THE CONTRIBUTION OF REALIZED COVARIANCE MODELS TO THE ECONOMIC VALUE OF VOLATILITY TIMING

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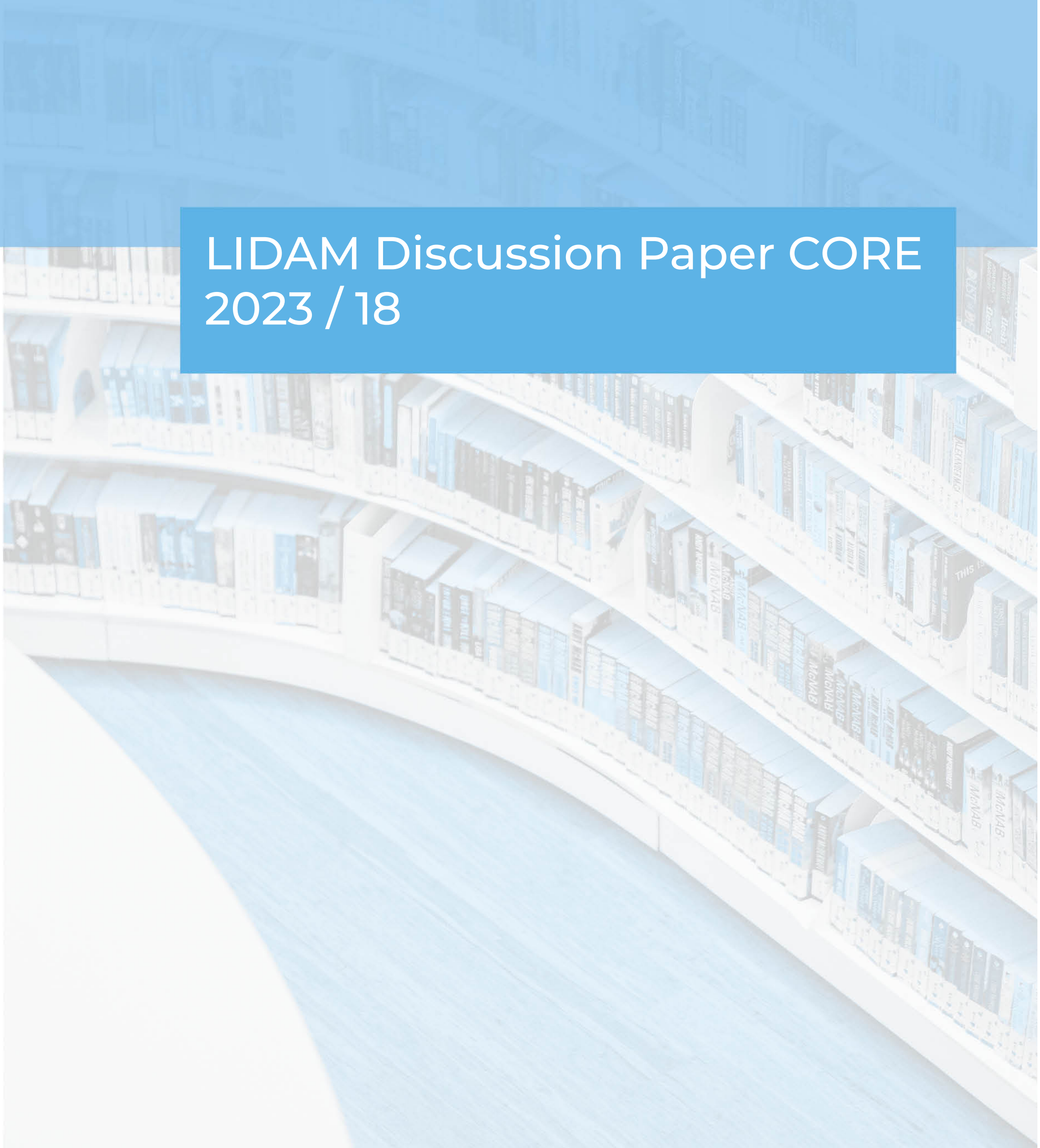

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The contribution of realized covariance models to the economic value of volatility timing

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Abstract

Realized covariance models specify the conditional expectation of a realized covariance matrix as a function of past realized covariance matrices through a GARCH-type structure. We compare the forecasting performance of several such models in terms of economic value, measured through economic loss functions, on two datasets. Our empirical results indicate that the (HEAVY-type) models that use realized volatilities yield economic value and significantly surpass the (GARCH) models that use only daily returns for daily and weekly horizons. Among the HEAVY-type models, for a dataset of twenty-nine stocks, those that are specified to capture the heterogeneity of the dynamics of the individual conditional variance processes and to allow these to differ from the correlation processes (namely, DCC-type models) are more beneficial than the models that impose the same dynamics to the variance and covariance processes (namely, BEKK-type models), whereas for the dataset of three assets, the different models perform similarly. Finally, using a directly rescaled intra-day covariance to estimate the full-day covariance provides more economic value than using the overnight returns, as the latter tend to yield noisy estimators of the overnight covariance, impairing their predictive capacity.

Keywords: volatility timing, realized volatility, high-frequency data, forecasting.

JEL Classification: G11, G17, C32, C58.

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1 Introduction

Forecasting the volatility (in the sense of the covariance matrix) of daily returns is important for risk management. The current consensus, based on a large variety of empirical studies, is that, to this end, the use of a realized measure of the daily volatility, based on intra-daily returns, is more beneficial than the exclusive use of daily data. The intuitive explanation of this difference is that, under suitable but realistic conditions, a realized covariance matrix is a more precise measure of the daily volatility than a measure constructed using only daily returns.

The benefits gained thanks to the use of a realized covariance matrix are evaluated through measures of forecasting performance, which can be of statistical or economic nature. In an influential paper, Fleming, Kirby, and Ostdiek (2003) provide a method to measure "the economic value of volatility timing using realized volatility" and show, empirically, that the economic value of switching from daily to intra-daily returns can be substantial. A large number of authors have adopted the method of Fleming, Kirby, and Ostdiek (2003), using different models and data.[1]

Whether the basic data is daily or intra-daily, the volatility forecasts are based on a time series econometric model. Such a model specifies the conditional covariance (i.e., the conditional expectation of the volatility) matrix of the daily return vector as a function of past data. When the data is daily, the conditional covariance of day t is typically specified as a moving average of the outer-products of the past returns; this can be achieved indirectly through a multivariate GARCH model. When the data is intra-daily, the conditional covariance of day t can be specified as a moving average of the past realized covariances. This is what Fleming et. al. (2003) did explicitly, with a parsimonious way to define and estimate the coefficients of the moving average terms as functions of a scalar parameter.

Several more sophisticated models for forecasting realized covariance matrices have been developed and used since 2003. The primary contribution of this paper is to evaluate empirically whether such "realized covariance models" are more beneficial than the simple, but successful, models used by Fleming, Kirby, and Ostdiek (2003).

[1] A Google search, conducted on June 25, 2023, using the title of the paper of Fleming, Kirby, and Ostdiek (2003), identified 778 citations;, among which e.g., Marquering and Verbeek (2004) and Bollerslev, Hood, Huss, and Pedersen (2018). Fleming, Kirby, and Ostdiek (2003) extends Fleming, Kirby, and Ostdiek (2001) where the authors show the economic value of switching from unconditionally efficient static portfolios to portfolios based upon volatility models based on daily returns.

An important and influential development of realized covariance models has consisted in substituting a realized measure of volatility for the outer-product of the past return vector in the conditional variance equation of a multivariate GARCH model, in particular with the high frequency based volatility (HEAVY) class of models.[2] This development was implemented progressively, with the HEAVY-BEKK model of Noureldin, Shephard, and Sheppard (2012), the HEAVY β-Factor-GARCH model of Sheppard and Xu (2019), and the HEAVY-DCC (dynamic conditional correlation) of Bauwens and Xu (2022).[3]

A common empirical finding of these papers is that the HEAVY models have better forecasting performances of the conditional covariance of daily returns than the traditional multivariate GARCH models. When statistical performance measures are used, they differ, at least partially, between papers, in terms of loss functions or statistical tests: in Noureldin, Shephard, and Sheppard (2012) and Sheppard and Xu (2019), the forecasts of different models are compared by equal predictive accuracy (EPA) Diebold-Mariano tests, enabling only pairwise comparisons, while Bauwens and Xu (2022) use the model confidence set (MCS) approach of Hansen, Lunde, and Nason (2011) which enables to compare jointly a set of models. Regarding forecast performance comparisons based on an economic criterion, Sheppard and Xu (2019) compare the relative performance of pairs of models (by EPA tests) in terms of marginal expected shortfall, which is the expected loss of an asset conditional on a factor (such as a market index) being in a state of stress, see Acharya, Pedersen, Philippon, and Richardson (2017). Bauwens and Xu (2022) use the MCS testing procedure to rank jointly a set of models in terms of optimal (global minimum variance, and minimum variance) portfolio performances. None of these studies uses the criterion of the economic value of volatility timing of Fleming, Kirby, and Ostdiek (2003).

Thus, there is plenty of scope for assessing the economic value of the HEAVY models, including a HEAVY version that we add of the simple model of Fleming, Kirby, and Ostdiek (2003). We implement the comparisons using two datasets. The first one is relative to futures contracts for S&P 500, Treasury bonds and gold, for the period 2003/07/01-2022/08/05, this being an update of the data used by Fleming, Kirby, and Ostdiek (2003). The trading of

[2]Two univariate models that share some similarity were first developed independently: the HEAVY model of Shephard and Sheppard (2010) Shephard and Sheppard (2010) and the realized GARCH model of Hansen, Huang, and Shek (2012).

[3]There exists a multivariate realized β-GARCH model, by Hansen, Lunde, and Voev (2014), but no realized-BEKK and realized-DCC versions that generalize the univariate realized GARCH model of Hansen, Huang, and Shek (2012).

these contracts occurs almost without interruption during each trading calendar day, so that it is possible to measure the daily volatility of a complete day through a realized covariance measure.

The second dataset is relative to twenty-nine stocks belonging to the DJIA (Dow Jones Industrial Average) index, for the period 2001/01/03-2018/04/16. On this type of market, the trading period corresponds to a few hours of a day. Since a realized covariance matrix uses the intra-daily returns available during the trading period, it underestimates the complete day covariance matrix. A bias correction method is required. We adopt two correction methods: the first one transforms the trading period realized covariance matrix so that it has the same sample average as the average of the outer products of the daily (close-to-close) returns; the second one adds the overnight variation (i.e., the outer product of the overnight return) to the trading period realized matrix, and then transforms it in the same way as the first method to match the sample average of the outer products of the daily (close-to-close) returns. We compare empirically these two methods over the models we use for assessing their economic significance and we find that the first correction method is more valuable; this is primarily due to the noisy estimator of the overnight covariance produced by the second method.

In addition to the previous finding, our empirical results mainly confirm that the (HEAVY-type) models that use realized volatilities yield economic value and significantly surpass the (GARCH) models that use only daily returns; this is at least the case for daily and weekly forecasts. Among the HEAVY-type models, those that are specified to capture the heterogeneity of the dynamics of the individual conditional variance processes and to allow these to differ from the correlation processes (namely, DCC-type models) are more beneficial than the models that impose the same dynamics to the variance and covariance processes (namely, BEKK-type models); this is the case for the dataset relative to the twenty-nine stocks, whereas for the dataset of three assets, the different models perform similarly.

The remainder of the paper is divided in five sections. Section 2 presents the data, the definition of the realized covariance measure and the bias correction methods used for the second dataset. Section 3 defines the realized covariance models used in the empirical applications and discusses the estimation results. Section 4 expounds the economic criteria used to compare the models through out-of-sample forecasts and the related statistical methods. Section 5 contains the empirical results. Section 6 concludes. Additional technical and empirical details are provided in a set of appendices.

2 Realized covariance: definitions, bias correction, and data

2.1 Definitions and bias correction

Let $r_t = (r_{1t}, r_{2t} \ldots r_{kt})'$ denote the $k \times 1$ (close-to-close) daily return vector of day t corresponding to k assets and $r_{(j)t} = (r_{(j)1t}, r_{(j)2t} \ldots r_{(j)kt})'$ the corresponding j-th intra-daily return vector at time j on day t, where $j = 1, 2, ..., N$. The simplest realized covariance measure for the k assets on day t is the $k \times k$ matrix defined as

$$RC_t = \sum_{j=1}^{N} r_{(j)t} r'_{(j)t}. \tag{1}$$

Other types of realized covariance estimators which are somewhat robust to noise have been considered, for example, the realised kernel of Barndorff-Nielsen, Hansen, Lunde, and Shephard (2011).

Assuming that $N > k$, RC_t is positive definite (PD). Denote by v_t the $k \times 1$ realized variance vector of day t, consisting of the diagonal elements of RC_t, and by RL_t the realized correlation matrix of day t, defined as

$$RL_t = \{\mathrm{diag}(RC_t)\}^{-1/2} RC_t \{\mathrm{diag}(RC_t)\}^{-1/2}, \tag{2}$$

where $\mathrm{diag}(RC_t)$ is the diagonal matrix obtained by setting the off-diagonal elements of RC_t equal to zero, and the exponent $-1/2$ transforms each diagonal element into the inverse of its square root. Thus, the off-diagonal elements of RL_t are the realized correlation coefficients for the asset pairs, and its diagonal elements are equal to unity.

The realized covariance measure RC_t defined above uses the intra-daily returns available during the trading period, i.e. between the opening and closing times. The overnight variation of returns is not taken into account. Hence RC_t underestimates the whole day covariance. A bias correction method is required. We adopt two correction methods to transform RC_t into a daily covariance.

1. Rescaling RC_t to satisfy the condition that the average value (over the sample period of T observations) of the realized covariance matrices is the same as the average value of the outer products of the daily returns. We use the transformation formula of Sheppard and Xu (2019), as it guarantees a PD covariance matrix:

$$RC2_t = \Lambda RC_t \Lambda', \tag{3}$$

where $\Lambda = \bar{\Sigma}^{1/2}\bar{M}^{-1/2}, \bar{\Sigma} = T^{-1}\sum_{t=1}^{T} r_t r_t'$ and $\bar{M} = T^{-1}\sum_{t=1}^{T} RC_t$. For a symmetric PD matrix A, its symmetric square root, denoted by$A^{1/2}$, is defined as $UL^{1/2}U'$, where U is a matrix containing the eigenvectors of A, and $L^{1/2}$ is a diagonal matrix containing the square roots of the eigenvalues of A.

2. Adding to RC_t the outer product $r_{t,on}r_{t,on}'$ of the overnight return $r_{t,on}$ (the close-to-open return preceding the trading period) as an estimate of the overnight covariance, and rescaling the sum as in the previous case:

$$RC1_t = \Lambda_1(r_{t,on}r_{t,on}' + RC_t)\Lambda_1', \tag{4}$$

where $\Lambda_1 = \bar{\Sigma}^{1/2}\bar{M_1}^{-1/2}$ with $\bar{M_1} = T^{-1}\sum_{t=1}^{T}(r_{t,on}r_{t,on}' + RC_t)$.

The rescaling is used when the non-trading period corresponds to a large part of the day, or if the addition of the overnight variation is not sufficient to ensure that the average of the rescaled RC_t is close to the average of the outer products of the daily returns.

2.2 Data

We use two datasets for the empirical analyses. The first is a dataset about the same futures contracts as studied by Fleming, Kirby, and Ostdiek (2001): S&P 500 futures (STOCK, hereafter), Treasury bond futures (BOND), and Gold futures (GOLD); it is named SBGF (for Stock-Bond-Gold Futures) hereafter. The sample period is July 1, 2003 to August 5, 2022, with a total of 4864 trading days. We choose July 1, 2003 as starting date, as trading occurs both in the daytime and in the evening (e.g., from 7:20 to 16:00 and from 17:00 to 7:20 for BOND) from that day. This was not the case in the dataset of Fleming, Kirby, and Ostdiek (2001), since the trading occurred only during the daytime. Appendix A provides the exact trading times of the three futures contracts and explains the computation of the realized covariances from the high-frequency data. Since the realized covariance matrices of this dataset are computed using five-minute spaced intra-daily returns based on (approximately) 23 hours per day, they should be close to the whole day covariances. In Table 16 (in Appendix A), it can be checked that the average realized variances are close to the average daily squared returns, so that no bias correction method is applied.

The second dataset consists of realized covariance matrices for twenty-nine stocks belonging to the Dow Jones Industrial Average (DJIA) index; it is named DJ29 hereafter. The sample period is January 3, 2001 to April 16, 2018, with a total of 4092 trading days. The same dataset is used by Bauwens and Xu (2022) and Bauwens and Otranto (2022), where more information is provided about the data source and construction from high-frequency data. The realized covariance matrices are computed using synchronized five-minute returns in the trading session that extends from 9:30 Eastern Standard Time (EST) to 16:00 EST. Overnight returns are computed accordingly as the differences between the opening trade log-price at 9:30 EST and the previous close log-price at 16:00 EST.

Descriptive statistics for the original and bias-corrected covariance matrices are reported in Appendix B. The realized variances (v_t, the diagonal of RC_t) do not account for the overnight variation, hence their time series means, reported in column 3 of Table 17 under the header v), are at about 50 to 60 percent of the corresponding average squared returns (column 2). The time series means ($v1n$, in column 4 of the same table) of the realized variances augmented by the squared overnight returns but not rescaled by Λ_1 as in (4) are much closer to the average squared returns (r^2). The corrected realized variances ($v1_t$ and $v2_t$, diagonal of $RC1_t$ and $RC2_t$) are on average equal to the average squared returns by construction (column 2 of the table).

The time series standard deviations of the squared returns (r^2), realized variances (v), and bias-corrected realized variances ($v1$ and $v2$) satisfy the following inequalities: $r^2 > v1 > v2 > v$ – only the inequality $v1 > v2$ being (marginally) reversed in six cases. The time series of the daily squared returns are generally featuring more extreme peaks than the realized variances, explaining the larger time series means and standard deviations of the squared returns. The smaller time series means and standard deviations of the trading period are obviously due to the fact that the trading period is a fraction of the day, but the addition of the overnight return to the realized variance of that period and the rescaling by Λ_1 inflates the standard deviations more than just rescaling by (Λ_2), because the overnight variations has also some specific extremes.

Concerning the realized covariances and correlations, the bias corrections increase generally the means and standard deviations (see Tables 18 and 19) with respect to the uncorrected realized measures, but the differences between the average realized correlations ($Rcor$), rescaled overnight return augmented realized correlations ($RCor1$) and scaled realized correlations ($RCor2$) are much smaller than for the corresponding average realized variances and covariances.

3 Realized covariance models and estimation results

3.1 Definitions of models

Table 1 – BEKK, FKO and DCC model equations

BEKK-	Equation	Restrictions
GARCH	$G_t = (1-\alpha_G-\beta_G)\bar{H} + \alpha_G r_{t-1}r'_{t-1} + \beta_G G_{t-1}$	$\alpha_G, \beta_G \geq 0, \alpha_G + \beta_G < 1$
HEAVY-H	$H_t = (1-\beta_H)\bar{H} - \alpha_H \bar{M} + \alpha_H RC_{t-1} + \beta_H H_{t-1}$	$\alpha_H, \beta_H \geq 0, \beta_H < 1$
HEAVY-M	$M_t = (1-\alpha_M-\beta_M)\bar{M} + \alpha_M RC_{t-1} + \beta_M M_{t-1}$	$\alpha_M, \beta_M \geq 0, \alpha_M + \beta_M < 1$
FKO-	Equation	Restrictions
GARCH	$G_t = \exp(-\alpha_G)G_{t-1} + \alpha_G \exp(-\alpha_G) r_{t-1}r'_{t-1}$	$\alpha_G \geq 0$
HEAVY-H	$H_t = \exp(-\alpha_H)H_{t-1} + \alpha_H \exp(-\alpha_H) RC_{t-1}$	$\alpha_H \geq 0$
HEAVY-M	$M_t = \exp(-\alpha_M)M_{t-1} + \exp(-\alpha_M) RC_{t-1}$	$\alpha_M \geq 0$
DCC-	Variance equation of asset i	Restrictions
GARCH	$g_{t,i} = \omega_{G,i} + A_{G,i} r^2_{t-1,i} + B_{G,i} g_{t-1,i}$	$A_{G,i}, B_{G,i} \geq 0, A_{G,i} + B_{G,i} < 1$
HEAVY-H	$h_{t,i} = \omega_{H,i} + A_{H,i} v_{t-1,i} + B_{H,i} h_{t-1,i}$	$A_{H,i}, B_{H,i} \geq 0, A_{H,i} + B_{H,i} < 1$
HEAVY-M	$m_{t,i} = \omega_{M,i} + A_{M,i} v_{t-1,i} + B_{M,i} m_{t-1,i}$	$A_{M,i}, B_{M,i} \geq 0, A_{M,i} + B_{M,i} < 1$
	Correlation equation	
GARCH	$Q_t = (1-\alpha_Q-\beta_Q)\bar{R} + \alpha_Q u_{t-1}u'_{t-1} + \beta_Q Q_{t-1}$	$\alpha_Q, \beta_Q \geq 0, \beta_Q < 1$
HEAVY-H	$R_t = (1-\beta_R)\bar{R} - \alpha_R \bar{P} + \alpha_R RL_{t-1} + \beta_R R_{t-1}$	$\alpha_R, \beta_R \geq 0, \beta_R < 1$
HEAVY-M	$P_t = (1-\alpha_P-\beta_P)\bar{P} + \alpha_P RL_{t-1} + \beta_P P_{t-1}$	$\alpha_P, \beta_P \geq 0, \alpha_P + \beta_P < 1$

r_t: daily return vector; RC_t: realized covariance matrix of day t, see (1); $\mathcal{F}_t^{LF} = \{r_s \,\text{for}\, s < t\}$ (past daily returns); $\mathcal{F}_t^{HF} = \{RC_s, r_s \,\text{for}\, s < t\}$ (past realized covariance and daily returns).
-For BEKK and FKO equations: $G_t = E(r_t r'_t|\mathcal{F}_{t-1}^{LF})$, $H_t = E(r_t r'_t|\mathcal{F}_{t-1}^{HF})$, $M_t = E(RC_t|\mathcal{F}_{t-1}^{HF})$, $\bar{H} = E(r_t r'_t) \approx \sum_{t=1}^T r_t r'_t/T$, $\bar{M} = E(RC_t) \approx \sum_{t=1}^T RC_t/T$.
-For DCC-variance equations: $r_{t,i}$: daily return of asset i (i-th element of r_t); $v_{t,i}$: realized variance of asset i (i-th diagonal element of RC_t); $g_{t,i} = E(r^2_{t,i}|\mathcal{F}_{t-1}^{LF})$; $h_{t,i} = E(r^2_{t,i}|\mathcal{F}_{t-1}^{HF})$; $m_{t,i} = E(v_{t,i}|\mathcal{F}_{t-1}^{HF})$.
-For DCC-correlation equations: u_t: vector of degarched returns, with i-th element $u_{t,i=}r_{t,i}/g_{t,i}$; RL_t: realized correlation matrix, see (2); $Q_t = E(u_t u'_t|\mathcal{F}_{t-1}^{LF})$, transformed into a correlation matrix by transforming each element into a correlation coefficient; $R_t = E(RL_t|\mathcal{F}_{t-1}^{HF})$; $P_t = E(M_t|\mathcal{F}_{t-1}^{HF})$; $\bar{R} = E(u_t u'_t) \approx \sum_{t=1}^T u_t u'_t/T$; $\bar{P} = E(RL_t) \approx \sum_{t=1}^T RL_t/T$.

We use the term 'realized covariance model' in the broad sense of a model that specifies the conditional expectation of a covariance matrix (conditional covariance) as a function of past realized covariance matrices. The conditional expectation in question can be that of the covariance matrix of a (daily or other) return vector or of a realized covariance matrix. For the former case, we consider the BEKK-HEAVY-H model of Noureldin, Shephard, and Sheppard (2012), a model of Fleming, Kirby, and Ostdiek (2003) – see their equation (15)– which we call FKO-HEAVY-H, and the DCC-HEAVY-H model of Bauwens and Xu (2022); the suffix -H indicates that these models concern the conditional covariance of daily returns, usually named H_t. For the realized covariance case, we consider the BEKK-HEAVY-M of Noureldin, Shephard, and Sheppard (2012), the DCC-HEAVY-M of Bauwens and Xu (2022), to which we add a FKO-HEAVY-M model. In Table 1, we provide the equations defining these models and the restrictions imposed on their parameters to ensure the positive-definitness of the conditional covariance matrices. The table also defines the usual BEKK-GARCH and DCC-GARCH models, to which we add the FKO-GARCH model, which is similar to the equation (14) in Fleming, Kirby, and Ostdiek (2003). All models are in scalar version, i.e. each equation of a covariance or correlation matrix depends on scalar parameters.

A BEKK model has one parameter more than the corresponding FKO model. Each of these models implies that the conditional covariance matrix V_t (where V_t is one of the matrices G_t, H_t, M_t) is a moving average of the past observed matrices O_t ($r_t r_t'$ for GARCH, RC_t for HEAVY-H and -M), i.e. $V_t = \bar{V} + \sum_{j=1}^{\infty} w_j O_{t-j}$, where $\bar{V}$ is a constant matrix and w_j are scalar declining weights. For FKO, $\bar{V} = 0$ and $w_j = \alpha \exp(-j\alpha)$ (where α is α_G, α_H or α_M). For BEKK, $\bar{V} \neq 0$ and $w_j = \alpha\beta^{j-1}$. For FKO, α determines the first weight, $\alpha \exp(-\alpha)$, and the rate of decline of the weights, $\exp(-\alpha)$; for BEKK, α is the first weight and the rate of decline is β. A DCC model has more parameters, allowing each conditional variance to have its own weights, different from the identical weights of the realized correlations; the weights of each conditional covariance depend on the parameters of the corresponding variances and of the correlation process.

3.2 Full sample estimation results

The lagged RC_t matrix appears in the HEAVY equations of the BEKK and FKO models, and the lagged realized variances and correlation matrix RL_t in the equations of the DCC models. Using $RC2_t$ instead of RC_t does not change the estimated models since they differ only by a scaling matrix that is constant, but using $RC1_t$ changes the models and their estimated parameters. To distinguish the HEAVY models estimated with the DJ29 dataset, we use the suffixes H1 (instead of H) or M1 (instead of M) when $RC1_t$ is used, and H2 or M2 when $RC2_t$ is used. For the SBGF dataset, only RC_t is used, so we have HEAVY-H and HEAVY-M only.

The models are estimated by the quasi-maximum likelihood (QML) method explained in Appendix C. The full sample parameter estimates are reported in the tables of Appendix D.

For the SBGF data, the estimates of the parameters of the conditional variance equations of BOND and GOLD of the DCC models are close within each type of model. For STOCK, the $A_\bullet$ coefficient (with $\bullet =$ G, H or M) is much higher than for the other two assets, and the $B_\bullet$ one is smaller; this corresponds to a stronger sensitivity of the STOCK volatility to its lagged realized variance and a less smooth volatility. The HEAVY-H correlation process is more sensitive to the lagged realized correlation than the HEAVY-M, but the reverse holds for the HEAVY-H covariance process of the BEKK model (though not of FKO). The HEAVY-H first weight (w_1) of the moving average form is equal to 0.25 for FKO, 0.27 for BEKK, and the respective decline ratios (ρ) are 0.78 and 0.68. For HEAVY-M, these values are $w_1 = 0.19$, $\rho = 0.83$ for FKO, $w_1 = 0.37$, $\rho = 0.61$ for BEKK. Another noticeable feature is the quasi-identical estimates of $A_\bullet$ and $B_\bullet$ of HEAVY-H and HEAVY-M for STOCK, which is not the case for BOND and GOLD.

For the DJ29 data, in each model class (DCC, BEKK, KKO), the M1 and H1 models have rather close estimates, the M2 and H2 models also, but the estimates of a H1 (or H2) model are different from those of the corresponding M1 (or M2) model. In M2 and H2 models, the α (β) estimates are larger (smaller) than in the corresponding M1 and H1 models, indicating more sensitivity of the conditional moment (variance or correlation) to the lagged realized value and a less smooth conditional process. These differences indicate that the bias correction method used to transform the trading period realized matrix to the whole day matrix matters. Adding the overnight return and rescaling (method 1), instead of directly rescaling (method 2), yields a less reactive and smoother estimated conditional process.

Comparing a specific model across the three classes, the HEAVY-H2 first weight and decline ratio of the moving average form are equal to 0.038 and 0.96 for FKO, 0.13 and 0.86 for BEKK, 0.047 and 0.95 for the DCC correlation process, but (using the median estimates) 0.39 and

0.60 for the DCC variance process. So, the FKO covariance process is the least sensitive and smoothest process, followed by the DCC correlation process, the BEKK process, and the DCC variance processes. This ranking is the same for the other four types of models.

Further insight can be gained from the partial log-likelihood (PLL) value and Bayesian information criterion (BIC). To obtain comparable measures between the GARCH and HEAVY models, we define, similarly to Hansen, Huang, and Shek (2012), a partial log-likelihood (PLL) function for the time series of daily returns, which is based on the assumption that r_t is Gaussian, with zero mean and covariance matrix V_t, where V_t is G_t for a GARCH model, H_t for a HEAVY-H, and M_t for a HEAVY-M. The PLL function formula is thus

$$PLL_V = -\frac{1}{2}\sum_{t=1}^{T}\left(\log|V_t| + r_t'V_t^{-1}r_t\right) \text{ for } V = G, H, M. \tag{5}$$

From the results reported in Tables 2 and 3 we observe that i) in the FKO and DCC classes, HEAVY-H (or -H2) has the lowest BIC (i.e., best fit) in both datasets (HEAVY-H2 for DJ29 is the same as HEAVY-H for SBGF); ii) in the BEKK class, HEAVY-H2 has the best fit for DJ29 data, but HEAVY-M has the best fit for SBGF data; iii) in each model category (column of the table), the DCC model has a better fit, despite having a larger number of parameters, than the BEKK and FKO models, and the BEKK is better than FKO (with one minor exception).

4 Model evaluation criteria

For comparing the economic value of the models defined in Section 3, we use three economic loss functions (defined in section 4.1) and compute them for a set of out-of-sample forecasts. Because the loss values may differ between models due to the inevitable estimation imprecision inherent to the use of a finite sample of data, we employ the model confidence set (MCS) statistical method (briefly exposed in section 4.2) to assess the significance of the loss differences between the models. In addition, we use the measure of economic value proposed by Fleming et al. (2001, 2003) for comparing two models, i.e., the return an investor (with a given risk aversion) would be willing to sacrifice to switch from one model to another. Like Bollerslev, Patton, and Quadvlieg (2018), we assess the statistical significance of the sacrificed return via the reality check of White (2000).

4.1 Economic loss functions

We adopt the widely used economic loss functions of global minimum variance (GMV) and minimum variance (MV) portfolio; see e.g. Engle and Colacito (2006), and Engle and Kelly (2012). We denote by V_t the covariance matrix forecast of a model for day t of the forecast period, and by T_s the number of days of the forecast period. For each day, we compute the optimal GMV portfolio weight vector w_t that is the solution of the minimization of $\hat{w}_t' V_t \hat{w}_t$ under the constraint that the weights add up to 1, allowing short sales, and we compute the implied optimal GMV portfolio return $w_t' r_t$ (where r_t is the observed return). Next we compute the forecast variance of the T_s returns, which is the GMV loss function for the considered model. A superior model according to this loss function produces an optimal portfolio with lower variance. To compute the MV loss function, we proceed in the same way, except that the optimal weight vector minimizes the same function as above, under the additional constraint that $\hat{w}_t' r_t$ be larger than a fixed threshold that we set at 10 percent (annually). To compute the optimal MV weights, we need an estimate of the expected return, which we fix at the sample mean of the observed returns of the forecast period.

For each model and portfolio allocation criterion, we compute three characteristics of the optimal weight vectors $w_t = (w_{t,1} w_{t,2} \dots w_{t,k})'$ of the forecast period. These characteristics, introduced by Bollerslev, Patton, and Quadvlieg (2018), are the portfolio concentration CO_t, the portfolio total short positions SP_t, and the portfolio turnover TO_t from day t to day $t+1$, defined as

$$CO_t = (\sum_{i=1}^{k} w_{t,i}^2)^{1/2}, \tag{6}$$

$$SP_t = \sum_{i=1}^{k} w_{t,i} 1_{\{w_{t,i}<0\}}, \tag{7}$$

$$TO_t = \sum_{i=1}^{k} \left| w_{t+1,i} - w_{t,i} \frac{1 + r_{t,i}}{1 + w_t' r_t} \right|. \tag{8}$$

Portfolios with less concentration and short positions may be of interest for practical implementation and reducing transaction costs. The turnover measure is relevant if transaction costs are proportional to it, in which case the portfolio return is reduced and becomes

$$r_{wt}(c) = w_t' r_t - c\, TO_t. \tag{9}$$

We compare the models by the MCS procedure using the forecast sample average of each characteristic as "loss" function.

To compare the economic significance of the different forecasting models, we also consider the utility-based framework of Fleming et al. (2001, 2003), also used in Bollerslev, Patton, and Quadvlieg (2018) and many other papers. Assuming that an investor has a quadratic utility function with absolute risk aversion parameter γ, the realized daily utility generated by the optimal portfolio based on the covariance forecasts from model a, is defined as

$$U\left(r_{wt}^{(a)}(c),\gamma\right) = \left(1 + r_{wt}^{(a)}(c)\right) - \frac{\gamma}{2(1+\gamma)}\left(1 + r_{wt}^{(a)}(c)\right)^2, \tag{10}$$

where $r_{wt}^{(a)}(c)$ is the optimal (GMV or MV) portfolio return of day t obtained with model a for transaction cost c. We use as additional loss function the forecast sample average of (10) multiplied by -1.

Following Fleming, Kirby, and Ostdiek (2003), the economic value of using model b instead of model a can be measured by solving for Δ_γ the equation

$$\sum_{t=1}^{T_s} U\left(r_{wt}^{(a)}(c),\gamma\right) = \sum_{t=1}^{T_s} U\left(r_{wt}^{(b)}(c) - \Delta_\gamma(c),\gamma\right), \tag{11}$$

where $\Delta_\gamma(c)$ can be interpreted as the return the investor with absolute risk aversion γ would be willing to sacrifice to switch from using model a to using model b. The above equation can be solved analytically as explained in Appendix E.

4.2 The model confidence set statistical procedure

The MCS procedure allows reasearchers to compare a set of models through a loss function based on the forecasts of the models of the considered set – in our setup these forecasts are the covariance matrix forecasts. The procedure compares the models jointly, without the need to choose arbitrarily a reference model. It has been developed by Hansen, Lunde, and Nason (2003) and further elaborated by Hansen, Lunde, and Nason (2011); it has become a widespread tool for model comparison.[4]

For a chosen loss function, and a initial set of models $\mathcal{M}_0$, the loss difference between each pair of models in the set is computed (at every time point $t = 1, \ldots, T_s$ of the forecast period), so that for models a and b, we get $D_{t,ab} = L_{t,a} - L_{t,b}$, where $L_{t,\cdot}$ is the chosen loss function

[4]A Google search on "the model confidence set" found 1993 citations on June 27, 2023.

(e.g., the GMV loss). At each step of the procedure, the null hypothesis of equal predictive accuracy, $E[D_{t,ab}] = 0$, is tested for $\forall a > b \in \mathcal{M}$, a subset of models $\mathcal{M} \subset \mathcal{M}_0$, with $\mathcal{M} = \mathcal{M}_0$ at the initial step. If H_0 is rejected at a chosen significance level α (e.g., 0.05 or 5 percent), the worst performing model is removed from $\mathcal{M}$; hence, a model is removed from $\mathcal{M}$ only if it is significantly inferior to the other models. This procedure is continued until a set of models includes no model that can be rejected at the level α, the resulting set being the model confidence set at the $1 - \alpha$ confidence level. The procedure does not necessarily select a single best model, as it may end with a set models of equal forecasting ability, which may be the initial set $\mathcal{M}_0$.

The test statistic for the null hypothesis $E[D_{t,ab}] = 0$ is the range statistic of Hansen, Lunde, and Nason (2011):

$$\max_{a,b \in \mathcal{M}} \frac{|\ \overline{D}_{ab}\ |}{\sqrt{\widehat{\mathrm{Var}}(\overline{D}_{ab})}}, \tag{12}$$

where $\overline{D}_{ab} = \frac{1}{T_s} \sum_{t=1}^{T_s} D_{t,ab}$, and the estimated variance $\widehat{\mathrm{Var}}(\overline{D}_{ab})$ is obtained by a bootstrap approach (Hansen, Lunde, and Nason (2003)), with 5,000 replications and block length of 22.

5 Empirical results: comparisons of out-of-sample forecasts

To compute out-of-sample forecasts, each model is first estimated on the sample of 3,000 observations starting at the first date of the available data, and out-of-sample forecasts for the next five observations (3001-3005) are computed from the estimated model. Next, each model is re-estimated on the window of 3,000 observations from observation 6 to 3005, and the five forecasts (3006-3010) are computed. The procedure is continued until the end of the sample and results in a total of $T_s = 1092$ out-of-sample forecasts for $s = 1$, 1088 for $s = 5$, and 1071 for $s = 22$ for the DJ29 dataset, and $T_s = 1864$ out-of-sample forecasts for $s = 1$, 1860 for $s = 5$, and 1,843 for $s = 22$ for the SBGF dataset.

5.1 SBGF data

5.1.1 Loss functions and portfolio features (Tables 4-6)

For the GMV and MV loss functions (see the StDev columns in the tables), at horizons 1 and 5, in each case the six realized covariance models form the MCS95 (i.e., the model confidence set, at the 95% level of confidence, obtained by comparing the nine models); thus, the three models that use only daily data (DCC-GARCH, BEKK-GARCH, FKO) are excluded. At horizon 22, for GMV, the MCS95 consists only of the four BEKK and DCC realized covariance models; for MV, the three DCC models, the BEKK-HEAVY-H, and the simple FKO model form the MCS95, this being the single setup in which two models that use only daily data are in MCS95. Considering all these results, we can conclude that i) the realized covariance models are superior to the GARCH models, and ii) the HEAVY-M and HEAVY-H models perform similarly.

Using the (opposite of the) utility function as loss criterion (see the Utility columns) with risk aversion parameter γ equal to 1 and transaction cost set to zero, the nine models are included in the MCS95, whatever the horizon and portfolio type, with the exception, at horizon 1 for the MV utilities, that the models that use only daily data (DCC-GARCH, BEKK-GARCH, FKO) are not in the MCS95.

Considering the average return values (see the Return columns), the nine models are in each MCS95, except the models that use only daily data (DCC-GARCH, BEKK-GARCH, FKO) at horizon 22 for MV portfolio returns. The higher level of MV returns than of GMV returns is due to the imposed constraint on the expected returns in the MV optimization.

The average concentration (column CO in the tables) and the the importance of short positions (SP column, in absolute value) of the HEAVY-M portfolio is lower than or equal to the value for the corresponding HEAVY-H. For both criteria, each of the six MCS95 includes from one to four models (except for SP of GMV at horizon 22, where the SP values are all very close to zero so that eight models are in the MCS95). Concerning the average turnover (TO column), the nine models are in each of the six MCS95, and the values do not differ much between the models (except one outlier).

5.1.2 Economic value of model switching (Tables 7-9)

The second column of each table reports the economic gain $\Delta_\gamma(c)$ – see (11) – of using the HEAVY-H model instead of the HEAVY-M and the GARCH model of the same class, when the risk aversion parameter γ is equal to 1 and the transaction cost is set to zero; for example in the DCC class, for horizon 1 and GMV portfolio, using HEAVY-H instead of GARCH, is worth 10.9 basis points, which is a statistically significant value at the 5% level based on the reality check of White (2000), while switching from HEAVY-M to HEAVY-H entails an insignificant loss of 3.1 points. Considering the three model classes, three horizons and two types of portfolio, the results confirm i) the positive economic value of each realized covariance (HEAVY-H) model with respect to the corresponding model that uses only daily data (with two exceptions, at horizon 22, for FKO), this being statistically significant in ten among the eighteen comparisons, and ii) the equivalence of the HEAVY-M and HEAVY-H models, since the gains or losses are small and insignificant.

The last three columns of each table provide a view of the impact of increasing the risk aversion parameter from 1 to 10, and the transaction cost from 0 to 0.01.

For a given value of the transaction cost, the increase of the risk aversion parameter amplifies the economic gain or loss of using the HEAVY-H model instead of the HEAVY-M and the GARCH model of the same class; for example, at horizon 1, for $\gamma = 10$ and $c = 0$, the gain of switching from DCC-GARCH to DCC-HEAVY-H is 90.4 instead of 10.9 for $\gamma = 1$ and $c = 0$. The number of significant gains or losses increases slightly when the risk aversion increases (from 10 to 13 for the changes from GARCH to HEAVY-H, and from 0 to 3 for HEAVY-M to HEAVY-H).

For a given value of the risk aversion parameter, the increase of the transaction cost (from 0 to 0.01) has a small impact on the economic gains or losses: in almost all comparisons, a gain remains a gain, a significant gain remains a significant gain, and likewise for losses. The differences of values of the gains or losses are very small, with the single exception at horizon 1, for MV portfolio, of switching from BEKK-GARCH to BEKK–HEAVY-H: for $\gamma = 1$, the gain is multiplied by almost 4, for $\gamma = 10$, by 2.

In each table, and for each portfolio type, the last block (of three rows) shows the results for switches between the HEAVY-H models of the three classes (BEKK, DCC, FKO), since these are the best performing models in their own class. Like for the comparisons commented above, the gains or losses are stable with respect to the transaction cost, and they are boosted by the higher risk aversion. Nevertheless, the differences between the models are mostly small and insignificant for horizons 1 and 5, whereas at horizon 22 they are big and significant, especially for $\gamma = 10$. Except for the monthly horizon where the FKO model performs better than DCC and BEKK, the three HEAVY-H models can be considered as equivalent.

5.2 DJ29 data

5.2.1 Loss functions and portfolio features (Tables 10-12)

About the two ways to transform the realized covariance of the trading period into a daily covariance, no model (HEAVY-H1 or -M1) that uses the first correction ($RC1_t$) is included in one of the MCS95 for the GMV and MV loss functions (see the StDev columns in the tables). In contrast, the models that use the second correction ($RC2_t$) often belong to the MCS95; in particular the DCC-HEAVY-M2 and -H2 models are in each of the six MCS95; the FKO-HEAVY-M2 model is in three of them, FKO-HEAVY-H2 in five, BEKK-HEAVY-M2 in four, and BEKK-HEAVY-H2 in one. In brief, i) the models (H2 and M2) using the $RC2_t$ realized covariances perform better than the models (H1 and M1) using $RC1_t$, and ii) the DCC models perform better than the FKO and BEKK models; this second finding is not much surprising for models of dimension 29, since DCC is more flexible than BEKK and FKO, by allowing different parameters in the individual variance processes and in the latter with respect to the correlation process.

The models that use only the daily returns are rarely included in the (StDev) GMV and MV MCS95, except the FKO model (included in the three GMV MCS95 and one MV), which has smaller losses than DCC-GARCH and BEKK-GARCH.

Using the (opposite of the) utility function as loss criterion (see the Utility columns) with risk aversion parameter γ equal to 1 and transaction cost set to zero, the fifteen models are considered as equivalent at horizon 22 and for MV loss at horizon 5; at horizon 1, the included models are the M2 and H2 models, i.e., the same as for the corresponding StDev losses (with one exception: BEKK-HEAVY-H2 is included for GMV Utility, not for StDev). Considering the average return values (see the Return columns), the fifteen models are in each MCS95.

The average concentration (column CO in the tables) and the importance (in absolute value) of short positions (SP column) of the HEAVY-M2 and -H2 portfolios are lower than or equal to the values for the corresponding HEAVY-M1 and -H1 models. For each measure, the six MCS95 contain the DCC-HEAVY-H2 model, plus two other models at most. Concerning the average turnover (TO column), the fifteen models are all in each of the six MCS95, and the values do not differ much between the models.

5.2.2 Economic value of model switching (Tables 13-15)

The second column of each table reports the economic gain $\Delta_\gamma(c)$ – see (11) – of using the HEAVY-H2 model instead of each other model of the same class; for example, in the FKO class, for horizon 1 and MV portfolio, using HEAVY-H2 instead of HEAVY-H1 is worth 76.8 basis points, which is a significant value at the 5% level based on the reality check of White (2000), and switching from HEAVY-M2 to HEAVY-H2 entails a n insignificant loss of 2.9 basis points. The main findings from these economic gain comparisons are summarized in three items:

i) The HEAVY-H2 model entails a gain with respect to the HEAVY-H1 and -M1 models of the same class in 34 cases (over 36); only two losses occur (at horizon 22, for GMV, BEKK), but they are not statistically significant (at the 5% level); on the contrary, several gains are statistically significant (12 at horizon 1, 8 at horizon 5, and 5 at horizon 22). This set of results confirms the better performance of the models using $RC2_t$ with respect to the models using $RC1_t$, already mentioned for the GMV and MV loss functions.

ii) Switching from HEAVY-M2 to HEAVY-H2 provides no significant gain or loss in the six possible comparisons at horizon 1, one significant loss at horizon 5 (for GMV, BEKK), and one significant gain at horizon 22 (for GMV, DCC). The performances of these two types of models can be considered as broadly equivalent.

iii) Switching from the DCC-GARCH model to the DCC-HEAVY-H2 corresponding model entails a statistically significant gain in the six possible comparisons, i.e. at each horizon and for each loss function; the gain values (Δ_1) are important, being between 40.5 and 108.3 basis points. Switching from BEKK-GARCH to BEKK-HEAVY-H2 creates a significantly positive gain in four cases, and switching from FKO to FKO-HEAVY-H2 entails insignificant gains or losses. Using realized covariances as inputs in the DCC class is more valuable than in the other two classes.

The last three columns of each table provide a view of the impact of increasing the risk aversion parameter from 1 to 10, and the transaction cost from 0 to 0.01.

For a given value of the transaction cost, the increase of the risk aversion parameter amplifies the economic gain or loss of using the HEAVY-H2 model instead of the other HEAVY models (M2, M1, H1) and the GARCH model of the same class; for example, at horizon 22, for $\gamma = 10$

and $c = 0$, the gain of switching from DCC-GARCH to DCC-HEAVY-H2 is 265.7 basis points instead of 40.5 for $\gamma = 1$ and $c = 0$. The significant gains or losses for $\gamma = 1$ remain significant for $\gamma = 10$ in most cases.

For a given value of the risk aversion parameter, the increase of the transaction cost (from 0 to 0.01) has in many comparisons a strong effect on the economic gains or losses: there are sign switches, but less so when both values are significant. The differences of values of the gains or losses vary a lot, and several are important.

In each table, and for each portfolio type, the last block (of three rows) shows the results for switches between the HEAVY-H2 models of the three classes (BEKK, DCC, FKO), since these are the best performing models in their own class. Like for the comparisons commented above, the gains or losses are amplified by the higher transaction cost and risk aversion parameter. Switching from BEKK to DCC creates significant gains when they are positive; losses occur only for horizon 5 and transaction cost 0.01, but they are statistically insignificant. Switching from FKO to BEKK results in 19 losses (out of 24 cases), among which only six are significant; none of the five gains is significant. Switching from DCC to FKO entails losses in all cases but one (at horizon 5, GMV portfolio, $\gamma = 1$ and $c = 0.01$) but they are statistically insignificant (except at horizon 5, GMV portfolio, $\gamma = 10$ and $c = 0$). In brief, the BEKK-HEAVY-H2 is less valuable than the corresponding DCC and FKO models,, and FKO seems more valuable than DCC, although there is no statistical support for this conclusion.

5.3 Synthesis of empirical findings

The main findings for both datasets can be summarized in three items:

1. The models utilizing the high-frequency intra-day data, namely, the HEAVY-M and HEAVY-H models for the SBGF dataset and the HEAVY-M2 and HEAVY-H2 models for the DJ29 dataset, exhibit a significant improvement in forecasting performance for the daily covariance when the forecasting horizon is short (i.e., daily or weekly). However, for forecasting monthly-ahead, the gains or losses are marginal and statistically insignificant. This observation aligns with the findings in univariate volatility forecasting, as, for example, in Lyocsa, Molnar, and Vyrost (2021).

2. For the DJ29 dataset, the models using the first bias correction method (i.e., HEAVY-M1

and HEAVY-H1), which employs the overnight returns to construct the full-day covariance, are not as effective as the models using the second bias correction method (i.e., HEAVY-M2 and HEAVY-H2), where the covariance of the intraday returns is directly rescaled to the full-day covariance. The primary reason for this is that the overnight returns yield noisy estimators of the overnight covariance.

3. For the small portfolios of three assets of the SBGF dataset, the DCC, BEKK, and FKO models, exhibit similar performances. Conversely, for the larger portfolio of twenty-nine assets of the DJ29 dataset, the DCC model outperforms the BEKK and FKO models. The reason is that the the DCC model is able to capture the heterogeneity of the individual variance processes, which is more important for twenty-nine assets than for three.

In brief, the most promising model for a large cross section of assets appears to be the HEAVY-H2 model, which exclusively uses intraday returns and directly models the daily covariance matrix employing a GARCH model structure, but with the lagged realized covariance matrix as driving variable.

6 Conclusions

It is known, since Fleming, Kirby, and Ostdiek (2003), that realized covariance (RC) models, i.e., multivariate volatility models that employ high-frequency data to measure the multivariate volatility, yield significant additional economic value with respect to GARCH models that employ only daily data. We show empirically that this holds also for more recently developed RC models with respect to the models used by Fleming, Kirby, and Ostdiek (2003). For relatively homogenous data, the different types of models (DCC, BEKK, FKO) provide similar results, but for heterogenous data, DCC-type RC models have an edge.

For the datasets we use, we find that the benefit of the RC models is important for one to five days ahead forecast horizons, not for a monthly horizon. Attention should be paid to the way by which the full day covariance matrix is constructed, when the data do not allow to measure the latter completely from the trading period intraday returns. Adding the outer product of overnight returns to correct the trading period covariance matrix before rescaling the latter does not seem advantageous, with respect to directly rescaling the trading period covariance to the full day one. More empirical research is needed, based on different data, is needed to add further evidence on such issues.

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Table 2 – PLL and BIC on full sample of SBGF data

	p	GARCH	HEAVY-H	HEAVY-M
		Partial log-likelihood (PLL)		
DCC-	11	-17138	-16712	-16837
BEKK-	2	-17266	-16915	-16845
FKO-	1	-17497	-16900	-16920
		Bayesian information criterion (BIC)		
DCC-		<u>7.050</u>	<u>**6.875**</u>	<u>6.927</u>
BEKK-		7.103	6.959	**6.930**
FKO-		7.196	**6.951**	6.959

p: number of estimated parameters; PLL: see eq. (5). Bold values: minimum BIC in row. Underlined values: minimum BIC in column.

Table 3 – PLL and BIC on full sample of DJ29 data

	p	GARCH	HEAVY-M1	HEAVY-H1	HEAVY-M2	HEAVY-H2
			Partial log-likelihood (PLL)			
DCC-	89	-187724	-188090	-186778	-186205	-184187
BEKK-	2	-191475	-188526	-188269	-186711	-185452
FKO-	1	-193771	-190610	-190592	-189158	-188699
			Bayesian information criterion (BIC)			
DCC-		<u>91.76</u>	<u>91.93</u>	<u>91.29</u>	<u>91.01</u>	<u>**90.03**</u>
BEKK-		93.59	92.15	92.02	91.26	**90.65**
FKO-		94.71	93.16	93.15	92.45	**92.23**

p: number of estimated parameters; PLL: see eq. (5). Bold values: minimum BIC in row. Underlined values: minimum BIC in column.

Table 4 – Loss function comparisons at daily horizon (s=1) for SBGF data

	GMV portfolio					
	TO	CO	SP	Return	StDev	Utility
DCC-GARCH	**0.724**	0.682	-0.056	**2.018**	6.919	**0.707**
DCC-HEAVY-M	**0.753**	0.665	**-0.009**	**2.375**	**6.692**	**0.710**
DCC-HEAVY-H	**0.928**	0.692	-0.083	**2.388**	**6.738**	**0.710**
BEKK-GARCH	**0.682**	0.685	-0.069	**1.742**	6.960	**0.705**
BEKK-HEAVY-M	**0.805**	**0.653**	-0.026	**2.268**	**6.743**	**0.709**
BEKK-HEAVY-H	**0.790**	0.687	-0.079	**1.828**	**6.787**	**0.708**
FKO	**0.941**	0.682	-0.073	**1.617**	6.976	**0.705**
FKO-HEAVY-M	**0.656**	**0.653**	**-0.012**	**2.077**	**6.742**	**0.709**
FKO-HEAVY-H	**0.832**	0.654	-0.018	**2.149**	**6.739**	**0.709**
	MV portfolio					
	TO	CO	SP	Return	StDev	Utility
DCC-GARCH	**1.628**	**0.947**	**-0.191**	**9.872**	14.571	0.559
DCC-HEAVY-M	**1.660**	0.951	**-0.186**	**10.944**	**14.279**	**0.569**
DCC-HEAVY-H	**1.848**	0.960	**-0.192**	**10.954**	**14.315**	**0.568**
BEKK-GARCH	**2.613**	**0.943**	-0.213	**9.392**	14.704	0.553
BEKK-HEAVY-M	**2.041**	**0.945**	-0.198	**10.339**	**14.263**	**0.568**
BEKK-HEAVY-H	**2.709**	0.964	-0.243	**10.459**	**14.302**	**0.568**
FKO	**2.247**	0.963	-0.225	**9.043**	14.788	0.551
FKO-HEAVY-M	**1.800**	0.952	-0.205	**10.465**	**14.325**	
FKO-HEAVY-H	**2.327**	0.954	-0.204	**10.319**	**14.293**	

TO: average (over forecast sample) turnover;
CO: average portfolio concentration;
SP: average of short positions;
Return: average of optimal portfolio annualized returns for transaction cost = 0;
StDev: corresponding standard deviation;
Utility: mean of utilities defined by (10), with $\gamma = 1$, $c = 0$ and (a) the model indicated in the first column.
Bold values in columns 2-7 identify the models included in the 95% MCS computed for the nine models of each portfolio type.

Table 5 – Loss function comparisons at weekly horizon (s=5) for SBGF data

			GMV portfolio			
	TO	CO	SP	Return	StDev	Utility
DCC-GARCH	**0.885**	0.682	-0.057	**1.921**	6.933	**0.706**
DCC-HEAVY-M	**0.773**	**0.653**	**-0.009**	**2.520**	**6.741**	**0.710**
DCC-HEAVY-H	**1.456**	0.689	-0.067	**2.228**	**6.768**	**0.709**
BEKK-GARCH	**0.728**	0.685	-0.061	**1.876**	6.869	**0.707**
BEKK-HEAVY-M	**0.853**	0.655	-0.025	**2.358**	**6.744**	**0.710**
BEKK-HEAVY-H	**0.989**	0.688	-0.065	**2.082**	**6.765**	**0.709**
FKO	**0.979**	0.685	-0.065	**1.690**	6.916	**0.706**
FKO-HEAVY-M	**0.827**	**0.653**	-0.014	**2.489**	**6.828**	**0.709**
FKO-HEAVY-H	**0.982**	**0.653**	-0.018	**2.516**	**6.825**	**0.709**
			MV portfolio			
	TO	CO	SP	Return	StDev	Utility
DCC-GARCH	**1.925**	0.935	**-0.193**	**10.233**	14.799	**0.553**
DCC-HEAVY-M	**1.587**	0.933	**-0.193**	**10.967**	**14.519**	**0.562**
DCC-HEAVY-H	**1.692**	0.947	-0.203	**10.944**	**14.566**	**0.561**
BEKK-GARCH	**2.031**	**0.933**	-0.208	**9.700**	14.896	**0.549**
BEKK-HEAVY-M	**9.086**	**0.929**	**-0.191**	**10.833**	**14.473**	**0.563**
BEKK-HEAVY-H	**1.901**	0.945	-0.230	**11.030**	**14.414**	**0.565**
FKO	**1.636**	0.939	-0.210	**10.107**	14.867	**0.551**
FKO-HEAVY-M	**1.560**	0.948	**-0.202**	**11.617**	**14.555**	**0.563**
FKO-HEAVY-H	**1.613**	0.948	-0.202	**11.578**	**14.545**	**0.563**

For explanations, see note below Table 4.

Table 6 – Loss function comparisons at monthly horizon (s=22) for SBGF data

GMV portfolio

	TO	CO	SP	Return	StDev	Utility
DCC-GARCH	**1.288**	0.688	**-0.001**	**1.723**	7.160	**0.703**
DCC-HEAVY-M	**0.972**	0.673	**0.000**	**2.107**	**7.037**	**0.705**
DCC-HEAVY-H	**0.641**	0.710	**0.000**	**1.875**	**7.013**	**0.705**
BEKK-GARCH	**1.610**	0.687	**-0.001**	**1.706**	7.314	**0.700**
BEKK-HEAVY-M	**0.822**	0.665	**0.000**	**2.710**	**7.068**	**0.706**
BEKK-HEAVY-H	**1.765**	0.692	**0.000**	**2.587**	**7.044**	**0.706**
FKO	**1.519**	0.686	-0.004	**1.995**	7.351	**0.700**
FKO-HEAVY-M	**1.705**	**0.653**	**0.000**	**2.965**	7.574	**0.699**
FKO-HEAVY-H	**1.709**	**0.653**	**0.000**	**2.965**	7.574	**0.699**

MV portfolio

	TO	CO	SP	Return	StDev	Utility
DCC-GARCH	**2.168**	0.882	-0.167	8.170	**14.613**	**0.554**
DCC-HEAVY-M	**2.660**	0.890	-0.177	**10.011**	**14.625**	**0.557**
DCC-HEAVY-H	**3.331**	0.905	-0.192	**9.959**	**14.683**	**0.556**
BEKK-GARCH	**1.799**	0.880	-0.164	8.197	14.598	**0.555**
BEKK-HEAVY-M	**1.584**	**0.869**	**-0.143**	**10.160**	14.722	**0.555**
BEKK-HEAVY-H	**1.854**	0.876	-0.170	**10.383**	**14.515**	**0.561**
FKO	**1.659**	0.902	-0.171	8.175	**14.648**	**0.553**
FKO-HEAVY-M	**2.704**	0.911	-0.164	**10.606**	14.863	**0.552**
FKO-HEAVY-H	**2.826**	0.911	-0.164	**10.606**	14.863	**0.552**

For explanations, see note below Table 4.

Table 7 – Economic values of switching between models for two risk aversion parameters γ and two transaction costs c, at daily horizon (s=1) for SBGF data

	γ=1		γ=10	
	c=0	c=0.01	c=0	c=0.01
GMV portfolio				
from DCC-GARCH to DCC-HEAVY-H	<u>10.9</u>	<u>11.9</u>	<u>90.4</u>	<u>87.8</u>
from DCC-HEAVY-M to DCC-HEAVY-H	-3.1	-3.4	<u>-37.1</u>	<u>-39.7</u>
from BEKK-GARCH to BEKK-HEAVY-H	8.7	7.5	<u>86.0</u>	<u>84.9</u>
from BEKK-HEAVY-M to BEKK-HEAVY-H	-7.4	-7.5	<u>-39.5</u>	<u>-39.2</u>
from FKO to FKO-HEAVY-H	<u>13.6</u>	<u>14.1</u>	<u>101.2</u>	<u>101.9</u>
from FKO-HEAVY-M to FKO-HEAVY-H	1.1	0.6	3.0	0.8
from BEKK-HEAVY-H to DCC-HEAVY-H	9.1	8.7	39.2	37.2
from FKO-HEAVY-H to BEKK-HEAVY-H	-6.6	-6.5	-42.2	-41.2
from DCC-HEAVY-H to FKO-HEAVY-H	-2.5	-2.4	-3.2	-1.7
MV portfolio				
from DCC-GARCH to DCC-HEAVY-H	<u>27.6</u>	<u>27.1</u>	<u>253.7</u>	<u>250.7</u>
from DCC-HEAVY-M to DCC-HEAVY-H	-5.5	-6.5	<u>-143.4</u>	<u>-145.4</u>
from BEKK-GARCH to BEKK-HEAVY-H	<u>37.4</u>	<u>134.6</u>	<u>322.7</u>	<u>658.6</u>
from BEKK-HEAVY-M to BEKK-HEAVY-H	-5.2	-9.1	-148.7	-152.6
from FKO to FKO-HEAVY-H	<u>53.7</u>	<u>53.6</u>	<u>401.2</u>	<u>398.6</u>
from FKO-HEAVY-M to FKO-HEAVY-H	4.2	3.1	63.4	55.0
from BEKK-HEAVY-H to DCC-HEAVY-H	3.2	8.3	-30.0	19.8
from FKO-HEAVY-H to BEKK-HEAVY-H	1.0	-4.0	-22.9	-73.6
from DCC-HEAVY-H to FKO-HEAVY-H	-3.2	-5.2	40.4	32.3

The numerical values are the $\Delta_\gamma(c)$ as defined by (11) for γ and c indicated in the headers, i.e. the economic gain (if positive) or loss (if negative) of switching from one model to another.
Underlined values indicate statistical significance at the 5% level.

Table 8 – Economic values of switching between models for two risk aversion parameters γ and two transaction costs c, at weekly horizon (s=5) for SBGF data

	γ=1		γ=10	
	c=0	c=0.01	c=0	c=0.01
GMV portfolio				
from DCC-GARCH to DCC-HEAVY-H	$\underline{9.6}$	5.3	$\underline{64.8}$	$\underline{58.4}$
from DCC-HEAVY-M to DCC-HEAVY-H	-5.0	$\underline{-6.3}$	-24.2	$\underline{-34.6}$
from BEKK-GARCH to BEKK-HEAVY-H	4.1	3.7	41.9	39.2
from BEKK-HEAVY-M to BEKK-HEAVY-H	-4.2	-4.7	-19.3	-21.1
from FKO to FKO-HEAVY-H	$\underline{6.2}$	$\underline{6.1}$	$\underline{31.8}$	$\underline{13.5}$
from FKO-HEAVY-M to FKO-HEAVY-H	0.5	0.2	2.5	0.2
from BEKK-HEAVY-H to DCC-HEAVY-H	1.3	0.4	-0.4	-7.1
from FKO-HEAVY-H to BEKK-HEAVY-H	-0.5	-0.4	37.0	37.0
from DCC-HEAVY-H to FKO-HEAVY-H	-1.2	-0.2	-44.6	-36.7
MV portfolio				
from DCC-GARCH to DCC-HEAVY-H	$\underline{14.7}$	15.6	$\underline{211.6}$	$\underline{212.7}$
from DCC-HEAVY-M to DCC-HEAVY-H	-7.7	-8.6	-143.5	-145.7
from BEKK-GARCH to BEKK-HEAVY-H	$\underline{54.6}$	$\underline{54.7}$	$\underline{398.6}$	$\underline{399.1}$
from BEKK-HEAVY-M to BEKK-HEAVY-H	10.9	37.4	111.5	$\underline{254.3}$
from FKO to FKO-HEAVY-H	$\underline{41.3}$	$\underline{41.8}$	$\underline{318.6}$	$\underline{317.4}$
from FKO-HEAVY-M to FKO-HEAVY-H	1.2	1.1	24.4	24.0
from BEKK-HEAVY-H to DCC-HEAVY-H	-24.2	$\underline{-23.8}$	-143.9	35.9
from FKO-HEAVY-H to BEKK-HEAVY-H	14.7	14.5	196.7	93.6
from DCC-HEAVY-H to FKO-HEAVY-H	10.1	10.0	52.9	54.4

For explanations, see note below Table 7.

Table 9 – Economic values of switching between models for two risk aversion parameters γ and two transaction costs c, at monthly horizon (s=22) for SBGF data

	γ=1		γ=10	
	c=0	c=0.01	c=0	c=0.01
GMV portfolio				
from DCC-GARCH to DCC-HEAVY-H	12.1	14.3	15.0	17.2
from DCC-HEAVY-M to DCC-HEAVY-H	-0.8	0.2	15.5	19.3
from BEKK-GARCH to BEKK-HEAVY-H	28.3	30.4	14.6	28.1
from BEKK-HEAVY-M to BEKK-HEAVY-H	0.5	-2.0	16.9	2.0
from FKO to FKO-HEAVY-H	-7.2	-6.0	-132.9	-123.3
from FKO-HEAVY-M to FKO-HEAVY-H	-0.2	-0.2	-0.3	-0.3
from BEKK-HEAVY-H to DCC-HEAVY-H	-4.9	-2.3	16.5	31.4
from FKO-HEAVY-H to BEKK-HEAVY-H	35.3	34.9	268.2	266.3
from DCC-HEAVY-H to FKO-HEAVY-H	-30.3	-32.7	-272.3	-278.7
MV portfolio				
from DCC-GARCH to DCC-HEAVY-H	7.3	7.9	-104.2	-94.0
from DCC-HEAVY-M to DCC-HEAVY-H	-10.0	-11.6	-105.8	-111.4
from BEKK-GARCH to BEKK-HEAVY-H	34.4	35.5	151.4	160.3
from BEKK-HEAVY-M to BEKK-HEAVY-H	33.9	33.3	270.1	267.6
from FKO to FKO-HEAVY-H	-9.4	-11.9	-255.4	-263.9
from FKO-HEAVY-M to FKO-HEAVY-H	-1.8	-1.0	-1.1	-5.2
from BEKK-HEAVY-H to DCC-HEAVY-H	-30.5	-33.7	-236.0	-245.8
from FKO-HEAVY-H to BEKK-HEAVY-H	51.4	53.3	379.9	386.8
from DCC-HEAVY-H to FKO-HEAVY-H	-21.6	-20.9	-241.6	-238.3

For explanations, see note below Table 7.

Table 10 – Economic value comparisons at daily horizon (s=1) for DJ29 data

	GMV portfolio					
	TO	CO	SP	Return	StDev	Utility
DCC-GARCH	**4.134**	0.499	-0.423	**1.106**	11.640	0.618
DCC-HEAVY-M1	**4.316**	0.502	-0.473	**-1.084**	11.451	0.618
DCC-HEAVY-H1	**5.608**	0.467	-0.404	**-1.290**	11.241	0.622
DCC-HEAVY-M2	**3.410**	**0.451**	-0.366	**1.333**	**10.822**	**0.637**
DCC-HEAVY-H2	**3.715**	**0.441**	**-0.348**	**1.596**	**10.793**	**0.638**
BEKK-GARCH	**3.883**	0.482	-0.411	**-3.086**	11.111	0.621
BEKK-HEAVY-M1	**4.502**	0.511	-0.496	**-2.584**	11.350	0.617
BEKK-HEAVY-H1	**5.125**	0.513	-0.481	**-3.023**	11.308	0.617
BEKK-HEAVY-M2	**3.242**	0.470	-0.407	**-0.535**	**10.765**	**0.634**
BEKK-HEAVY-H2	**4.221**	0.510	-0.480	**-2.757**	10.990	**0.625**
FKO	**7.240**	0.452	-0.428	**0.296**	**10.848**	**0.634**
FKO-HEAVY-M1	**9.461**	0.512	-0.527	**-2.127**	11.453	0.616
FKO-HEAVY-H1	**4.144**	0.508	-0.516	**-2.032**	11.431	0.616
FKO-HEAVY-M2	**2.832**	0.463	-0.415	**0.348**	**10.833**	**0.634**
FKO-HEAVY-H2	**2.857**	**0.452**	-0.392	**1.037**	**10.875**	**0.635**
	MV portfolio					
	TO	CO	SP	Return	StDev	Utility
DCC-GARCH	**3.314**	0.524	-0.520	**9.059**	12.253	0.619
DCC-HEAVY-M1	**5.299**	0.541	-0.599	**7.135**	12.182	0.617
DCC-HEAVY-H1	**3.660**	0.500	-0.511	**5.820**	11.896	0.621
DCC-HEAVY-M2	**3.120**	**0.492**	-0.498	**10.102**	**11.510**	**0.638**
DCC-HEAVY-H2	**4.002**	**0.478**	**-0.466**	**8.838**	**11.337**	**0.640**
BEKK-GARCH	**7.261**	0.513	-0.524	**3.288**	12.473	0.602
BEKK-HEAVY-M1	**5.895**	0.557	-0.624	**7.233**	12.052	0.620
BEKK-HEAVY-H1	**4.600**	0.557	-0.611	**6.815**	11.981	0.621
BEKK-HEAVY-M2	**4.569**	0.516	-0.541	**8.504**	**11.501**	**0.635**
BEKK-HEAVY-H2	**3.961**	0.551	-0.608	**6.738**	**11.628**	**0.629**
FKO	**4.176**	0.491	-0.539	**7.977**	11.461	**0.635**
FKO-HEAVY-M1	**5.472**	0.561	-0.652	**7.605**	12.150	0.619
FKO-HEAVY-H1	**3.948**	0.558	-0.646	**7.578**	12.116	0.619
FKO-HEAVY-M2	**4.661**	0.512	-0.536	**9.154**	**11.633**	**0.634**
FKO-HEAVY-H2	**3.401**	0.501	-0.518	**9.850**	**11.710**	**0.633**

For explanations, see note below Table 4.

Table 11 – Economic value comparisons at weekly horizon (s=5) for DJ29 data

	TO	CO	SP	Return	StDev	Utility
GMV portfolio						
DCC-GARCH	**2.997**	0.485	-0.409	**-0.042**	11.466	**0.620**
DCC-HEAVY-M1	**5.143**	0.504	-0.463	**-1.101**	11.430	0.618
DCC-HEAVY-H1	**3.476**	0.472	-0.378	**0.463**	11.244	**0.626**
DCC-HEAVY-M2	**3.087**	0.447	-0.329	**0.630**	**11.004**	**0.631**
DCC-HEAVY-H2	**3.594**	**0.430**	**-0.312**	**1.468**	**10.948**	**0.634**
BEKK-GARCH	**3.145**	0.482	-0.409	**-3.643**	11.174	**0.619**
BEKK-HEAVY-M1	**4.811**	0.511	-0.492	**-1.713**	11.450	0.617
BEKK-HEAVY-H1	**3.607**	0.512	-0.476	**-1.870**	11.418	0.617
BEKK-HEAVY-M2	**2.648**	0.471	-0.397	**0.077**	**11.074**	**0.629**
BEKK-HEAVY-H2	**2.881**	0.505	-0.459	**-1.864**	11.231	**0.621**
FKO	**3.243**	0.451	-0.428	**-0.452**	**10.986**	**0.629**
FKO-HEAVY-M1	**4.118**	0.512	-0.527	**-1.467**	11.550	0.615
FKO-HEAVY-H1	**4.626**	0.508	-0.516	**-1.397**	11.527	0.616
FKO-HEAVY-M2	**3.734**	0.462	-0.413	**1.330**	**11.164**	**0.629**
FKO-HEAVY-H2	**3.813**	0.452	-0.393	**1.775**	**11.081**	**0.632**
MV portfolio						
	TO	CO	SP	Return	StDev	Utility
DCC-GARCH	**3.763**	0.513	-0.512	**7.558**	12.372	**0.613**
DCC-HEAVY-M1	**3.674**	0.547	-0.591	**9.672**	12.122	**0.623**
DCC-HEAVY-H1	**3.963**	0.517	-0.516	**9.330**	11.917	**0.627**
DCC-HEAVY-M2	**3.718**	0.485	-0.470	**10.590**	**11.858**	**0.631**
DCC-HEAVY-H2	**4.638**	**0.473**	**-0.451**	**9.895**	**11.682**	**0.634**
BEKK-GARCH	**4.368**	0.515	-0.533	**5.115**	12.039	**0.616**
BEKK-HEAVY-M1	**8.211**	0.558	-0.631	**8.893**	12.265	**0.618**
BEKK-HEAVY-H1	**4.441**	0.558	-0.617	**8.810**	12.219	**0.619**
BEKK-HEAVY-M2	**3.813**	0.518	-0.547	**10.021**	**11.930**	**0.628**
BEKK-HEAVY-H2	**3.916**	0.548	-0.601	**9.009**	12.048	**0.624**
FKO	**3.359**	0.494	-0.546	**8.039**	11.649	**0.631**
FKO-HEAVY-M1	**4.455**	0.564	-0.662	**8.938**	12.354	**0.616**
FKO-HEAVY-H1	**3.932**	0.561	-0.656	**8.865**	12.318	**0.617**
FKO-HEAVY-M2	**3.116**	0.515	-0.544	**10.758**	12.060	**0.627**
FKO-HEAVY-H2	**3.185**	0.504	-0.528	**11.172**	**12.000**	**0.629**

For explanations, see note below Table 4.

Table 12 – Economic value comparisons at monthly horizon (s=22) for DJ29 data

	TO	CO	SP	Return	StDev	Utility
GMV portfolio						
DCC-GARCH	**2.848**	0.462	-0.357	**-0.855**	11.482	**0.618**
DCC-HEAVY-M1	**2.971**	0.507	-0.424	**-1.339**	11.416	**0.618**
DCC-HEAVY-H1	**2.959**	0.483	-0.355	**-0.338**	11.342	**0.622**
DCC-HEAVY-M2	**3.837**	0.458	-0.331	**-2.094**	**11.198**	**0.622**
DCC-HEAVY-H2	**2.463**	**0.445**	**-0.289**	**-0.924**	**11.114**	**0.626**
BEKK-GARCH	**4.929**	0.483	-0.402	**-4.913**	11.603	**0.607**
BEKK-HEAVY-M1	**4.764**	0.511	-0.479	**-1.114**	11.574	**0.615**
BEKK-HEAVY-H1	**4.008**	0.512	-0.461	**-1.264**	11.557	**0.615**
BEKK-HEAVY-M2	**3.185**	0.478	-0.383	**-0.047**	11.484	**0.619**
BEKK-HEAVY-H2	**3.629**	0.507	-0.439	**-1.667**	11.562	**0.614**
FKO	**4.140**	0.451	-0.428	**-0.290**	**11.260**	**0.624**
FKO-HEAVY-M1	**5.061**	0.512	-0.526	**-0.735**	11.701	**0.613**
FKO-HEAVY-H1	**4.611**	0.509	-0.518	**-0.775**	11.682	**0.613**
FKO-HEAVY-M2	**5.244**	0.460	-0.408	**2.284**	11.537	**0.623**
FKO-HEAVY-H2	**3.234**	**0.453**	**-0.393**	**2.807**	**11.379**	**0.627**
MV portfolio						
	TO	CO	SP	Return	StDev	Utility
DCC-GARCH	**2.794**	0.491	-0.481	**7.615**	**12.192**	**0.618**
DCC-HEAVY-M1	**3.246**	0.550	-0.567	**9.941**	12.253	**0.621**
DCC-HEAVY-H1	**3.083**	0.528	-0.515	**9.425**	12.130	**0.623**
DCC-HEAVY-M2	**4.888**	0.488	-0.475	**9.586**	**12.096**	**0.624**
DCC-HEAVY-H2	**2.636**	**0.481**	**-0.445**	**9.478**	**11.933**	**0.627**
BEKK-GARCH	**4.008**	0.517	-0.546	**5.095**	12.601	**0.603**
BEKK-HEAVY-M1	**4.602**	0.557	-0.634	**10.474**	12.476	**0.616**
BEKK-HEAVY-H1	**3.948**	0.556	-0.621	**10.376**	12.460	**0.616**
BEKK-HEAVY-M2	**5.741**	0.521	-0.554	**10.600**	12.383	**0.619**
BEKK-HEAVY-H2	**4.591**	0.548	-0.610	**10.092**	12.425	**0.617**
FKO	**4.433**	0.497	-0.552	**8.957**	**11.899**	**0.627**
FKO-HEAVY-M1	**5.163**	0.566	-0.672	**10.550**	12.596	**0.613**
FKO-HEAVY-H1	**4.306**	0.564	-0.667	**10.411**	12.567	**0.614**
FKO-HEAVY-M2	**3.627**	0.516	-0.549	**11.778**	12.536	**0.617**
FKO-HEAVY-H2	**4.765**	0.508	**-0.537**	**12.375**	12.375	**0.622**

For explanations, see note below Table 4.

Table 13 – Economic values of switching between models for two risk aversion parameters γ and two transaction costs c, at daily horizon (s=1) for DJ29 data

	γ=1		γ=10	
	c=0	c=0.01	c=0	c=0.01
GMV portfolio				
from DCC-GARCH to DCC-HEAVY-H2	98.5	164.1	496.2	671.7
from DCC-HEAVY-M1 to DCC-HEAVY-H2	77.4	152.3	152.9	532.0
from DCC-HEAVY-H1 to DCC-HEAVY-H2	98.5	198.9	255.7	675.6
from DCC-HEAVY-M2 to DCC-HEAVY-H2	5.9	11.5	33.7	93.8
from BEKK-GARCH to BEKK-HEAVY-H2	16.7	-366.0	104.9	-509.7
from BEKK-HEAVY-M1 to BEKK-HEAVY-H2	37.6	-128.8	228.6	-166.4
from BEKK-HEAVY-H1 to BEKK-HEAVY-H2	37.9	-173.5	249.7	-165.7
from BEKK-HEAVY-M2 to BEKK-HEAVY-H2	-26.7	-323.1	-279.7	-365.8
from FKO to FKO-HEAVY-H2	4.5	513.6	-23.9	713.7
from FKO-HEAVY-M1 to FKO-HEAVY-H2	91.4	114.7	379.8	417.0
from FKO-HEAVY-H1 to FKO-HEAVY-H2	94.8	591.7	390.6	756.4
from FKO-HEAVY-M2 to FKO-HEAVY-H2	2.4	472.2	-45.1	298.7
from BEKK-HEAVY-H2 to DCC-HEAVY-H2	64.6	424.9	195.2	539.8
from FKO-HEAVY-H2 to BEKK-HEAVY-H2	-50.6	-336.0	-179.6	-365.5
from DCC-HEAVY-H2 to FKO-HEAVY-H2	-14.6	0.4	-136.8	-19.8
MV portfolio				
from DCC-GARCH to DCC-HEAVY-H2	107.4	113.1	592.4	604.7
from DCC-HEAVY-M1 to DCC-HEAVY-H2	96.0	210.0	440.5	749.3
from DCC-HEAVY-H1 to DCC-HEAVY-H2	117.4	283.6	567.0	930.8
from DCC-HEAVY-M2 to DCC-HEAVY-H2	8.5	-15.8	186.6	42.8
from BEKK-GARCH to BEKK-HEAVY-H2	135.9	267.3	564.5	883.4
from BEKK-HEAVY-M1 to BEKK-HEAVY-H2	42.0	-114.6	309.0	-249.4
from BEKK-HEAVY-H1 to BEKK-HEAVY-H2	46.8	-25.7	349.3	-136.7
from BEKK-HEAVY-M2 to BEKK-HEAVY-H2	-33.5	-196.7	-185.2	-149.4
from FKO to FKO-HEAVY-H2	-11.6	111.6	-154.2	388.0
from FKO-HEAVY-M1 to FKO-HEAVY-H2	72.3	199.9	371.2	727.6
from FKO-HEAVY-H1 to FKO-HEAVY-H2	76.8	139.5	390.1	545.8
from FKO-HEAVY-M2 to FKO-HEAVY-H2	-2.9	41.3	-154.9	199.5
from BEKK-HEAVY-H2 to DCC-HEAVY-H2	55.3	224.6	286.6	794.7
from FKO-HEAVY-H2 to BEKK-HEAVY-H2	-21.6	-219.4	83.8	-257.5
from DCC-HEAVY-H2 to FKO-HEAVY-H2	-35.5	-21.3	-154.2	-188.5

For explanations, see note below Table 7.

Table 14 – Economic values of switching between models for two risk aversion parameters γ and two transaction costs c, at daily horizon (s=5) for DJ29 data

	γ=1		γ=10	
	c=0	c=0.01	c=0	c=0.01
GMV portfolio				
from DCC-GARCH to DCC-HEAVY-H2	72.5	-199.8	359.3	-324.2
from DCC-HEAVY-M1 to DCC-HEAVY-H2	42.8	169.5	238.6	475.0
from DCC-HEAVY-H1 to DCC-HEAVY-H2	78.8	-187.2	183.6	-325.0
from DCC-HEAVY-M2 to DCC-HEAVY-H2	14.6	-179.3	65.3	-324.3
from BEKK-GARCH to BEKK-HEAVY-H2	11.6	24.3	-44.6	48.1
from BEKK-HEAVY-M1 to BEKK-HEAVY-H2	21.1	41.1	153.9	218.5
from BEKK-HEAVY-H1 to BEKK-HEAVY-H2	23.3	90.7	174.4	395.2
from BEKK-HEAVY-M2 to BEKK-HEAVY-H2	-36.9	118.6	-270.7	240.3
from FKO to FKO-HEAVY-H2	11.8	25.6	-119.9	-138.3
from FKO-HEAVY-M1 to FKO-HEAVY-H2	81.3	86.6	188.7	309.1
from FKO-HEAVY-H1 to FKO-HEAVY-H2	84.7	71.5	153.8	257.1
from FKO-HEAVY-M2 to FKO-HEAVY-H2	13.8	66.5	87.7	307.1
from BEKK-HEAVY-H2 to DCC-HEAVY-H2	64.2	-121.8	198.1	-224.4
from FKO-HEAVY-H2 to BEKK-HEAVY-H2	-44.8	-11.3	-44.8	93.3
from DCC-HEAVY-H2 to FKO-HEAVY-H2	-11.7	179.2	-234.4	149.9
MV portfolio				
from DCC-GARCH to DCC-HEAVY-H2	108.2	68.6	517.5	394.2
from DCC-HEAVY-M1 to DCC-HEAVY-H2	34.6	-37.4	253.8	-209.7
from DCC-HEAVY-H1 to DCC-HEAVY-H2	56.7	-27.1	384.5	-110.5
from DCC-HEAVY-M2 to DCC-HEAVY-H2	15.4	-64.9	201.8	-298.0
from BEKK-GARCH to BEKK-HEAVY-H2	38.1	72.8	25.4	244.7
from BEKK-HEAVY-M1 to BEKK-HEAVY-H2	24.1	44.0	200.5	274.3
from BEKK-HEAVY-H1 to BEKK-HEAVY-H2	28.6	483.4	237.2	667.5
from BEKK-HEAVY-M2 to BEKK-HEAVY-H2	-25.7	144.7	-162.7	183.6
from FKO to FKO-HEAVY-H2	-12.5	-29.1	-141.2	-186.6
from FKO-HEAVY-M1 to FKO-HEAVY-H2	63.1	8.0	330.6	-79.8
from FKO-HEAVY-H1 to FKO-HEAVY-H2	67.4	29.2	352.9	99.3
from FKO-HEAVY-M2 to FKO-HEAVY-H2	12.1	-61.4	98.7	-187.0
from BEKK-HEAVY-H2 to DCC-HEAVY-H2	53.9	-23.7	342.2	-146.9
from FKO-HEAVY-H2 to BEKK-HEAVY-H2	-28.3	26.7	-94.8	290.4
from DCC-HEAVY-H2 to FKO-HEAVY-H2	-27.4	-3.3	-141.1	-196.3

For explanations, see note below Table 7.

Table 15 – Economic values of switching between models for two risk aversion parameters γ and two transaction costs c, at daily horizon (s=22) for DJ29 data

	γ=1		γ=10	
	c=0	c=0.01	c=0	c=0.01
GMV portfolio				
from DCC-GARCH to DCC-HEAVY-H2	40.5	16.9	265.7	152.4
from DCC-HEAVY-M1 to DCC-HEAVY-H2	19.6	-3.8	179.2	42.2
from DCC-HEAVY-H1 to DCC-HEAVY-H2	37.8	11.7	230.4	88.0
from DCC-HEAVY-M2 to DCC-HEAVY-H2	21.1	27.4	88.0	94.6
from BEKK-GARCH to BEKK-HEAVY-H2	37.2	136.3	70.1	472.9
from BEKK-HEAVY-M1 to BEKK-HEAVY-H2	-4.7	54.6	-9.4	267.3
from BEKK-HEAVY-H1 to BEKK-HEAVY-H2	-4.2	67.5	8.5	313.6
from BEKK-HEAVY-M2 to BEKK-HEAVY-H2	-25.1	-36.2	-131.0	-251.8
from FKO to FKO-HEAVY-H2	17.5	105.5	-224.0	277.5
from FKO-HEAVY-M1 to FKO-HEAVY-H2	70.4	136.1	269.4	455.0
from FKO-HEAVY-H1 to FKO-HEAVY-H2	72.1	308.1	280.1	506.4
from FKO-HEAVY-M2 to FKO-HEAVY-H2	23.4	91.8	155.0	392.9
from BEKK-HEAVY-H2 to DCC-HEAVY-H2	57.4	47.4	311.4	238.6
from FKO-HEAVY-H2 to BEKK-HEAVY-H2	-66.0	-77.1	-268.7	-305.8
from DCC-HEAVY-H2 to FKO-HEAVY-H2	7.4	29.0	-224.0	-65.6
MV portfolio				
from DCC-GARCH to DCC-HEAVY-H2	51.3	55.8	276.9	282.8
from DCC-HEAVY-M1 to DCC-HEAVY-H2	25.5	153.7	223.1	652.6
from DCC-HEAVY-H1 to DCC-HEAVY-H2	35.9	47.8	310.7	327.7
from DCC-HEAVY-M2 to DCC-HEAVY-H2	20.0	102.1	193.5	483.6
from BEKK-GARCH to BEKK-HEAVY-H2	72.9	291.4	238.4	931.0
from BEKK-HEAVY-M1 to BEKK-HEAVY-H2	2.0	-21.9	59.4	-198.1
from BEKK-HEAVY-H1 to BEKK-HEAVY-H2	3.1	7.3	81.2	102.3
from BEKK-HEAVY-M2 to BEKK-HEAVY-H2	-11.4	36.0	-152.1	234.8
from FKO to FKO-HEAVY-H2	-26.7	330.6	-129.1	474.1
from FKO-HEAVY-M1 to FKO-HEAVY-H2	45.6	14.9	249.5	-6.8
from FKO-HEAVY-H1 to FKO-HEAVY-H2	47.0	47.4	272.8	224.2
from FKO-HEAVY-M2 to FKO-HEAVY-H2	27.8	-21.2	217.4	-152.8
from BEKK-HEAVY-H2 to DCC-HEAVY-H2	55.8	105.0	412.9	522.6
from FKO-HEAVY-H2 to BEKK-HEAVY-H2	-30.3	-11.7	-151.7	83.3
from DCC-HEAVY-H2 to FKO-HEAVY-H2	-28.4	-98.4	-128.9	-177.5

For explanations, see note below Table 7.

Appendices

A SGBF data description

The data are obtained from TickData, Inc. for the futures contracts S&P 500 futures (ES: CME GROUP), Treasury bond futures (US:CCBOT/CME GROUP), and gold futures (GC: COMEX/CME GROUP). The sample period is July 1, 2003 to August 5, 2022.

The trading close times (US central standard times) of the three futures contracts are as follows: 1) Gold futures contract closes at 13:30 between 01/07/2003 and 03/12/2006, and closes at 17:00 between 04/12/2006 and 05/08/2022; 2) Bond futures contract closes at 16:00 between 01/07/2003 and 05/08/2022; 3) Stock futuresA contract closes at 15:15 between 01/07/2003 and 17/11/2012, and closes at 16:00 between 18/11/2012 and 05/08/2022. Therefore, we assume that portfolios are rebalanced at 13:30 each day between 01/07/2003 and 03/12/2006, at 15:15 each day between 04/12/2006 and 17/11/2012, and at 16:00 each day between between 18/11/2012 and 05/08/2022. The last transaction prices before the chosen close time as the close price. This procedure is the same as in Fleming, Kirby, and Ostdiek (2003).

Table 16 – Descriptive statistics - SBGF data

	r^2			v			Mean Cov. & *Mean Cor.*		
	Mean	StDev	Max	Mean	StDev	Max	STOCK	BOND	GOLD
STOCK	3.544	14.931	453	3.571	9.240	205	*1*	*-0.271*	*0.022*
BOND	1.059	2.286	69.6	1.163	1.313	28.6	-0.599	*1*	*0.118*
GOLD	3.081	8.742	294	3.347	4.299	62.2	0.171	0.191	*1*

Column 1: asset type; Columns 2 and 5: time-series (ts) means; Columns 3 and 6: ts standard deviations; Columns 4 and 7: ts maximum; r^2: squared returns; v: realized variances; Columns 8-10: ts means of realized covariances and (in *italics*) realized correlations. The means, standard deviations and maximum are annualized values, in percentage, i.e., multiplied by 252 and by 100.

To compute the realized measures, we follow the method of Fleming, Kirby, and Ostdiek (2003). We use the five-minute returns from close time on day t-1 to close time on day t: We construct the realized variances using all of these returns. For the realized covariances, however, we can use only the returns that are contemporaneous across markets. For example, the returns after 1:30 pm in the stock and bond markets cannot be used to compute the realized covariances with gold because the gold market is closed. Since this reduces the number of available observations, we construct the realized covariances using the the two-step procedure of Fleming, Kirby, and Ostdiek (2003). First, we use the contemporaneous returns to compute a preliminary set of realized variances and covariances, from which we compute the realized correlations. Second, we convert these correlations back into covariances using the realized variances based on the entire set of five-minute returns.

Table 16 provides summary descriptive statistics of the data. The realized variances are computed based on 23 hours (approximately, depending on the years) of intra-daily returns, hence they should be close to the daily squared return, which can be observed from Table 16. However, one can notice that that average values of the realized variances (v) are slightly higher than the average of the daily squared returns. On the contrary, their standard deviations are much smaller, the reason is that large squared returns are very often more extreme than large realized variances. This can be seen on Figure 1, which shows the squared returns and realized variances of each asset during two periods: 2008, a year of low volatility until September and extreme volatility afterwards (the 'subprime mortgage' crisis), and 2021/08/06-2022/08/05, a period of low and intermediate volatility level.

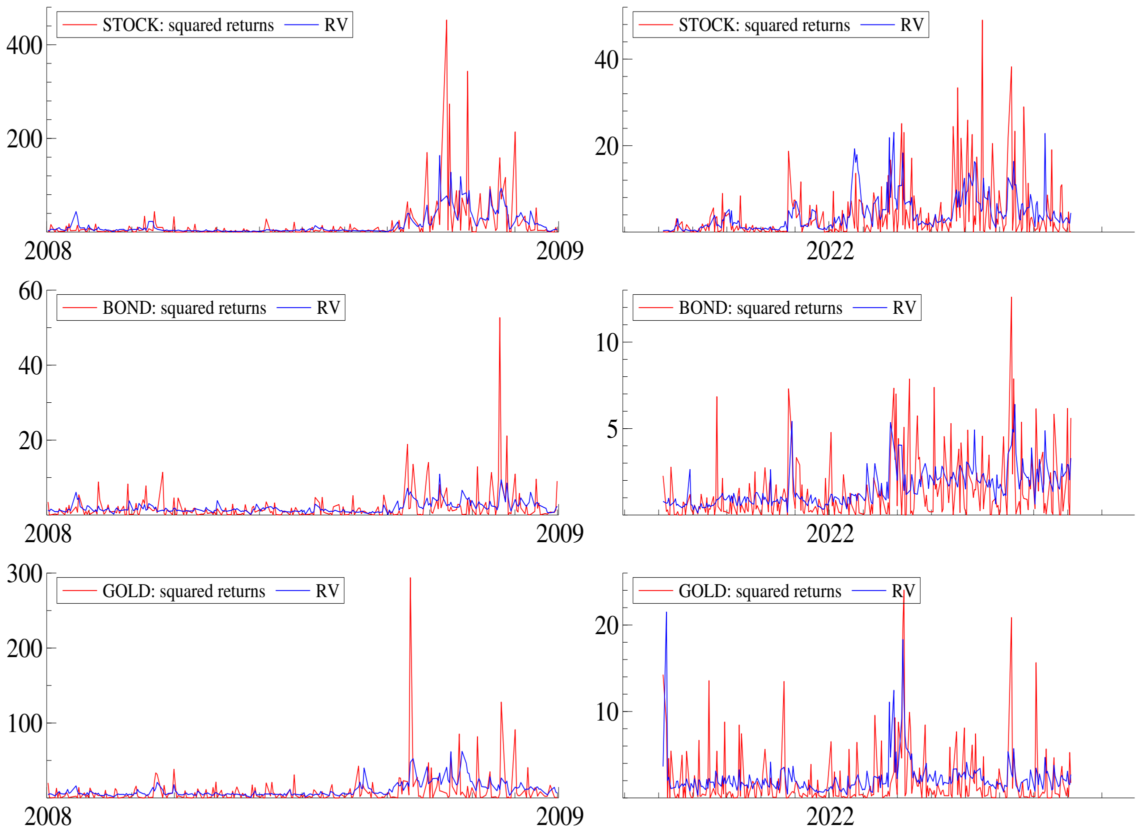

Figure 1 – Annualized realized variances (RV) and squared returns

B DJ29 data description

Table 17 – Descriptive statistics of realized variances - DJ29 data

	Means			Standard deviations			
Stock	$r^2, v1, v2$	v	$v1n$	r^2	v	$v1$	$v2$
AAPL	14.234	8.245	13.282	38.982	13.685	30.774	23.518
AXP	13.076	6.954	10.287	47.122	16.866	33.420	31.377
BA	8.840	4.778	7.224	25.816	7.509	25.357	14.339
CAT	10.185	5.533	8.659	26.481	9.073	19.289	17.456
CSCO	14.258	6.757	12.111	49.812	11.748	35.429	24.869
CVX	6.626	3.821	5.406	24.317	7.842	12.517	14.494
DIS	8.780	4.982	7.794	29.172	9.612	27.002	16.823
DWDP	12.212	4.472	8.055	39.113	7.083	29.715	19.246
GE	9.325	5.443	8.657	31.200	13.038	23.675	22.089
GS	13.384	7.237	11.184	53.465	24.863	49.495	42.384
HD	8.693	5.126	7.617	25.575	8.983	18.212	15.473
IBM	5.962	3.294	5.428	17.718	6.665	14.236	11.880
INTC	12.664	6.639	11.069	39.711	10.013	32.934	19.395
JNJ	3.312	2.200	3.311	15.043	4.162	13.106	6.472
JPM	16.069	8.037	12.266	65.176	20.037	38.165	39.965
KO	3.670	2.386	3.421	12.768	4.270	6.619	6.682
MCD	4.934	3.325	4.949	14.781	6.226	9.708	9.314
MMM	5.010	3.082	4.412	13.578	6.715	9.726	10.740
MRK	7.651	4.036	6.697	44.891	11.360	41.629	21.284
MSFT	8.476	4.386	7.077	26.000	6.698	18.103	13.108
NKE	8.030	4.239	6.666	29.404	6.486	23.422	12.436
PFE	5.966	3.700	6.003	19.551	5.469	16.303	9.377
PG	3.303	2.166	3.140	9.719	3.919	6.032	6.195
TRV	8.294	4.426	6.892	37.886	11.525	25.514	21.335
UNH	9.918	5.063	7.641	49.433	8.985	22.767	18.015
UTX	6.889	3.589	5.333	40.373	6.405	19.416	12.782
VZ	5.795	3.841	5.374	16.791	7.173	10.420	10.989
WMT	4.651	3.066	4.408	13.598	5.453	8.455	8.223
XOM	5.906	3.594	5.084	21.656	8.155	11.951	13.775
Min	3.303	2.166	3.140	9.719	3.919	6.032	6.195
Max	16.069	8.245	13.282	65.176	24.863	49.495	42.384
Med	8.294	4.386	6.892	26.481	7.842	19.416	14.494

Column 1: stock tickers. Columns 2-4: time-series averages; Columns 5-9: time-series standard deviations; r^2: squared returns; $v1$: rescaled realized variances with overnight squared returns, see (4); $v2$: rescaled realized variances, see (3) ; v: realized variances, see (1); $v1n$: realized variances with overnight squared returns, not rescaled. Rows "Min", "Max", and "Med" report the minimum, maximum, and median across the stocks. All values are annualized, in percentage.

Table 18 – Descriptive statistics of realized covariances - DJ29 data

	Means		Standard deviations		
Stock	RC	$RC1$ & $RC2$	RC	$RC1$	$RC2$
AAPL	2.135	3.775	4.589	9.113	8.202
AXP	2.239	5.139	5.573	13.015	12.448
BA	1.754	3.691	4.096	10.709	8.598
CAT	2.040	4.267	4.806	9.586	9.966
CSCO	2.156	4.454	4.535	9.753	9.283
CVX	1.625	3.314	4.638	8.566	9.357
DIS	1.806	4.095	4.241	10.782	9.402
DWDP	1.867	4.468	4.319	10.330	10.191
GE	2.061	4.272	5.096	10.816	10.374
GS	2.210	4.935	6.009	14.036	12.724
HD	1.934	3.759	4.649	9.486	8.966
IBM	1.631	3.167	3.963	7.066	7.545
INTC	2.200	4.450	4.321	9.261	8.733
JNJ	1.114	2.049	2.798	5.497	5.301
JPM	2.398	5.466	5.811	13.017	13.202
KO	1.184	2.073	3.020	5.175	5.405
MCD	1.295	2.204	3.354	5.800	5.846
MMM	1.592	3.162	3.755	6.958	7.459
MRK	1.433	2.726	3.618	7.020	7.106
MSFT	1.847	3.765	3.847	7.907	7.858
NKE	1.520	3.201	3.595	7.932	7.443
PFE	1.453	2.840	3.396	7.117	6.823
PG	1.130	1.988	2.909	5.177	5.294
TRV	1.544	3.670	4.191	11.009	9.499
UNH	1.422	2.949	4.020	8.846	8.422
UTX	1.658	3.738	4.091	9.888	8.876
VZ	1.515	2.775	3.975	6.823	7.222
WMT	1.379	2.359	3.205	5.676	5.544
XOM	1.636	3.231	4.635	8.274	9.007
Min	1.114	1.988	2.798	5.175	5.294
Max	2.398	5.466	6.009	14.036	13.202
Med	1.636	3.670	4.096	8.846	8.598

Column 1: stock tickers. Columns 2-4: means of the time series averages of the realized covariances of each stock with the other 28 stocks; Columns 5-7: standard deviations of the time series averages of the realized covariances of each stock with the other 28 stocks; RC: realized covariances, see (1); $RC1$: rescaled realized covariances with overnight cross-products of returns, see (4); $RC2$: rescaled realized covariances, see (3). Rows "Min", "Max", and "Med" report the minimum, maximum, and median across the stocks. All values are annualized, in percentage.

Table 19 – Descriptive statistics of realized correlations - DJ29 data

	Means			Standard deviations		
Stock	$RCor$	$RCor1$	$RCor2$	$RCor$	$RCor1$	$RCor2$
AAPL	0.326	0.349	0.326	0.152	0.213	0.152
AXP	0.447	0.462	0.447	0.145	0.183	0.145
BA	0.378	0.396	0.378	0.159	0.203	0.159
CAT	0.398	0.422	0.398	0.156	0.198	0.156
CSCO	0.367	0.405	0.367	0.148	0.192	0.148
CVX	0.377	0.393	0.377	0.174	0.217	0.174
DIS	0.422	0.447	0.422	0.160	0.196	0.160
DWDP	0.385	0.416	0.385	0.157	0.204	0.157
GE	0.426	0.441	0.426	0.150	0.189	0.150
GS	0.418	0.425	0.418	0.143	0.184	0.143
HD	0.375	0.399	0.375	0.156	0.198	0.156
IBM	0.382	0.418	0.382	0.154	0.193	0.154
INTC	0.382	0.413	0.382	0.146	0.188	0.146
JNJ	0.349	0.380	0.349	0.162	0.206	0.162
JPM	0.429	0.438	0.429	0.143	0.182	0.143
KO	0.318	0.350	0.318	0.160	0.203	0.160
MCD	0.303	0.330	0.303	0.163	0.210	0.163
MMM	0.435	0.460	0.435	0.160	0.194	0.160
MRK	0.304	0.347	0.304	0.162	0.210	0.162
MSFT	0.382	0.419	0.382	0.149	0.188	0.149
NKE	0.336	0.370	0.336	0.162	0.204	0.162
PFE	0.347	0.386	0.347	0.160	0.204	0.160
PG	0.324	0.354	0.324	0.160	0.203	0.160
TRV	0.397	0.413	0.397	0.159	0.200	0.159
UNH	0.284	0.313	0.284	0.162	0.212	0.162
UTX	0.435	0.453	0.435	0.155	0.191	0.155
VZ	0.326	0.351	0.326	0.164	0.205	0.164
WMT	0.317	0.352	0.317	0.159	0.196	0.159
XOM	0.395	0.413	0.395	0.170	0.210	0.170
Min	0.284	0.313	0.284	0.143	0.182	0.143
Max	0.447	0.462	0.447	0.174	0.217	0.174
Med	0.378	0.405	0.378	0.159	0.200	0.159

Column 1: stock tickers. Columns 2-4: means of the time series averages of the realized correlations of each stock with the other 28 stocks;
Columns 5-7: standard deviations of the time series averages of the realized correlations of each stock with the other 28 stocks; $RCor$: realized correlations (2) from realized covariances (1); $RCor1$: correlations from rescaled realized covariances (4); $RCor2$: correlations from rescaled realized covariances (3). Rows "Min", "Max", and "Mean" report the minimum, maximum, and mean across the stocks. .

C Quasi-maximum likelihood estimation

To define a quasi-likelihood function for the GARCH and HEAVY-H conditional covariance equations, we add the assumption that the conditional distribution of the return vector is multivariate Gaussian, $N_k(0, V_t)$, where V_t is G_t for a GARCH equation and H_t for a HEAVY-H equation. The quasi-log-likelihood function for T observations, given initial values, is

$$QLN_V(\theta_V) = -\frac{1}{2}\sum_{t=1}^{T}\left(\log|V_t| + r_t'V_t^{-1}r_t\right), \tag{C1}$$

where θ_V is the parameter vector of the corresponding model. This function can be maximized numerically with respect to θ_V for the BEKK and FKO models. For the DCC models, a two-step estimation procedure is used, whereby the parameters of the variance equations are estimated in the first step, and the parameters of the correlation equation are estimated in the second step, see Engle (2002).

For the HEAVY-M equations, we assume that the conditional distribution of RC_t is a central Wishart distribution of dimension k , $W_k(\nu, M_t/\nu)$, where ν is the degrees of freedom parameter (restricted by $\nu > k-1$) and M_t is the conditional mean of RC_t. The quasi-log-likelihood function for a sample of T observations, given initial conditions, is

$$QLW_M(\theta_M) = -\frac{\nu}{2}\sum_{t=1}^{T}\left[\log|M_t| + \mathrm{tr}(M_t^{-1}RC_t)\right] \tag{C2}$$

This function can be maximized numerically with respect to θ_M (independently of the value of ν, which can be set equal to one) in a single step for the BEKK and FKO models. For the DCC models, a two-step estimation procedure is used, similarly to the two-step procedure for the Gaussian – see Bauwens and Xu (2022).

D Estimation results on full samples

Table 20 – Parameter estimates (and robust t-statistics) for SBGF data

	GARCH		HEAVY-H		HEAVY-M	
DCC	Conditional Variance					
	A_G	B_G	A_H	B_H	A_M	B_M
STOCK	0.140	0.833	0.606	0.379	0.601	0.384
BOND	0.044	0.946	0.110	0.876	0.235	0.728
GOLD	0.043	0.949	0.154	0.813	0.304	0.673
	Conditional Correlation					
	α_Q	β_Q	α_R	β_R	α_P	β_P
	0.033	0.949	0.435	0.564	0.222	0.758
	(6.32)	(99.14)	(6.78)	(8,80)	(24.53)	(73.00)
BEKK	α_G	β_G	α_H	β_H	α_M	β_M
	0.057	0.934	0.272	0.684	0.372	0.606
	(11.90)	(130.49)	(8.38)	(17.42)	(36.46)	(13.26)
FKO	α_G		α_H		α_M	
	0.034		0.253		0.186	
	(10.52)		(13.57)		(26.22)	

Table 21 – Parameter estimates and robust t-statistics) for DJ29 data

	GARCH		HEAVY-M1		HEAVY-H1		HEAVY-M2		HEAVY-H2	
DCC	Conditional Variance									
	A_G	B_G	A_M	B_M	A_H	B_H	A_M	B_M	A_H	B_H
Min	0.025	0.826	0.055	0.000	0.057	0.381	0.174	0.139	0.296	0.508
Med	0.068	0.918	0.294	0.675	0.233	0.738	0.401	0.568	0.387	0.597
Max	0.123	0.970	0.995	0.937	0.538	0.939	0.794	0.788	0.479	0.683
	Conditional Correlation									
	α_Q	β_Q	α_P	β_P	α_R	β_R	α_P	β_P	α_R	β_R
	0.003	0.988	0.053	0.913	0.019	0.976	0.089	0.866	0.047	0.942
	(9.30)	(475.08)	(5.19)	(43.85)	(25.67)	(881.07)	(10.74)	(53.78)	(29.79)	(439.64)
BEKK	α_G	β_G	α_M	β_M	α_H	β_H	α_M	β_M	α_H	β_H
	0.009	0.988	0.049	0.935	0.033	0.961	0.191	0.755	0.134	0.856
	(25.96)	(1893.80)	(4.13)	(54.06)	(192.59)	(7.79)	(14.02)	(37.81)	(137.42)	(30.21)
FKO	α_G		α_M		α_H		α_M		α_H	
	0.0055		0.018		0.016		0.086		0.038	
	(17.05)		(14.71)		(8.11)		(23.81)		(6.71)	

For DCC Conditional Variance: Min is the minimum of the 29 estimates, Med the median, and Max the maximum.

E Solving equation (10)

We solve (10) after dividing both sides by T_s. For lighter notations, we use the symbol Y for Δ_γ , we set $a = 1$, $b = 2$, we denote by U_1 the utility function divided by T_s on the left side of (10), and by $U_2(Y)$ the function on the right side divided by T_s. So, we must find Y such that $U_1 = U_2(Y)$. By direct developments, we get $U_1 = 1 - A + (1 - 2A)S_1 - AV_1$, and

$U_2 = 1 - A + (1-2A)S_2 - AV_2 + (2A - 1 + 2AS_2)Y - AY^2$, where $A = 0.5\gamma/(1+\gamma)$, $S_i = \sum_{t=1}^{T_s} r_{it}$, and $V_i = \sum_{t=1}^{T_s} r_{it}^2$, for $i = 1, 2$, r_{it} being the optimal portfolio return of period t for model i, i.e., r_{it} stands for $r_{wt}^{(i)}$ of (10).

By subtracting $U_2(Y)$ from U_1 using the expressions above, we get the quadratic equation $AY^2 + BY + C = 0$, where A is defined above, $B = 1 - 2A(1 + S_2)$, and $C = (1 - 2A)(S_1 - S_2) + A(V_2 - V_1)$. This equation has two real roots if $B^2 - 4AC \geq 0$. The solution of interest as measure of economic gain (the maximum fee an investor would sacrifice to switch from model 1 to model 2) is the smallest root, if it is positive. If the smallest root is negative, its opposite is the gain of switching from model 2 to 1.