level.

This paper first aims at providing an estimate of the societal EROI using a simple macroeconomic model with two sectors, an energy sector and a final sector aggregating the rest of the economy. For the year 2018 and at a worldwide level, we obtain a gross EROI of 9.4, and a net EROI of 8.5. The estimation of the Net Energy Ratio (NER), a second indicator more comprehensive than the EROI, allows assessing the energy embodied in the intermediate and capital consumptions of the entire economy. The NER calculation reveals that only 39% of the final energy produced contributes to total consumption (private and public) and economic growth, the remaining 61% being consumed within the economy. These intermediate energy consumptions (direct and indirect) are unevenly distributed between the two sectors: 11% of the production goes to the energy sector, while no less than 50% is consumed within the final sector.

These figures proved to be very insensitive to variations in the value of estimated model parameters.

*Corresponding author: elise.dupont@uclouvain.be

Introduction

Easy access to abundant and affordable primary energy resources has allowed human societies to develop and prosper. The quality of these resources refers to the ease with which the energy system can extract and transform them into a form useful to society. A widely used concept for measuring this quality is the Energy Return on Investment (EROI), defined as the ratio between the energy produced by a technology or energy system and all the energy that had to be invested, directly or indirectly, from the extraction of raw materials to the dismantling of the site at the end of its lifetime. Provided that the same boundaries and time period (i.e. life time vs 1 year) are used, which is the subject of numerous publications (Murphy et al. (2011); Brandt and Dale (2011); Murphy et al. (2016); Raugei (2019a); Brockway et al. (2019); Raugei (2019b)), this indicator makes it possible to compare the energy profitability of different energy production processes.

This ratio can be applied to the study of a specific site, a given sector, or even to the entire energy sector of an economy. While there are many studies devoted to calculating the EROI of a site or a sector, those at the level of an economy or a society are much rarer. This study is at the most aggregated level since it aims, among other things, at estimating the EROI of the world economy.

The few publications concerning the study of the *societal EROI* (to use the same terminology as Hall et al. (2009)) mainly concern the estimation of a lower bound below which a prosperous lifestyle would not be sustainable. Hall et al. (2009) estimate that the main fuels would need to offer an EROI around 5 to maintain "all that we call civilization", but this figure is the result of an educated guess, not a calculation. Using an econometric equation linking growth rates, energy expenditures, capital formation and population for the United States, Court and Fizaine (2016) estimate the minimum EROI necessary to maintain economic growth at 11. A recent study by Court (2019) shows that the minimum sustainable societal EROI at the global level has declined from just over 20 in 1900 to stabilize at around 6 since the 1970s. This evolution is the result of significant improvements to the energy efficiency of the economy, which have however slowed down in recent decades. Brandt (2017) uses an input/output model of a fictive 4-sectors economy whose intersectoral technical coefficients are estimated from US data and expressed in orders of magnitude (i.e. in powers of 10). This model leads to a minimum societal EROI of around 5 since below this threshold, the needs, especially in terms of capital, of the energy sector become too heavy for the economy to bear.

Overall, it can be seen that estimates of the minimum societal EROI vary significantly from one contribution to another. This may be the result of differences in (i) the methodology, (ii) the boundaries used for the EROI calculation, and (iii) the objective that this lower bound is supposed to reflect. As a result, the estimates are hardly comparable.

On top of that, all these estimates are valid for an energy system similar to the current one, i.e. based on a mixture of conventional fuels, whose conversion efficiencies are limited. As mentioned by Raugei (2019a), an energy system based mainly on renewable energies could probably be viable with relatively lower EROI levels¹.

¹However, according to the authors, these improvements will certainly be limited, given that the transition to renewable energies corresponds to an abrupt shift to a capital intensive energy system. The most significant part of the energy inputs to a fully renewable energy system are linked to raw materials extraction and processing. It will be difficult to reduce the energy needs for the industrial processes needing really high temperatures, and the impacts of metal depletion on the cost of extraction of those materials will have a negative impact on the energy inputs (as highlighted for example by Fizaine and Court (2015)).

A methodology for estimating the EROI at a country level was proposed by Lambert et al. (2014). The national societal EROI is calculated as the amount of energy that the country can produce or import with a unit of energy invested in its economy. More precisely, (i) the numerator is the amount of energy produced or imported with one dollar of gross domestic product (GDP), and (ii) the denominator is the amount of energy needed by the country to produce that dollar. The denominator corresponds to the energy intensity of the country, i.e. the ratio between energy consumption and GDP. Linking the estimated societal EROI and the Human Development Index (HDI) for a large sample of countries, Lambert et al. (2014) have shown that (i) societal EROI and HDI are positively correlated² and (ii) with few exceptions, countries with a high level of development (HDI above .75) are characterised by a societal EROI above 20.

Like the previous contribution, the MEDEAS model (Capellán-Pérez et al. (2019, 2020)) aims at describing, among other things, the evolution of the actual societal EROI (and not a lower bound). The EROI of the energy system is calculated endogenously, based on a detailed review of the life cycle analyses of the different energy sources, and including the ancillary facilities necessary to manage the intermittency of renewable energies (storage, oversizing of the production capacities and adaptation of the network). For 2015, using aggregated data at the world level, they found an EROI equal to 12.

Brockway et al. (2019) have recently calculated EROIs for fossil fuels at both primary and final energy stages, for the time period 1995-2011. They use IEA data for direct energy inputs, and they develop a methodology, based on the work of Brand-Correa et al. (2017), using input-output tables to estimate the indirect energy inputs associated with production of fossil fuels. For 2011 they found a net EROI at primary energy conversion stage of 29, which reduces to only 6.1 at the final energy stage. Their results suggest that current EROIs of fossil fuels may not be so different from the EROIs of renewables.

Using a simple macroeconomic model, we propose in this paper an estimate of the societal EROI at the global level, and we assess its robustness to variations in the estimated parameters of the model. Considering that this societal EROI is calculated using annual energy flows, the indicator calculated in this paper corresponds to a Power Return On Investment (PROI) as defined by Dale (2019). However, as the terminology EROI is much more widespread, it was decided to use the acronym EROI throughout this work. In fact, existing estimates of the EROI for a whole sector or a specific region always correspond to a PROI, as it is not possible to accurately take into account the service lifetime of all the installations in operation. Given its modelling approach, the paper bears similarity to the contributions of Capellán-Pérez et al. and Brockway et al. However, unlike MEDEAS, where only energy flows are used to estimate the EROI, our model uses both economic and energy flows.

The EROI metric only takes into account the direct (intermediate energy consumption) and indirect (non-energy intermediate consumption and capital consumption) consumptions of the energy sector. Consequently, it does not allow the calculation of the final energy actually available for final demand (sum of private and public consumptions and net investment) because it ignores the intermediate and capital consumptions of the rest of the economy. This is why we propose to estimate for the first time a second indicator introduced by Fagnart and Germain (2016), called the "Net Energy Ratio" (NER) of the economy³. This concept is broader than the EROI, in that it takes into account the inter-

²The relationship is, however, non-linear (in the sense that the HDI increases less and less rapidly with societal EROI) and is characterized by a saturation effect.

³This concept, which measures the share of the total energy produced that is available for final demand,

mediate and capital consumptions of the entire economy. By doing so, it becomes possible to determine the fraction of the final energy produced by the energy sector that is "really useful", i.e. that contributes either to consumption (private and public) or to economic growth.

The structure of the article is as follows. Section 1 presents the model. Section 2 describes how the model is calibrated, and presents the societal EROI and NER estimates for 2018. Given the uncertainty in some of the model parameters, a sensitivity analysis is conducted in Section 3 to evaluate the robustness of the estimates obtained in the previous section.

1 The model

1.1 Main features of the model

The model (illustrated in Figure 1) is that of a closed economy, with two sectors of activity: (i) the energy sector (ES) which produces final energy for the rest of the economy and (ii) the final sector (FS) which produces a general-purpose good, called the final good, used for consumption and investment. The output of the ES is distributed between intermediate consumptions of the two sectors and deliveries for final consumption (*final* referring to private (household) as well as public (government) consumption). Both sectors use three factors of production: capital, final energy and non-energy intermediate consumption (NEIC). The capital stock of the economy is shared between the two sectors according to their respective needs.

1.2 The equations

The energy and final sectors are identified by the indices e and f , respectively. The model is dynamic in the sense that the variables are time-dependent. However, to simplify the writing, time-dependency is omitted in the equations.

Sectoral outputs and factors consumptions are linked by the following relationships :

$$Y_i = \frac{K_i}{v_i} = \frac{E_i}{q_i} = \frac{X_i}{x_i} \quad (1)$$

where:

- Y_i , K_i , E_i and X_i refer to the production, capital stock, energy consumption and non-energy intermediate consumption (NEIC) of sector i , respectively. Energy production and consumption are measured in flows of energy (i.e. J/year), final good production and NEIC in flows of final goods (i.e. \$/year), and capital stock in units of final good (i.e. \$).
- v_i , q_i and x_i refer to the capital, energy, and NEIC intensities of sector i , respectively, i.e. the quantities of these inputs per unit of output. These intensities are likely to change over time as technologies evolve.

The total capital stock is shared between the two sectors :

$$K = K_e + K_f \quad [\$] \quad (2)$$

is to be differentiated from the NER as defined by Brandt and Dale (2011), which is a variant of the EROI which measures the ratio between the net energy produced and the energy invested.

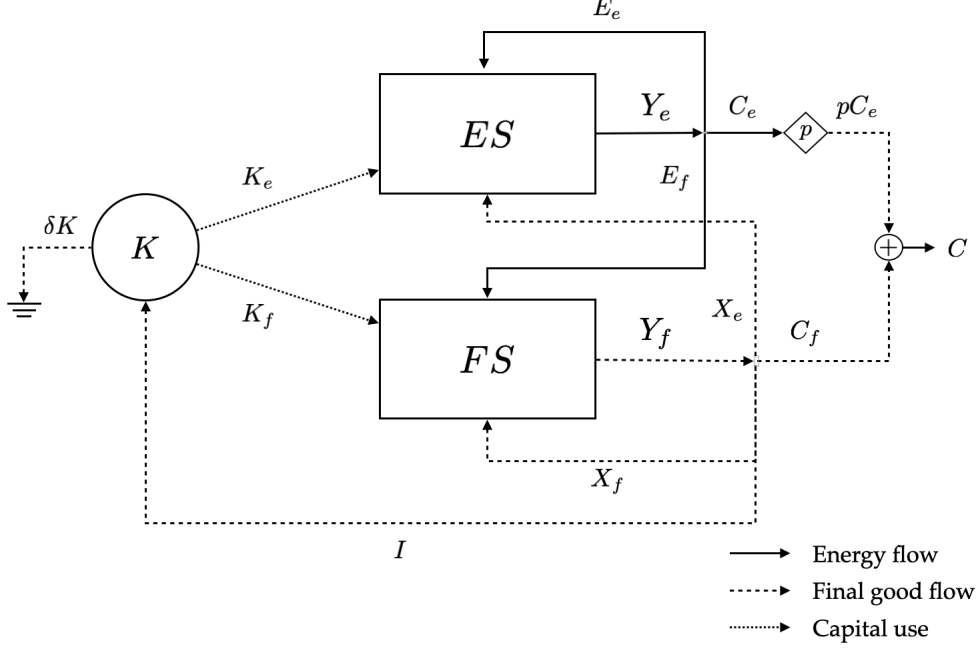


Figure 1: Closed economy, with two sectors of activity: the ES producing final energy, and the FS producing final goods. Y_i represent the final output flows, E_i represent the energy input flows, X_i the non-energy intermediate consumption input flows, K_i the stock of capital used by each sector.

The capital intensity of the economy is defined as the ratio between the total capital stock and gross domestic product (GDP) :

$$v = \frac{K}{GDP} \quad [\$ / (\$/y)] \quad (3)$$

Capital accumulation is described by :

$$\Delta K = I - \delta K \quad [\$ / y] \quad (4)$$

where I is the gross investment (in $\$/y$). δ refers to the average depreciation rate of the capital ($0 < \delta < 1$). It is a weighted average of the sectoral depreciation rates :

$$\delta = \frac{\delta_e K_e + \delta_f K_f}{K} \quad [1/y] \quad (5)$$

The production of the FS is divided into sectoral NEIC, deliveries for final consumption and investment :

$$Y_f = X_e + X_f + C_f + I \quad [\$ / y] \quad (6)$$

The output of the ES is divided between sectoral intermediate consumptions and final energy consumption (defined as private and public consumption of energy, similarly as final consumption of final good):

$$Y_e = E_e + E_f + C_e = E_e + E \quad [J/y] \quad (7)$$

where E refers to the net production of the ES (i.e. excluding self-consumption).

The price of the FS is chosen as the numeraire and equals to 1. With p being the real price of energy (in $\$/J$), the added value of each sector is calculated as:

$$\begin{aligned} AV_f &= Y_f - X_f - pE_f = (1 - x_f - pq_f)Y_f & [\$/y] \\ AV_e &= p(Y_e - E_e) - X_e = (p(1 - q_e) - x_e)Y_e & [\$/y] \end{aligned} \quad (8)$$

Total consumption is defined as the sum of final consumptions (of final goods and energy) :

$$C = C_f + pC_e \quad [\$/y] \quad (9)$$

By definition, the GDP (in $\$/y$) is the sum of the sectors' added value, i.e.:

$$\begin{aligned} GDP &= AV_f + AV_e \\ &= (1 - x_f - pq_f)Y_f + (p(1 - q_e) - x_e)Y_e \end{aligned} \quad (10)$$

The second equality derives from (8) and splits the GDP in terms of sectoral deliveries to final demand.

The macroeconomic savings rate is defined by:

$$s = \frac{I}{GDP} \quad (11)$$

It represents the share of the GDP invested in capital production, the remaining going to final consumption.

1.3 The EROI

The calculation of the EROI follows the approach of Fagnart and Germain (2016). It is defined as follows:

$$EROI = \frac{\text{Gross final energy produced}}{\text{Direct} + \text{Indirect final energy inputs}}$$

In order to produce an amount of gross final energy Y_e , the ES consumes (i) E_e as own energy consumption (in J/y) (i.e. direct final energy inputs), (ii) X_e as NEIC (in $\$/y$), and (iii) $\delta_e K_e$ as capital consumption (in $\$/y$). Since the EROI only accounts for energy flows, it is necessary to estimate the embodied energy included in the intermediate consumption flow X_e and capital flow $\delta_e K_e$, i.e. all the energy that was required to produce, install, maintain and dismantle the fraction of the ES capital stock consumed during the period under consideration (i.e. the indirect energy inputs to the energy sector).

Let E_{X_e} and $E_{\delta_e K_e}$ (both in $[J/y]$) be the energy embodied in the flows measuring the NEIC X_e and capital depreciation $\delta_e K_e$. Since (i) final production is assumed to be homogeneous (meaning that the model does not differentiate between energy vectors or end-uses) and (ii) it takes $E_f J$ to produce $Y_f \$$, this embodied energy flow can be estimated by:

$$E_{X_e} + E_{\delta_e K_e} = \frac{E_f}{Y_f}(X_e + \delta_e K_e) = q_f(x_e + \delta_e v_e)Y_e \quad (12)$$

given (1)⁴. For the followings, we define:

$$w_i = x_i + \delta_i v_i \quad (i = e, f) \quad (13)$$

These new technical coefficients represent the sectoral intermediate and capital consumption (ICC), per unit of output (i.e. w_f in [\$/\\$] and w_e in [\$/J]). The EROI then becomes:

$$EROI = \frac{Y_e}{E_e + E_{X_e} + E_{\delta_e K_e}} = \frac{Y_e}{E_e + q_f(x_e + \delta_e v_e)Y_e} = \frac{1}{q_e + q_f w_e} \quad (14)$$

given the previous equation and (1). As already mentioned in the introduction, given the fact that annual flows are used, the EROI calculated here at the level of the energy sector corresponds to a PROI as defined by Dale (2019).

Note that in this paper we chose for the EROI boundaries the concept of GEER (gross external energy ratio) as defined by Brandt and Dale (2011). The energy produced and directly consumed by the process is excluded from both the numerator and denominator of the GEER. A distinction is therefore made between final energy self-consumed within the energy sector (accounted for) and that self-consumed within a particular production site (not accounted for).

For processes with a significant fraction of self-consumed energy, the GEER is significantly greater than the GER (gross energy ratio), where this self-consumed energy is found in both the numerator and the denominator⁵. The GEER is chosen here for the EROI calculation because (i) it is consistent with the IEA definition of the energy industry own use and (ii) it is more relevant for assessing the capacity of a process to generate a surplus of energy available for the economy.

A ‘net’ societal EROI can also be calculated, similarly to the $EROI_{FIN}$ from Brockway et al. (2019), or the NEER from Brandt and Dale (2011):

$$EROI_{net} = \frac{\text{Net final energy produced}}{\text{Direct} + \text{Indirect final energy inputs}}$$

The net final energy produced is defined as the gross energy produced Y_e , minus the direct energy inputs E_e . Equation (14) then becomes:

$$\begin{aligned} EROI_{net} &= \frac{Y_e - E_e}{E_e + E_{X_e} + E_{\delta_e K_e}} = \frac{Y_e(1 - q_e)}{E_e + q_f(x_e + \delta_e v_e)Y_e} \\ &= \frac{1 - q_e}{q_e + q_f w_e} = (1 - q_e)EROI \end{aligned} \quad (15)$$

1.4 Energy losses in the economy

The output of the economy that is considered as "useful" is that which serves for total consumption and economic growth. It is measured by the net domestic product (NDP),

⁴This estimate of the embodied energy is based on the energy content of the share of the investment (at time t) used to replace the capital that has become obsolete (at time t). An alternative approach would have been to estimate the embodied energy in the capital stock based on the energy content of past investments. Since q_f decreases over time, this approach would lead to a higher estimate of the embodied energy contained in the flow measuring capital depreciation.

⁵For example, for global power generation, Kis et al. (2018) obtain an estimate of 7.4 for GER and 11.3 for GEER.

equal to GDP minus capital consumption, or equally to the sum of total consumption and *net* investment ($C + I_{net}$, where $I_{net} = I - \delta K = \Delta K$). We write:

$$\begin{aligned} NDP &= GDP - \delta K \\ &= C + I_{net} \\ &= C_f + pC_e + \Delta K \quad [$/y] \end{aligned} \quad (16)$$

Following the approach of Fagnart and Germain (2016), $NDP_{max} = \frac{Y_e}{q_f}$ is defined as the theoretical *maximum* useful output, which would be obtained if the totality of the ES output could be devoted to final consumption C_f or net investment I_{net} . The following relationship between the actual NDP and NDP_{max} is established in Appendix (5.1) :

$$NDP = \left((1 - q_e - \zeta)(1 - w_f) - q_f w_e + p\zeta q_f \right) NDP_{max} \quad (17)$$

where $\zeta = C_e/Y_e$ represents the fraction of output Y_e that directly goes to final consumption, and w_i ($i = e, f$), are defined by (13).

We see that only a fraction of the theoretical quantity NDP_{max} contributes to the actual NDP, because there are "losses" (in the sense of reduced useful output) in the economy.

- the first loss comes from the ICC of both sectors. The direct needs of the ES corresponds to a fraction q_e of its output, while the indirect needs are measured by the technical coefficient w_e . The needs of the FS correspond to a fraction $\frac{w_f}{q_f}$ of the incoming energy flow (i.e. $1 - q_e$). In total the fraction of NDP lost is measured by $L_1 = q_e + q_f w_e + (1 - q_e)w_f$.
- the second part is related to C_e , the amount of energy production delivered directly to final consumption that is not available for the FS. The resulting loss of final goods production equals $\frac{C_e}{q_f}(1 - w_f)$. Since $C_e = \zeta Y_e$, the *fraction* of $\frac{Y_e}{q_f}$ lost via C_e equals $(1 - w_f)\zeta$. However, the loss in final goods production is (partly) offset by pC_e , the value of direct energy deliveries. Measured as a fraction of $\frac{Y_e}{q_f}$, this compensation is worth $pq_f\zeta$. Consequently, the lost fraction of NDP_{max} related to C_e is ultimately measured by $L_2 = (1 - w_f - pq_f)\zeta$.

Given the above, (17) becomes :

$$NDP = (1 - L_1 - L_2)NDP_{max}$$

In the particular case where $\zeta = 0$, $L_2 = 0$ and we find the result of Fagnart and Germain (2016). They define the value $1 - L_1$ as the Net Energy Ratio (NER) of the economy, i.e. the fraction of the energy produced that can be transformed into useful final goods. This concept is broader than the EROI as it takes into account the losses related to the ICC of *the whole economy*, whereas the EROI only takes into account the losses related to the ICC of the ES.

Finally, if we look at the case where $w_f = 0$ (no ICC in the FS) and $\zeta = 0$ (no direct private or public energy consumption), then (17) is reduced to:

$$\begin{aligned} NDP &= (1 - q_e - q_f w_e)NDP_{max} \\ &= \left(1 - \frac{1}{EROI} \right) NDP_{max} \end{aligned}$$

Only in this very special case does the EROI alone determine the useful fraction of NDP_{max} .

The above results are listed in Table 1. The first column shows the losses considered (in ascending order) and the second column shows the corresponding NDP values. NDP_{max} , NDP_{EROI} and NDP_{NER} are theoretical values, while the last row shows the actual NDP. The different losses are measured by the difference between 2 successive NDPs (for example, $NDP - NDP_{NER}$ measures the loss of NDP due to C_e).

Losses	NDP
Nil	$NDP_{max} = \frac{Y_e}{q_f}$
ICC ES	$NDP_{EROI} = \left(1 - \frac{1}{EROI}\right) \frac{Y_e}{q_f}$
ICC ES+FS	$NDP_{NER} = (1 - L_1) \frac{Y_e}{q_f}$
ICC ES+FS + C_e	$NDP = (1 - L_1 - L_2) \frac{Y_e}{q_f}$

Table 1: Nature of losses and corresponding NDP

2 The reference calibration

First, we calibrate the model based on global observations for a so-called reference year. Next, a sensitivity analysis is performed on the model parameters that need to be estimated, and whose values are therefore characterised by some uncertainty. It will be shown that the reference calibration is robust to variations in these parameters because the current energy system mainly relies on direct energy inputs.

2.1 Observed data

Final good flows are expressed in billions of 2010 US dollars (GUS\$2010), while energy flows are expressed in exajoules (EJ). Time dependency in the units is deliberately omitted as the focus is on a particular year. The reference year is 2018, the latest year for which the IEA provides public data on global energy production and consumption. The model only records final energy flows.

Energy data: Regarding final energy production and consumptions, IEA statistics provide the following data worldwide:

- Total final consumption (including non-energy uses): 416.1 EJ
- Energy industry own use (EIOU): 35.1 EJ
- Losses in energy distribution, transmission and transport: 9.3 EJ
- Non-energy uses: 38.4 EJ
- Households and public consumption: $C_e = 144.1$ EJ , considering that the direct final energy consumption by households includes all residential energy consumption (88.3 EJ) and 46.1% of the energy dedicated to transport (121 EJ).⁶

⁶See Ribeiro et al. (2007) for the breakdown by mode of transport of the energy dedicated to transport at the global level.

Two major assumptions are made here regarding: (i) losses in energy distribution, transmission and transport and, (ii) non-energy uses:

- Similarly to Brockway et al. (2019), the distribution of final energy is assumed to be part of the ES. The losses in energy distribution, transmission and transport from the IEA data are therefore added as direct energy inputs to the ES. Indeed it corresponds to an additional flow of final energy that need to be produced by the ES to ensure a certain flow of final energy to the FS.
On the other hand, some may argue that these losses should not be considered at all in the model, as they can be assimilated to losses in the transformation processes. Therefore, in order to be as consistent as possible, the two results will be mentioned (societal EROI with and without losses).
- The fraction of non-energy uses in final consumption was 9.2% in 2018. Similarly to Capellán-Pérez et al. (2019), non-energy uses are not accounted for in energy production and are assumed to be produced by the final sector. As a result, the data provided by the IEA require slight adjustments. Non-energy uses are removed from final energy consumption, and self-consumption of the ES is split between the energy requirements of the ES and the FS (i.e. between E_e and E_f) in proportion to the fraction of non-energy uses in final energy consumption.

With these observed data and the above assumptions, the energy flows of the model are computed: $Y_e = 422.1$ EJ (= final consumption + own energy use + losses - non-energy uses), $E_e = 41.2$ EJ (= (1-0.092) (own energy uses) + losses), $E = 380.9$ EJ (= $Y_e - E_e$), and $E_f = 236.8$ EJ (= $E - C_e$).

This directly translates into the energy intensity of the ES: $q_e = \frac{E_e}{Y_e} = 9.8\%$. These energy data indicate that 62% of the energy produced E (i.e. Y_e minus self-consumption E_e) is allocated to deliveries to the FS, with the remaining 38% going to direct private and public consumption.

Notes on EIOU IEA data: Two important observations can be underlined here concerning the EIOU IEA data:

- First, it is good to point out that self-consumption in the sense of the IEA includes all final energy consumption of the ES industries for heating, lighting and the operation of the various types of equipment used for the extraction and transformation of primary energy (mainly fossil fuels) into final energy. However, other consumptions are excluded, e.g. coal burned in power stations at extraction sites, and gas burned during extraction. In these two cases, it is a self-consumption of *primary* energy which is not included in the accounts since it has never left the production sites. This fits well with the GEER concept chosen for the EROI (see Section 1.3).
- Second, the IEA EIOU data may suffer some limitations. Indeed there is a high level of aggregation as these data were produced from country data so there might be some inconsistencies among the reported data. However, despite these limitations, these data are globally relevant and have been used by several other authors King et al. (2015); Brand-Correa et al. (2017); Brockway et al. (2019); Capellán-Pérez et al. (2019).

Economic Data: The World Bank provides the following data:

- World GDP, excluding taxes and subsidies (i.e. corresponding to the indicator *gross value added at basic prices*) : $GDP_{2018} = 73014$ GUS\$2010 and $GDP_{2017} = 71344$ GUS\$2010. The growth rate of the economy is therefore given by:

$$g_{2018} = \frac{GDP_{2018} - GDP_{2017}}{GDP_{2017}} = 2.29\%$$

- Savings rate : $s = 25.12\%$.

2.2 Estimated parameters

From the model variables defined in Section 1, several interesting parameters can be calculated, namely (i) the capital stock allocation between the two sectors, (ii) the share of the ES's added value in GDP, (iii) the share of the private and public budget that is devoted to energy expenditures, and (iv) the total intermediate consumption of the FS. Unlike the savings rate, these parameters are not directly observable. But they can be estimated from the literature and from published economic data. The estimation of these 4 parameters, in addition to the depreciation rate of the capital in both sectors, allows to calibrate the model and therefore to calculate all the variables presented in Section 1, and in particular the EROI and the NER.

In the following paragraphs, reasonable ranges of values for each of these parameters are proposed. This will allow a sensitivity analysis (around the central value of the interval) to be performed in Section 3 to assess how sensitive the model calibration is to these variations.

Capital depreciation rate: As (4) indicates, capital depreciates exponentially. However, the depreciation rate differs according to the sector (see (5)). The depreciation rate δ_i of sector i can be approximated by :

$$\delta_i \approx \frac{1}{T_i} \tag{18}$$

where T_i is the average lifetime of capital in sector i . Estimating this lifetime is not an easy task, as the capital stock is composed of long-life equipment (e.g. buildings) and very short-life equipment (e.g. computers).

Regarding the FS, the European Central Bank estimates T_f around 20 years (European Central Bank (2006)). For Canada, T_f is estimated at (i) about 25 years for buildings and (ii) between 11 and 15 years for machinery and equipment (Statistics Canada (2007)). This leads us to choose 20 years as the reference value for T_f , with a range of variation equal to $[15, 25]$ years.

Although some models consider a homogeneous capital stock (for example Dale et al. (2012) assume that $T_e = T_f = 20$ years), it can be argued that the infrastructure of the ES has a longer lifetime. For Canada, the average lifetimes of oil refineries and natural gas processing plants are estimated to be 23 and 25 years respectively. For machinery and equipment related to power generation, values range from 23 to 37 years for the largest components (steam generators, turbines, engines) and 15 years for smaller equipment . This leads us to choose 25 years as the reference value for T_e , with a range of variation equal to $[20, 30]$ years.

Share of the capital stock devoted to the ES: This share is defined as:

$$\mu = \frac{K_e}{K} \quad (19)$$

The GEMBA model from Dale et al. (2012) is used to estimate μ . The calibration of this model provides, for 2010 and expressed in terms of energy content, 4000 EJ for the industrial sector capital stock and 280 EJ for the energy sector capital stock, which gives $\mu = 6.54\%$. An interval of $[3, 10]\%$ will be tested in the sensitivity analysis.

To estimate the last three parameters necessary for the calibration of the model, we rely among others on the input-output tables published by the OECD (OECD (2015)). These tables give, by sector and by country, and expressed in current US dollars, intermediate consumption, value added and total output. By aggregating the data by country and sector, we compute worldwide estimates of various indicators for the year 2015, which is the last year for which these tables are currently available.

Share of the energy sector in GDP Let α denote the ES's share in GDP:

$$\alpha = \frac{AV_e}{GDP} = \frac{pE - X_e}{GDP} \quad (20)$$

where the second equality results from (8).

Using the input-output tables, we find an estimate of 6% for 2015. To do this, the following 3 sectors of activity are associated to the ES: *Mining and extraction of energy producing products*, *Coke and refined petroleum products*, and *Electricity, gas, water supply, sewerage, waste and remediation services*, while the GDP corresponds to the sum of all sectors' added values. Point estimates between 3% and 4% are found for the United States, Japan, and some European countries (Germany, France, Italy, and the United Kingdom). Significantly higher values are found for example for China (6 %), Canada (9 %) or Russia (13.5 %).

Kümmel et al. (2015) gives an estimate of 5 % worldwide, while Ayres et al. (2013) gives a range between 4 and 8 %. Estimates of Court and Fizaine (2016) for the world are also in this range.

Given these estimates, the reference value chosen for α is 6%, with a range of variation equal to $[4, 8]\%$. Energy, although essential to any economic activity, only represents a small fraction of the added value of the economy.

The estimate of α gives a lower bound for the energy price p , as the added value of ES is necessarily lower than the product pE :

$$p > p_{min} = \frac{AV_e}{E} = \frac{\alpha GDP}{E} \quad (21)$$

Private and public energy expenditures: η is defined as the share of private and public consumption devoted to energy expenditures:

$$\eta = \frac{pC_e}{C} \quad (22)$$

The input-output tables provide a breakdown of final consumption by economic sector. By identifying the sectors related to the energy sector and removing all taxes and subsidies,

an estimate of 3.8% for η is found for the year 2015. However, this value is slightly lower than that of previous years (around 4.2%), so we will use a reference value of 4% for the calibration, with a range of variation equal to [3, 6]%. For the United States and China, where energy is comparatively cheaper than in other countries, η is estimated at around 3%.

Given the lower bound on the price of energy (21), η must satisfy the following condition:

$$\eta > \eta_{min} = \frac{p_{min}C_e}{C} = \frac{\alpha C_e}{(1-s)E}$$

With the estimate of 6% for α , and the observed values for C_e , E and s , it comes that $\eta_{min} = 3\%$.

Intermediate consumption of the FS: The ratio between the intermediate consumption of the FS (in final goods and energy) and its output is defined as:

$$z_f = \frac{IC_f}{Y_f} = \frac{X_f + pE_f}{Y_f} = x_f + pq_f \quad (23)$$

Using the input-output tables again, we find for z_f an estimate of 50% at the global level for 2015. It means that to produce 1 unit of output the FS needs to invest 0.5 units in intermediate consumption.

To perform the calculation, the sectors associated with the ES (see the estimate of α) are removed from the intermediate consumption and output. For the United States, the value obtained is 43% which corresponds to the values of other sources (42.9% on FRED (2018)). For China, "*the world's factory*", this value increases to 61%.

The estimated value for z_f used in the reference calibration is therefore 50%, with a range of variation equal to [40, 60]%.

2.3 Calibrated parameters

With the estimates of T_i and μ , the average depreciation rate δ (the weighted average of the sectoral depreciation rates) is calculated. (5) implies :

$$\delta = \frac{\frac{K_e}{T_e} + \frac{K_f}{T_f}}{K} = \frac{\mu}{T_e} + \frac{1-\mu}{T_f} \quad (24)$$

With the estimate of η and based on the observations of the energy consumption C_e and total consumption $C = (1-s)GDP$, the energy price p and total consumption C_f are estimated:

$$p = \frac{\eta C}{C_e}$$

$$C_f = C - pC_e = (1-\eta)C$$

The estimate of α and the observed value of GDP make it possible to calculate the added values of the two sectors and the NEIC of the ES:

$$AV_e = \alpha GDP$$

$$\begin{aligned}
AV_f &= (1 - \alpha)GDP \\
X_e &= pE - AV_e
\end{aligned} \tag{25}$$

The technical coefficient x_e is then directly computed, given (1): $x_e = \frac{X_e}{Y_e}$.

The estimate of the technical coefficient z_f , gives output Y_f . Indeed we have:

$$Y_f = AV_f + IC_f = \frac{AV_f}{1 - z_f}$$

E_f being an observed data, the following values can be computed: $q_f = \frac{E_f}{Y_f}$, $x_f = z_f - pq_f$ via (23), and X_f via (1).

Based on the growth rate of the economy, its capital intensity (v , defined by (3)) can be calculated. Indeed, v and g , the GDP growth rate, are linked by the following relationship (see Appendix 5.2) :

$$g = g_k - g_v = \frac{s}{v} - \delta - g_v \tag{26}$$

Where, for any variable x , $g_x = \frac{\Delta x}{x}$. It is empirically verified that v has been approximately constant over the last decades, i.e. $g_v \approx 0$ (see Appendix 5.3). Since in this case $g_k = g$, (26) implies :

$$v = \frac{s}{g_k} + \delta \tag{27}$$

Knowing v and the GDP, the capital stock K can be calculated via (3). With the estimate of μ , the capital stocks of the two sectors are then determined (K_e and K_f).

Knowing Y_e (observed), Y_f , K_e and K_f (calibrated above), the technological parameters v_f and v_e are directly calculated via (1). With the set of technical parameters (v_i , q_i , x_i , and δ_i , $i = e, f$), one finally finds the values of the societal EROI via (14) and of the different losses described in subsection 1.4.

2.4 Results

Results for the reference calibration ($T_e = 25$ years, $T_f = 20$ years, $\mu = 6.5\%$, $\alpha = 6\%$, $\eta = 4\%$ and $z_f = 50\%$) are included in Table 2.

v_f	q_f [MJ/US\$2010]	x_f	v_e [US\$2010/GJ]	q_e	x_e [US\$2010/GJ]	v	EROI	NER
1.72	1.7	0.47	39.1	0.1	3.31	3.45	9.4	0.39

Table 2: Results for the reference calibration (2018)

2.4.1 Parameters

Table 2 provides the following information :

- The ES self-consumption parameter (q_e) indicates that 10% of the final energy produced is self-consumed by the ES (see Section 2.1).
- Given the energy intensity of the FS (q_f), it takes 1.7 MJ to produce one US\$2010 of final goods (i.e. 0.5 kWh/US\$2010).

- Regarding the NEIC of the ES, x_e indicates that 3.3 US\$2010 of final goods are needed per GJ produced.
- Regarding the NEIC of the FS, x_f indicates that 47% of the FS output is self-consumed.
- The capital intensity of the ES (v_e) indicates that 39.1 US\$2010 of installed capital is required to produce a GJ of final energy.
- The FS capital intensity (v_f) indicates that it takes 1.72 US\$2010 of installed capital to produce 1 US\$2010 of final goods.
- The capital intensity of the economy (v) indicates that it takes about 3.5 US\$2010 of installed capital to produce 1 US\$2010 of GDP. This value, consistent with the literature, is directly derived from the observed values of the growth rate, the savings rate and the estimated value of the capital depreciation rate (cfr. 26).

2.4.2 Estimation of the societal EROI

Given the reference values of the different parameters, we find a societal EROI of 9.4. Looking at (14) and Table 2, one can see that it is mainly the value of q_e that determines the EROI of the current energy system. Indeed, q_e is around 10% while the product $q_f w_e$ equals 0.85%. The total energy consumption of the ES (i.e. the denominator of the EROI) is distributed as follows:

- 92.1% in the direct form of final energy consumption
- 5.4% in the form of energy indirectly contained in the NEIC X_e
- 2.5% in the form of energy indirectly contained in capital. The energy consumption linked to the infrastructures thus represents only a small fraction of the total.

This distribution corresponds to the estimates found in the literature.

Dale et al. (2012) postulates in their GEMBA model that final energy production from fossil fuels requires 90% of energy inputs in direct form. It is also consistent with the work of Gagnon et al. (2009), which studies the energy return from oil and gas production worldwide. Their results indicate that indirect energy inputs accounted for 11% of total energy inputs in 1992 and 9% in 2006. To obtain these figures, they use production data from oil and gas wells in the United States and the United Kingdom, and oil company expenditures on exploration, development, extraction and production. Brandt (2017) also assumes that 10% of the energy produced by fossil fuels is directly self-consumed, and 1% in indirect form. This corresponds to 91% of the inputs in direct form, and 9% in indirect form.

Putting our EROI equal to 9.4 on the function linking societal EROI and HDI (adjusted for inequality) developed by Lambert et al. (2014) ($HDI = 0.1827 \ln(EROI_{soc} + 0.0999)$) leads to a global HDI of 0.51. This figure is close to 0.596, the average HDI observed at the global level in 2018.

Finally, we note that the production of the FS is distributed as follows: 48.4% for ICCs (47.4% goes to the FS and only 1% to the ES), 38.2% to final consumption and 13.4% to investment.

As discussed in Section 2.1, one could also calculate the societal EROI obtained excluding losses at both the numerator and denominator. It gives a value of 11.7, showing that including losses has a significant impact on the estimated EROI. This figure is close to the EROI of 12 for the current energy system calculated by Capellán-Pérez et al. (2019). Finally, the net societal EROI given by equation (15) equals 8.5. This value is higher than the value of 6.1 found by Brockway et al. (2019) for the EROI of fossil fuels at final energy stage for the year 2011, the main differences between the two methodologies being the estimation of the indirect energy inputs, and the scope of the calculation (Brockway et al. (2019) consider only fossil fuels).

2.4.3 Estimation of the NER

L_1	61.1
=ICC ES	10.6
+ICC FS	50.5
L_2	14.1
=Losses of final goods	15
$-pC_e$	0.9
$L_1 + L_2$	75.2

Table 3: Losses estimation for the reference calibration [%].

Estimates of the various losses described in Section 1.4 are shown in Table 3. The losses related to the ICC of the two sectors amount to 61.1%. These losses are unevenly distributed between the two sectors: 10.6% for the ES, 50.5% for the FS. Thus, limiting oneself to the concept of EROI (which accounts for the ICC of the ES alone) implies ignoring the significant losses related to the ICC of the FS. The NER ($= 1 - L_1$) equals 39%. Because of the ICC of the two sectors, a little less than 2/5 of the final energy can potentially be used for the production of final goods (i.e. either for consumption or for economic growth).

This potential is far from being reached. Table 3 shows that non-negligible losses are related to the energy consumption C_e . Starting from the ratio $\zeta = C_e/Y_e$, equal to 34.1% in 2018, and after taking into account the ICC of the FS (measured by $(1 - w_f)\frac{Y_e}{q_f}$), the loss of useful production of final goods reaches 14.1% of the total. This loss is only very slightly compensated by the value of the private and public energy consumption pC_e (1% of NDP_{max}).

This last observation suggests that the NDP would be higher if (at least part of) the energy consumed by households were instead allocated to the production of final goods. At a given net investment, total consumption C would also be higher (since $C = C_f + pC_e = NDP - I_{net}$).

By analogy with the definition of the EROI, which is the ratio between the output and the ICC of the ES, the ratio between NDP and ICC of the two sectors can be defined here. This gives a ratio of 1.64 ($=1/0.61$). At the level of the economy, it is thus necessary to invest one unit of final good to produce 1.64 *useful* units.

3 Sensitivity analysis

As mentioned above, there exists uncertainty regarding the estimated parameters μ , α , η , z_f , T_e and T_f . It is therefore important to know whether or not the calibration of the previous section is sensitive to a variation in these estimated parameters.

	Reference	Min	Max
T_f [years]	20	15	25
T_e [years]	25	20	30
α [/100]	6	4	8
η [/100]	4	3	5
z_f [/100]	50	40	60
μ [/100]	6.5	3	10

Table 4: Intervals of variation of the estimated parameters.

The ranges of variation of the estimated parameters with their reference values are given in Table 4. All the variations given by this table have been tested, but to ease the presentation of the results only the estimated parameters which have a significant influence on the calibrated parameters are included. Table 5 therefore only includes the variations of η and z_f , and ignores those of T_e , T_f , α and μ .

Remember that all observed quantities remain unchanged through the sensitivity analysis.

Variations of z_f and η Regarding the various calibrated parameters, the following observations can be made:

- p increases with η because $p = \eta C / C_e$, and C and C_e are observed.
- x_f increases with z_f because $x_f = z_f - pq_f$, p being insensitive to the variations of z_f and q_f being of low weight.
- x_e increases with η because $x_e = X_e / Y_e = p(1 - q_e) - \alpha GDP / Y_e$ varies only through p , the other quantities remaining either observed or fixed.
- q_f decreases if z_f increases because $q_f = \frac{E_f}{Y_f} = \frac{E_f(1-z_f)}{(1-\alpha)GDP}$, where E_f and GDP are observed and α is fixed.
- v_f decreases if z_f increases because $v_f = K_f / Y_f$: (i) $K_f = (1 - \mu)vGDP$ where μ is fixed, GDP is observed and v remains unchanged (cfr. infra) and (ii) we saw in the previous point that Y_f increases with z_f .
- v_e remains invariable because $v_e = \frac{K_e}{Y_e} = \frac{\mu v GDP}{Y_e}$ where μ is fixed, GDP and Y_e are observed and v remains unchanged.

Crossing the variations of z_f and η , the societal EROI varies between 8.7 and 10, with a reference value of 9.4. We observe that the estimated parameter that has the most influence on the EROI is η , but that the EROI is very insensitive to changes in z_f . This is due to the fact that it is mainly determined by q_e , which is observed and therefore does not depend on the estimated parameters (see Section 2.4.2). Now q_e equals 9.8%, while the product

η [/100]	3			4			5		
z_f [/100]	40	50	60	40	50	60	40	50	60
p [US\$2010/GJ]	11.38	11.38	11.38	15.18	15.18	15.18	18.97	18.97	18.97
q_f [MJ/US\$2010]	2.07	1.72	1.38	2.07	1.72	1.38	2.07	1.72	1.38
x_f	0.38	0.48	0.58	0.37	0.47	0.58	0.36	0.47	0.57
x_e [US\$2010/GJ]	0	0	0	3.31	3.31	3.31	6.74	6.74	6.74
v	3.45	3.45	3.45	3.45	3.45	3.45	3.45	3.45	3.45
v_f	2.06	1.72	1.37	2.06	1.72	1.37	2.06	1.72	1.37
v_e [US\$2010/GJ]	39.07	39.07	39.07	39.07	39.07	39.07	39.07	39.07	39.07
$q_f w_e$ [/100]	0.3	0.25	0.2	1.01	0.84	0.67	1.72	1.43	1.15
<i>EROI</i>	9.94	9.99	10.04	9.28	9.43	9.58	8.71	8.93	9.17
<i>EROI_{net}</i>	8.97	9.01	9.06	8.38	8.51	8.65	7.86	8.06	8.27
<i>NER</i>	46.67	38.9	31.12	46.67	38.9	31.12	46.67	38.9	31.12
L_1	53.33	61.1	68.88	53.33	61.1	68.88	53.33	61.1	68.88
= <i>ICC ES</i>	10.06	10.01	9.96	10.77	10.6	10.44	11.48	11.19	10.91
+ <i>ICC FS</i>	43.26	51.09	58.92	42.55	50.5	58.45	41.85	49.91	57.98
L_2	16.97	14.14	11.31	16.97	14.14	11.31	16.97	14.14	11.31
= <i>Losses of final goods</i>	17.77	14.81	11.85	18.04	15.03	12.03	18.31	15.26	12.21
- pC_e	0.8	0.67	0.54	1.07	0.89	0.71	1.34	1.12	0.89
$L_1 + L_2$	70.29	75.25	80.2	70.29	75.25	80.2	70.29	75.25	80.2

Table 5: Calibration results for different values of η and z_f (reference and extreme values).

$q_f w_e$ is worth at most 1.72% (for the maximum value of η and minimum value of z_f). This product varies mainly because of the variations in the NEIC intensity of the energy sector x_e . But in any case, it remains low and q_e remains predominant in the determination of EROI. This observation is again inherent to the current structure of the energy sector.

We also notice that the losses L_1 and L_2 change significantly with z_f , but little with η . Consequently, the only parameter that has a significant influence on the NER is z_f .

Table 5 also shows that the capital intensity of the economy v is almost insensitive to the variation of the estimated parameters. This is because v is calculated from (27), where (i) s and $g_k = g$ are observed and do not depend on η and z_f and (ii) δ varies very little with the estimated parameters.

Variations of α and μ The variations of the α parameter have a slight influence on the EROI (it goes from 8.95 to 10 when α varies between 4 and 8%), but have much less impact on the NER (which goes from 38.4 to 39.4 %). The only parameter which varies significantly with α is the technical parameter x_e which goes from 6.8 to 0 US\$2010/GJ (corresponding to an extreme case without NEIC in the ES) when α increases from 4 to 8%. Indeed, if α increases, the added value of the ES increases. The price p being fixed by the value of η and the observed parameters, an increase of AV_e leads to a decrease of X_e via (25).

Changes in μ have no influence on the NER (which increases from 38.8 to 39% when μ increases from 3 to 10%), and only slightly changes the EROI (which decreases from 9.6

to 9.3). The EROI decreases when μ increases as more of the available capital has to be devoted to the energy sector. At a given energy production Y_e , it follows that the capital intensity of the energy sector v_e increases (from 18 to 60 US\$2010/GJ).

4 Conclusion

This article begins by developing a simple two-sectors macroeconomic model. The energy sector produces a final energy flow, some of which is delivered directly to final consumption (private and public) and the remainder to the final sector. The final sector produces a general-purpose good used either for final consumption, or for the intermediate consumptions of both sectors, or to renew (and possibly increase) the capital stock available to both sectors. Based on the calibration of the technical parameters (sectoral energy, capital, and non-energy intermediate consumption intensities, and capital depreciation rates), two indicators are then calculated: (i) the EROI, defined as the ratio between the production and the (direct and indirect) consumptions, and (ii) the NER, which measures the fraction of the final energy produced that is ultimately available, i.e. after deduction of the energy needs (direct and indirect) of the two sectors.

The model calibrated for the world and the reference year of 2018 leads to a gross societal EROI of 9.4, and a net societal EROI of 8.5. This means that 11% ($=1/\text{EROI}$) of the final energy produced is consumed by the energy sector itself, while the remaining 89% ($=1-1/\text{EROI}$) are available for the rest of the economy. Our estimate of the EROI is higher than most of the (few) *floor* values proposed in the literature for this indicator. The NER is estimated at 39%. It follows that the (direct and indirect) consumptions of the final sector represents 50% ($=89-39$) of the final energy produced.

Analysis of the EROI denominator reveals that 92% of the total energy inputs to the energy sector are in direct form. This result, in line with the literature, illustrates the fact that the current global energy system is mainly based on fossil fuels, whose exploration, extraction and transformation processes require significant amounts of direct energy.

A sensitivity analysis is then carried out on the model parameters that had to be estimated and whose values are somewhat uncertain. Namely, the capital stock allocation between the two sectors, the share of the energy sector's added value in GDP, the share of the private and public budget that is devoted to energy expenditures, and the total intermediate consumption of the final sector.

Overall, the results of the model are not very sensitive to variations in these parameters, especially the EROI and the NER which depend mainly on observed energy data.

Future work could exploit this model to assess how the EROI and NER would evolve with a transition towards an energy system based mainly on intermittent renewable energy sources. Indeed, such a transition is likely to change profoundly the structure of the energy system, as well as the capital needs of the economy.

Acronyms

CC capital consumption.

EROI Energy Return On Investment.

ES energy sector.

FS final sector.

GDP gross domestic product.

ICC intermediate and capital consumption.

NDP net domestic product.

NEIC non-energy intermediate consumption.

NER Net Energy Ratio.

PROI Power Return On Investment.

5 Appendices

5.1 Development of (17)

Unlike in Fagnart and Germain (2016), there are in the present model deliveries of the ES to private and public consumption ($C_e \neq 0$). In the following, we develop a relationship between the useful output of the economy and the energy production Y_e . The first approach is based on the definition of final consumption and establishes its relationship with Y_e , while the second approach consists in directly calculating the fraction of Y_e contained in the ICCs of both sectors.

Link between final consumption and Y_e : (1), (4), (5), (6), (7) lead to the following equations linking sectoral outputs Y_i and final consumptions C_i ($i = e, f$):

$$\begin{aligned} Y_e &= q_e Y_e + q_f Y_f + C_e \\ Y_f &= (x_e + \delta_e v_e) Y_e + (x_f + \delta_f v_f) Y_f + C_f + \Delta K \\ &= w_e Y_e + w_f Y_f + C_f + \Delta K \end{aligned}$$

We define D_f as the final demand (sum of consumption of final goods and net investment), and isolate both C_e et D_f to find:

$$C_e = (1 - q_e) Y_e - q_f Y_f \quad (28)$$

$$D_f \triangleq C_f + \Delta K = (1 - w_f) Y_f - w_e Y_e \quad (29)$$

The first equation gives an expression for Y_f that we incorporate in the second equation to write the following relationship between Y_e , C_e and D_f :

$$D_f = (1 - w_f) \frac{(1 - q_e) Y_e - C_e}{q_f} - w_e Y_e$$

Let $\zeta = C_e/Y_e$ be the fraction of energy production sold for final consumption. Then the previous equation becomes:

$$\begin{aligned} D_f &= \left((1 - w_f)(1 - q_e - \zeta) - q_f w_e \right) \frac{Y_e}{q_f} \\ &= \left((1 - q_e - \zeta) - \left(q_f w_e + (1 - q_e - \zeta) w_f \right) \right) \frac{Y_e}{q_f} \end{aligned} \quad (30)$$

This equation gives the quantity of goods that can be delivered to final demand (sum of consumption and net investment) from a certain amount of final energy. The theoretical maximal output is given by the ratio $\frac{Y_e}{q_f}$. The fraction that actually reaches the FS is given by $(1 - q_e - \zeta)$, it corresponds to the energy flow E_f . Then the first loss is the fraction of Y_e consumed by the NEIC and capital consumption (CC) of the ES, measured by $q_f w_e$, and the second loss is the fraction of Y_e consumed by the NEIC and CC of the FS, measured by $(1 - q_e - \zeta) w_f$. These losses can be further distributed between C_e and the rest of the output Y_e (see section 1.4).

Link between the NEICs and CCs of both sectors and Y_e :

$$\begin{aligned} \text{NEICs} + \text{CCs} &= X_e + \delta_e K_e + X_f + \delta_f K_f \\ &= \left(q_f w_e + \frac{Y_f q_f}{Y_e} w_f \right) \frac{Y_e}{q_f} \\ &= \left(q_f w_e + (1 - q_e - \zeta) w_f \right) \frac{Y_e}{q_f} \end{aligned}$$

We indeed find the last terms of (30).

Given the definition of the net domestic product (16), equation (30) gives an expression for the NDP as a function of the technical parameters, the fraction ζ , and the production Y_e :

$$\begin{aligned} NDP &= D_f + p C_e \\ &= \left((1 - w_f)(1 - q_e) - q_f w_e + \zeta(1 - w_f + p q_f) \right) \frac{Y_e}{q_f} \end{aligned}$$

which corresponds to equation (17).

5.2 Link between v , g and g_k

Using (4), (11) and (3) we write:

$$\Delta K = s GDP - \delta K = \left(\frac{s}{v} - \delta \right) K$$

Hence the capital growth rate g_k is given by :

$$g_k = \frac{\Delta K}{K} = \frac{s}{v} - \delta$$

Be g the GDP growth rate : $g = \frac{\Delta GDP}{GDP}$. Starting from (3), one can write at first order $g_v = \frac{\Delta v}{v} = \frac{\Delta K}{K} - \frac{\Delta GDP}{GDP}$, which implies equation (26) :

$$g = g_k - g_v = \frac{s}{v} - \delta - g_v$$

GDP growth is decomposed into the growth of the capital stock and the decline in the capital intensity of the economy.

5.3 Validation of $g_v = 0$

In the reference calibration, the capital intensity of the economy ($v = \frac{K}{GDP}$) was assumed to be constant, and therefore g_v was zero. This assumption can be validated as follows. One can do the exercise of estimating v for the years 1990 to 2018 via the previous equations. A posteriori, we observe that this parameter is more or less constant, *regardless of the hypothesis that we postulated a priori* for its rate of variation g_v . From then on, the starting postulate $g_v = 0$ is the only valid one.

References

- R. U. Ayres, J. C. J. M. van den Bergh, D. Lindenberger, and B. Warr. The underestimated contribution of energy to economic growth. *Struct. Chang. Econ. Dyn.*, 27:79–88, 2013. ISSN 0954349X. doi: 10.1016/j.strueco.2013.07.004. URL <http://dx.doi.org/10.1016/j.strueco.2013.07.004>.
- L. Brand-Correa, P. E. Brockway, C. L. Copeland, T. J. Foxon, A. Owen, and P. G. Taylor. Developing an Input-Output Based Method to Estimate a National-Level Energy Return on. *Energies*, 10(534):1–21, 2017. doi: 10.3390/en10040534.
- A. R. Brandt. How Does Energy Resource Depletion Affect Prosperity ? Mathematics of a Minimum Energy Return on Investment (EROI). *Biophys. Econ. Resour. Qual.*, 2(1):1–12, 2017. ISSN 2366-0120. doi: 10.1007/s41247-017-0019-y.
- A. R. Brandt and M. Dale. A General Mathematical Framework for Calculating Systems-Scale Efficiency of Energy Extraction and Conversion: Energy Return on Investment (EROI) and Other Energy Return Ratios. *Energies*, 4:1211–1245, 2011. doi: 10.3390/en4081211.
- P. E. Brockway, A. Owen, L. Brand-correa, and L. Hardt. Article : Estimation of global final stage energy-return-on-investment for fossil fuels with comparison to renewable energy sources. *Nat. Energy*, 4(7):612–621, 2019.
- I. Capellán-Pérez, C. de Castro, L. Javier, and M. González. Dynamic Energy Return on Energy Investment (EROI) and material requirements in scenarios of global transition to renewable energies Energy Return On energy Invested Energy Stored On energy Invested. *Energy Strateg. Rev.*, 26(July):100399, 2019. ISSN 2211-467X. doi: 10.1016/j.esr.2019.100399. URL <https://doi.org/10.1016/j.esr.2019.100399>.
- I. Capellán-Pérez, I. de Blas, J. Nieto, C. de Castro, L. J. Miguel, O. Carpintero, M. Mediavilla, L. F. Lobejon, N. Ferreras-alonso, P. Rodrigo, F. Frechoso, and D. Alvarez-Antelo. MEDEAS : a new modeling framework integrating global biophysical and socioeconomic constraints. *Energy Environ. Sci.*, (February), 2020. doi: 10.1039/C9EE02627D.
- V. Court. An Estimation of Different Minimum Exergy Return Ratios Required for Society. *Biophys. Econ. Resour. Qual.*, 4(3):1–13, 2019. ISSN 2366-0120. doi: 10.1007/s41247-019-0059-6. URL <https://doi.org/10.1007/s41247-019-0059-6>.
- V. Court and F. Fizaine. Energy expenditure , economic growth , and the minimum EROI of society. *Energy Policy*, 95(June 2018):172–186, 2016. ISSN 0301-4215. doi: 10.1016/j.enpol.2016.04.039. URL <http://dx.doi.org/10.1016/j.enpol.2016.04.039>.
- M. Dale, S. Krumdieck, and P. Bodger. Global energy modelling - A biophysical approach (GEMBA) Part 2: Methodology. *Ecol. Econ.*, 73:158–167, 2012. ISSN 09218009. doi: 10.1016/j.ecolecon.2011.10.028. URL <http://dx.doi.org/10.1016/j.ecolecon.2011.10.028>.
- M. C. Dale. When is EROI Not EROI ? *Biophys. Econ. Resour. Qual.*, 4(16):1–4, 2019. ISSN 2366-0120. doi: 10.1007/s41247-019-0065-8. URL <https://doi.org/10.1007/s41247-019-0065-8>.

- European Central Bank. Estimates of the Euro Area capital stock. *ECB Mon. Bull.*, (May):45–47, 2006.
- J.-F. Fagnart and M. Germain. Net energy ratio , EROEI and the macroeconomy. *Struct. Chang. Econ. Dyn.*, 37:121–126, 2016. doi: 10.1016/j.strueco.2016.01.003.
- F. Fizaine and V. Court. Renewable electricity producing technologies and metal depletion: A sensitivity analysis using the EROI. *Ecol. Econ.*, 110:106–118, 2015. ISSN 09218009. doi: 10.1016/j.ecolecon.2014.12.001. URL <http://dx.doi.org/10.1016/j.ecolecon.2014.12.001>.
- FRED. Intermediate inputs dynamics, 2018. URL <https://fredblog.stlouisfed.org/2018/06/intermediate-input-dynamics/>.
- N. Gagnon, C. A. S. Hall, and L. Brinker. A Preliminary Investigation of Energy Return on Energy Investment for Global Oil and Gas Production. (September), 2009. doi: 10.3390/en20300490.
- C. A. S. Hall, S. Balogh, and D. J. R. Murphy. What is the minimum EROI that a sustainable society must have? *Energies*, 2009. ISSN 19961073. doi: 10.3390/en20100025.
- C. W. King, J. P. Maxwell, and A. Donovan. Comparing World Economic and Net Energy Metrics , Part 2 : Total Economy Expenditure Perspective. *Energies*, 8:12975–12996, 2015. doi: 10.3390/en81112347.
- Z. Kis, N. Pandya, and R. H. Koppelaar. Electricity generation technologies: Comparison of materials use, energy return on investment, jobs creation and CO₂emissions reduction. *Energy Policy*, 120(May):144–157, 2018. ISSN 03014215. doi: 10.1016/j.enpol.2018.05.033. URL <https://doi.org/10.1016/j.enpol.2018.05.033>.
- R. Kümmel, D. Lindenberger, and F. Weiser. The economic power of energy and the need to integrate it with energy policy. *Energy Policy*, 86:833–843, 2015. doi: <https://doi.org/10.1016/j.enpol.2015.07.026>.
- J. G. Lambert, C. A. S. Hall, S. Balogh, A. Gupta, and M. Arnold. Energy, EROI and quality of life. *Energy Policy*, 64:153–167, 2014. ISSN 03014215. doi: 10.1016/j.enpol.2013.07.001.
- D. J. Murphy, C. A. S. Hall, M. Dale, and C. Cleveland. Order from chaos: A preliminary protocol for determining the EROI of fuels. *Sustainability*, 2011. ISSN 20711050. doi: 10.3390/su3101888.
- D. J. Murphy, M. Dale, and D. Moeller. Comparing Apples to Apples : Why the Net Energy Analysis Community Needs to Adopt the Life-Cycle Analysis Framework. *Energies*, 9(917), 2016. doi: 10.3390/en9110917.
- OECD. OECD.stat, 2015. URL <https://stats.oecd.org/>.
- M. Raugei. Net energy analysis must not compare apples and oranges. *Nat. Energy*, 4: 86–88, 2019a.
- M. Raugei. Energy Return on Investment : Setting the Record Straight. *Joule*, 3(8): 1810–1811, 2019b. ISSN 2542-4351. doi: 10.1016/j.joule.2019.07.020. URL <https://doi.org/10.1016/j.joule.2019.07.020>.

K. Ribeiro, S. Kobayashi, M. Beuthe, J. Gasca, D. Greene, D. S. Lee, Y. Muromachi, P. J. Newton, S. Plotkin, D. Sperling, R. Wit, P. J. Zhou, O. R. Davidson, P. R. Bosch, R. Dave, and U. Kingdom. Transport and its infrastructure. In *Clim. Chang. 2007 Mitigation. Contrib. Work. Gr. III to Fourth Assess. Rep. Intergov. Panel Clim. Chang. [B. Metz, O.R. Davidson, P.R. Bosch, R. Dave, L.A. Meyer (eds)]*. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA., 2007.

Statistics Canada. Depreciation Rates for the Productivity Accounts. *Can. Product. Rev.*, 2007.