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CORE

DISCUSSION PAPER

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Abstract

We consider the problem of assigning agents to slots on a line, where only one agent can be served at a slot and each agent prefers to be served as close as possible to his target. Our focus is on utilitarian methods, i.e., those that minimize the total gap between targets and assigned slots. We first consider deterministic assignment of agents to slots, and provide a direct method for testing if a given deterministic assignment is utilitarian. We then consider probabilistic assignment of agents to slots, and make use of the previous method to propose a utilitarian modification of the classic random priority method to solve this class of problems. We also provide some logical relations in our setting among standard axioms in the literature on assignment prob- lems.

Keywords: probabilistic assignment, random priority, utilitarianism, sd-efficiency, bottleneck.

JEL Classification: C78, D61, D63

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1 Introduction

This paper is concerned with assigning agents to slots in a bottleneck facility where each slot can serve only one agent. Each agent has a preferred slot (target) and wants to be served as close as possible to it. Due to the geometric interpretation, we refer to the problem as one of assigning agents to a line. Our model has potential applications within a wide range of matching problems in which agents are served sequentially and only limited preference information (e.g., the preferred serving time) is available from the agents. For instance, assigning drivers to congested facilities, personnel to work shifts, applicants to job interviews, tenants to public housing projects, tennis players to courts, students to oral exams, etc.

Our main focus in this paper will be on *utilitarian* solutions, i.e, those that minimize the aggregate gap between agents' preferred slot and their allocated slot. In order to explore that issue, we first consider deterministic allocations, and obtain an auxiliary result saying that an allocation is *constrained utilitarian* (i.e., utilitarian with respect to allocations that share the same set of slots) if and only if there is no swap of slots between any two agents that reduces the aggregate gap. We use this result to focus on shifts of assignments to connected segments of slots, and whether they give rise to reductions in the aggregate gap. From here, we obtain a direct characterization of utilitarianism and an intuitive algorithm for checking it, which is tailor-suited to the problem.[1]

Deterministic assignment is usually criticized on equity grounds. If the assigned objects are indivisible then all deterministic assignments will violate any fundamental notion of fairness. A usual course of action in normative economics to deal with this problem is to resort to randomization. Probabilistic assignment, which is commonly used in practice, arises when randomizing over deterministic allocations. The first probabilistic assignment solution that comes to mind is the so-called *random priority* (RP) solution, which assigns objects based on a random ordering of the agents.[2]

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[1]The procedure is thereby an alternative to the standard approach in Combinatorial Optimization, where utilitarianism can be checked by solving the classical linear sum assignment problem, identifying costs with gaps, and then seeing if the aggregate gap of the optimal solution is the same as for the allocation at hand. This is a well-known problem in that literature, typically referred as the *optimality condition for the minimum weight matching of bipartite graphs* (e.g., Schrijver, 2003).

[2]In particular, the first agent in the random order receives his preferred slot.

RP obeys in our setting a fundamental principle of fairness known as *equal treatment of equals*, as well as the basic incentive-compatibility principle of *strategy-proofness*. It is, however, in conflict with efficiency (and, in the case of our setting, even with weak forms of it). An alternative to RP is the so-called *probabilistic serial* (PS) solution, which assigns probability distributions over objects by means of a natural constructive argument (the so-called "eating algorithm") in which each indivisible object is considered as a divisible object of probability shares. PS is sd-efficient and sd-envy-free, although only weakly sd-strategy-proof.[3] As we shall show later, it is not utilitarian in our context; in fact, a utilitarian solution cannot be sd-envy-free.

We suggest in this paper a modified version of RP satisfying equal treatment of equals, which is not only sd-efficient but also utilitarian. Intuitively, the new solution behaves as RP by ordering agents randomly and then assigning them to slots sequentially, but with the important modification of doing so in a way that will ensure that the allocation following each step in the algorithm will be utilitarian among those agents already assigned. The construction of this new solution is partly based on our algorithm for checking utilitarianism of deterministic assignments.

The rest of the paper is organized as follows. In Section 2, we summarize the related literature to this paper on assignment problems. In Section 3, we set the preliminaries of our model, deal with deterministic assignments and present our algorithm to check utilitarianism. In Section 4, we move to probabilistic assignments and introduce our proposed (sd-efficient and utilitarian) solution for this context. In Section 5, we provide some logical relations in our setting among standard axioms in the literature. We conclude in Section 6. We defer some proofs of auxiliary results to an Appendix.

2 Related literature

Assignment problems have long been analyzed in the operations research literature, mostly taking a classical combinatorial optimization approach (see, for instance, Burkard et al., (2012) and the literature cited therein). In the economics literature, however, the interest on assignment problems has mostly relied on issues of efficiency, incentive compatibility and fairness.[4]

[3] As it will be explained later, the *sd* prefix refers to stochastic dominance.

[4] Another concept that might lie at the intersection of both literatures is the notion of stability, which has also received considerable attention since Gale and Shapley (1962) and their analysis of the so-called marriage problem. See also Roth and Sotomayor (1990)

For instance, Hylland and Zeckhauser (1979) deal, in an early an influential contribution, with the general problem of achieving efficient allocations in matching problems in which individuals must be allocated to positions with limited capacities, and propose an algorithm for it based on market-clearing prices, which also guarantees that assignments are *envy-free*, i.e., no agent would prefer the assignment of some other agent. Their algorithm, however, is not *strategy-proof*, i.e., it does not always give agents the incentive to report their true preferences. It turns out, as shown by Zhou (1990), that no solution in such setting satisfies strategy-proofness, (ex-ante) efficiency, and a notion of fairness weaker than no-envy. Zhou (1990) also discussed the RP solution mentioned above (known as *serial dictatorship* by Abdulkadiroglu and Sönmez (1998), among others), which had played the role of a folk solution for a long time, although it had not been considered in the economics literature before.

Bogomolnaia and Moulin (2001) can be considered as the seminal work that sparked the recent interest within the economic literature on assignment problems. They restrict attention to strict preferences and ordinal solutions (i.e., solutions that rely only on ordinal rankings of objects) and introduce the notion of *sd-efficiency* (*ordinal efficiency* in their paper).[5] They characterize, building on the contribution of Cres and Moulin (2001) in a related setting, all sd-efficient assignments and show that sd-efficiency is incompatible with sd-strategy-proofness, and equal treatment of equals.[6] They single out PS as the canonical sd-efficient solution, and show that it is sd-envy-free but not sd-strategy-proof, whereas the opposite holds for RP. As a matter of fact, they show that, for more than three agents, no solution satisfies the two axioms together, along with equal treatment of equals.[7]

PS is nowadays a prominent solution for assignment problems. Besides being popularized by Bogomolnaia and Moulin (2001), it has been characterized, in related settings to theirs, by Bogomolnaia and Moulin (2002, 2004), Katta and Sethuraman (2006), Kesten (2009), Manea (2009), Yilmaz (2009), Che and Kojima (2010), Kojima and Manea (2010), Hashimoto

and the literature cited therein.

[5]As we shall define later, a probabilistic assignment is sd-efficient if it is not stochastically dominated with respect to individual preferences over certain objects.

[6]Abdulkadiroglu and Sönmez (2003) provide an alternative characterization of sd-efficiency. See also McLennan (2002) and Manea (2008) for the relationship between sd-efficiency and ex-ante efficiency.

[7]Such negative result also holds for the restricted domain of single-peaked preferences (e.g., Kasajima, 2012), but not for the restricted domain in which agents have the same preferences, except for the relative position in the rankings of “not being assigned” (e.g., Bogomolnaia and Moulin, 2002).

and Hirata (2011), Heo (2011), Bogolmonaia and Heo (2012) and Kasajima (2012), among others.

In this paper, we study a slightly different version of the model considered by Bogomolnaia and Moulin (2001) and their followers. Not only we allow for weak preferences, as in Katta and Sethuraman (2006), but we actually assume that preferences are *single-peaked*, as in Kasajima (2012), and with the important proviso that they are symmetric with respect to the peak (target). As such, our model is reminiscent of the so-called division problem with single-peaked preferences, initiated by Sprumont (1991). It also touches some recurrent topics in queueing problems (e.g., Dolan, 1978).

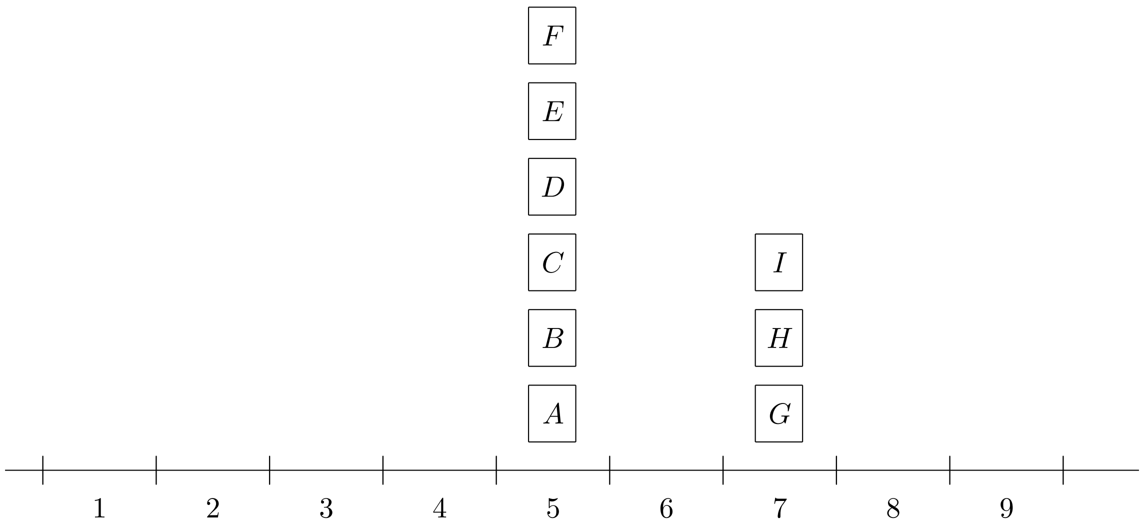
3 Deterministic Assignments

Imagine a facility with a fixed service capacity that can serve one agent at each *slot*. We use the term "slot" which can refer to both a point in time (i.e., a time slot) or a location (i.e., a physical slot) arranged on a line. In the latter case, we could imagine that the situation is one where the agents are showing up at a particular location (slot) before a given deadline, and then they are going to be served as close as possible to the slot they arrived at.

The set of slots is identified by the set of integers. It is assumed that slots are equidistant (in time, or in space).

Agents are labeled by letters $A, B, \ldots$, with generic elements i and j. Each agent i has a preferred slot t_i which we refer to as the agent's *target*. We label the agents so that $t_A \leq t_B \leq \ldots$ A *problem of assigning agents to a line* (in short, a *problem*), consists of a finite number of agents and the list of their corresponding targets (i.e., slots at which they would liked to be served). In general, a problem can be depicted graphically, as in the following example.

Example 1: Six agents wish to be served at slot 5, and three agents wish to be served at slot 7.



We add two modeling assumptions from the outset. First, the identity of agents is irrelevant for solving problems. Second, our methods for solving problems will not depend on how we label the slots.[8]

The two assumptions together permit the use of a concise notation, in which only the number of agents preferring (consecutive) targets are considered. For instance, the problem of Example 1 would be described by the notation $[6, 0, 3]$. Note that 0 is inserted to indicate that there is indeed a slot between the target of six agents and the target of three agents, which itself is not the target of any agent.

An *allocation* (of a problem) is a deterministic assignment of agents to slots. Denote by x_i the slot assigned to agent i in allocation x. We shall often refer to x_i as agent i's outcome (in allocation x).

For each allocation x of a problem we define the *gap* of agent i as $g_i(x) = |t_i - x_i|$, to be interpreted as the disutility of agent i from being assigned the slot x_i while having target t_i. We assume that only the size of the gap matters, i.e., agents' preferences are symmetric around the target.

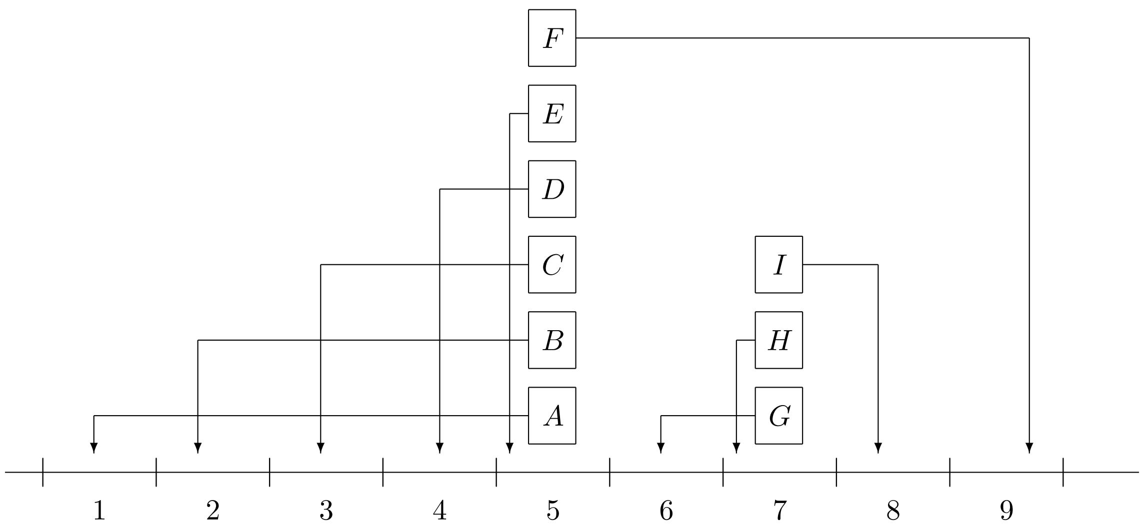
[8] For instance, the problem of Example 1, in which six agents share slot 5 as their target, and three more agents share slot 7 as their target, is solved as the problem in which six agents share slot 9 as their target, and three more agents share slot 11 as their target, except for the corresponding shift in four slots.

An allocation x for a given problem is *utilitarian* if it minimizes the aggregate gap of the agents, $\sum_i g_i(x)$. Similarly, an allocation x for a given problem is *constrained utilitarian* if it minimizes the aggregate gap of the agents, subject to the constraint that only slots assigned in x can be used. As shown by the following example, Pareto efficiency does not even imply constrained utilitarianism.

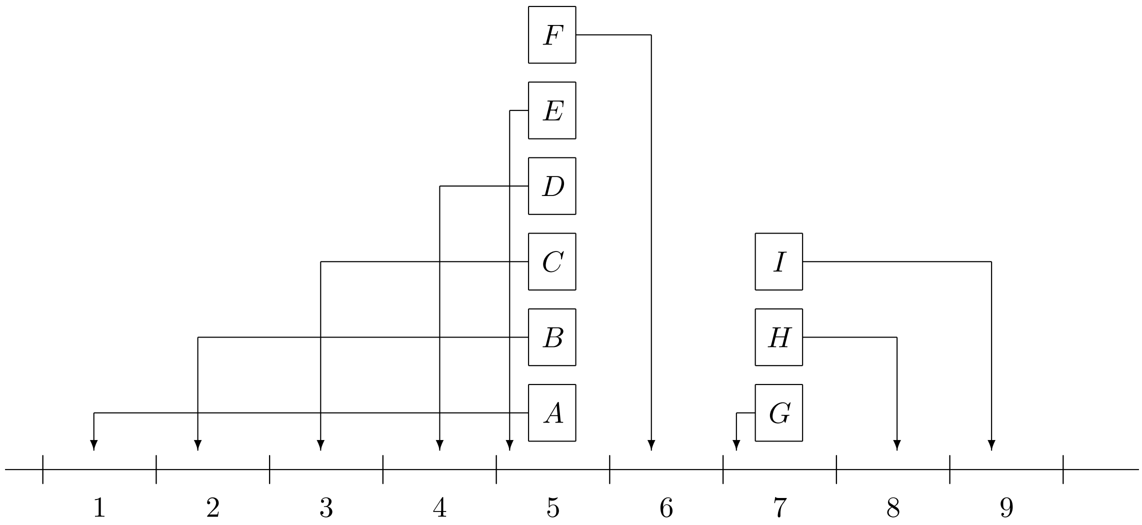
Consider the following two allocations for the problem of Example 1:

Example 1 (cont'd):

Allocation x



Allocation y



We have $\sum_i g_i(x)$ =16 and $\sum_i g_i(y)$=14 . However both assignments are Pareto efficient.

3.1 Utilitarian Check Algorithm

In what follows, we often make use of the following preliminary observations:[9]

For a given allocation x, any other allocation using the same slots can be obtained by a sequence of pairwise swaps of outcomes (e.g., Burkard et al., 2012). In particular, a constrained utilitarian allocation can be obtained from x by sequentially conducting pairwise swaps where each swap is ordering outcomes like the targets. Note also that each such swap is aggregate gap-reducing or aggregate gap-neutral.

We record a few further results which will be useful in the following.

Lemma 1. *Let x be an allocation where assigned slots are ordered like targets. Then, the following statements hold:*

[9] These observations follow from applications of general results for assignment problems satisfying the so-called *Monge property*, (see Burkard et al., 2012, p. 150).

1. *x is constrained utilitarian (and thus any pairwise swap of outcomes is either aggregate gap-neutral or aggregate gap-increasing).*

2. *Any constrained utilitarian allocation using the same slots as in x can be obtained from x by a sequence of aggregate gap-neutral pairwise swaps.*

Lemma 2. *A given allocation y is constrained utilitarian if and only if there does not exist a pairwise aggregate gap-reducing swap of outcomes between two agents.*

Lemma 2 suggests the intuitive algorithm for checking utilitarianism defined below. For the algorithm we further need the following definition: Given an allocation x, a *connected segment* of slots is a set of occupied slots with empty neighbor slots in both ends.

Utilitarian Check Algorithm:
Step 1: Order all outcomes like the targets through some sequence of pairwise swaps (if necessary). If any such pairwise swap is aggregate gap-reducing then conclude that the allocation is not utilitarian. Otherwise, go to Step 2.

Step 2: Given the target ordered allocation produced in Step 1, check all connected segments using the following procedure:

Step 2.A: Label slots in the m-slot connected segment from left to right by $a, b, c, \ldots, m$, and let $A, B, C \ldots, M$ be the agents with outcomes $a, b, c, \ldots, m$ respectively.

Start counting from the left: Define c_i recursively as follows,

$c_1 = -1$ if $t_A \geq a$ and $c_1 = 1$ otherwise

$c_2 = c_1 - 1$ if $t_B \geq b$ and $c_2 = c_1 + 1$ otherwise

$\vdots$

If $c_i > 0$, for some $i \in \{a, \ldots, m\}$, then conclude that the allocation is not utilitarian. Otherwise, go to Step 2.B.

Step 2.B: Perform a procedure similar to that in Step 2.A but this time counting from the right to the left. Formally, define d_i recursively as follows,

$d_1 = -1$ if $t_m \leq m$ and $d_1 = 1$ otherwise

$d_2 = c_1 - 1$ if $t_{m-1} \leq m - 1$ and $d_2 = d_1 + 1$ otherwise

$\vdots$

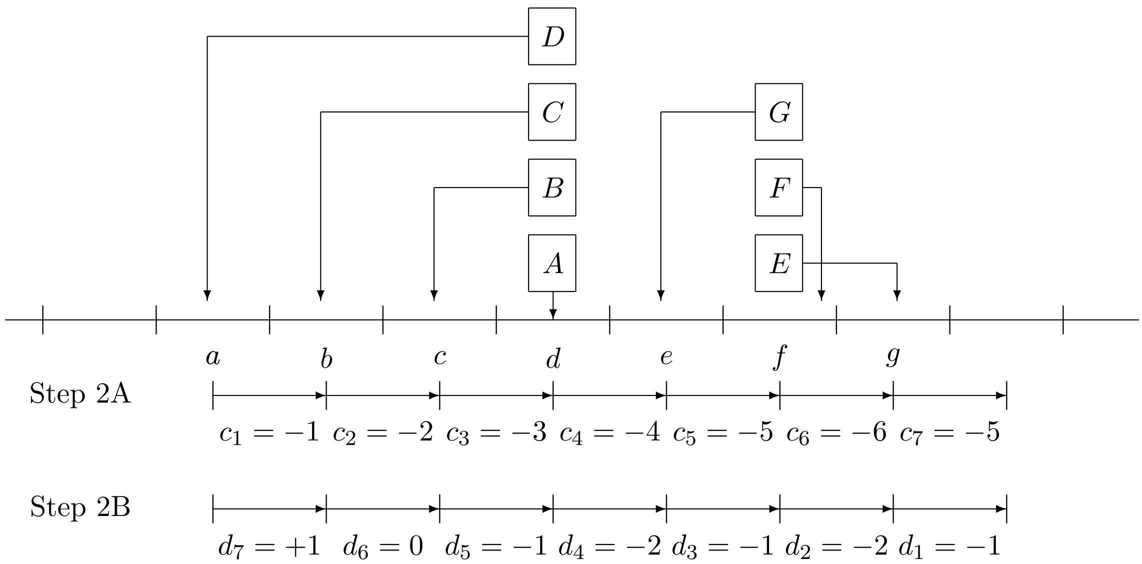
If $d_i > 0$, for some $i \in \{a, \ldots, m\}$, then conclude that the allocation is not utilitarian. Otherwise, go to Step 3.

Step 3: Repeat the process described in Step 2 for any other (non-visited) connected segment.

$\vdots$

Step k: If no further connected segments exist, and the process did not terminate in any of the previous steps, conclude the allocation is utilitarian.

Illustration of the Utilitarianism Check Algorithm for the problem $[4, 0, 3]$



It is worth stressing that our algorithm just described is an alternative to the standard approach in Combinatorial Optimization, where utilitarianism can be checked by solving the classical linear sum assignment problem, identifying costs with gaps, and then seeing if the aggregate gap of the optimal solution is the same as for the allocation at hand. Our algorithm provides a simpler and more intuitive way to check utilitarianism. Furthermore, as we shall see later, the intuition underlying our algorithm can be used to propose a new (probabilistic) utilitarian solution arising from the classical random priority solution.

Theorem 1: *The Utilitarian Check Algorithm works: it finishes in a finite number of steps and concludes that an allocation is utilitarian if and only if this is in fact the case.*

Proof: Suppose a given allocation x has survived the algorithm and denote by y the corresponding allocation after ordering outcomes from x as targets by a sequence of pairwise swaps (cf. Step 1).

Our aim is to show that y, and hence x, is utilitarian.

First, we observe that, for y, all agents with outcomes in a connected segment have targets within the segment. Indeed, if some agent has his target outside the segment there would be an aggregate gap-reducing shift involving this agent and all agents, either before, or after, in the segment, towards the target. This is not the case, as y survived the algorithm.

Now, by contradiction, assume that there exists an alternative allocation z that reduces the aggregate gap compared to y. If z involves several unused slots compared to y, then there also exists an aggregate gap-reducing allocation z' involving a single unused slot. Indeed, we can go from y to z by a number of sequences of reallocations, where each such sequence involves the use of a single slot unused at y.

Without loss of generality, we can assume that the agent being reallocated to an unused slot moves to a neighbor slot to his segment. Indeed, this can be assumed because in any other case the agent that moves to the new slot could benefit from a unilateral change to such an unused neighbor slot of his own segment, which is closer to his target.

Moreover, without loss of generality, we can further assume that all agents involved in the sequence of reallocations leading from y to z' have targets within the same segment. Otherwise, we could obtain at least the same aggregate gap reduction by eliminating agents with targets outside the segment from the sequence.

Now, order the outcomes in z' like the targets (as in y) by a sequence of pairwise swaps and denote this allocation z''. Note that, as all such swaps are either aggregate gap-neutral or aggregate gap-reducing the allocation obtained is also aggregate gap-reducing compared to y. However, it then follows that y would not survive the algorithm, as the allocation z'' corresponds to a shift of outcomes in the relevant connected segment of y; a contradiction. Q.E.D.

4 Probabilistic Assignments

We now move to focus on random (probabilistic) assignments, partially building onto the analysis of deterministic assignments from the previous section. Formally, a *probabilistic assignment* is a specification of marginal probabilities for each agent over slots. Our convention regarding probabilistic assignments will be to disregard *inactive slots*, i.e., slots used with zero probability. Furthermore, if the number of *active slots* (i.e., slots assigned to some agents with a positive probability) is greater than the number of agents, *dummy agents* can be introduced to match the total number of slots. In doing so, a probabilistic assignment will always be represented by a bistochastic matrix.[10]

For a given problem, a *lottery* is a probability distribution over deterministic allocations. It is clear that a lottery induces a probabilistic assignment. Conversely, by the classical Birkhoff-von Neumann decomposition theorem, any probabilistic assignment is consistent with some lottery, although, typically, it is not uniquely determined (e.g., Bapat and Raghavan, 1977).[11]

In this paper, we endorse the approach in the seminal contribution of Bogomolnaia and Moulin (2001) and assume that each agent cares only about his marginal distribution over outcomes (gaps, in our framework).

For any given probabilistic assignment, let p_i^j be the probability that agent i obtains a gap j. A vector $p_i = (p_i^0, p_i^1, ..., p_i^m)$ is a marginal distribution over gaps if $p_i^j \geq 0$ and $\sum p_i^j = 1$. A marginal distribution $p_i = (p_i^0, p_i^1, ...)$ is (first order) *stochastically dominated* by a marginal distribution $s_i = (s_i^0, s_i^1, ..., s_i^n)$ if $\sum_{h=1}^k p_i^k \leq \sum_{h=1}^k s_i^k$ for each $k \leq \max\{m, n\}$, with the convention that $p_i^h = 0$ if $m < h$ and $s_i^h = 0$ if $n < h$.[12] We say that stochastic dominance is *strict* if the inequalities hold with at least one strict inequality. A probabilistic assignment P is stochastically dominated by S if, for each agent, the marginal distribution in S stochastically dominates that in P.

A probabilistic assignment is utilitarian if there exists a lottery consistent with it such that each allocation in its support is utilitarian.[13]

[10]Note that a deterministic assignment is, formally speaking, a one-to-one correspondence between the set of agents and the set of objects. Thus, it could also be convenient to think of a deterministic assignment as a $0-1$ matrix, with rows indexed by agents and columns indexed by objects, and containing exactly one 1 in each row and each column.

[11]See Budish et al., (2012) for further discussion on this issue.

[12]Equivalently, p_i is first order stochastically dominated by s_i if it is possible to obtain p_i from s_i by moving probability mass from larger to smaller gaps.

[13]Equivalently, a probabilistic assignment is utilitarian if, for each lottery consistent with it, each allocation in its support is utilitarian (see Hougaard et al., 2012, for details).

A *solution* is a mapping from problems to probabilistic assignments. We say that a solution satisfies one of the above properties if each probabilistic assignment it gives rise to satisfies that property. In what follows, we only consider solutions satisfying *equal treatment of equals*, which says that agents with the same target should get the same probability distribution over slots, *slot invariance*, meaning that the solution does not depend on the labeling of slots (as mentioned in Section 3), and *symmetry*, which says that if we "flip" the problem from left to right, the corresponding solution flips too.

4.1 A canonical solution

We now introduce a probabilistic assignment obtained from a central solution in the literature: the so-called *EPS algorithm* (e.g., Bogolmonaia and Moulin, 2001; Katta and Sethuraman, 2006). Such algorithm (adapted to our framework) works as follows.[14]

1. Give each agent a probability share of his most preferred slot, as large as possible, given the constraint that each agent must receive the same probability share. (Thus, the slots that are the most popular targets will be completely shared). Remove those slots that have been completely shared.

2. Give each agent a probability share of his most preferred slots among those not removed, as large as possible, given the constraint that each agent must receive the same total probability share. Remove slots that have been completely shared.

$\vdots$

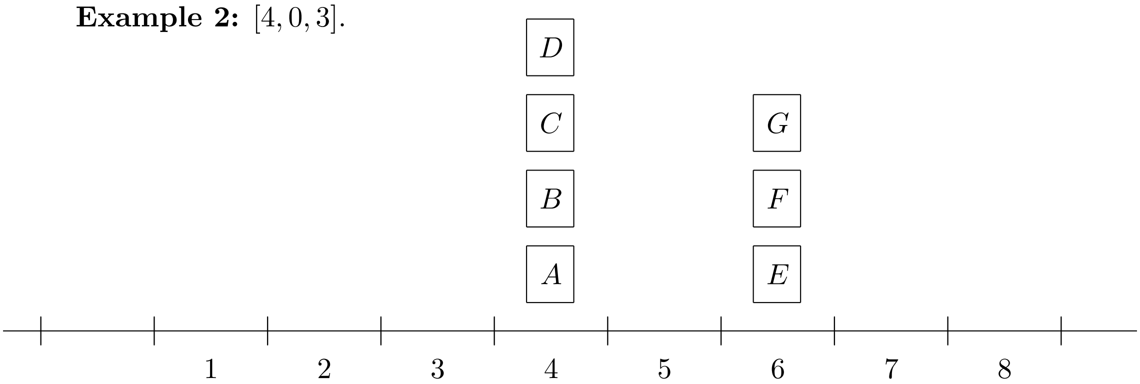
The algorithm stops when all agents have obtained marginal probabilities over slots (that sum to 1).[15]

The following example shows that this canonical (sd-efficient) solution fails to be utilitarian.

[14] We refer to Katta and Sethuraman (2006) for a more formal treatment of the EPS algorithm. The algorithm is similar to the older ones used in processor-sharing (e.g., Kleinrock, 1967).

[15] In general, each step can be handled by solving an appropriately defined max flow problem, as described by Katta and Sethuraman (2006).

Example 2: $[4, 0, 3]$.



The EPS algorithm yields the following probabilistic assignment for this example:

agent / slot	1	2	3	4	5	6	7	8
A	3/84	1/4	1/4	1/4	9/42			
B	3/84	1/4	1/4	1/4	9/42			
C	3/84	1/4	1/4	1/4	9/42			
D	3/84	1/4	1/4	1/4	9/42			
E					1/21	1/3	1/3	6/21
F					1/21	1/3	1/3	6/21
G					1/21	1/3	1/3	6/21

Hence, with probability 1/7, an allocation occurs that does not minimize aggregate gap. More precisely, the canonical sd-efficient solution - the EPS solution - is not utilitarian, i.e., it does not minimize the average gap (over all agents) for each possible realization.

4.2 A new utilitarian solution

Motivated by the observation that the EPS solution (the canonical sd-efficient solution) fails to be utilitarian, we introduce in this section a solution which is not only sd-efficient but also utilitarian.

A simple and intuitive way of assigning agents to slots probabilistically is by means of the random priority (RP) solution: First, pick an agent randomly and assign him to his most preferred slot. Then, pick another agent randomly (among the remaining agents) and assign him to a preferred slot (among those still free), and so on until all agents have been assigned to slots. Note that, in our model, the RP solution fails to be even ex-post efficient (think, for instance, of the example [2,1]).[16] However, we will provide a modification which is not only ex-post efficient but also utilitarian (and hence also sd-efficient). Intuitively, the *modified random priority* solution behaves as RP by ordering agents randomly and then assigning them to slots sequentially, but with the important modification that it does so in a way that will ensure that the allocation following each step in the algorithm will be utilitarian among those agents already assigned.

More precisely, the modified RP works as follows: Order agents randomly. Assign the first agent his target. Assign the second agent a most preferred free slot; if there are two such slots pick one of them with equal probability, ... Suppose that $k-1$ agents have been assigned, and consider now the next agent k in the order. If his target is free, assign him the target and go to the next agent. If his target is not free, we consider two allocations constructed as follows:

(a) Assign k to the slot furthest to the right that is occupied by some agent i with target to the left of k and who is assigned to an outcome to the right of his own target (if no such outcome exists, assign k to the first free slot to the left of his target, and go to (b)). Assign agent i to the slot furthest to the right that is occupied by some agent j with target to the left of i and who is assigned to an outcome to the right of his own target (if no such outcome exists, assign j to the first free slot to the left of his target, and go to (b)), etc. until some agent has been assigned the first free slot to the left.

(b) We now consider the symmetric counterpart construction involving agents having target to the right of k's target that stops when an agent has been assigned the first free slot to the right of k's target.

Now, if the two allocations in (a) and (b) respectively give rise to the same aggregate gap among assigned agents $1, 2, ..., k$, choose one of them with equal probability. Otherwise, choose the one with lowest aggregate gap among assigned agents $1, 2, ..., k$. Formally,

Modified Random Priority Algorithm:

Let N denote the set of all agents in a given problem.

Step 1: Pick an agent randomly with equal probability among N, and denote him agent A_1. Let $x^1_{A_1} = t_{A_1}$.

[16] In the original setting of Bogomolnaia and Moulin (2001), in which agents have strict preferences, the RP solution is always ex-post efficient.

$\vdots$

Step k: Suppose that the agents $A_1, ..., A_{k-1}$, have already been chosen randomly in steps 1,..., $k-1$ and assigned to slots $x^{k-1}_{A_1}, ...x^{k-1}_{A_{k-1}}$. Pick an agent randomly from the set $N\backslash\{A_1, ..., A_{k-1}\}$ with equal probability, and denote him A_k.

Let α be the unoccupied slot closest to t_{A_k} for which $\alpha \leq t_{A_k}$ and let γ be the unoccupied slot closest to t_{A_k} for which $t_{A_k} \leq \gamma$.

If $\alpha = \gamma$ let $x^k_{A_k} = t_{A_k}$, and $x^k_i = x^{k-1}_i$ for each $i \in \{A_1, ..., A_{k-1}\}$ and go to step $k+1$.

Otherwise, let $z_{A_k} = \max\{z \mid z = \alpha$, or $x^{k-1}_i = z, t_i < t_{A_k}, t_i < x^{k-1}_i$, for some $i \in \{A_1, ..., A_{k-1}\}\}$. Let l_1 denote the agent in $\{A_1, ..., A_{k-1}\}$ for which $x^{k-1}_{l_1} = z_{A_k}$. Let $z_{l_1} = \max\{z|z = \alpha$, or $x^{k-1}_i = z, t_i < t_{l_1}, t_i < x^{k-1}_i$, for some $i \in \{A_1, ..., A_{k-1}\}\backslash\{l_1\}\}$. Let l_2 denote the agent for which $x^{k-1}_{l_2} = z_{l_1}$, define $z_{l_2} = \max\{z|z = \alpha$, or $x^{k-1}_i = z, t_i < t_{l_1}, t_i < x^{k-1}_i$, for some $i \in \{A_1, ..., A_{k-1}\}\backslash\{l_1, l_2\}\}$ etc., until we have defined agents, $l_1, \ldots, l_r$ such that $t_{A_k} > t_{l_1} > t_{l_2} > \ldots$, with $z_{l_r} = \alpha$. Now, define

$$g(l_h) = \begin{cases} z_{l_{h-1}} - z_{l_h}, & \text{if } t_{l_h} \leq z_{l_h} \\ (t_{l_h} - z_{l_h}) - (z_{l_{h-1}} - t_{l_h}), & \text{if } t_{l_h} > z_{l_h}, \end{cases}$$

with the convention that $z_{l_0} = z_{A_k}$. Intuitively, $g(z_{l_h})$ measures the gain (possibly negative) that l_h gets from being reallocated from $z_{l_{h-1}}$ to z_{l_h}. Define $G = \sum_{h=1,\ldots,r} g(l_h)$, with the convention that $G = 0$ if $z_{A_k} = \alpha$.

As a natural mirror-image of the above definitions, we also define the following. Let $q_{A_k} = \min\{z|z = \gamma$, or $x^{k-1}_i = z, t_i > t_{A_k}, t_i > x^{k-1}_i$, for some $i \in \{A_1, ..., A_{k-1}\}\}$. Let m_1 denote the agent in $\{A_1, ..., A_{k-1}\}$ for which $x^{k-1}_{m_1} = q_{A_k}$. Let $q_{m_1} = \min\{z|z = \gamma$, or $x^{k-1}_i = z, t_i > t_{m_1}, t_i > x^{k-1}_i$, for some $i \in \{A_1, ..., A_{k-1}\}\backslash\{m_1\}\}$. Let m_2 denote the agent for which $x^{k-1}_{m_2} = q_{m_1}$, define $q_{m_2} = \min\{z|z = \gamma$, or $x^{k-1}_i = z, t_i > t_{m_1}, t_i > x^{k-1}_i$,for some $i \in \{A_1, ..., A_{k-1}\}\backslash\{m_1, m_2\}\}$ etc., until we have defined agents, $m_1, ..., m_s$ such that $t_{A_k} < t_{m_1} < t_{m_2} < ..$, with $q_{m_s} = \gamma$. Now, define

$$g(m_h) = \begin{cases} q_{m_h} - q_{m_{h-1}}, & \text{if } z_{m_h} \leq t_{m_h} \\ (q_{m_h} - t_{m_h}) - (q_{m_h} - q_{m_{h-1}}), & \text{if } t_{m_h} < z_{m_h}, \end{cases}$$

with the convention that $q_{m_0} = q_{A_k}$. Intuitively, $g(m_h)$ measures the gain (possibly negative) that m_h gets from being reallocated from $q_{l_{h-1}}$ to q_{l_h}. Define $H = \sum_{h=1,\ldots,s} g(m_h)$, with the convention that $H = 0$ if $z_{A_k} = \gamma$.

Now, we consider three cases:

(1) If $|z_{A_k} - t_{A_k}| - G > |z_{A_k} - t_{A_k}| - H$, let $x^k_{A_k} = z_{A_k}$ and $x^k_{l_h} = z_{l_h}$, for each $h = 1, ...r$, and $x^k_i = x^{k-1}_i$ for $i \in \{A_1, ..., A_{k-1}\}\backslash\{l_1, ..., l_r\}$.

(2) If $|z_{A_k} - t_{A_k}| - G < |z_{A_k} - t_{A_k}| - H$, let $x^k_{A_k} = q_{A_k}$ and $x^k_{l_h} = q_{m_h}$, for each $h = 1, ...s$, and $x^k_i = x^{k-1}_i$ for $i \in \{A_1, ..., A_{k-1}\} \backslash \{m_1, ..., m_s\}$.

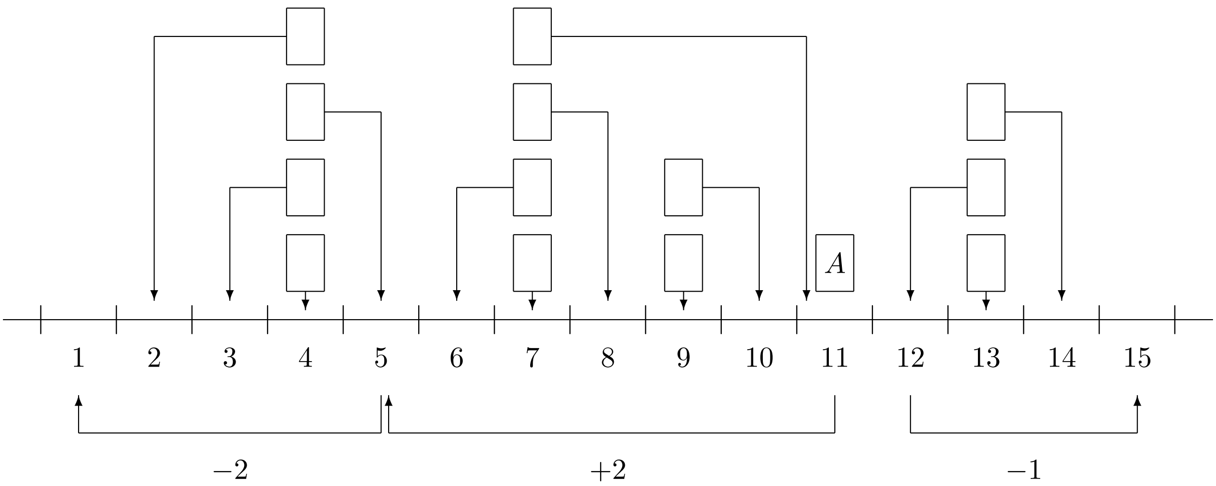
(3) If $|z_{A_k} - t_{A_k}| - G = |z_{A_k} - t_{A_k}| - H$ choose either the allocation in (1) or (2) with equal probability.

Go to step $k + 1$.

Stop the algorithm when all agents have been assigned to an outcome.

The next figure illustrates a situation where $k = 14$, i.e., 13 agents have been assigned to slots by way of the algorithm. A new agent, A, with target at 11, has to be assigned to a slot. There are two options, either assign him to target 11 and push two agents to the left, or assign him to slot 12 and push an agent to the right. The arrows below the lines show how they are going to be pushed according to the algorithm and the associated gain/loss from doing so. Clearly, the outcome in this example will be that the agent is assigned to slot 11.

Illustration of the Modified *RP* algorithm



It is clear from the construction of the modified RP algorithm that the solution it induces satisfies the three basic assumptions that we imposed from the outset; namely, equal treatment of equals, slot invariance and symmetry. As desired, it is also utilitarian:

Theorem 2: *The Modified RP solution is utilitarian.*

Proof: We need to show that any given allocation $x = (x_1, \ldots, x_n)$ obtained from applying the Modified RP algorithm is utilitarian.

Note first that allocation $x^1_{A_1} (= t_{A_1})$ is utilitarian for the single-agent problem consisting of one agent A_1 with target t_{A_1}.

Next, suppose that $x^{k-1}_{A_1}, ..., x^{k-1}_{A_{k-1}}$ obtained in Step $k-1$ is utilitarian with respect to the set of agents $\{A_1, ..., A_{k-1}\}$ (with targets $t_{A_1}, ..., t_{A_{k-1}}$ respectively). We aim to show that then $x^k_{A_1}, ..., x^k_{A_k}$ obtained in Step k is utilitarian with respect to the set of agents $\{A_1, ..., A_k\}$ (with targets $t_{A_1}, ..., t_{A_k}$ respectively).

For this, we first argue that $x^k_{A_1}, ..., x^k_{A_k}$ is constrained utilitarian. Indeed, suppose that $x^k_{A_1}, ..., x^k_{A_k}$ fails to be constrained utilitarian. Then by application of Lemma 2 there are agents i, j in $\{A_1, ..., A_k\}$ such that $t_i < t_j$, $x_j < x_i, t_i < x_i$ and $x_j < t_j$. However, by construction of step k in the algorithm this cannot happen. Thus, $x^k_{A_1}, ..., x^k_{A_k}$ is constrained utilitarian.

Next, we observe that if a utilitarian allocation can be obtained from using the slots already occupied in $x^{k-1}_{A_1}, ..., x^{k-1}_{A_{k-1}}$ and then either the slot α or the slot γ, the allocation $x^k_{A_1}, ..., x^k_{A_k}$ is utilitarian. For this note that if a utilitarian allocation can be obtained from using the slots already occupied in $x^{k-1}_{A_1}, ..., x^{k-1}_{A_{k-1}}$ and the slot α, but not the slot γ, we have $|z_{A_k} - t_{A_k}| - G \geq |z_{A_k} - t_{A_k}| - H$ and as $x^k_{A_1}, ..., x^k_{A_k}$ is constrained utilitarian, it is also utilitarian. Likewise, if a utilitarian allocation can be obtained from using the slots already occupied in $x^{k-1}_{A_1}, ..., x^{k-1}_{A_{k-1}}$ and the slot γ, but not the slot α, we have $|z_{A_k} - t_{A_k}| - G \leq |z_{A_k} - t_{A_k}| - H$ and, as $x^k_{A_1}, ..., x^k_{A_k}$ is constrained utilitarian, it is also utilitarian. Finally, if a utilitarian allocation can be obtained from using the slots already occupied in $x^{k-1}_{A_1}, ..., x^{k-1}_{A_{k-1}}$ and either the slot α or γ, we have $|z_{A_k} - t_{A_k}| - G = |z_{A_k} - t_{A_k}| - H$ and, as $x^k_{A_1}, ..., x^k_{A_k}$ is constrained utilitarian, it is also utilitarian.

It remains to argue that a utilitarian allocation can be obtained from using the slots already occupied in $x^{k-1}_{A_1}, ..., x^{k-1}_{A_{k-1}}$ and then either the slot α or the slot γ.

For this, let $y_{A_1}, ..., y_{A_k}$ be a utilitarian allocation and consider two cases:

Case 1. y_{A_k} is not one of the slots already occupied in $x^{k-1}_{A_1}, ..., x^{k-1}_{A_{k-1}}$. Obviously, we then have that $(x^{k-1}_{A_1}, ..., x^{k-1}_{A_{k-1}}, y_{A_k})$ is utilitarian, i.e., a utilitarian allocation can be obtained from using the slots already occupied in $x^{k-1}_{A_1}, ..., x^{k-1}_{A_{k-1}}$ and then some other unoccupied slot. But then clearly this slot is either α or γ and we are done.

Case 2. y_{A_k} is one of the slots already occupied in $x_{A_1}^{k-1},...,x_{A_{k-1}}^{k-1}$. Then, we can obtain $y_{A_1},...,y_{A_k}$ from $x_{A_1}^{k-1},...,x_{A_{k-1}}^{k-1}$ by a sequence of interchanges as follows: The first interchange assigns agent A_k to y_{A_k}, then agent i for which $x_i^{k-1}=y_{A_k}$ gets y_i instead, etc., until some agent j gets a slot y_j that is unoccupied at $x_{A_1}^{k-1},...,x_{A_{k-1}}^{k-1}$. Next, if the final allocation obtained is not $y_{A_1},...,y_{A_k}$, pick an agent h which does not have y_h yet, move the agent k who currently has that slot to the slot y_k, etc., and continue with such sequences of interchanges until the final allocation $y_{A_1},...,y_{A_k}$ is obtained. Note that all interchanges except for the first sequence could have been carried out given the allocation $x_{A_1}^{k-1},...,x_{A_{k-1}}^{k-1}$ with the same gain in aggregate gap. This contradicts that $x_{A_1}^{k-1},...,x_{A_{k-1}}^{k-1}$ is utilitarian, and we are done. Q.E.D.

Example 2 (cont'd). The modified RP solution induces the following probabilistic assignment for the problem $[4,0,3]$:

agent / slot	1	2	3	4	5	6	7	8
A		1/4	1/4	1/4	1/4			
B		1/4	1/4	1/4	1/4			
C		1/4	1/4	1/4	1/4			
D		1/4	1/4	1/4	1/4			
E						1/3	1/3	1/3
F						1/3	1/3	1/3
G						1/3	1/3	1/3

Hence, it never happens that an allocation occurs that does not minimize aggregate gap.

We should acknowledge that our proposal is, by no means, the only one to construct a utilitarian solution. For instance, one could consider the (simpler) alternative in which any deterministic utilitarian allocation is first set and, afterwards, agents sharing target permute. The resulting mechanism would also be utilitarian. Nevertheless, a well-defined (probabilistic) solution implementing that idea would require to define a rule for the selection of the initial deterministic utilitarian allocation. Numerous rules could be considered, which, in contrast to our proposal, not only renders the resulting mechanism somewhat arbitrary, but also in conflict with the notion of symmetry that we have imposed from the outset.[17]

[17] Consider, for instance, the problem $[3,0,3]$. A symmetric probabilistic solution (obeying equal treatment of equals) should yield each agent the same probability distribution over gaps. This cannot be achieved with the alternative mentioned above.

5 Further insights

We have concentrated so far on the property of utilitarianism. As we have seen, the canonical sd-efficient rule (EPS) fails to be utilitarian, whereas the proposed modification of RP is utilitarian. Utilitarianism has been a main focus within the operations research literature on assignment problems. The economic literature on assignment problems, however, has mostly focused on Pareto-like efficiency conditions, as well as strategic and fairness notions. In this section, we aim to provide a bridge between both approaches, upon exploring the logical relation between those concepts in our setting. In order to do so, let us first formally introduce the concepts we are going to consider.

A lottery is *ex-post efficient* if any allocation that occurs with positive probability is Pareto efficient. A probabilistic assignment is ex-post efficient if it is consistent with an ex-post efficient lottery.

A probabilistic assignment is *sd-efficient* if it is not stochastically dominated by any other probabilistic assignment.

A probabilistic assignment is *sd-envy-free* if the probability distribution over gaps for an agent i stochastically dominates the probability distribution over gaps induced from obtaining instead the distribution over outcomes that any other agent j gets.

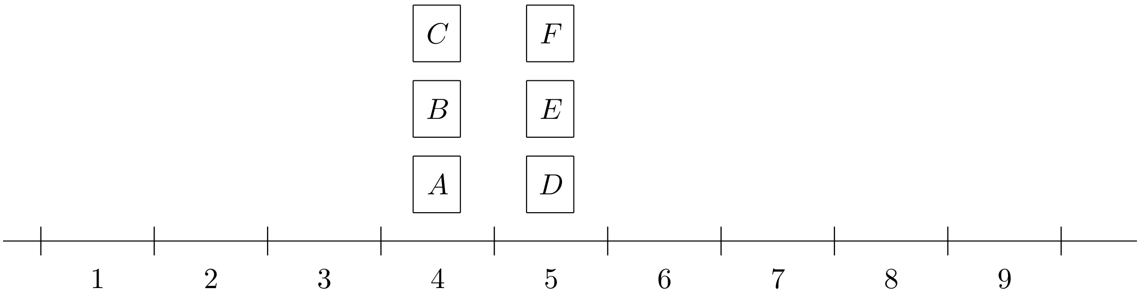
A solution is *sd-strategy-proof* if the probability distribution over gaps for an agent i stochastically dominates the probability distribution over gaps induced from misrepresenting his target.

Theorem 3. The following statements hold in our framework:

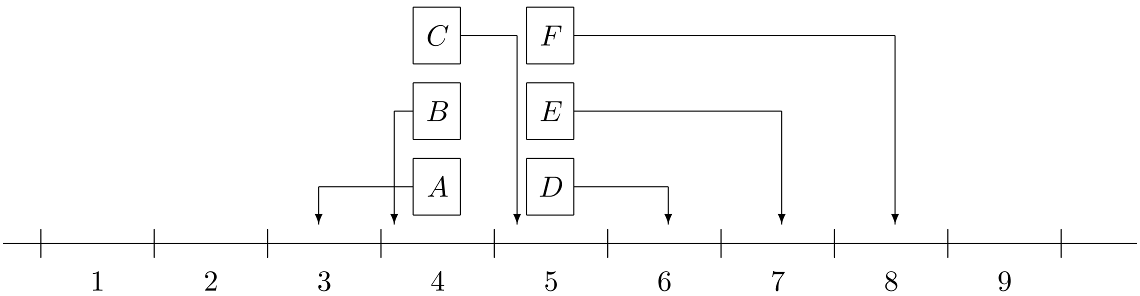
- Sd-efficiency implies ex-post efficiency, but ex-post efficiency does not imply sd-efficiency.
- Utilitarianism implies sd-efficiency, but sd-efficiency does not imply utilitarianism.
- Utilitarianism is incompatible with sd-envy-free.
- Utilitarianism is incompatible with sd-strategy-proofness

Proof. The first part of the first statement is straightforward. The second part of it is shown by means of the following example:

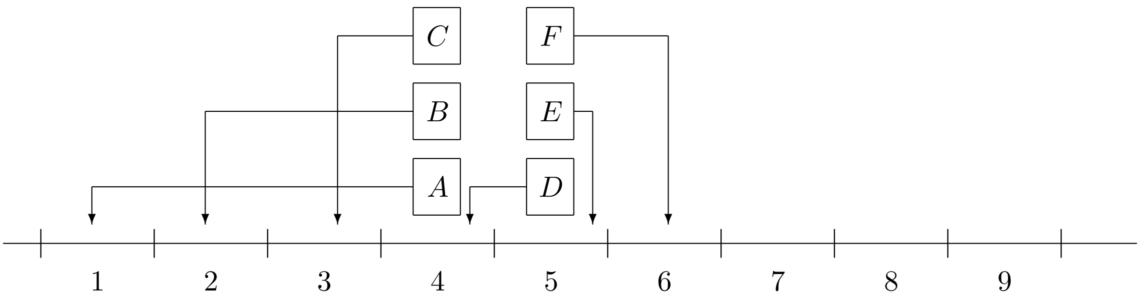
Example 3: $[3, 3]$



Allocation x



Allocation y

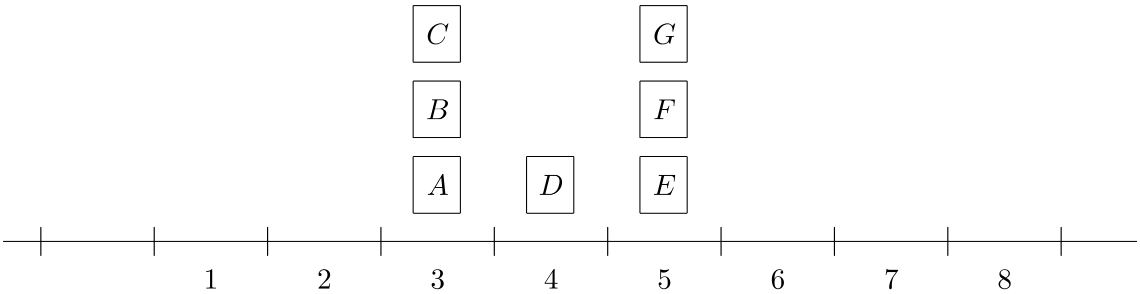


Note that both assignment x and assignment y are efficient. Moreover, any lottery (satisfying equal treatment of equals) that gives positive probability to both assignments x and y, fails to be sd-efficient, as agent C gets slot 5 with positive probability and agent D gets slot 4 with positive probability; hence it is possible to swap probabilities between these two agents for slots 4 and 5 making both agents better off ex ante (while keeping the other agents unaffected). This concludes the proof of the first statement.

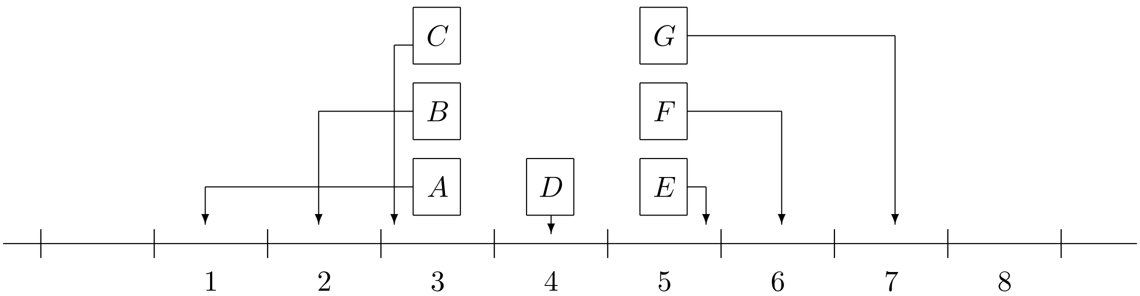
The second part of the second statement has been shown above (with the EPS solution). As for the first part of it, suppose, by contradiction, that the average expected gap is minimized in an allocation, but that the allocation is not sd-efficient. If so, some probability mass can be moved towards strictly preferred alternatives for at least one agent. Thus, the expected gap cannot be minimized, a contradiction.

The third statement is proved by means of the following example:

Example 4: [3,1,3]



Allocation x

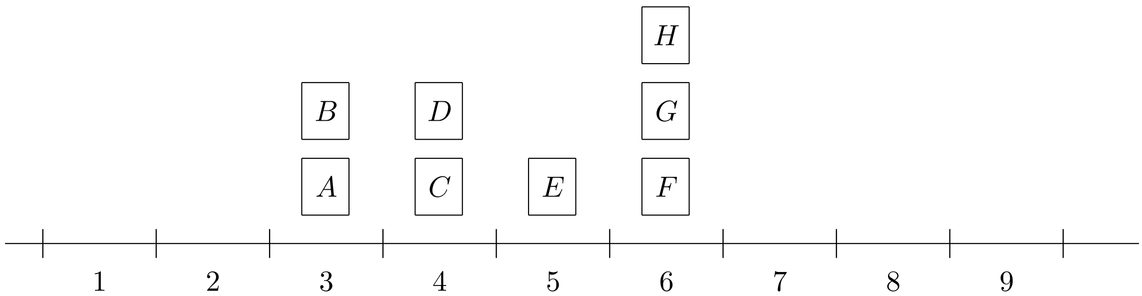


In this problem, a utilitarian solution assigns agents A, B, and C to outcomes 1, 2 and 3, respectively, agent D to 4, and agents E, F and G to 5, 6 and 7, respectively (as allocation x in the picture). Note that the other agents do not face a probability distribution of outcomes that first-order (stochastically) dominates agent D's assignment. Thus, the solution fails to be sd-envy-free.

Finally, as for the fourth statement, let us reconsider Example 2, i.e., $[4, 0, 3]$. In such example, by equal treatment of equals, all agents sharing target 3 will be assigned to a slot with distance 2 or more from the target, with positive probability. Now, if one of those agents misrepresents his preferences and states slot 4 as his target, he will be assigned to slot 4 (and hence will face a gap of 1) with certainty. Thus, the probability distribution over gaps for an agent with slot 3 from misrepresenting preferences cannot be stochastically dominated by the resulting distribution from truth-telling. Q.E.D.

Theorem 3 shows, in particular, that utilitarianism is in conflict with the focal notion of sd-strategy-proofness. It turns out that, in our context, such a conflict also exists for the EPS solution, as shown by the following example.

Example 5: $[2, 2, 1, 3]$



agent / slot	1	2	3	4	5	6	7	8	9
A	1/6	1/3	1/2						
B	1/6	1/3	1/2						
C	1/6	1/6		1/2	1/6				
D	1/6	1/6		1/2	1/6				
E					2/3			1/4	1/12
F						1/3	1/3	1/4	1/12
G						1/3	1/3	1/4	1/12
H						1/3	1/3	1/4	1/12

Here, an agent with target at slot 6 may benefit from claiming slot 5 as his target since in that case he would never be assigned to slot 9. Given the above example, it remains as an open question whether an sd-efficient and sd-strategy-proof solution exists in our framework.

6 Conclusion

We have studied in this paper a specific assignment problem of agents to a facility (represented by slots on a line), where only one agent can be served at a time, and each one wants to be served as close as possible to his preferred slot. This kind of problem arises in multiple daily-life situations such as assigning drivers to congested facilities, or personnel to work shifts, tennis players to courts, etc.

We provide a new solution that arises from the classical random priority solution. Our solution is not only sd-efficient, but also utilitarian (i.e., it minimizes the aggregate gap between agents' preferred slot and their allocated slot). Our construction builds onto a tailor-suited mechanism for testing whether a given deterministic assignment is utilitarian.

There are at least three natural ways to generalize our work:

The first extension refers to the possibility of assuming that more than one agent can be served at each slot, instead of one at a time. It is not difficult to see that, with straightforward modifications, each result in this paper can be extended to that setting.

The second extension refers to assuming single-peaked preferences (but not necessarily symmetric around the peak). Such a model has been considered recently by Kasajima (2012) to provide a deeper insight into the incompatibility of equal treatment of equals, sd-efficiency and sd-strategy-proofness. It turns out, however, that the results of this paper do not extend to such a more general model. To illustrate that, consider the three-agent problem, in which the three share 3 as a peak, but differ in the disutility they get from being assigned slots 1 and 2. More precisely, A would have a utility of 0, 3, and 4, in each of the corresponding slots, whereas B would have -10, 1, and 4, and C would have 0, 1, and 5. Then, it is straightforward to show that the assignment in which A gets slot 1, B gets slot 2, and C gets slot 3, does not permit a bilateral swap improving the overall utility of the group, whereas the overall utility of the group can indeed be improved by reassigning the whole group, so that A gets slot 2, B gets slot 3, and C gets slot 1.

The third one refers to consider assignment on a circle, rather than a line. In other words, the first and last feasible slot are neighbor slots. A plausible case fitting this setting is that in which one agent can be served each hour and agents need to be served every day. Algorithms for finding minimum cost assignments within this model have been considered in Karp and Li (1975) and Aggerwal et al., (1992), among others.

7 Appendix

Proof of Lemma 1: Suppose y is a constrained utilitarian allocation using the same slots as in x. Then, by a sequence of pairwise swaps, where each is aggregate gap-neutral, an allocation y' can be obtained where slots are ordered like the targets (indeed if two outcomes are not ordered like the targets, a pairwise swap is either gap-neutral or gap-reducing and as y is constrained utilitarian it cannot be gap-reducing). Note that y' and x are then identical allocations up to pairwise swaps between agents with shared targets, and hence the two statements follow. Q.E.D.

Proof of Lemma 2: Clearly, if y is constrained utilitarian there does not exist an aggregate gap-reducing pairwise swap. Thus, we prove the converse implication.

Suppose that there does not exist an aggregate gap-reducing pairwise swap and, by contradiction, that the allocation is not constrained utilitarian.

By the observation prior to Lemma 1, there exists a sequence of pairwise swaps, each ordering outcomes according to targets, leading to gap reduction. This sequence only contains aggregate gap-neutral or aggregate gap-reducing pairwise swaps. By assumption, this sequence starts with one or more aggregate gap-neutral pairwise swaps. It remains to be shown that whenever such a sequence exists, an aggregate gap-reducing pairwise swap exists from the initial allocation.

First, we claim that no aggregate gap-neutral pairwise swap between two agents A and B can lead to a subsequent aggregate gap-reducing pairwise swap between any of these two agents, and a third agent C, unless such an aggregate gap-reducing pairwise swap was possible from the initial allocation. Indeed, consider three agents A, B, and C with targets t_A, t_B and t_C, where a swap between the outcomes of agents A and B is aggregate gap-neutral. As the case $t_A = t_B$ is trivial, we will assume, without loss of generality, that $t_A < t_B$.

Consider an allocation where A and B have outcome either x_k or x_h, where $x_k < x_h$, and C has outcome x_C. Note that, as we have assumed that a swap between the outcomes of agents A and B is aggregate gap-neutral, we have either $t_B \leq x_h, x_k$ or $x_h, x_k \leq t_A$. As the two cases can be treated in a symmetric way, we assume for the rest of the proof that $t_B \leq x_h, x_k$. We consider four cases:

Case 1. $t_C \leq t_B$. In this case, $x_C \leq t_B$, as otherwise any sequence of pairwise swaps is aggregate gap-neutral. If $t_C \leq t_A$, then either outcomes are ordered like the targets (in which case no sequence of pairwise swaps reduce the aggregate gap), or the swap between A and B orders outcomes like targets (in which case it is not possible to reduce the aggregate gap from a subsequent swap involving C). If $t_A < t_C \leq t_B$, then $t_C \leq x_C \leq t_B$ (otherwise there exists an aggregate gap-reducing pairwise swap between A and C from the beginning). Thus, a swap between A and C is aggregate gap-neutral regardless of whether A has swap with B or not and a swap between C and B can only increase the aggregate gap regardless of a swap between A and B.

Case 2. $t_B < t_C \leq x_k$. If $x_C \geq t_C$ then all swaps are aggregate gap-neutral. If $x_C < t_C$ then C can make a gap-reducing swap with either A or B from the beginning.

Case 3. $x_k < t_C \leq x_h$. If $x_C \geq x_h$ then either outcomes are ordered like the targets or the swap between A and B orders outcomes like targets. If $x_C < x_k$ then there exists an aggregate gap reducing pairwise swap between A and C from the beginning. If $x_k \leq x_C < x_h$ then

i) if $x_k \leq x_C < t_C$ there exists an aggregate gap-reducing swap initially by swapping with the agent assigned x_h, and

ii) if $t_C \geq x_C < t_h$ the allocation is efficient.

Case 4. $t_C > x_h$.Then either there exists an aggregate gap-reducing swap from the beginning or the swap is weakly increasing the gap.

Now, as the sequence of an aggregate gap-neutral pairwise swap followed by an aggregate gap-reducing pairwise swap cannot exist, unless there exists an aggregate gap-reducing pairwise swap from the beginning, we get, by an induction argument, that there cannot exist any finite sequence of aggregate gap-neutral switches followed by an aggregate gap-reducing swap unless there exists an aggregate gap-reducing pairwise swap from the beginning. Q.E.D.

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