

On the usefulness of intraday price ranges to gauge liquidity in cap-based portfolios

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Abstract

We find that easy-to-observe price ranges are useful for estimating intraday liquidity. Following the literature on range-based volatility estimators, we go beyond the use of the closing price only and rely on the full range of prices. Based on high, low, opening, *and* closing (HLOC) prices, we show that a greater intensity in the price discovery process (as measured by the opening-close range) and a higher level of price uncertainty (as captured by the high-low range) lower ex-ante liquidity for small, mid, and large caps. Realized volatility (RV) fails to capture these effects. Although order books have become increasingly difficult to treat, there is some good news: it has never been easier to look at price ranges.

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1 Introduction

The quick and accurate estimation of liquidity has always been a particular challenge in finance. In early research, liquidity was reduced to immediacy, i.e. the immediate conversion of an asset into cash at the best available price (Demsetz 1968). As the literature on market microstructure expanded, a more comprehensive definition of liquidity was proposed. For example, Harris (2003) defines liquidity as ‘the ability to trade large size quickly, at low cost, when you want to trade’. In this definition, four liquidity dimensions can be identified: immediacy, width, depth and resiliency. Given the multi-dimensional feature of liquidity, it has become particularly challenging to obtain a reliable snapshot of the dynamics of liquidity. An impressive body of research has indeed demonstrated that liquidity dynamics can be complex, all the more so in stressful market conditions when there is strong price uncertainty.

In the extant literature on high-frequency data, price uncertainty is traditionally measured by realized volatility (RV) which is considered as a highly efficient volatility proxy. Realized volatility is nothing more than the sum of squared high-frequency returns over a given sampling period. These squared returns are based on closing prices (or, better, on closing mid-quotes, to avoid the bid-ask bounce bias). We combine the literature on range-based volatility estimators with the literature on intraday liquidity to show that the inclusion of the highest, lowest, opening and closing (HLOC) prices observed during the day gives additional information on liquidity dynamics in the order book, beyond what realized volatility brings as explanatory power.

Reinvestigating the relationship between liquidity and volatility is worthwhile for the obvious reason that if price uncertainty is misestimated, market participants cannot evaluate the (liquidity) risk premium correctly and make wrong investment decisions. It is more likely to be the case when price uncertainty is only based on changes in closing prices because the variation in closing prices ignores two sources of uncertainty: the variations between the highest and the lowest price, on the one hand, and the variation between the last closing price and the next opening price, on the other.¹ As suggested by Fiess and MacDonald (2002), HLOC prices are particularly appropriate to characterize price ranges, such as the magnitude of total price fluctuations or the occurrence of price gaps. The literature on range-based volatility estimators, that we briefly review in the next section, also confirms the usefulness of HLOC prices.

¹Uncertainty is not defined in the sense of Knight (1921), i.e. a risk that is not measurable, but by the amplitude of total price variation between the low, high, open, and and close prices.

Range-based price movements include: the Open-Close (OC) range, the High-Low (HL) range, and an interaction variable (OCHL), i.e. the ratio of the OC to the HL ranges. All else equal, a large OC range indicates that the price discovery process is driven by strong buying or selling pressure, so that the price moves towards a new fundamental value. The HL range measures the total price movement. For a given OC range, a large HL range indicates that price uncertainty has been strong, without being necessarily captured by volatility which relies on closing prices only. The OCHL ratio measures the *relative* amplitude of the price movement beyond the Open-Close range. When the OCHL ratio is close to zero, the price discovery process is very much polluted by uncertainty that prevails around the true stock value. We investigate whether these three variables contain more information than price returns or traditional volatility measures with respect to both the price discovery process and the behavior of market participants. OC and HL ranges may indeed convey additional information than the simple return, which is computed on the basis of closing prices only. The HL range is also a measure of total price variation and is not equivalent to realized volatility, which is again strictly based on closing prices or mid-quotes. Finally, the OCHL ratio compares the magnitude of the price discovery process (OC range) to the total price movements (HL range): the lower the OCHL ratio, the lower the proportion of total price uncertainty that can be justified by the price discovery process.

The objective of this paper is to revisit the liquidity-volatility relationship by quantifying the information content of these price ranges for estimating liquidity. By testing several hypotheses, we show that these range-based measures of price dynamics are significantly related to liquidity. We address two questions in particular: How is intraday liquidity affected by the price discovery process (i.e. the OC range), the total price variation (i.e. the HL range), and the proportion of total price uncertainty that can be justified by the price discovery process (i.e. the OCHL ratio)? And does realized volatility encompass these range-based measures of price dynamics? Liquidity in the limit order book is measured by several book-based and trade-based proxies. Ex-ante (or book-based) liquidity is measured by the relative spread, the depth, slope, and dispersion in the order book. Ex-post (or trade-based) liquidity is measured by the number of buyer and seller-initiated trades, the total number of trades, the average trade size, and the trading volume. As control variables, we also include dummy variables related to the occurrence of zero returns, the bullish or bearish movement during the interval, and the occurrence of price gaps.²

We use Euronext intraday data on 300 stocks belonging to three different market capital-

²Price gaps occur when the previous high (low) is below (above) the current low (high).

1 ization classes (i.e. small, mid and large caps). The literature has provided valuable evidence
2 showing that market capitalization has a direct impact on liquidity: Less liquid stocks often
3 belong to the small-cap segment of the market. For each of these three cap-based portfolios,
4 we study the sensitivity of each of the liquidity proxies to all the price movement variables
5 defined above. We estimate the regressions on 15-minute intervals by OLS with adjusted
6 standard errors. We further address this relationship by implementing the robust and median
7 regression techniques that deal with the presence of outliers in the sample. We also investigate
8 endogeneity issues by estimating a model of simultaneous equations. As a robustness check,
9 we use 10-minute and 20-minute time intervals.

HLOC price ranges are found to provide additional information on the behavior of buyers
10 and sellers and on the way liquidity evolves, even after adding realized volatility as explanatory
11 variable. The results suggest that, whatever the liquidity dimension, HLOC price movements
12 are informative to characterize liquidity dynamics. Positive changes in price ranges for both HL
13 and OC ranges are related to negative variations in book-based (or ex-ante) liquidity proxies.
14 All else equal, liquidity is further reduced when the OCHL ratio decreases, meaning that
15 liquidity further decreases when the price discovery process is not smooth and accompanied by
16 ‘excess’ uncertainty (i.e. by price variations that go beyond the OC range). Inversely, liquidity
17 improves when the total variation observed during the interval does not differ much from the
18 OC price range. In such a case, the price discovery is not plagued by ‘excess’ uncertainty.
19 We also confirm the positive link between trade-based proxies (i.e. trading activity) and price
20 uncertainty, in accordance with the literature on the volume-volatility relationship. We do
21 not insist much on these findings since they are well-known. All in all, we conclude that the
22 information content of HLOC price movements for intraday liquidity estimation is undeniably
23 substantial, including for the small-cap portfolio.

The remainder of the paper is organized as follows. In Section 2, we describe the sample
24 and the model specification. We also define all the liquidity and price movement variables
25 used in the empirical section. We report the empirical findings and the robustness checks in
26 Section 3. The final section concludes.

27 **2 Data and methodology**

28 We use tick-by-tick Euronext data for 61 trading days from February 1, 2006 to April 30, 2006.
29 The database of 300 stocks is divided into three cap-based portfolios. Large, mid, and small

1 caps respectively represent those companies with a market capitalization larger than EUR
 2 1 billion, between EUR 150 millions and EUR 1 billion, and below EUR 150 millions. We
 3 include 100 stocks in each category based on their market cap at the beginning of the sample.

The use of this dataset presents two key advantages. First, we have information on the full
 4 order book, including undisclosed data on hidden orders and market members' ID. By using
 5 market members' ID, we are able to disentangle buyer-initiated and seller-initiated trades
 6 without any error margin. In many market microstructure studies, the Lee and Ready (1991)
 7 algorithm is used to categorize buyer and seller-initiated trades. Although this algorithm has
 8 proved to be relatively efficient, misclassification still occurs. In our dataset, there is none since
 9 we know the order that initiates the transaction. Second, we avoid the volume shift and market
 10 fragmentation that have been occurring since the implementation phase of MiFID. As today's
 11 trading environment is much more decentralized than before, more recent datasets are often
 12 less representative and less reliable. In a number of recent studies, there is often insufficient
 13 information on the level of trading activity that prevails on the competing Multilateral Trading
 14 Facilities (MTFs) and dark pools.

In the following two sections, we give more details on the various proxies that we use to
 15 characterize liquidity. By drawing insights from the literature on range-based volatility esti-
 16 mators, we also motivate the use of range-based price movements to better estimate liquidity.
 17 Finally, we state six hypotheses that will be tested in the empirical Section.

18 2.1 Liquidity proxies

19 At the end of each trading interval, we calculate the relative (quoted) spread (RS) and the
 20 quantities outstanding at the five best limits (i.e. depth) for the ask side (QA), the bid side
 21 (QB), and the sum of the bid and ask (Q).

Finally, we use dispersion and slope measures. The dispersion measure indicates how
 22 remote limit orders are from one another. It is computed as follows:

$$Dispersion_{i,t} = \frac{1}{2} \left(\frac{\sum_{j=1}^5 w_{i,j,t}^{Bid} Dst_{i,j,t}^{Bid}}{\sum_{j=1}^5 w_{i,j,t}^{Bid}} + \frac{\sum_{j=1}^5 w_{i,j,t}^{Ask} Dst_{i,j,t}^{Ask}}{\sum_{j=1}^5 w_{i,j,t}^{Ask}} \right), \quad (2.1)$$

where $Dst_{i,j,t}^{Bid} = (Price_{i,j-1,t}^{Bid} - Price_{i,j,t}^{Bid})$ and $Dst_{i,j,t}^{Ask} = (Price_{i,j,t}^{Ask} - Price_{i,j-1,t}^{Ask})$ for secu-
 23 rity i , interval t , and the j^{th} price limit. For the distance at the first best limit, the midquote

1 is used instead of the price at the next limit. $w_{i,j,t}$ are the quantities displayed at the offer or
 2 bid size, normalized by the total depth at the five best limits. As Kang and Yeo (2008) outline,
 3 dispersion is small under fierce competition since each trader wants to gain price priority. By
 4 gaining price priority, market members reduce the spread and the distance between each price
 5 limit, increasing the density of the order book.

As defined in Næs and Skjeltorp (2006), the slope measure is computed by averaging the
 6 price elasticities of quantities available at the five best quotes:

$$Slope_{i,t} = \frac{DE_{i,t} + SE_{i,t}}{2}, \quad (2.2)$$

where $DE_{i,t}$ and $SE_{i,t}$ are respectively the demand and supply elasticities. They are computed
 7 as follows:

$$DE_{i,t} = \frac{1}{5} \left(\frac{v_1^B}{|p_1^B/p_0 - 1|} + \sum_{\tau=1}^4 \frac{v_{\tau+1}^B/v_\tau^B - 1}{|p_{\tau+1}^B/p_\tau^B - 1|} \right), \quad (2.3)$$

$$SE_{i,t} = \frac{1}{5} \left(\frac{v_1^A}{|p_1^A/p_0 - 1|} + \sum_{\tau=1}^4 \frac{v_{\tau+1}^A/v_\tau^A - 1}{|p_{\tau+1}^A/p_\tau^A - 1|} \right). \quad (2.4)$$

p_τ^B and p_τ^A are the bid and ask prices displayed at quote τ . p_0 denotes the quoted midpoint.
 8 Finally, v_τ^B and v_τ^A are the natural logarithm of accumulated total depth at limit τ for the bid
 9 and the ask respectively.³

A steep slope indicates that volumes in the order book are concentrated around a given
 10 price limit: because traders agree about the value of the security, accumulated depth quickly
 11 increases when a small price variation occurs (high elasticity). A gentle slope suggests that vol-
 12 umes in the order book are not aggregated at a given limit: accumulated depth only increases
 13 for larger price variations because traders have different views on the price of the security
 14 (low elasticity). The slope is therefore positively related to liquidity and negatively correlated
 15 with volatility, as outlined by Glosten (1994), Goldstein and Kavajecz (2004) and Næs and
 16 Skjeltorp (2006).

We also analyze trade-based measures (i.e. ex-post liquidity proxies). We first look at the
 17 number of buyer and seller-initiated trades, NB and NS respectively. Using our database,

³By accumulated, we mean the sum of the quantities outstanding at that limit as well as the sum of all quantities outstanding at each better quote.

1 we find the sign of the transaction by matching the orders that generate the trade and by
2 comparing the submission time of the orders. With this method, all trades are correctly
3 categorized without any error margin. We also analyze the total number of trades ($Ntrades$),
4 the average trade size (ATS), and the total volume of traded shares ($Volume$).

Descriptive statistics for both book-based and trade-based liquidity proxies defined above
5 are given in the first panel of Table 1.

As expected, the distributions of the liquidity proxies are heavily skewed and leptokurtic.
6 Interestingly, we observe significant differences between the cap-based portfolios, suggesting
7 that the characterization of liquidity may be different across these subsamples. In general, both
8 skewness and kurtosis decrease when we move to large caps. The average percentage change
9 for each liquidity proxy is however very low. This comes from the narrow time intervals that we
10 use (15 minutes). For instance, dispersion changes on average by 1.68% for small caps, 8.21%
11 for mid caps and 4.57% for large caps. Trading activity measures display negative signs for the
12 first subsample, -0.1119 and -0.1143 for ΔNB or ΔNS , respectively. This is the consequence
13 of some zero volume intervals that display a ΔNB or ΔNS equal to -1.

14 2.2 Price Ranges

15 In this Section, we motivate the use of price ranges in the estimation of liquidity by drawing
16 insights from the literature on range-based volatility estimators. We then define the selected
17 price ranges and state the hypotheses that will be tested in the empirical analysis. Let H_t , L_t ,
18 O_t , and C_t , denote, respectively, the high, low, opening, and closing prices at time t . Let us
19 consider the following basic stochastic differential equation to introduce the key range-based
20 volatility estimators.

$$dS_t = \mu S_t dt + \sigma S_t dW_t, \quad 0 \leq t \leq T \quad (2.5)$$

21 where S_t is the asset price at time t , μ is the instantaneous drift, σ is volatility, and W_t is a
22 standard Brownian motion.

Table 1: Descriptive statistics

	Small Caps				Mid Caps				Large Caps			
	Mean	STD	Skewness	Kurtosis	Mean	STD	Skewness	Kurtosis	Mean	STD	Skewness	Kurtosis
$\Delta Dispersion$	0.0168	0.25	33.777	3593.65	0.0821	25.63	557.47	310832	0.0457	0.42	34.12	4031.1
ΔRS	0.173	1.47	21.754	971.18	0.2923	2.16	98.54	27449.9	0.2671	1.4	51.295	10777.63
$\Delta Slope$	0.1067	0.84	13.23	336.67	0.1768	1.04	9.431	170.28	0.2009	0.92	6.281	139.17
ΔQB	0.0889	15.61	455.764	229267.8	1.2765	658.4	557.278	310684.6	0.1416	3.38	327.787	128235.5
ΔQA	0.0657	7.13	313.611	116066	1.245	610.29	553.733	307922.4	0.1383	4.69	503.639	268874.2
ΔQ	0.0543	9.17	373.24	163288.7	1.212	634.56	556.677	310226.4	0.0715	2.49	417.121	198502.2
$\Delta Ntrades$	0.0809	2.62	21.106	1089.19	0.3168	2.48	28.606	3535.27	0.2866	1.7	25.564	2085.72
ΔNB	-0.1119	2.72	36.546	2822.02	0.1666	2.37	12.552	405.49	0.4431	2.27	15.048	588.61
ΔNS	-0.1143	2.46	24.225	1477.76	0.1637	2.56	54.463	9487.19	0.3957	2.02	13.056	489.82
$\Delta Volume$	10.8568	854.78	222.373	57637.53	5.5898	198.91	158.774	30340.2	1.9077	46.97	113.547	17665.67
ΔATS	6.1549	442.76	185.764	38062.76	1.9356	34.23	86.631	10257.07	0.5125	10.52	105.565	15265.35
ΔOC_t	-0.0832	2.09343	16.5353	530.26	0.17121	2.73511	14.6255	575.32	0.34819	2.19259	7.2744	142.79
$\Delta OCHL_t$	0.03881	1.4436	19.0019	872.42	0.17727	1.92326	15.3708	602.45	0.14639	1.38583	5.6033	78.22
ΔHL_t	0.05388	2.35805	16.129	438.41	0.28748	3.16102	30.1193	2588.7	0.29835	1.69443	26.0014	2738.66
ΔRV_t	0.28273	2.91399	16.3488	572.21	0.47407	3.06253	23.6848	1899.85	0.22444	1.42489	19.7264	1134.58

This table presents the descriptive statistics for *changes* in all the book-based, trade-based, and price-based variables used in the empirical analysis. The variables are presented in their first differences for each cap-based portfolio. QB and QA are the sums of the quantities outstanding at the five best limits, on bid and ask sides, respectively. Q is the sum of the two. $Slope$ denotes the slope of the book computed following Næs and Skjeltorp (2006) while $Dispersion$ stands for the dispersion measure computed following Kang and Yeo (2008). RS denotes the relative spread. NB and NS respectively denote the number of buy and sell trades. $Ntrades$ is the total number of trades and ATS is the average trade size of these trades. $OCHL_t = \frac{|Close_t - Open_t|}{High_t - Low_t}$ is the percentage of net variation on total fluctuation. HL_t denotes the High-Low range of the interval. The RV_t variable is the realized volatility.

It can be shown that the traditional, or close-to-close, estimator of variance for a driftless security (i.e. $\mu = \sigma^2/2$) is given by:

$$\sigma_{cc}^2 = \frac{1}{n} \sum_{t=1}^n \left(\ln \frac{C_t}{C_{t-1}} \right)^2, \quad n \geq 1 \quad (2.6)$$

The intraday version that we use in the empirical analysis is the realized volatility which is equal to the sum of the corresponding intradaily squared returns based on closing prices (or midpoints):

$$RV_t(\Delta) \equiv \sum_{j=1}^n (r_{t-j+1}(\Delta))^2 \equiv \sum_{j=1}^n \left(\ln \frac{C(t-j+1)}{C(t-j\Delta)} \right)^2, \quad n \geq 1 \quad (2.7)$$

where n is the number of intraday periods used to compute realized volatility at time t and $r_t(\Delta)$ is the discretely sampled Δ -period return at time t .

The first range-based volatility estimator was introduced by Parkinson (1980) who calls it the ‘extreme-value’ estimator because it is based on the highest and lowest prices observed during a time interval. For a driftless security, it is given by:

$$\sigma_P^2 = \frac{1}{4n \ln 2} \sum_{t=1}^n \left(\ln \frac{H_t}{L_t} \right)^2, \quad n \geq 1 \quad (2.8)$$

It can be shown that if the stock price follows Equation (2.6) with $\mu = \sigma^2/2$, then σ_P^2 is about five times more efficient than the traditional estimator. Under these assumptions, Garman and Klass (1980) construct a minimum variance unbiased estimator that simultaneously uses the opening, closing, high, and low prices:

$$\sigma_{GK}^2 = \frac{1}{n} \sum_{t=1}^n \left(0.511 \left(\ln \frac{H_t}{L_t} \right)^2 - 0.019 \left[\ln \left(\frac{C_t}{O_t} \right) \ln \left(\frac{H_t L_t}{O_t^2} \right) - 2 \ln \left(\frac{H_t}{L_t} \right) \ln \left(\frac{L_t}{O_t} \right) \right] - 0.383 \left(\ln \frac{C_t}{O_t} \right)^2 \right), \quad n \geq 1 \quad (2.9)$$

This estimator is theoretically more than 7 times more efficient than the traditional estimator. The absence of drift is relaxed by Rogers and Satchell (1991) whose estimator is independent of the drift. They also propose an adjustment to take into account the fact that the stock price may not be continuously monitored. Finally, Yang and Zhang (2000) also pro-

1 poses a minimum-variance unbiased estimator that is independent of the drift. Their estimator
2 does not rely on low and high prices but includes overnight variance, contrary to the previous
3 estimators. More recently, price ranges have also been shown to be useful in the estimation of
4 volatility (see Alizadeh et al. 2002, Bali and Weinbaum 2005, or Shu and Zhang 2006, among
5 others).

Based on these findings, we attempt to improve the estimation of liquidity by extracting as
6 much information as possible from these price ranges, while still considering realized volatility
7 as a key proxy for price uncertainty. For each of the 300 stocks, we rebuild HLOC prices
8 over the whole sample period. As tick data are not appropriate for HLOC analysis, we use
9 15-minute intervals, leading to 34 intervals per day.⁴ HLOC price dynamics are captured in
10 different ways.

Firstly, we use the absolute value of the price variation during interval t , i.e. $OC_t =$
11 $\frac{|Close_t - Open_t|}{RangeMid_t}$, where $RangeMid_t = \frac{High_t + Low_t}{2}$. We take the absolute value in order to control
12 for the bullish or bearish pressure that occurs during the interval. As outlined by Chakravarty
13 and Holden (1995), and Bloomfield et al. (2005), a small difference between opening and closing
14 prices may be related to a more liquid order book. When the fundamental value of the stock
15 is located inside the spread, informed traders act less aggressively, take less liquidity in the
16 book, and may even become dealers. As informed trading and *ex-ante* liquidity demand are
17 expected to be lower when the OC range is small, we expect OC_t to be negatively associated
18 with liquidity. Therefore, all the liquidity proxies should exhibit a negative sign, except for
19 the spread and dispersion variables.⁵ The first hypothesis that we test is stated as follows:

20 *H1: The wider the OC range implied by the price discovery process, the lower the liquidity.*

Secondly, we include the High-Low range. It is computed as $HL_t = \frac{High_t - Low_t}{RangeMid_t}$, where
21 $RangeMid_t = \frac{High_t + Low_t}{2}$. The HL range measures the total price movement. For a given
22 OC range, a large HL range indicates that price uncertainty has been high, without being
23 necessarily captured by volatility which relies on closing prices only.⁶ We therefore expect
24 HL_t and liquidity to be negatively related.

⁴This interval length is often used in the literature, e.g. Fiess and MacDonald (2002). In the robustness check section, we investigate whether our results are sensitive to the interval length and we apply the same methodology on 10-minute and 20-minute intervals.

⁵The relationship is expected to be positive in these two cases. The larger the open-close range, the larger the expected spread, and the larger the expected dispersion.

⁶The negative relationship between volatility and liquidity is documented in Benston and Hagerman (1974), Stoll (1978b), Stoll (1978a), Amihud and Mendelson (1980), Ho and Stoll (1981), Copeland and Galai (1983), Admati and Pfleiderer (1988), Foster and Viswanathan (1990), Pastor and Stambaugh (2003), and Harris (2003), among others

1 The second hypothesis is formulated as follows:

2 *H2: The wider the HL range (for a given OC range), the more uncertain the price discovery process, and the lower the liquidity.*

3 Thirdly, we include an interaction term between the two previous variables by dividing the
4 absolute value of the OC range by the HL range, i.e. $OCHL_t = \frac{|Close_t - Open_t|}{High_t - Low_t}$. This measure
5 is equal to 1 when there is no price variation beyond the opening and closing prices. In this
6 case, the two ranges coincide and the price discovery process is rather smooth, unidirectional,
7 and not much affected by uncertainty. All else equal, liquidity is expected to improve in this
8 environment. It is the opposite when the ratio is close to 0. When the price discovery process
9 implies very large fluctuations relative to the open-close price change observed during the
10 interval, liquidity is expected to deteriorate. All else equal, a significantly positive coefficient
11 for the interaction variable would indicate that liquidity increases (decreases) when the OCHL
12 ratio increases (decreases). While controlling for the variation in both the OC and HL ranges,
the OCHL variable is therefore positively related to liquidity. The third hypothesis is:

13 *H3: The lower the OCHL ratio (for a given OC and HL ranges), the higher the negative impact of price uncertainty on liquidity.*

14 Descriptive statistics for the price movement variables are given in the second panel of
15 Table 1. As for liquidity proxies, these descriptive statistics clearly outline strong differences
between the different quartiles.

16 Finally, we add four dummy variables and test three additional hypotheses. The “Zero
17 Return” dummy (ZR_t) controls for the presence of zero returns. It is equal to 1 when there
18 is a zero return and 0 otherwise. On the one hand, Lesmond et al. (1999) argue that zero
19 returns are associated with high transaction costs: Informed traders do not trade because the
20 value of the new information does not exceed the cost to trading. On the other hand, zero
21 returns may characterize moments in time when there is precisely less informed trading and
22 more liquidity, as argued by Chakravarty and Holden (1995), and Bloomfield et al. (2005). If
23 informed traders do not take liquidity to move price towards their fundamental value, they
24 behave as dealers, earn the spread, and supply liquidity, which in turn produces a zero return.
We can state the fourth hypothesis as follows.

25 *H4: When there is a zero return, liquidity is higher provided that informed traders supply liquidity.*

The “Upward Window” dummy (UW_t) controls for the presence of a price gap between

1 the highest price observed during interval $t - 1$ and the lowest price observed during interval t .
2 The dummy is equal to 1 when there is an upward window, and 0 otherwise. The “Downward
3 Window” dummy (DW_t) controls for the occurrence of a price gap between the lowest price
4 observed during interval $t - 1$ and the highest price observed during interval t . The dummy
5 is equal to 1 in case of a downward window and 0 otherwise. The “Upward Window” and
6 “Downward Window” dummies are expected to be negatively related to liquidity as the for-
7 mation of a window is typically linked to an illiquid state of the order book, as outlined by
8 Boudt and Petitjean (2014) among others. However, they point to a rather strong resilience
9 in liquidity after the occurrence of a gap.

10 *H5: When there has been a price gap, liquidity is lower provided that resilience remains limited.*

The last dummy is a “Direction” dummy (D_t) which is equal to 1 when there is a positive
11 price movement and 0 otherwise. This dummy is added to control for the link between returns
12 and liquidity. In comparison with negative returns, we expect positive returns to lead to a
13 decrease in both depth and order imbalances.⁷ In the case of a positive return, QB becomes
14 higher since many market participants want to participate to the upward discovery process led
15 by informed traders. QA drops as sellers do not want to trade with better informed traders.
16 Furthermore, since liquidity is abundant on the other (buy) side, sellers will submit aggressive
17 marketable orders that generate more seller-initiated trades, i.e. NS is higher and NB is
18 lower.

19 *H6: When there is an upward price movement, both depth and order imbalances decrease.*

20 2.3 Model specification

21 The fixed-effect panel specification that we estimate is the following:

$$\begin{aligned} \Delta L_{i,t} = & \alpha_0 + \alpha_1 \Delta OC_{i,t} + \alpha_2 \Delta OCHL_{i,t} + \alpha_3 \Delta HL_{i,t} \\ & + \alpha_4 UW_{i,t} + \alpha_5 DW_{i,t} + \alpha_6 D_{i,t} + \sum_{i=2}^{100} \beta_i S_i + \nu_{i,t} \end{aligned} \quad (2.10)$$

22 where $L_{i,t}$ is the liquidity proxy measured at interval t for stock i , and Δ indicates the
proportional change of the variable, i.e. $\Delta L_{i,t} = \frac{L_{i,t} - L_{i,t-1}}{L_{i,t-1}}$. All the explanatory variables are

⁷Order imbalance is equal to the number of buyer minus seller-initiated trades, i.e. $NB - NS$. Depth imbalance means the quantities outstanding at the five best limits for the ask side (QA) minus the quantities outstanding at the five best limits for the bid side (QB).

1 defined in the previous section. The $n - 1$ S_i dummies denote the stocks' fixed effects that are
2 present in each portfolio. As outlined in Chordia et al. (2000), we use first differences in order
3 to deal with econometric issues that affect liquidity variables, such as high persistence and/or
4 non-stationarity.

We first estimate the model by using the clustered standard errors approach that is de-
5 scribed in Petersen (2009) to correct for the bias in the OLS standard errors that arises for
6 panel estimation. The standard errors produced by this method are robust to within cluster
7 correlation. As outlined by Petersen (2009), this method produces unbiased standard errors
8 when there is a firm-specific effect and is superior to White, Newey-West, and Fama-MacBeth
9 correction methods in such a case. We split the sample by market capitalization and run
10 the analysis separately on each of the three portfolios. We also employ robust and median
11 regression techniques in order to deal with some of the limits of the traditional OLS technique,
12 among which non-normality.

In order to control for volatility in the analysis, we run the same regressions with an
13 additional variable, RV_t , which is the realized volatility. Widely recognized as an efficient
14 estimate of volatility, we compute the realized volatility by aggregating 1-minute returns over
15 15 minutes. The objective of this specification is to check whether HLOC dynamics remain
16 significant when the effect of volatility on liquidity is controlled for. This new model is specified
17 as follows:

$$\begin{aligned} \Delta L_{i,t} = & \alpha_0 + \alpha_1 \Delta OC_{i,t} + \alpha_2 \Delta OCHL_{i,t} + \alpha_3 \Delta HL_{i,t} \\ & + \alpha_4 UW_{i,t} + \alpha_5 DW_{i,t} + \alpha_6 D_{i,t} + \alpha_7 RV_{i,t} + \sum_{i=2}^{100} \beta_i S_i + \nu_{i,t} \end{aligned} \quad (2.11)$$

We also apply a model of simultaneous equations to control for the endogeneity issue
18 between liquidity and price movements. Price movements lead to changes in liquidity, but
19 the opposite may also be true. Prices are also expected to move after changes in the state of
20 the limit order book. Prices are indeed likely to be affected by the (dis)equilibrium between
21 informed and noise traders or between liquidity providers and liquidity takers. The following
22 system of simultaneous equations is estimated for each cap-based portfolio in order to check
23 whether the panel-based results still hold when endogeneity is controlled for.

$$\left\{ \begin{array}{l} \Delta L_{i,t} = \alpha_{10} + \alpha_{11}\Delta OC_{i,t} + \alpha_{12}\Delta HL_{i,t} + \alpha_{13}\Delta OCHL_{i,t} \\ \quad + \alpha_{14}UW_{i,t} + \alpha_{15}DW_{i,t} + \alpha_{16}D_{i,t} + \sum_{i=2}^{100} \beta_i S_i + \nu_{i,t}, \\ \Delta OC_{i,t} = \alpha_{20} + \alpha_{21}\Delta L_{i,t} + \alpha_{22}\Delta HL_{i,t} + \alpha_{23}\Delta OCHL_{i,t} \\ \quad + \alpha_{24}UW_{i,t} + \alpha_{25}DW_{i,t} + \alpha_{26}D_{i,t} + \sum_{i=2}^{100} \beta_i S_i + \nu_{i,t}, \\ \Delta HL_{i,t} = \alpha_{30} + \alpha_{31}\Delta OC_{i,t} + \alpha_{32}\Delta L_{i,t} + \alpha_{33}\Delta OCHL_{i,t} \\ \quad + \alpha_{34}UW_{i,t} + \alpha_{35}DW_{i,t} + \alpha_{36}D_{i,t} + \sum_{i=2}^{100} \beta_i S_i + \nu_{i,t}, \\ \Delta OCHL_{i,t} = \alpha_{40} + \alpha_{41}\Delta OC_{i,t} + \alpha_{42}\Delta HL_{i,t} + \alpha_{43}\Delta L_{i,t} \\ \quad + \alpha_{44}UW_{i,t} + \alpha_{45}DW_{i,t} + \alpha_{46}D_{i,t} + \sum_{i=2}^{100} \beta_i S_i + \nu_{i,t}, \end{array} \right.$$

where $L_{i,t}$ is the analyzed liquidity proxy for stock i at time t , S_i are fixed-effect dummies
1 and ν_{it} is a white noise disturbance term. The price movement variables in these equations
2 have been presented in Section 2.2. The inclusion of Equations 2 to 4 controls for the presence
3 of endogeneity in the contemporaneous relationship. The 2SLS estimation method is used and
4 the instrumental variables are the lagged values of the right-hand side endogenous variables
5 in each equation (not shown to keep the equations readable).

As multicollinearity may be an issue, Table 2 presents Pearson correlation coefficients
6 between the variables included in these specifications. We observe that correlation never goes
7 above 0.60.

Table 2: Correlation Matrix - first differences

	ΔOC_t	$\Delta OCHL_t$	ΔHL_t	ΔRV_t
ΔOC_t	1.00			
$\Delta OCHL_t$	0.50	1.00		
ΔHL_t	0.60	-0.04	1.00	
ΔRV_t	0.37	0.01	0.51	1.00

This table presents the correlation between the first differences of the numeric variables of the model. OC_t denotes the relative absolute value of the net movement, $OCHL_t$ denotes the ratio of net movement to total fluctuation and HL_t is the High-Low range. RV_t is the realized volatility.

3 Empirical analysis

Before running the panel regression models in the next section, we conduct an explanatory and non-parametric study of the HLOC price movement variables by zooming on the first, fifth and tenth decile of their distribution. The first decile includes the narrowest price movements while the tenth decile includes the largest price movements. For each of these three deciles, we compute all the liquidity proxies defined in the previous section. We then conduct non parametric median tests to assess whether the three subsamples display significantly different medians for each liquidity proxy. Under the null hypothesis, medians are equal. The goal is to compare the state of liquidity when HLOC price variations are significantly different. We carry out this analysis for the three portfolios of small, mid and large caps.

Table 3 reports the median scores. All the results of the median tests are highly significant at a 1% significance level (which we do not indicate in the table to avoid redundancy). Clearly, liquidity deteriorates as price ranges increase. In general, the median scores of (il)liquidity proxies drop (rise) when the price range becomes larger. As an illustration, we show in Figure 1 the change in median scores for the relative spread across the three portfolios and the three deciles.

For large caps, the relative spread more than doubles when narrow and wide HLOC price variations are compared. A wide OC range indicates that the price discovery process has led to a new market value that is significantly different from the market value that prevailed before. When compared to a narrow OC range, there is a significant widening of the spread for large caps. The same result is found when we look at the effect of total price variation on the relative spread. In comparison to narrow HL ranges, wide HL ranges increase the relative spread significantly.

The dispersion for large caps is very much impacted by the intensity of the price discovery process. When narrow and wide OC ranges are compared, the dispersion goes from 0.1986 to 0.5925. Slope for large caps follow the opposite pattern than the spread, confirming that liquidity worsens when HLOC price variations are wider. We observe that depth is much less affected by variations in the HLOC prices than the previous (il)liquidity proxies. It decreases with the intensity of the price discovery process but slightly rises with total price variation. Regarding the number of trades and the average trade size, there is a sharp and undeniable rise when total price variation increases, which explains the rise in (total) volume.

For mid caps, the overall picture is very similar: liquidity drops when HLOC price variations
1 become wider.

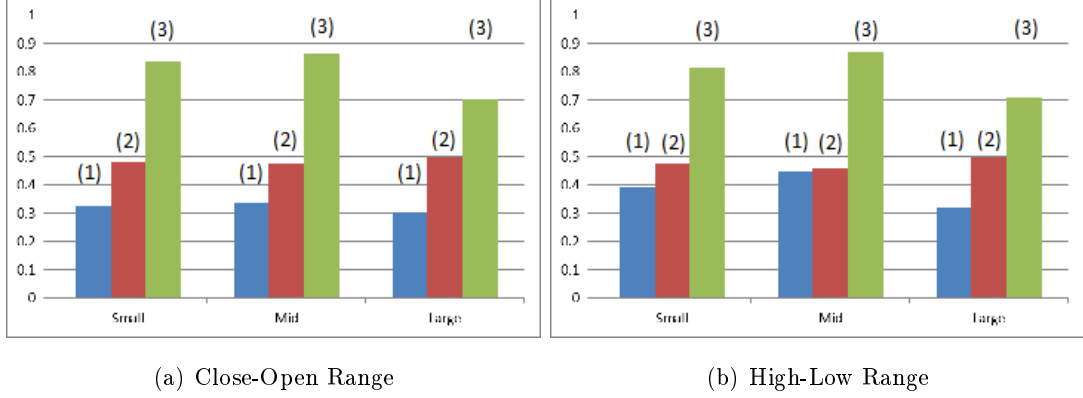
For small caps, the relative spread, dispersion, and slope point to a sharp deterioration in
2 liquidity when HLOC price changes widen. This rise in illiquidity is accompanied by a surge
3 in trading volume (due to both the number of trades and the average trade size), which is
4 especially noticeable when changes in total price variation are considered.

Table 3: A first look at liquidity and HLOC price movements

Range size		OC_t			HL_t		
Proxy		Small	Mid	Large	Small	Mid	Large
RS	Narrow	0.3285	0.3397	0.3042	0.3947	0.4447	0.3174
	Medium	0.4792	0.4745	0.4994	0.4735	0.4606	0.4968
	Wide	0.8376	0.8643	0.7001	0.8167	0.8704	0.7081
Dispersion	Narrow	0.3552	0.3935	0.1986	0.8063	0.5573	0.5452
	Medium	0.4887	0.5003	0.5261	0.4826	0.4897	0.4821
	Wide	0.5354	0.6038	0.5925	0.3328	0.5248	0.5982
Slope	Narrow	0.6588	0.6627	0.7008	0.6033	0.5425	0.6699
	Medium	0.5216	0.5218	0.5012	0.5262	0.5359	0.5066
	Wide	0.1687	0.1631	0.2894	0.1875	0.1707	0.2773
Q	Narrow	0.1483	0.4730	0.5071	0.1336	0.3958	0.3931
	Medium	0.5134	0.5056	0.5066	0.5150	0.5073	0.5231
	Wide	0.7443	0.4819	0.4399	0.7463	0.5455	0.4218
Ntrades	Narrow	0.4589	0.4775	0.4507	0.3488	0.2311	0.1640
	Medium	0.5062	0.5012	0.4894	0.5051	0.5162	0.5147
	Wide	0.4911	0.5128	0.6340	0.6106	0.6392	0.7183
ATS	Narrow	0.1454	0.4318	0.5753	0.1383	0.3642	0.3372
	Medium	0.5062	0.4968	0.4874	0.4954	0.4921	0.5160
	Wide	0.7864	0.6046	0.5259	0.7842	0.6837	0.5320
Volume	Narrow	0.3602	0.4688	0.4901	0.3225	0.2759	0.1974
	Medium	0.5066	0.4990	0.4883	0.5056	0.5083	0.5198
	Wide	0.5867	0.5394	0.6036	0.6327	0.6581	0.6446

This table presents the outcomes of the non-parametric median test for the three market capitalization portfolios: Small, mid, and large. For each variable, Narrow, Medium, and Wide respectively denote the first 10 percentiles, the mid of the sample, and the last 10 percentiles of the subsamples based on the range size. QB and QA are the sums of the quantities outstanding at the five best limits, on bid and ask sides, respectively. Q is the sum of the two. *Slope* denotes the slope of the book computed following Næs and Skjeltorp (2006) while *Dispersion* stands for the dispersion measure computed following Kang and Yeo (2008). *RS* denotes the relative spread. *NB* and *NS* respectively denote the number of buy and sell trades. *Ntrades* is the total number of trades and *ATS* is the average trade size of these trades.

Figure 1: HLOC Price Movements and Relative Spread



This figure presents the mean score of the median tests for both Close-Open and High-Low ranges for the relative spread. For each capitalization group (Small, Mid and Large), the three bars represent from the left to the right, the mean scores of the first decile (1), the mid of the sample (2) and the last decile (3).

3.1 General panel regressions

For each liquidity proxy defined in Section 2.1, we further study the relationship between liquidity and price dynamics by estimating the fixed-effect panel regression model defined in Section 2.3. The main advantage of this parametric approach is to allow for the inclusion of control variables in the analysis, which was not possible in the explanatory framework described in the previous section.

We validate our three hypotheses on the relationship between HLOC price dynamics and liquidity. The clustered OLS results indeed suggest that there is a statistically relevant relationship between the two (see Table 4). Although the magnitude of this relationship somewhat varies among the liquidity dimensions, the results are very similar and only differ in the significance level in most cases.

In the discussion below, we check whether each of the six hypotheses stated in Section 2.2 is verified or not.

Table 4: Regressions with clustered standard errors

ΔL_t	ΔOC_t	ΔHL_t	$\Delta OCHL_t$	ZR_t	UW_t	DW_t	D_t
Panel A : Large Caps							
ΔRS	0.037***	0.142***	-0.043***	-0.111***	0.109***	-0.015	0.033***
$\Delta Dispersion$	0.015***	0.026***	-0.015***	-0.018***	0.093***	0.054***	0.003
$\Delta Slope$	-0.023***	-0.075***	0.057***	-0.023**	0.124***	0.197***	-0.021***
ΔQB	0.000	-0.005	0.008*	-0.071***	-0.076***	0.012	0.034***
ΔQA	0.002	-0.006**	0.005	-0.025***	-0.031	-0.097***	-0.055***
ΔQ	-0.001	-0.007***	0.005**	-0.020***	-0.072***	-0.057***	-0.010***
ΔNB	0.022	0.329***	-0.075***	0.248***	0.068	-0.050	-0.538***
ΔNS	-0.004	0.295***	-0.054***	-0.270***	-0.080*	-0.018	0.469***
$\Delta Ntrades$	0.019*	0.246***	-0.052***	-0.026**	0.083*	-0.002	-0.009
ΔATS	-0.009*	0.092***	0.002	0.090	-0.049	0.057	-0.022
$\Delta Volume$	-0.040	0.632***	-0.028	0.183	0.304	-0.062	-0.064
Panel B : Mid Caps							
ΔRS	0.070***	0.066**	-0.061***	-0.001	0.029	0.108*	0.054**
$\Delta Dispersion$	0.037	0.009*	-0.043	0.003	-0.216	-0.033	-0.407
$\Delta Slope$	-0.021***	-0.009	0.041***	-0.020	0.233***	0.079**	-0.024**
ΔQB	0.004	0.003	-0.001	-0.052***	-0.005	0.093***	0.082***
ΔQA	0.005*	0.001	0.006	0.048***	0.163***	-0.016	-0.120***
ΔQ	0.002	0.001	0.003	0.008	0.041***	0.015	-0.021***
ΔNB	0.089***	0.056	-0.088***	0.306***	0.998***	-0.513***	-0.584***
ΔNS	0.027*	0.134***	-0.054***	-0.266***	-0.400***	0.729***	0.610***
$\Delta Ntrades$	0.023**	0.119***	-0.045***	0.040	0.376***	0.199***	-0.005
ΔATS	-0.000	0.010	-0.034***	-0.149	-0.034	0.112	0.084
$\Delta Volume$	0.053*	0.249***	-0.145***	-0.042	0.553***	0.237	-0.027
Panel C : Small Caps							
ΔRS	-0.001	0.179***	0.011	0.246***	-0.003	0.138*	0.056***
$\Delta Dispersion$	0.009**	0.016***	-0.002	0.031***	0.079***	0.085***	0.005*
$\Delta Slope$	0.001	-0.026***	0.034***	-0.011	0.163***	0.030	0.004
ΔQB	0.006	0.009**	0.000	-0.001	0.002	0.099***	0.075***
ΔQA	0.006**	0.004**	-0.000	0.087***	0.116***	-0.036*	-0.080***
ΔQ	0.004**	0.004**	-0.001	0.039***	0.022	0.011	-0.010***
ΔNB	0.143***	0.297**	-0.068***	0.637***	2.648***	-0.625***	-0.524***
ΔNS	0.015	0.195***	-0.014	0.109	-0.013	1.642***	0.513***
$\Delta Ntrades$	0.076***	0.199***	-0.043***	0.344***	1.161***	0.612***	-0.039
ΔATS	-0.004	-0.012	-0.025**	0.155	-0.091	0.001	-0.430
$\Delta Volume$	0.184***	0.271***	-0.129***	1.503**	1.839***	0.727**	-0.285**

This table presents the different models that are estimated with clustered standard errors. Panels A, B and C display the estimates for small, mid and large caps, respectively. The first column, L_t , indicates the dependant variable and the following columns are the exogenous variables. QB and QA are the sums of the quantities outstanding at the five best limits, on bid and ask sides, respectively. $Q5$ is the sum of the two. $Slope$ denotes the slope of the book computed following Næs and Skjeltorp (2006) while $Dispersion$ stands for the dispersion measure computed following Kang and Yeo (2008). RS denotes the relative spread. NB and NS respectively denote the number of buy and sell trades. $Ntrades$ is the total number of trades and ATS is the average trade size of these trades. $OC_t = \frac{|Close_t - Open_t|}{RangeMid_t}$ is the absolute value of the net movement. $OCHL_t = \frac{|Close_t - Open_t|}{High_t - Low_t}$ is the percentage of net variation on total fluctuation. HL_t denotes the High-Low range of the interval. ZR_t is a dummy variable that control for the presence of zero returns. UW_t is equal to 1 when an upward window occurs on the chart and 0 otherwise, while DW_t is equal to 1 when a downward window appears and 0 otherwise. The D_t dummy equals 1 when the net price movement is positive and 0 otherwise. The fixed effect dummies are not presented here.

1 *H1: The wider the OC range implied by the price discovery process, the lower the liquidity.*

Liquidity drops when the OC range increases, validating the first hypothesis. When significant, the relative spread, dispersion, and slope estimates have all the expected signs. The large cap portfolio exhibits the most robust results: All three proxies are significant. The relative spread for mid caps is particularly affected by a rise in the OC range. Interestingly, liquidity in small caps seems to be less sensitive to the intensity of the price discovery process. However, depth responds to changes in the OC range to a larger extent for the small cap portfolio than for the other two. Given that small caps are typically less followed by analysts, we conjecture that a more intense price discovery process is viewed by traders as an indication that the market is more active, encouraging traders to submit a higher number of limit orders. Trading activity measures point in the same direction.

Overall, a more intense price discovery for small caps and mid caps leads to a larger number of trades and a higher trading volume, while the average trade size remains unaffected. For large caps, such evidence is much weaker. All else equal, the intensity of the price discovery process lowers liquidity, the relationship being the strongest for large and mid caps.

14 *H2: The wider the HL range (for a given OC range), the more uncertain the price discovery process, and the lower the liquidity.*

The second hypothesis is also verified since HL_t is negatively correlated with liquidity for all portfolios, as clearly indicated by the relative spread and the dispersion measures. As shown in Table 4, the most significant results are found for the large and small cap portfolios. Overall, the level of liquidity for small caps is found to be much more sensitive to the total price variation than to the price discovery process itself. For small caps, Hypothesis 2 is indeed verified across a higher number of liquidity proxies than Hypothesis 1. There is no big difference between Hypotheses 1 and 2 with respect to depth and trading activity, except maybe for large caps. More uncertainty in the price discovery process of large caps increases all trade-based variables. All else equal, a wider total price variation deteriorates liquidity, while it increases trading volume and the number of trades.

24 *H3: The lower the OCHL ratio (for a given OC and HL ranges), the higher the negative impact of price uncertainty on liquidity.*

The third hypothesis is validated for all liquidity proxies. When OC and HL ranges variations are controlled for, the interaction variable (measuring ‘relative price certainty’) displays a positive relationship to liquidity. Although it holds true for any of the three portfolios, large

1 caps display the most convincing results. The relative spread, dispersion, and slope measures
2 for the large cap portfolio are all three significant at 1%, with the expected sign. Although
3 the OCHL interaction term has very weak influence on depth, it is clearly the opposite with
4 respect to trading activity. In all three cap-based portfolios, the number of trades increases
5 when ‘relative’ price uncertainty increases (i.e. when OCHL decreases). ATS and Volume
6 follow the same pattern for the small and mid caps.

*H4: When there is a zero return, liquidity is higher provided that informed traders supply
7 liquidity.*

Evidence for the fourth hypothesis is mixed. Supporting results are found for large caps.
8 The relative spread and the dispersion decreases when there is a zero return for large caps.
9 However, the slope of the book decreases, pointing to a lower elasticity and less attractive
10 liquidity conditions. A zero return leads to lower available quantities and a lower number
11 of trades, indicating that there are fewer trades and available quantities when a zero return
12 occurs. ATS and Volume increases but the effect is not significant. Liquidity for mid caps
13 does not seem to be sensitive to the occurrence of a zero return, in contrast to the small cap
14 portfolio. Dispersion, spread, depth, and number of trades all increase when a zero return
15 occurs for small caps, while it was the opposite for large caps. As indicated in Table 4, the
16 sum of quantities over the two sides, ΔQ , shows a significant positive estimate (0.039**) which
17 suggests that depth increases on average for small caps when a zero return occurs. Overall,
18 these empirical results seem to be more in accordance with the propositions of Chakravarty
19 and Holden (1995) and Bloomfield et al. (2005), than with the findings of Lesmond et al.
20 (1999). The evidence is nevertheless not strong enough to close the debate.

*H5: When there has been a price gap, liquidity is lower provided that resilience remains
21 limited.*

The fifth hypothesis seems to be validated. Liquidity seems to be negatively affected by
22 the occurrence of a price gap, especially for large caps where the relative spread and dispersion
23 increase significantly. The slope estimate is nevertheless positive and significant, suggesting
24 that volumes in the order book are more concentrated at a given limit after the occurrence of a
25 price gap. Although both the dispersion and the relative spread increase, results for the slope
26 indicate that the agreement among traders about the value of the security would be higher
27 after the occurrence of a price gap (than without). Depth proxies indicate that the variation
28 in depth is dependent on the direction of the window. When an upward window occurs, QB
29 seems to drop while QA seems to rise. When a downward window occurs, QA seems to fall

1 while QB seems to increase. As expected, trading activity estimates are significant for both
2 NB_t and NS_t and the signs are opposed to the corresponding depth proxies. For instance,
3 depth at the bid drops and the number of buyer-initiated trades is higher when an upward
4 window occurs, pointing to higher trading aggressiveness. Correspondingly, depth at the ask
5 drops and the number of seller-initiated trades is higher in the case of a downward window.

6 *H6: When there is an upward price movement, both depth and order imbalances decrease.*

The sixth hypothesis is validated. First, we observe that QB (QA) is higher (lower) when
7 there is an upward price movement, implying that depth imbalance decreases. Second, NB
8 also drops while NS increases, reducing the order imbalance. Such a result is explained by
9 the increase in aggressive trading when liquidity is abundant on the other side of the book.

Overall, there is strong support for the six hypotheses stated in Section 2.2. We first observe
10 that liquidity drops when OC or HL increases. All else equal, liquidity is positively related
11 to the $OCHL$ ratio. Trading activity measures are also related to the evolution of HLOC
12 dynamics. Although the study has proven to be relevant for all the market capitalization
13 portfolios, supporting evidence is stronger for large caps than for small or mid caps. Results
14 are indeed more significant when we move from small or mid caps to the large cap portfolio.

15 **3.2 Controlling for volatility**

16 The results obtained when the realized volatility is included in the regressions are presented
17 in Table 5.

Even when volatility is controlled for, the parameter estimates of the HLOC price move-
18 ment variables remain significant. All these results suggest that the dynamics of HLOC price
19 variations capture changes in liquidity that cannot be explained by the realized volatility. Over-
20 all, all dimensions of liquidity are significantly affected by HLOC price movements. HLOC
21 price dynamics may therefore contain additional information when it comes to explaining
22 liquidity on the stock market.

23 Let us study the six hypotheses again in more details to identify the most striking changes.

Table 5: Controlling for volatility

ΔL_t	ΔOC_t	ΔHL_t	$\Delta OCHL_t$	ZR_t	UW_t	DW_t	D_t	ΔRV_t
Panel A : Small Caps								
ΔRS	0.032	0.173***	-0.031**	-0.152***	-0.133*	0.050	0.090**	-0.034
$\Delta Dispersion$	0.015***	0.008**	-0.010***	-0.019**	0.067***	0.079***	0.007	0.001
$\Delta Slope$	-0.006	-0.052***	0.029**	-0.045	0.074	-0.070	-0.020	0.045**
ΔQB	0.006	0.008	-0.004	-0.123***	-0.012	0.026	0.166***	-0.000
ΔQA	0.003	0.005	-0.004**	0.066***	0.000	-0.038	-0.201***	-0.005**
ΔQ	0.003*	0.006**	-0.002	0.008	-0.018	0.002	-0.029***	-0.003*
ΔNB	0.115***	0.237**	-0.098***	0.343***	1.963***	-0.961***	-1.008***	0.129*
ΔNS	0.006	0.136***	-0.061***	-0.739***	-0.384**	1.103***	0.970***	0.052**
$\Delta Ntrades$	0.061***	0.159***	-0.076***	-0.144**	0.734***	0.287***	-0.082	0.050
ΔATS	-0.002	-0.008	-0.026**	0.154	-0.065	0.020	-0.418	-0.011
$\Delta Volume$	0.155***	0.197**	-0.186***	0.737	1.065**	0.180	-0.457*	0.108
Panel B : Mid Caps								
ΔRS	0.069***	0.085***	-0.067***	-0.126***	-0.082	-0.005	0.084***	0.023*
$\Delta Dispersion$	0.028*	-0.010	-0.049	-0.024**	-0.436	-0.161	-0.598	0.074
$\Delta Slope$	-0.022***	-0.013*	0.038***	-0.055***	0.193***	0.077**	-0.031**	-0.012***
ΔQB	0.004	0.002	-0.003	-0.099***	-0.036**	0.066***	0.106***	0.006
ΔQA	0.004	0.001	0.005	0.038***	0.133***	-0.022	-0.163***	0.002
ΔQ	0.001	0.001	0.003	-0.003	0.025*	0.012	-0.030***	0.001
ΔNB	0.060***	0.072***	-0.089***	0.149***	0.660***	-0.775***	-0.755***	0.087***
ΔNS	0.014	0.105***	-0.069***	-0.608***	-0.617***	0.448***	0.754***	0.060***
$\Delta Ntrades$	0.009	0.139***	-0.053***	-0.170***	0.150***	-0.010	-0.020	0.030***
ΔATS	0.001	0.020*	-0.034***	-0.147	-0.024	0.118	0.086	-0.015
$\Delta Volume$	0.028	0.323***	-0.159***	-0.465*	0.122	-0.173	-0.067	0.011
Panel C : Large Caps								
ΔRS	0.032***	0.093***	-0.037***	-0.113***	-0.077**	-0.155***	0.032***	0.119***
$\Delta Dispersion$	0.014***	0.019***	-0.014***	-0.018***	0.058***	0.027	0.003	0.021***
$\Delta Slope$	-0.021***	-0.066***	0.054***	-0.026***	0.170***	0.239***	-0.022***	-0.029***
ΔQB	0.000	-0.002	0.007*	-0.073***	-0.068***	0.022	0.035***	-0.007*
ΔQA	0.003	-0.002	0.004	-0.027***	-0.015	-0.081***	-0.057***	-0.012***
ΔQ	-0.001	-0.002	0.005**	-0.021***	-0.053***	-0.041***	-0.010***	-0.013***
ΔNB	0.020	0.282***	-0.072***	0.240***	-0.100	-0.163**	-0.542***	0.093***
ΔNS	-0.005	0.277***	-0.054***	-0.281***	-0.142***	-0.075	0.473***	0.034***
$\Delta Ntrades$	0.019*	0.236***	-0.053***	-0.035**	0.043	-0.028	-0.009	0.016
ΔATS	-0.010*	0.088***	0.003	0.091	-0.063	0.046	-0.022	0.010
$\Delta Volume$	-0.038	0.633***	-0.031	0.174	0.305	-0.048	-0.064	-0.016

This table presents the different models that are estimated with clustered standard errors with a control variable for the realized volatility. Panels A, B and C display the estimates for small, mid and large caps, respectively. The first column, L_t , indicates the dependant variable and the following columns are the exogenous variables. QB and QA are the sums of the quantities outstanding at the five best limits, on bid and ask sides, respectively. $Q5$ is the sum of the two. $Slope$ denotes the slope of the book computed following Næs and Skjeltorp (2006) while $Dispersion$ stands for the dispersion measure computed following Kang and Yeo (2008). RS denotes the relative spread. NB and NS respectively denote the number of buy and sell trades. $Ntrades$ is the total number of trades and ATS is the average trade size of these trades. $OC_t = \frac{|Close_t - Open_t|}{RangeMid_t}$ is the absolute value of the net movement. $OCHL_t = \frac{|Close_t - Open_t|}{High_t - Low_t}$ is the percentage of net variation on total fluctuation. HL_t denotes the High-Low range of the interval. ZR_t is a dummy variable that control for the presence of zero returns. UW_t is equal to 1 when an upward window occurs on the chart and 0 otherwise, while DW_t is equal to 1 when a downward window appears and 0 otherwise. The D_t dummy equals 1 when the net price movement is positive and 0 otherwise. The fixed effect dummies are not presented here.

1 *H1: The wider the OC range implied by the price discovery process, the lower the liquidity.*

 There is no significant change in the results between Tables 4 and 5, confirming that ‘the
2 intensity of the price discovery process lowers liquidity, the relationship being the strongest
3 for large and mid caps’.

*H2: The wider the HL range (for a given OC range), the more uncertain the price discovery
4 process, and the lower the liquidity.*

 All else equal, a wider total price variation deteriorates liquidity, as previously outlined.
5 In Table 5, results are even slightly strengthened since the slope for mid caps is negative and
6 statistically significant (while it was not in Table 4).

*H3: The lower the OCHL ratio (for a given OC and HL ranges), the higher the negative
7 impact of price uncertainty on liquidity.*

 Results in Tables 4 and 5 are very similar. The validation of H3 is even stronger for the
8 small cap portfolio since the relative spread and the dispersion are both negative and significant
9 at 1% (while they were not before).

*H4: When there is a zero return, liquidity is higher provided that informed traders supply
10 liquidity.*

 Evidence for the fourth hypothesis in Table 5 is stronger than before. Controlling for
11 realized volatility, both the relative spread and the dispersion are now significant and display
12 the expected signs for all three portfolios. In Table 4, H4 was validated for large caps mostly.

*H5: When there has been a price gap, liquidity is lower provided that resilience remains
13 limited.*

 Evidence in support of the fifth hypothesis is weaker than before. The relative spread is
14 now negative and significant for small and large caps, showing evidence of resiliency in liquidity.
15 This finding is somewhat counterbalanced by the slope variable which is no longer positive and
16 significant in the small cap portfolio. The debate with respect to post-gap resiliency remains
17 therefore open.

18 *H6: When there is an upward price movement, both depth and order imbalances decrease.*

19 There is no difference between Tables 4 and 5. The sixth hypothesis is validated.

1 3.3 Robust and median regressions

2 We apply the robust and median regression techniques to our panel models in order to identify
3 whether the results are affected by the presence of outliers. These types of regressions are
4 useful when some hypotheses of the ordinary least squares estimation are potentially violated.
5 These two specifications also provide an interesting snapshot to circumvent the mean as a
6 measure of central tendency for the estimates. The outcomes are presented in Table 6 and 7,
7 respectively. We report the results with realized volatility as a control variable since it was
8 significant in many cases in Table 5.

The use of the robust regression technique does not change the overall picture. There are
9 nevertheless some points worth mentioning.

The validation of H1 in Table 6 is stronger for small caps when the robust regression
10 technique is used instead of the method based on clustered standard errors and used in Table
11 5. The slope variable becomes positive and significant. Depth for small caps also become
12 negative and significant, as it was (and still is) the case for mid and large caps. Finally, the
13 variables related to trading activity (i.e. volume, number of trades, and average trade size)
14 become more (positively) significant than before, whatever the market capitalization of the
15 stocks.

Regarding H2, mid caps display better results than before: The slope is significant at 1%
16 (instead of 10%) and the dispersion measure is now positive and significant.

Results for H3 are very much unchanged. They are slightly reinforced for mid caps as the
17 dispersion measure turns out to be negatively significant while it was insignificant before.

We confirm that results for H4 are rather equivocal. The relative spread is positive and
18 significant for small and large caps. The occurrence of a zero return has also a clear negative
19 impact on trading activity. However, the slope variable turns out to be positive and significant
20 for mid and large caps.

We confirm that evidence of resiliency is found after the occurrence of price gaps, especially
21 for large caps. Post-gap resiliency is nevertheless weaker for mid caps.

22 The validation of H6 is unchanged.

Empirical results obtained by running median regressions and reported in Table 7 are very
23 much in line with Table 5.

Table 6: Robust regressions with realized volatility

ΔL_t	ΔOC_t	ΔHL_t	$\Delta OCHL_t$	ZR_t	UW_t	DW_t	D_t	ΔRV_t
Panel A : Small Caps								
ΔRS	-0.000	0.064***	-0.011***	0.044***	-0.034	0.034	0.004	-0.012***
$\Delta Dispersion$	0.006***	0.006***	-0.005***	0.003	0.024**	0.027***	0.007*	-0.002**
$\Delta Slope$	-0.005**	-0.035***	0.012***	0.006	0.033*	-0.015	-0.017*	0.005**
ΔQB	-0.003**	0.004**	0.001	0.000	-0.013	-0.012	0.063***	-0.003*
ΔQA	-0.003**	0.004***	0.001	0.056***	-0.032**	-0.054***	-0.070***	-0.004***
ΔQ	-0.002*	0.004***	0.000	0.031***	-0.018*	-0.028**	-0.016***	-0.004***
ΔNB	0.021***	0.070***	-0.025***	0.145***	0.405***	-0.549***	-0.442***	0.015***
ΔNS	0.023***	0.039***	-0.038***	-0.321***	-0.495***	0.552***	0.467***	0.014***
$\Delta Ntrades$	0.020***	0.133***	-0.034***	-0.058***	0.075***	0.161***	0.007	0.007**
ΔATS	0.006**	0.004	-0.002	0.002	0.027	-0.034	-0.023**	0.007**
$\Delta Volume$	0.032***	0.091***	-0.034***	-0.045**	0.129***	0.122***	-0.016	0.012***
Panel B : Mid Caps								
ΔRS	0.013***	0.039***	-0.019***	0.001	-0.088***	0.005	0.005	0.002
$\Delta Dispersion$	0.005***	0.002***	-0.005***	0.000	0.016**	0.030***	0.006**	0.001*
$\Delta Slope$	-0.011***	-0.027***	0.019***	0.014**	0.048***	0.014	-0.014***	-0.003**
ΔQB	-0.001	0.001	-0.001	-0.013***	-0.035***	-0.008	0.046***	-0.004***
ΔQA	-0.002**	0.000	0.001	0.038***	-0.025***	-0.017**	-0.049***	-0.004***
ΔQ	-0.000	0.001	-0.000	0.013***	-0.022***	-0.013**	-0.007***	-0.003***
ΔNB	0.020***	0.050***	-0.034***	0.092***	0.238***	-0.450***	-0.352***	0.017***
ΔNS	0.010***	0.077***	-0.033***	-0.240***	-0.401***	0.254***	0.335***	0.008***
$\Delta Ntrades$	0.016***	0.126***	-0.038***	-0.055***	0.010	-0.002	-0.004	0.006***
ΔATS	0.004***	0.007***	-0.008***	-0.039***	-0.023	-0.035**	-0.013**	0.005***
$\Delta Volume$	0.027***	0.085***	-0.042***	-0.075***	-0.016	-0.030	-0.017*	0.009***
Panel C : Large Caps								
ΔRS	0.007***	0.052***	-0.015***	0.011***	-0.082***	-0.106***	0.009***	0.012***
$\Delta Dispersion$	0.006***	0.021***	-0.009***	-0.001	-0.014***	-0.031***	0.004***	0.010***
$\Delta Slope$	-0.009***	-0.040***	0.015***	0.035***	0.032***	0.086***	-0.011***	-0.031***
ΔQB	-0.005***	-0.005***	0.002*	0.002	-0.091***	-0.022**	0.020***	-0.017***
ΔQA	-0.004***	-0.004***	0.002	0.023***	-0.074***	-0.074***	-0.035***	-0.019***
ΔQ	-0.005***	-0.003***	0.003***	0.011***	-0.079***	-0.046***	-0.009***	-0.018***
ΔNB	0.011***	0.195***	-0.031***	0.133***	-0.115***	-0.191***	-0.336***	0.006***
ΔNS	0.005***	0.189***	-0.027***	-0.187***	-0.205***	-0.058***	0.308***	-0.001
$\Delta Ntrades$	0.007***	0.236***	-0.026***	-0.016***	-0.129***	-0.065***	0.002	0.003**
ΔATS	0.003***	0.066***	-0.010***	-0.025***	-0.085***	-0.074***	-0.016***	-0.007***
$\Delta Volume$	0.005***	0.308***	-0.027***	-0.031***	-0.197***	-0.149***	-0.013***	-0.012***

This table presents the different models that are estimated with the robust regression technique. Panels A, B and C display the estimates for small, mid and large caps, respectively. The first column, L_t , indicates the dependant variable and the following columns are the exogenous variables. QB and QA are the sums of the quantities outstanding at the five best limits, on bid and ask sides, respectively. $Q5$ is the sum of the two. $Slope$ denotes the slope of the book computed following Næs and Skjeltorp (2006) while $Dispersion$ stands for the dispersion measure computed following Kang and Yeo (2008). RS denotes the relative spread. NB and NS respectively denote the number of buy and sell trades. $Ntrades$ is the total number of trades and ATS is the average trade size of these trades. $OC_t = \frac{|Close_t - Open_t|}{RangeMid_t}$ is the absolute value of the net movement. $OCHL_t = \frac{|Close_t - Open_t|}{High_t - Low_t}$ is the percentage of net variation on total fluctuation. HL_t denotes the High-Low range of the interval. ZR_t is a dummy variable that control for the presence of zero returns. UW_t is equal to 1 when an upward window occurs on the chart and 0 otherwise, while DW_t is equal to 1 when a downward window appears and 0 otherwise. The D_t dummy equals 1 when the net price movement is positive and 0 otherwise. The fixed effect dummies are not presented here.

Table 7: Median regressions with realized volatility

ΔL_t	ΔOC_t	ΔHL_t	$\Delta OCHL_t$	ZR_t	UW_t	DW_t	D_t	ΔRV_t
Panel A : Small Caps								
ΔRS	0.002	0.102***	-0.010**	0.005	0.007	0.044**	0.004	-0.016***
$\Delta Dispersion$	0.007***	0.009***	-0.006***	0.002	0.026**	0.032***	0.002	-0.002
$\Delta Slope$	-0.002	-0.036***	0.013***	0.005	0.007	-0.038**	-0.006*	0.009***
ΔQB	-0.002	0.005**	0.000	-0.006	0.003	0.019	0.032***	-0.002
ΔQA	0.000	0.003*	-0.001	0.023***	-0.006	-0.009	-0.042***	-0.003**
ΔQ	-0.001	0.005***	0.000	0.014***	-0.002	-0.003	-0.012***	-0.004***
ΔNB	0.031***	0.120***	-0.028***	0.173***	0.615***	-0.531***	-0.506***	0.020*
ΔNS	0.020***	0.101***	-0.036***	-0.398***	-0.476***	0.825***	0.584***	0.016**
$\Delta Ntrades$	0.035***	0.176***	-0.040***	-0.061***	0.148***	0.219***	-0.014	0.008
ΔATS	0.008***	0.003	-0.003	0.018	0.031	-0.045	-0.040***	0.012***
$\Delta Volume$	0.044***	0.192***	-0.037***	-0.010	0.221***	0.139**	-0.032	0.029**
Panel B : Mid Caps								
ΔRS	0.020***	0.059***	-0.023***	-0.003**	-0.073***	0.002	0.001	-0.000
$\Delta Dispersion$	0.008***	0.003**	-0.006***	0.002	0.021***	0.032***	0.000	0.001
$\Delta Slope$	-0.013***	-0.024***	0.020***	0.008***	0.059***	-0.004	-0.001	-0.002**
ΔQB	0.001	0.001	-0.000	-0.010***	-0.007	0.011	0.022***	-0.001
ΔQA	0.001	-0.001	-0.000	0.016***	0.003	0.001	-0.026***	-0.002**
ΔQ	0.001	0.000	-0.001	0.002	-0.003	-0.000	-0.003***	-0.002***
ΔNB	0.022***	0.088***	-0.034***	0.129***	0.327***	-0.509***	-0.427***	0.029***
ΔNS	0.011**	0.116***	-0.033***	-0.296***	-0.477***	0.345***	0.411***	0.022***
$\Delta Ntrades$	0.015***	0.152***	-0.038***	-0.070***	0.044**	-0.005	-0.009	0.017***
ΔATS	0.005**	0.011***	-0.007***	-0.039***	-0.020	-0.014	-0.013*	0.008***
$\Delta Volume$	0.018***	0.212***	-0.036***	-0.079***	0.009	-0.062**	-0.029***	0.017***
Panel C : Large Caps								
ΔRS	0.001***	0.000***	-0.001***	-0.001***	-0.003***	0.001*	0.002***	0.000***
$\Delta Dispersion$	0.008***	0.018***	-0.010***	-0.003***	0.007	-0.018**	0.003***	0.013***
$\Delta Slope$	-0.011***	-0.027***	0.017***	0.007***	0.035***	0.078***	-0.006***	-0.023***
ΔQB	-0.004***	-0.004**	0.002	-0.009***	-0.063***	-0.007	0.020***	-0.012***
ΔQA	-0.004***	-0.003*	0.003**	0.013***	-0.053***	-0.061***	-0.030***	-0.015***
ΔQ	-0.004***	-0.003**	0.004***	0.007***	-0.061***	-0.035***	-0.008***	-0.013***
ΔNB	0.005*	0.251***	-0.028***	0.152***	-0.142***	-0.212***	-0.365***	0.037***
ΔNS	-0.001	0.248***	-0.024***	-0.193***	-0.192***	-0.085***	0.330***	0.011**
$\Delta Ntrades$	0.007***	0.259***	-0.027***	-0.015***	-0.112***	-0.051**	0.000	0.019***
ΔATS	0.002	0.076***	-0.009***	-0.022***	-0.075***	-0.086***	-0.018***	-0.002
$\Delta Volume$	-0.002	0.407***	-0.025***	-0.023***	-0.193***	-0.137***	-0.020***	0.008

This table presents the different models that are estimated in the framework of a median regression. Panels A, B and C display the estimates for small, mid and large caps, respectively. The first column, L_t , indicates the dependant variable and the following columns are the exogenous variables. QB and QA are the sums of the quantities outstanding at the five best limits, on bid and ask sides, respectively. $Q5$ is the sum of the two. $Slope$ denotes the slope of the book computed following Næs and Skjeltorp (2006) while $Dispersion$ stands for the dispersion measure computed following Kang and Yeo (2008). RS denotes the relative spread. NB and NS respectively denote the number of buy and sell trades. $Ntrades$ is the total number of trades and ATS is the average trade size of these trades. $OC_t = \frac{|Close_t - Open_t|}{RangeMid_t}$ is the absolute value of the net movement. $OCHL_t = \frac{|Close_t - Open_t|}{High_t - Low_t}$ is the percentage of net variation on total fluctuation. HL_t denotes the High-Low range of the interval. ZR_t is a dummy variable that control for the presence of zero returns. UW_t is equal to 1 when an upward window occurs on the chart and 0 otherwise, while DW_t is equal to 1 when a downward window appears and 0 otherwise. The D_t dummy equals 1 when the net price movement is positive and 0 otherwise. The fixed effect dummies are not presented here.

We reassert the validation of H1, with a noticeable improvement: The coefficient of the
1 dispersion measure for mid caps turns out to be significant at 1% (versus 10% before). Finally,
2 variables related to trading activity for mid caps (i.e. volume, number of trades, and average
3 trade size) as well as depth variables for large caps are now all significant (while they were not
4 before).

The validation of H2 is reinforced for mid caps for two reasons. Firstly, the coefficient of
5 the dispersion variable displays the expected sign and becomes significant at 5% (while it was
6 not in Table 5). Secondly, the significance of the slope variable is improved, lowering from
7 10% to 1%. We confirm that liquidity indisputably rises, for large caps in particular.

8 H3 is unaffected by the use of the median regression technique.

Results for H4 rather point to increased liquidity, especially for large caps. The slope
9 variable turns out to be positive and significant for mid and large caps. H4 is nevertheless not
10 confirmed with respect to liquidity of small caps: The relative spread, the dispersion, and the
11 slope are not significant. Overall, the testing of H4 is rather sensitive to the chosen estimation
12 method. There is insufficient evidence to close the debate on the link between liquidity and
13 zero returns.

H5 is especially verified for small caps, as there is no evidence of resiliency. The relative
14 spread and the slope variables are positive and even statistically significant when the median
15 regression technique is used. We confirm that large caps display the strongest evidence of
16 resilience illiquidity, although trading activity (characterized by volume, average trade size,
17 and number of trades) turns out to be negatively affected by the occurrence of price gaps. The
18 other change is the relative spread which is positive and significant at 10% when there is a
19 downward price gap (while it was negative and significant in table 5). Mid caps display mixed
20 results. The dispersion variable becomes positive and significant at 1%, pointing to lower
21 liquidity, but the relative spread and the dispersion variables exhibit negative and significant
22 signs when there has been an upper window, showing evidence of resiliency.

23 Results for H6 are again unchanged.

1 3.4 Simultaneous equations

2 The results for the system of simultaneous equations are presented in Table 8. To save space,
3 only the first equation of each system is shown.⁸

The results provided in Table 8 clearly indicate that liquidity is still affected by HLOC
4 price movements after controlling for endogeneity in the contemporaneous relationship. These
5 outcomes are very much similar to the panel regressions presented above. In other words, the
6 conclusions drawn from panel regressions (corrected for within correlation in standard errors)
7 are still valid when endogeneity is controlled for.

H1 is verified for large caps, irrespective of the liquidity variable considered. Mid caps
8 display similar results, except for the dispersion variable which is not significant. The weakest
9 results are found for small caps. Overall, the intensity of the price discovery process leads to
10 an increase in the number of trades.

H2 is confirmed for all cap-based portfolios. The dispersion variable for mid-caps is the
11 only liquidity variable being not statistically significant. A higher total price variation also
12 leads to increased trading activity, as captured by the number of transactions and the trading
13 volume.

H3 is also supported by the empirical results. Large caps display the most convincing
14 results: The three liquidity variables have all significant coefficients with the expected signs.
15 The only unexpected result is obtained for the relative spread of mid caps.

The testing of the three additional hypotheses related to the control variables delivers
16 similar results than those obtained previously. H4 is confirmed for large caps, inconclusive for
17 mid caps, and invalidated for small caps. Again, it shows that the link between liquidity and
18 zero returns requires further investigation. We confirm that H5 is mostly verified for small
19 caps, as there is weak evidence of resiliency after a price gap. The only unexpected sign is
20 obtained for the slope after the occurrence of an upward price gap. For mid caps, the only
21 relevant result is obtained for the slope variable which points to increased liquidity after a
22 gap in price. Large caps display contrasting evidence, pointing in both directions. While the
23 dispersion variable suggests lower liquidity, the slope variable indicates the opposite. Finally,
24 H6 is validated, especially with respect to order imbalance.

⁸The parameter estimates of the control equations are available upon request.

Table 8: Simultaneous Equations

ΔL_t	ΔOC_t	ΔHL_t	$\Delta OCHL_t$	ZR_t	UW_t	DW_t	D_t
Panel A : Small Caps							
ΔRS	-0.004	0.176***	0.001	0.234***	-0.033	0.114**	0.045**
$\Delta Dispersion$	0.008***	0.016***	-0.003**	0.030***	0.074***	0.080***	0.003
$\Delta Slope$	-0.002	-0.028***	0.027***	-0.022	0.150***	0.023	-0.007
ΔQB	0.004	0.009***	-0.001	-0.009	-0.001	0.098***	0.073***
ΔQA	0.005***	0.005***	-0.003	0.086***	0.109***	-0.023	-0.088***
ΔQ	0.003***	0.004***	-0.002	0.039***	0.018	0.014	-0.013***
ΔNB	0.138***	0.301***	-0.070***	0.589***	2.761***	-0.518***	-0.605***
ΔNS	0.010	0.194***	-0.020**	-0.007	0.104	1.746***	0.535***
$\Delta Ntrades$	0.071***	0.201***	-0.046***	0.274***	1.245***	0.687***	-0.064**
ΔATS	0.006	-0.037	-0.045	0.090	-0.156	-0.192	-0.406
$\Delta Volume$	0.185***	0.271***	-0.131*	1.381***	2.063***	0.911	-0.264
Panel B : Mid Caps							
ΔRS	0.068***	-0.065***	0.065***	0.029	0.014	0.077	0.050**
$\Delta Dispersion$	0.031	0.007	-0.038	-0.007	-0.318	-0.118	-0.400
$\Delta Slope$	-0.022***	-0.009***	0.039***	-0.008	0.221***	0.074***	-0.027***
ΔQB	0.014	-0.005	-0.006	0.382	-0.041	0.052	0.087
ΔQA	0.040	-0.028	-0.007	1.410	0.054	-0.149	-0.100
ΔQ	0.019	-0.013	-0.003	0.699	-0.015	-0.052	-0.010
ΔNB	0.086***	0.060***	-0.088***	0.269***	1.048***	-0.439***	-0.623***
ΔNS	0.025***	0.137***	-0.056***	-0.319***	-0.344***	0.793***	0.613***
$\Delta Ntrades$	0.021***	0.122***	-0.048***	-0.003	0.416***	0.256***	-0.018
ΔATS	0.007	-0.000	-0.048	-0.197	-0.170	-0.073	0.109
$\Delta Volume$	0.059	0.247***	-0.152***	-0.193	0.584	0.275	-0.025
Panel C : Large Caps							
ΔRS	0.034***	0.140***	-0.049***	-0.077***	0.079***	-0.072***	0.032***
$\Delta Dispersion$	0.015***	0.026***	-0.016***	-0.016***	0.085***	0.040***	0.003
$\Delta Slope$	-0.026***	-0.076***	0.053***	0.007	0.103***	0.158***	-0.020***
ΔQB	-0.002	0.006	-0.007	-0.091***	-0.076	0.024	0.014
ΔQA	-0.000	-0.004*	0.005**	-0.013*	-0.025	-0.092***	-0.056***
ΔQ	-0.003	-0.007**	0.005*	-0.021***	-0.073***	-0.055**	-0.014***
ΔNB	0.028***	0.318***	-0.081***	0.199***	0.039	-0.108**	-0.534***
ΔNS	0.003	0.287***	-0.061***	-0.300***	-0.125***	-0.076**	0.468***
$\Delta Ntrades$	0.025***	-0.057***	0.243***	-0.054***	0.055*	-0.039	-0.005
ΔATS	-0.003	0.082***	-0.002	0.022	-0.104	-0.035	-0.019
$\Delta Volume$	0.042	0.549***	-0.131	-0.542	0.014	-0.509	0.242

This table presents the equations systems that are estimated in the framework of simultaneous equations. Panels A, B and C display the estimates for small, mid and large caps, respectively. The first column, L_t , indicates the dependant variable and the following columns are the exogenous variables. QB and QA are the sums of the quantities outstanding at the five best limits, on bid and ask sides, respectively. $Q5$ is the sum of the two. $Slope$ denotes the slope of the book computed following Næs and Skjeltorp (2006) while $Dispersion$ stands for the dispersion measure computed following Kang and Yeo (2008). RS denotes the relative spread. NB and NS respectively denote the number of buy and sell trades. $Ntrades$ is the total number of trades and ATS is the average trade size of these trades. $OC_t = \frac{|Close_t - Open_t|}{RangeMid_t}$ is the absolute value of the net movement. $OCHL_t = \frac{|Close_t - Open_t|}{High_t - Low_t}$ is the percentage of net variation on total fluctuation. HL_t denotes the High-Low range of the interval. ZR_t is a dummy variable that control for the presence of zero returns. UW_t is equal to 1 when an upward window occurs on the chart and 0 otherwise, while DW_t is equal to 1 when a downward window appears and 0 otherwise. The D_t dummy equals 1 when the net price movement is positive and 0 otherwise. The fixed effect dummies are not presented here.

1 3.5 Time intervals

2 In order to check the robustness of our results even further, we focus on different interval lengths
3 but keep the sample unchanged regarding the stocks and dates. The results are presented
4 in Table 9 and Table 10 for 10-minute and 20-minute intervals, respectively. Following the
5 recommendations of Petersen (2009), we estimate the same regressions by OLS with clustered
6 standard errors and control for within correlation across stocks.

7 These tables show that the results are very similar to the core analysis (Table 5). The
8 highlighted dynamics seem to be valid when we consider longer or shorter time frames.

9 The estimates are even more significant for 20-minute intervals than for 10-minute intervals
10 but this may be attributed to the increase in noise that affects smaller time frames, especially
11 for small caps.

12 There are very few differences in the significance levels but the signs remain consistent
13 in general. For instance, the slope displays more significant estimates, for each portfolio, in
14 both samples in comparison to Table 5. Depth proxies show significant estimates that are
15 inconsistent with H1 for small and mid caps in the 20-minute sample but remain consistent
16 for large caps. To sum up, the conclusions are very much unchanged when the interval length
17 is modified.

15 4 Conclusion

16 Liquidity measurement is not an easy task for two reasons. First, liquidity is typically specified
17 in several dimensions. Second, very few theoretical models (if any) can be applied in real-time
18 by practitioners. In this context, efficient intraday liquidity estimation remains a thorny issue
19 in finance. Not surprisingly, the art of making accurate and quick estimations of intraday
20 liquidity is still in its infancy and represents a serious challenge for both practitioners and
21 academics. In this paper, we ask the following question: Is it possible to improve real-time
22 liquidity measurement by looking at HLOC price ranges on an intraday basis? Although price
23 movements are recognized as useful for evaluating volatility, trading activity and (ex-ante)
24 liquidity, no study has investigated the information content of HLOC intraday price ranges for
25 liquidity evaluation. We therefore revisit the liquidity-volatility relationship to quantify the
26 information content of price ranges, while controlling for the level of realized volatility (among
27 others).

Table 9: Robustness check : 10-minute interval regressions with clustered standard errors

ΔL_t	ΔOC_t	ΔHL_t	$\Delta OCHL_t$	ZR_t	UW_t	DW_t	D_t	ΔRV_t
Panel A : Small Caps								
ΔRS	0.033	0.169***	-0.048**	-0.088*	-0.015	0.065	0.060	-0.034
$\Delta Dispersion$	0.017***	0.006*	-0.013***	-0.014*	0.062***	0.045***	0.013*	-0.000
$\Delta Slope$	-0.016**	-0.029***	0.045***	-0.082***	-0.015	-0.037	-0.073***	0.005
ΔQB	0.004	0.011	0.002	-0.143***	-0.007	0.039	0.164***	-0.003
ΔQA	0.007	0.005	-0.006*	0.064***	0.060	-0.023	-0.183***	-0.007*
ΔQ	0.004	0.005*	-0.004*	-0.010	0.017	0.010	-0.021**	-0.005**
ΔNB	0.053	0.100***	-0.075***	0.176***	1.066***	-0.648***	-0.913***	0.031
ΔNS	0.040*	0.109***	-0.089***	-0.920***	-0.532***	1.080***	1.034***	0.039
$\Delta Ntrades$	0.056***	0.113***	-0.084***	-0.240***	0.500***	0.324***	-0.012	0.005
ΔATS	0.005	0.001	-0.019***	0.163	-0.022	0.165	-0.034	0.001
$\Delta Volume$	0.073*	0.199***	-0.151***	-0.215	0.890*	1.150*	-0.273	-0.007
Panel B : Mid Caps								
ΔRS	0.011	0.174***	-0.038***	-0.130***	-0.124*	-0.126*	-0.012	0.004
$\Delta Dispersion$	0.017***	0.005*	-0.011***	-0.020***	0.047***	0.056***	0.010**	0.004
$\Delta Slope$	-0.017***	-0.030***	0.041***	-0.075***	0.153***	0.094***	-0.018	-0.003
ΔQB	0.001	0.008***	0.007	-0.087***	-0.015	0.037*	0.120***	0.001
ΔQA	0.007	0.009**	-0.003	0.024**	0.117***	-0.026	-0.157***	-0.006
ΔQ	0.002	0.003**	0.003	-0.009	0.020	-0.005	-0.017***	-0.002
ΔNB	0.078***	0.069***	-0.102***	0.214***	0.584***	-0.767***	-0.787***	0.071***
ΔNS	0.031**	0.104***	-0.087***	-0.612***	-0.623***	0.440***	0.806***	0.055***
$\Delta Ntrades$	0.033**	0.120***	-0.076***	-0.174***	0.142***	0.027	-0.009	0.033***
ΔATS	-0.002	0.059**	0.010	0.012	-0.121	0.592	-0.352***	-0.020
$\Delta Volume$	0.018	0.471***	-0.142***	-0.077	0.334	0.078	-0.692***	-0.015
Panel C : Large Caps								
ΔRS	0.031***	0.141***	-0.032***	-0.096***	-0.069	-0.138***	0.038***	0.063***
$\Delta Dispersion$	0.013***	0.025***	-0.010***	-0.015***	0.059***	0.028***	0.005*	0.014***
$\Delta Slope$	-0.018***	-0.080***	0.052***	-0.031***	0.187***	0.246***	-0.025***	-0.013***
ΔQB	0.003	0.000	0.004	-0.073***	-0.110***	-0.014	0.045***	-0.003
ΔQA	0.008***	-0.000	0.000	-0.013*	-0.010	-0.082***	-0.057***	-0.012***
ΔQ	0.002	-0.002	0.002	-0.016***	-0.068***	-0.058***	-0.007**	-0.008***
ΔNB	-0.001	0.393***	-0.074***	0.239***	-0.059	-0.098	-0.645***	0.048***
ΔNS	-0.004	0.314***	-0.071***	-0.355***	-0.030	-0.060	0.572***	0.032***
$\Delta Ntrades$	0.013*	0.289***	-0.054***	-0.044***	0.108***	0.013	0.001	0.012
ΔATS	-0.013***	0.070***	-0.014	-0.014	-0.136	0.131	-0.165	0.025
$\Delta Volume$	-0.066**	0.680***	-0.043	0.018	0.097	0.399	-0.228	0.033

This table presents the different models that are estimated by OLS with the clustering approach for a 10-minute intervals sample. Panels A, B and C display the estimates for small, mid and large caps, respectively. The first column, L_t , indicates the dependant variable and the following columns are the exogenous variables. QB and QA are the sums of the quantities outstanding at the five best limits, on bid and ask sides, respectively. Q is the sum of the two. $Slope$ denotes the slope of the book computed following Næs and Skjeltorp (2006) while $Dispersion$ stands for the dispersion measure computed following Kang and Yeo (2008). RS denotes the relative spread. NB and NS respectively denote the number of buy and sell trades. $Ntrades$ is the total number of trades and ATS is the average trade size of these trades. $OC_t = \frac{|Close_t - Open_t|}{RangeMid_t}$ is the absolute value of the net movement. $OCHL_t = \frac{|Close_t - Open_t|}{High_t - Low_t}$ is the percentage of net variation on total fluctuation. HL_t denotes the High-Low range of the interval. ZR_t is a dummy variable that control for the presence of zero returns. UW_t is equal to 1 when an upward window occurs on the chart and 0 otherwise, while DW_t is equal to 1 when a downward window appears and 0 otherwise. The D_t dummy equals 1 when the net price movement is positive and 0 otherwise. The fixed effect dummies are not presented here.

Table 10: Robustness check : 20-minute interval regressions with clustered standard errors

ΔL_t	ΔOC_t	ΔHL_t	$\Delta OCHL_t$	ZR_t	UW_t	DW_t	D_t	ΔRV_t
Panel A : Small Caps								
ΔRS	0.050**	0.111***	-0.064***	-0.168***	-0.283**	0.032	0.149***	0.034
$\Delta Dispersion$	0.016***	0.007***	-0.013***	-0.012	0.062***	0.039*	0.015**	0.000
$\Delta Slope$	-0.019***	-0.029***	0.052***	-0.157***	0.095*	0.033	-0.007	0.005
ΔQB	0.006*	0.004	0.004	-0.128***	-0.015	0.041	0.170***	-0.001
ΔQA	0.011***	-0.002	-0.000	0.084***	0.019	-0.075***	-0.174***	0.000
ΔQ	0.006**	0.000	0.001	0.010	-0.004	-0.017	-0.013	-0.001
ΔNB	0.085***	0.137***	-0.113***	0.227***	1.327***	-0.593***	-0.882***	0.072**
ΔNS	0.027	0.074***	-0.100***	-0.837***	-0.390*	0.949***	0.969***	0.104***
$\Delta Ntrades$	0.050***	0.117***	-0.098***	-0.285***	0.580***	0.275***	0.028	0.067
ΔATS	0.009	-0.001	-0.036***	-0.134	-0.054	-0.109	0.003	0.001
$\Delta Volume$	0.076*	0.156***	-0.219***	-0.637***	0.518	-0.023	-0.077	0.163
Panel B : Mid Caps								
ΔRS	0.051***	0.087***	-0.050***	-0.172***	-0.081	-0.000	0.074***	0.013
$\Delta Dispersion$	0.012***	0.005***	-0.007***	-0.036***	0.060***	0.046***	0.012**	0.002
$\Delta Slope$	-0.020***	-0.022***	0.035***	-0.062***	0.132***	0.047*	-0.041**	-0.001
ΔQB	0.005**	0.002	-0.004*	-0.101***	-0.040*	0.007	0.107***	0.006
ΔQA	0.006***	0.000	-0.004**	0.030**	0.151***	-0.025	-0.153***	-0.000
ΔQ	0.003***	-0.000	-0.002	-0.007	0.027*	-0.003	-0.027***	0.001
ΔNB	0.082***	0.090***	-0.103***	0.179***	1.114***	-0.802***	-0.784***	0.094***
ΔNS	0.055***	0.085***	-0.093***	-0.647***	-0.381***	0.653***	0.794***	0.072***
$\Delta Ntrades$	0.056***	0.098***	-0.084***	-0.182***	0.516***	0.048	-0.020	0.040***
ΔATS	0.001	0.016	-0.017	-0.325	-0.144	0.810	0.134	0.005
$\Delta Volume$	0.071	0.331***	-0.145**	-0.459	1.266**	0.963	-0.031	0.127**
Panel C : Large Caps								
ΔRS	0.015***	0.134***	-0.016***	-0.107***	-0.139***	-0.152***	0.034***	0.024
$\Delta Dispersion$	0.007***	0.026***	-0.007***	-0.024***	0.017	0.022	0.002	0.006**
$\Delta Slope$	-0.011***	-0.071***	0.032***	-0.037***	0.232***	0.202***	-0.026***	-0.000
ΔQB	0.001	-0.007**	0.005*	-0.073***	-0.075***	-0.018	0.033***	-0.003
ΔQA	0.002	-0.006	0.004	-0.033***	-0.029	-0.095***	-0.059***	-0.004
ΔQ	0.000	-0.008**	0.003*	-0.022***	-0.066***	-0.067***	-0.012***	-0.006*
ΔNB	0.038**	0.294***	-0.077***	0.192***	0.965***	-0.034	-0.490***	0.111***
ΔNS	0.016*	0.240***	-0.057***	-0.277***	0.462***	0.500***	0.434***	0.104***
$\Delta Ntrades$	0.034***	0.214***	-0.057***	-0.056***	0.638***	0.293***	-0.008	0.078***
ΔATS	0.001	0.057***	-0.011***	-0.007	-0.073	0.023	-0.029***	-0.008*
$\Delta Volume$	0.015	0.421***	-0.065***	0.003	0.774***	0.393	-0.065***	0.125***

This table presents the different models that are estimated by OLS with the clustering approach for a 20-minute intervals sample. Panels A, B and C display the estimates for small, mid and large caps, respectively. The first column, L_t , indicates the dependant variable and the following columns are the exogenous variables. QB and QA are the sums of the quantities outstanding at the five best limits, on bid and ask sides, respectively. Q is the sum of the two. $Slope$ denotes the slope of the book computed following Næs and Skjeltorp (2006) while $Dispersion$ stands for the dispersion measure computed following Kang and Yeo (2008). RS denotes the relative spread. NB and NS respectively denote the number of buy and sell trades. $Ntrades$ is the total number of trades and ATS is the average trade size of these trades. $OC_t = \frac{|Close_t - Open_t|}{RangeMid_t}$ is the absolute value of the net movement. $OCHL_t = \frac{|Close_t - Open_t|}{High_t - Low_t}$ is the percentage of net variation on total fluctuation. HL_t denotes the High-Low range of the interval. ZR_t is a dummy variable that control for the presence of zero returns. UW_t is equal to 1 when an upward window occurs on the chart and 0 otherwise, while DW_t is equal to 1 when a downward window appears and 0 otherwise. The D_t dummy equals 1 when the net price movement is positive and 0 otherwise. The fixed effect dummies are not presented here.

Using Euronext order book data, we define three cap-based portfolios of 100 stocks each and compute several proxies for liquidity and price movements. The dependent variables characterize the different dimensions of liquidity. We include several proxies for order-based measures: Depth, spread, slope and dispersion. We also test trade-based proxies: Number of trades, mean size of trades, and trading volume. We estimate the regression using clustered standard errors that control for within correlation across stocks. In a second step, we include a proxy for the realized volatility in order to control for volatility dynamics that may affect price movements. We also conduct robust and median regressions to control for the presence of outliers in the sample as well as systems of simultaneous equations to control for endogeneity. As a further robustness check, we test our models at the 10-minute and 20-minute intervals.

Our findings suggest that price ranges help better characterize liquidity dynamics whatever the liquidity dimension considered. Firstly, we conclude that the intensity of the price discovery process (that we characterize by the OC range) is accompanied by a decrease in (ex-ante) liquidity (all else equal, of course). Secondly, a wider total price variation that results from higher price uncertainty (i.e. a higher HL range) is also associated with a fall in (ex-ante) liquidity (ceteris paribus). Thirdly, controlling for the intensity of the price discovery process and the level of price uncertainty, we find that higher ‘relative price certainty’ (that we measure by the interaction variable of the OC range divided by the HL range) displays a positive relationship to liquidity. Fourthly, liquidity seems to be higher when there is a zero return, noticing that the inclusion of realized volatility as a control variable has a positive effect on this relation. The results are nevertheless relatively sensitive to the estimation method and the debate on the link between zero returns and liquidity remains open. Fifthly, we cannot conclude that liquidity is lower when there has been a price gap because some evidence of resiliency is found for mid and large caps. Sixthly, both depth and order imbalances decrease when there is an upward price movement. Finally, we confirm the positive link between trade-based proxies and volatility, that is well-documented in the literature on the volume-volatility relationship.

All in all, positive changes in price ranges for both Close-Open and High-Low ranges are related to negative variations in book-based liquidity proxies, whether or not realized volatility is controlled for. All else equal, book-based liquidity is higher when the two ranges coincide (i.e. when the total price variation during the interval is justified by the price discovery process), pointing to a high level of ‘relative price certainty’. Not only realized volatility does not capture these effects but all these results are also very much confirmed across the three cap-based portfolios. The large cap portfolio nevertheless displays the most convincing results.

We conclude that the investigation of the dynamics of HLOC prices at the intraday level
1 would help market participants characterize the state of the limit order book more efficiently
2 than by trying to measure each liquidity dimension in turn. In this respect, HLOC price ranges
3 constitute a useful source of information, notwithstanding the fact that they can be displayed
4 in any trading platform around the world.

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