

Stochastic Programming for Valuing Energy Storage Providing Primary Frequency Control

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Abstract—This paper introduces a novel modelling of the economics of energy storages providing frequency containment reserves. The focus lies both on the practical operation of day-ahead markets and the stochastic and autocorrelation characteristics of the grid frequency as they are practical concerns for any energy storage operator. Based on yearly series of grid-frequency measurements we put in light seasonal trends in the grid frequency. After having removed these trends we build an ARMA-GARCH model with fat-tail feature to explain the remaining autocorrelation. The overall model can be used to generate scenarios of 15-minute time-step frequency deviations and we explain how these scenarios can be used along with stochastic programming to dispatch and compute the revenues of a storage providing frequency containment reserves. By considering two different cases, battery storage and variable-speed pumped-storage, the paper emphasizes the importance of the technical characteristics of energy storages providing frequency containment reserves.

Index Terms—Energy storage, Frequency Containment Reserves, Grid-frequency modelling, Stochastic Programming.

I. INTRODUCTION

The recent liberalisation and the ongoing energy transition have profoundly transformed European power systems by providing incentives for new entrants but also new technologies. The development of renewables as wind power and photovoltaics leads to a decrease in grid inertia and an increase in grid-frequency volatility because these technologies are generally connected to the grid through power electronics converters. In the meantime, renewable generation is pushing traditional generation as gas-fired power plants out of the market, which decreases the availability of classical providers of ancillary services. These are incentives for the development of energy storage systems.

Table I summarises several studies on the economics of energy storage. These studies focus on day-ahead price arbitrage and/or on the provision of ancillary services, mostly frequency containment reserves (FCRs), or reserves assimilable to frequency restoration or replacement reserves (FRRs or RRs);

The authors would like to thank Elia, the Belgian transmission system operator, for the provision of 2012 and 2013 frequency measurements.

TABLE I
THE ECONOMICS OF ENERGY STORAGE IN POWER SYSTEMS.

Price arbitrage only	FCR only	Price arbitrage and FRR/RR
[1], [4], [7], [8]	[2], [3], [6]	[5], [9], [10]

those reserves were formerly known as respectively primary, secondary and tertiary. Valuation techniques vary from one paper to another but typically are heuristics [1]–[6], linear programming [7], [8], mixed integer linear programming [9], mixed integer nonlinear programming [10]. The provision of FCRs is the ancillary service with the most requiring technical and operational constraints as FCR providers must always be running and able to vary their power injections/offtakes. It is thus not surprisingly considered as the most valuable application for energy storages [11].

An energy storage providing FCRs faces expected and unexpected energy deviations, respectively because of its cycle efficiency and because mean frequency deviations generally differ from zero over a given period of time. The most obvious way to deal with those energy deviations is to regularly buy electrical energy on the day-ahead market. However, when assessing the revenues of a storage providing FCRs, [2] and [3] do not take into account the practical operation of the day-ahead market as they consider that one can buy and sell energy whenever needed. Instead, energy requirements should be forecasted the day ahead in order to enable the storage operator to bid on the day-ahead market. In [6] it is considered that the energy storage is coupled to a power plant, of which part of the nominal power is reserved to charge the storage. However this case should be valued as if the charging power was bought on the market. Indeed the opportunity cost of the power plant is directly linked to the market prices of electricity.

When sizing and valuing a storage providing FCRs, [2] uses a particular realization of frequency deviations over one month, April 2005, in the union for the co-ordination of transmission of electricity (UCTE). Although it has the advantage to use real data, this approach, rather deterministic, does not account for the diversity of frequency deviations happening in reality.

In this paper we propose to model the revenues of an energy storage taking into account the actual operation of the day-ahead market and the characteristics of the grid frequency, seasonality, autocorrelation, stochastic and fat-tail features, as they are practical concerns of the storage operator. The outline of the paper is as follows. Section II gives some insights on the frequency dynamics and emphasizes its fat-tail feature. Section III shows how to model frequency deviations with seasonality trends and an ARMA-GARCH model. Section IV shows how to model the revenues of a storage, first doing price arbitrage and second providing FCRs.

II. FAT-TAILNESS OF THE GRID FREQUENCY

Each power system is characterized by a specific frequency, e.g. 50 Hz for the continental Europe (CE) grid and 60 Hz for grids of Northern America. The frequency can be considered uniform over a synchronous region. It is directly related to the rotational speed of synchronous generating machines and varies with time according to mismatches between electricity generation and consumption. Frequency deviations can be modelled generalizing the swing equation to a whole synchronous power system:

$$J \frac{d\omega}{dt} = \frac{P_{mec} - P_{el}}{\omega}$$

with J [kg.m²] the total moment of inertia, $\omega = 2\pi f$ [rad/s] the angular speed with f the grid-frequency and P_{mec} and P_{el} [W] respectively the mechanical and electrical powers. According to this relation, a decrease in the grid frequency happens when the mechanical power is lower than the electrical power. Inversely, an increasing frequency is linked to too much mechanical power compared to the electrical power.

In practice several mechanisms are put in place to constantly adjust P_{el} in order for the frequency to remain near its nominal value. These mechanisms, namely FCRs, FRRs and RRs, are very effective as illustrated on Fig. 1, showing the probability density function obtained from frequency measurements at a 10-second time step over the year 2012. Several comments can be made on these data. First, approximately 95% of frequency realizations are included in the band [49.95, 50.05] Hz. Second, although the frequency distribution may look normally distributed to the reader, it actually does not pass the Anderson-Darling normality test as the associated p-value is less than 2.2e-16. Third, using the maximum-likelihood estimation (MLE), we observe that a student distribution, $t(50, 0.0207, 12)$, better fits the frequency data than a normal distribution, $\mathcal{N}(50, 0.0226)$. This comes from the fact that the grid frequency is characterized by a fat-tailed distribution, as can be emphasized by a quantile-quantile (Q-Q) plot. As shown on Fig. 1, frequency deviations of either less than one or more than two standard deviations have a greater probability of occurrence than predicted by the normal distribution. It is the opposite for frequency deviations from one to two standard deviations. The conclusions are the same for 2013.

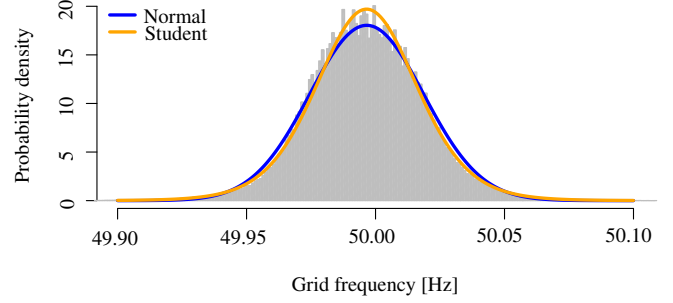


Fig. 1. The probability density of the grid-frequency exhibits fat-tailness.

While fat-tailness is not a well known characteristic of the grid frequency, it is a common topic in the financial industry [12], [13], e.g. when dealing with asset returns, and it is a practical concern for the modeller. Building realistic time-series models requires to make a distributional hypothesis. In order to capture fat-tails, a popular choice in the financial industry is the student distribution. It includes the normal distribution as a special case, when its shape parameter tends to infinity, and can therefore be seen as an extension of the classical Gaussian framework. In the following section we will come back to the fat-tail feature of the grid frequency when modelling energy deviations over 15-minute time steps.

III. MODELLING 15-MINUTE ENERGY DEVIATIONS

We are looking at time steps of around 15 minutes. Over such time steps the mean of frequency deviations is generally different from zero, which makes FCR providers as storages face unexpected energy deviations, i.e. unexpected changes in their stored energy. According to the 2013 frequency measurements, there is a 9% probability for one-hour-averaged frequency deviations to induce more than 0.1 hour of energy deviation for each megawatt of contracted FCR. We want to model such frequency deviations in order to take them into account in the operation of the storage, i.e. the objective is to build a model for generating credible 15-minute time-step series of frequency deviations. First we average over 15-minute time-steps the series of 10-second time-step frequency deviations. Now the next thing to do before fitting a model to this new 15-minute time-step series is to remove its seasonal components.

A. Seasonality removal

As emphasized on Fig. 2(a), the partial autocorrelation function (PACF) of the 15-minute time-step series has a high value at lag 4. A fast Fourier transform (FFT) of this series, not displayed in the paper, shows a component of 1-hour period, which indicates an hourly pattern in frequency deviations. This result is consistent since electrical energy is bought at constant power for time steps of one hour while the load is

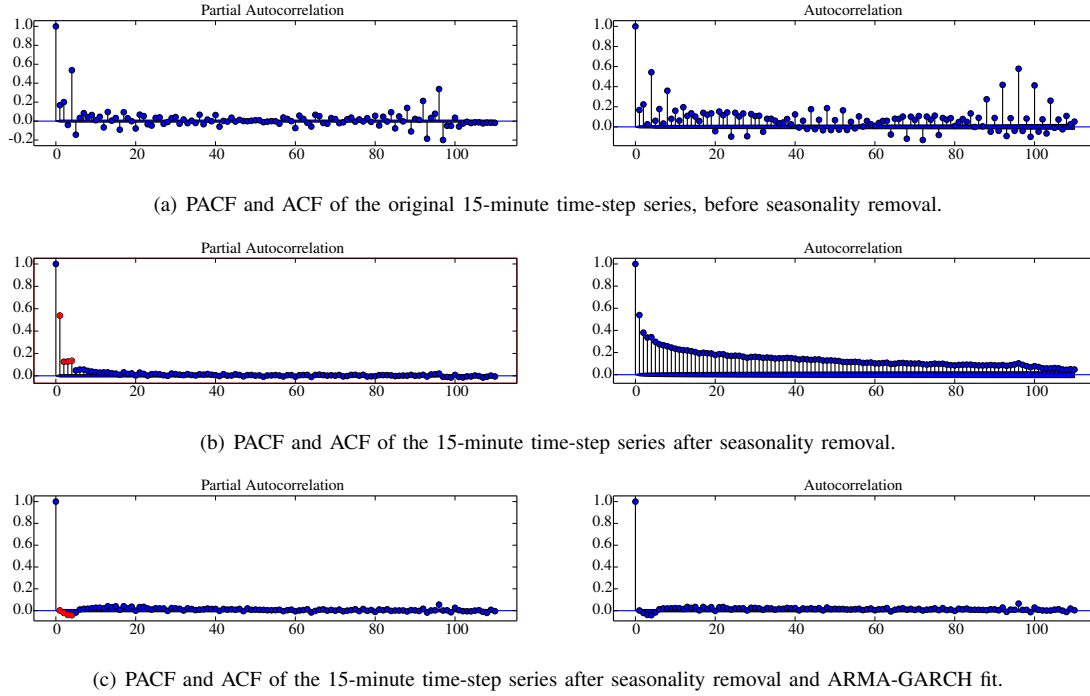


Fig. 2. The autocorrelation in the 15-minute time-step series is explained by seasonality trends and an ARMA-GARCH model.

constantly varying, increasing in the morning and decreasing in the evening. We also observe high values of the PACF around lag 96, i.e. around a 24-hour lag, while the FFT exhibits high values around a frequency of 0.01 Hz, corresponding to signals of 24-hour period. As emphasized by the ACF, PACF and FFT, our series contains at least hourly and daily trends. The first step before fitting a model is to remove them.

We consider a 24×4 matrix, 24 hours $\times$ 4 quarters, in which we will compute the mean of 15-minute frequency deviations over one year, differentiating by quarter of the day. We see that when the first column of the matrix is positive, the last one is generally negative and vice versa. In other words, positive frequency deviations at the beginning of an hour are generally followed by negative deviations at the end of the hour and vice versa. Subtracting those means from the 15-minute-averaged time series decreases the lag-4 coefficient of approximately 0.2. More broadly the shape of the partial autocorrelation function improves and the 0.25 Hz component of the FFT also gets reduced. Now if we repeat the process but with a $12 \times 24 \times 4$ matrix, i.e. differentiating by month and quarter of the day, we are able to reduce the lag-4 coefficient to 0.2 and even a bit less than 0.2 adding another differentiation between week and week-ends. Lag-1 coefficient increases, lag-2 coefficient decreases while lag-3 coefficient increases. The other coefficients move toward zero including around lag 96.

To sum up, we find that the autocorrelation in our original time series is mainly affected by monthly and hourly seasonality. Instead of fitting an arbitrary function, e.g. a cosine, to remove seasonality, we have computed it as means, differentiating by month, by week/week-end, by hour and by quarter. Those means, which can be stored in

a $12 \times 2 \times 24 \times 4$ matrix, are then removed from the original 15-minute time-step series. As indicated on Fig. 2, the results are PACF and ACF exhibiting less autocorrelation for the series with seasonality removed compared to the original 15-minute time-step series. Note that removing the computed 2012 seasonality from the 2013 data does also significantly improve the PACF and ACF, although not as well as for the 2012 data. This check, which is important to do, shows that (i) the seasonality observed in 2012 generalizes well for the 2013 data and (ii) the proposed methodology to derive seasonality does not lead to overfitting. The importance of seasonality in the data can be better appreciated noting that the computed means in the $12 \times 2 \times 24 \times 4$ matrix range from zero up to 0.049 Hz while the standard deviations, associated to each mean, remain roughly around 0.01 Hz.

B. ARMA(4,0)-GARCH(1,1) model

In the previous subsection we have seen that removing seasonality reduces the autocorrelation in the data. Some unexplained autocorrelation is however remaining, in particular from lag-1 to lag-4 as indicated in red on the PACF of Fig. 2(b). This autocorrelation can be taken into account by considering in our model an autoregressive moving average process ARMA(p,q). The shapes observed on the PACF and ACF of Fig. 2(b) can be identified as characteristic of an ARMA(4,0) process. After fitting an ARMA(4,0) model, we notice two things. First the residuals still exhibit a fat-tail feature, i.e. they are not normally distributed, which is a problem as most tools for fitting ARMA models use MLE with the hypothesis of normally distributed residuals. Second,

TABLE II
FITTED PARAMETERS OF THE ARMA(4,0)-GARCH(1,1) MODEL WITH
RESIDUALS FOLLOWING A STUDENT DISTRIBUTION.

	Estimate	Std. Error	t value	p value	Significance
φ_1	4.171e-01	5.562e-03	74.99	$< 2e-16$	***
φ_2	7.127e-02	5.870e-03	12.14	$< 2e-16$	***
φ_3	7.274e-02	5.809e-03	12.52	$< 2e-16$	***
φ_4	1.461e-01	5.314e-03	27.49	$< 2e-16$	***
σ	1.482e-05	1.272e-06	11.65	$< 2e-16$	***
α	9.943e-02	6.076e-03	16.37	$< 2e-16$	***
β	7.451e-01	1.743e-02	42.74	$< 2e-16$	***
$shape$	7.860e+00	3.016e-01	26.06	$< 2e-16$	***

the ACF and PACF of the squared residuals show that residuals are characterized by a variable volatility, also referred to as volatility clustering. Although this effect seems of low magnitude, volatility clustering explains part of the fat-tail feature of the residuals and we therefore decide to take it into account using a generalized autoregressive conditional heteroskedastic (GARCH) model. As it is usually done in finance [14], [15], we fit a GARCH(1,1) process and specify a *student* distribution for the residuals. The overall model is thus given by (1), (2) and (3):

$$X_t = \mu + \sum_{i=1}^4 \varphi_i X_{t-i} + \sigma_t \varepsilon_t \quad (1)$$

$$\varepsilon_t \sim t_\nu \quad (2)$$

$$\sigma_t^2 = \sigma + \beta \sigma_{t-1}^2 + \alpha \varepsilon_{t-1}^2 \quad (3)$$

while the fitted parameters are given in Table II. All those parameters are significantly different from zero, at the 1% level, as emphasized by the p-value and significance columns. Note that the *shape* parameter is related to the student distribution fitted to the standardized residuals, i.e. $t(0, 1, 7.86)$. Considering non-standardized residuals, the student distribution becomes $t(0, 0.0099, 6.54)$. A Q-Q plot of those residuals allows to show that they actually follow this distribution. The values obtained on the PACF and ACF of the residuals are close to zero, as indicated on Fig. 2(c), showing that nearly no autocorrelation is remaining in the residuals.

With our model, an ARMA(4,0)-GARCH(1,1) process plus seasonality terms, we are able to generate scenarios of 15-minute time-step frequency deviations and it has been checked that series generated from our model are similar to the initial 15-minute time-step series. Given the presented methodology to build the model and the performed analysis of the residuals, we can conclude that our model is able to reproduce the dynamics of frequency deviations. A Python package implementing the model has been released and can be installed using `pip`. The code, opensource and freely available on GitHub¹, is based on RTE's 2015 frequency data.

¹http://github.com/thmercier/grid_toolkit

Sets

H the set of hours h

Parameters

$Price_h \forall h$, spot prices in €/MW

η the half-cycle efficiency

E_{max} storage limit in hours

Variables

$P_h^{off}, P_h^{inj} \forall h$, binary variables to choose the pump/turbine mode

$0 \leq E_h \leq E_{max} \forall h$, the stored energy in hours

Model

$$\max \sum (\eta P_h^{inj} - P_h^{off}) * Price_h$$

$$\begin{aligned} s.t. \quad & P_h^{inj} + P_h^{off} \leq 1 \quad \forall h \\ & E_{h+1} = E_h - P_h^{inj} + \eta P_h^{off} \quad \forall h \mid h < \text{card}(H) \\ & E_1 = E_{max}/2 \end{aligned}$$

Fig. 3. Revenues maximization of price-taker PSH.

IV. ECONOMICS OF STORAGE: PRICE ARBITRAGE VERSUS PROVISION OF FCR

A. Price arbitrage

When studying the economics of energy storage in power systems, the most widely considered source of revenues is certainly price arbitrage (see Table I) on short-term electricity markets. Although there are two possible markets by country, the day-ahead market (DAM) and the continuous intraday market (CIM), economic studies generally focus on the former since the latter is considered not liquid enough. Most European DAMs are coupled but limited interconnection capacities imply price divergences between national markets and national prices should therefore be used.

In order to derive an upper bound of the achievable revenues in the DAM, a commonly used framework is the perfect-forecasting price-taker storage. By considering that prices are perfectly known and that the storage is sufficiently small compared to the market not to influence the price, one can compute an upper bound of the revenues with a simple LP formulation [8]. We consider this framework for two different technologies: fixed-speed pumped-storage hydropower (PSH) and battery storage. Following the methodology used in [8], the revenues maximization of a PSH unit on the DAM can be formulated as in Fig. 3. Binary variables are used because we consider that fixed-speed PSH does not enable power variations. At each time step one has to choose between one of the following modes: pump, turbine or rest. In the case of a battery storage, the need for binary variables disappears as power injections/offtakes become variable. Both complexity and time to solve are thus reduced for the battery storage while revenues slightly increase.

The MILP formulation of Fig. 3 can be run for a one-year horizon in order to derive revenues in €/MW/year but another option is to run it for several shorter periods, e.g. of two weeks. Arguments for the second option are that (i) storage operators should not have reliable price forecasts for long time horizons

TABLE III
AVERAGE BELGIAN FCR VOLUMES AND PRICES IN 2015 [18].

Delivery 2015	Symmetric200 MW-€/MW/h	Symmetric100 MW-€/MW/h	Downward MW-€/MW/h	Upward MW-€/MW/h
January	83 - 36.26			
February	32 - 25.62	28 - 40.72	23 - 11.80	23 - 5.60
March	41 - 22.60	21 - 51.36	23 - 15.60	21 - 5.80
April	33 - 29.00	28 - 40.11	22 - 11.40	22 - 4.93
May	32 - 26.56	28 - 54.95	23 - 10.50	23 - 4.17
June	31 - 31.05	26 - 84.93	26 - 9.60	26 - 3.79
July	30 - 31.86	27 - 66.09	26 - 9.60	26 - 3.59
August	30 - 32.70	30 - 32.28	23 - 12.60	23 - 3.14
September	83 - 28.94			
October	39 - 34.58	22 - 21.63	22 - 10.50	22 - 3.19
November	33 - 22.26	25 - 26.64	25 - 9.95	25 - 10.57
December	45 - 26.40	19 - 21.60	19 - 7.75	19 - 10.39

while a two-week horizon allows the storage to take advantage of weekly seasonality in prices [8], and (ii) slicing the yearly horizon enables to decrease the computational time.

B. Provision of FCRs

Revenues can be generated from the provision of FCRs if the energy storage is able to continuously vary its power injections/offtakes, within a predefined range, according to a local grid-frequency measurement. FCR providers should be able to fully activate in 30 seconds and sustain the offered power for at least 15 consecutive minutes [16]. The largest market for FCRs includes Germany, Switzerland, Austria and the Netherlands, which currently represents 783 MW, i.e. 26.1% [17] of the total FCR requirement for the CE grid. Since July 2011, the market is organized as weekly tenders with average prices of respectively 16.4, 17.6, 20.9 and 22.4 €/MW/h for 2012, 2013, 2014 and 2015 (until end October).

Another FCR market is the Belgian one. The national TSO, Elia, has divided it into four different products:

- Symmetric200: the classical FCR that must be fully activated for frequency deviations of ± 200 mHz.
- Symmetric100: similar to Symmetric200 but with a full activation for frequency deviations of ± 100 mHz.
- Downward: the contracted reserves are active in the 50.1 to 50.2 Hz band, therefore only for power offtakes.
- Upward: this reserve is only active in the 49.9 to 49.8 Hz frequency band and thus only for power injections.

Since 2015, monthly tenders are organized to match the ENTSO-E requirement of having 83 MW of FCRs. These 83 MW can either be contracted through the Symmetric200 product and/or a combination of the other products. Table III shows the results of the monthly tenders for 2015. Given these FCR prices, it seems a priori more lucrative for an energy storage to provide FCRs than to do price arbitrage. For each megawatt of FCR, the storage can earn at least 20 €/MW/h while arbitrage requires price spreads of at least 40 € to make the same revenue. However, providing FCRs leads to technical challenges since the power output should constantly be varied.

An energy storage providing FCRs faces discharges because of the provided service; so one way or another, it has to buy electrical energy. The most obvious possibility is to do it on the DAM, preferably when prices are low. In this case, the

FCR provision is somewhat linked to price arbitrage on the DAM. It should however be emphasized that this energy has to be bought the day-ahead, which implies forecasting energy requirements. Making such forecasts is difficult because of the uncertainty on future frequency deviations and thus on the storage's future state of charge. Frequency deviations can lead the storage to charge/discharge more quickly/slowly than expected. When its upper/lower limit is reached and no energy has been sold/bought on the DAM, its only way to keep providing FCRs is to sell/buy energy "in real-time", i.e. on the imbalance market, by offsetting its power injection/offtake.

In this paper we propose to use the ARMA(4,0)-GARCH(1,1) model we have built and generate scenarios of 15-minute time-step frequency deviations, so as to match the imbalance market's time step, and then use these scenarios in two-stage stochastic programs in order to derive, as precisely as possible, the revenues of a perfect-forecasting price-taker storage providing FCRs. The formulation of our stochastic program is given in Fig. 4 for a variable-speed PSH unit. The variable-speed enables the storage to vary its power injections/offtakes, although in a limited range, e.g. from 60% to 100% of the rated power. This means that the storage has to be constantly operating, either in pump or in turbine mode, for example at 80% of its rated power, in order to keep providing FCRs. The PSH unit is more constrained than the battery storage and the former can provide less FCRs for a same rated power, and must be running to provide it. The revenues are thus expected to be lower while complexity and time to solve are higher because of the binary variables. Note that in order for stochastic programming to improve the valuation of energy storage providing FCRs, the precise characteristics of the considered storage should be known.

The problem of Fig. 4 is much more complex than the one of Fig. 3 as frequency-deviation scenarios, depending on the number of scenarios, introduce a huge number of additional binary variables. As it is classically done in economic dispatch [19], one might need to use scenarios reduction techniques in order to get a problem of manageable complexity.

Taking into account the practical operation of the day-ahead market, we propose to run the stochastic program of Fig. 4 for two-week horizons and to implement the first day of the resulting dispatch, along with the realization of frequency deviations. At the end of the first day, the stochastic program should be run for another two weeks, etc to cover one year.

V. CONCLUSIONS

The recent developments in the power industry have created incentives for the development of grid-connected energy storages. Investment in these should target high-value market niches, which includes FCRs provision. Nevertheless, providing this ancillary service is a challenge for storages since it makes them face unexpected energy deviations which have to be compensated for on short-term electricity markets. Based on frequency measurements, an ARMA-GARCH model is built in order to generate series of 15-minute time-step

Sets

H the set of hours h in a two-week period
 N the set of subperiods n
 T a set of time periods t such that $\text{card}(T) = \text{card}(N) \text{ card}(H)$
 S the set of frequency scenarios s , with $s = 0$ the no frequency deviation scenario

Variables

$P_h \forall h$, the binary variables for choosing the turbine/pump mode
 $0 \leq E_{s,t} \leq E_{\max} \forall s, \forall t$, the stored energy in hours
 $IB_{s,t}^{inj}, IB_{s,t}^{off} \forall s, \forall t$, the binary variables for respectively choosing the injection and offtake imbalance

Model

$$\begin{aligned} \max \sum_h (\eta P_h - 1 + P_h) \text{Price}_h - \frac{IBCost}{M \text{card}(S)} \sum_{s,h,n} (IB_{s,Mh+n}^{inj} - IB_{s,Mh+n}^{off}) (1 + \eta) \\ E_{s,Mh+n+1} = E_{s,Mh+n} - (P_h + \eta - \eta P_h) (1 + P_{s,Mh+n}^{FCR}) / M \\ - (IB_{s,Mh+n}^{inj} - IB_{s,Mh+n}^{off}) (1 + \eta) / M \quad \forall s, \forall n, \forall h \mid (h, n) \neq (\text{card}(H), M) \\ IB_{s,Mh+n}^{inj} + IB_{s,Mh+n}^{off} \leq 1 \quad \forall s, \forall n, \forall h \\ E_{s,0} = E_{\max}/2 \quad \forall s \\ E_{\max}/2 - E_{\max}/100 \leq E_{s,Mh+n} \leq E_{\max}/2 + E_{\max}/100 \quad \forall s, \forall h, \forall n \mid (h, n) = (\text{card}(H), M) \end{aligned}$$

Parameters

$\text{Price}_h \forall h$, spot prices in €/MW
 $IBCost$ the average imbalance price
 η the half-cycle efficiency
 $E_{\max}$ the storage limit in hours
 $P_{s,t}^{FCR} \forall s, \forall t$, energy adjustments due to the FCR provision
 $M = \text{card}(N)$

Fig. 4. Revenues maximization of price-taker PSH providing FCRs.

frequency deviations. The model accounts for important grid-frequency features, seasonality, autocorrelation, stochasticity and fat-tailness, and it is implemented in a freely available Python package, enabling the quick generation of scenarios. The paper explains, by posing theoretical bases, how to use such scenarios in two-stage stochastic programs in order to dispatch and value energy storages providing FCRs. Compared to existing valuation techniques, our model is closer to the actual operation of electricity markets and makes use of some important insights regarding the grid frequency.

Further investigations will apply the proposed valuation technique to test cases, including several storage technologies and their precise technical characteristics, as it would enable to better assess its interest. Attention should be paid to the complexity of the problem and scenarios reduction techniques could be considered to manage this complexity.

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