Comparison of speed-density models in the age of Connected and Automated Vehicles

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Abstract

Fundamental Diagrams (FDs) present the relationship between flow, speed, and density and give some valuable information about traffic features such as capacity, congested and uncongested situations, etc. On the other hand, high accurate speed-density models can produce more efficient FDs. Although numerous speed-density models are presented in the literature, there are very few models for Connected and Autonomous Vehicles (CAVs). One of the recent spend-density models that take into account the penetration rate of CAVs is provided by Lu et al. (*1*). However, the estimation power of this model hasn't been tested against other speed-density models, and it hasn't been applied to high-speed networks like freeways. Thus, this paper made a comparison between the Lu speed-density model and a well-known speed-density model (Papageorgiou) in freeway and grid networks. Different CAVs behaviors (aggressive, normal, and conservative) are evaluated in this comparison. The comparison has been made between two speed-density models using the Mean Absolute Percentage Error (MAPE) and a T-test. The MAPE and T-test results show that differences between the two speed-density models are not significant in two case studies and that Lu is a powerful speed-density model to estimate speed compared with a well-known speed-density model. For the sake of comparing the above-mentioned models, this paper investigates the impact of CAVs on capacity based on FDs. The results suggest that the magnitude of the impacts of CAVs on road capacity (capacity increment percentage) which are obtained from two speed-density models are very close to each other. Also, the extent to which CAVs affect road capacity is highly dependent on their behavior.

Keywords: speed-density models, fundamental diagram (FD), connected and automated vehicles (CAVs), capacity, SUMO.

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1. Introduction

Fundamental Diagrams (FDs), which represent the relationship between speed, density, and flow, play a vital role in traffic science, and with the help of FDs, some useful information such as density, speed, capacity, uncongested and congested situations, etc., can be obtained. For example, for the assessment of the quality of highway services, the speed-flow relationship can be used, and speed-density can indicate the change in traffic flow (*2*). Moreover, Macroscopic traffic flow models perform better when a suitable speed-density relationship is chosen (*3*). Therefore, speed-density models are essential for analyzing traffic flow patterns.

For decades many studies have tried to develop accurate and robust speed-density models, and these speed-density models have different forms such as linear, logarithmic, exponential, etc. (*4-11*). Some of the earlier studies developed linear forms of speed-density models. That general form of the linear speed-density model was developed by May and Keller (*6*). This model has two shape parameters, and some other parameters include free-flow speed and jam density. Some other linear speed-density models are special forms of this model. For example, in models which are developed by Pipes (*8*) and Drew (*7*), one of two shape parameters is equal to 1, and in the model developed by Greenshield, both shape parameters are equal to 1. Another form of the speed-density model is logarithmic. A logarithmic speed-density model has been developed by Greenberg (*9*), and this model has two parameters: (*i*) optimum speed and (*ii*) jam density. This model has no free-flow speed parameter, and this model generates infinite speed. Other forms of speed-density models are exponential models, which these models fulfill flow-speed-density properties and represent empirical data robustly (*3*). For instance, Papageorgiou et al. (*4*) developed an exponential speed-density model which has free-flow speed and critical density parameters, and this model has a shape parameter. In addition, there are some other exponential speed-density models, such as Drake et al. (*10*) and Underwood (*11*), which are special forms of the Papageorgiou speed-density model. However, these types of speed-density models did not pay attention to the effect of new technologies such as Connected and Automated Vehicles (CAVs).

The rapid increase in the contribution of technologies in transportation provides the opportunity to reduce some of the traditional transportation problems. One of these technologies is CAVs which are expected to positively impact various aspects of transportation, including capacity and safety(*12-16*). The CAVs' technology has two main features: automation and connectivity. Automation refers to the ability to be driven by the combinations of human and machine decision-making and control systems. The Society of Automotive Engineers (SAE) (*17*) defines six levels of driving automation ranging from 0 (fully manual) to 5 (fully autonomous). These levels of automation describe the sharing between humans and machines for controlling vehicles. This study investigates the impact of fully automated vehicles on traffic flow which automated vehicles are capable of performing all aspects of dynamic driving under any environmental and road conditions (*17*). The other feature (connectivity) can be considered as a feature that allows the vehicle to communicate with infrastructures (V2I), other vehicles (V2V), and pedestrians (V2P) (*13*).

CAVs (due to automation and connectivity features) have different behavior in terms of traffic operation than Human Drive Vehicles (HDVs). In other words, CAVs have different microscopic features based on the level of automation, connectivity, V2I, and V2V communication. For example, fully connected and automated vehicles with aggressive driving behavior can benefit from smaller headway and smaller distance between vehicles (13). Consequently, CAVs can affect FDs and some related components such as capacity, critical density, and congested and uncongested situations. However, none of the aforementioned speed-density models consider CAVs' effects, such as penetration rates of CAVs. To address this issue, Lu et al. (*1*) developed a novel semiparametric speed-density model in which penetration rates of CAVs are considered, and they applied this speed-density model in urban networks. However, this model is not applied in high-speed networks such as freeways, and its estimation power has not been evaluated against other popular speed-density models, such as the Papageorgiou et al. (*4*) speed-density model. Moreover, Lu et al. (*1*) only consider one type of driving behavior of CAVs which is based on aggressive driving behavior. To fill these gaps, this paper aims to:

1. Compare the estimation power of the Lu et al. (*1*) speed-density model with a well-known speed-density model (Papageorgiou speed-density model).
2. Explore the power of Lu et al.' s (*1*) speed-density model in freeways. The main motivation for this contribution is that the Lu model has only been applied to urban areas.
3. For comparison, investigate the impact of CAVs on the capacity of both freeways and urban areas using microsimulation and FDs. This paper first defines the driving behavior of HDVs and CAVs using microsimulation and demines the capacity using FDs. Because a majority of studies look at the effect of CAVs on capacity only based on one type of network (*14; 18-20*), this paper investigates the impact of CAVs on the capacity of a low-speed network which is a grid network, and a high-speed network which is a freeway network.
4. Investigate the impact of CAVs on the capacity of freeways and urban roads based on three driving behavior of CAVs. This study defines the CAVs' driving behavior based on aggressive, normal, and conservative driving behavior.

The remainder of the paper is organized as follows. In section 2, The study framework of this paper is presented. Section 3 describes the results, and finally, section 4 presents the conclusions and suggestions for future studies.

2. Study framework

To generate FDs diagrams and investigate the impact of CAVs on the capacity, it is necessary to determine the differences HDVs and CAVs based on microscopic driving behavior (car-following and lane-changing models). Thus, this section has two parts: the first describes microsimulations (the car-following model and lane-changing model), and the second describes FDs (which are generated either directly from simulation data or by feeding simulation data into speed-density models). Figure 1 shows the framework of this study. Using microsimulation, the Krauss car-following model and LC2013 lane-changing model (Simulation of Urban Mobility (SUMO) simulator lane-changing model) were applied to show HDVs and CAVs driving behavior. Car-following model and lane-changing model parameters were modified to demonstrate the difference between CAVs and HDVs driving behavior. To generate FDs, two models have been applied by feeding simulation data into speed-density models: i. the Lu et al. (*1*)'s speed-density model and ii. the Papageorgiou speed-density model (*4*). By implementing these two speed-density models and the simulation data, FDs are specified, and thus their estimation power is compared. Moreover, the capacities of roads under different penetration rates of CAVs are calculated from FDs.

Figure 1. Study framework

2.1. Microsimulation

There is yet no practical use of CAVs in transportation networks. Therefore, simulation is a principal tool to determine how CAVs affect traffic flow in mixed traffic. Microscopic traffic simulation makes it possible to define the driving characteristics of both CAVs and HDVs. A variety of microsimulation software such as VISSIM, PARAMICS, AIMSUN, and SUMO are employed by researchers to analyze driving behavior. SUMO, the open-source microscopic traffic simulation software used in this study (*21*), allows us to modify driving behaviors and obtain desired information. This section describes the driving behavior of CAVs and HDVs utilizing the Krauss car-following model (*22*) and the LC2013 lane-changing model (*23*).

2.1.1. Krauss Car-following model

For analyzing the longitudinal movement of vehicles, the Krauss car-following (*22*) model is employed (*22*), which describes the safe speed (*24*). This car-following model is the default car-following model in the SUMO simulator. Unlike the Wiedemann car-following model (*25*) in VISSIM software, which has an acceleration function for each traffic regime and is a psychophysiological model, the Krauss model is a space-continuous car-following model with an emphasis on maintaining the safe speed. In terms of parameter selection, longitudinal movement, acceleration, deceleration, and gap acceptance were taken into account (*1*). Thus, Krauss car-following parameters for mapping the longitudinal movement of HDVs and CAVs must be modified. Time headway, acceleration, and minimum gap for HDVs are based on the default value in SUMO simulator (*27*), and CAVs parameters are taken from Atkins Ltd (*13*), and driver imperfection value (sigma) for HDVs and CAVs is taken from Lu et al. (*1*). These parameters are shown in Table 1. There are two important reasons for choosing CAVs' driving behavior from Atkins's (2016) study (*13*). First, this study suggests car-following values for urban and high-speed networks. Since this study investigates the impact of CAVs on the flow of both urban and freeway networks, car-following values based on Atkins's (2016) study (*13*) are reasonable. Second, Atkins's (2016) (*13*) study presents different driving behavior styles, from concretive to aggressive. On the other hand, one of our study's goals is to investigate the impact of CAVs on the capacity of transportation networks when the driving behavior of CAVs is based on three different driving behavior includes aggressive, normal, and conservative driving behavior. Thus, it can be realized that car-following model parameters, which are based on Atkins's (2016) study (*13*), can show conservative, normal, and aggressive driving behavior. Atkins's (2016) study (*13*) presents nine levels of CAVs' driving behavior, from conservative (level = 1) to aggressive driving (level = 9) behavior. Thus, this study uses level 1 for conservative driving behavior, level 5 for normal driving behavior, and level 9 for aggressive driving behavior. It is reasonable that conservative driving behavior has higher headway, minimum gap, and lower acceleration while normal and aggressive driving behavior have smaller headway, minimum gap and higher acceleration.

In addition, in order to avoid the collision, deceleration and emergency deceleration are set equal for HDVs and CAVs (*1*). Also, sigma shows driver imperfection and can vary between 0 and 1; the 0 value shows perfect driving. In this study simulation environment, CAVs have perfect driving, and the value for HDVs is set to the SUMO simulator default value. It should be pointed out that, the road capacity offered by this study for HDVs may be different from the capacity offered by HCM (2300-2400 veh/hr) when the speed limit is 100 km/hr (*26*) as microscopic parameters are based on the SUMO simulator car-following and lane-changing model.

Table 1. HDVs and CAVs car-following parameters

Vehicle type	tau	Mingap	Accel	Decel	Emergency decel	Sigma
HDV	1	2.5	2.6	4.5	8	0.5
CAV (aggressive)	0.5	0.5	3.9	4.5	8	0
CAV (normal)	0.9	1.5	3.5	4.5	8	0
CAV (conservative)	2.1	2.5	3.1	4.5	8	0

tau: The driver's desired (minimum) time headway (s)
Mingap: Minimum Gap when standing (m)
Accel: The acceleration ability of vehicles of this type (m/s^2)
Decel: The deceleration ability of vehicles of this type (m/s^2)
Emergency decel: The maximum deceleration ability of vehicles of this type in case of emergency (m/s^2)
Sigma: The driver imperfection (0-1, 0 denotes perfect driving)

2.1.2. Lane-changing

It is important to note that one of the measures that show the vehicle's movement in microsimulation is lane-changing, which explains the lateral movement of vehicles (*28*). This study uses LC2013 lane-changing (*27*), a lane-changing model in the SUMO simulator. Three motivations for lane-changing in SUMO are illustrated in Figure 2 (strategic, cooperative, and tactical lane-changing). The first motivation is strategic lane-changing, which describes changing lanes when there is no connection between the current lane and the next edge on the route. Another motivation is cooperative lane-changing. There are times when vehicles perform lane-changing maneuvers solely to assist another vehicle in changing lanes, and this behavior is modeled in sumo lane-changing named cooperative lane-changing. The vehicles are informed about being blocked followers by other vehicles in this mode of lane-changing. Thus, the ego vehicle may change lanes in either direction unless a strategic reason prevents vehicles from lane-changing to clear a space for the blocked vehicle (*23*). The third motivation is tactical lane-changing which allows the vehicle to change lanes when it follows a slower leader. Its most important parameter is modified to demonstrate how CAVs and HDVs behave based on the LC2013 lane-changing model.

To determine differences between CAVs and HDVs in lane-changing behavior, Mintsis et al. (*29*) and Lücken et al. (*30*) studies' results are used because these studies determined the differences between lane-changing behavior of HDVs and CAVs based on SUMO simulator lane-changing concepts. The most important parameter of the lane-changing model for CAVs and HDVs is derived from these two studies. The study conducted by Mintsis et al. (*29*) determined which parameter among several lane-changing model parameters would have the strongest influence on driving behavior from a lane-changing perspective. They found that the most effective parameter is *lcAssertive* which describes the "Willingness to accept lower front and rear gaps on the target lane" (*27*). When *lcAssertive* values are higher, a vehicle's behavior toward shorter gaps is more aggressive and vehicles accept lower gaps for lane-changing, and when *lcAssertive* values are lower, the opposite is true and vehicles accept more gaps for lane-changing (*29*). Referring to the results of the Mintsis et al. (*29*) study, the *lcAssertive* for HDVs is 1.3, while for CAVs, Lücken et al. (*30*) showed that this parameter could vary from 0.6 to 0.8 (see (*29; 30*) for lane-changing value selection). In this research, the value of *lcAssertive* is set to 0.8 for aggressive driving behavior, 0.7 for normal driving behavior, and 0.6 for conservative driving behavior. All other parameters are set to SUMO's default values.

Figure 2. Hierarchy of lane-change logic (*29*)

2.2. Fundamental Diagram

FD is one of the key concepts in traffic flow theory and exhibits the relation between speed, density, and flow. FD determines the capacity (the maximum flow rate) and critical density (density at capacity point). This capacity divides the traffic situation into two regimes: 1. Traffic is in an uncongested state for densities lower than the critical density 2. Higher densities result in a congested state (*31*). Figure 3 shows an example of FDs.

Figure 3. An example of fundamental diagrams

The following equation describes the relationship between flow, density, and speed in FD:

$$Q_i\,(p_i\,) = V\,(p_i\,).p_i \tag{1}$$

Where $Q_i\,(p_i\,)$ is flow (vehicle/h), p_i is average density (veh/km), V is speed (km/h). To generate FDs directly from simulation, some data such as density and speed are needed. Thus, the data were collected from the edge-based output in the SUMO simulator during simulations. The edge-based data includes some information about edges, such as link density, sample size, and average speed.

In the next two subsections, the Lu speed-density model and Papageorgiou speed-density model that has been compared with each other in this study are discussed.

2.2.1. Lu speed-density models

Regarding the penetration rate of vehicle automation, Lu et al. (*1*) developed a semiparametric speed-density model. This model calculates the speed as the following equation:

$$V(r,p) = a + s(p) + \beta.r + \gamma.rp \tag{2}$$

In equation 2 (Lu et al. (*1*) speed-density equation), a is average speed under free-flow conditions, $s(p)$ is a non-parametric smooth function, a, β and γ are model parameters, and r is vehicle automation penetration rate. In this model, vehicle automation penetration rates have been considered that affect free-flow average speed and the slope of the speed-density function. This model has two important features. First, it considers vehicle automation penetration rate, and secondly, no assumptions need to be made about the baseline functional form of the speed-density curve. As a result, these features make this model powerful for estimation, and it is more straightforward to use for analysis of different penetration rates. In this model, semiparametric regression specification is estimated with a generalized additive model (GAM), originally a generalized linear model (GLM). The GLM describes a linear relationship between the unknown parameter and the mean of the dependent variable (*1; 32*). The form of GLM is:

$$g(\mu_{y|x}) = \beta_0 + \sum_{j=1}^{p} \beta_j x_j \tag{3}$$

In which x_j is the independent variable; β_0 and β_j are model parameters. In GAM, $\sum_{j=1}^{p} \beta_j x_j$ is replaced with $\sum_{j=1}^{p} s_j(x_j)$. Where $s_j(x_j)$ is a smooth function and is used to summarize the trend of a dependent variable Y as a function of a number of independent variables (see (*32*) for more information about GAM).

2.2.2. Papageorgiou speed-density model

The second speed-density model is developed by Papageorgiou et al. (*4*), as follow:

$$V(k) = v_f exp\left[\frac{-1}{c}(\frac{p}{p_m})^c\right] \tag{4}$$

In equation 4 (Papageorgiou et al. (*4*) speed-density equation), V is speed, v_f is a function of free-flow speed, p_m is critical density, p is density, and c is the model shape parameter. The difference between this model and the previous model is that the previous model considers the penetration rate of vehicle automation, but this model does not take into account the penetration rate of vehicle automation. To generate FDs, the speed-density models (equation (2) and equation (4)) should be substituted in equation (1). The effect of CAVs on capacity is determined by the FDs.

3. Results

This section describes the results, which show the comparison between two speed-density models and determine the impact of CAVs on two case studies. Firstly, the case studies and their features are introduced. Then the modeling results are surveyed.

3.1. Case Studies

Two networks are simulated (shown in Figure 4). The first network is a basic two-lane freeway with a speed limit of 100 km/h and a length of 5 km. The second network is a grid network with 49 intersections with a speed limit of 50 km/h. All links of the grid network are bidirectional one-lane with a length of 400 meters. All traffic lights have a fixed cycle time of 90 seconds. To simulate the uncongested condition in the basic freeway network, the demand has been increased from zero to the capacity point. After this point, for creating congested conditions, variable speed signs (VSS) are used to create congestion (*33*). Ultimately, the demand declined to zero smoothly. This method is useful for both congested and uncongested situations and for generating FDs. The vehicles are routed randomly in the grid network by implementing *Randomtrip.py* (SUMO build-in tool) (*27*). As a result, the generated random trips are more flexible and homogenous in comparison with a demand model with fixed routes for the grid network (*1*). Moreover, in the case of a large network (such as the grid network in this study), the inserted vehicles are distributed randomly over the entire network, given a binomial distribution of inserted vehicles along each edge. This gives a reasonable approximation to the Poisson distribution (*27*).

Figure 4. Case studies

3.2. Results of First Case Study (Basic freeway)

In this section comparison between two speed-density is made, and the impact of CAVs on the capacity of the basic freeway (first case study) based on FDs is provided. To evaluate the impacts of CAVs, six scenarios were considered. In each scenario, the penetration rate of CAVs increased by 20 percent (starting by 0 percent). In the freeway network, for each scenario, data were obtained and aggregated based on 1 minute time interval (120 minutes in total). Ten iterations were considered for this case study.

3.2.1. Lu and Papageorgiou speed-density model

The GAM has been used to estimate the Lu speed-density model's coefficients. Table 2 shows a summary of coefficients estimations. In this table, R^2, the goodness of fit measure, is 0.89 for aggressive driving behavior, 0.91 for normal driving behavior, and 0.92 for conservative driving behavior, and these values demonstrate that this model is validated to estimate the relationship between speed, CAVs penetration rate, and density. To show that the null hypothesis is rejected and there is a relationship between speed and CAVs' penetration rate and density, the t-value should be far from zero, and the p-value should be small (*1*), which is the case for this model. Hence, the model accurately describes the relationship between speed, penetration rate, and density of CAVs under different driving behavior. Table 2 shows some information about the impact of CAVs on average speed. Generally, parameter α shows average speed under free-flow condition (*1*), which has a higher value for conservative driving behavior compared to aggressive and normal driving behavior. On the other hand, β and γ are coefficients that show the impact of the introduction of CAVs on average speed and speed-density function slope respectively. Table 2 demonstrates that β is positive for aggressive and normal behavior and it is negative for conservative behavior which means when CAVs have normal or aggressive driving behavior, they have a positive impact on average speed, and when CAVs have conservative driving behavior, they have a negative impact on average speed and the main reason is that HDVs behave more aggressively than CAVs with conservative behavior and the introduction of CAVs decreases average speed. On the other hand, γ shows how the introduction of CAVs influence the impact of increasing density on average speed so that if γ is positive, CAVs can reduce the negative impact of increasing density on average speed (*1*). Table 2 shows that γ is positive for CAVs with aggressive and normal behavior and it can be realized that the negative impact of increasing density on average speed has reduced as a result of the introduction of CAVs with aggressive and expected behavior. Moreover, γ is negative for CAVs with conservative behavior which shows that CAVs with conservative driving behavior can strengthen the negative impact of increasing density on average speed. After Lu et al. (*1*) speed-density model, this section reports the results obtained from the Papageorgiou speed-density model using an exponential curve. The statistical analysis of this model is reported in Table 3. Based on this table, R^2 varies between 0.89 and 0.91, which indicates that this model can accurately estimate the relationship between density and speed.

Table 2. GAM regression for the basic freeway (first case study)

Driving behavior	Coefficient	Estimate	Std. Error	t value	Pr(>|t|)	R2
Aggressive	α	59.604	0.226	264.140	0.000	0.89
	β	0.022	0.007	3.367	0.001	
	γ	0.005	0.000	36.106	0.000	
	s(p)	-	-	-	0.000	
Normal	α	66.382	0.180	368.396	0.000	0.91
	β	0.043	0.006	7.307	0.000	
	γ	0.002	0.000	13.167	0.000	
	s(p)	-	-	-	0.000	
Conservative	α	77.757	0.169	459.989	0.000	0.92
	β	-0.070	0.007	-9.769	0.000	
	γ	-0.004	0.000	-17.985	0.000	
	s(p)	-	-	-	0.000	

Table 3. Papageorgiou speed-density model regression analysis for the basic freeway (first case study)

Driving behavior	PR	Estimation			Std. Error			T-value			R^2
		V_f	p_m	c	V_f	p_m	c	V_f	p_m	c	
Aggressive	0	94.18	2.40	38.34	0.68	0.38	0.06	138.69	99.70	38.77	0.89
	20	93.78	2.40	42.78	0.66	0.46	0.06	141.18	92.56	37.98	0.89
	40	93.53	2.47	47.97	0.61	0.46	0.06	154.32	104.51	40.55	0.89
	60	93.02	2.61	54.36	0.55	0.46	0.06	169.36	118.89	43.17	0.9
	80	92.13	2.76	63.12	0.49	0.46	0.06	188.41	137.71	45.97	0.91
	100	91.35	3.06	72.90	0.43	0.45	0.06	211.13	162.13	47.42	0.91
Normal	0	94.18	38.34	2.40	0.68	0.38	0.06	138.69	99.70	38.77	0.89
	20	94.11	40.43	2.33	0.69	0.46	0.06	136.01	88.44	37.46	0.89
	40	93.90	42.52	2.41	0.65	0.43	0.06	144.37	99.10	39.51	0.9
	60	93.74	44.99	2.46	0.61	0.40	0.06	153.96	111.13	42.43	0.9
	80	93.17	47.38	2.51	0.58	0.39	0.06	161.39	120.66	44.09	0.91
	100	92.88	49.50	2.55	0.56	0.39	0.06	166.75	125.59	44.67	0.91
Conservative	0	94.18	2.40	38.34	0.68	0.38	0.06	138.69	99.70	38.77	0.89
	20	94.27	2.29	36.28	0.72	0.39	0.06	130.99	93.29	38.64	0.89
	40	94.20	2.23	34.07	0.73	0.34	0.06	129.26	99.50	40.48	0.89
	60	94.15	2.11	31.96	0.76	0.34	0.05	123.17	94.05	39.39	0.89
	80	94.24	2.03	29.73	0.80	0.33	0.05	117.75	89.40	38.45	0.9
	100	94.66	1.97	27.80	0.82	0.33	0.05	112.26	88.27	37.32	0.91

The FDs, generated by both microsimulation data, the Lu speed-density model, and the Papageorgiou speed-density model, are illustrated in Figures 5, 6, and 7. Figure 5 illustrates that the aggressive driving behavior of CAVs can increase the capacity of the freeway substantially. Figure 6 shows the impact of CAVs on the capacity of freeways based on normal driving behavior, and this figure demonstrates that CAVs can increase the capacity of the freeway network. Figure 7 shows the impact of CAVs on FDs based on conservative driving behavior. This figure illustrates that the conservative driving behavior of CAVs can reduce the capacity of the freeway network.

Figure 5. FDs (speed-density relationship) based on aggressive driving behavior of CAVs (First case study)

Figure 6. FDs (speed-density relationship) based on normal driving behavior of CAVs (First case study)

Figure 7. FDs (flow-density relationship) based on conservative driving behavior of CAVs (First case study)

In detail, Table 4 shows the impact of different driving behavior of CAVs on the capacity of freeways. This table confirms that aggressive driving behavior of CAVs increases the capacity of the freeway network up to 101%, the normal behavior of CAVs can increase the capacity of freeway up to 30%, and the conservative behavior of CAVs can reduce the capacity of the freeway up to 34%.

Table 4. Basic freeway capacity change based on simulation, the Lu model, and the Papageorgiou model

Driving behavior	PR	Capacity (veh/h/lane)			Percentage of Capacity change compare to zero PR		
		Simulation	Lu	Papageorgiou	Simulation	Lu	Papageorgiou
Aggressive	0	2576	2592	2381	-	-	-
	20	2887	2870	2644	12.11	10.71	11.04
	40	3216	3241	2994	24.85	25.00	25.74
	60	3669	3663	3446	42.45	41.30	44.70
	80	4386	4123	4046	70.30	59.03	69.90
	100	5013	4616	4801	94.63	78.07	101.61
Normal	0	2576	2492	2381	-	-	-
	20	2624	2586	2477	1.89	3.75	4.02
	40	2791	2681	2636	8.36	7.58	10.68
	60	2880	2779	2810	11.80	11.51	18.01
	80	3051	2879	2964	18.46	15.53	24.48
	100	3158	2982	3106	22.61	19.66	30.44
Conservative	0	2576	2291	2381	-	-	-
	20	2353	2119	2208	-8.63	-7.50	-7.28
	40	2136	1990	2049	-17.07	-13.14	-13.96
	60	1961	1866	1874	-23.86	-18.54	-21.31
	80	1788	1748	1713	-30.58	-23.69	-28.05
	100	1592	1636	1583	-38.17	-28.60	-33.54

Generally, based on the driving behavior of CAVs the impact of CAVs on the capacity of the freeway can vary between -34% and 101%. It is worth mentioning that this range of change is based on microscopic features discussed in the car-following and lane-changing section. Also, when the penetration rate is 0, the capacity is higher than HCM capacity. This is reasonable because the driving behavior of HDVs is based on Krauss car-following parameters values, and based on the different car-following models , a different capacity is obtained, and the value of capacity when PR=0 is near to the HCM value for a 100 km/h speed limit (2300-2400 veh/h) (*26*).

3.2.2. Comparison of Speed-Density models and capacity results

Table 4 illustrates a comparison between capacities that are obtained from simulation (based on the relationship between speed, density, and flow), Lu et al.(*1*) speed-density model, and Papageorgiou speed-density model. This table shows that differences between capacities are not significant. To prove that the differences between capacities are not significant, a T-test is used. T-tests are done between simulation and Lu capacities, Lu and Papageorgiou capacities, and simulation and Papageorgiou capacities. P-values show differences between the two models are not significant. To compare the performance of the two speed-density models, the Mean Absolute Percentage Error (MAPE) and T-test for both models per penetration rate are calculated (Table 5). MAPE is calculated as following Equation (5):

$$MAPE = \left(\frac{1}{N}\sum_{t=1}^{N}\left|\frac{S_t - E_t}{S_t}\right|\right) * 100 \quad (5)$$

Where N is the number of observed speeds or flows, S_t is the simulation value, E_t is the speed-density model value. Generally speaking, based on MAPE, the Papageorgiou model shows better performance in estimating speed compared to the Lu model. However, the difference between two models' speeds and two models' flows values is neglectable. To prove the fact that the differences between the two models' speeds and flows values are not significant, the T-test is used. Based on the P-values of the T-test, which are shown in Table 5, all values are much higher than 0.05, which shows that differences between the two models' speeds and flow values are not significant. Therefore, it can be concluded that, although there are some differences between MAPE's value of the two speed-density models, the Lu model shows an acceptable performance in comparison with the well-known Papageorgiou model.

Table 5 MAPE results of Lu and Papageorgiou Speed-Density models

Driving behavior	MAPE Speed Lu Speed-Density model	MAPE Speed Papageorgiou Speed-Density model	P- value
Aggressive	9.06	5.06	0.27
Normal	5.02	5.02	0.34
Conservative	3.91	3.53	0.47

3.3. Results of Second case study (Grid network)

In this section, the comparison between two speed-density models is made, and the impact of CAVs on the capacity of the grid network is evaluated. Similar to the first case study, six scenarios have been defined. In each scenario penetration rate of CAVs increased by 20%. Data were obtained and aggregated in the grid network for each scenario based on 1 minute time interval (45 minutes in total). Moreover, each simulation is repeated ten times (10 iterations).

3.3.1. Lu and Papageorgiou speed-density model

To survey Lu's speed-density model statistically, Table 6 shows a summary of the estimation of coefficients. In this model, all of the R^2 are over 0.95, which indicates this model can explain the relationship between speed, CAVs penetration rate, and density with high accuracy.

Table 6. GAM regression for grid network (Second case study)

Driving behavior	Coefficient	Estimate	Std. Error	t value	Pr(>|t|)	R2
Aggressive	α	29.16	0.086	338.11	0.000	0.98
	β	0.032	0.002	14.27	0.000	
	γ	0.001	0.000	9.99	0.000	
	s(p)	-	-	-	0.000	
Normal	α	28.008	0.067	420.74	0.000	0.99
	β	0.023	0.002	13.77	0.000	
	γ	0.000	0.000	3.70	0.000	
	s(p)	-	-	-	0.000	
Conservative	α	26.570	0.065	410.108	0.000	0.97
	β	-0.002	0.0002	-2.955	0.033	
	γ	0.000	0.000	-4.730	0.000	
	s(p)	-	-	-	0.000	

Table 6 illustrates that in aggressive driving behavior of CAVs, the parameter α has a higher value than conservative and normal driving behavior. In addition, the parameter β is positive for aggressive and normal behavior and is negative for conservative behavior, which means that CAVs with normal and aggressive driving behavior have a positive impact on average speed and CAVs with conservative driving behavior have a negative impact on average speed. Moreover, Table 6 shows that the parameter γ is positive for aggressive and it is zero for conservative behavior and normal behavior. As mentioned before, γ shows whether CAVs can change the impact of increasing density on average speed. Consequently, CAVs with aggressive driving behavior can decrease the negative impacts of increasing density on speed. However, CAVs with normal and conservative driving behavior cannot mitigate the negative effect of density increment on speed. Table 7 provides the results of the Papageorgiou model's statistical analysis, which demonstrates that speed can be accurately estimated using this model (R^2 varies between 0.96 and 0.99; all t-values are far from zero, and all p-values are equal to zero).

Table 7. Papageorgiou speed-density model regression analysis for grid network (Second case study)

Driving behavior	PR	Estimation			Std Error			T-value			
		V_f	p_m	c	V_f	p_m	c	V_f	p_m	c	R^2
Aggressive	0	46.33	26.84	1.20	0.27	0.30	0.02	174.30	89.43	62.01	0.99
	20	46.41	28.18	1.18	0.27	0.35	0.02	171.08	80.32	58.40	0.99
	40	46.64	30.84	1.18	0.24	0.38	0.02	195.65	82.04	62.66	0.98
	60	46.70	31.20	1.22	0.25	0.44	0.02	190.30	70.55	54.34	0.97
	80	46.61	32.15	1.31	0.19	0.42	0.02	244.34	75.77	58.90	0.97
	100	46.26	32.22	1.46	0.22	0.60	0.04	209.00	53.98	40.40	0.99
Normal	0	46.33	26.84	1.20	0.27	0.30	0.02	174.30	89.43	62.01	0.99
	20	46.60	26.28	1.19	0.24	0.27	0.02	195.68	96.98	68.97	0.98
	40	46.41	26.44	1.27	0.19	0.22	0.02	243.86	122.71	80.97	0.97
	60	47.03	28.79	1.18	0.24	0.32	0.02	199.68	88.76	65.89	0.98
	80	47.30	29.91	1.19	0.26	0.40	0.02	178.81	75.69	56.60	0.97
	100	47.36	30.35	1.23	0.28	0.46	0.02	172.01	65.41	49.64	0.96
Conservative	0	46.33	26.84	1.20	0.27	0.30	0.02	174.30	89.43	62.01	0.97
	20	47.15	27.33	1.11	0.36	0.43	0.02	130.19	63.87	48.92	0.98
	40	47.42	27.43	1.10	0.32	0.37	0.02	148.61	73.64	56.61	0.98
	60	47.61	26.57	1.13	0.34	0.37	0.02	139.27	71.95	53.12	0.97
	80	48.35	25.21	1.10	0.37	0.37	0.02	131.44	67.90	51.26	0.96
	100	48.66	24.50	1.11	0.38	0.37	0.02	128.46	66.95	50.02	0.98

To illustrate the impact of CAVs on capacity FDs are used; however, because there are a lot of routes and flows in the grid network, the average flow, speed, and density of the whole network are used to generate FDs. Thus, Macroscopic fundamental diagrams (MFDs) are used to explore the impact of CAVs on the capacity of the grid network. The MFDs, generated by microsimulation data, Lu speed-density model, and Papageorgiou speed-density model, are illustrated in Figures 8,9, and 10. Figure 8 illustrates the aggressive driving behavior of CAVs increases the capacity of the grid network. Moreover, Figure 9 demonstrates the impact of CAVs on the capacity of the grid network based on normal driving behavior, and this figure shows that CAVs increase the capacity of the grid network. Finally, Figure 10 shows the impact of CAVs on MFDs based on conservative driving behavior. This figure illustrates that the conservative driving behavior of CAVs decreases the capacity of the grid network.

Figure 8. MFDs (speed-density relationship) based on aggressive driving behavior of CAVs (Second case study)

Figure 9. MFDs (speed-density relationship) based on normal driving behavior of CAVs (Second case study)

Figure 10. MFDs (flow-density relationship) based on conservative driving behavior of CAVs (Second case study)

In detail, Table 8 illustrates the impact of different styles of driving behavior of CAVs on the capacity of the grid network. This table shows that the aggressive driving behavior of CAVs increases the capacity of the grid network up to 18%, the normal behavior of CAVs increases the capacity of the grid network up to 43%, and the conservative behavior of CAVs decreases the capacity of the grid network by up to 14%. Generally, based on the driving behavior of CAVs the impact of CAVs on the grid network capacity can vary between -14% and 43%. It is worth mentioning that this range of change is based on microscopic features discussed in the car-following and lane-changing section.

Table 8. Grid network capacity change based on simulation, the Lu model, and the Papageorgiou model

Driving behavior	PR	Capacity (veh/h/lane)			Percentage of Capacity change compare to zero PR		
		Simulation	Lu	Papageorgiou	Simulation	Lu	Papageorgiou
Aggressive	0	550	534	540	-	-	-
	20	558	567	561	1.54	6.02	3.80
	40	607	604	615	10.44	13.00	13.74
	60	654	647	644	18.95	21.11	19.11
	80	697	701	697	26.83	31.16	29.04
	100	757	764	752	37.69	42.97	39.20
Normal	0	550	522	540	-	-	-
	20	520	538	528	-5.47	3.06	-2.38
	40	564	555	557	2.57	6.36	3.08
	60	588	574	579	6.95	10.06	7.22
	80	620	595	611	12.69	14.06	13.09
	100	617	618	638	12.14	18.36	18.08
Conservative	0	550	514	540	-	-	-
	20	515	509	522	-6.35	-0.87	-3.39
	40	528	505	522	-4.02	-1.72	-3.36
	60	516	500	520	-6.13	-2.59	-3.68
	80	485	496	492	-11.76	-3.38	-8.98
	100	473	492	484	-13.90	-4.14	-10.51

3.3.2. Comparison of Speed-Density models and capacity results

Table 8 compares the capacities obtained from simulation, the Lu speed-density model, and the Papageorgiou speed-density model. As shown in this table, the three approaches give similar results for evaluating the capacity of the grid network. Based on MAPE results, the Papageorgiou model shows superior performance in three styles of driving behavior of CAVs. Based on the results of the T-test (Table 9), it can be claimed that there is no significant difference between the two models. Therefore, it can be concluded that the Lu model shows powerful performance compared with the well-known Papageorgiou model.

Table 9. MAPE results of Lu and Papageorgiou Speed-Density models

Driving behavior	MAPE Speed Lu Speed-Density model	MAPE Speed Papageorgiou Speed-Density model	P- value
Aggressive	3.07	2.27	0.44
Normal	2.83	1.91	0.42
Conservative	3.71	3.25	0.49

4. Conclusion

In this paper, a comparison between a novel speed-density model which considers CAVs' penetration rates and the Papageorgiou speed-density model has been made, and the impact of CAVs based on three different driving behavior on the capacity of transportation networks has been investigated. On the one hand, many different speed-density models did not consider the effect of CAVs in their models, and on the other hand, the estimation of the new speed-density model, developed by Lu et al. (*1*), which considers the CAVs' penetration rate is not compared with other speed-density models to realize its estimation power. In addition, the Lu et al. (*1*) speed-density model is not tested for high-speed networks like basic freeways. Moreover, Lu et al. (*1*) only consider the aggressive driving behavior of CAVs. Therefore, to fill these gaps, in this paper, the Lu et al. (*1*) speed-density model is tested for the basic freeway and grid networks, and this model is compared with the Papageorgiou speed-density model (*4*). In addition, this study defines the driving behavior of CAVs based on aggressive, normal, and conservative driving behavior.

The microsimulation is carried out by implementing the SUMO simulator. To show the difference in driving behavior between CAVs and HDVs, Krauss car-following parameters and LC2013 lane-changing model parameters are modified. To investigate the impact of CAVs on the capacity of roads, six scenarios are considered. In every scenario penetration rate of CAVs increases by 20% (starting by 0%). Then, the microsimulation data are fed into two speed-density models, namely the Lu and Papageorgiou, to generate FDs. The FDs, which are extracted directly from simulation, and the FDs, generated by simulation data and the two speed-density models are compared.

A comparison is performed between the two speed-density models, which reveals the better performance of the Papageorgiou model based on MAPE in the basic freeway network. However, the T-test shows that the difference between Papageorgiou and Lu models is small. It can be realized that Lu is also a powerful model to estimate speed in the basic freeway network. In the grid network, the comparison results between two speed-density models show that the Papageorgiou model performs better. But, the T-test indicates that Papageorgiou and Lu have small differences in speed estimation. It is concluded that Lu can estimate the speed with high accuracy compared with the Papageorgiou speed-density model, which is a well know speed-density model in the grid network. Also, the results confirmed that capacities obtained from the two models are close to each other, and differences between capacities are not significant. Moreover, the results confirm that CAVs have the ability to increase the capacity of basic freeway up to 101 % based on aggressive driving behavior, and CAVs can reduce the capacity of freeway up to 34% based on conservative driving behavior respectively. Also, they can increase the capacity of the grid network up to 43% when CAVs' movements are based on aggressive driving behavior, and CAVs can decrease the capacity of the grid network up to 14% based on conservative driving behavior. It should be noted that the results of the impact of CAVs on capacity are based on aggressive, conservative, and normal driving behavior and microscopic features, which have been discussed in sections 2.1.1 and 2.1.2. If the behavior of CAVs with regards to lane-changing and car-following behavior varies, the results will change.

Future research should pay attention to comparing the Lu model with other speed-density models using real data. Also, future studies should investigate the lane-changing model more deeply and modify more lane-changing parameters. Other car-following models like the intelligent driver model (IDM) and cooperative adaptive cruise control (CACC) can also be tested. Moreover, it can be more realistic to calibrate Krauss car-following model parameters for HDVs driving behavior based on real traffic data. Given that the Lu model is a novel model, future work can focus on improving this model. Also, future studies should investigate the impact of CAVs on more complex networks such as Weaving and Merge freeway networks. Based on the SUMO simulator setting, this study did not explore the effect of reaction time on driving behavior performance, thus future studies can investigate the impact of reaction time on the behavior of HDVs and CAVs.

5. Author Contributions

The authors confirm contribution to the paper as follows: Study conception and design: A.H. Karbasi, B. Bamdad Mehrabani, L. Sgambi, M. Saffarzadeh; Data collection: A.H. Karbasi, B. Bamdad Mehrabani; Analysis and interpretation of results: A.H. Karbasi, B. Bamdad Mehrabani, M. Cools. L. Sgambi; Draft manuscript preparation: A.H. Karbasi, B. Bamdad Mehrabani, M. Cools. All authors reviewed the results and approved the final version of the manuscript. The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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