ICTAM 2024

THE 26TH INTERNATIONAL CONGRESS OF THEORETICAL AND APPLIED MECHANICS

Choking of parallel compressible streams with shear and wall friction

30 August 2024

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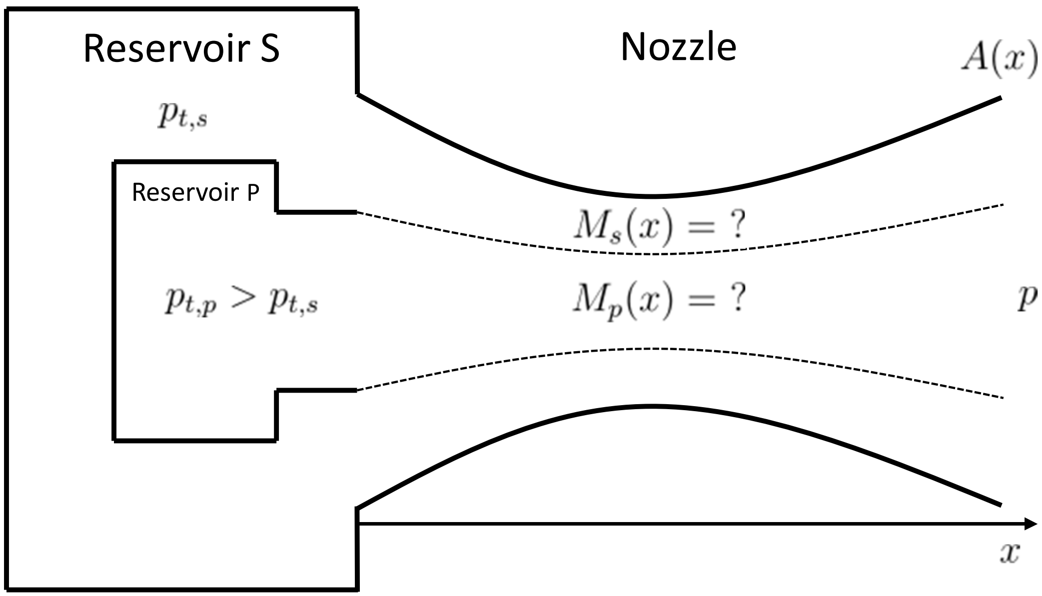
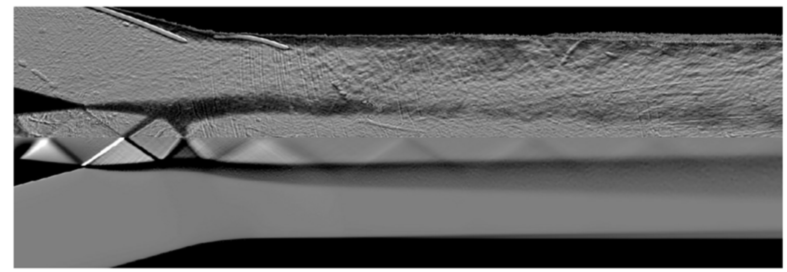
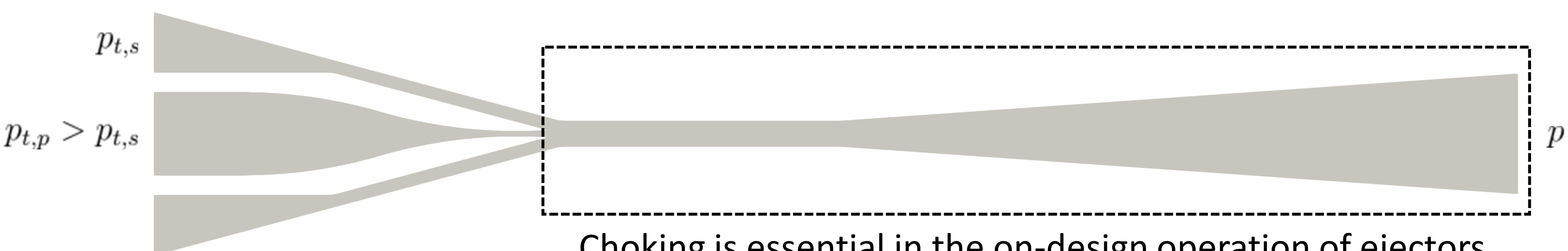
Choking of parallel streams





This problem is inherently 2/3D. [1]

Can the streams and their choking be described with cross-stream averaged quantities (1D)?

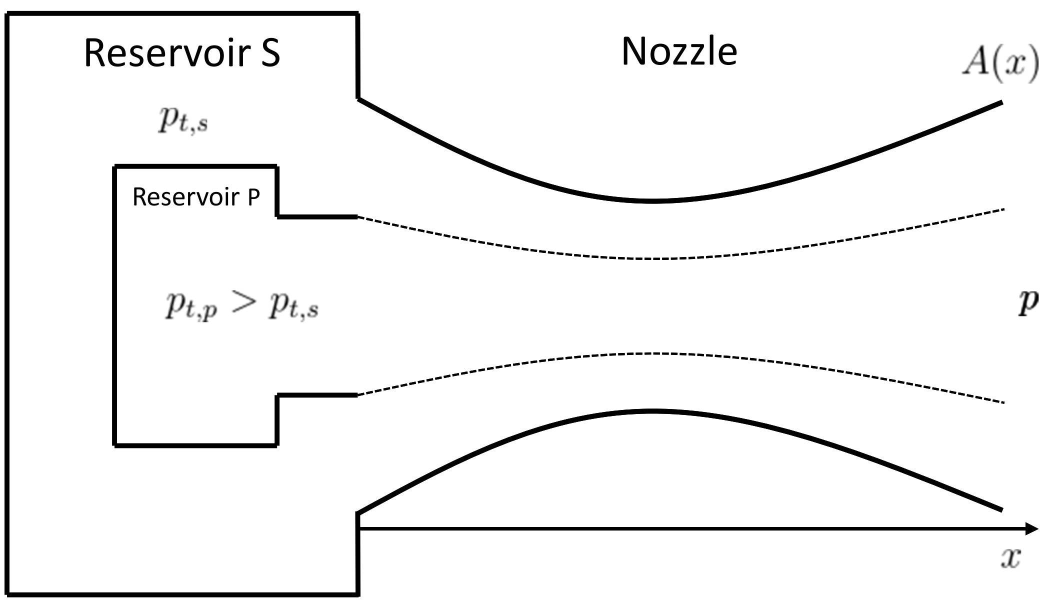


Choking is essential in the on-design operation of ejectors.

Outline

- 1D model definition
- Choking mechanism
- Numerical solution
- Axisymmetric RANS for comparison
- Results
- Conclusion and ongoing work

1D model definition



An extension of the compound flow theory with friction between the streams and at the wall

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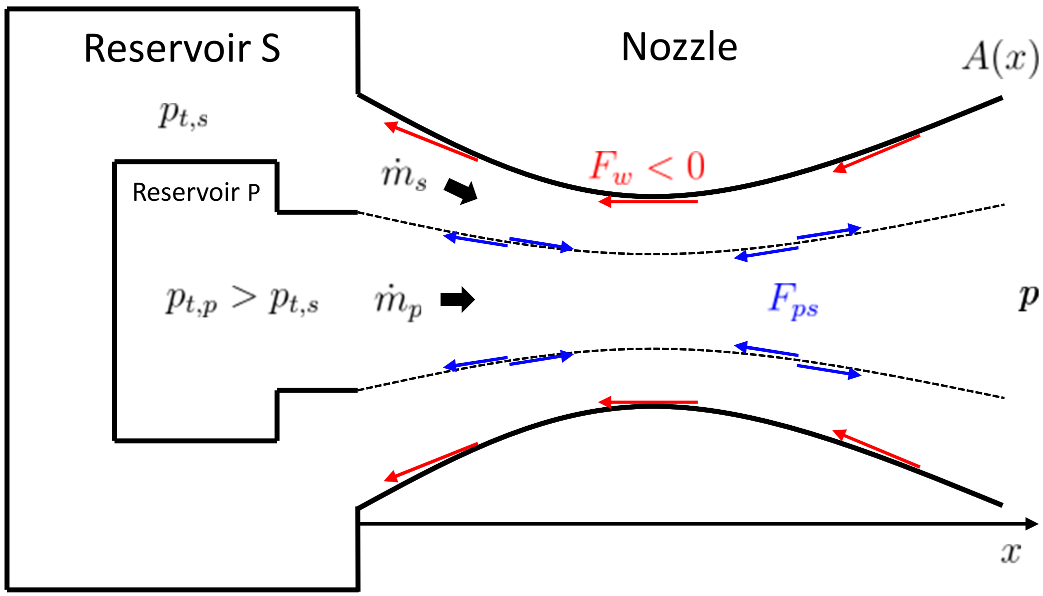
(In revision for JFM)

1D model definition



8 unknowns with 7 equations, so we need an additional constraint.

Conservation equations in each stream:

$$\frac{d}{dx}(\dot{m}_i) = 0$$

$$\frac{d}{dx}(\dot{m}_i u_i) = -A_i \frac{dp}{dx} + F_i$$

$$\frac{d}{dx}(\dot{m}_i h_{t,i}) = 0$$

Definition of the forces:

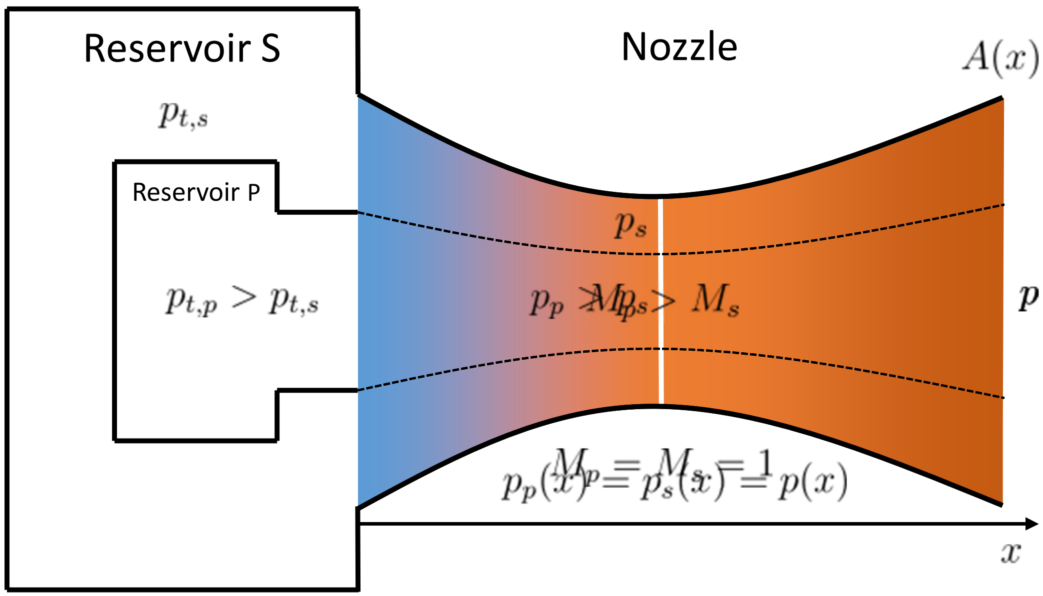
$$F_w = -\frac{1}{2} f_w \left(\rho_s u_s^2\right) l_w$$

$$F_{ps} = \frac{1}{2} f_{ps} \left(\frac{\rho_p + \rho_s}{2} (u_p - u_s)^2\right) l_{ps}$$

Constraint on the cross-sections:

$$A_p(x) + A_s(x) = A(x)$$

1D model definition: additional constraint



The solution for a single stream is invalid.

$$M_p = M_s = 1 \;\Rightarrow\; p_p > p_s$$

The static pressure is uniform across the streams.

$$p_p = p_s \;\Rightarrow\; M_p > M_s$$

There is no section where both streams are sonic.

➡ How do parallel streams choke?

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From the conservation equations to explicit ODEs

Conservation equations in each stream:

$$\begin{cases} \dfrac{d}{dx}(\dot{m}_i) = 0 \\ \dfrac{d}{dx}(\dot{m}_i u_i) = -A_i \dfrac{dp}{dx} + F_i \\ \dfrac{d}{dx}(\dot{m}_i h_{t,i}) = 0 \end{cases} \qquad i \in [p, s]$$

Definition of the forces:

$$\begin{cases} F_w = -\dfrac{1}{2} f_w \left(\rho_s u_s^2\right) l_w \\ F_{ps} = \dfrac{1}{2} f_{ps} \left(\dfrac{\rho_p + \rho_s}{2} (u_p - u_s)^2\right) l_{ps} \end{cases}$$

Constraints:

$$\begin{cases} A_p(x) + A_s(x) = A(x) \\ p_p(x) = p_s(x) = p(x) \end{cases}$$

➡ Choking?

In each stream:

$$\begin{cases} \dfrac{dA_i}{dx} = \left(A_i \dfrac{1 - M_i^2}{\gamma M_i^2}\right) \dfrac{1}{p} \dfrac{dp}{dx} \\ \dfrac{dp_{t,i}}{dx} = p_{t,i} \dfrac{F_i}{pA_i} \\ \dfrac{dT_{t,i}}{dx} = 0 \end{cases} \qquad i \in [p, s]$$

➡ Dividing streamline

For the uniform static pressure:

$$\frac{1}{p} \frac{dp}{dx} = \frac{1}{\beta} \left(\frac{dA}{dx} + N_w + N_{ps} \right)$$

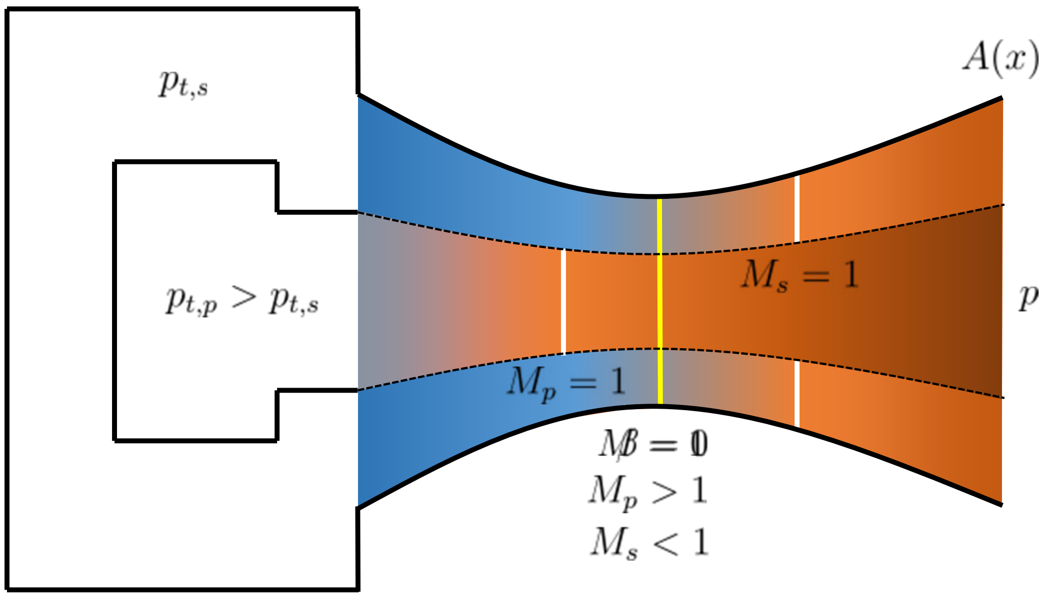
$$\beta = \sum_i A_i \frac{1 - M_i^2}{\gamma M_i^2}$$

"Compound choking indicator"
Singular if $\beta = 0$.

$$N_w = -\frac{f_w l_w}{2} \left(1 + (\gamma - 1) M_s^2\right)$$

$$N_{ps} = \frac{f_{ps} l_{ps}}{2} \left(\frac{M_p^2 - M_s^2}{\gamma M_p^2 M_s^2} \right) \left(\frac{\rho_p + \rho_s}{2} (u_p - u_s)^2 \right) \frac{1}{p}$$

How do parallel streams choke?



Parallel streams can choke with a subsonic and a supersonic component.

Single stream:

$$\frac{1}{p}\frac{dp}{dx} = \frac{1}{1-M^2}\left(\gamma M^2 \frac{1}{A}\frac{dA}{dx}\right) = \frac{1}{A\frac{1-M^2}{\gamma M^2}}\frac{dA}{dx}$$

Sonic: $M = 1 \qquad \frac{dA}{dx} = 0$

Double stream:

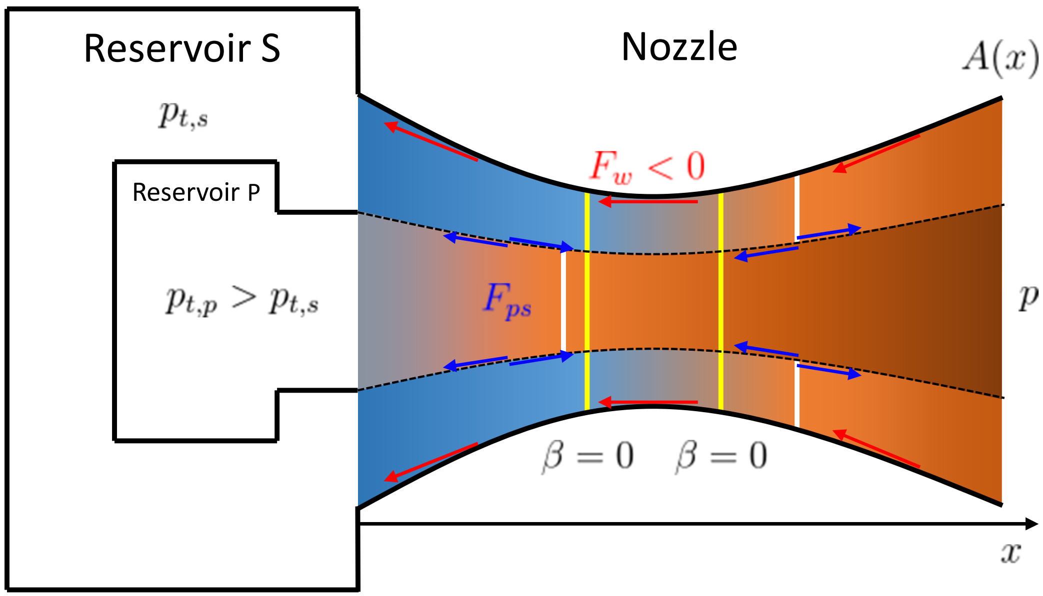
$$\frac{1}{p}\frac{dp}{dx} = \frac{1}{\beta}\left(\frac{dA}{dx} + N_w + N_{ps}\right)$$

Sonic: $\beta = \sum_i A_i \frac{1-M_i^2}{\gamma M_i^2} = 0 \qquad \frac{dA}{dx} = -N_w - N_{ps}$

Possible if: $M_p = M_s = 1$ (single stream)
Or: $M_p > 1 \quad M_s < 1$

[3] Pearson H., Holiday J.B., Smith S.F. A Theory Of the Cylindrical Ejector Propelling Nozzle. The Journal of the Royal Aeronautical Society, 1958.
[4] Bernstein A., Heiser W.H., Hevenor C. Compound-compressible nozzle flow. Journal of Applied Mechanics, 1967.

Effect of the friction forces



$$\frac{1}{p}\frac{dp}{dx} = \frac{1}{\beta}\left(\frac{dA}{dx} + N_w + N_{ps}\right)$$

Sonic: $\beta = \sum_i A_i \frac{1 - M_i^2}{\gamma M_i^2} = 0 \quad \frac{dA}{dx} = -N_w - N_{ps}$

$$N_w = -\frac{f_w l_w}{2}\left(1 + (\gamma - 1) M_s^2\right) \quad < 0$$

$$N_{ps} = \frac{f_{ps} l_{ps}}{2}\left(\frac{M_p^2 - M_s^2}{\gamma M_p^2 M_s^2}\right)\left(\frac{\rho_p + \rho_s}{2}(u_p - u_s)^2\right)\frac{1}{p} \quad > 0$$

Dominant wall friction:

$|N_w| > |N_{ps}| \quad ➡ \quad \frac{dA}{dx} > 0 \quad (N_w < 0)$

(single stream with friction: $N_w < 0$, $N_{ps} = 0$)

Dominant friction between the streams:

$|N_w| < |N_{ps}| \quad ➡ \quad \frac{dA}{dx} < 0$

Parallel streams can become sonic in a convergent section due to friction between the streams.

[2] Van den Berghe J., Mendez M.A., Bartosiewicz Y. An extension of the compound flow theory with shear and friction. In revision for JFM, 2024.

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Numerical solution

Set of ordinary differential equations in space:

$$\frac{1}{p}\frac{dp}{dx} = \frac{1}{\beta}\left(\frac{dA}{dx} + N_w + N_{ps}\right) \qquad \frac{1}{p_{t,i}}\frac{dp_{t,i}}{dx} = \frac{F_i}{pA_i} \qquad \frac{dT_{t,i}}{dx} = 0 \qquad \frac{dA_i}{dx} = \left(A_i\frac{1-M_i^2}{\gamma M_i^2}\right)\frac{1}{p}\frac{dp}{dx}$$

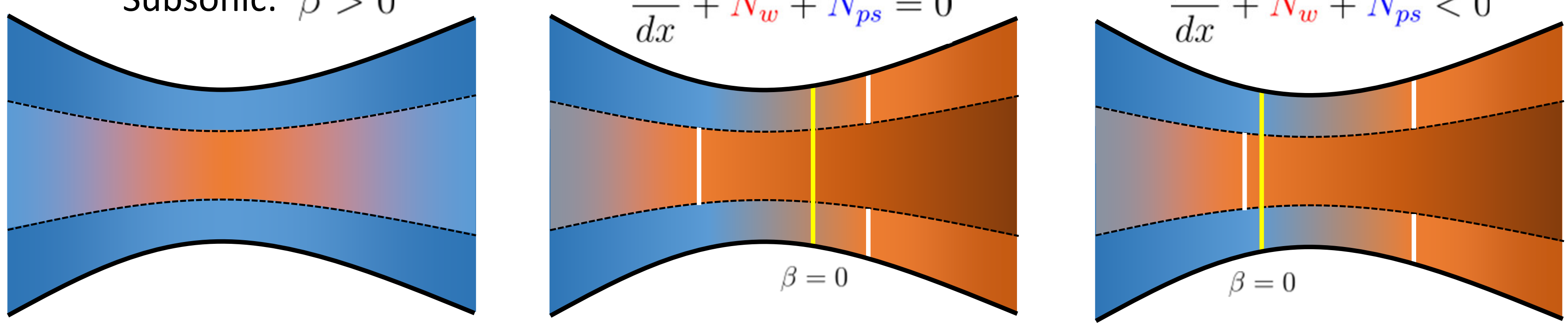
Boundary conditions:

$$[p_{t,p}, p_{t,s}, T_{t,p}, T_{t,s}, A_p, A_s]\,(x=0)$$

p is the only unknown at the inlet!

Solve with a shooting method:

Subsonic: $\beta > 0$ | $\frac{dA}{dx} + N_w + N_{ps} = 0$ | $\frac{dA}{dx} + N_w + N_{ps} < 0$



$p(x=0)$ too high | $p(x=0)$ correct | $p(x=0)$ too low

The singularity of the pressure equation if $\beta = 0$ is avoided using Taylor expansions. [2]

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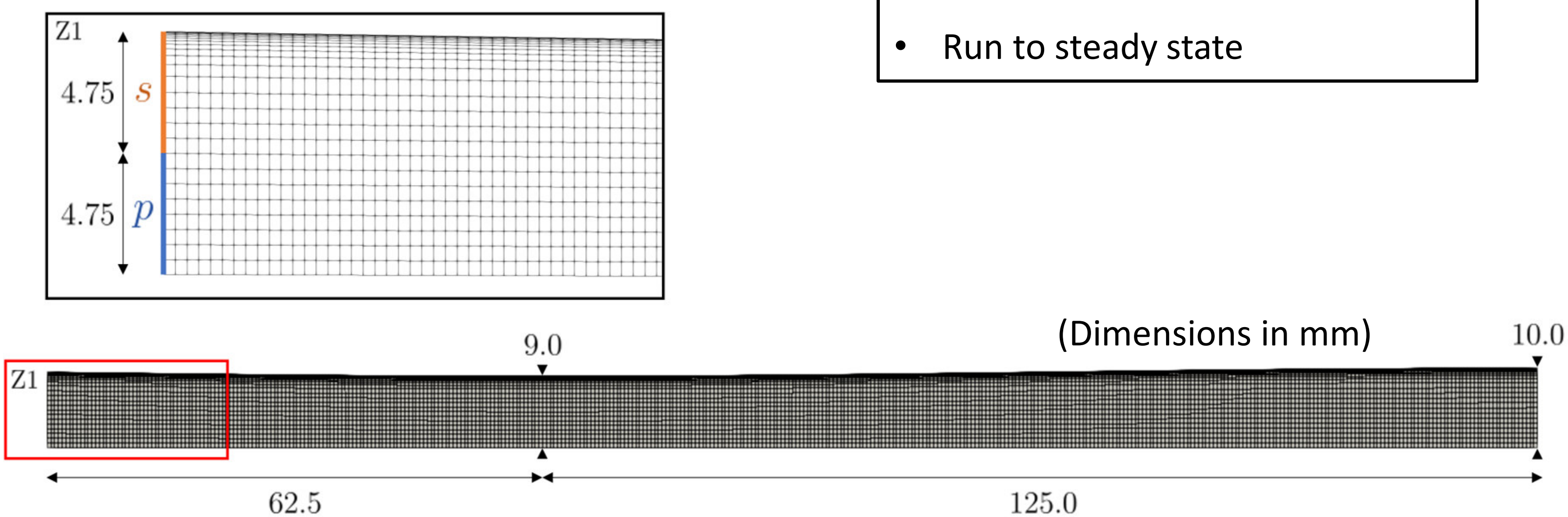
Axisymmetric RANS for comparison

Domain

- Cosine shaped between $[\pi/2\,, 2\pi]$
- Axisymmetric
- Structured mesh (Gmsh)
- Wall-resolved ($y^+ < 1$)

Solver

- rhoCentralFoam, OpenFoam v9
- k-ω SST turbulence model
- No swirl
- Second order in space
- First order in time
- Run to steady state



[2] Van den Berghe J., Mendez M.A., Bartosiewicz Y. An extension of the compound flow theory with shear and friction. In revision for JFM, 2024.

Axisymmetric RANS for comparison

Boundary conditions

Case	$p_{t,p}$ [bar]	p_p [bar]	$T_{t,p}$ [K]	$p_{t,s}$ [bar]	$T_{t,s}$ [K]	TI [%]	l_t [m]	Wall	Comments
1	3.0	1.2	300	1.5	300	n/a	n/a	slip	inviscid
2	3.0	1.2	300	1.5	300	1	$6.65 \cdot 10^{-4}$	slip	viscous
3	3.0	1.2	300	1.5	300	1	$6.65 \cdot 10^{-4}$	no-slip	viscous

Supersonic primary inlet $M_p = 1.2$

Subsonic secondary inlet

Wavetransmissive outlet

$$\begin{cases} F_w = 0 & F_{ps} = 0 \\ F_w = 0 & F_{ps} \neq 0 \\ F_w \neq 0 & F_{ps} \neq 0 \end{cases}$$

Post-processing

1. Dividing streamline from the mass flow rate: $\dot{m}_p(x) = \int_0^{r_d(x)} \rho(x,r)u(x,r)\, 2\pi r\, dr$
2. Averaging over the two cross-sections:

$$\hat{\rho}_i(x) = \frac{1}{A_i}\int_{A_i} \rho\, dA_i \qquad \hat{u}_i(x) = \frac{1}{\hat{\rho}_i A_i}\int_{A_i} \rho u\, dA_i \qquad \hat{e}_{t,i}(x) = \frac{1}{\hat{\rho}_i A_i}\int_{A_i} \rho e_t\, dA_i$$

3. Equivalent Mach number: $M_{eq} = \left(\gamma \frac{\beta}{A} + 1\right)^{-\frac{1}{2}}$ (unity if β = 0)

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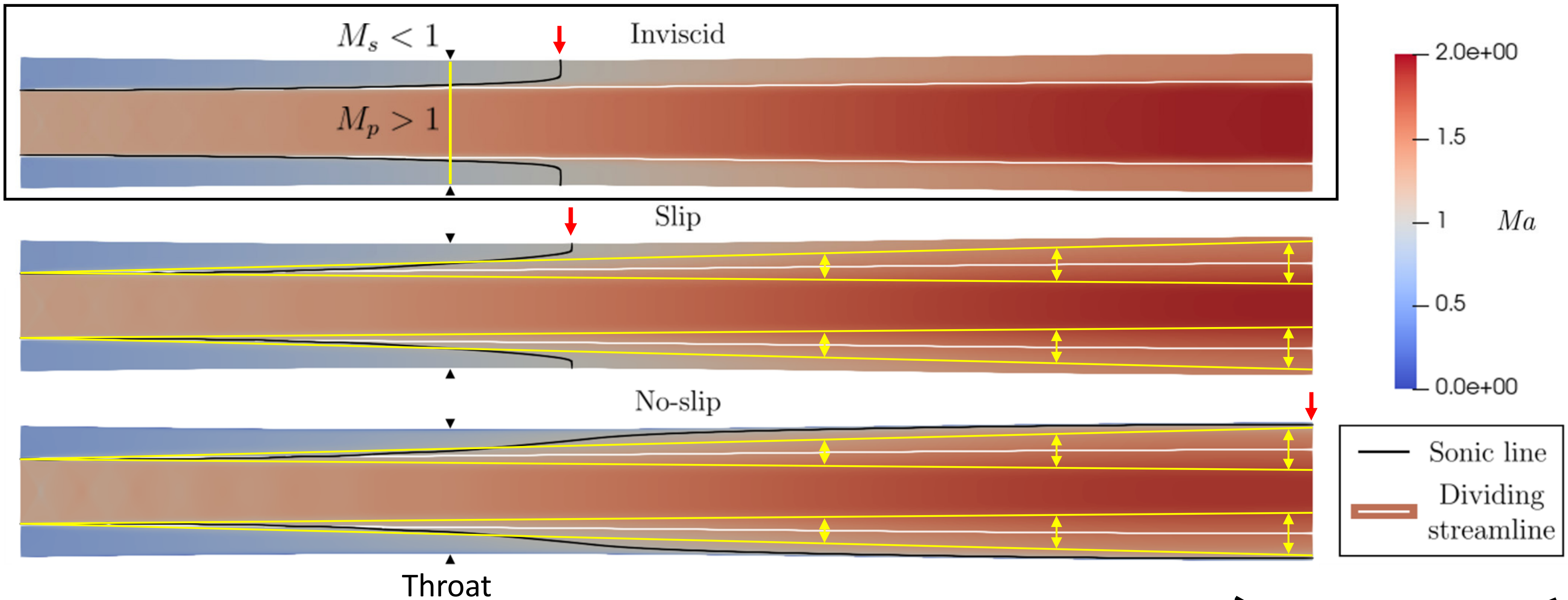
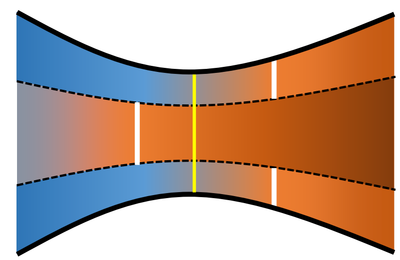
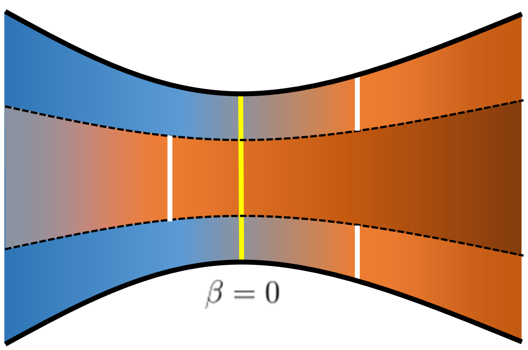
2D flow fields



- The flow field matches our qualitative prediction
- Developing shear layer in the viscous cases
- Developing boundary layer in the no-slip case



1D results: inviscid case

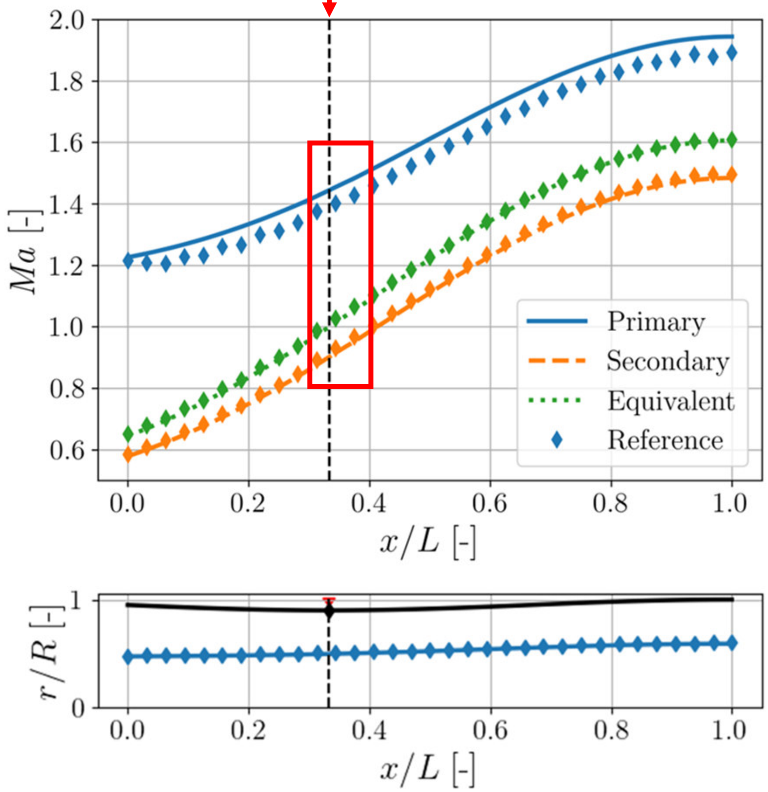


$$\frac{1}{p}\frac{dp}{dx} = \frac{1}{\beta}\left(\frac{dA}{dx} + \cancel{N_w} + \cancel{N_{ps}}\right)$$

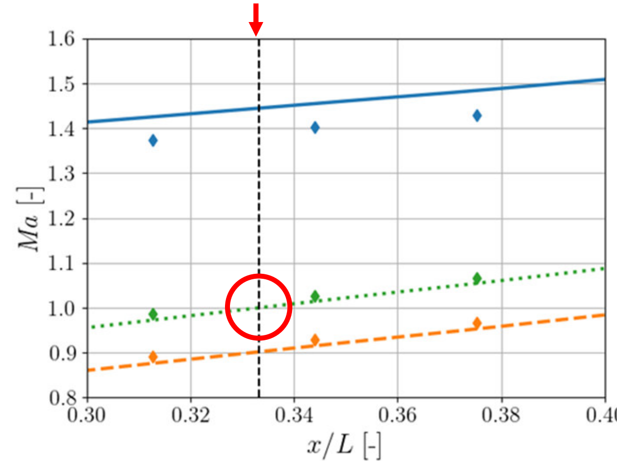
Sonic: $\beta = 0$ $\quad \frac{dA}{dx} = 0$

Sonic section ---

Throat

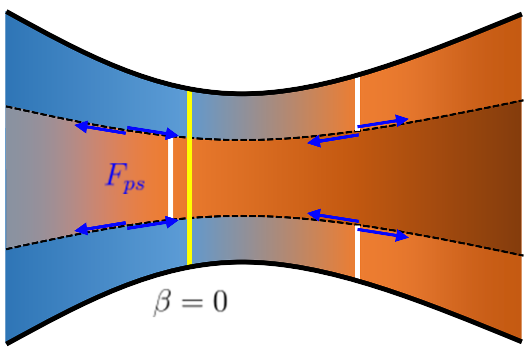


Throat



This confirms the classic isentropic compound choking theory. [3,4]

1D results: viscous case, slip condition

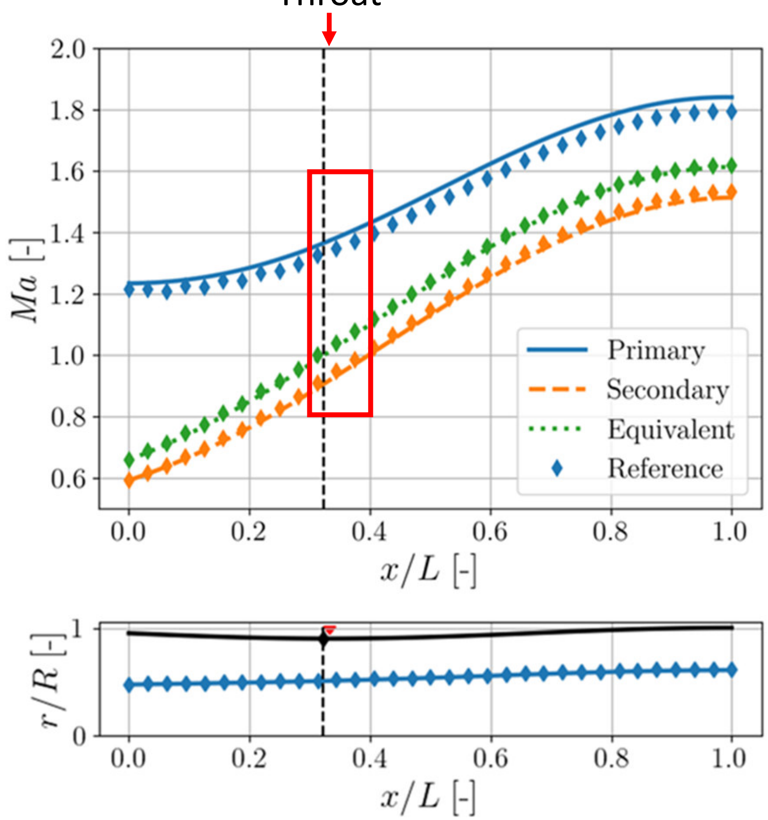


$$\frac{1}{p}\frac{dp}{dx} = \frac{1}{\beta}\left(\frac{dA}{dx} + \cancel{N_w} + N_{ps}\right)$$

Sonic: $\beta = 0$ $\quad \frac{dA}{dx} < 0$

Sonic section ---

Throat



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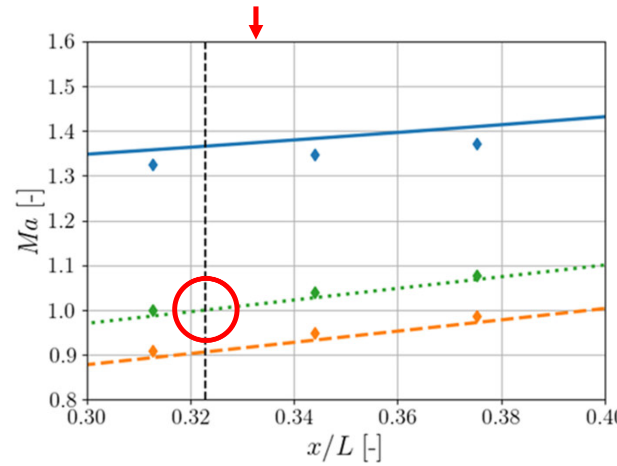
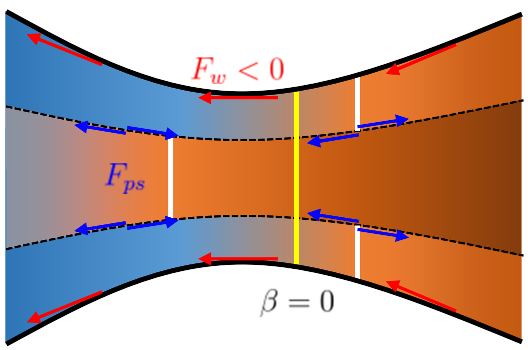


The sonic section moves upstream due to friction between the streams. [2,5]

[5] Lamberts O. et al. The compound-choking theory as an explanation of the entrainment limitation in supersonic ejectors. Energy 2018.

1D results: viscous case, no-slip condition

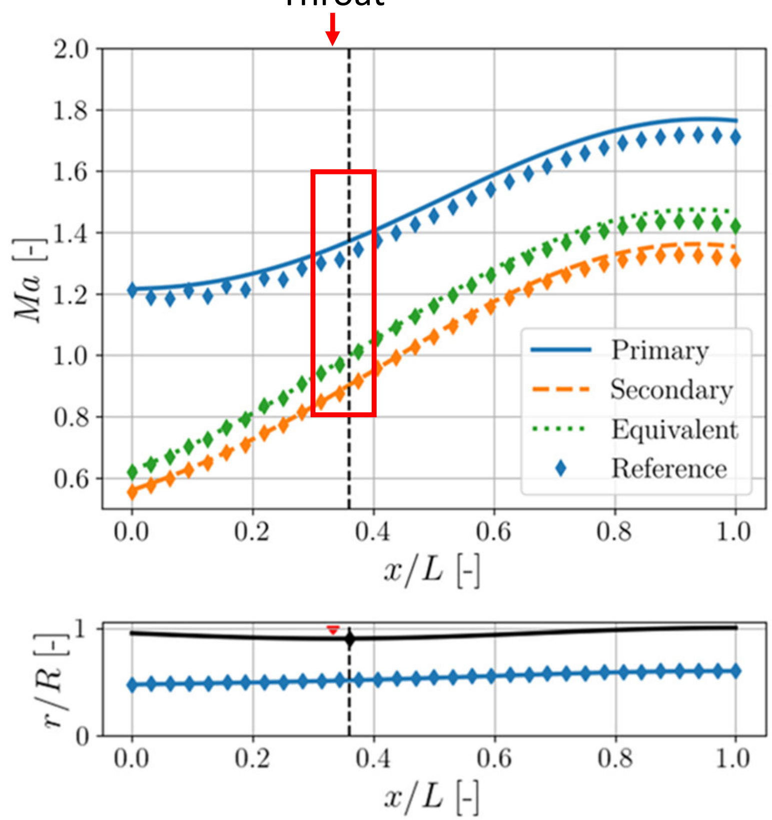


$$\frac{1}{p}\frac{dp}{dx} = \frac{1}{\beta}\left(\frac{dA}{dx} + N_w + N_{ps}\right)$$

$$\beta = 0 \qquad \frac{dA}{dx} = -N_w - N_{ps}$$

Sonic section ---

Throat



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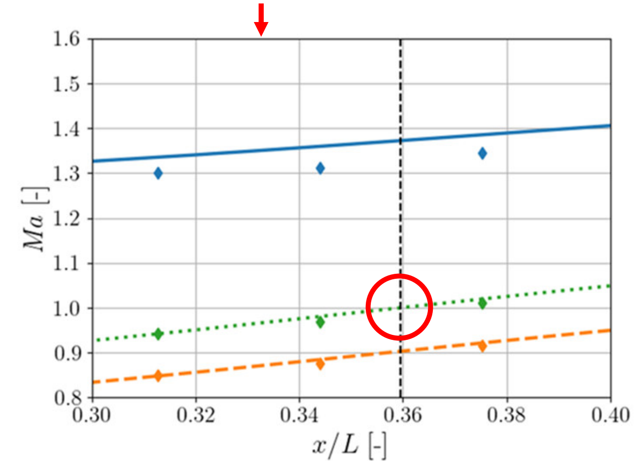
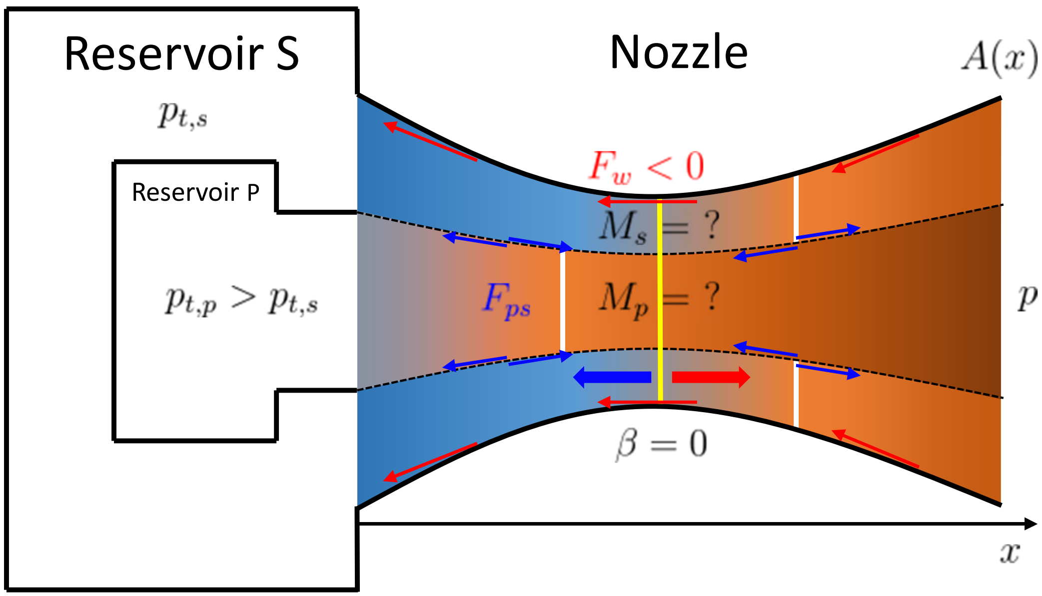


The sonic section moves downstream due to friction at the wall. [2,6]

[6] Kracik J., Dvorak V. Effect of wall roughness on secondary flow choking in supersonic air ejector with adjustable motive nozzle. IJHFF 2023.

Conclusion



Can the streams and their choking be described with cross-stream averaged quantities (1D)?

- Uniform static pressure
- Inter-stream and wall friction

$$\Rightarrow \frac{1}{p}\frac{dp}{dx} = \frac{1}{\beta}\left(\frac{dA}{dx} + N_w + N_{ps}\right)$$

Sonic: $\beta = 0$ $\quad \frac{dA}{dx} = -N_w - N_{ps}$

Choking of parallel streams

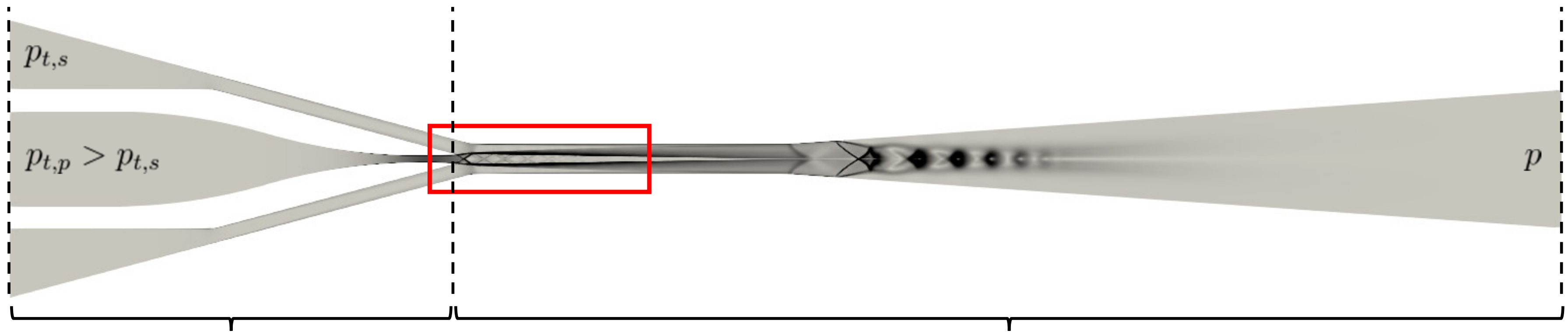
- Both streams are sonic
- A supersonic and a subsonic stream

Displacement of the sonic section

- Upstream due to F_{ps} (inter-stream)
- Downstream due to F_w (wall)

Ongoing work

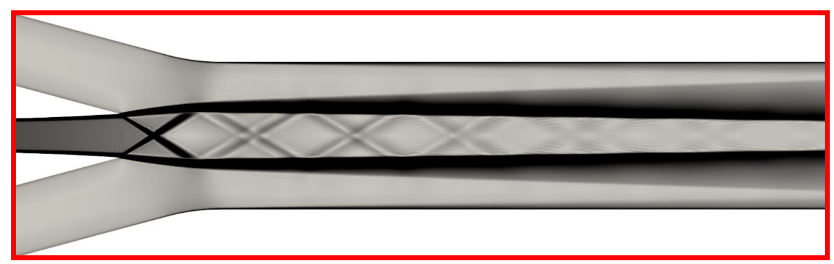
Application to ejector modelling



Two 1D inlets (single stream) 1D compound flow (double stream)

Challenges:

- Shocks at the primary outlet and in the diffuser
- Calibration of the friction forces



References

1. Lamberts O., Chatelain P., Bartosiewicz Y.: New methods for analyzing transport phenomena in supersonic ejectors. International Journal of Heat and Fluid Flow 64: 23-40, 2017.
2. Van den Berghe J., Mendez M.A., Bartosiewicz Y.: An extension of the compound flow theory with shear and friction. In revision for Journal of Fluid Mechanics, 2024.
3. Pearson H., Holiday J.B., Smith S.F.: A Theory Of the Cylindrical Ejector Propelling Nozzle. The Journal of the Royal Aeronautical Society 62: 746–751, 1958.
4. Bernstein A., Heiser W.H., Hevenor C.: Compound-compressible nozzle flow. Journal of Applied Mechanics 34: 548-554, 1967.
5. Lamberts O., Chatelain P., Bartosiewicz Y.: The compound-choking theory as an explanation of the entrainment limitation in supersonic ejectors. Energy 158: 524–536, 2018.
6. Kracik J., Dvorak V.: Effect of wall roughness on secondary flow choking in supersonic air ejector with adjustable motive nozzle. International Journal of Heat and Fluid Flow 103: 109168, 2023.

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