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Competition for leadership in teams

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ABSTRACT

We analyze a model of information centralization in teams where players can only exchange information through an endogenous network. Over several periods each player can either pass or not pass her information to her neighbors. Once one player has centralized all the information, all players receive some payoff. The winner who collects all the information gets an additional reward. Since each player discounts payoffs over time, she faces the dilemma of either letting another player centralizing all the information fast, or trying to collect all the information by herself and to overtake the leadership. We find that there is always a single winner who centralizes the information at equilibrium and we characterize the winner and the duration for any network and for any discount factor. We show that the star network is always pairwise stable. More surprisingly, we find that even networks in which the centralization takes a long time can be pairwise stable.

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1. Introduction

The objective of the paper is to develop a model of information centralization in teams where team members can only exchange information through a communication network and to predict which communication networks could emerge in the long run. There are n players arranged in a connected network and each player belonging to the team has a unique piece of information. The task is to centralize all the information. The team must centralize all the information facing some exogenously given deadline. In each period, each player decides either to hold on her information items or to pass on to her neighbors in the communication network. Players take their decisions sequentially following some decision order or hierarchical ranking. The timing in each period is as follows. Player 1 takes her decision first. Next, each player chooses her action knowing the choices made by the players preceding her in the ranking. After all players have selected an action, pass on or hold on, the information is passed according to their decisions. The decision order or hierarchical ranking can be interpreted as the leadership structure within the team. Hence, player 1 is the current leader, and so players, beside collaborating for collecting all the information, compete for becoming the winner and the new team leader and getting the reward. If there is no winner after the items have been transmitted, the next period starts. As soon as at least one player has centralized all the information the game ends and all players receive some positive payoff. The player who collects all

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the information gets an additional reward. If several players centralize all the information at the same time they split the reward equally. The players have time preferences and so the utility of each player depends on the time it takes until one player centralizes all the information and whether she gets the reward or not.

We find that there is always a single winner who centralizes the information at equilibrium. When, compared to the reward, the common discount factor is low, the player with the lowest eccentricity is the winner, where the eccentricity of a player is the geodesic distance from this player to the player furthest away from her in the communication network. All players want the information to be centralized as fast as possible. The player with the lowest eccentricity can centralize the information fastest and so she wins. The duration is given by the number of periods it takes the winner to centralize the information and is then equal to the minimal eccentricity (i.e. the radius) of the network. When the discount factor becomes large, players care less about the duration and focus more on becoming the new leader and getting the additional reward. Obviously, player 1 has an advantage over the other players since she takes her decision first. She can then enforce the other players to pass on their information since otherwise they will not be able to centralize the information at all. However, the leader can decide not to centralize all the information herself if it is beneficial for her that another player collects all items faster. Another reason why the leader may not end up being the player who centralizes all the information is that some player blocks her from winning. Some players may have incentives to hold on their items long enough making it impossible for the leader to win. For instance, some player may hold on her information first to avoid that the leader, who has a much higher eccentricity than her, wins. As soon as the leader cannot win anymore, she passes on her information. This is the only type of situations where, at equilibrium, the duration is not equal to the eccentricity of the winner.

Beside characterizing the winner and the duration for any network and for any discount factor, we also predict which communication networks are stable. We assume there is an infinitesimally small cost to form a link. Under this assumption only minimally connected networks can be pairwise stable[1] and players will add a link to each other if the duration for centralizing all the information becomes shorter. We also find conditions that exclude some networks from being pairwise stable without having to check all possible additional links. For example, networks in which two players, in equilibrium, play such that they block each other from winning are never pairwise stable. Hence, the duration is equal to the eccentricity of the winner in any pairwise stable network. In addition, we provide more information about network structures which are always (star or symmetric star-like networks) or never (line networks with an even number of players) pairwise stable. One interesting result is that even with endogenous network formation it can take the winner a long time to centralize all information, e.g. in some line networks with an odd number of players. Finally, we show that once players become farsighted when forming and cutting links, only Pareto efficient networks like the star network can be stable.

A similar setting was first introduced in Hagenbach (2011) but with an exogenously given communication network.[2] The players are arranged in a network and want to centralize the information, but they decide simultaneously whether to pass or to hold on their information. This game of information transmission has multiple equilibrium outcomes. We rather adopt a sequential decision order yielding a unique equilibrium outcome. This decision order or ranking allows us not only to make more precise statements about the winner and the duration, but also to endogenize the communication network.[3]

One motivation for our model is the process of finding a spokesman, a promoter or a coordinator for a research project. All players contribute a part to the success of the project, but in the end only one player coordinates and promotes the project to the outside world. The additional reward can be either some additional funding the spokesman of a project gets or some benefits in terms of scientific reputation. Other examples include R&D joint ventures, political agreements, international agreements for fighting terrorism or climate change negotiations. Another example are the Panama Papers[4] investigations in 2015 where many different journalists and newspapers worked together, but the project is now promoted by only a small subset of the people who worked on it. The German journalist Bastian Obermayer was first to obtain all the raw data. He and his newspaper (Süddeutsche Zeitung) realized they could not analyze all the data by themselves. They contacted the International Consortium of Investigative Journalists and started the team project to retrieve information from the data. So, in our model Bastian Obermayer would be designated as the first player or leader, while the process of working through all the documents represents the centralization of (useful) information. In this example we can find most of our assumptions satisfied. Obviously, the entire team had a strong incentive to finish the project early as they wanted to stop the damage that was dealt by the firms and people they investigated. This corresponds to our assumption that the players discount over time. In this application, all newspapers and journalists were rewarded with the Pulitzer Prize and so they all got the same payoff. At the same time Bastian Obermayer was the one that represented the project in many interviews and on television, which gave him additional publicity. Clearly, the International Consortium of Investigative Journalists can be represented as a network. Even though, all newspapers might be able to share information with each other, this does not hold for all journalists in the newspapers and especially not for the external specialists that were hired directly by individual newspapers. These specialists worked on certain tasks and sent their results to the newspaper who hired them.

[1] See Jackson and Wolinsky (1996).

[2] Bonacich (1990) conducted experiments on communication in networks where each player receives a different subset of letters from a quotation and could pass on the information to her neighbors or hold on the information private. The first player who completes the quotation receives a reward.

[3] Our game can be seen as a war of attrition with discrete time Hendricks and Wilson (1985). Wars of attrition usually have a multiplicity of equilibria. One type of equilibrium corresponds to ours: one player waits, while other players pass their items Kornhauser et al. (1989). We select between the different equilibria by introducing a decision order for the players.

[4] All information available at https://panamapapers.icij.org/

Then, the information was shared by this newspaper with other journalists. Even the assumption that all the players have unique information is closely approximated in this example. Everyone had access to the more than 2.6 Terabyte of data, but the tasks were split between the journalists. The people working on this project completed their aim to go through all the information, because they did not want anyone to get away with what they had done.

We now turn to the related literature. Radner (1993) studies the efficiency of hierarchies in a model where players process information. He compares different structures and shows which hierarchical structures are efficient for a given set of variables. Another approach of information centralization is done by Jehiel (1999). In his setting a decision maker needs to gather some information to decide about a project. The decision maker's future employment depends on the outcome of the project. The decision maker gets fired if she selects a bad project. All other players get a certain share of the surplus of the project. Jehiel (1999) states conditions for optimal communication structures from the players and from the decision maker perspective. For strategic information transmission networks and its related literature we refer to Galeotti et al. (2013). They study a model of cheap-talk on networks and show how the players' welfare increases with more truthful messages. In addition, they find that in larger communities the communication decreases. Recently, Bloch et al. (2018) study strategic communication of rumors in a network with biased and unbiased agents that derive payoffs from a collective decision. Unbiased agents prefer that the decision correctly matches the true state of the world while biased agents want to enforce a specific decision regardless of the true state. They show that the social network can act as a filter and that biased agents may have incentives to limit their number.

The idea of leadership has been neglected in economics for a long time. Recently, Dewan and Squintani (2017) propose a model where a leader is selected and then this leader receives cheap-talk messages from the agents. To trust the messages the leader must first build a network of trustworthy associates. They show that the quality of the leadership depends on the judgment and wisdom of the people surrounding the leader.[5] In economics the literature on teams deals mainly with the comparison of individual decision making and team decision making and the positive effects of teams for individuals (see Cooper and Kagel (2005); Kocher et al. (2006); Holmstrom (1982); among others).

The paper is organized as follows. In Section 2 we introduce the model. In Section 3 we show that there is always a single winner. In Section 4 we characterize the winner and the duration for any network and for any discount factor, and we also predict which communication networks could emerge in the long run. In Section 5 we conclude and discuss the robustness. All proofs can be found in the appendix.

2. Model

Let $N = \{1, \ldots, n\}$ be a set of finite players, which are arranged in an undirected network g. A link in g between player i and player j is denoted by $ij \in g$. Let $N(g) = \{i \in N \mid \exists\ j \in N \text{ such that } ij \in g\}$ be the set of players who have at least one link in g. Let $N_i(g) = \{j \in N \mid ij \in g\}$ be the set of neighbors of player i in g. The player's degree counts her number of neighbors, i.e. $d_i(g) = |N_i(g)|$. A player i who has only one neighbor, $d_i(g) = 1$, is called a loose end. A path in g from i to j is a sequence of distinct links, which connect player i and player j. The length of a path is equal to the number of links on the path. A network g is said to be connected if there exists at least one path between all players $i, j \in N(g)$ with $i \neq j$. A network g is minimally connected if for all players $i, j \in N(g)$ with $i \neq j$, there exists exactly one path that connects player i and player j. Given a connected network g, the geodesic or shortest distance between player i and player j is the length of the shortest path between them and is denoted by $d_{ij}(g)$.[6]

Definition 1 (eccentricity and radius)**.** Let g be connected. The eccentricity of player i is $e_i(g) = \max_{j \in N} d_{ij}(g)$. The radius of the graph g is $r(g) = \min_{i \in N} e_i(g)$.

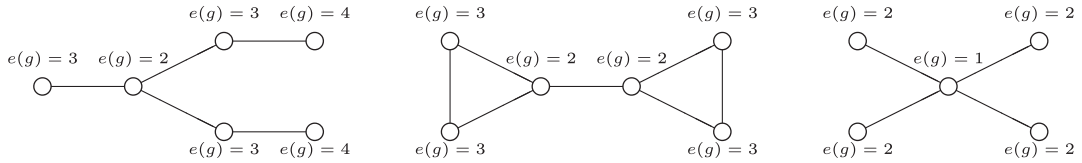
The eccentricity of player i is the geodesic distance from player i to the player furthest away from her. Given a connected network g, the radius is the minimal eccentricity. Fig. 1 illustrates the notions of eccentricity and radius for three different networks.

At the beginning of the game each player belonging to the team is in the possession of a unique piece of information. We label the information items such that player i has item i in her possession, while player j has item j in her possession. There is a finite number of periods T given. In each period the players can either pass on (P) all their information to their neighbors or hold on (H) the information private. When player i passes on her information, she passes copies of all the information items in her possession to all her neighbors $j \in N_i(g)$. The information set of each player is non-decreasing over time t. Players only pass on copies of their items to their neighbors, but they also keep the information in their possession.[7] Once a player has collected all the information, the game ends and the players stop passing information. The payoff of each player equals 1, except for the player who centralizes all the information. This player receives an additional reward R. If there are several players who have collected all information in the same period, those players share the reward equally. We

[5] The literature has mainly focused on why players should follow a leader Hermalin (1998) or why there should be a leader Komai et al. (2007). For a survey on leadership, see Ahlquist and Levi (2011).

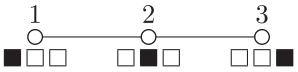
[6] Throughout the paper if we do not mention $N(g)$ for a connected network g, we implicitly presume that $N(g) = N$.

[7] Each player either passes on all her information to her neighbors or holds on. That is, she is not able to split her information into different information items. This is consistent with R&D collaborations in chemistry or biology where after adding item A to item B it is only possible to give information about the resulting item AB, but not about both components.

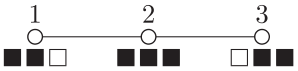


(a) min. connected, $r(g) = 2$ (b) not min. connected, $r(g) = 2$ (c) star, min. connected, $r(g) = 1$

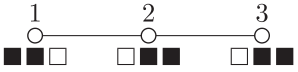
Fig. 1. Eccentricity and radius.



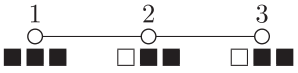
(a) Players and their information at $t = 1$



(b) Players and their information after $t = 1$, with $a_1^1 = a_2^1 = a_3^1 = P$



(c) Players and their information after $t = 1$, with $a_1^1 = H$ and $a_2^1 = a_3^1 = P$



(d) Players and their information after $t = 2$, with $a_1^1 = a_1^2 = H$ and $a_2^1 = a_2^2 = a_3^1 = P$

Fig. 2. A three-player line network.

call the player(s) who get a share of the reward the winner(s). For every network g, the set of winners of the game with deadline T is denoted by $W(g, T)$. The duration of the game is given by the number of periods it takes the winner(s) to centralize the information, and is denoted by $\tau(g, T)$. If no player centralizes all the information on time, all players get a payoff of 0. Players have time preferences with constant discount factor, $\delta \in (0, 1)$, and the utility function of player i is given by

$$u_i(g,T) = \begin{cases} 0 & \text{if } W(g,T) = \emptyset \\ \delta^{\tau(g,T)-1} \cdot 1 & \text{if } W(g,T) \neq \emptyset \text{ and } i \notin W(g,T) \\ \delta^{\tau(g,T)-1} \cdot (1 + R \cdot (1/|W(g,T)|)) & \text{if } i \in W(g,T). \end{cases}$$

The effects of the reward R and the discount factor δ are similar in terms of players' incentives to pass or hold on their information. If the discount factor or the reward is large, then each player focuses on getting the reward for herself and cares less about the duration it takes for centralizing all the information. On the contrary, if the discount factor or the reward is small, then each player tends to prefer that all information is centralized as soon as possible.

In each period t, the players sequentially choose their action $a_i^t \in \{P, H\}$, $i \in N$. There is a fixed decision order or current leadership ranking $\{1, 2, ..., n\}$ that determines the order of who is making a choice along the sequence. The timing in each period t is as follows. The current leader, player 1, starts and takes her decision, which is then observed by all other players. Afterwards, player 2 decides, already knowing what player 1 did. Then, player 3 chooses her action knowing the choices made by both players preceding her in the ranking. When player n has to pick an action, she takes into account the decisions of all other players. After all players have selected an action, pass on (P) or hold on (H), the information is passed according to their decisions. If there is no winner after the items have been forwarded, the next period starts. If a player is indifferent between passing and holding on, we simply assume that she passes on the information. The game is solved backwards looking for subgame perfect Nash equilibria.

In Fig. 2 we look at one possible way to arrange three players in a line. Each box represents a piece of information. A box is filled if the player has that piece of information in her possession. If we face a short deadline of $T = 1$, the only equilibrium outcome is that player 1 and player 3 pass on their information and player 2 is the unique winner as displayed in (b). If player 1 or player 3 would deviate, all players would obtain a utility of 0, so they prefer to pass on all their information. The action of player 2 makes no difference for the utility, because the other players cannot get all items within

one period. As soon as we extend the deadline to $T = 2$, the analysis becomes more complex and the equilibrium now depends on δ. If players are sufficiently impatient, $\delta \leq 1/(1+R)$, the equilibrium is as before: player 1 and player 3 pass on their information in the first period and player 2 wins. But, if players are patient, $\delta > 1/(1+R)$, player 1 has incentives to hold on her information during both periods forcing the other two players to pass on their information to her (see (c) and (d)). The intuition works as follows. For low values of δ, the players rather focus more on centralizing all the information, while for higher values of δ, the reward becomes more important than the duration for centralizing all the information. If player 1 holds on her information in the first period, player 3 is indifferent and so she passes on her information to player 2. In the second period, player 1 can enforce player 2 to pass on her information by holding on her information once again. The complete game tree for this example can be found in the appendix.

If the network g is not connected, it is impossible for any player to centralize all the information. Otherwise, the shortest time for player i to centralize all the information is equal to player i's eccentricity, $e_i(g)$. Only if the deadline T is larger or equal to the eccentricity of player i, player i can win the game. From now on, we set $T = |N| - 1$ and we write $W(g)$ for $W(g, |N| - 1)$, $u_i(g)$ for $u_i(g, |N| - 1)$ and $\tau(g)$ instead of $\tau(g, |N| - 1)$. It implies that each player can potentially be the winner of the game. If we would set a shorter deadline, some players could not ex-ante win the game. For any deadline larger than $|N| - 1$, the current leader, namely player 1, would even have a stronger advantage over the other players.

3. One winner and minimally connected networks

We show that, given any connected network, only a single player will get the reward at equilibrium. This result does not depend on the discount factor δ, the reward R or on the network g as long as g is connected. Since no player can centralize all the information if the network g is not connected, we only consider connected networks.

Proposition 1. *There is a unique winner at equilibrium, i.e.* $|W(g)| = 1$.

The easiest way to understand the intuition behind Proposition 1 is to consider the game with just two connected players. No matter the deadline $T > 1$, player 1 always wins after a single period. If both players hold on their information before the last period T, player 1 still holds on her information in T forcing player 2 to pass on her information. Otherwise, both players would be worse off (ending with a payoff of 0). By backward induction, in the second to last period $T - 1$, player 2 has incentives to pass on her information since she anticipates that if she holds on, she will pass on in period T, and she ends up worse off because of time preferences. Repeating the above argument until we reach period $t = 1$, we get that player 2 already passes on her information during the first period, and player 1 is the single winner. We denote by w the player who is the single winner.

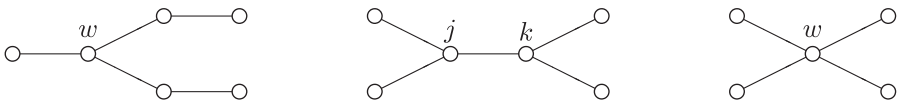
Beside looking for who is going to be the winner, we are also interested in understanding which networks are likely to arise when players need to centralize all the information. To do so, we need to define a notion which captures the stability of a network. We restrict our analysis to pairwise deviations and we suppose that there are positive but infinitesimally small costs to forming links. In other words, the cost of any link is always less important than any payoff from centralizing all the information. Hence, as in Goyal and Joshi (2003) or Mauleon et al. (2014), we use a strict version of Jackson and Wolinsky (1996) notion of pairwise stability.[8] A network is pairwise stable if no player has an incentive to delete one of her links and no two players strictly benefit from adding a link between them. Formally, a network g is pairwise stable if (i) for all $ij \in g$, $u_i(g) > u_i(g - ij)$ and $u_j(g) > u_j(g - ij)$, and (ii) for all $ij \notin g$, if $u_i(g) < u_i(g + ij)$ then $u_j(g) \geq u_j(g + ij)$.

As soon as a network is not minimally connected, it cannot be pairwise stable, because there is always at least one link that can be deleted while the duration remains constant. Even if players are connected by two or more paths, the information just flows through one path. Since links are costly, the players prefer to delete links until the network is minimally connected. Given any minimally connected network, players will never delete a link and move to a non-connected network since it would be impossible to centralize all the information, and all players would end up with zero utility. Thus, we only have to consider additional links that players may want to form. Two players will add a link between them if they both benefit from the link. Since at equilibrium only one player is the winner, the only possibility for both players to benefit from an additional link, is that this link decreases the time it takes until the information is centralized. It then follows that a network that differs from a pairwise stable network by just one link cannot have a shorter duration than the duration of the pairwise stable network.

4. Winner, duration and stability

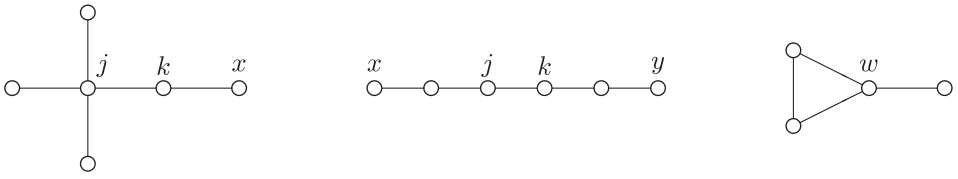
In this section we characterize the winner, the time it takes for the winner to centralize all the information and the stable networks. We show that, for any number of players and for any discount factor, there always exist pairwise stable networks. From the preceding section we already know that the winner and the duration for collecting all the information are affected by the value of the discount factor δ and the reward R.

[8] See Mauleon and Vannetelbosch (2016) for an overview on network formation games.



(a) $r(g) = 2$, Winner: w (b) $r(g) = 2$, Winner: $\min j, k$ (c) $r(g) = 1$, Winner: w

Fig. 3. Pairwise stable networks for $\delta \leq 1/(1+R)$.



(a) $r(g) = 2$, Winner: $\min j, k$ (b) $r(g) = 3$, Winner: $\min j, k$ (c) $r(g) = 1$, Winner: w

Fig. 4. Non pairwise stable networks for $\delta \leq 1/(1+R)$.

4.1. Low discount factor

For $\delta \leq 1/(1+R)$ the players prefer to decrease the duration for centralizing all the information by one period over becoming the winner at a later time.

Proposition 2. *Assume $\delta \leq 1/(1+R)$ holds. The winner is $w = \min_{e_i = r(g)} i$ and the duration is $\tau(g) = r(g) = e_w(g)$.*

Proposition 2 tells us that, for low discount factors ($\delta \leq 1/(1+R)$), the player with the lowest eccentricity is the winner. All players prefer to centralize the information as fast as possible. The player with the lowest eccentricity can centralize the information fastest and so she wins. If there are several players with the lowest eccentricity, then the one who is first in the decision order wins. This player can hold on her information to avoid that other players win. Since all the other players want to centralize the information as fast as possible, they will pass on their information to her. The duration is then equal to the radius of the network.

Proposition 3. *Assume $\delta \leq 1/(1+R)$ holds. A minimally connected network g is pairwise stable if and only if there is no link $ij \notin g$ that decreases the radius, i.e. $\forall ij \notin g : r(g+ij) = r(g)$.*

The duration for centralizing all the information is equal to the radius. Two players will only form an additional link, if they both benefit from this new link. But, only one of them can be the winner. Hence, this new link must decrease the duration in order to offset the infinitesimally small cost for forming links. Fig. 3 shows us three different network structures that are pairwise stable. In (a) and (c) there is a single player with the lowest eccentricity, while in (b) two players have the lowest eccentricity. In none of the networks it is possible to add a single link that decreases the radius. Hence, these three networks are pairwise stable. On the contrary, in network (a) of Fig. 4, players have incentives to form links. In this network, depending on the decision order, either player j or k wins. Player j has an incentive to link with player x, because it decreases the duration by one period and player j would become the winner even if she makes her decision after player k. In network (b) the two players (j and k) who are most central have incentives to form links with the players who are the most far away (j to y or k to x). With such additional link the radius decreases to two and so the duration decreases by one period. The network (c) is not minimally connected and so it cannot be pairwise stable.

4.2. Low and high discount factor

When the discount factor becomes large, players care less about the duration and focus more on becoming the winner and getting the additional reward.

Given the network g, the deadline T and the actions in period $s = 1, \ldots, t-1$, $(a_j^s)_{j \in N}$, we denote by $\Delta_i(g, t, T, (a_j^s)_{j \in N}^{s=1,\ldots,t-1})$ the remaining time needed from period t so that player i centralizes all the information. At the beginning of the game, the minimum time needed for player i to centralize all the information is equal to her eccentricity, that is $\Delta_i(g, 1, T, \emptyset) = e_i(g)$. In any minimally connected network, a player who is a loose end plays no role once she has passed on her information. We can remove such player from the network and look at the reduced network instead. Given the minimally connected network g and the actions $(a_j^s)_{j \in N}^{s=1,\ldots,t-1}$, let $(\widehat{g}^s)^{s=1,\ldots,t-1}$ be the sequence of networks where $\widehat{g}^s = \widehat{g}^{s-1} \setminus \{ij \in g \mid a_i^s = P \text{ and } i \text{ is a loose end in } \widehat{g}^{s-1}\}$ (with $\widehat{g}^0 = g$). The network $\widehat{g}^{t-1}$ is the reduced network obtained from g. For a minimally connected network g, the remaining time needed from period t so that player i can central-

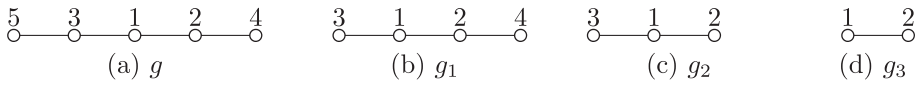


Fig. 5. A line-network g and its reduced networks g_1, g_2, g_3.



Fig. 6. Blocking and mutual blocking.

ize all the information, $\Delta_i(g, t, T, (a_j))$, is equal to the eccentricity of player i on the reduced network $\widehat{g}^{t-1}$ with deadline $T-(t+1)$. It means that $\Delta_i(g,t,T,(a_j)_{j\in N}^{s=1,\dots t-1}) = \Delta_i(\widehat{g}^{t-1}, 1, T-(t+1), \emptyset) = e_i(\widehat{g}^{t-1})$, and $\Delta_i(g,1,T,(a_j)_{j\in N}^{s=1,\dots t-1}) = t-1+\Delta_i(\widehat{g}^{t-1}, 1, T-(t+1), \emptyset) = t-1+e_i(\widehat{g}^{t-1})$.

In Fig. 5 we illustrate the notion of remaining time and reduced network, and how we can remove players who are loose ends from the network once they have passed on their information. Starting from network g we obtain that the remaining time so that player 1 can centralize all the information is equal to $\Delta_1(g,1,4,\emptyset) = e_1(g) = 2$. Suppose first that $a_5^1 = P$ and $a_4^1 = H$. We can remove player 5 from g and we get the reduced network g_1. Then, $\Delta_1(g,2,4,a_5^1 = P, a_4^1 = H) = \Delta_1(g_1,1,3,\emptyset) = e_1(g_1) = 2$. Suppose instead that $a_4^1 = a_5^1 = P$, then we can remove both players 4 and 5 from g and we get the reduced network g_2 with $\Delta_1(g,2,4,a_4^1 = a_5^1 = P) = \Delta_1(g_2,1,3,\emptyset) = e_1(g_2) = 1$. Suppose next that $a_4^1 = a_5^1 = P$ and $a_3^2 = P$. Then, we can first remove both players 4 and 5 and next player 3 from g and we get the reduced network g_3 with $\Delta_1(g,3,4,a_4^1 = a_5^1 = P, a_3^2 = P) = \Delta_1(g_3,1,2,\emptyset) = e_1(g_3) = 1$.

Definition 2 (blocking)**.** We say player k has the power and incentive to block player j in favor of player i at time t_k^j if there exists $\rho_k^j \in \{1,\dots,T\}$ such that

1. $\Delta_j(g, t_k^j, T, (a_l)) + \rho_k^j > T - t_k^j$ and
2. for $i \neq k$ we have $\Delta_i(g, t_k^j, T, (a_l)) + \rho_k^j < \Delta_j(g, t_k^j, T, (\widehat{a}_l))$

with $a_k^t = H\ \forall t < t_k^j$ and $\widehat{a}_k^t \neq a_k^t$ for some $t < t_k^j$.

Definition 2 tells us that player k blocks player j if this decreases the duration it takes to centralize all the information. The first condition ensures that if player k holds on her information for ρ_k^j periods, player j cannot win. The second condition gives the incentives for player k to block player j in favor of player i. The time it takes for player i to collect all the information once player k holds on her information during ρ_k^j periods (a_k^t) is less than the time it would take for player j in case that k passes on ($\widehat{a}_k^t \neq a_k^t$).

Definition 3 (mutual blocking)**.** Let player k have the power and incentive to block player j and let player j have the power and incentive to block player k. There is mutual blocking if the periods in which player j and player k block each other are identical, i.e. $t_k^j = t_j^k$ and $\rho_k^j = \rho_j^k$.

Let ρ_{mutual} denote the number of mutual blocking periods. Formally,

$$\rho_{mutual} = \left| \bigcup_{x=1}^{|N|} \left(\bigcup_{y=1}^{|N|} \left(\left\{ t_x^y, \dots, t_x^y + \rho_x^y - 1 \right\} \cap \left\{ t_y^x, \dots, t_y^x + \rho_y^x - 1 \right\} \right) \right) \right|$$

with t_x^y and ρ_x^y as in Definition 2. The number of mutual blocking periods is the union of all mutual blocking periods of player x with other players, which is given by the intersection of blocking periods of player x and player y.

Fig. 6 illustrates the notions of blocking and mutual blocking. Suppose that $\delta > (1/(1+R))^{1/2}$. In the left-hand network (a) if player 1 hold on her information in the first period, player 4 would have an incentive to block her in favor of player 2. Indeed, player 4 would hold on in the first period and by that she could achieve that player 1 is unable to win (because she would then need 5 periods to centralize all the information and the deadline is 4 periods) and that player 2 wins after three periods (instead of letting player 1 to win after 4 periods, in case player 4 would have passed the information). In the right-hand network (b), player 2 has incentives to block player 1 in favor of player 3, and player 1 has incentives to block player 2 in favor of player 3. In the first period player 1 and player 2 hold on their information and hence prevent that one of them wins (because they will need 5 periods to centralize all the information and the deadline is 4 periods) making player 3 the winner after three periods (instead of letting player 1 or player 2 to win after four periods, in case of passing their information).

The next two propositions characterize the winner and the duration for any value of the discount factor, $\delta \in (0,1)$.

Proposition 4. *Player $i \in N$ wins if and only if*

1. $\forall j < i$ *we have* $e_j(g) > e_i(g)$, *and either*



Fig. 7. Two networks with player 1 as winner.

1.1. $\delta \leq (1/(1+R))^{1/(e_j(g)-e_i(g))}$ *or*
1.2. *there is some* $k \in N$ *who has the power and incentive to block* j *in favor of* i,
2. $\forall j > i$ *either*
2.1. $e_j(g) \geq e_i(g)$ *or*
2.2. $e_j(g) < e_i(g)$, $\delta > (1/(1+R))^{1/(e_i(g)-e_j(g))}$ *and there is no* $k \in N$ *who has the power and incentive to block* i *in favor of* j.

Proposition 4 tells us that several conditions have to be satisfied for making player i the winner. First, it must take more time for all players who decide before i to centralize all the items (Condition 1) and these players must either prefer that i wins, because the discount factor is small (Condition 1.1) or they must be blocked from winning (Condition 1.2). Second, all players who decide after player i either cannot centralize all the information faster (Condition 2.1) or can centralize all the information faster but the discount factor is such that i prefers to win and i is not blocked from winning (Condition 2.2).

Proposition 5. *Let player* $i \in N$ *be the winner. The duration* $\tau(g, T)$ *is given by* $e_i(g) + \rho_{mutual}$.

Proposition 5 tells us that the duration is equal to the eccentricity of the winner plus the number of mutual blocking periods.

By means of Fig. 6 we provide some intuition behind these results. Suppose that $\delta > (1/(1+R))^{1/2}$. In the left-hand network (a) player 1 is blocked from winning, but there is no mutual blocking period, so that player 2 wins after two periods at equilibrium. In the first period player 1 and player 4 pass on their information, while in the second period player 3 and player 5 pass on. The other actions have no impact on the equilibrium outcome. The reason that player 1 passes on in the first period is that, if she would deviate and hold on her information, player 4 would have an incentive to block her. The first period of blocking would increase the duration by one, so player 1 does not deviate and no blocking happens at equilibrium. In the right-hand network (b), there is a mutual blocking period, which blocks player 1 and player 2 from winning at equilibrium. After that period of mutual blocking, both player 1 and player 2 pass on their information and player 3 wins after three periods. Both players who block each other have incentives to do so since they will end up with a utility of δ^2, while if they deviate and pass on their information in the first period their utility would be only δ^3. However, if we increase the deadline T by an additional period, player 2 would have to hold on two periods to avoid that player 1 wins. Then, player 2's utility would be the same whether player 1 wins after four periods or whether player 3 wins after $2+2$ periods. By assumption she would pass on, making player 1 the winner. This last example emphasizes that increasing the deadline gives even more advantage to the current leader, namely player 1.

For small discount factors, Proposition 2 already helps us to find the winner easily. For larger discount factors, Proposition 4 enables us to get some corollaries that simplify searching for the winner.

Corollary 1. *Assume* $\delta > 1/(1+R)$ *holds. If player 1's eccentricity is lower or at most* $(i-1)$ *higher than player* i*'s eccentricity, i.e.* $\forall i \neq 1 : e_i(g) + i - 1 \geq e_1(g)$, *then player* 1 *is the unique winner at equilibrium. The duration is equal to the eccentricity of player* 1, $\tau(g) = e_1(g)$.

When the discount factor is high, the advantage of taking the decision first is large. As long as player 1's eccentricity is just slightly larger than the eccentricities of the other players, player 1 centralizes all the information. For each player ranked further away in the decision order, the difference between the eccentricity of this player and the eccentricity of player 1 can be larger, while player 1 still remains the winner. Fig. 7 looks at two networks where player 1 is the winner. In both networks, the eccentricity of player 2 is $e_1(g) - 1$ and the eccentricity of player 3 is $e_1(g) - 2$, but it is still player 1 who wins at equilibrium because the difference between the eccentricities is relatively small.

Corollary 2. *Assume* $\delta > 1/(1+R)$ *holds. Player* i *wins after* e_i *periods if*

1. $\forall j < i : e_i(g) < e_j(g) - 1$ *and* $\delta \leq (1/(1+R))^{1/(e_j(g)-e_i(g))}$, *and*
2. $\forall j > i$ *either*
2.1. $e_j(g) \geq e_i(g) - 1$ *or*
2.2. $e_j(g) < e_i(g) - 1$ *and* $\delta > (1/(1+R))^{1/(e_i(g)-e_j(g))}$.

For intermediate values of the discount factor δ, results are similar. If δ is low enough and $e_i(g) + 1$ is smaller than the eccentricity of all the players that take their decision before player i and the same does not hold for a player who makes her decision after player i, then player i is the winner who centralizes all the information. In Fig. 8 we look at three networks where Corollary 2 holds. In the network (a) player 1 prefers that player 2 wins since the discount factor is too low. Even though the neighbors of player 2 would prefer to win, they cannot win, because player 2 makes her decision before them.

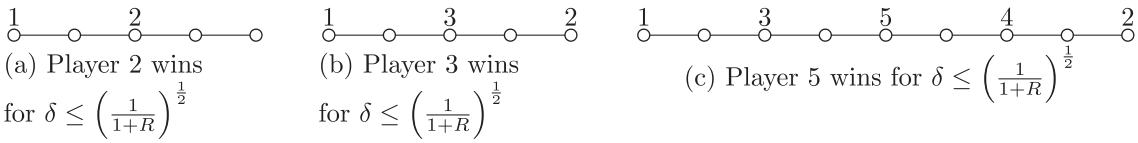


Fig. 8. Three networks where player 1 is not the winner.

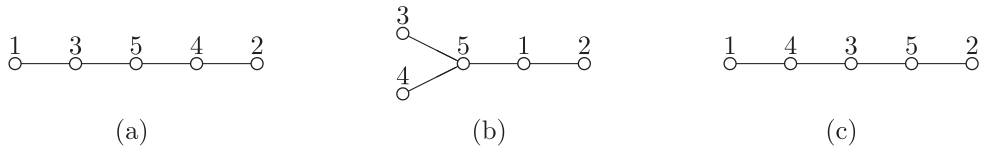


Fig. 9. Pairwise stable networks depending on the discount factor δ.

The analysis from networks (b) is similar. Player 1 and player 2 prefer that player 3 wins, since player 3 can win in two periods while player 1 or player 2 cannot. In network (c), even though player 3 can win faster than player 1 and player 2, there is another player, namely player 5, who can centralize all the information even faster. Indeed, Condition 2.1 of Corollary 2 holds for player 3 but Condition 2.2 does not hold because of player 5.

Beside characterizing the winner and the duration for any network, our objective is to predict which networks could emerge in the long run. We already know that two players will add a link to each other only if the duration for centralizing all the information becomes shorter. Moreover, only minimally connected networks can be pairwise stable.

It is hard to characterize the set of pairwise stable networks since this set is neither increasing nor decreasing in δ. Some networks are pairwise stable for all values of δ, some are only pairwise stable for low values of δ, while others are only pairwise stable for high values of δ. Network (a) in Fig. 9 is pairwise stable for $\delta \leq 1/(1+R)$ since no additional link can decrease the duration (and the radius) from two periods to one period. For $\delta > 1/(1+R)$, player 1 wins after four periods. In that case all players except player 3 have an incentive to link with player 1 and by that decrease the time it takes for player 1 to centralize all the information. Thus, there are networks that are pairwise stable only for low values of δ. In network (b), player 1 wins after two periods when $\delta > 1/(1+R)$. The players have no incentive to add links, because even if player 2 and player 5 add the link 25, player 1 would remain the winner after two periods. In addition, it is not possible for player 1 to win after one period by just adding a single link. For $\delta \leq 1/(1+R)$, it is player 1 who wins after two periods. In that case player 2 and player 5 have incentives to form the link 25 so that the duration decreases by one period and player 5 becomes the winner. Thus, there are networks that are pairwise stable only for high values of δ. Network (c) is pairwise stable for $\delta \leq (1/(1+R))^{1/2}$. In that case player 3 wins after two periods. For $\delta > (1/(1+R))^{1/2}$, player 1 and player 2 hold on in the first period and block each other from winning. After that mutual blocking period, player 3 wins, but it takes her three periods to centralize all the information. Player 1 and player 2 or player 1 and player 5 have an incentive to link since then player 1 would win after two periods. Thus, even for $\delta > 1/(1+R)$, there are networks which are pairwise stable only for some δ, namely $\delta \in (1/(1+R), (1/(1+R))^{1/2}]$.

The next proposition gives us additional information about network structures which are always or never pairwise stable.

Proposition 6.

1. *If $n > 3$ then any star network is pairwise stable;*
2. *Any symmetric star-like network with at least three arms[9] is pairwise stable;*
3. *If $n > 2$ is even then no line network is pairwise stable;*
4. *In any pairwise stable network, the duration is equal to the eccentricity of the winner.*

In any star network where one central player is connected to all other players, only the players who are not the center of the star can form links, but no link can decrease the duration for centralizing all the information. Thus, the star network is pairwise stable. A similar argument holds for symmetric star-like networks with radius $r \geq 2$. In a star-like network the length of the arms can be longer. A star-like network is symmetric if all arms of the star have the same length. Fig. 10 provides some examples of symmetric and asymmetric star-like networks. While stars and symmetric star-like networks are pairwise stable, most line networks are not pairwise stable. The reason is that, if the number of players is even ($n > 2$), there always exists an additional link that decreases the radius. Thus, a line network can only be pairwise stable if the number of players is two or odd. But, if the discount factor is low ($\delta \leq 1/(1+R)$), using Proposition 3, we conclude that all line networks with more than five players are not pairwise stable.[10] Finally, given that no network in which mutual blocking

[9] An arm of a star-like network is a line network with the center of the star at one end of the line.

[10] For $n > 2$ odd and $\delta > 1/(1+R)$, line networks are pairwise stable since the unique centered player cannot decrease her eccentricity by forming a single link.

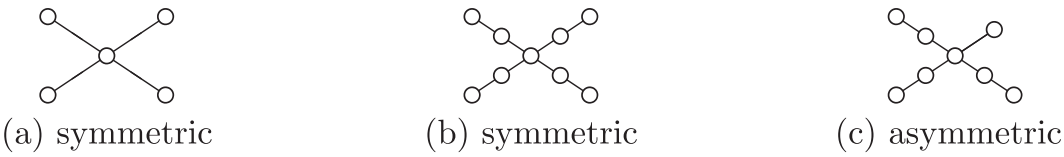


Fig. 10. Star-like networks.

occurs at equilibrium can be pairwise stable (Proposition 8 in the appendix), it follows from Proposition 5 that the duration for centralizing all the information is equal to the eccentricity of the winner in any pairwise stable network.

As shown in the right-hand network (b) of Fig. 6, mutual blocking occurs only when the discount factor is large ($\delta > (1/(1+R))^{1/2}$). The possibility of mutual blocking gives an additional aspect to the competition for leadership between the players. As in the example of Fig. 6, mutual blocking could occur between players with a good position in the decision order (like players 1 and 2 in the right-hand network (b)) but that occupy a bad position in the network regarding their eccentricity (players 1 and 2 are the loose ends in network (b) and then they have the largest eccentricity). When this is the case, the network under which mutual blocking takes place cannot be pairwise stable (as shown in Proposition 8) because the players who mutually block each other have incentives to form a link between them allowing player 1 to win after two periods instead the three periods that player 3 would need to centralize the information in case of mutual blocking.

5. Concluding discussion

We have proposed a model of information centralization in teams where players can only exchange information through an endogenous communication network. Players face a trade-off between cooperating for centralizing all the information fast and competing for being the one who collects all the information and becomes the team leader. Over several periods each player can either pass or not pass her information to her neighbors. Once one player has centralized all the information, all players receive some payoff. The winner who collects all the information gets an additional reward. Since each player discounts payoffs over time, she faces the dilemma of either letting another player centralizing all the information fast, or trying to collect all the information by herself and overtaking the leadership.

We have found that there is always a single winner who centralizes the information at equilibrium. We have characterized the winner and the duration for any network and for any discount factor, and we have predicted which networks emerge in the long run. Whether some player becomes the winner or not depends on her position in the network, her rank in the decision order and the discount factor. If the discount factor is low, the player with the lowest eccentricity wins. On the other hand, if the discount factor is high, a player first ranked in the decision order can outweigh a player with a low eccentricity. In addition, at equilibrium some players might hold on their information just to ensure that no player with an excessive eccentricity wins. Whether some minimally connected network is pairwise stable or not depends on the decision order and the discount factor. For instance, switching two players in a pairwise stable network may destabilize the network, because of the crucial role played by the decision order.

Collection of $n-1$ items or players with same items

What happens if players only need to collect at least $n-1$ items? Players whose information get centralized obtain 1 as before. The player who first centralizes at least $n-1$ items gets an additional reward R. If there are two (or more) players who first centralize, they share the reward equally. A player whose information is not centralized by some winner gets 0. All pairwise stable networks consist now of one isolated player and one minimally connected network connecting the remaining $n-1$ players. Obviously, for such network g connecting $n-1$ players, the results about the winner and the duration from the previous sections hold. However, even though the network g could be pairwise stable if we would restrict the analysis to $n-1$ players, the network g may not be pairwise stable since some players may have incentives to link with the isolated player.

What happens if two players have the same information? Assume that the players only need this information once for a successful project and that all players connected in the network get a payoff of 1 if the information is centralized by another player. Then, the pairwise stable networks consist either of one minimally connected network of n players or of one isolated player and one minimally connected component of size $n-1$. In the second case the isolated player is one of the players with a copy.

Farsighted players

What happens if players are farsighted while forming links? In subgame perfect equilibria all players are able to anticipate each others moves for each period. On the other hand, in the concept of pairwise stability players only look at the immediate change in the network that arises from deleting or forming a link and do not anticipate that other players may react to their changes. However, farsighted players can foresee the changes that follow from the deletion or formation of a link.[11] A farsighted stable set is the set of networks satisfying internal and external stability with respect to the notion of

[11] See Mauleon and Vannetelbosch (2016) for a comprehensive overview of the (myopic and farsighted) solution concepts for solving network formation games.

farsighted improving path Herings et al. (2009); Ray and Vohra (2015). A farsighted improving path from a network g to a network g' is a finite sequence of distinct networks $g_0, \ldots, g_L$ with $g_0 = g$ and $g_L = g'$ such that for every $\ell \in \{0, \ldots, L-1\}$ either (i) $g_{\ell+1} = g_\ell - ij$ for some $i, j \in N$ such that $u_i(g_L) > u_i(g_\ell)$; or (ii) $g_{\ell+1} = g_\ell + ij$ for some $i, j \in N$ such that $u_i(g_L) > u_i(g_\ell)$ and $u_j(g_L) > u_j(g_\ell)$. If there is a farsighted improving path from g to g', we write $g \to g'$. By $h(g) = \{g' \mid g \to g'\}$ we denote the set of networks that can be reached by a farsighted improving path from g.

Definition 4 (farsighted stability)**.** A set of networks V is a farsighted stable set if: (Internal stability) for every $g, g' \in V$, it holds $g' \notin h(g)$; and (External stability) for every $g \notin V$, it holds that $h(g) \cap V \neq \emptyset$.

This definition requires that between all pairs of networks belonging to the farsighted stable set, there is no farsighted improving path from one to the other (internal stability). In addition, from any network outside the farsighted stable set there is a farsighted improving path to at least one of the networks inside the farsighted stable set (external stability). Let $\mathcal{V}_i^\beta$ be the set of minimally connected networks in which player i is the winner and the duration is β periods; $\mathcal{V}_i^\beta = \{g \in \mathcal{G}_{min} \mid W(g) = i \text{ and } \tau(g) = \beta\}$. Let b_i be the minimal time in which player i can be the winner; $b_i = \min_{g|W(g)=i} \tau(g)$. If she never wins, we set $b_i = 0$. For any player i with $b_i > 0$ we have $\mathcal{V}_i^{b_i} \neq \emptyset$, and for all $\beta < b_i$, we have $\mathcal{V}_i^\beta = \emptyset$.

Proposition 7. *Take any player i such that $b_i > 0$. A set $V_i^{b_i} \subseteq \mathcal{V}_i^{b_i}$ is a farsighted stable set if and only if*

1. *for all $g, g' \in V_i^{b_i}$, for all $j \in N$, we have $d_j(g) = d_j(g')$;*
2. *there is no $g' \in \mathcal{V}_i^{b_i} \setminus V_i^{b_i}$ such that for all $j \in N$ we have $d_j(g) = d_j(g')$;*
3. *there is no g' such that $\tau(g') < b_i$ and $\delta \leq (1/(1+R))^{1/(b_i - \tau(g'))}$.*

A subset $V_i^{b_i}$ of $\mathcal{V}_i^{b_i}$ is a farsighted stable set if and only if: (Condition 1) any player i has the same number of links in all networks belonging to the set $V_i^{b_i}$; (Condition 2) there is no network belonging to $\mathcal{V}_i^{b_i}$, but not to $V_i^{b_i}$, where each player has the same number of links as in the networks in $V_i^{b_i}$; and (Condition 3) there is no network g' with a shorter duration such that player i prefers the outcome of g' over being the winner after b_i periods.

Corollary 3.

1. *$b_1 = 1$ and $V_1^1 = \{$star network with player 1 as center$\}$ is a farsighted stable set.*
2. *If $b_i = 1$, then $V_i^1 = \{$star network with player i as center $\}$ is a farsighted stable set.*
3. *For $\delta \leq 1/(1+R)$, V is a farsighted stable set if and only if $V = \{$star network$\}$.*
4. *The set $\mathcal{V}_i^{b_i}$ is a farsighted stable set if and only if for all $g, g' \in \mathcal{V}_i^{b_i}$ and all players $j \in N$ we have that $d_j(g) = d_j(g')$ and Condition 3 of Proposition 7 is satisfied.*

Independently of δ and R, player 1 is always the winner in the star network with her as the center. In that network it takes her only one period to centralize all information. This implies that V_1^1 always exists and is farsightedly stable. For all players who can win within one period, the set V_i^1 is always a farsighted stable set. The duration cannot be shorter than one period and the star network is the only minimally connected network in which a duration of one period is possible. As the set contains only a single element, Condition 1 of Proposition 7 is automatically satisfied. Furthermore, there is no other network in which all players have the same number of links, so Condition 2 holds as well. If the discount factor is low in the star network the center always centralizes all information within one period. Thus, for $\delta \leq 1/(1+R)$ we get $b_i = 1$ for all $i \in N$. This implies the third result of Corollary 3. For the last part of Corollary 3 we use the same arguing as for the second part. If we consider the entire set $\mathcal{V}_i^{b_i}$, Condition 2 of Proposition 7 always holds.

Farsightedness selects certain pairwise stable networks. For each possible winner w only the pairwise stable networks remain in which the duration is the lowest compared to other pairwise stable networks with the same winner w. This implies that farsightedness selects the pairwise stable networks that are Pareto efficient and eliminates the inefficient ones.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

Game tree of the example in Fig. 2. In Fig. 11 there is the game tree of the three player line example. For display reasons, we reduced the tree at those parts where both actions have the same effect. For example the action of player 2 (the center of the line) in the first period does not influence the outcome. For player 1 to get the item of player 3, player 2 need to pass again in the second period and vice versa for player 3 and the item of player 1.

The optimal action of each player (by backward induction) is highlighted by the bold print. Note, that an indifferent player decides to pass. In the first period, the action player 1 selects depends on the discount factor δ. For $\delta \leq 1+R$ she decides to pass, for $\delta > 1+R$ she holds.

Proof of Proposition 1. We first show that in equilibrium the project is always successful. We prove this by backward induction. Let us denote the items that player i has by I_i. In the last period, if the project is not successful yet we have for all $i \in N : |I_i((a_j^s)_{j\in N}^{s=1,\dots,T-1})| \neq n$. The last player to decide is player n. If the project is not successful yet player n cannot be the winner, because she misses some items and there is no player afterwards to pass these items to her. By passing player n can either complete the project or change nothing at all. In the first case the utility of player n after passing is $\delta^{T-1} \cdot 1$, while she gets 0 if she holds. Player n cannot be the winner in this case, otherwise the project would have been successful independent of the action of player n, because $I_n((a_j^s)_{j\in N}^{s=1,\dots,T-1}, (a_1^T, \dots, a_{n-1}^T, P)) = I_n((a_j^s)_{j\in N}^{s=1,\dots,T-1}), (a_1^T, \dots, a_{n-1}^T, H))$. In the second case player n is indifferent and so she will pass by assumption.

Player $n-1$ takes into account what player n will do. So when it is player $n-1$'s turn, she knows that if player n can finish the project by passing she will do so and in that case player $n-1$ gets a utility of at least $\delta^{T-1} \cdot 1$. If the success of the project depends on the action of player $n-1$ at time T she prefers passing (and utility δ^{T-1}) over holding (and utility 0). Repeating the same arguments for players $n-2$ to 1 and for periods $T-1$ to 1 completes this part of the proof.

It remains to show that there cannot be two winners. We prove this part by contradiction. Let player i and player j be winner, wlog. $i < j$ holds, i.e. player i always decides before player j. For both players to be a winner there have to exist a period t_i (respectively t_j) such that $a_i^{t_i} = P$ ($a_j^{t_j} = P$), otherwise player j would not get the item of player i and vice versa. We show that this contradicts the subgame perfectness.

In t_j player j (by backward reasoning) knows the outcome her two possible actions have. If she passes the information she will be a winner together with player i.

1. Case $t_i \leq t_j$, i.e. player i has already passed or announced to pass her information. Then player j's utility from passing is $u_j(a_j^{t_j} = P) = \delta^{\tau-1}(1 + R/2)$. If she holds her information, the duration does not change, because for all other players $k \neq i$, j the actions afterwards do not change, because their utility does not depend on the number of winners, but is always $u_k = \delta^{\tau-1}$. Therefore we get that the utility of holding is $u_j(a_j^{t_j} = H) = \delta^{\tau-1}(1+R) > \delta^{\tau-1}(1 + (1+R)/2)$. This implies that it cannot be subgame perfect for player j to pass.

2. Case $t_i > t_j$. Repeat the steps of the first case for player i instead of player j. □

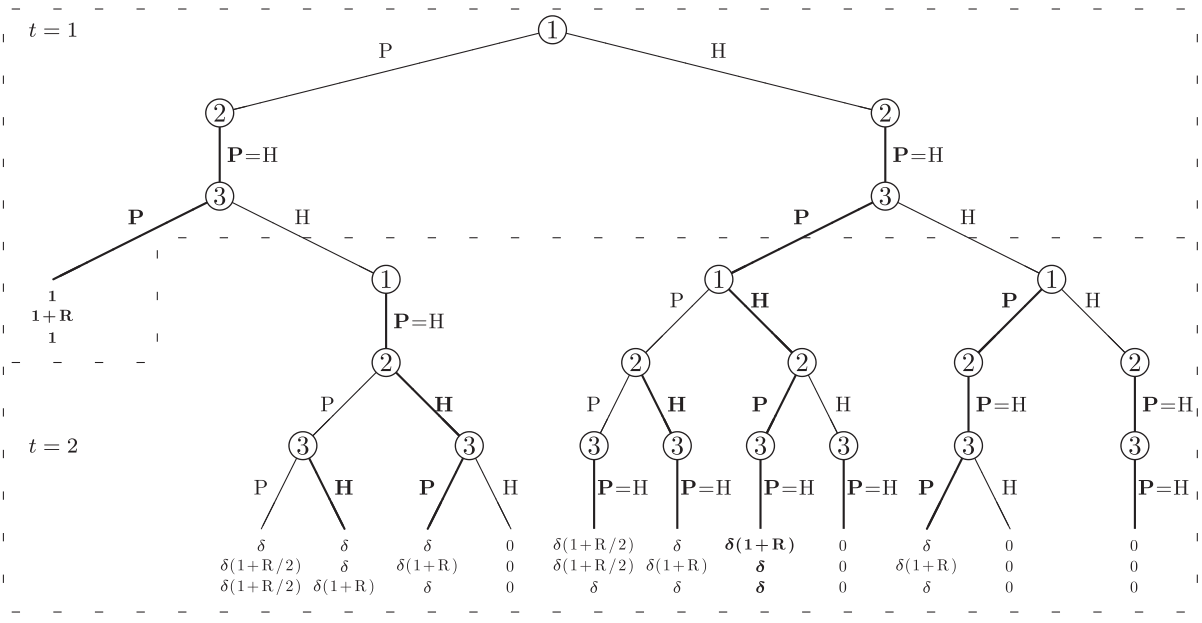


Fig. 11. Game tree.

Proof of Proposition 2. The minimal duration possible is $\tau(g) = r(g) = \min_{i\in N} e_i(g)$. One possibility to reach this duration is that all players pass in all periods, i.e. $\forall i \in N$ and $\forall t \in \{1,\ldots,r(g)\}: \ a_i^t = P$. The utility of player i in that case is

$$\delta^{\tau-1} \cdot \begin{cases} 1 & \text{if } e_i(g) \neq r(g) \\ (1 + (R/|W(g,T)|) & \text{if } e_i(g) = r(g)). \end{cases} \geq \delta^{\tau-1} \cdot 1.$$

If there is only a single winner, the proof is complete.

We show that the strategy profile described is not an equilibrium if there are two (or more) winners. First we show that all players $j \notin W(g,T)$ do not deviate: Consider all $j \in N$ with $e_j(g) > r(g)$. The only possibility that holding dominates passing in a period is that player j becomes the winner, but she can only win after $e_j(g)$ periods, i.e. her utility is at most $\delta^{e_j(g)-1} \cdot (1+R)$. The utility from passing (and having a winner after $r(g)$ periods) is $\delta^{r(g)-1} \cdot 1 \geq \delta^{e_j(g)-1} \cdot (1+R)$, because $\delta \leq 1/(1+R)$. In case of $e_j(g) = r(g)+1$, player j is indifferent and passes by assumption.

It remains to show that the players in the set of all possible winners $\{i \in N \mid e_i(g) = r(g)\}$ do not pass in each period: All these players know (by backward reasoning) that if they hold all other players still pass. Using the same arguments as in the proof of Proposition 1 yields that the first player i in the decision order with $e_i(g) = r(g)$ will deviate and become the single winner. □

Proof of Proposition 3. From Proposition 2 we know that the duration equals the radius. Proposition 1 states that there is always a unique winner at equilibrium. Combining these two results with the fact that only minimally connected networks can be pairwise stable, it follows that two players can only have incentives to form a link if this link decreases the radius and by that decreases the duration. □

Proof of Proposition 4. Let us first assume that there is no blocking. Following the same reasoning as in the proof of Proposition 2 we know that player i cannot be the winner if there is another player $j < i$ with $e_j(g) \leq e_i(g)$. This implies Condition 1: $\forall j < i$: $e_j(g) > e_i(g)$. A player i wants to win if she can centralize the information fast enough, i.e. if δ is such that she prefers winning over letting another player centralize the information faster. This implies that player i only wins if all $j < i$ prefer that player i wins and that she prefers winning over letting another player win.

The first condition is equivalent to: $\forall j < i$: $e_j(g) > e_i(g)$ (see above) and $u_j(j \text{ wins}) = \delta^{e_j(g)-1} \cdot (1+R) \leq \delta^{e_i(g)-1} \cdot 1 = u_j(i \text{ wins})$. Rewriting yields to $\delta \leq (1/(1+R))^{1/(e_j(g)-e_i(g))}$.

The second condition is that $\forall j > i$: either j cannot centralize the information faster, i.e. $e_j \geq e_i$ or i prefers to win, i.e. $u_i(i \text{ wins}) = \delta^{e_i(g)-1} \cdot (1+R) > \delta^{e_j(g)-1} \cdot 1 = u_i(j \text{ wins})$. Solving for δ results in $\delta > (1/(1+R))^{1/(e_i(g)-e_j(g))}$.

If we consider the possibility of blocking, then player i can win for the same reasons as without blocking but, in addition, she can also win if there is $j < i$ who wants to win (i.e. Condition 1.1. not required) but who is blocked from winning. Furthermore, there has to be no incentive for any player k to block player i from winning such that $j > i$ wins. □

Proof of Proposition 5. We show by contradiction that the duration can be neither larger nor smaller than the sum of the eccentricity of the winner and the number of mutual blocking periods. ($\not>$) Given that the strategies of the players are optimal, the duration cannot be longer. ($\not<$) Let ρ_k be the total number of mutual blocking periods, starting from period t_k. The same argument holds if several mutual blocking periods take place not successively. By the definition of mutual blocking, we get $\Delta_i(g, t_k, T, (a_l)) = \Delta_i(g, t_k + \rho_k, T, (a_l))$ for all $i \in N$. This implies that the duration cannot be lower than the eccentricity of the winner plus the number of mutual blocking periods. □

Lemma 1. *In no network, at equilibrium, three or more players block each other.*

Proof of Lemma 1. We show that three players have no incentive to mutually block each other. Without loss of generality, take player 1, player 2 and player 3. We first show that in a star-like network g with three arms, where player 1, player 2 and player 3 are the loose ends (i.e. the node at the end of each arm), they will not mutually block each other. A star-like network is one type of network architecture, but if the players do not mutually block each other in such type of network, they still do not block once we add additional links to the network.

Suppose that player 1, player 2 and player 3 block each other and that they all have the same eccentricity $e_1(g) = e_2(g) = e_3(g)$. Let β be the number of blocking periods and let player c be the center of the star-like network g. It follows that the duration $\tau(g)$ is equal to the sum of $e_c(g)$ and β:

$$\tau(g) = e_c(g) + \beta. \tag{1}$$

In addition, $\beta + e_1(g) = T + 1 = |N|$ has to hold since the number of blocking periods has to be large enough to prevent players 1, 2 and 3 from winning. Rewriting yields

$$\beta = |N| - e_1(g) \tag{2}$$

Plugging (2) into (1) gives us

$$\tau(g) = e_c(g) + |N| - e_1(g). \tag{3}$$

Players only block each other if the duration with blocking is lower than their eccentricity:

$$\tau(g) < e_1(g). \tag{4}$$

Merging (3) and (4) yields to

$$|N| + e_c(g) < 2 \cdot e_1(g). \tag{5}$$

In a symmetric star-like network, the eccentricity of the loose ends is twice the eccentricity of the center. Using this in (5) we get $|N| + e_c(g) < 4 \cdot e_c(g)$ which leads to

$$|N| < 3 \cdot e_c(g).$$

Since the eccentricity of the center of a symmetric star-like network with x arms is $e_c(g) = (|N| - 1)/x$, we get

$$|N| < (3/x) \cdot (|N| - 1).$$

This inequation never holds if there are at least three arms, $x \geq 3$. Thus, in a symmetric star-like network with at least three arms, the loose ends will never block each other. One can consider all other networks as symmetric star-like networks with additional players and links and then the same arguments hold. □

Proposition 8. *No network in which there occurs a mutual blocking period at equilibrium is pairwise stable.*

Proof of Proposition 8. From Lemma 1 we know that only two players j, k can block each other. Without loss of generality, assume $j < k$. The smallest number of blocking periods is one, which happens only on the line network. The loose ends block each other for one period and hence they achieve that a player with a lower eccentricity wins. In case of a line the eccentricity of player j and player k is $e_j(g) = e_k(g) = |N| - 1$ and the eccentricity of the winner is $e_w(g) = |N|/2$ if $|N|$ is even or $e_w(g) = (|N| - 1)/2$ if $|N|$ is odd. The time it takes the winner to centralize all the information is equal to the sum of her eccentricity plus the number of blocking periods, $e_w(g) + 1$. It follows that the network cannot be pairwise stable, because player j and player k would prefer to add the link jk. If g is a line, we get $e_j(g + jk) = e_k(g + jk) = |N|/2$ if $|N|$ is even or $e_j(g + jk) = e_k(g + jk) = (|N| - 1)/2$ if $|N|$ is odd. Then player j is the winner after $e_j(g + jk)$ periods which is strictly less than $e_w(g) + 1$. All other networks can be considered as adding players and links to the line network. For each player we add to the line, player j and player k have to block each other for an additional period. Let g' be the network where we have added x players to the line with $|N'|$ players, and so we have a tree with $|N'| + x$ players in total. The smallest eccentricity is $e_w(g') = |N'|/2$ if $|N'|$ is even and $(|N'| - 1)/2$ if $|N'|$ is odd. Therefore, we get that the duration of g' is $\tau(g') = e_w(g') + 1 + x$. Again player j and player k will link. The furthest the additional players can be away from each other corresponds to the situation where they are all linked to the center of the line. By this we get $e_j(g' + jk) \leq |N'|/2 + x$ if $|N'|$ is even and $e_j(g' + jk) \leq (|N'| - 1)/2 + x$ if $|N'|$ is odd. It follows that $e_j(g' + jk) \leq e_w(g') + 1 + x$ and so we have shown that the network g' cannot be pairwise stable. Notice that these arguments hold since it is assumed that in case of indifference the players pass on their information. □

Proof of Proposition 6. Part 1. Take $n > 3$. In any star network, the player who wins is either the center or player 1. If $\delta \leq 1/(1 + R)$, the center is the unique winner since all players want to centralize all the information as fast as possible. There is no incentive for an additional (costly) link since the players already centralize all the information in a single period. If $\delta > 1/(1 + R)$, then player 1 is the single winner. Suppose that player 1 is not the center of the star. Her optimal strategy is to hold on during all periods and forcing the other players to pass on their information to her. The duration is two periods. Player 1 cannot add just a single link to decrease her eccentricity from 2 to 1 since there are more than three players, $n > 3$. Thus, player 1 has no incentive to form a single additional link. The other players who are not the center cannot decrease the duration since an additional link would not change their eccentricity and player 1 (who is the first player to take her decision in each period) can stick to her optimal strategy. Using the property that two players i and j will add ij if and only if $g + ij$ has a shorter duration yields the result. Suppose now that player 1 is the center of the star. Then, we can use the same arguments as for $\delta \leq 1/(1 + R)$.

Part 2. The proof is analogue to the one of part 1. The center has no incentive to form any link and the remaining players cannot reduce their eccentricity by adding a link. In any symmetric star-like network with at least three arms, for any given player there always exist at least two other players to whom she has her maximal geodesic distance. Thus, a single additional link cannot decrease the duration. From Lemma 1 it follows that at equilibrium there is no blocking in any symmetric star-like network with at least three arms.

Part 3. Take $n > 2$ even. For each player i in any line network g there exists exactly one player j with the same eccentricity, $e_i(g) = e_j(g)$. When player i is the winner, it implies that $i < j$ for j such that $e_i(g) = e_j(g)$. Suppose that i and j are such that $e_i(g) = e_j(g)$, $i < j$ and $ij \notin g$. Both i and j have incentives to add the link ij since it does not change the winner but it decreases the duration for centralizing all the information. Suppose now i and j are such that $e_i(g) = e_j(g)$, $i < j$ and $ij \in g$. Then, there exists a single player k to whom the winner w has her maximal geodesic distance. Both w and k want to form a link since it decreases the duration, and moreover, this new link does not change who is the winner.

Part 4. It follows immediately from Proposition 5 and Proposition 8. □

Proof of Proposition 7. Internal stability.

In all networks $g \in V_i^{b_i}$, all players $j \in N$ have the same utility. For player i, we have $u_i = \delta^{b_i - 1}(1 + R)$, and for player $j \neq i$, we have $u_j = \delta^{b_i - 1}$. Condition 1 ensures that there is no other network in $V_i^{b_i}$ which has a the same duration, but in

which players have to pay for less links. So, for all $g \in V_i^{b_i}$ each player has to pay the same costs for links and gets the same payoff. Therefore, players are indifferent between the different networks in $V_i^{b_i}$ and have no incentive to start an improving path from $g \in V_i^{b_i}$ to $g' \in V_i^{b_i}$.

External stability.

(1) First, we show that for all $g' \notin \mathcal{V}_i^{b_i}$ there exists an improving path to some network $g \in V_i^{b_i}$ if and only if $\delta \leq (1/(1+R))^{1/(b_i - \tau(g'))}$ holds. By definition of b_i there can be no network g' with $W(g') = i$ and $\tau(g') < b_i$. So for $g' \notin \mathcal{V}_i^{b_i}$ we have to consider three cases: (i) $W(g') \neq i$ and $\tau(g') \geq \tau(g)$; (ii) $W(g') \neq i$ and $\tau(g') < \tau(g)$; (iii) $W(g') = i$ and $\tau(g') > \tau(g)$.

$\Rightarrow$ (i) There is an improving path from g' to g where first player i removes all her links. Then centralization cannot be successful so the players have an incentive to delete all links and end up in the empty network with $u_j = 0$ for all players j. From there the players move to $g \in V_i^{b_i}$, because the utility of each player is strictly positive if there is a winner. Player i has an incentive to start this improving path, because $u_i(g) > u_i(g')$. (ii) Analogue to (i), but player i only has an incentive to start the improving path if $u_i(g) = \delta^{b_i - 1}(1+R) > \delta^{\tau(g')-1} = u_i(g')$ holds. Rewriting this yields the condition on δ. (iii) Analogue to (i).

$\Leftarrow$ Assume there exists g' with $\tau(g') < b_i$ and $\delta \leq (1/(1+R))^{1/(b_i - \tau(g'))}$. It holds $g' \notin V_i^{b_i}$ and $W(g') \neq i$ by the definition of b_i. This implies $u_i(g') = \delta^{\tau(g')-1}$. Then player i has no incentive to initiate the improving path from g' to some $g \in V_i^{b_i}$, because $u_i(g) = \delta^{b_i - 1}(1+R) \leq \delta^{\tau(g')-1} = u_i(g')$. No other player $j \neq i$ has an incentive to deviate from g' to g, because $\tau(g') < \tau(g)$ implies $u_j(g') > u_j(g)$.

(2) Second, we show that for all $g' \in \mathcal{V}_i^{b_i} \setminus V_i^{b_i}$ there exists an improving path to some network $g \in V_i^{b_i}$. For all $g' \in \mathcal{V}_i^{b_i} \setminus V_i^{b_i}$, we have $W(g') = i$ and $\tau(g') = \tau(g)$ by definition.

$\Rightarrow$ Condition 2 implies that for every $g' \in \mathcal{V}_i^{b_i} \setminus V_i^{b_i}$ it holds that there exists a player j such that $d_j(g') \neq d_j(g)$. If $d_j(g') > d_j(g)$ holds, external stability follows immediately, because player j has an incentive to start an improving path from g' to g, because she then receives the same payoff, but has to pay for fewer links. On the other hand, if $d_j(g') < d_j(g)$ holds then there exists another player k with $d_k(g') > d_k(g)$ and she initiates the improving path. Player k always exists, because the networks g and g' are minimally connected and in each minimally connected network with N players, exactly $N-1$ links exist. If player j has fewer links, another player - namely player k - has to have more links.

$\Leftarrow$ The same arguments as for the proof for $g' \notin \mathcal{V}_i^{b_i}$ hold. □

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jebo.2020.05.026.

References

Ahlquist, J.S., Levi, M., 2011. Leadership: what it means, what it does, and what we want to know about it. Annu. Rev. Polit. Sci. 14, 1–24.
Bloch, F., Demange, G., Kranton, R., 2018. Rumors and social networks. Int. Econ. Rev. 59, 421–448.
Bonacich, P., 1990. Communication dilemmas in social networks: an experimental study. Am. Sociol. Rev. 55, 448–459.
Cooper, D., Kagel, J., 2005. Are two heads better than one? team versus individual play in signaling games. Am. Econ. Rev. 95 (3), 477–509.
Dewan, T., & Squintani, F. (2017). Leadership with trustworthy associates. Working paper Warwick University.
Galeotti, A., Ghiglino, C., Squintani, F., 2013. Strategic information transmission networks. J. Econ. Theory 148, 1751–1769.
Goyal, S., Joshi, S., 2003. Networks of collaboration in oligopoly. Games Econ. Behav. 43, 57–85.
Hagenbach, J., 2011. Centralizing information in networks. Games Econ. Behav. 72, 149–162.
Hendricks, K., Wilson, C., 1985. The war of attrition in discrete time. c.v. Starr Center for Applied Economics, New York University no. 85-32. Working papers
Herings, P.J.J., Mauleon, A., Vannetelbosch, V., 2009. Farsightedly stable networks. Games Econ. Behav. 67, 526–541.
Hermalin, B.E., 1998. Toward an economic theory of leadership: leading by example. Am. Econ. Rev. 88, 1188–1206.
Holmstrom, B., 1982. Moral hazard in teams. Bell J. Econ. 13 (2), 324–340.
Jackson, M., Wolinsky, A., 1996. A strategic model of social and economic networks. J. Econ. Theory 71, 44–74.
Jehiel, P., 1999. Information aggregation and communication in organizations. Manag. Sci. 45, 659–669.
Kocher, M., Strauss, S., Sutter, M., 2006. Individual or team decision-making - causes and consequences of self-selection. Games Econ. Behav. 56, 259–270.
Komai, M., Stegeman, M., Hermalin, B.E., 2007. Leadership and information. Am. Econ. Rev. 97, 944–947.
Kornhauser, L., Rubinstein, A., Wilson, C., 1989. Reputation and patience in the war of attrition. Economica 56, 15–24.
Mauleon, A., Sempere-Monerris, J.J., Vannetelbosch, V., 2014. Farsighted r&d networks. Econ. Lett. 125, 340–342.
Mauleon, A., Vannetelbosch, V., 2016. Network formation games. In: Galeotti, A., Bramoullé, Y., Rogers, B. (Eds.), The Oxford Handbook of the Economics of Networks. Oxford University Press, UK, pp. 167–190.
Radner, R., 1993. The organization of decentralized information processing. Econometrica 61, 1109–1146.
Ray, D., Vohra, R., 2015. The farsighted stable set. Econometrica 83 (3), 977–1011.