Efficient analysis of a 3D antenna installed on a finite ground plane

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***Abstract*—A fast spectral approach is proposed for the analysis of 3D antenna installed on a finite ground plane itself located on an infinite dielectric layer. The methodology presented here is twofold: on one hand the field radiated by the antenna is calculated using a decomposition into inhomogeneous plane waves, on the other hand the resulting current distribution on the ground plane is determined by applying the Method of Moments (MoM). It is validated for the SKA Log-periodic antenna (SKALA), under study for the Square Kilometer Array (SKA) radio-telescope. The calculated radiation patterns are compared to the ones obtained using the commercial software FEKO at 110 MHz.**

***Index Terms*—Inhomogeneous plane waves, Method of Moments, SKA, FEKO.**

I. Introduction

Methodologies have been developed for the accurate analysis of large planar antennas arrays, such as the HARP software which efficiently measures the effects of mutual coupling dedicated for the SKA low frequency array simulation [1]. These techniques, however, often consider an infinite PEC sheet when there is a ground plane beneath these antennas. This paper provides insight about the impact of a finite ground on the performance of the antenna, in particular the radiation pattern deformation. The field radiated by the antenna on the finite ground plane is computed thanks to a decomposition into inhomogeneous plane waves [2]. This spectral approach allows the description of the interactions with relatively few plane waves. Moreover, the number of inhomogeneous plane waves required to describe these interactions decrease with increasing distance between the antenna currents and the finite ground plane. These properties allow for building fast methods to compute the radiation pattern of a 3D antenna installed on a finite ground plane.

The proposed approach is tested for the case of the log-periodic antenna [3], proposed for the square kilometer array (SKA) low-frequency aperture array (LFAA), a next radio-telescope generation. The SKA aims to achieve a huge sensitivity by using base stations composed of hundreds of elements [4]. The implementation of the telescope is currently taking place in Australia and in South-Africa. The remainder of this paper is organized as follows: Section II gives the useful mathematical definitions. Section III is devoted to the reflection of inhomogeneous plane waves. Section IV explains how the different contibutions to the radiation pattern are calculated and compares them with the ones of the commercial software FEKO [5]. Finally, conclusions are drawn on section V.

II. Mathematical definitions

Let us consider a source radiating in a two semi-infinite medium, as described in Figure 1. The fields radiated by the source can be expressed in terms of the spectral components k_x, k_y and k_z as follows:

$$\begin{aligned}\overrightarrow{E} &= (-\hat{m}\ A_{TE} + \hat{e}\ A_{TM})\, e^{-jk_x x} e^{-jk_y y} e^{\pm jk_z z} \\ \overrightarrow{H} &= \frac{1}{\eta}\,(\hat{e}\ A_{TE} + \hat{m}\ A_{TM})\, e^{-jk_x x} e^{-jk_y y} e^{\pm jk_z z}\end{aligned} \tag{1}$$

where η is the free-space impedance and where the sign $\pm$ corresponds to a wave propagating in the $\mp z$ direction. The direction of propagation is defined as $\hat{u} = \overrightarrow{k}/||\overrightarrow{k}||$ with $\overrightarrow{k} = (k_x, k_y, k_z)$, the wavenumber.

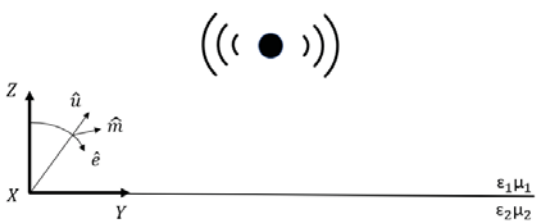

Fig. 1. Coordinates in a two semi-infinite medium.

By defining $\beta = \sqrt{{k_x}^2 + {k_y}^2}$, the norm of the wave vector $\overrightarrow{k}$ reads :

$$||\overrightarrow{k}|| \triangleq k = \sqrt{\beta^2 + {k_z}^2} \tag{2}$$

The $\hat{m}$ and $\hat{e}$ vectors, both orthogonal to the direction of propagation $\hat{u}$ express respectively the TE and TM wave direction of propagation and are defined in [6]. When writing the Green's function as the Weyl integral and convolving with the source currents [6], the radiated electric field can be written as

$$\begin{aligned}E_p(x,y,z) = \frac{-jk\eta}{(2\pi)^2} \iint F_p(k_x,k_y)\ \frac{e^{-j(k_x x + k_y y - k_z z)}}{2jk_z} \\ \left(1 + j\frac{\beta_I}{\beta_R}\right)\left(1 + j\frac{d\beta_I}{d\beta_R}\right)\ dk_{xR}\ dk_{yR}\end{aligned} \tag{3}$$

where p refers to either the TE or the TM component, the factors $\left(1 + j\frac{\beta_I}{\beta_R}\right)\left(1 + j\frac{d\beta_I}{d\beta_R}\right)$ have been introduced to allow for the integration only along the real parts of k_x and k_y while actually performing a path deformation in the complex β plane. $F_p(k_x, k_y)$ is the radiation pattern of the source written as

$$F_p(k_x, k_y) = \iiint_V \overrightarrow{J}(\overrightarrow{r}') \cdot \hat{e}_p \; e^{j(k_x x' + k_y y' - k_z z')} \; dV \tag{4}$$

for a source enclosed in a volume V with coordinates $\overrightarrow{r}' = (x', y', z')$ and with electric current density $\overrightarrow{J}$.
The relation between β_I and β_R corresponds to the complex contour deformation adopted in the decomposition into inhomogeneous plane waves approach. Depending on the distance between the source and the observation points, the choice of the contour can lead to different accuracies of the estimated field for the same number of inhomogeneous plane waves. In our case, the lowest parts of the antenna with respect to the ground plane are placed at a height smaller than $\lambda/15$ when operating in the SKALA frequency band (50 MHz - 350 MHz). At such heights, the well-known SDP contours [2], [7] may provide less accurate results than other contour deformations. In this work, we use the following complex contour deformation [8]:

$$\beta_I = A \; \beta_R \; e^{(\frac{-\beta_R}{k})} \tag{5}$$

where A is a well chosen constant.

III. Reflection of inhomogeneous plane waves

Let us first consider the SKALA above infinite soil as represented in Figure 2 assuming that the soil is a semi-infinite medium with homogeneous permittivity and that it is flat.

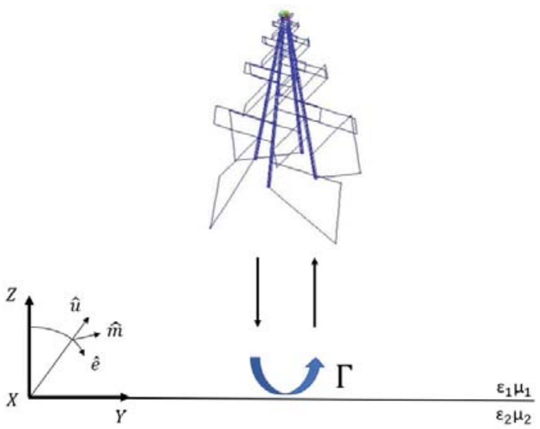

Fig. 2. SKALA above semi-infinite soil.

The fields radiated by the current distribution reflect on the soil and contribute to the total radiation pattern. Given the TE and TM decomposition described in Section II, it appears useful to characterize the waves reflection for these two polarizations.
The TE and TM reflection coefficients defined as the ratio between the reflected and the incoming waves read [9]:

$$\begin{aligned} \Gamma_{TE} &= \frac{ct_1\eta_2 - ct_2\eta_1}{ct_1\eta_2 + ct_2\eta_1} \\ \Gamma_{TM} &= \frac{ct_1\eta_1 - ct_2\eta_2}{ct_1\eta_1 + ct_2\eta_2} \end{aligned} \tag{6}$$

with 1 referring to the upper medium and 2 to the lower medium and

$$ct = \frac{k_z}{k} \tag{7}$$

IV. Evaluation of the different contributions of the radiation pattern and validation with FEKO

This section highlights the steps performed to compute the radiation pattern of the SKALA in presence of a finite ground plane. The total radiation pattern is built from three components:

1) The direct radiation from the antenna.
2) The radiation due to the waves reflected by the soil in the absence of the finite ground plane.
3) The direct radiation from the finite ground plane in the presence of the soil.

Let us first consider the isolated antenna. The direct radiation due to one of the antenna basis function can be expressed as:

$$\begin{aligned} F_{TE}(\hat{u}^+) &= \int \left(\overrightarrow{J}(\overrightarrow{r}') \cdot \hat{m}\right) e^{jkr'\hat{u}^+} dL \\ F_{TM}(\hat{u}^+) &= \int \left(\overrightarrow{J}(\overrightarrow{r}') \cdot \hat{e}\right) e^{jkr'\hat{u}^+} dL \end{aligned} \tag{8}$$

with $\hat{u}^+ = (k_x, k_y, k_z)/k$ taking into account all the waves with elevation angle $\theta \in [0, \frac{\pi}{2}]$. Secondly, the radiation pattern of the antenna above the infinite soil (Figure 2) can be completed by adding the contribution of the waves reflected by the soil in the absence of the ground plane. To account for these waves, equation (8) needs to be considered for directions described by $\hat{u}^- = (k_x, k_y, -k_z)/k$ and the TE,TM components need to be multiplied by their respective reflection coefficients, which are calculated with (6). The radiation pattern due to one of the antenna basis functions then reads:

$$F_p(\hat{u}^+) + \Gamma_p \; F_p(\hat{u}^-) \tag{9}$$

for polarization p.
The results of these equations are gathered for every basis function on the antenna and the resulting radiation pattern is shown and compared to results obtained with the commercial software FEKO in Figure 3.

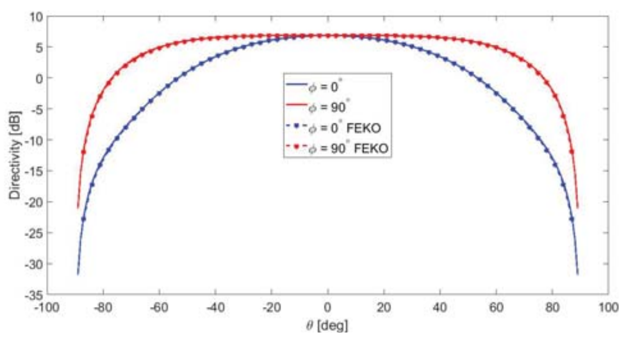

Fig. 3. Radiation pattern of the SKALA above infinite soil at 110 MHz.

The next step consists of putting a circular finite ground plane as represented in Figure 4 and of calculating the incident field on the latter for every current on the antenna. Hence, exploiting the spectral representation (3) for the incident field, the total field to be tested on the ground plane for polarization p (either TE or TM) reads:

$$E_{tp}(x,y,z) = \frac{-jk\eta}{(2\pi)^2} \iint \tilde{E}_p \; (1+\Gamma_p) \\ e^{-j(k_x x + k_y y)} \; dk_{xR} \; dk_{yR} \tag{10}$$

where

$$\tilde{E}_p = F_p(k_x,k_y) \; \frac{1}{2jk_z} \\ \left(1 + j\frac{\beta_I}{\beta_R}\right)\left(1 + j\frac{d\beta_I}{d\beta_R}\right) \tag{11}$$

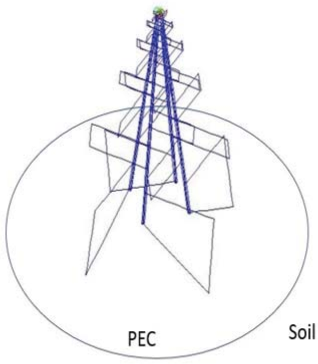

Fig. 4. SKALA antenna above a circular finite ground.

The current distribution on the finite ground plane is derived by solving a Method of Moment system of equations:

$$Z_{gg} i = v \tag{12}$$

where Z_{gg} is the MoM impedance matrix obtained for basis functions located at the interface between the two semi-infinite media. v is the excitation vector corresponding to the projection of the field derived in (11) on RWG basis functions defined on the ground plane. The antenna and the finite ground plane carry respectively $N_A = 1218$ and $N_{FGP} = 1273$ basis functions. The field induced by the antenna on the finite ground plane can be computed relatively fast thanks to the use of the decomposition into inhomogeneous plane waves. Starting from the antenna currents, the table shown in Figure 5 gives some simulation times needed to compute the excitation vector v when using a table of $n_k \times n_k$ inhomogeneous plane waves.

nk	Execution Time [s]
32	0.9
64	2.2
128	8.14
256	32.4

Fig. 5. Execution time for different number of inhomogeneous plane waves n_k when using MATLAB on a 3.2 GHz Intel i5 Core Processor.

The excitation vector v can be computed accurately with $n_k = 64$ when considering a frequency of 110 MHz. The radiation pattern of the ground plane is shown in Figure 6. The contributions of the antenna above the infinite soil and the finite ground plane can be gathered to compute the total radiation pattern. The total radiation pattern considering a 6 m diameter ground plane and corresponding to a frequency of 110 MHz is shown in Figure 7 where it is compared to the results obtained with FEKO. One can see that the results are quite consistent with those obtained with the commercial software.

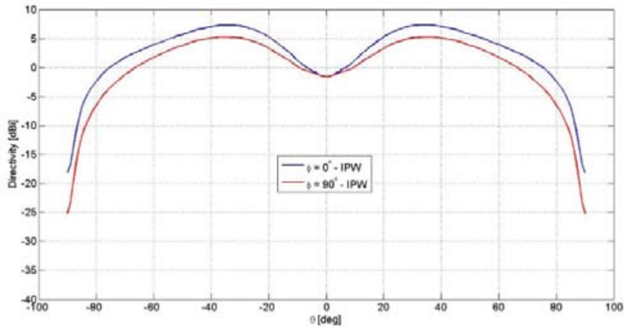

Fig. 6. Radiation pattern of the 6 m diameter ground plane at 110 MHz.

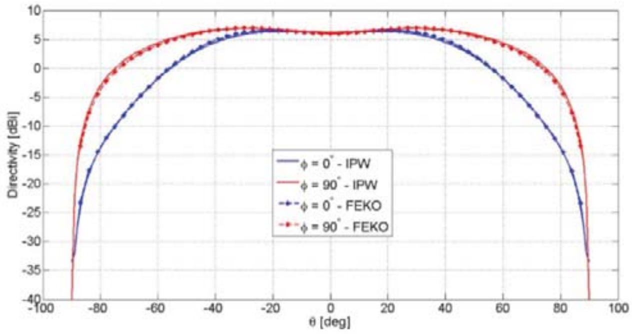

Fig. 7. Total radiation pattern at 110 MHz with a 6 m diameter ground plane.

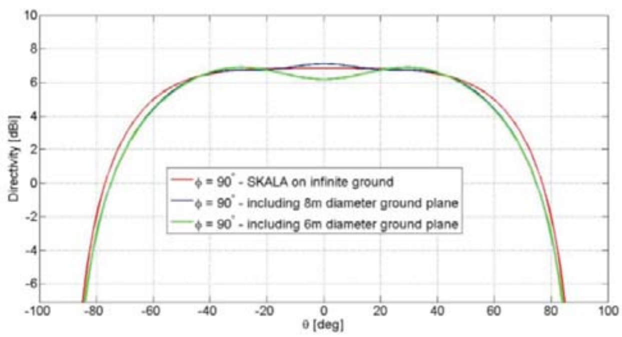

Fig. 8. Comparison between radiation pattern without finite ground, with 6 m diameter ground plane and with 8 m diameter ground plane at 110 MHz.

A comparison with the results considering a 8 m diameter ground plane is shown in Figure 8 considering a frequency of 110 MHz. One can see that the results not only differ when going from the infinite soil case and adding a finite ground plane but also when changing the size of the latter. The variation of the radiation pattern at zenith may vary from the order of one dB with changing diameter size of the finite ground plane.

V. Conclusion

We described a spectral approach to compute the radiation pattern of a 3D antenna above a finite ground plane. The results are consistent with those obtained with the EM commercial software FEKO and provide insight about the radiation pattern deformation due to the presence of a finite ground plane. Although this paper describes the method for a single antenna, it should be interesting to look forward its application for a complete base station of the SKA radio-telescope which is composed of hundreds of SKALA antennas installed on a finite ground plane.

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